Linking Scales in Earth's Sea Ice System

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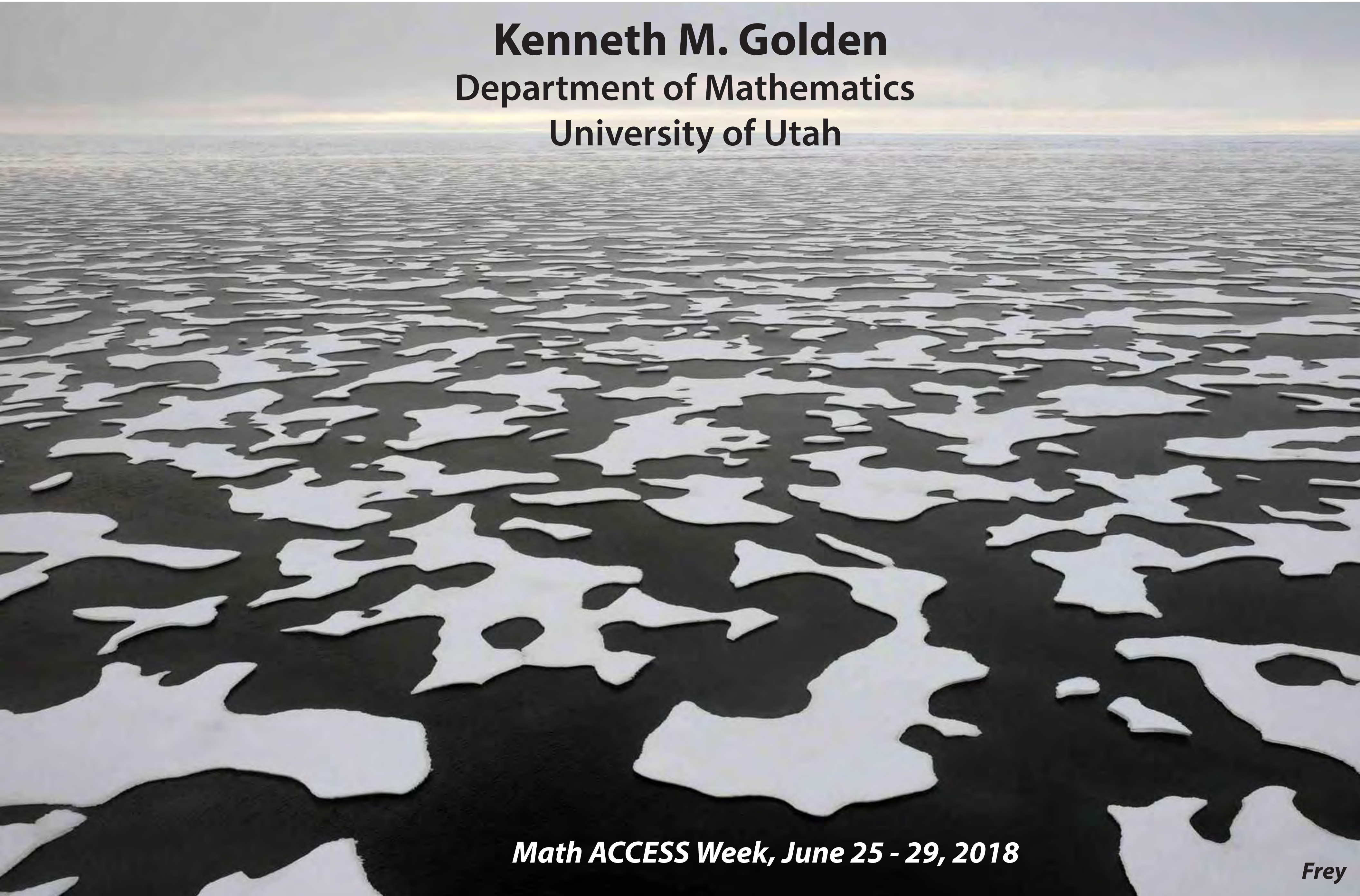
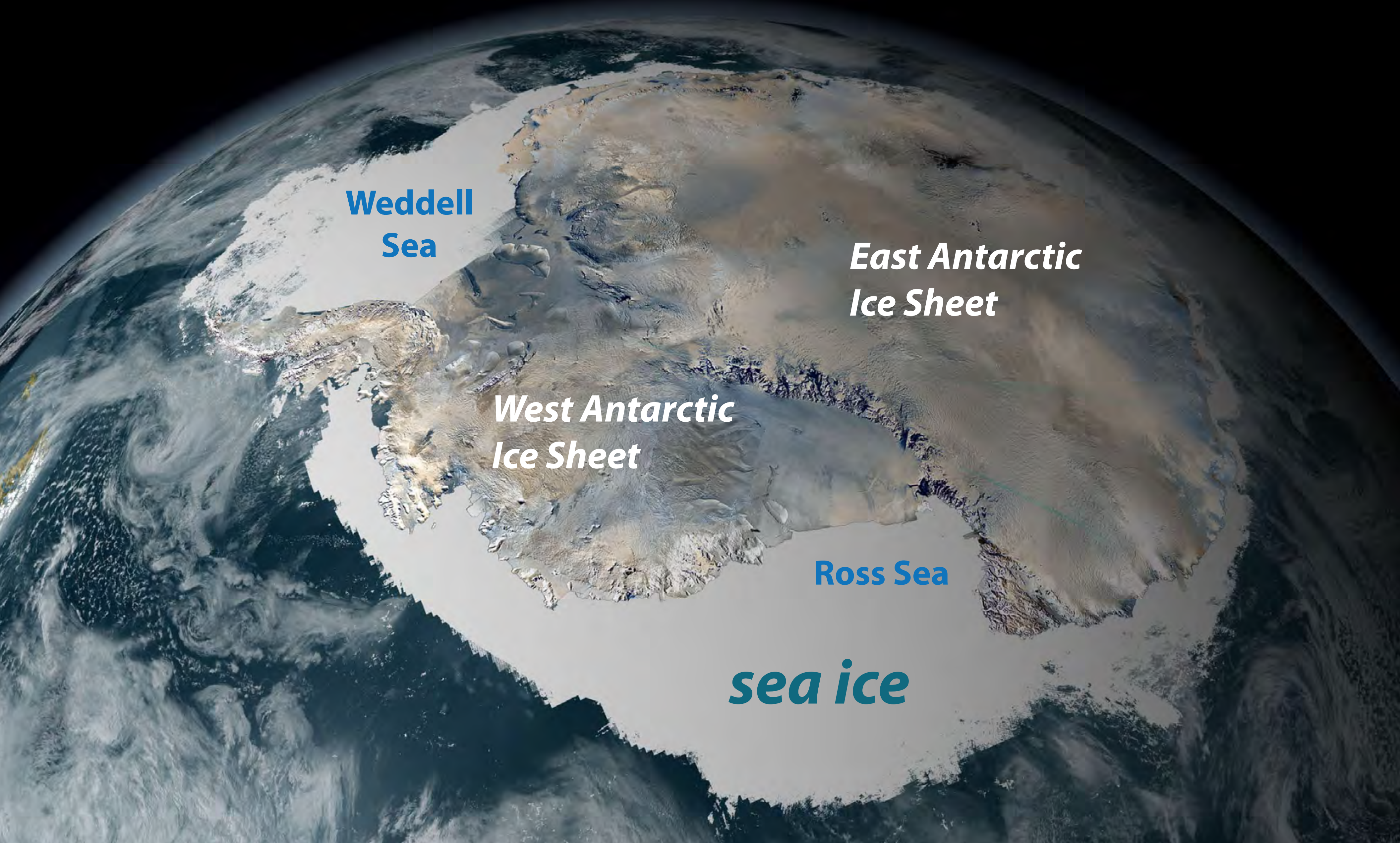
Math ACCESS Week, June 25 - 29, 2018



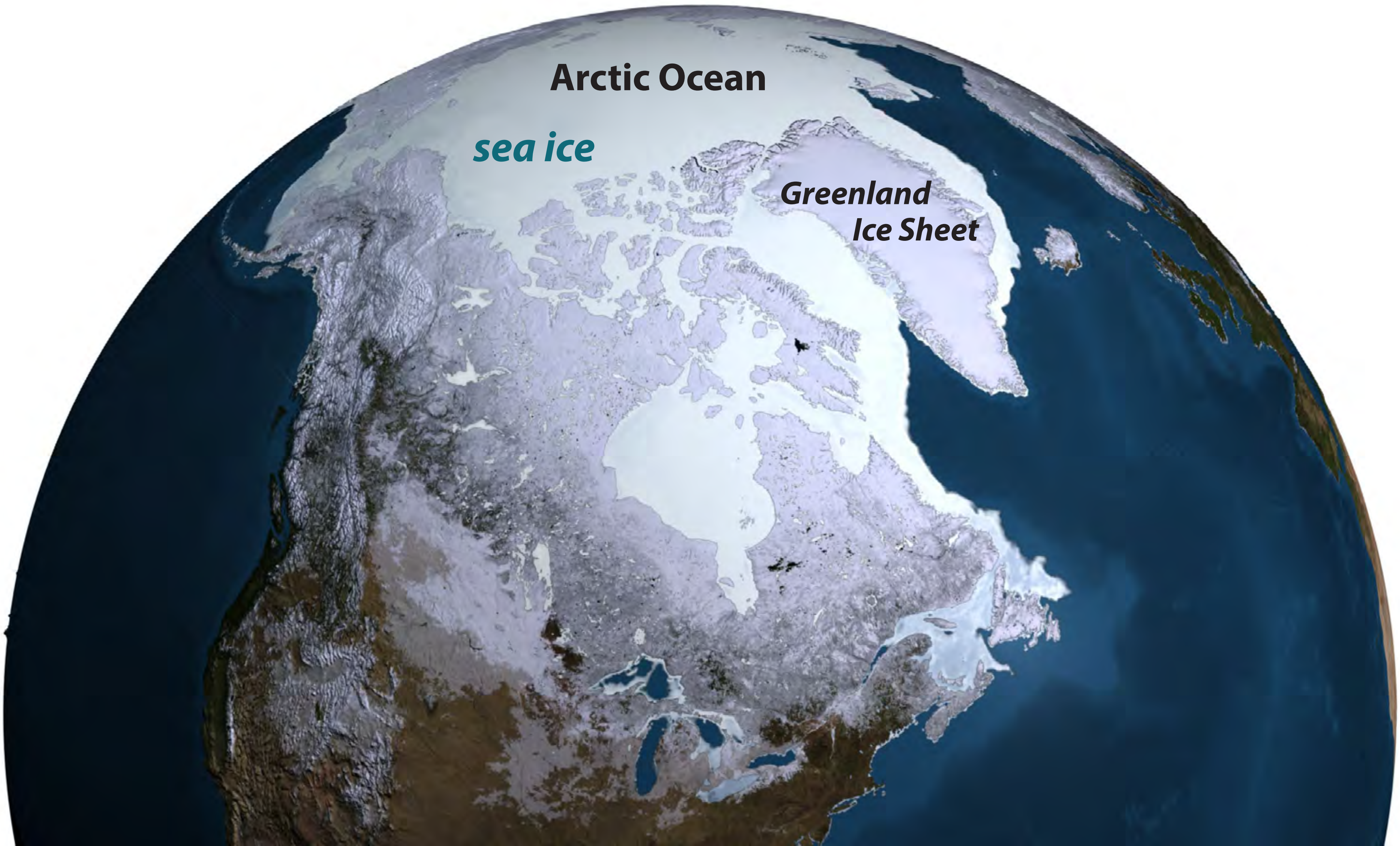
Frey

ANTARCTICA

southern cryosphere



THE ARCTIC
northern cryosphere

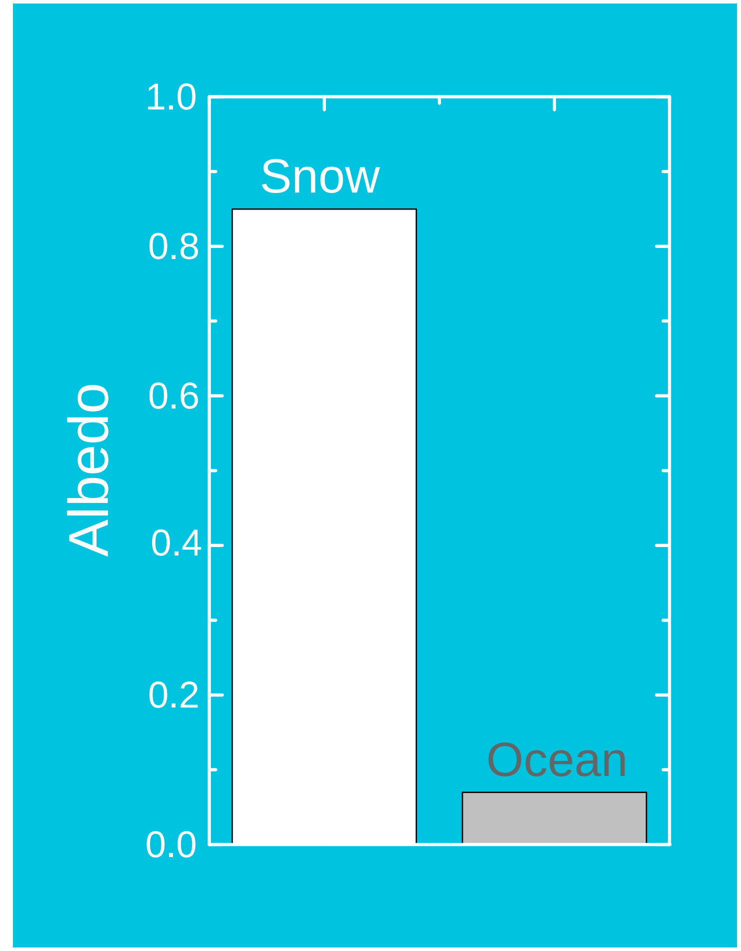


SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

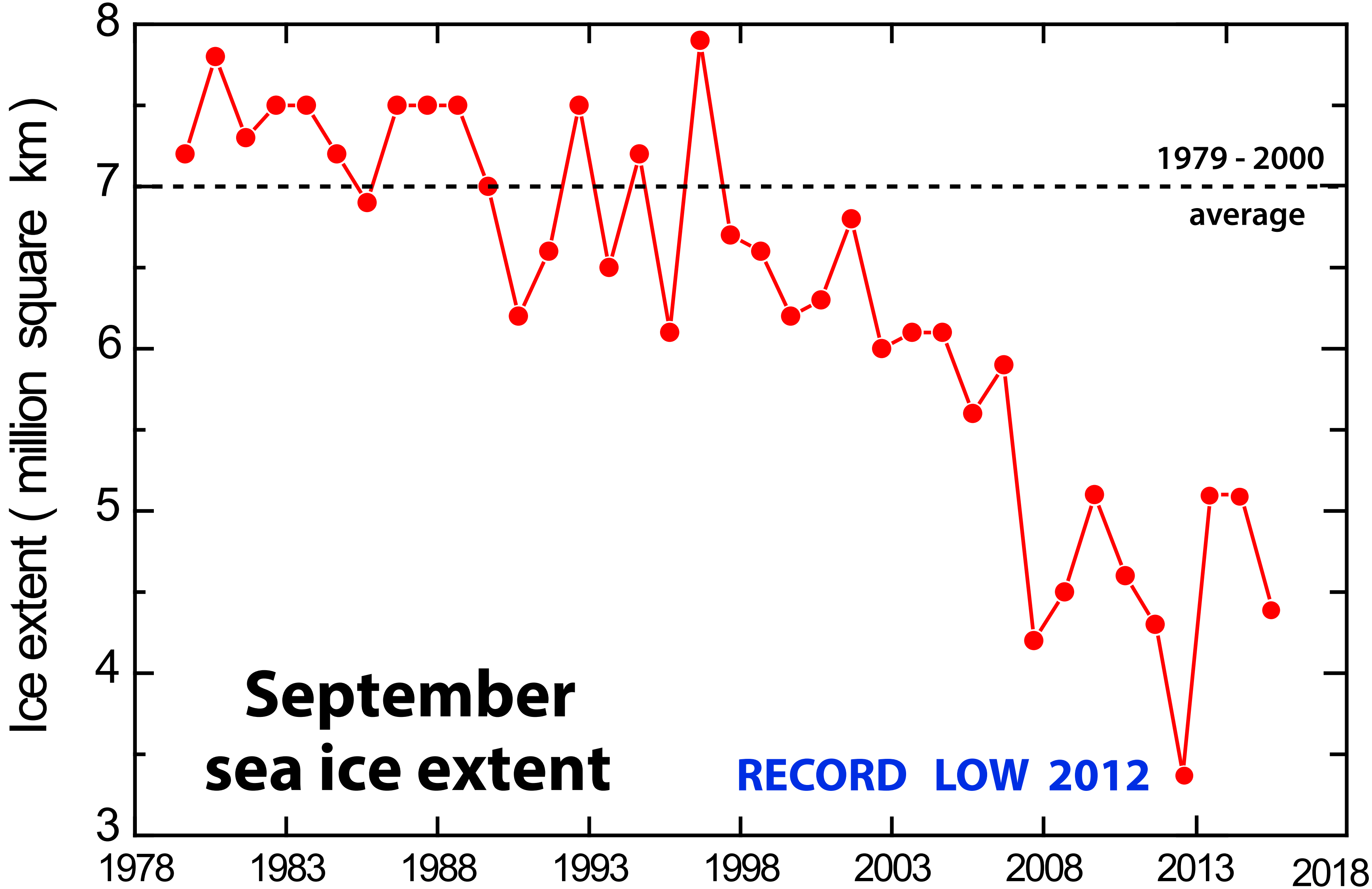
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

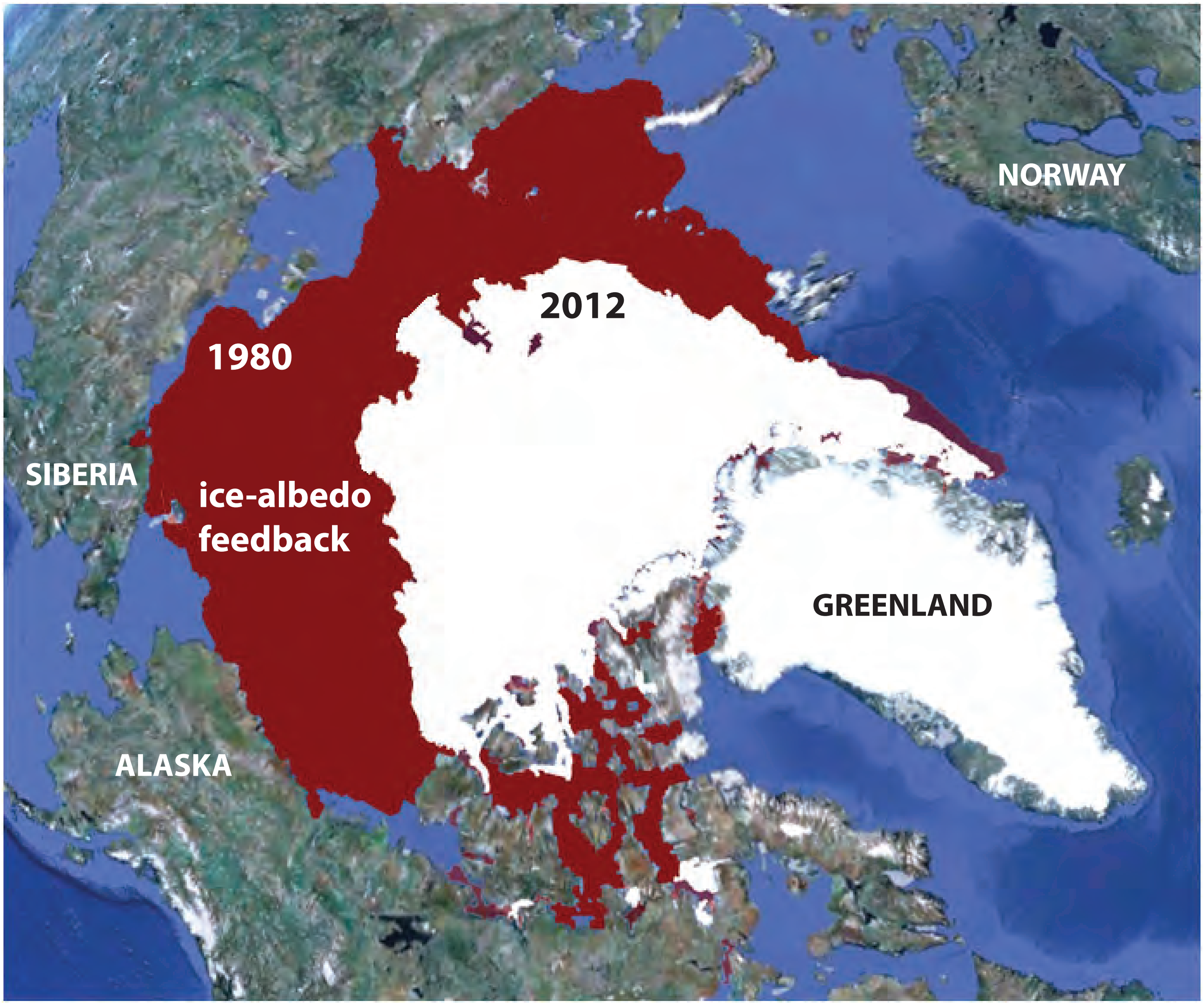
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



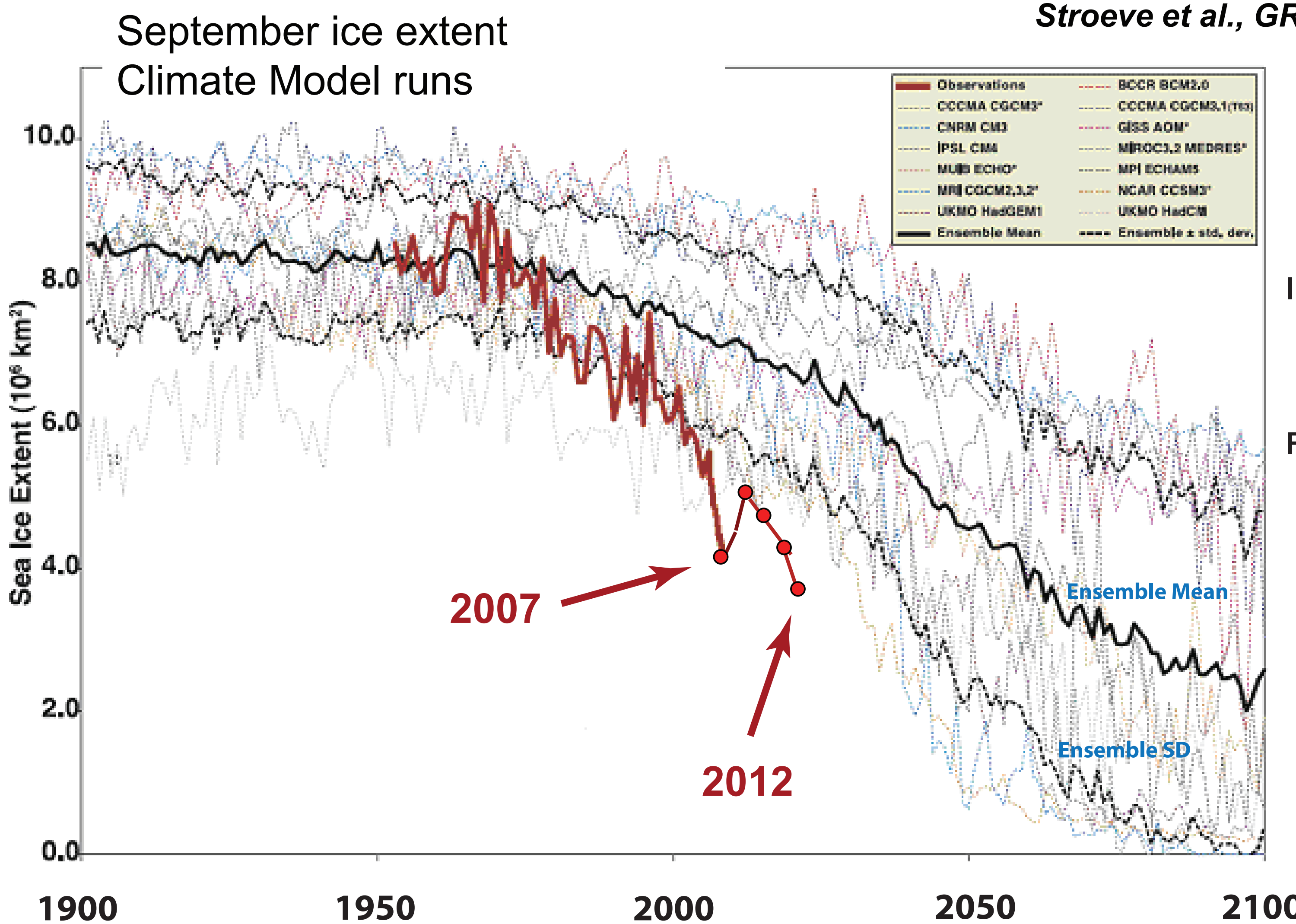
Perovich

recent losses
in comparison to
the United States

Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

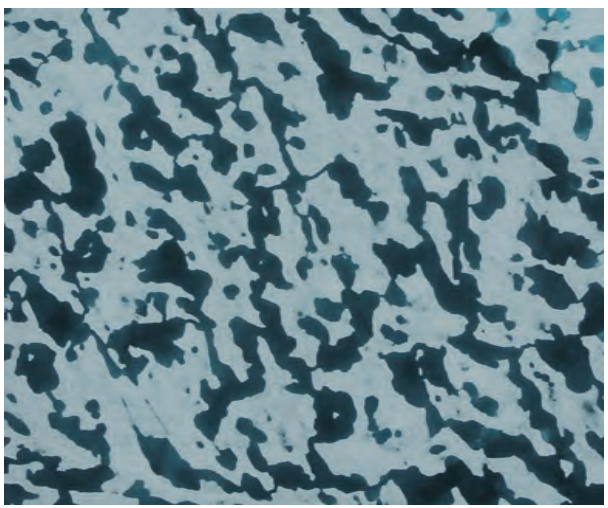
Fourth Assessment AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



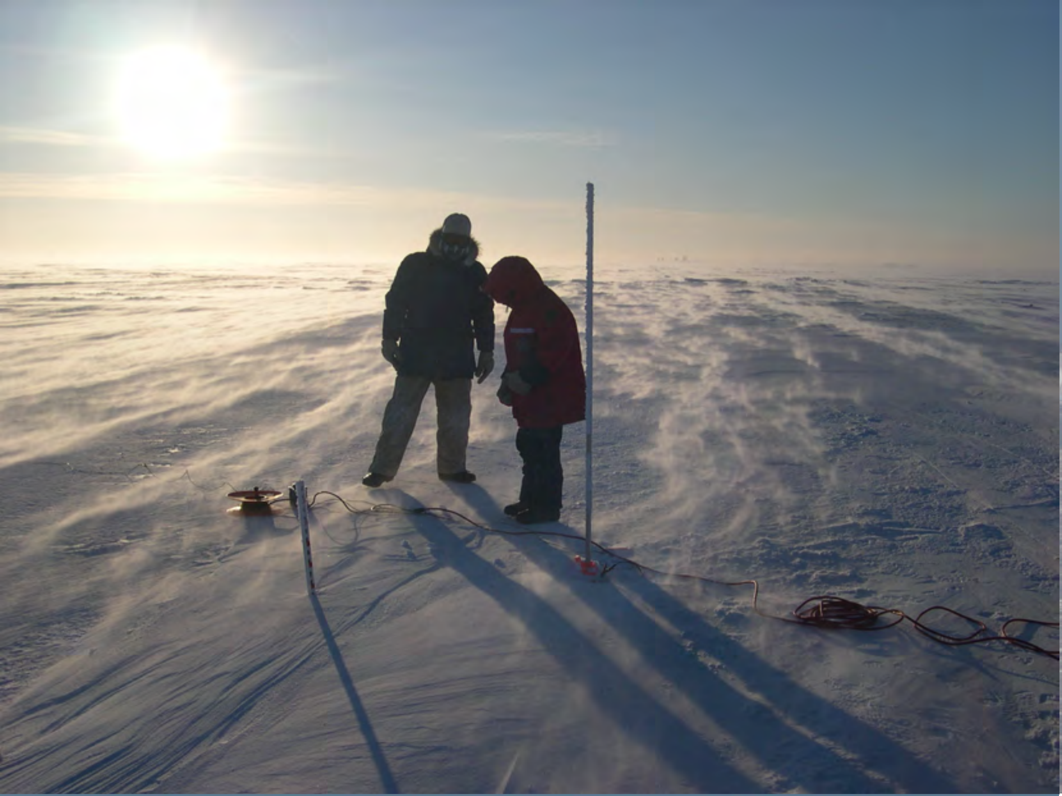
Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes

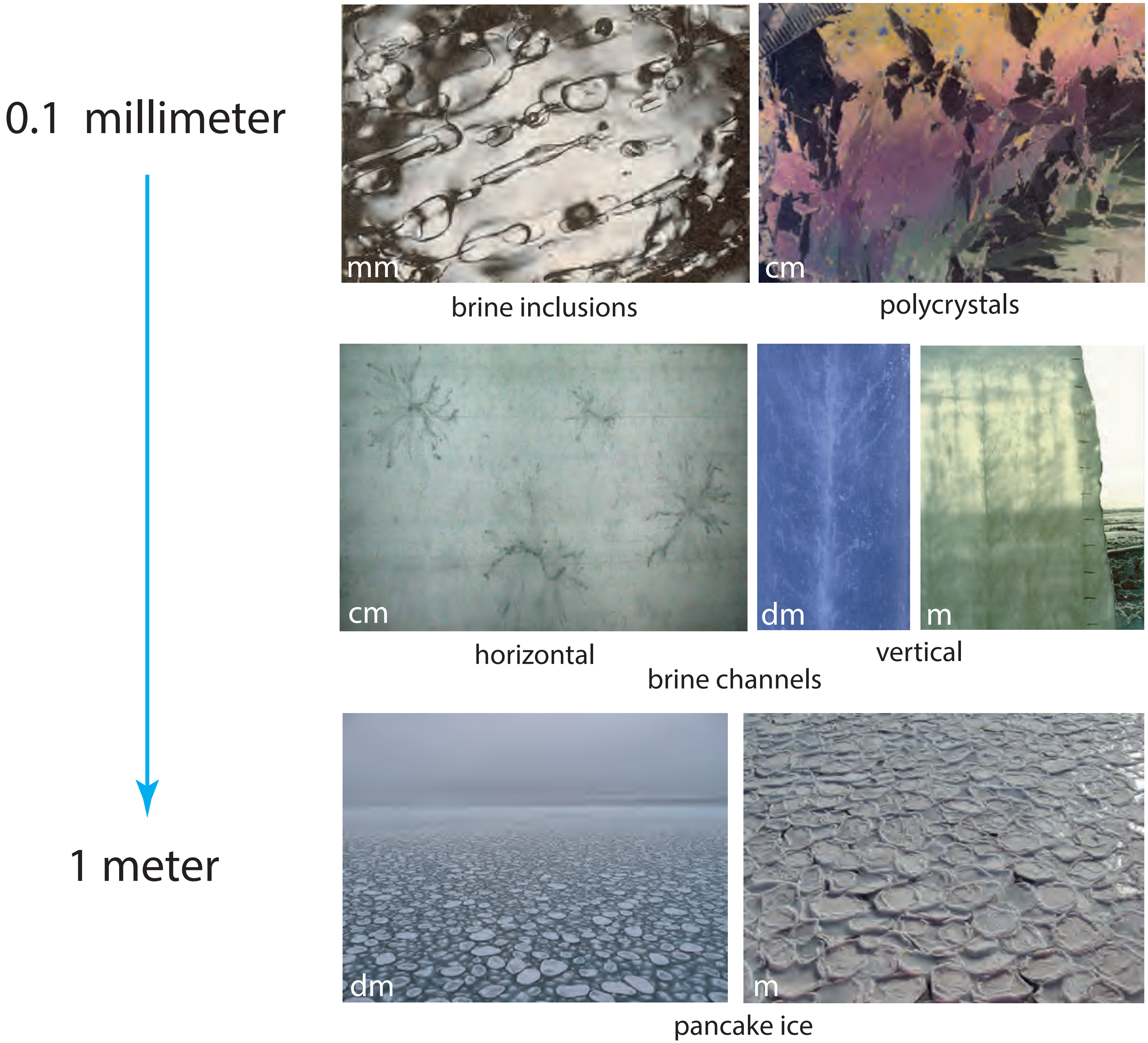
linkage of scales

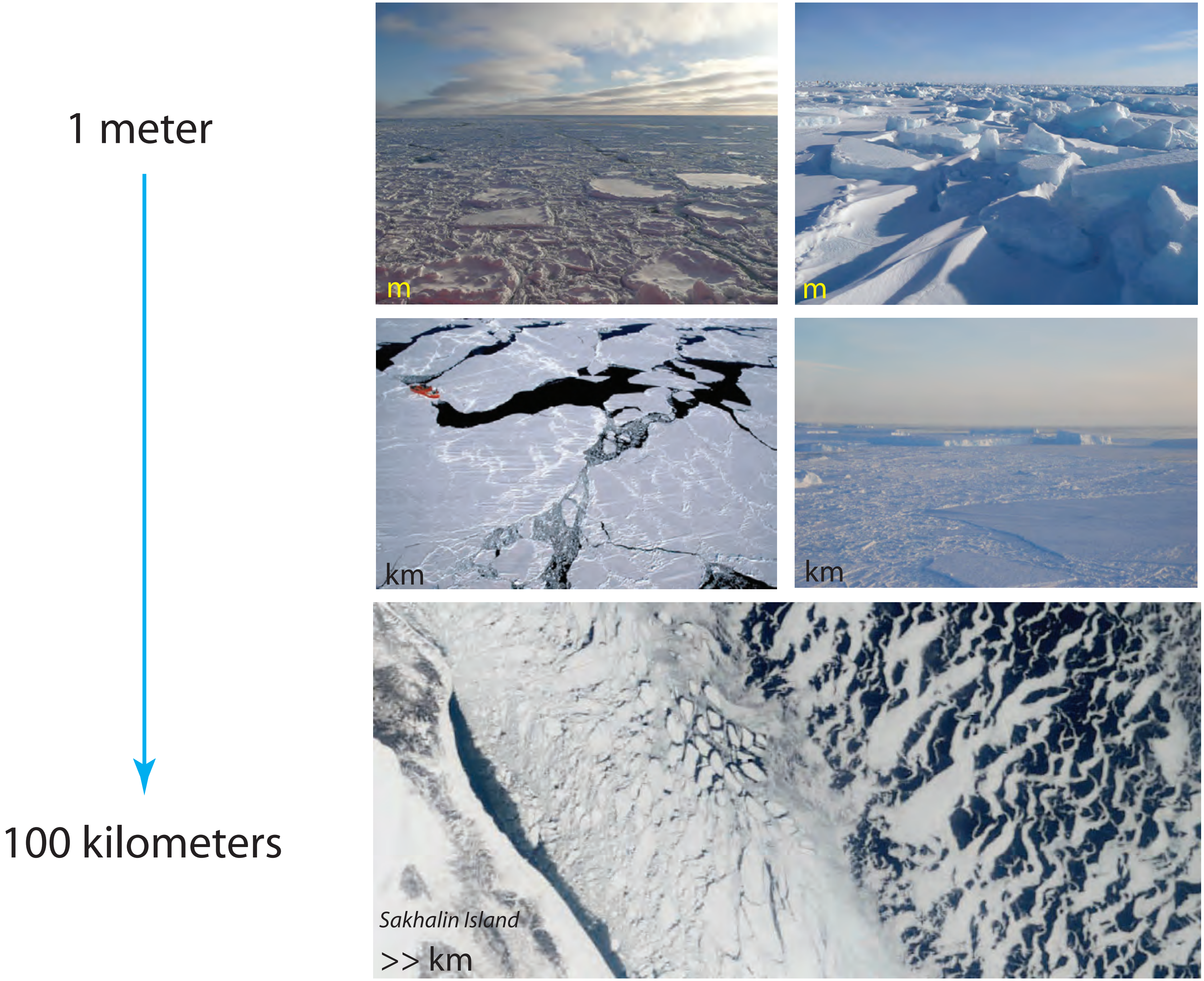


sea ice may appear to be a barren, impermeable cap ...

sea ice is a multiscale composite

displaying structure over 10 orders of magnitude



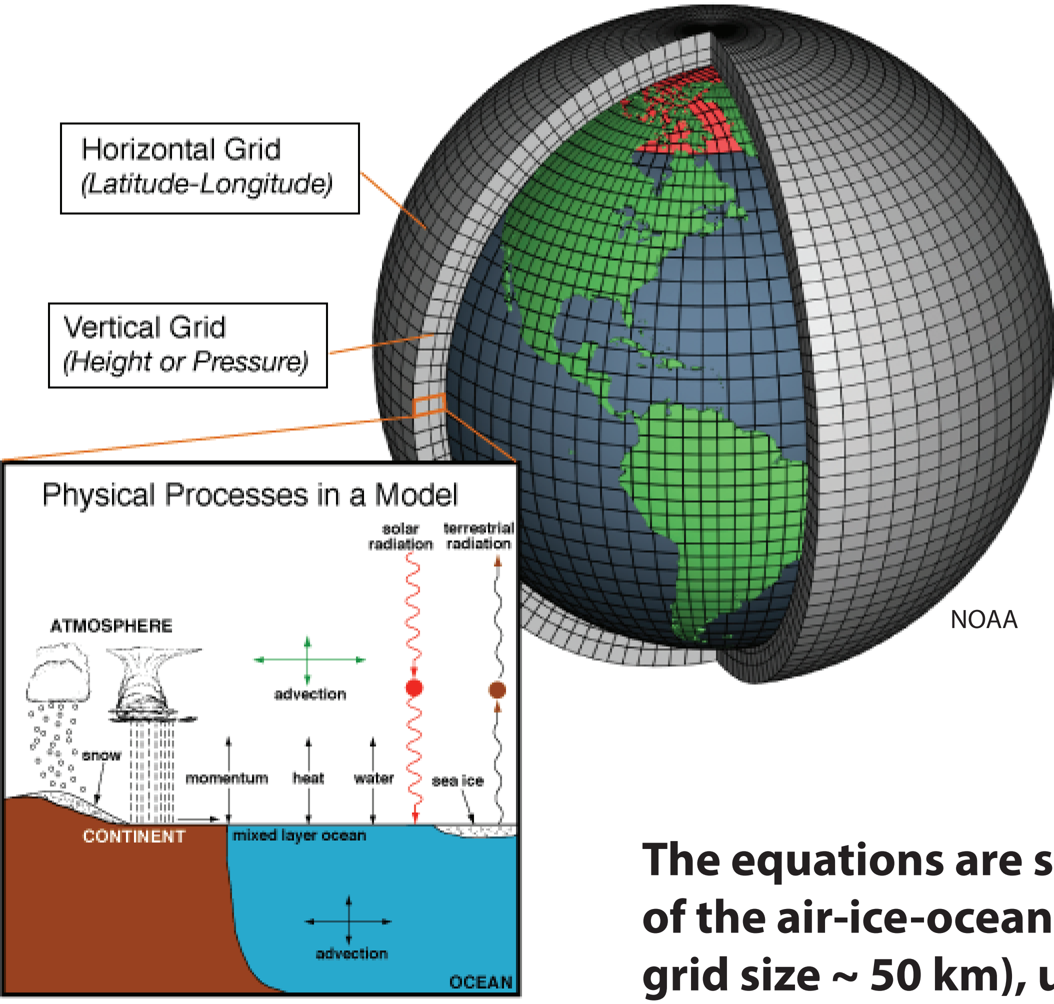

1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

What is this talk about? HOMOGENIZATION

Using mathematics of composite materials and statistical physics to LINK SCALES in the sea ice system and rigorously compute effective behavior ... to improve climate projections.

1. ***Climate models, tipping points***
2. ***Sea ice microphysics and flow thorugh porous media***
3. ***EM monitoring of sea ice, remote sensing***
4. ***Evolution of Arctic melt ponds, fractal geometry***

critical behavior ***cross - pollination***



Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

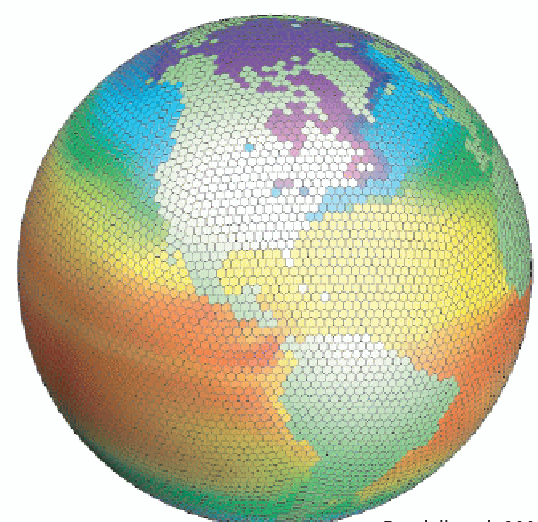
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 50 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales



Randall *et al.*, 2002

sea ice components of GCM's

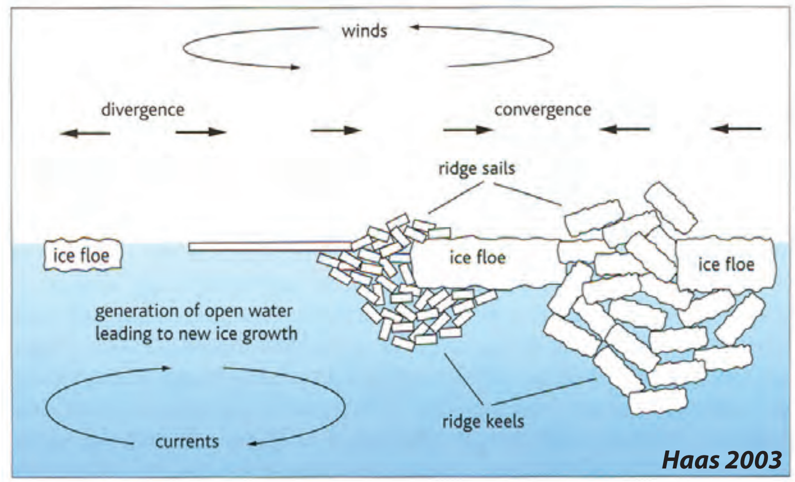
What are the key ingredients -- or ***governing equations***
that need to be solved on grids using powerful computers?

1. Ice thickness distribution $g(x, y, h, t)$ evolution equation

dynamics
+
thermodynamics

(Thorndike *et al.* 1975)

nonlinear PDE incorporating ice velocity field
ice growth and melting
mechanical redistribution
- ridging and opening



2. Conservation of momentum, stress *vs.* strain relation (Hibler 1979)

F = ma **for sea ice**

Coriolis, air and water drag,
floe - floe interactions, ...

dynamics

3. Heat equation for sea ice and snow

thermodynamics

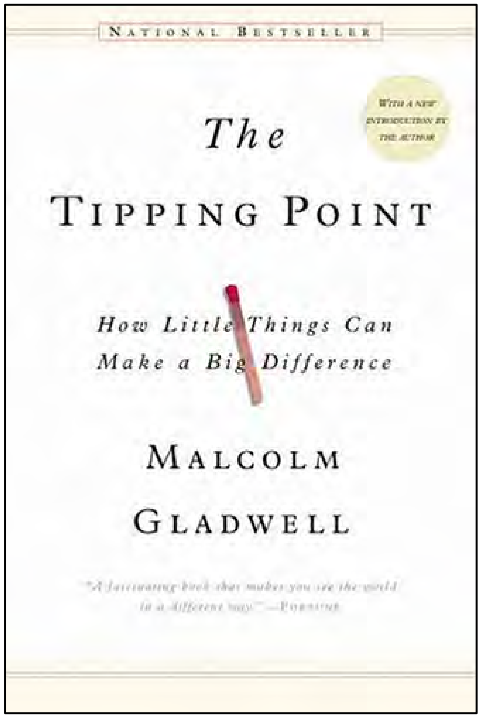
evolution of temperature field in ice
brine convection

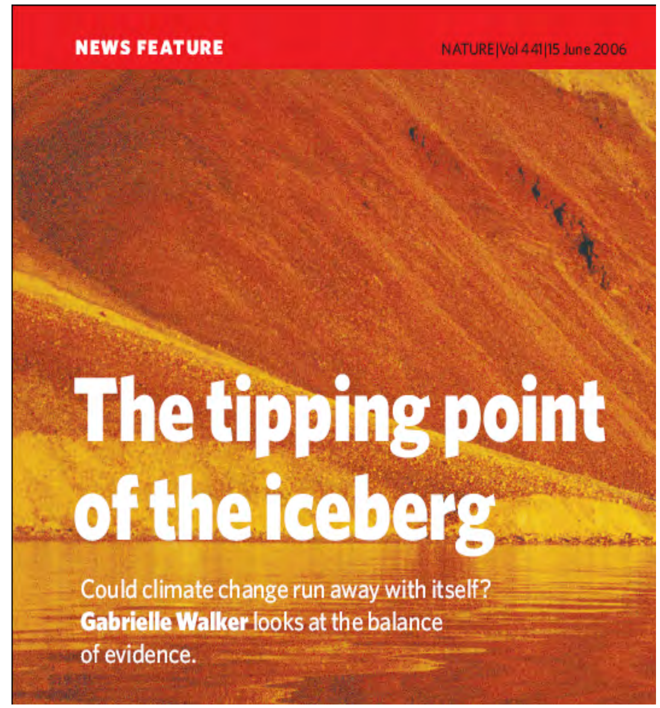
+ *balance of radiative and thermal fluxes on interfaces*

(Maykut and Untersteiner 1971)

tipping points in the mainstream

climate tipping points – September Arctic sea ice cover



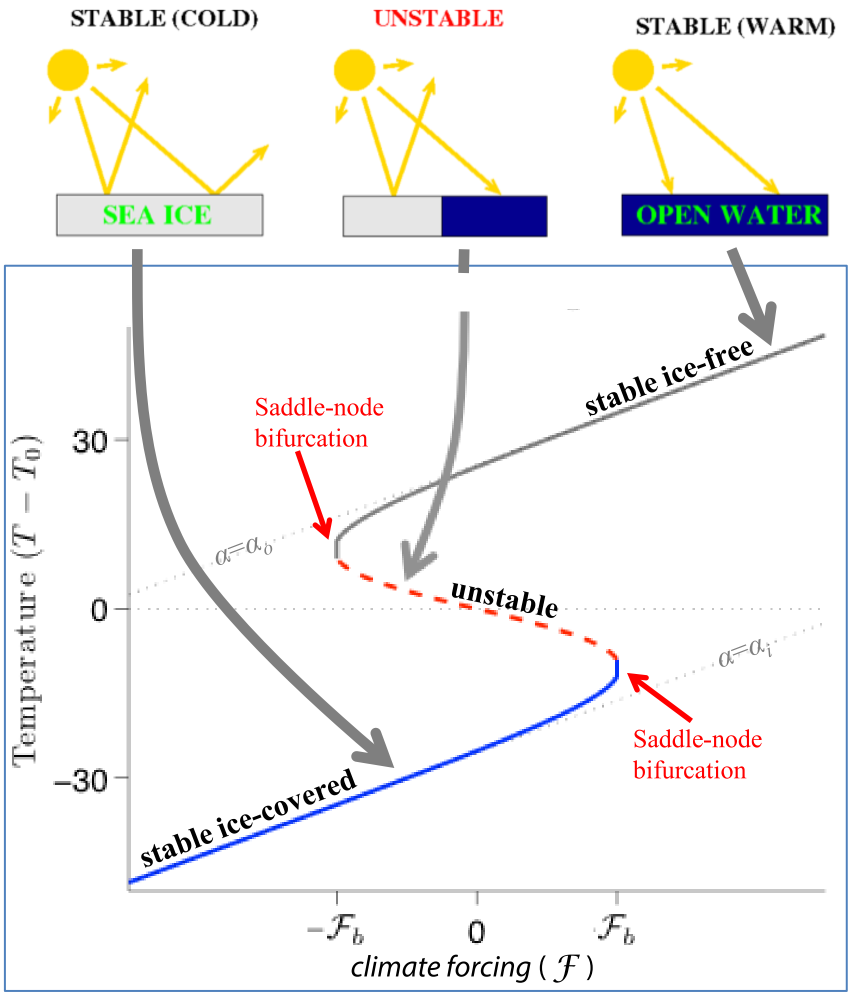


Melting of the Greenland ice sheet
Melting of the West Antarctic ice sheet
Permafrost and tundra loss, leading to the release of methane
Shutoff of N. Atlantic thermohaline conveyor (Gulf Stream) •••

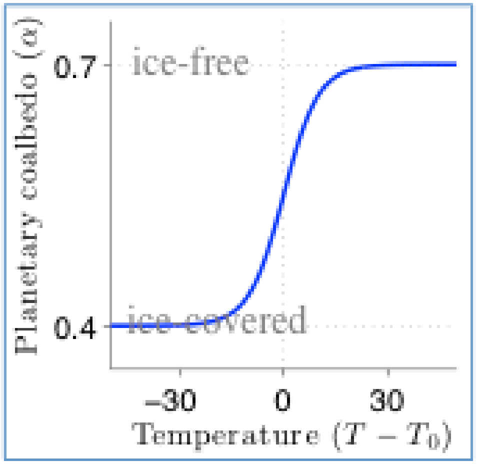
Lenton, et al., PNAS 2008

What would a tipping point in sea ice cover look like?

Bifurcation Diagram



nonlinear ice-albedo feedback

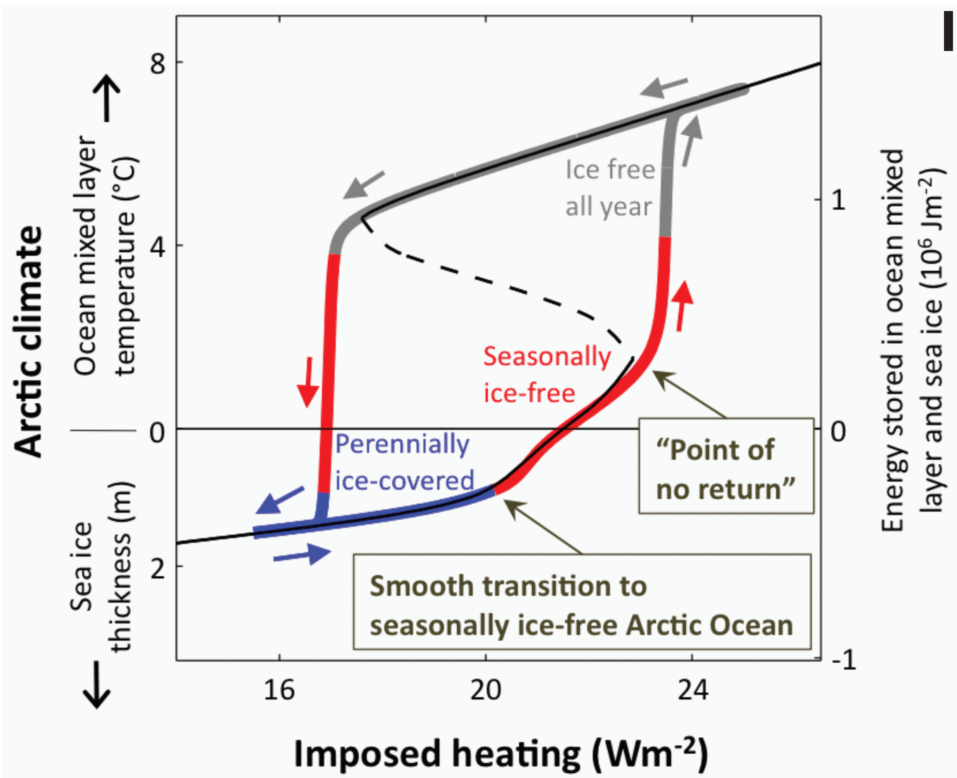


Eisenman

Has Arctic sea ice loss passed through a "tipping point"?

opposite pole from GCM: low order (toy) models of climate change

Eisenman, Wettlaufer, PNAS 2009: nonlinear ODE for energy in upper ocean

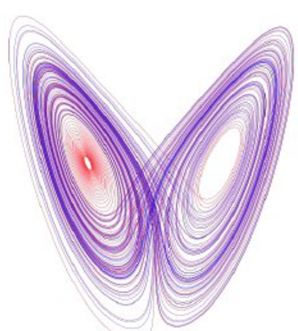


look for bifurcations, multiple equilibria

tipping point unlikely in loss of summer ice

Abbot, Silber, Pierrehumbert, JGR 2011

bifurcations with clouds, ice loss



Lorenz butterfly

Wagner and Eisenman, J Climate 2015

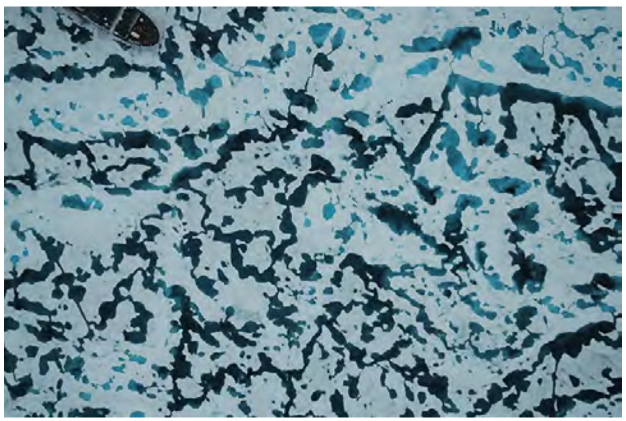
bridge gap between GCM and low order models

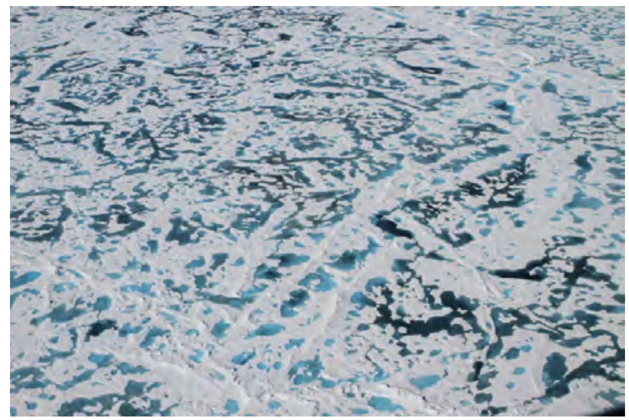
How do scales interact in the sea ice system?

basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds

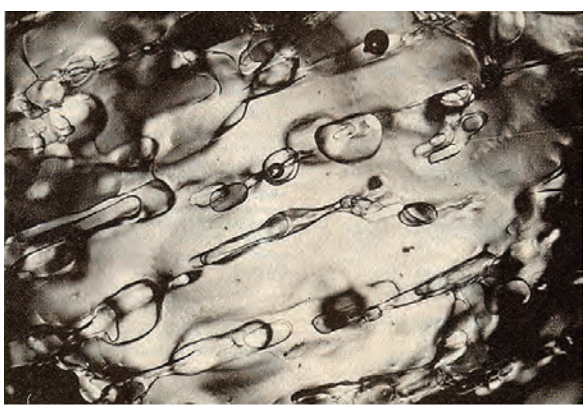




km
scale
melt
ponds

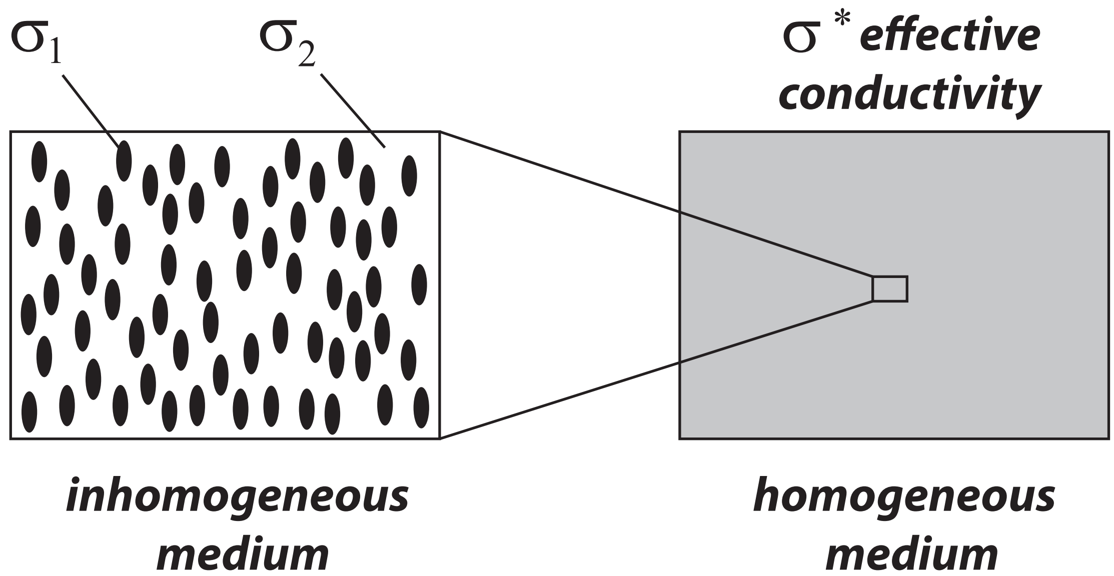
Linking Scales

mm
scale
brine
inclusions



meter
scale
snow
topography

HOMOGENIZATION - Linking Scales in Composites



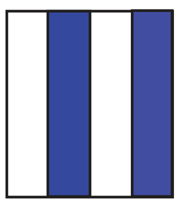
find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

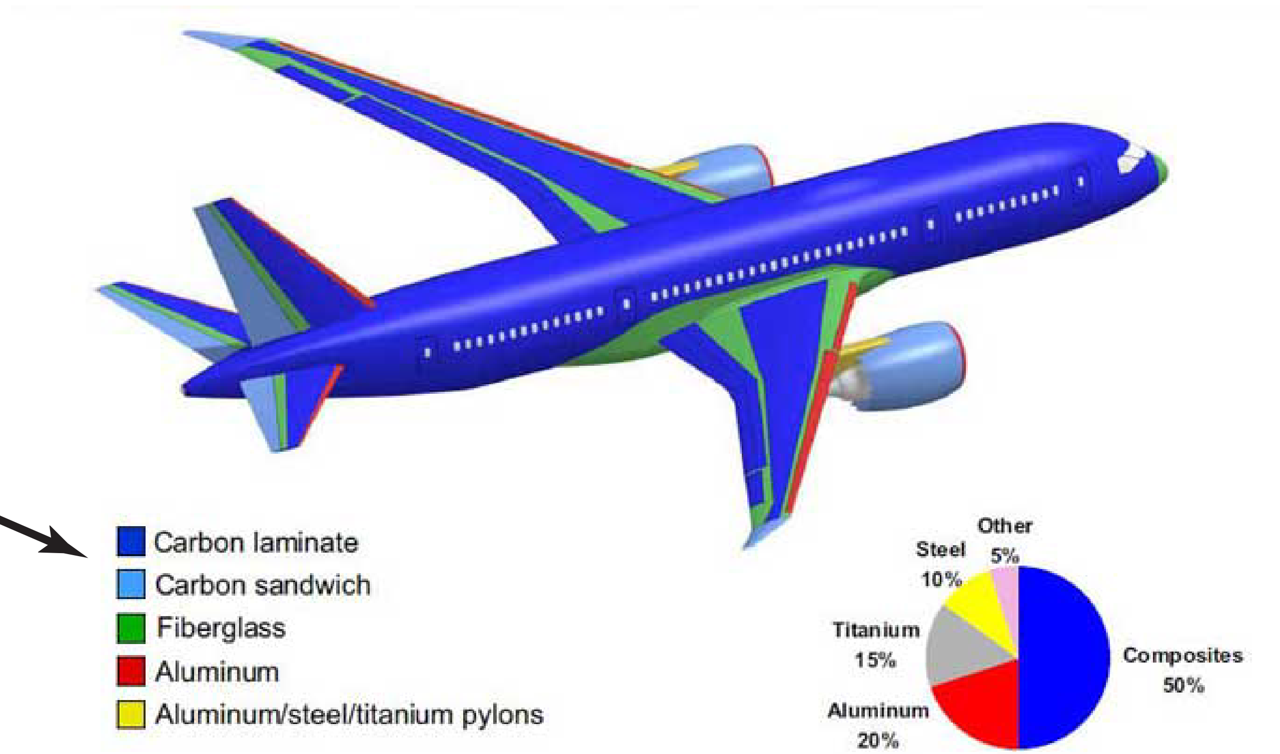
*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

Composite materials in the Boeing 787 Dreamliner



laminate structures
display optimal
properties



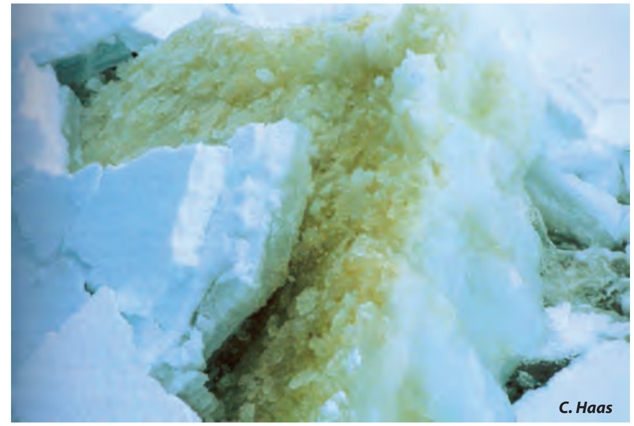
sea ice microphysics

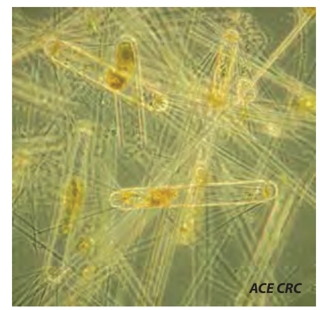
fluid transport

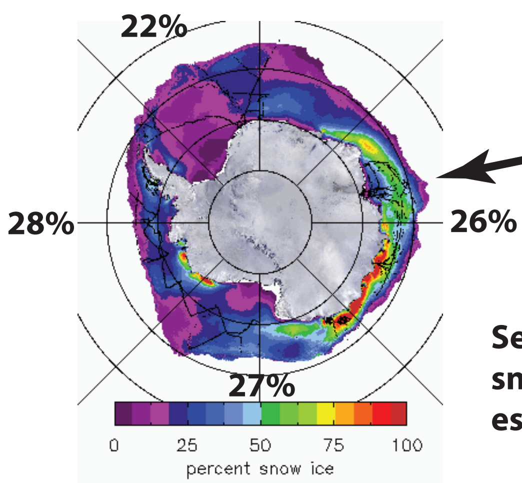
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

sea ice ecosystem

sea ice algae
support life in the polar oceans

fluid permeability k of a porous medium

porous concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium

averaged
fluid velocity

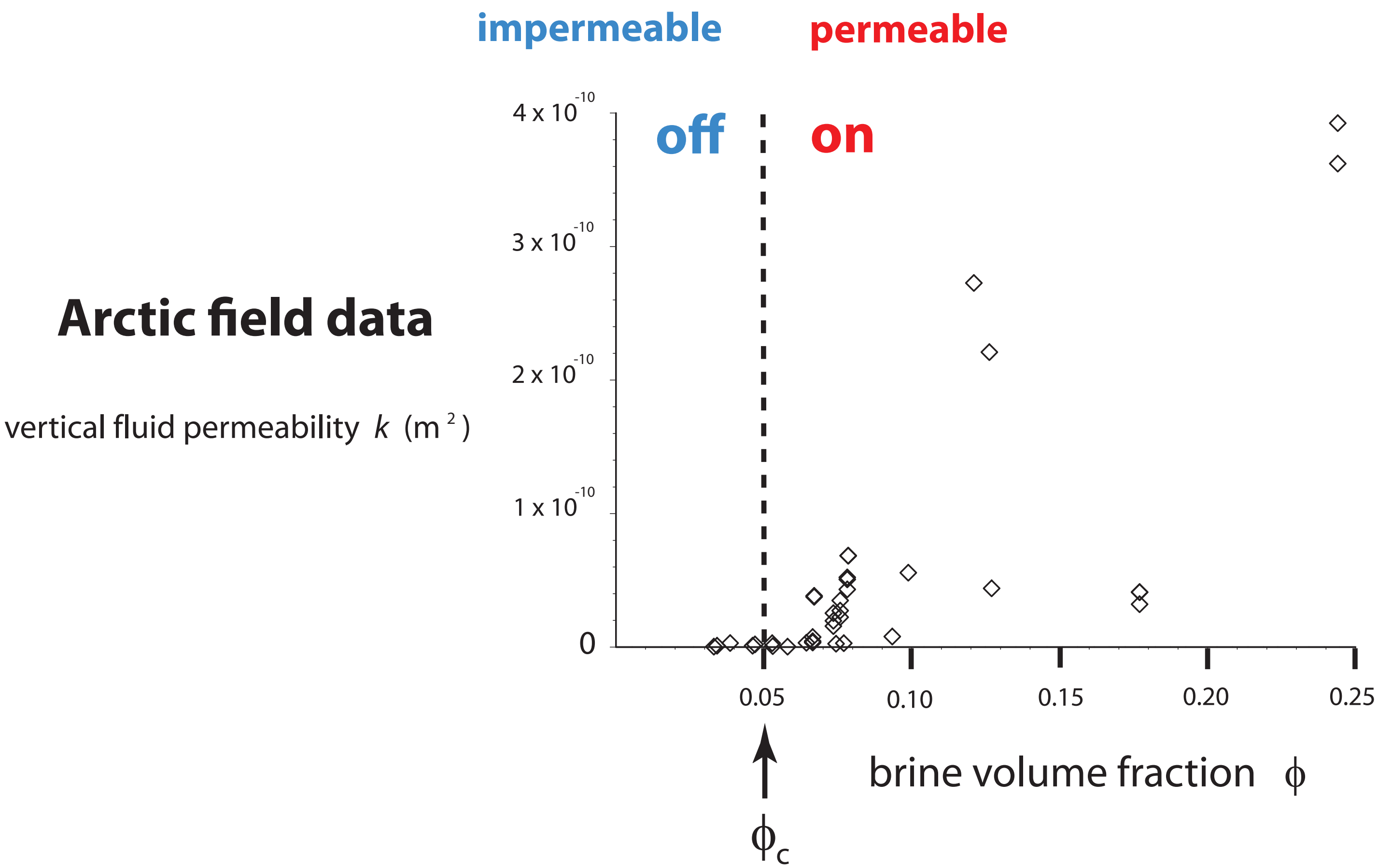
pressure
gradient

$$\mathrm{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

viscosity

$\mathbf{k}$ = fluid permeability tensor

Critical behavior of fluid transport in sea ice



on - off switch for fluid flow

$$\text{critical brine volume fraction} \quad \phi_c \approx 5\% \quad \longleftrightarrow \quad T_c \approx -5^\circ\text{C}, \; S \approx 5 \text{ ppt}$$

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

sea ice algal communities

D. Thomas 2004

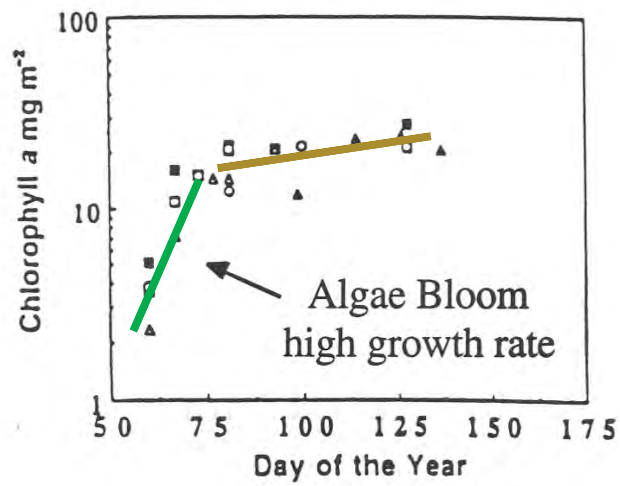
nutrient replenishment controlled by ice permeability

biological activity turns on or off according to ***rule of fives***

Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

critical behavior of microbial activity

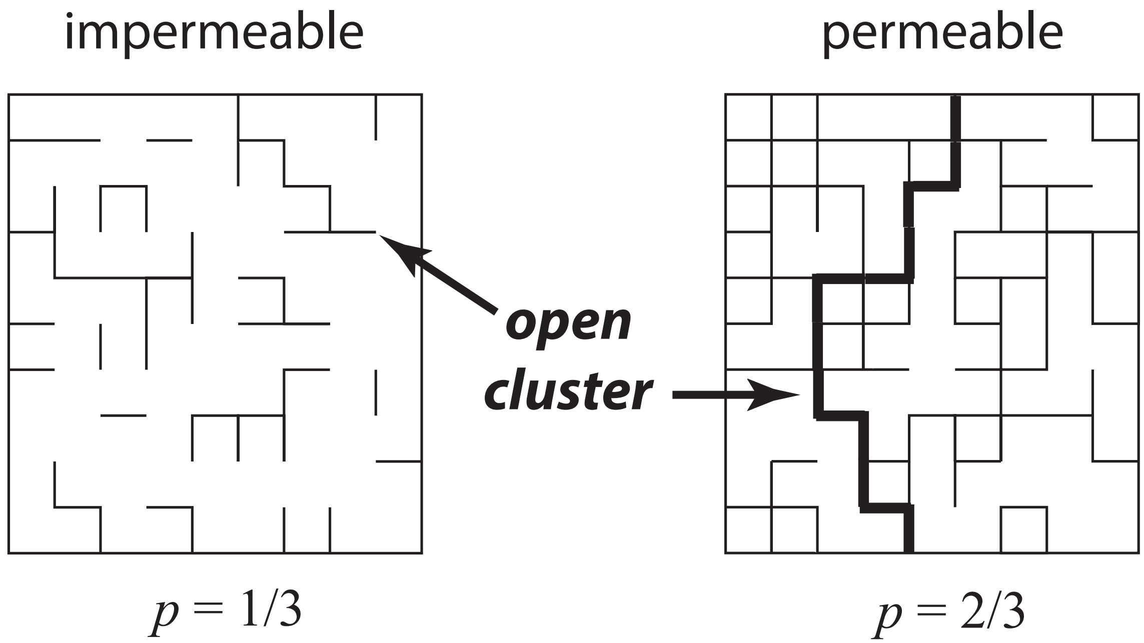


Convection-fueled algae bloom
Ice Station Weddell

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond → **open** *with probability* p
closed *with probability* $1-p$

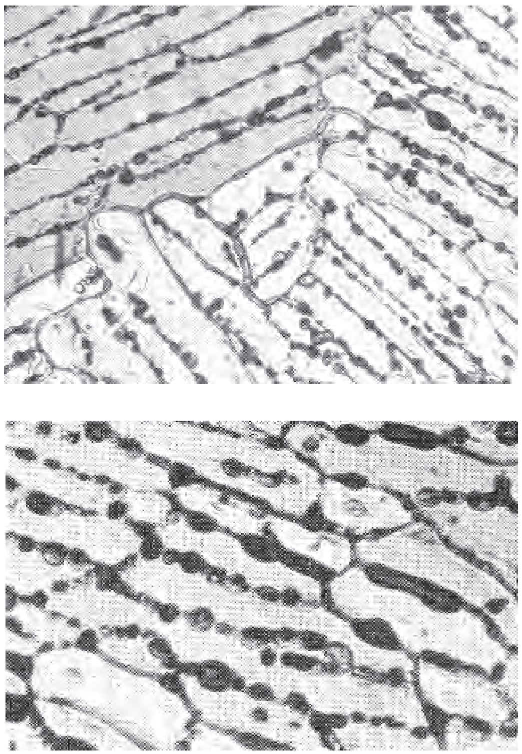
percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

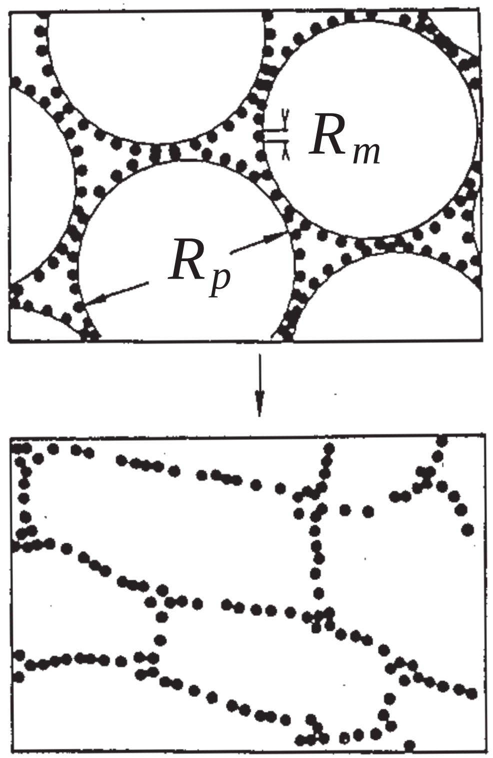
smallest *p* for which there is an infinite open cluster

Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

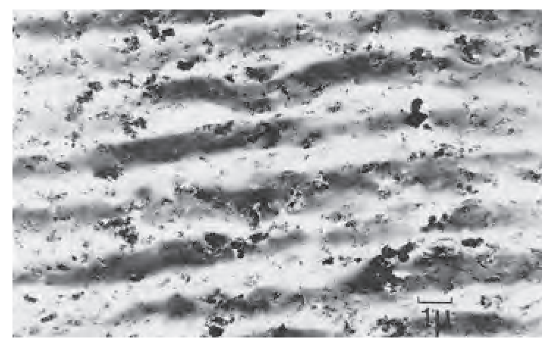
$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed powder



radar absorbing composite

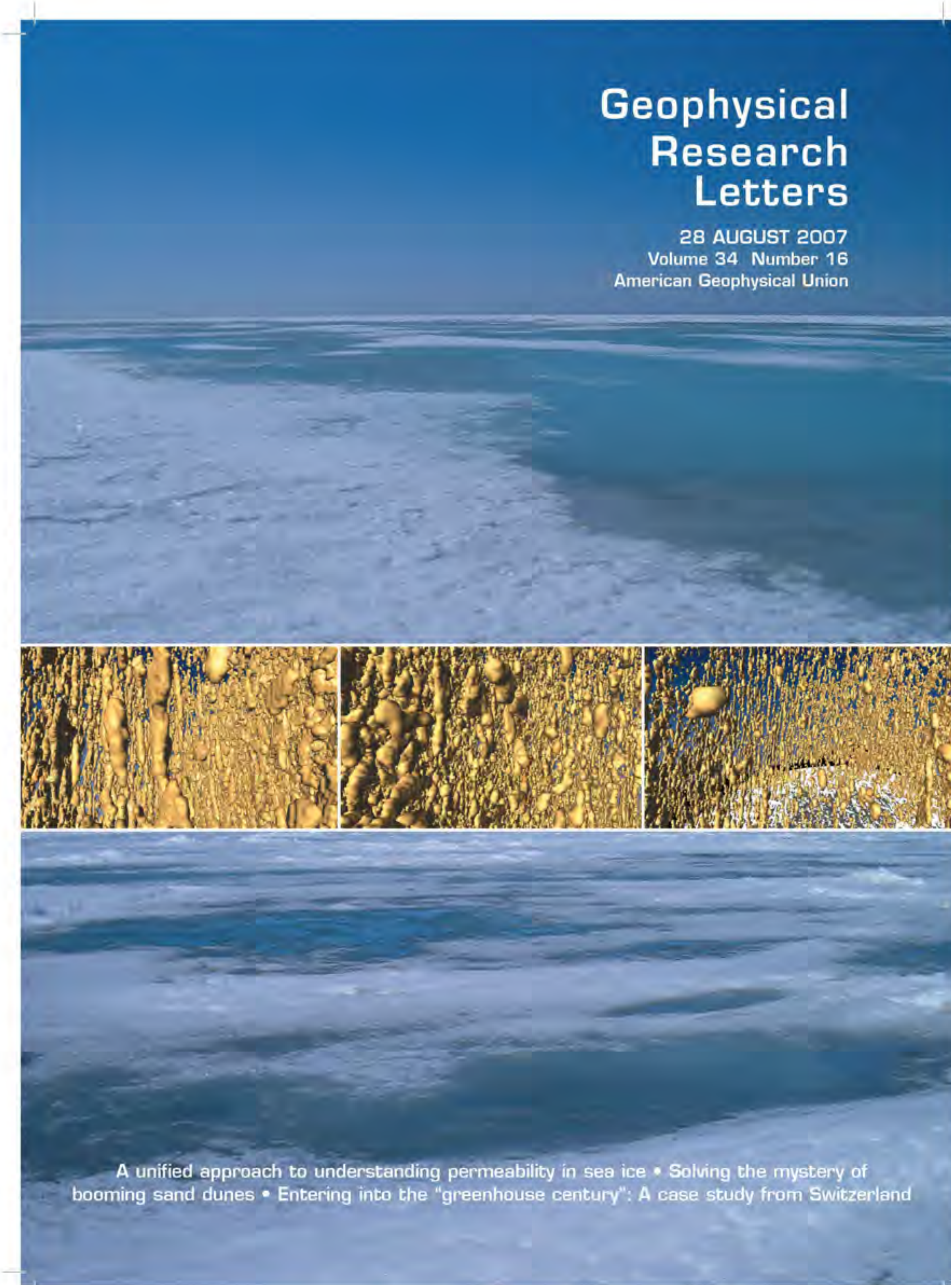
sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice ***Golden, Eicken, Heaton, Miner, Pringle, Zhu***

micro-scale

controls

macro-scale
processes



rigorous bounds
percolation theory
hierarchical model
network model

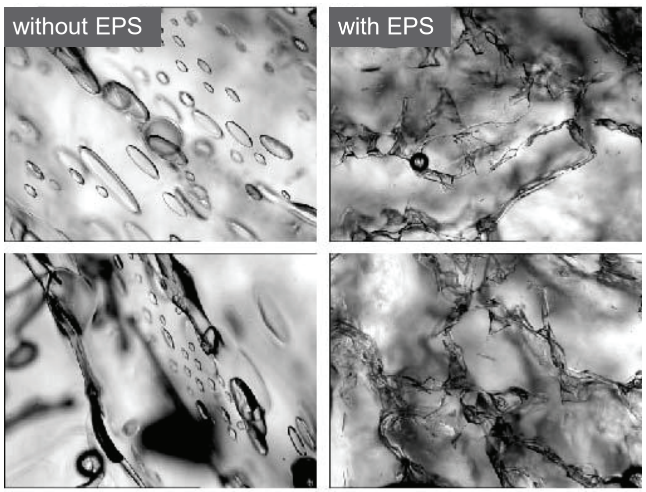
field data

X-ray tomography for brine inclusions

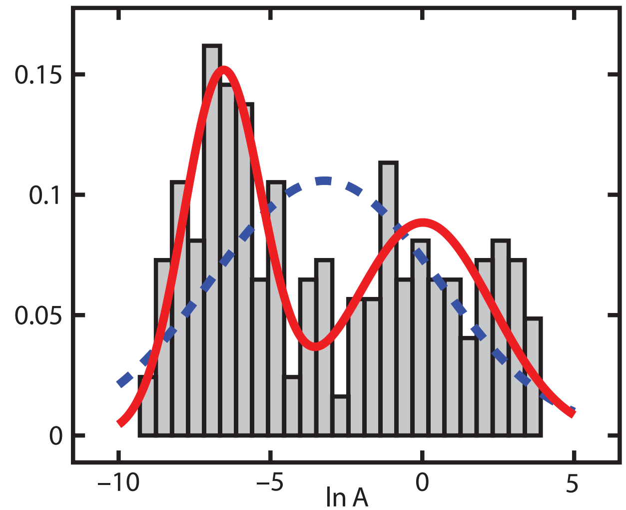
unprecedented look at thermal evolution of brine phase and its connectivity

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

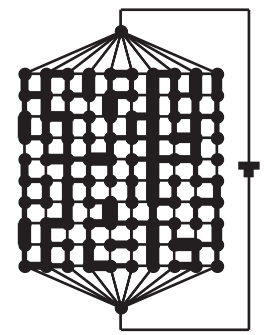
How does EPS affect fluid transport?

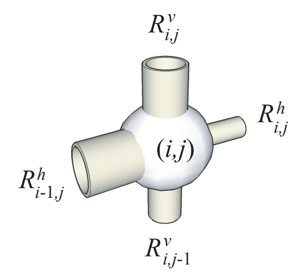


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





Zhu, Jabini, Golden, Eicken, Morris *Ann. Glac.* 2006

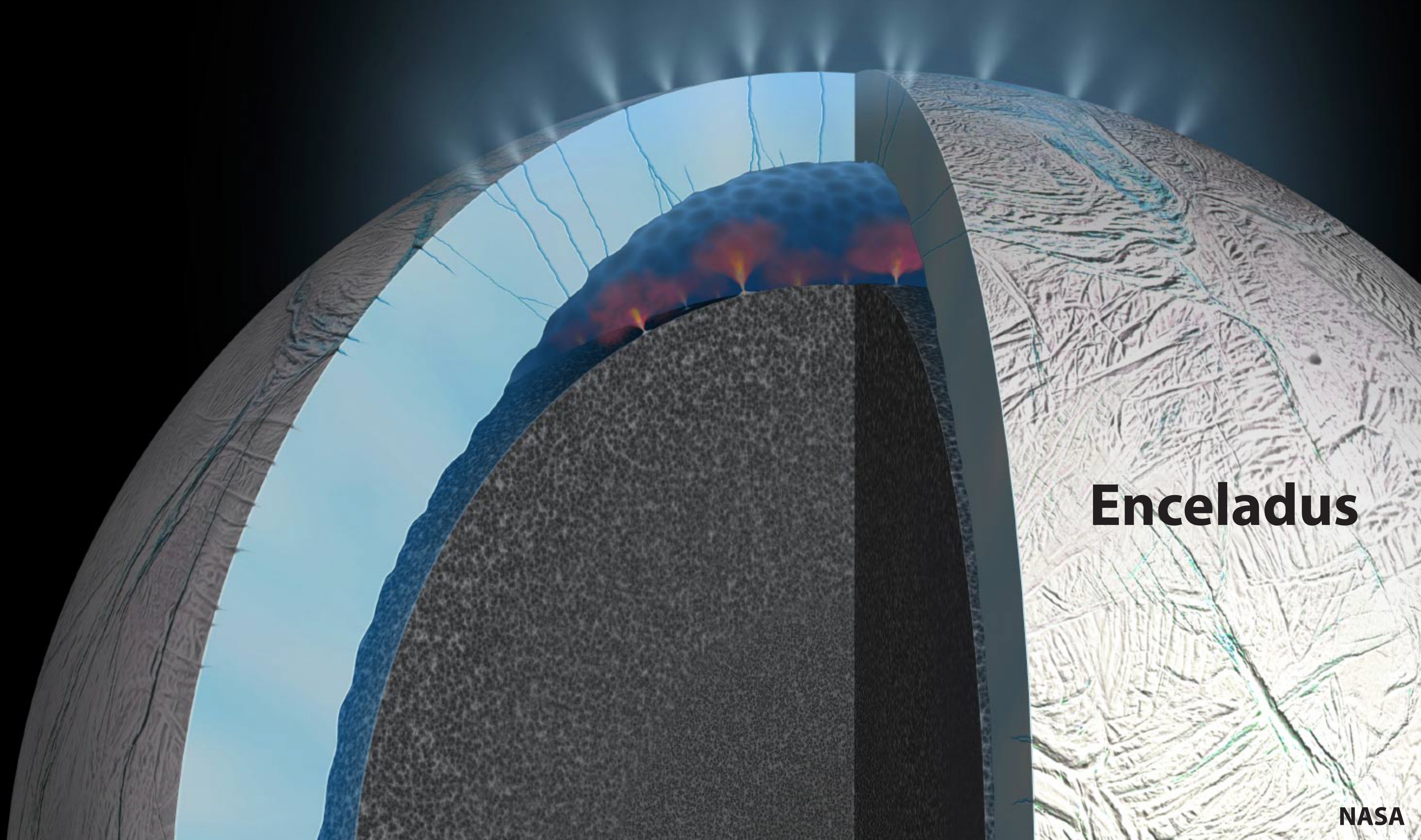
- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability *k*.
- **Rigorous bound on *k* for bimodal distribution of pore sizes**

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

How does the biology affect the physics?

Microbial Habitability in the Icy Moons of Jupiter and Saturn

Ruby Bowers
David Morison
Ken Golden



Remote sensing of sea ice

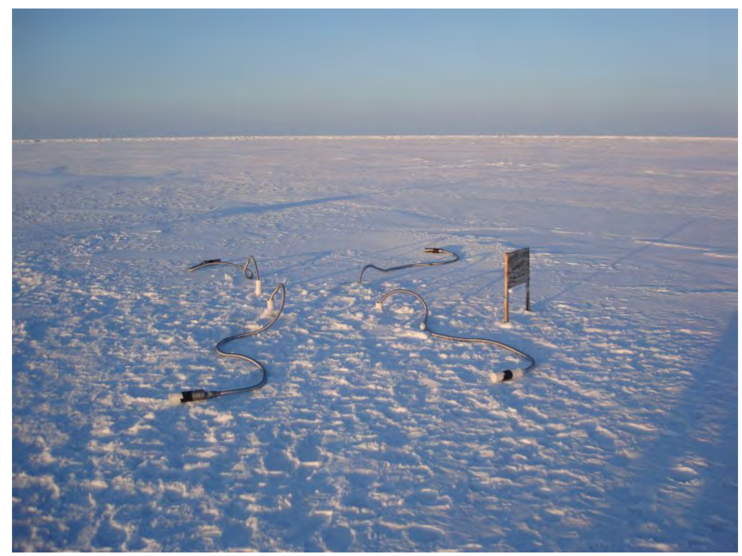
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

$$\varepsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)

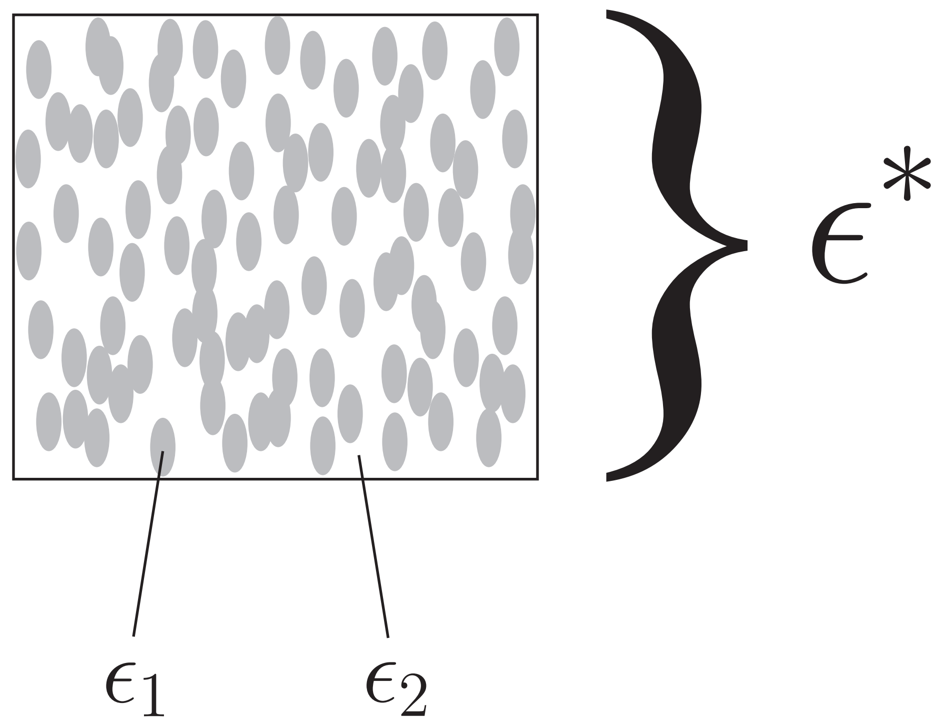


brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit

What are the effective propagation characteristics of an EM wave (radar, microwaves) in the medium?

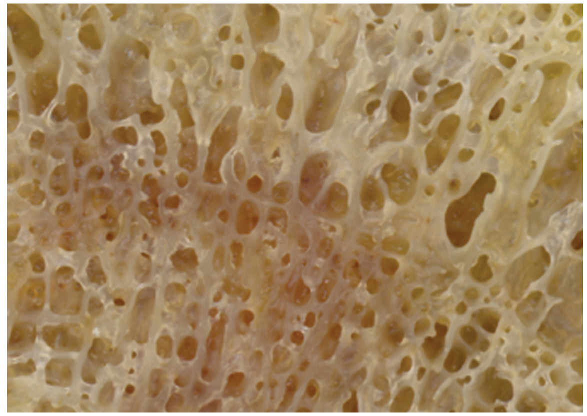
HOMOGENIZATION



p_1 , p_2 = volume fractions of brine and ice

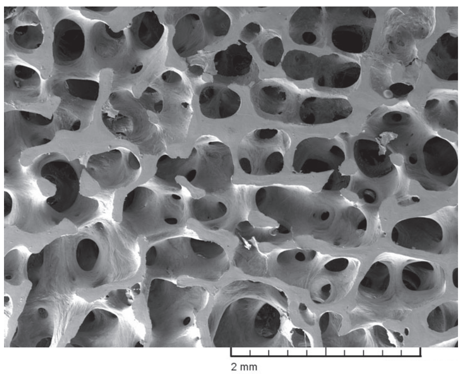
SEA ICE

HUMAN BONE

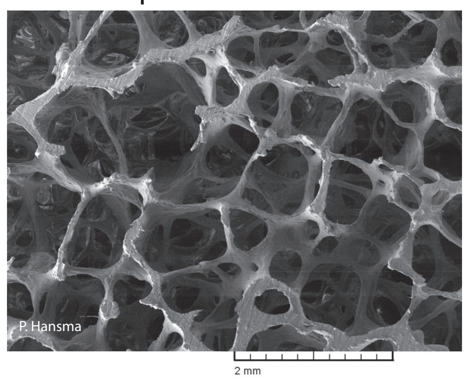


spectral characterization of porous microstructures in human bone

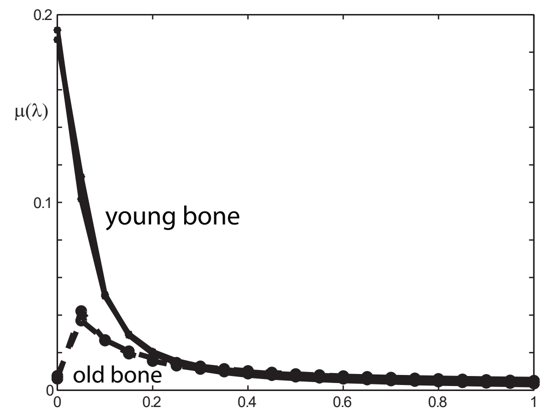
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures from complex permittivity data



use regularized inversion scheme

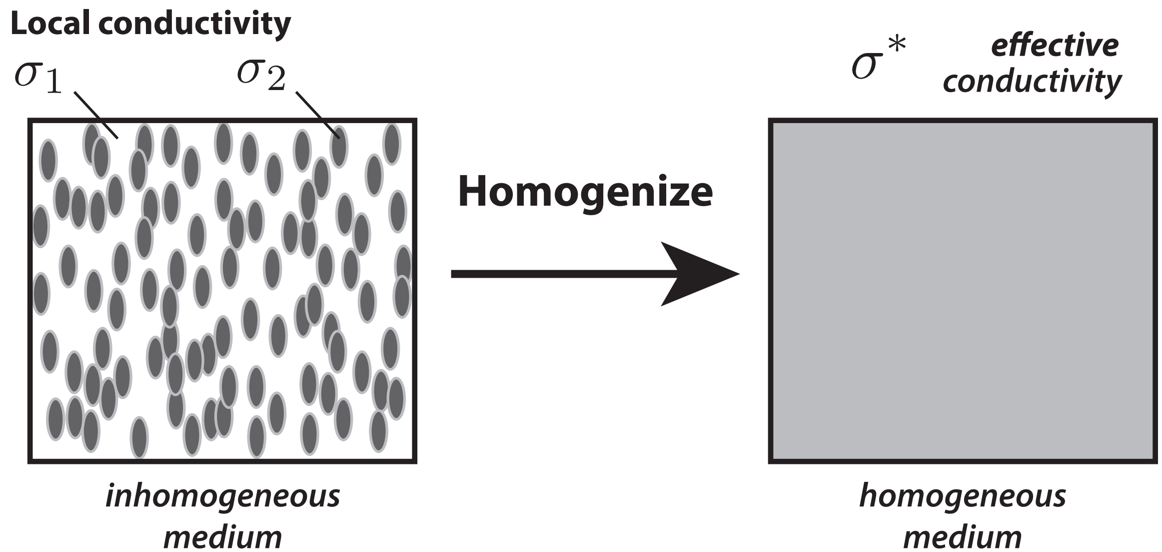
apply spectral measure analysis of brine connectivity and spectral inversion to electromagnetic monitoring of osteoporosis

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

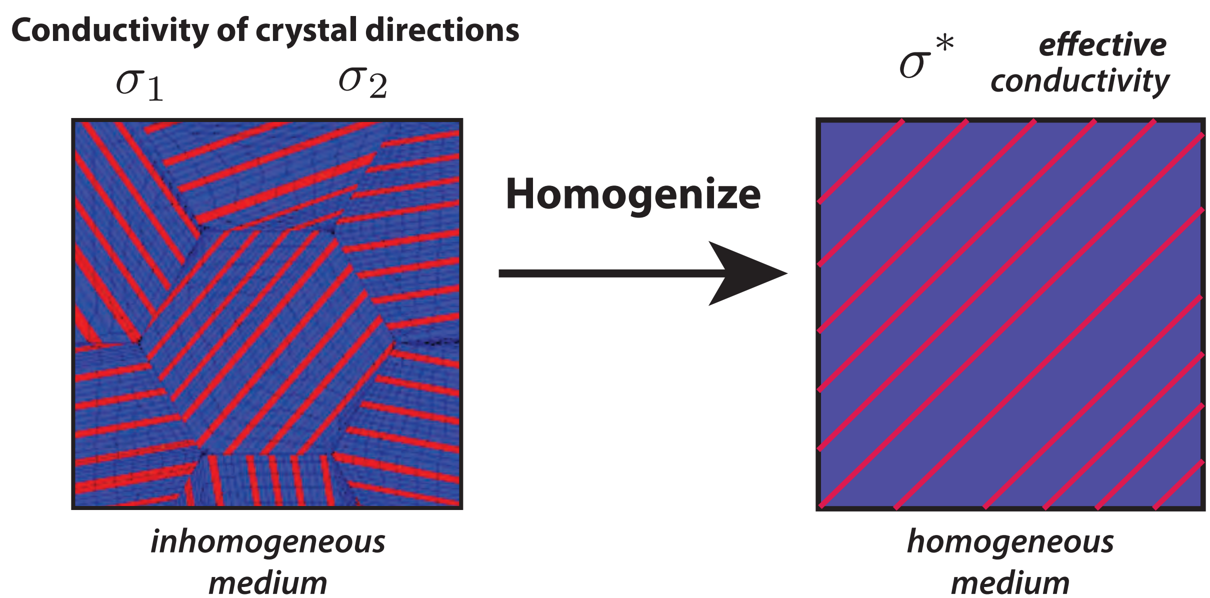
Homogenization for composite materials

Two-component composites



Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Polycrystalline media



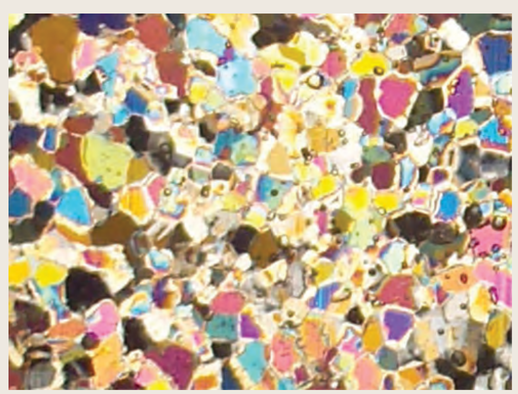
Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...

- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



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PROCEEDINGS A

350 YEARS OF SCIENTIFIC PUBLISHING

An invited review commemorating 350 years of scientific publishing at the Royal Society

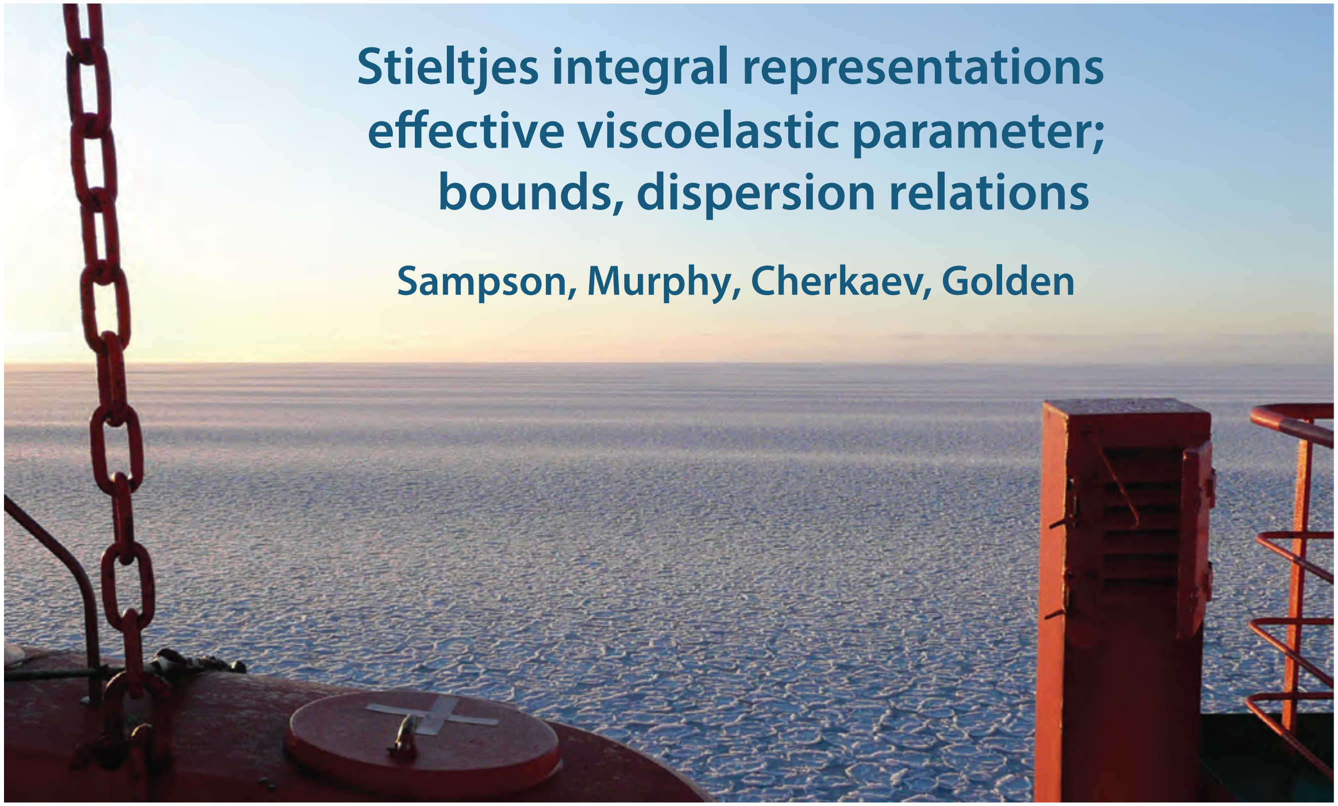
A method to distinguish between different types of sea ice using remote sensing techniques

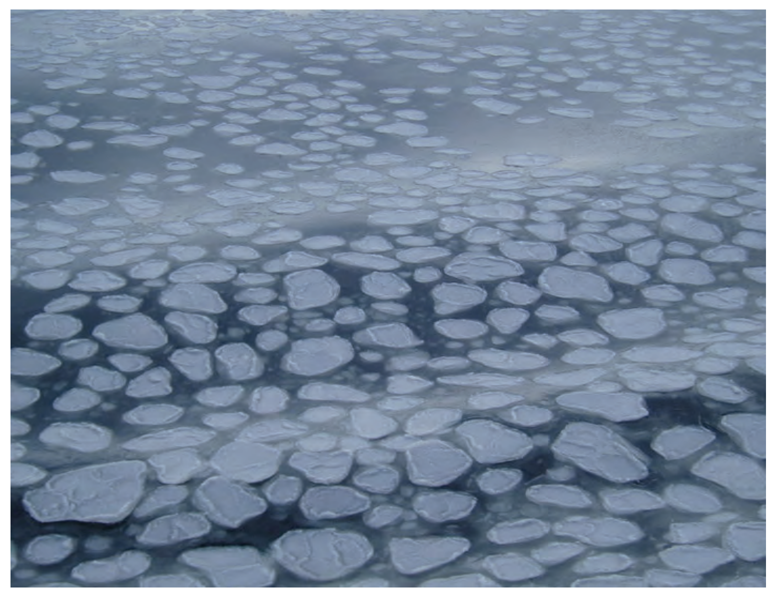
A computer model to determine how a human should walk so as to expend the least energy

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes

wave propagation in the marginal ice zone





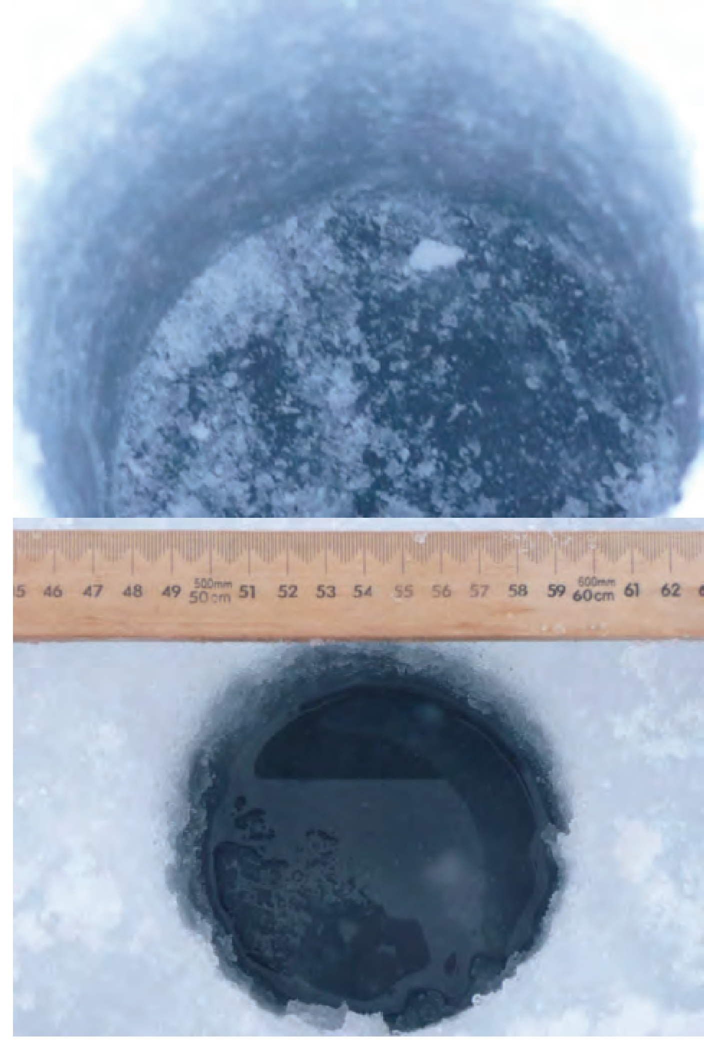
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

photo by Jan Lieser



measuring
fluid permeability
of Antarctic sea ice

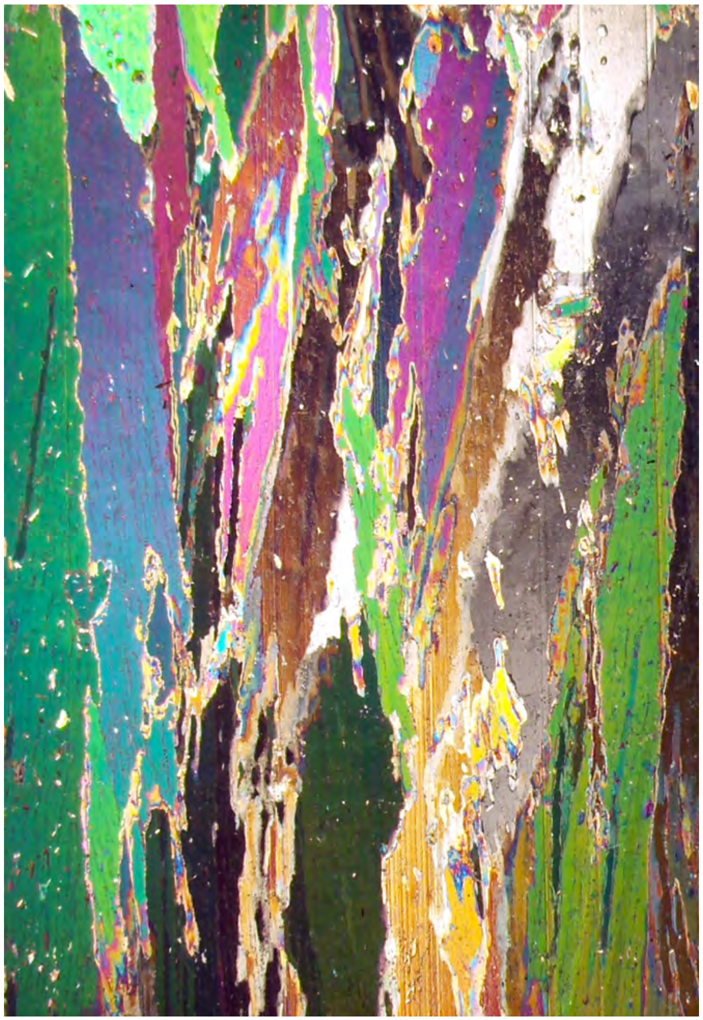
SIPEX 2007

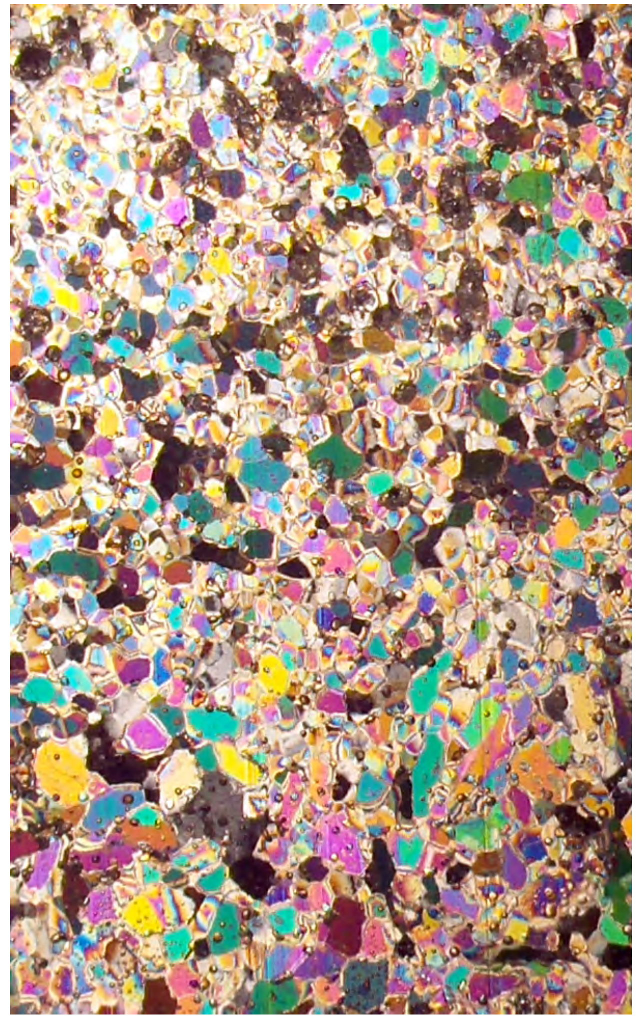
higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular

5%

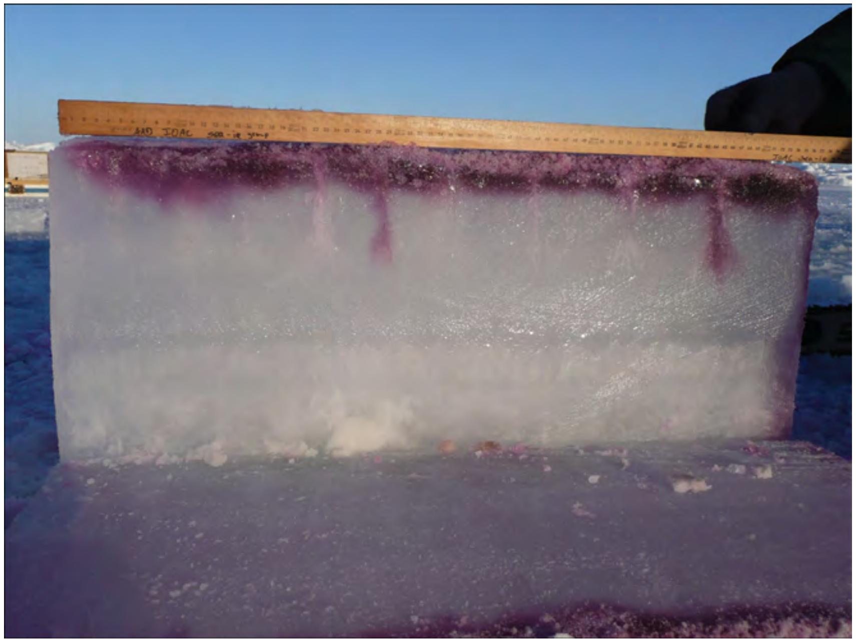


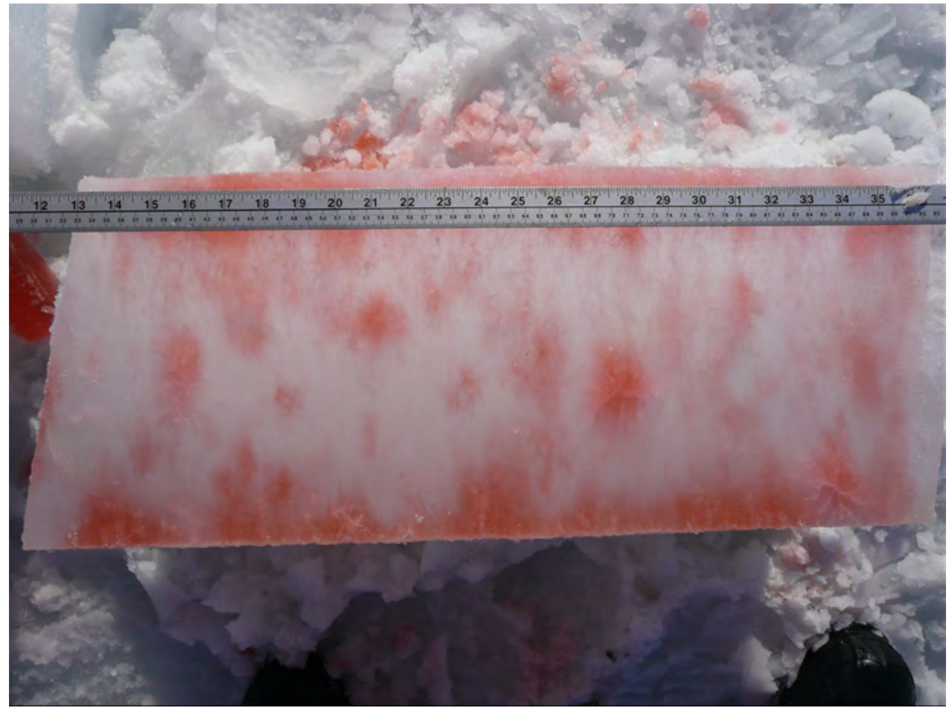


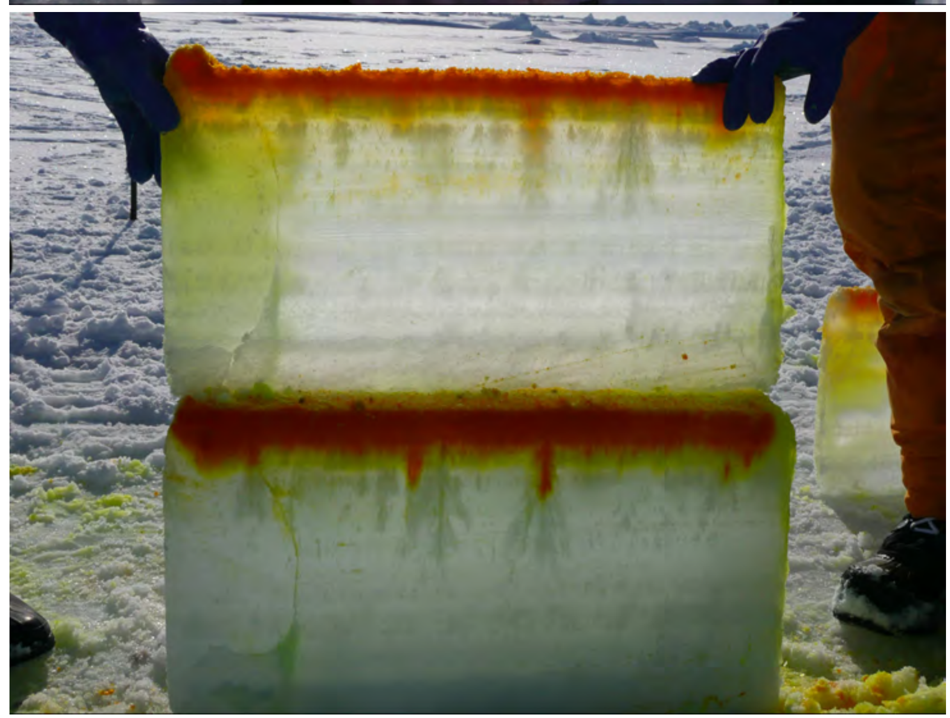
10%

Golden, Sampson, Gully, Lubbers, Tison 2018

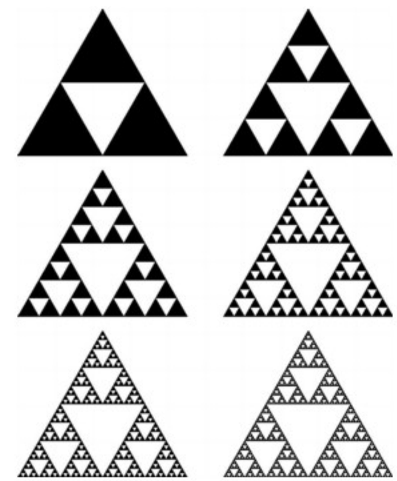
tracers flowing through inverted sea ice blocks

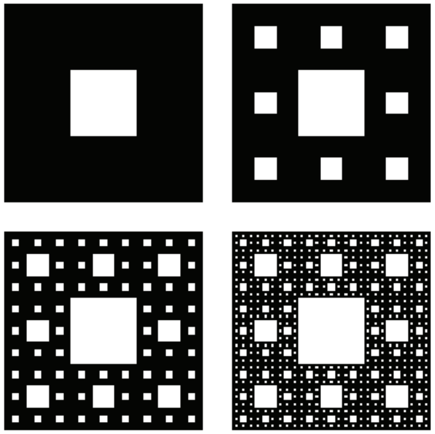


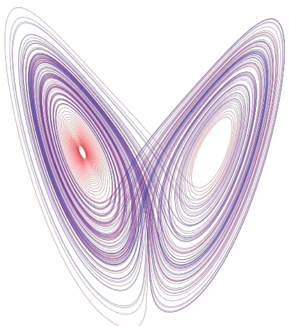




fractals and multiscale structure



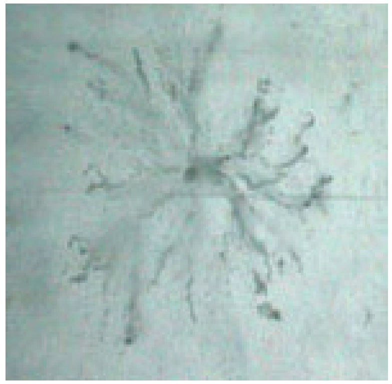




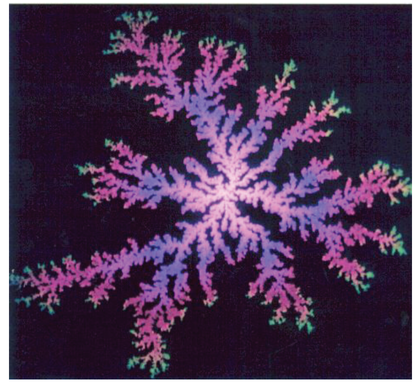
some
fractals

fractal microstructures

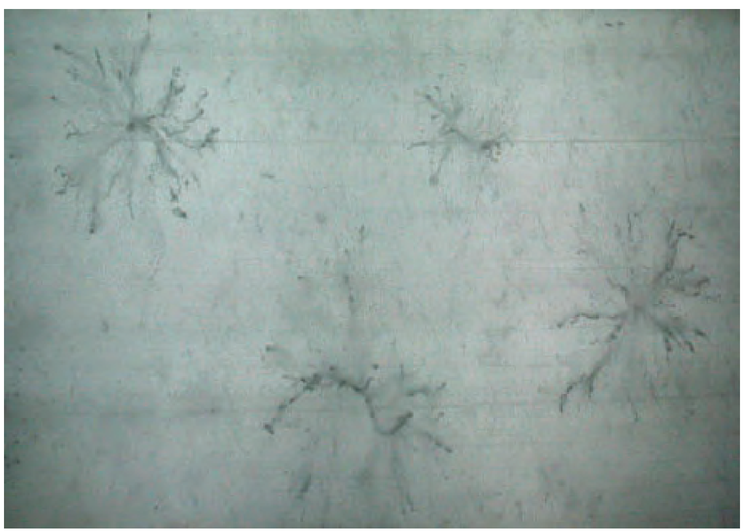
electrorheological fluid
with metal spheres

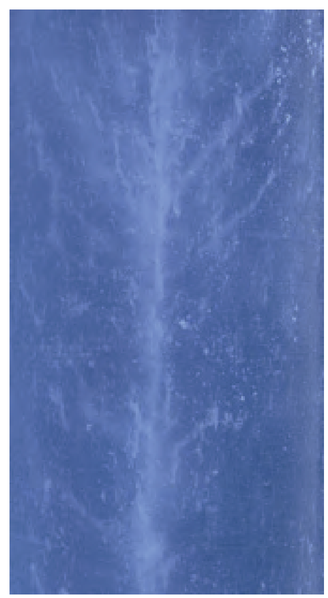


brine channel
in sea ice



diffusion limited
aggregation

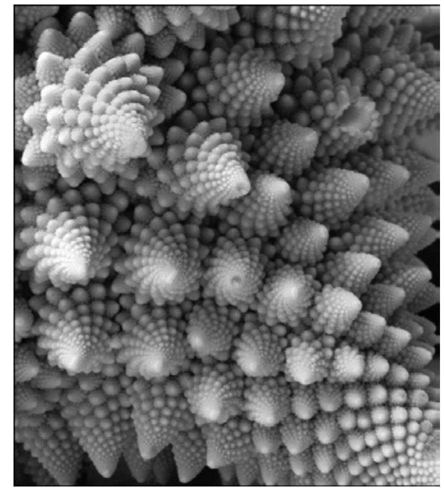


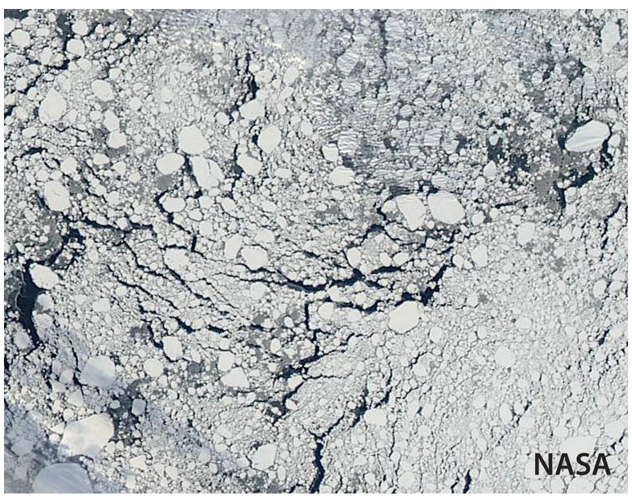


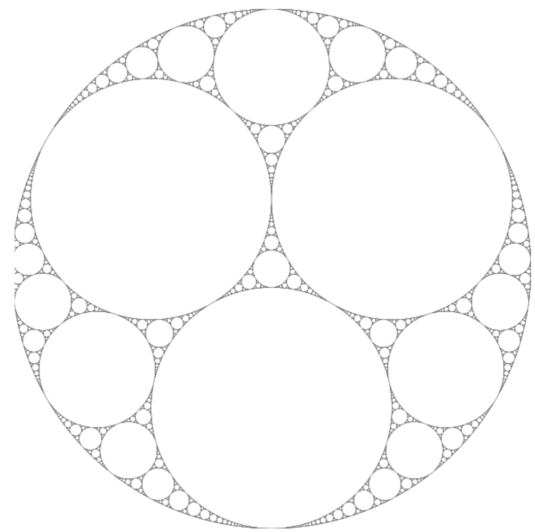
brine channels

the sea ice pack is a *fractal*

displaying self-similar structure on many scales





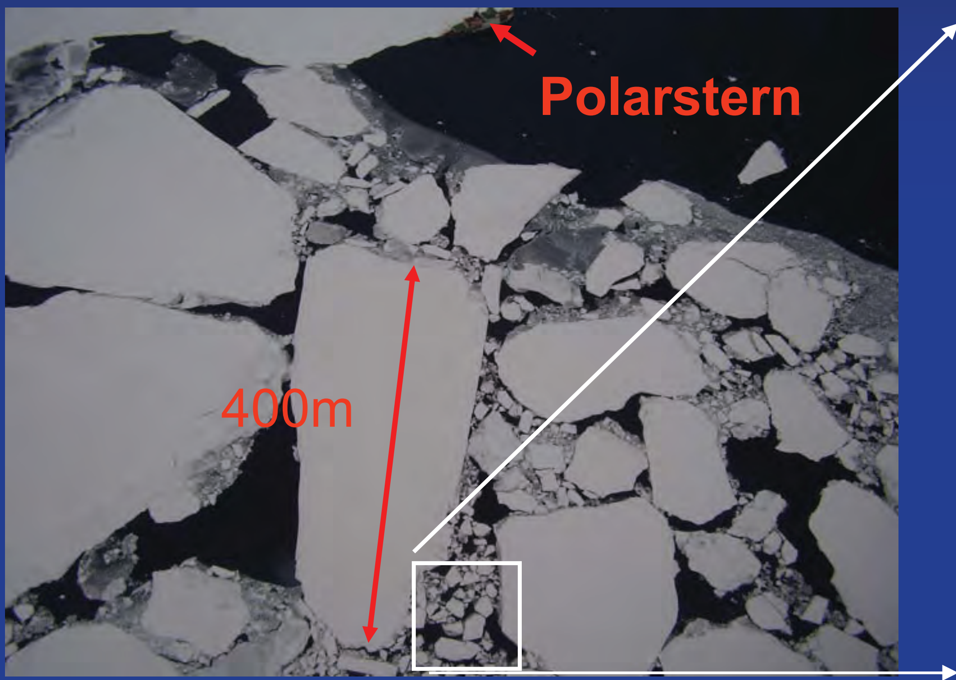


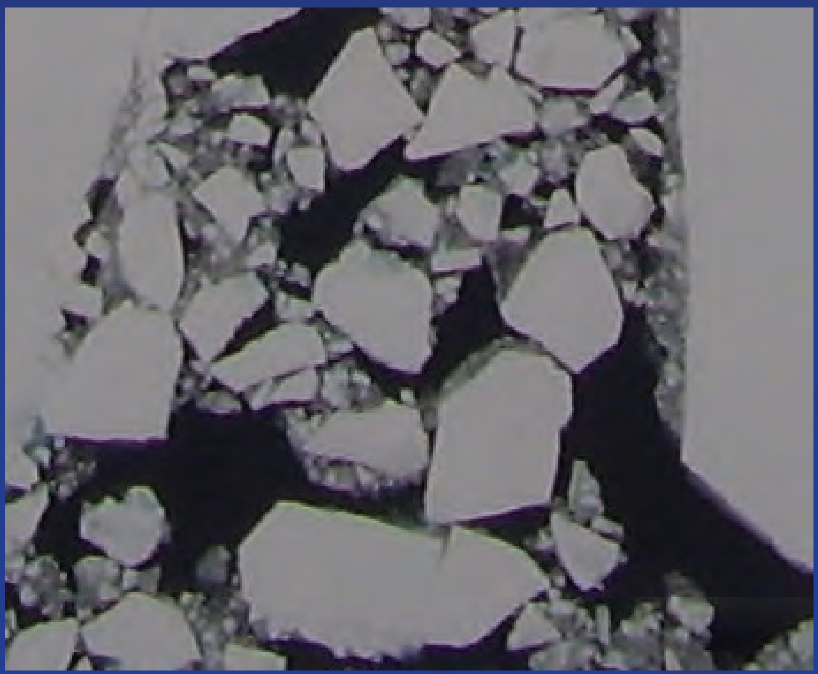
floe size distribution, area-perimeter relations, etc. important in dynamics (fracture), thermodynamics (melting)

Toyota, et al. Geophys. Res. Lett. 2006
Rothrock and Thorndike, J. Geophys. Res. 1984

Self-similarity of sea ice floes

Weddell Sea, Antarctica





fractal dimensions of Okhotsk Sea ice pack
smaller scales D~1.2, larger scales D~1.9

Toyota, *et al.* *Geophys. Res. Lett.* 2006
Rothrock and Thorndike, *J. Geophys. Res. 1984*

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

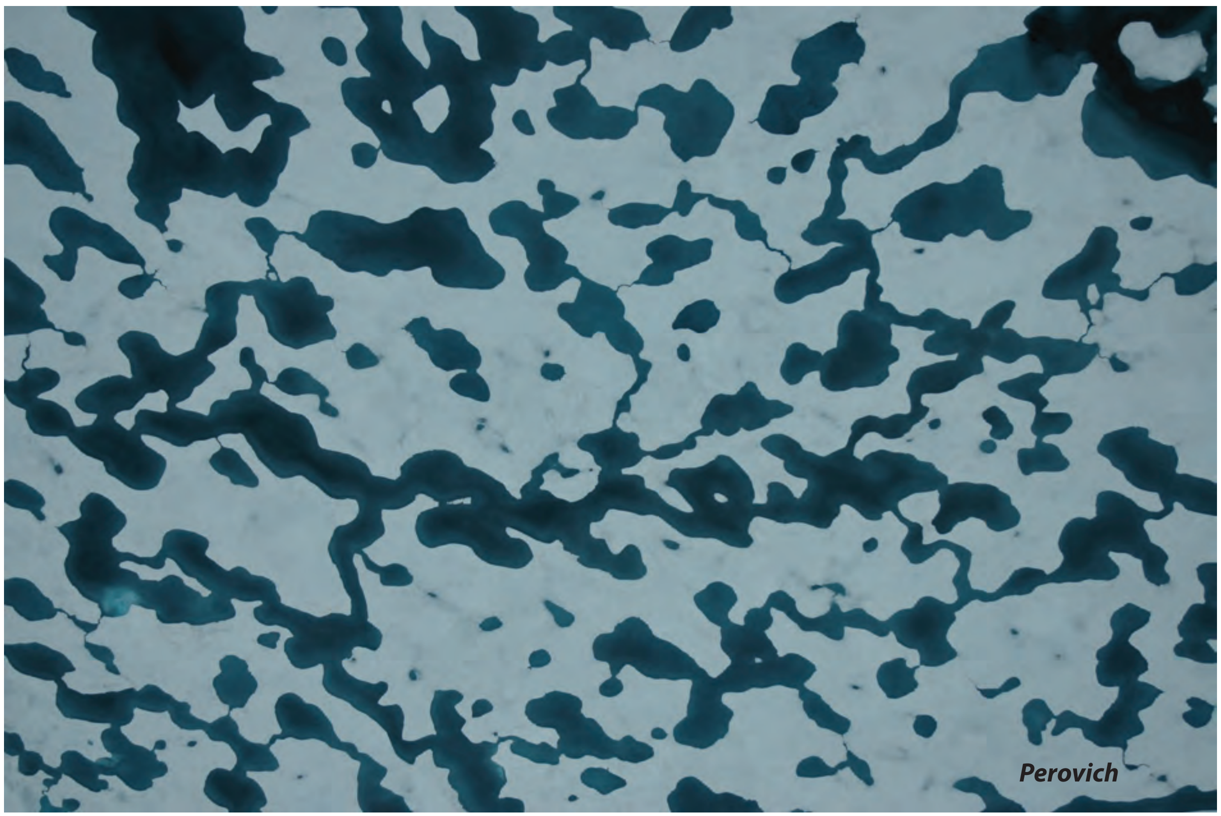
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

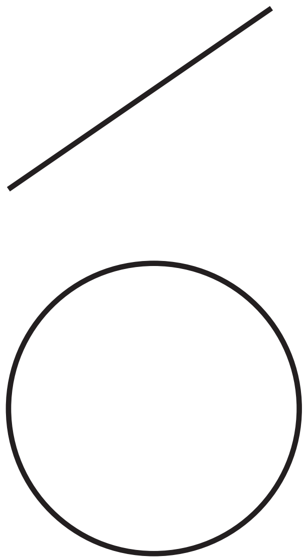
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

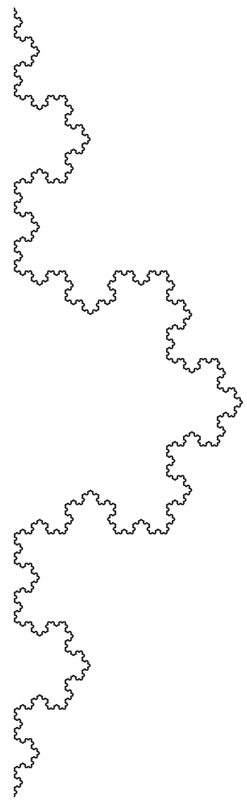
fractal curves in the plane

they wiggle so much that their dimension is >1



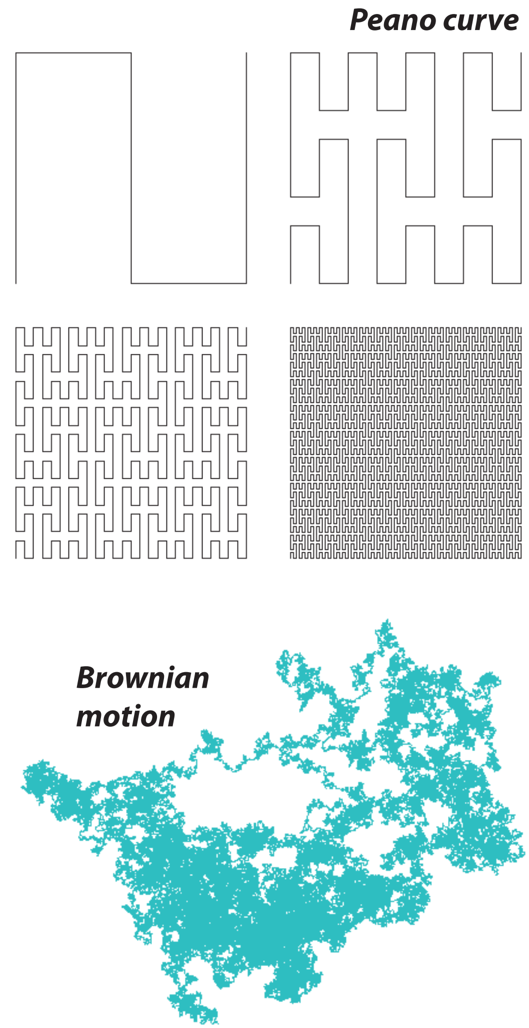
simple curves

$D = 1$



Koch snowflake

$D = 1.26$



space filling curves

$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

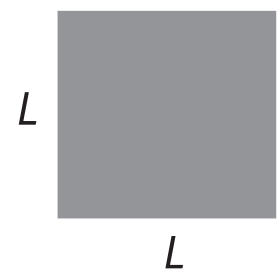
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

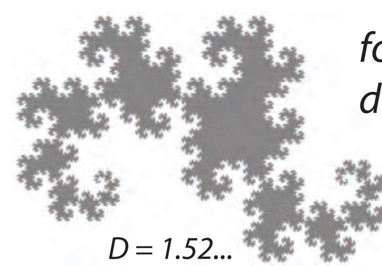
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

$$P \sim \sqrt{A}^{\,D}$$

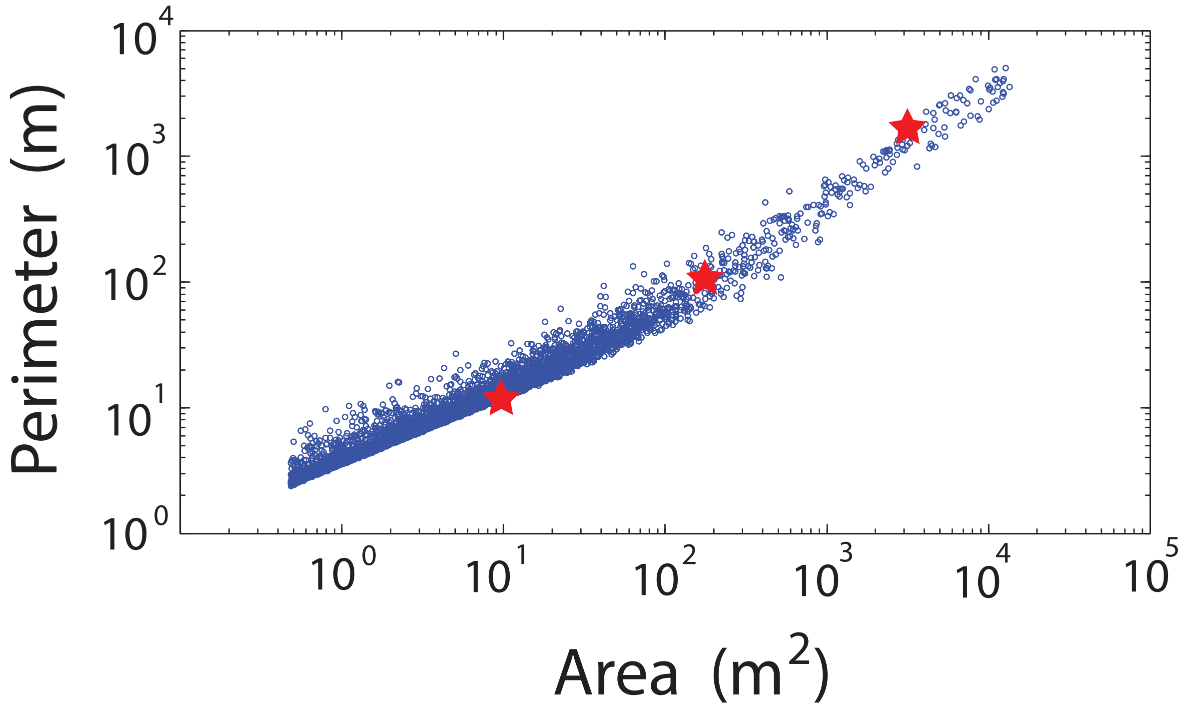


for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds

The Cryosphere, 2012

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden



data from 5269 Arctic melt ponds

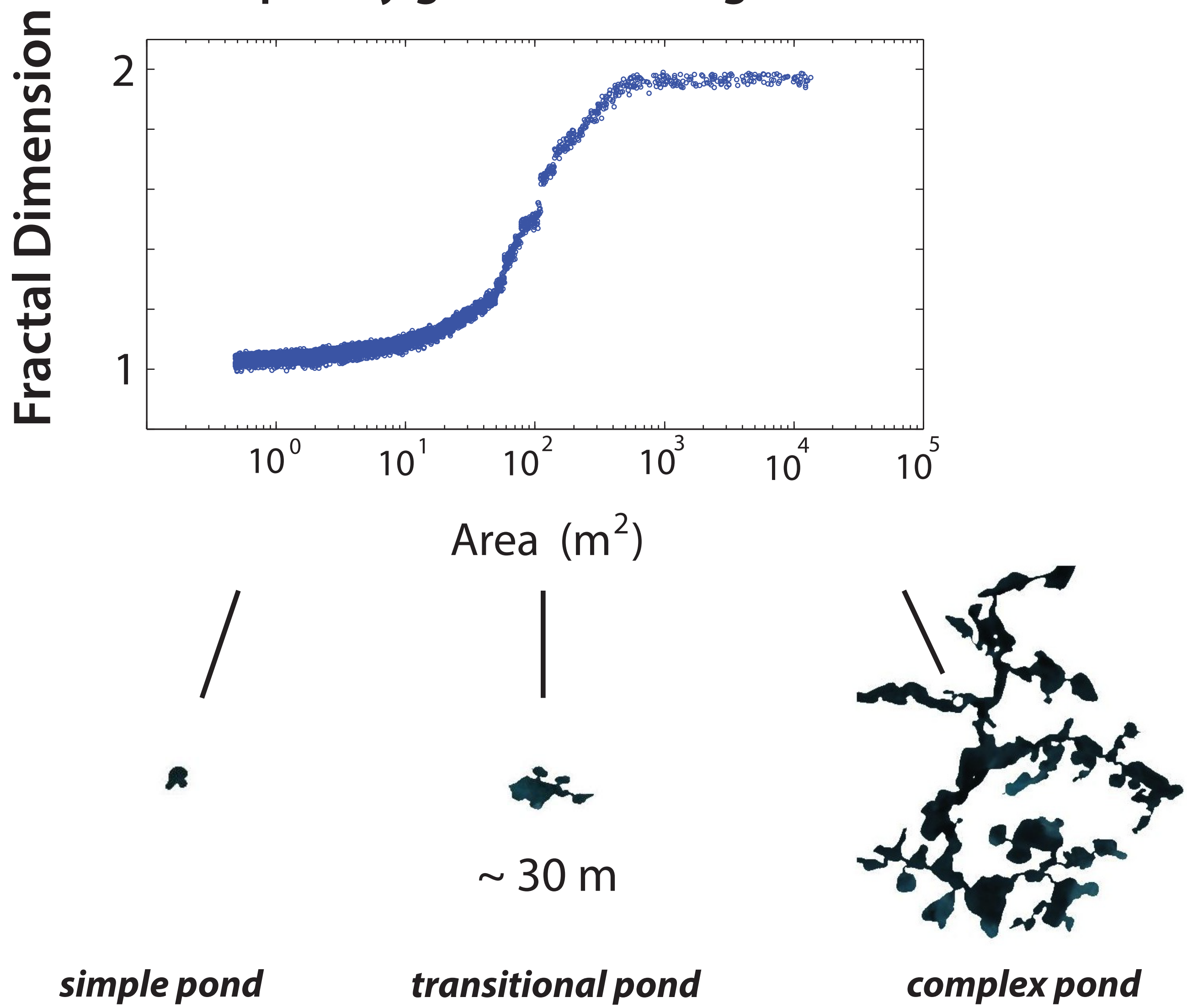
simple pond ***transitional pond*** ***complex pond***

Transition in the fractal geometry of Arctic melt ponds

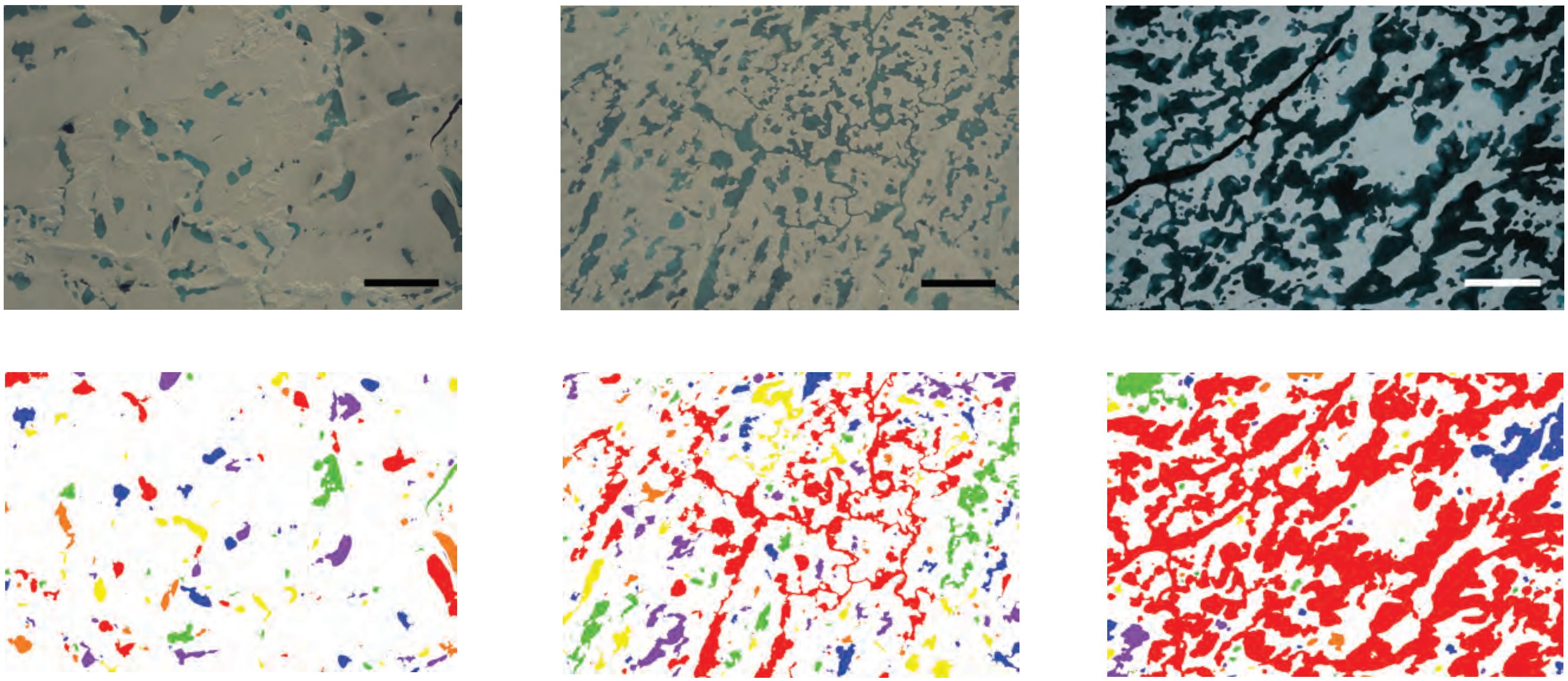
Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

complexity grows with length scale



small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

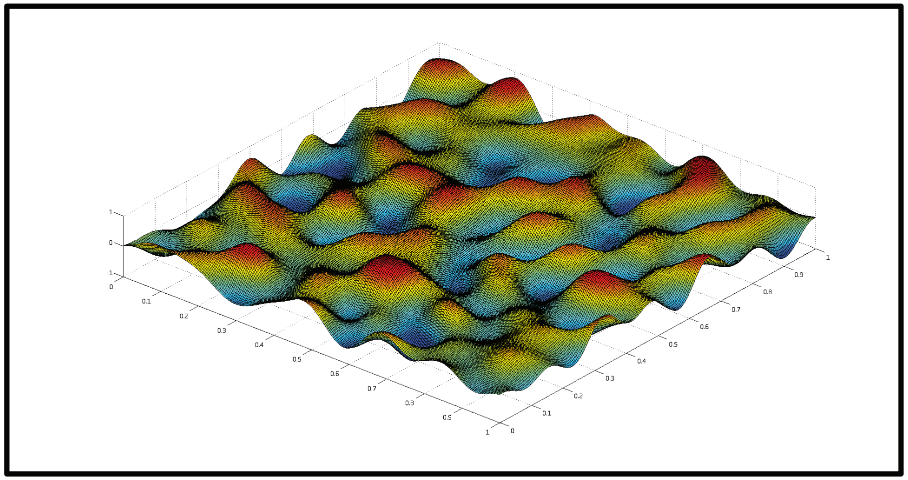
results on percolation threshold, correlation length, cluster behavior

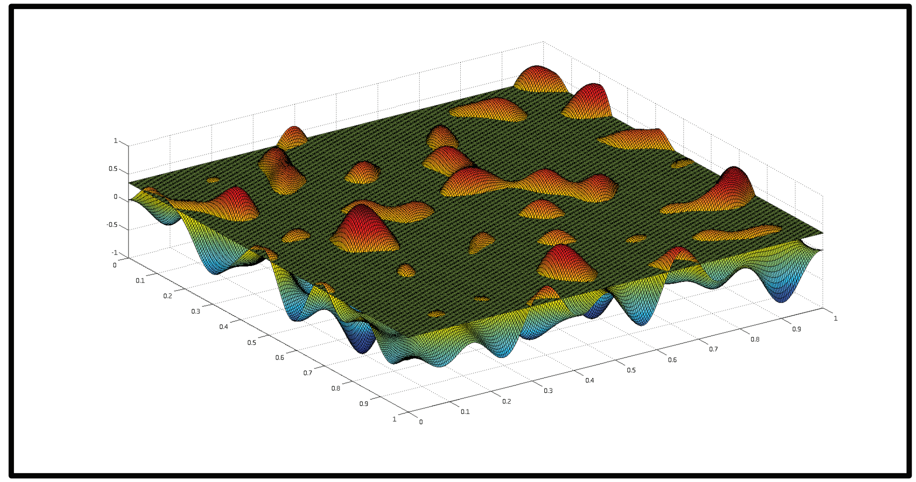
Anthony Cheng (Hillcrest HS), Dylan Webb (Skyline HS), Court Strong, Ken Golden

Continuum percolation model for melt pond evolution

level sets of random surfaces

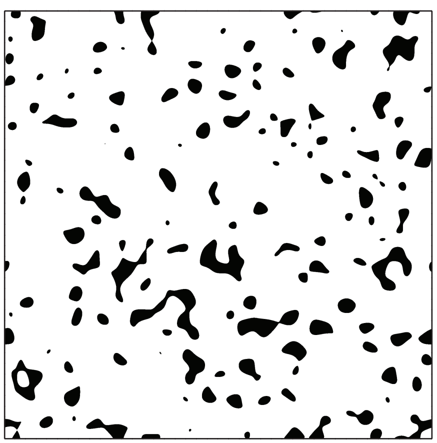
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

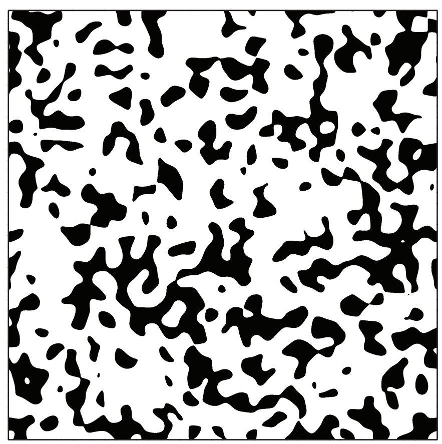


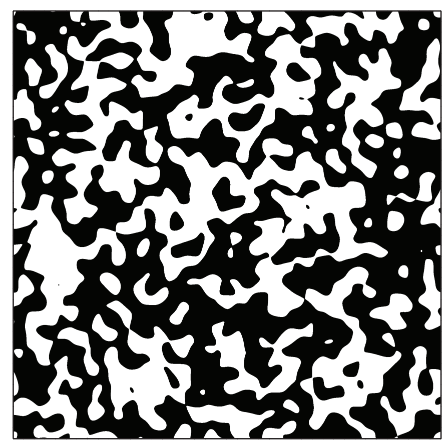


random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds





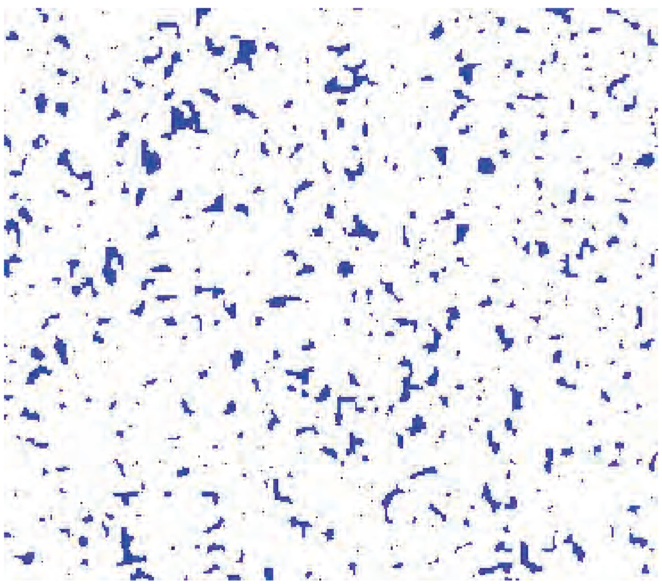


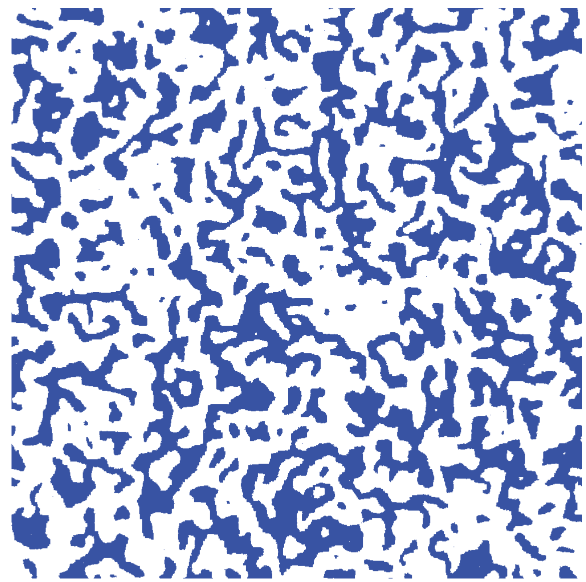
electronic transport in disordered media   *diffusion in turbulent plasmas*   *Isichenko, Rev. Mod. Phys., 1992*

Ising model for ferromagnets $\longrightarrow$ Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

magnetization $M = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$





"melt ponds" are clusters of magnetic spins that align with the applied field

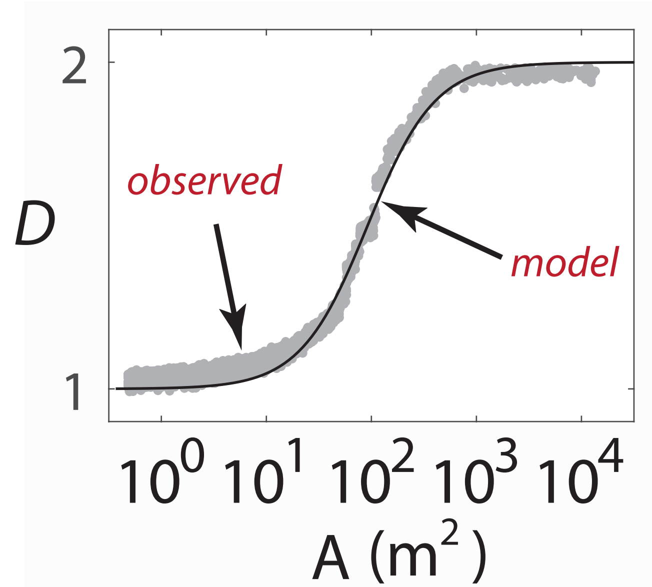
predictions of fractal transition, pond size exponent **Ma, Sudakov, Strong, Golden 2018**

Ising model results

Minimize Ising Hamiltonian energy

Random magnetic field represents snow topography; interaction term represents horizontal heat transfer.

*Melt ponds —
metastable islands of like spins
in our random field Ising model.*

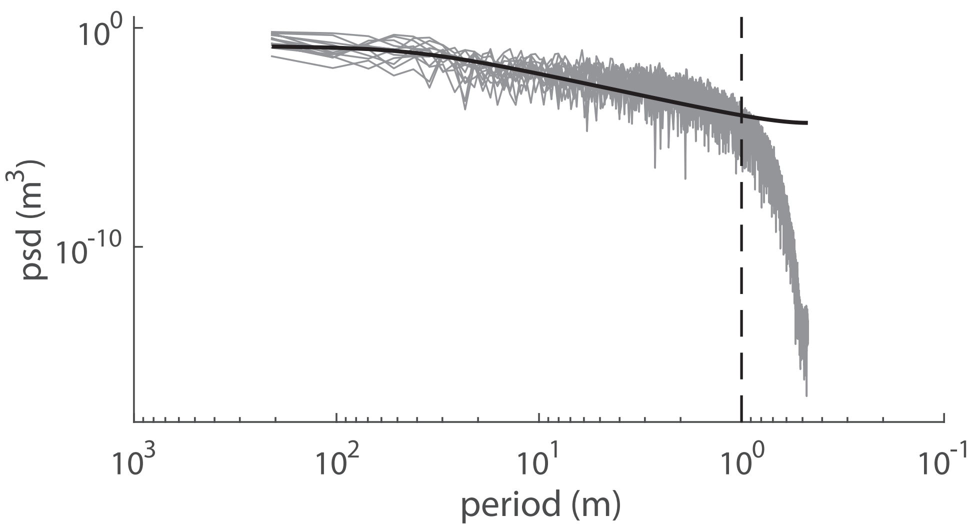


pond size distribution exponent

observed ***-1.5***
(Perovich, et al 2002)

model ***-1.58***

order out of disorder



The lattice constant *a* must be small relative to the 10-20 m length scales prominent in sea ice and snow topography. We set *a*=1 m as the length scale above which important spatially correlated fluctuations occur in the power spectrum of snow topography.

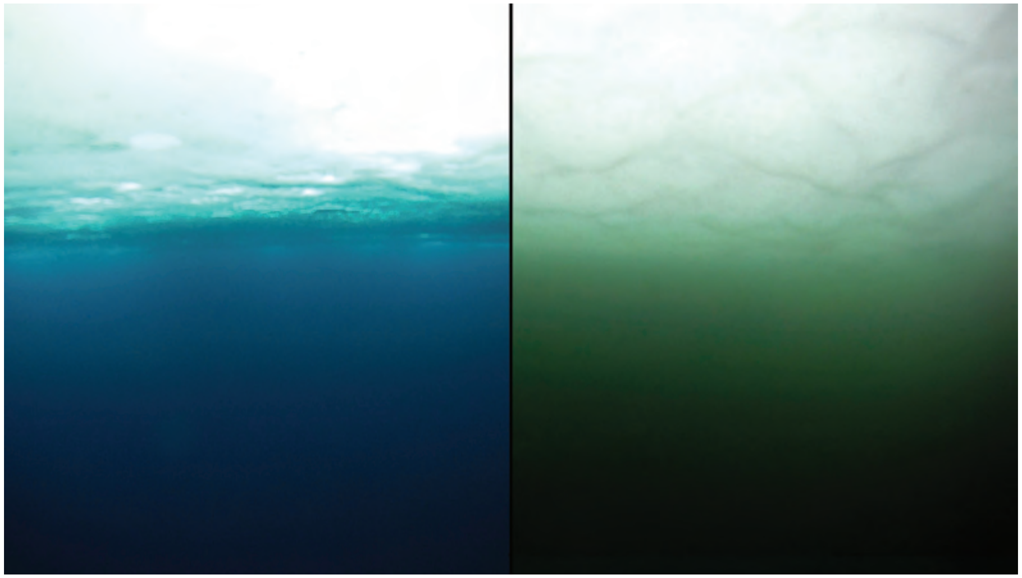
2011 massive under-ice algal bloom

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light through sea ice



no bloom bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

(2015 AMS MRC, Snowbird)

The Melt Pond Conundrum:

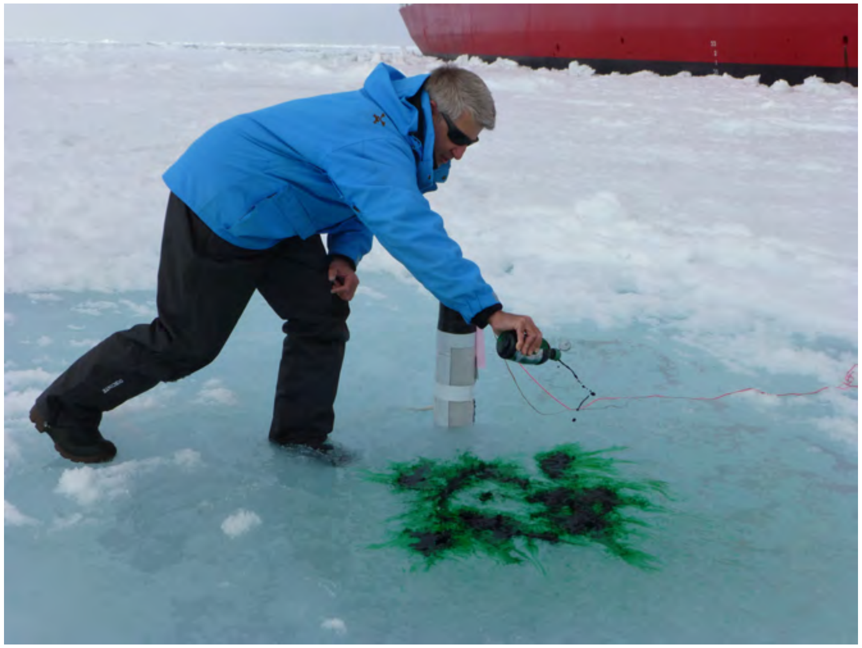
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy



Conclusions

1. Summer Arctic sea ice is melting rapidly, and melt ponds and other processes must be accounted for in order to predict melting rates.

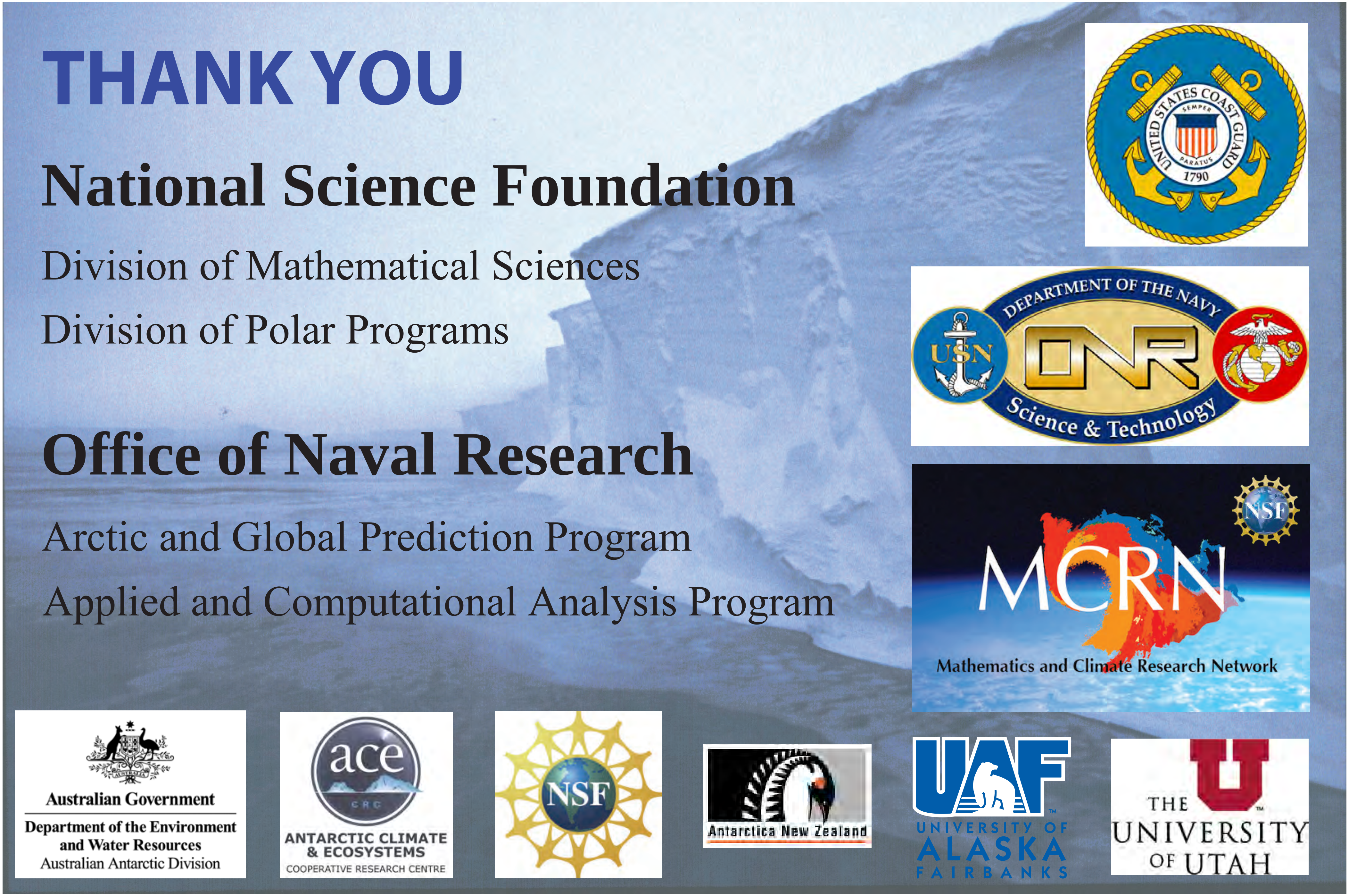
2. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.

3. Statistical physics and homogenization help *link scales*, provide rigorous methods for finding effective behavior, and advance how sea ice is represented in climate models.

4. Critical behavior (in many forms) is inherent in the climate system.

5. Field experiments are essential to developing relevant mathematics.

6. Our research will help to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.



Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999