

Special Issue on the Mathematics of Planet Earth

*In honor of Earth Day on April 22, we present articles on food security, sustainability, resource estimation and management, sea ice modeling, and more in this **special issue!***

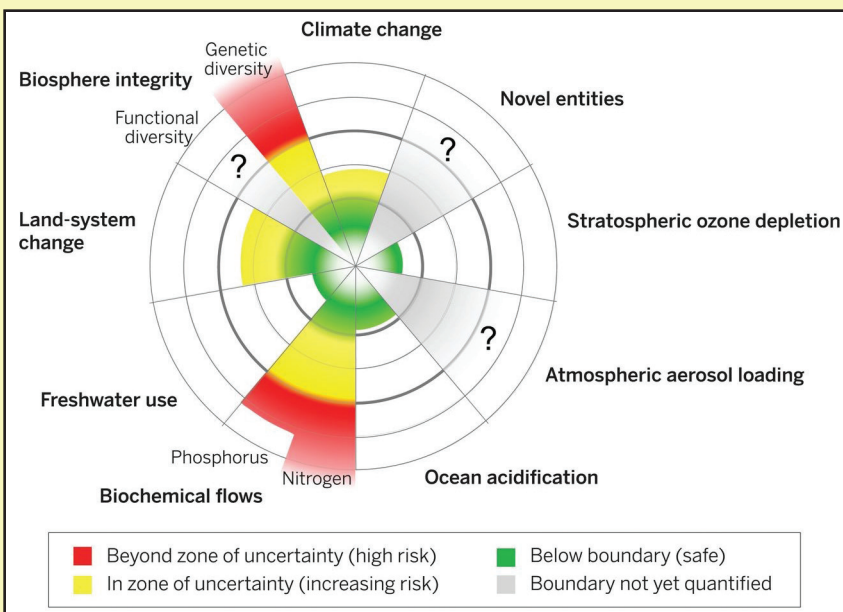


Figure 1. Estimated status of the control variables for seven of the planetary boundaries. Image courtesy of Steffen et al.

In an article on page 5, Hans Kaper and Mary Lou Zeeman illustrate how mathematical and computational skills can help model food systems.

Assessing Risks to Global Food Security

How Mathematicians Can Help

By James Case

The inaugural SIAM Conference on Mathematics of Planet Earth, held last September in Philadelphia, Pa., featured a public lecture by Molly Jahn of the University of Wisconsin, Madison (UW-Madison). Jahn, whose talk was entitled “Risks and Resilience in Global Food Systems: An Invitation for Mathematicians,” holds appointments in the Department of Agronomy, the Global Health Institute, and the Center for Sustainability and the Global Environment. She has served as dean of the university’s College of Agriculture and Life Sciences, director of the Wisconsin Agricultural Experiment Station, and Deputy and Acting Under Secretary of Research, Education, and Economics at the U.S. Department of Agriculture.

Jahn began her lecture by conceding that the current agricultural establishment (farmers, agribusinesses, and the agricultural research community) has been “stunningly successful” in improving agricultural productivity and efficiency. How else could we possibly be feeding a global population that

has grown from under 2 billion to over 7 billion in the last century? She hastened to add, however, that the existing food delivery system is by no means ideal. It leaves some 800 million people undernourished, while 1.5 billion are overweight or obese. Meanwhile, estimates indicate that 1.4 billion tons of food are wasted each year. Though this is a small fraction of the total quantity produced, it is still significant – more than enough to feed the 1.4 billion people subsisting on \$1.25 per day, or the 1.5 billion people who reside on degrading land. According to the Commission on Sustainable Agriculture and Climate Change (CSACC), more than 30 million acres of agricultural land are degraded each year due to overgrazing and other poor agricultural practices, climate change, groundwater depletion, urban sprawl, and additional human activities.

Cropland degradation, however, is not the only way in which current practices are overtaxing the planet. According to Jahn, the historic focus of research and intensive inputs

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Filling the Sea Ice Data Gap with Harmonic Functions

A Mathematical Model for the Sea Ice Concentration Field in Regions Unobserved by Satellites

*By Courtenay Strong and
Kenneth M. Golden*

Sea ice is frozen seawater that forms on the ocean surface in the Arctic basin and around the continent of Antarctica. Sea ice packs cover millions of square kilometers of our planet's surface and provide a habitat crucial to a diverse array of microorganisms, small crustaceans, marine birds, and mammals. Observed declines in sea ice amounting to approximately half a million square kilometers per decade are impacting global climate and ecosystems, and positive *sea ice-albedo feedback* is accelerating melting [2]. Sea ice has a very high albedo, meaning that it reflects most of the incoming sunlight. Declining ice coverage due to melting results in more solar energy entering the climate system, which leads to more warming and hence more melting. In fact, the September minimum of Arctic sea ice

extent dropped to about 3.4 million square kilometers in 2012, which is less than half of the 1979-2000 average value of approximately 7 million square kilometers.

Since 1972, the National Aeronautics and Space Administration has been monitoring sea ice using satellites that detect the small amounts of microwave radiation emitted by the ice. The satellites detect microwave emission through clouds during both day and night, and the resulting grids at 25-km horizontal resolution provide the most spatially-complete, long-term observational record of sea ice concentrations ($0 \leq c \leq 1$) over the polar regions in both hemispheres. Unfortunately, the orbit inclination and instrument swath of the passive microwave satellites leave a “polar data gap” around the North Pole where sea ice is not observed (see Figure 1). For many years, researchers assumed that this northernmost region of the Arctic was always covered

with sea ice. However, recent precipitous losses in the polar ice pack [1] call into question this assumption, which can significantly affect overall estimates of Arctic sea ice volume. By way of anecdotal evidence, the past two Decembers (2015 and 2016) have seen freakishly warm temperatures around the North Pole, with periods of almost 50 degrees Fahrenheit above average. Such dramatic changes motivate development of an *objective* method for estimating unobserved concentrations within the gap.

We propose [6] a partial differential equation-based model with tuned stochastic spatial heterogeneity to estimate the concentrations within a region Ω on Earth's surface:

$$f(\theta, \phi) = \psi(\theta, \phi) + W(\theta, \phi),$$

where θ is longitude and ϕ is latitude, or $f(\vec{r}) = \psi(\vec{r}) + W(\vec{r})$, where $\vec{r} \in \Omega$. We

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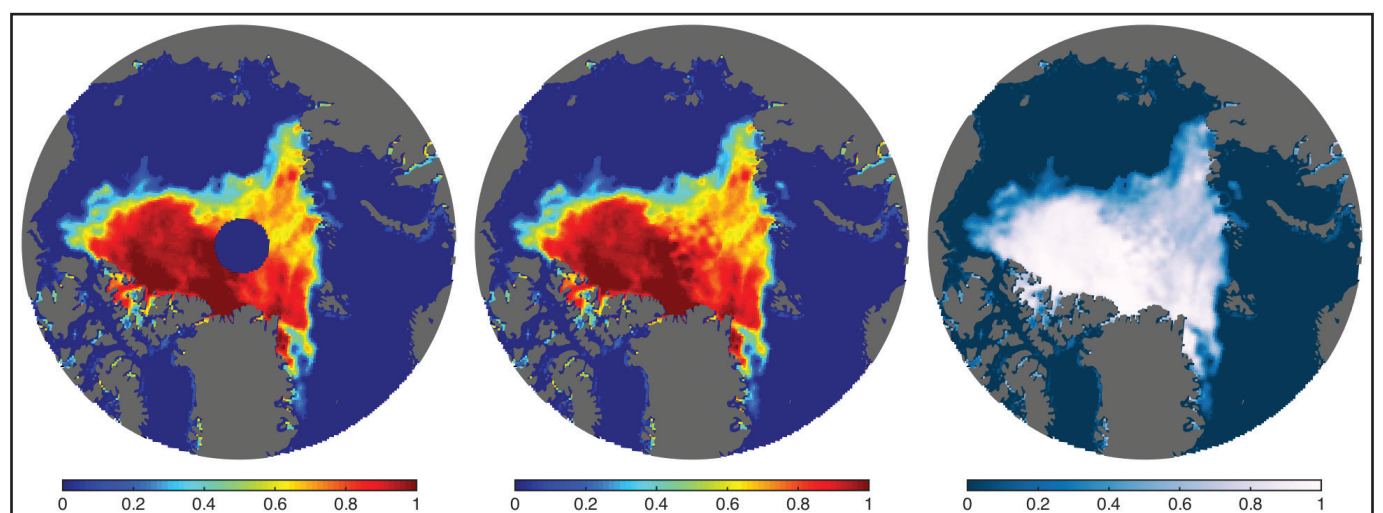


Figure 1. The left image is an example of the polar data gap (dark blue disc) on August 30, 2007, with shading outside the disc indicating concentration. The middle and right images show the data fill presented here; the color shading at right is similar to that used by the National Snow and Ice Data Center (<http://nsidc.org>). Image adapted from [6].