

# *Climate Change and the Melting Polar Ice Caps*

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*Department of Mathematics  
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# ANTARCTICA

southern cryosphere



Antarctic Peninsula

Weddell Sea

Ronne Ice Shelf

West Antarctic Ice Sheet

Ross Ice Shelf

Ross Sea

sea ice

Queen Maud Land

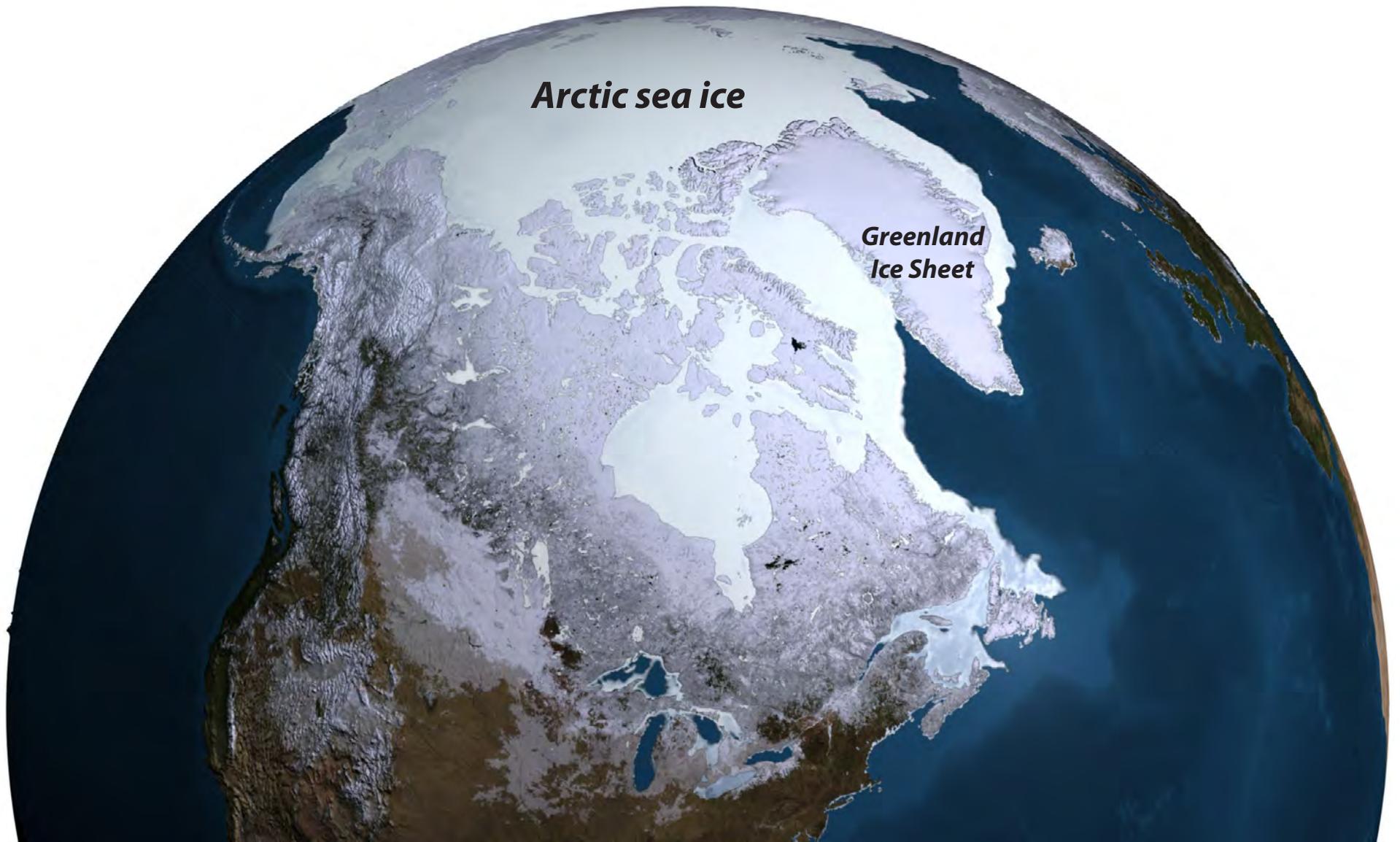
East Antarctic Ice Sheet

Wilkes Land

South America

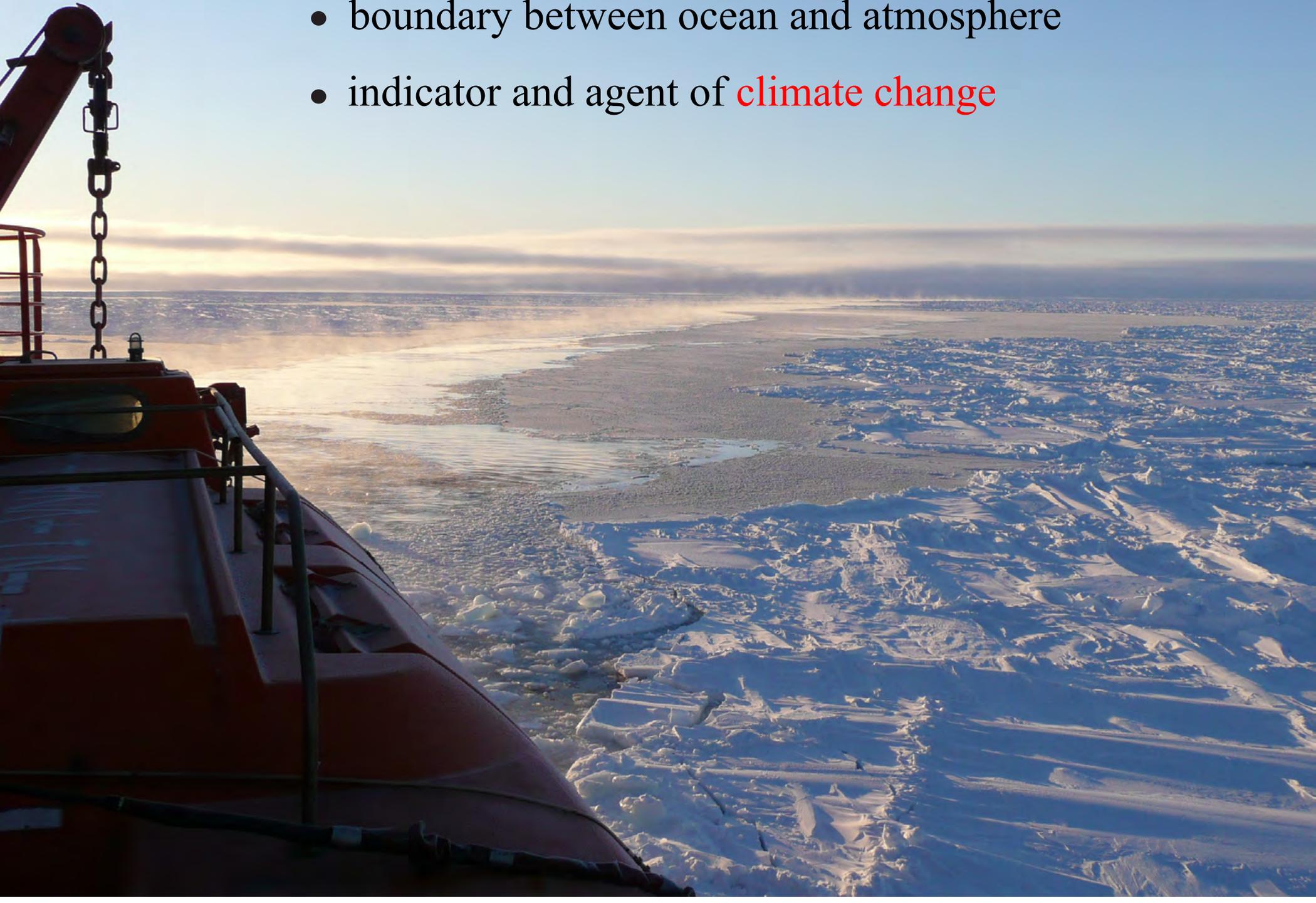
Australia

# northern cryosphere



SEA ICE covers 7 - 10% of earth's ocean surface

- boundary between ocean and atmosphere
- indicator and agent of **climate change**



# sea ice formation



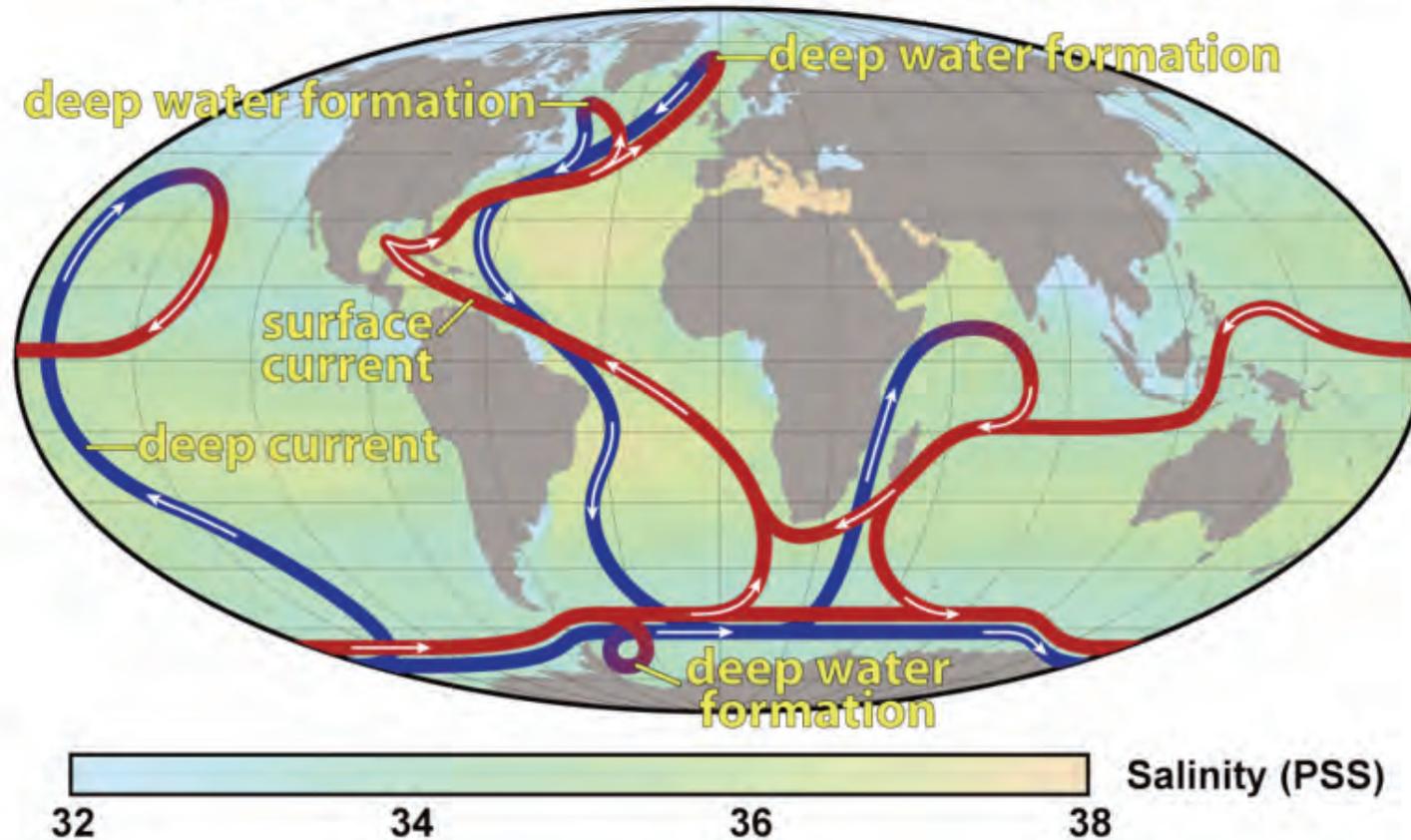
**brine expulsion from sea ice formation** results in water beneath the ice becoming cooler and saltier



**this denser water sinks rapidly to great depths**

**deep-water formation drives circulation in the world's oceans**

## Thermohaline Circulation

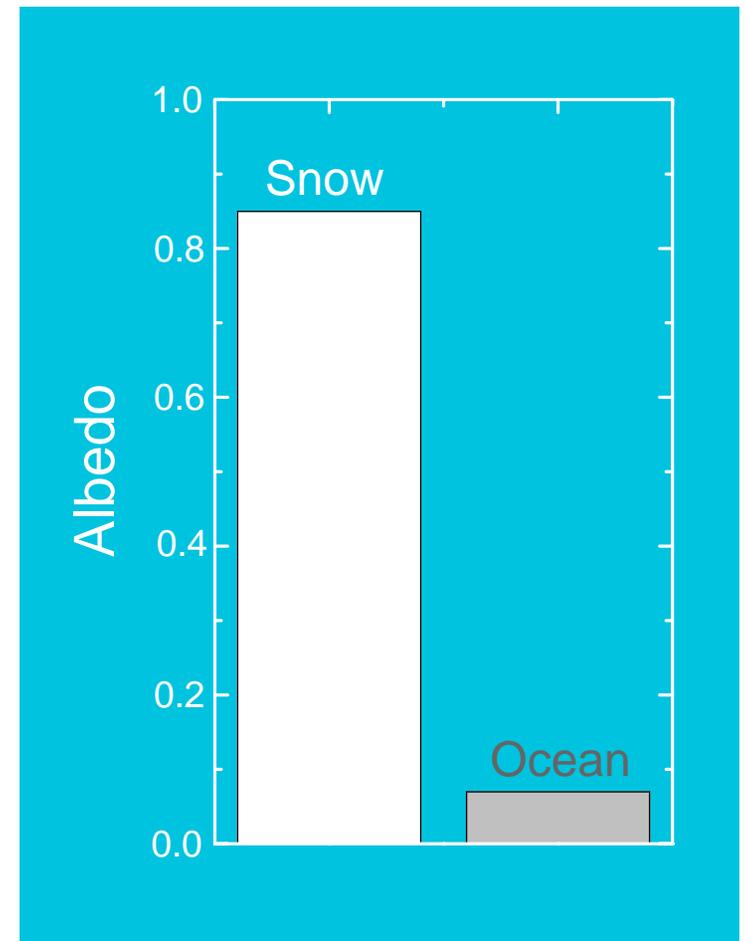


**GLOBAL THERMOHALINE CONVEYOR BELT**

polar ice caps critical to global climate  
in reflecting incoming solar radiation



white snow and ice  
reflect



dark water and land  
absorb

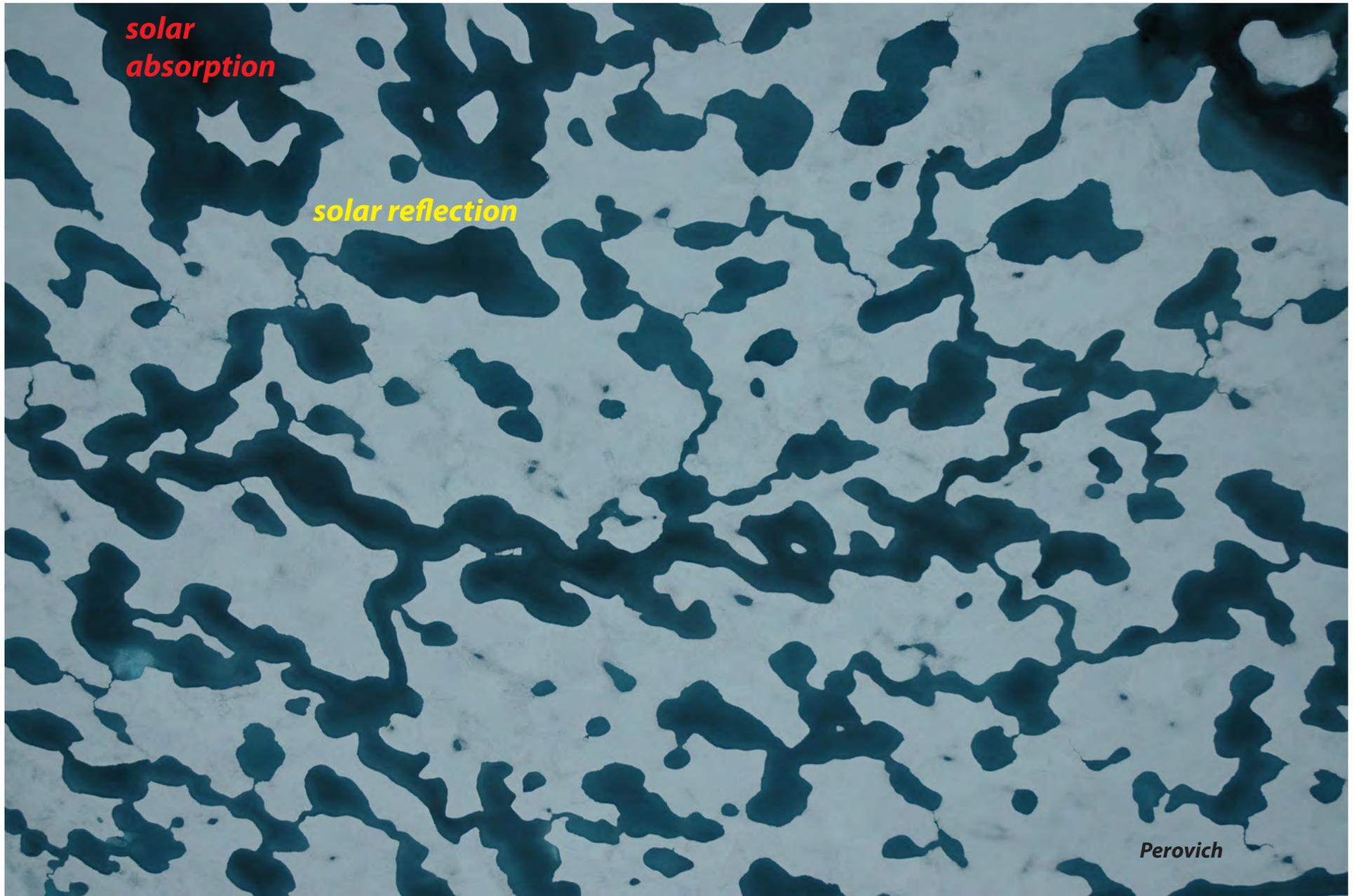
# *first year Antarctic sea ice*



*solar reflection*

*solar absorption*

# *Arctic melt ponds*



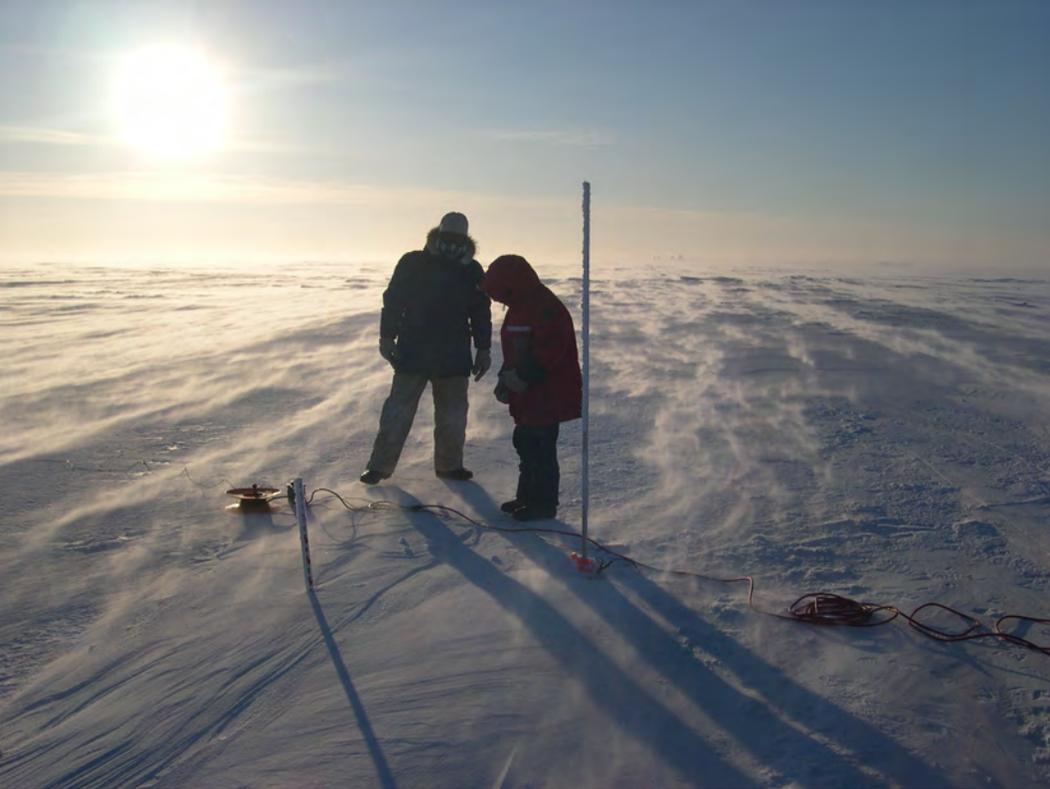
*solar  
absorption*

*solar reflection*

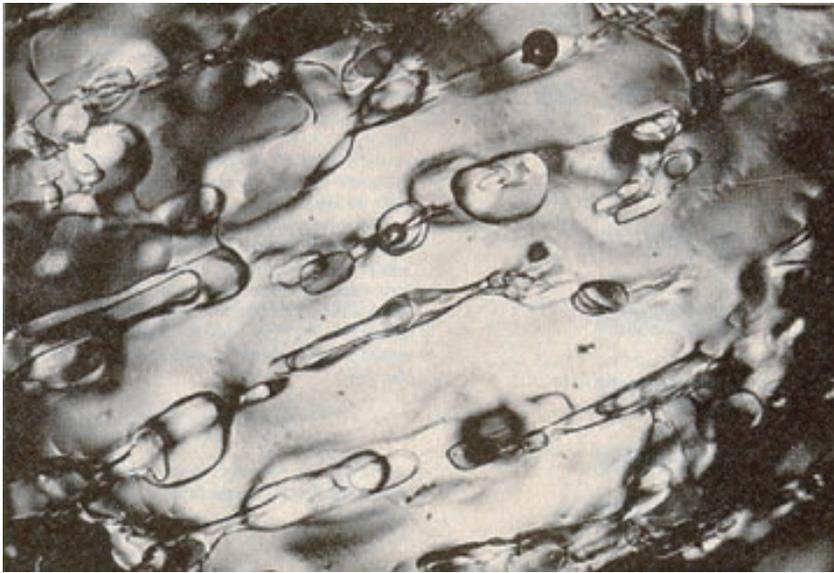
*Perovich*

*melt pond formation and albedo evolution -- major drivers in polar climate*

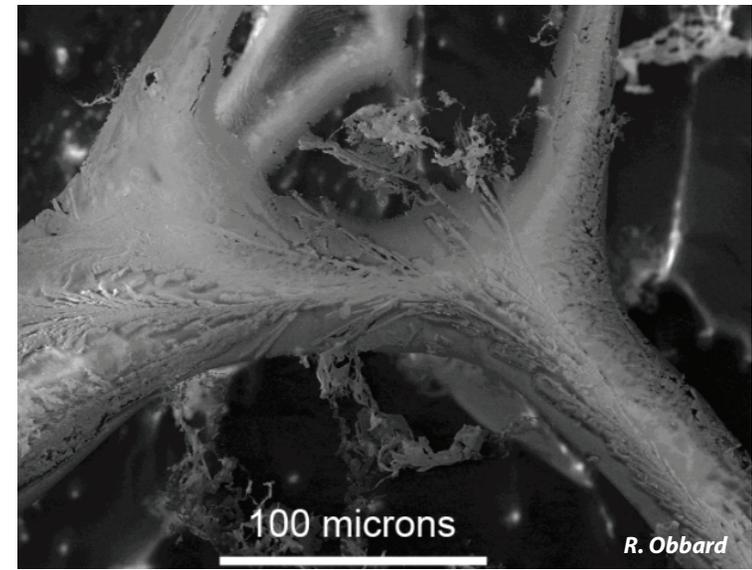
*key challenge for global climate models*



*sea ice may appear to be a barren, impermeable cap ...*



**brine inclusions in sea ice (mm)**



**micro - brine channel (SEM)**

***sea ice is a  
porous composite***

pure ice with brine, air, and salt inclusions

**brine channels (cm)**



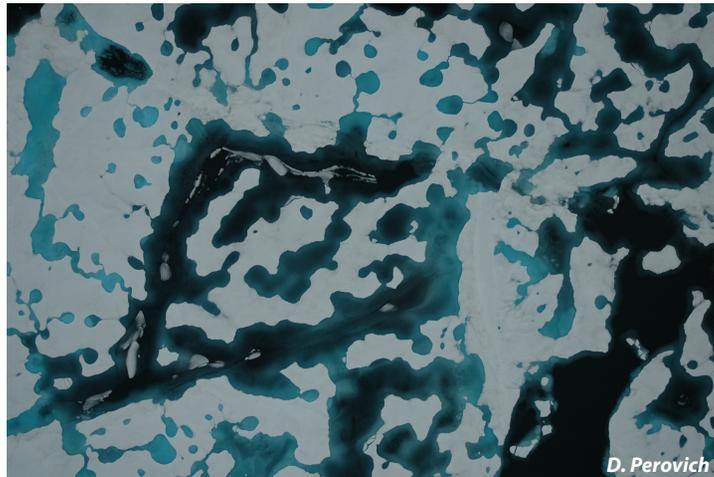
horizontal section



vertical section

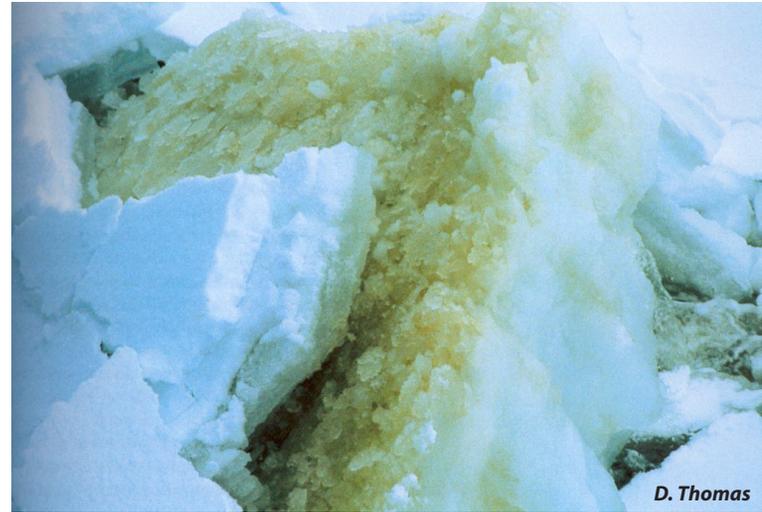
# ***fluid flow through porous sea ice mediates key processes in polar climate and ecosystems:***

evolution of Arctic melt ponds



*D. Perovich*

algal communities



*D. Thomas*



ACE CRC



C. Krembs

- ***formation and melting of sea ice***
- ***ocean-ice-atmosphere exchanges of heat, brine, CO<sub>2</sub>***
- ***growth and decline of microbial communities***

# ***What is this talk about?***

***... using science and math to study sea ice structures and processes which will improve our understanding and projections of climate change.***

1. ***Sea ice in the climate system*** - Arctic decline & global climate models
2. ***Multiscale structure*** -- fractal behavior of sea ice pack and melt ponds
3. ***Fluid flow through sea ice*** - geophysical and biological processes  
microphysics and percolation
4. ***Electromagnetic remote sensing*** - monitoring climate change and  
key sea ice transport processes
5. ***Antarctic experiments*** on fluid and electrical transport in sea ice

***video and photos from 2007 & 2010 Antarctic expeditions***

# ***mathematical themes of the talk***

***composite structures*** - sea ice structures and processes exhibit composite behavior on many scales

***critical behavior*** - small changes in system parameters can induce large changes in behavior  
***(phase transitions)***

***linkage of scales*** - how do smaller scales influence larger scale behavior

***the math doesn't care  
if a "composite" has microstructure  
on millimeter, meter, or kilometer scale***

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Kyle Steffen (Math)  
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## Australian Antarctic Division

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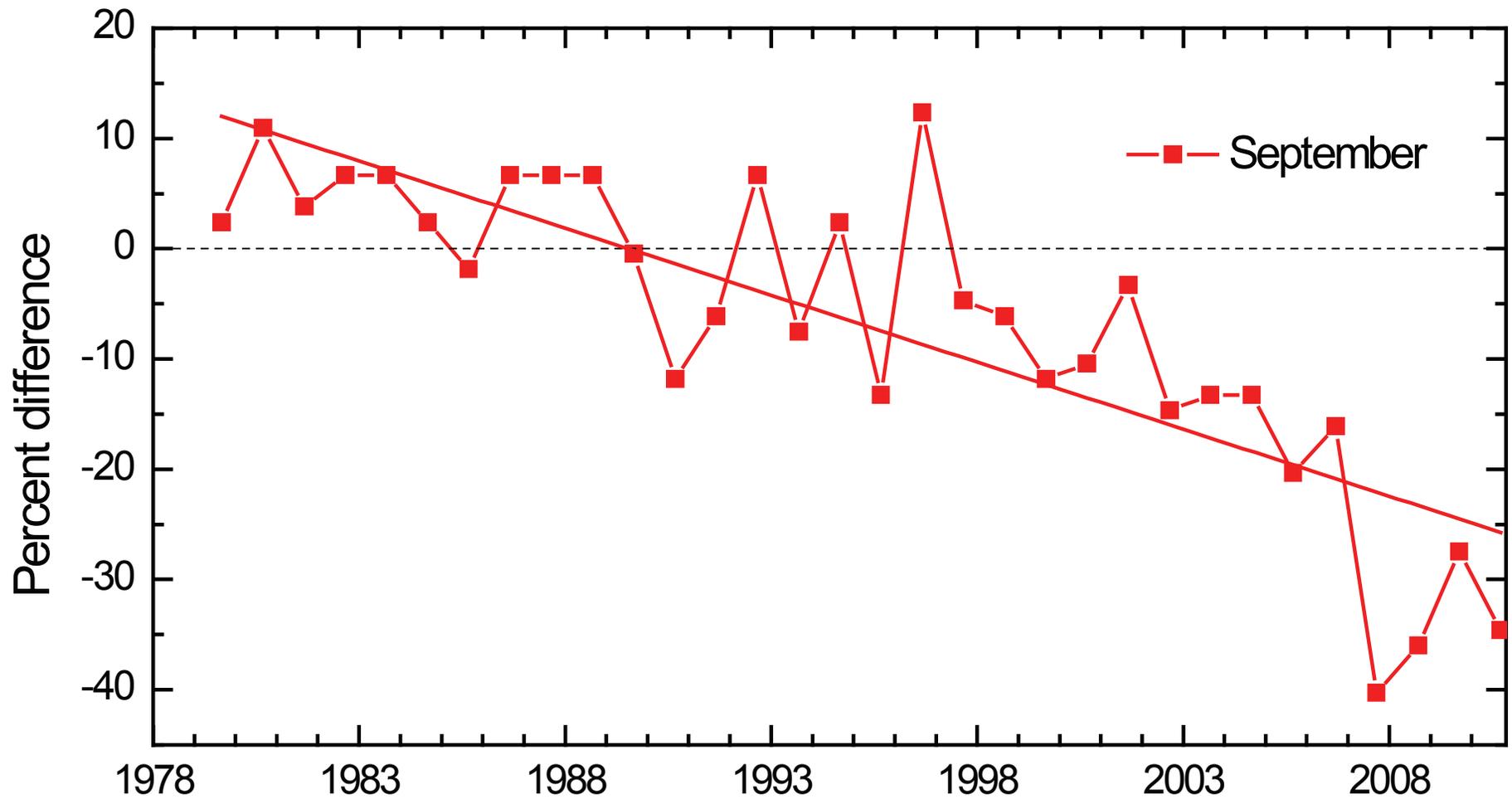
Don Perovich

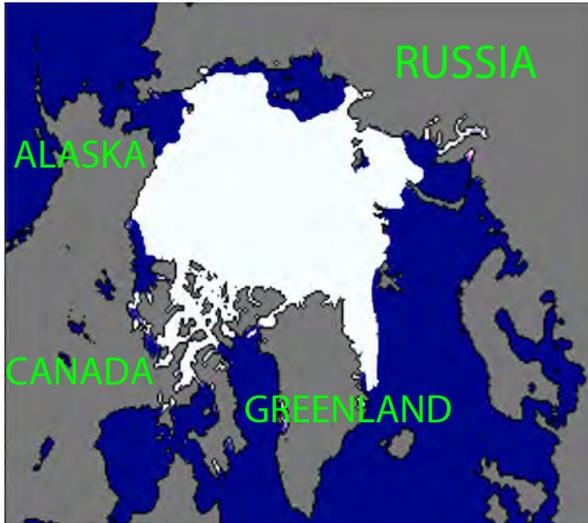




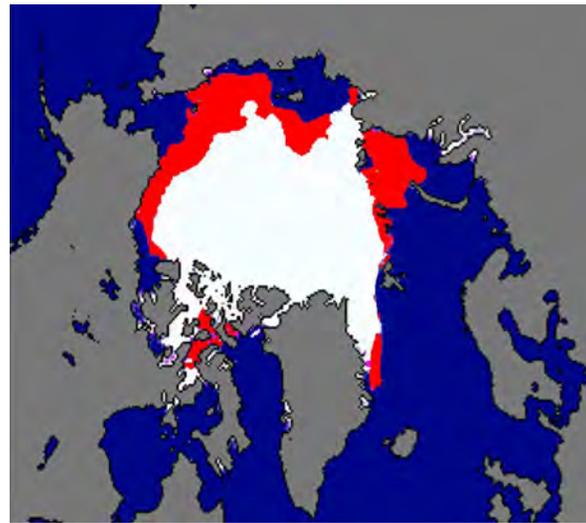
***Arctic meltdown and  
global climate models***

# Change in summer Arctic sea ice extent

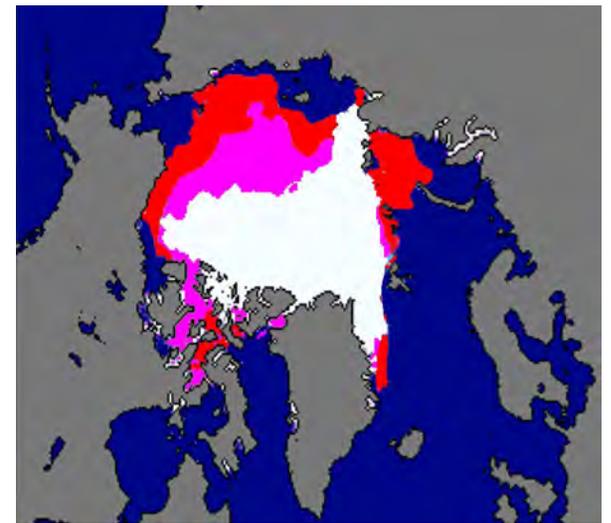




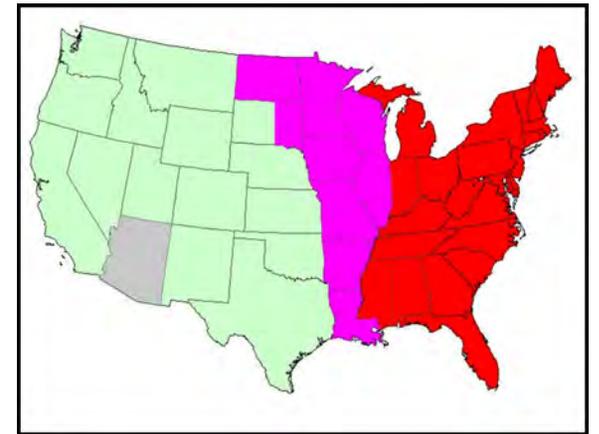
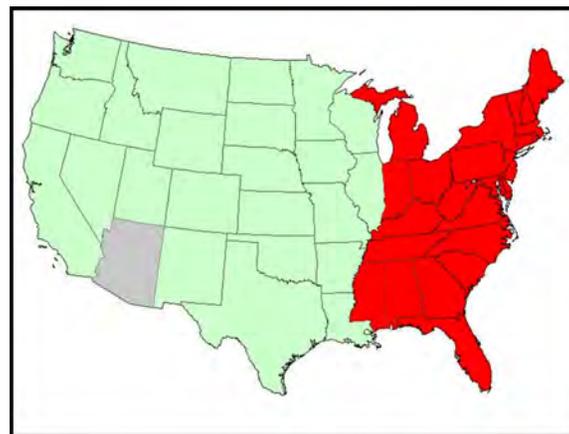
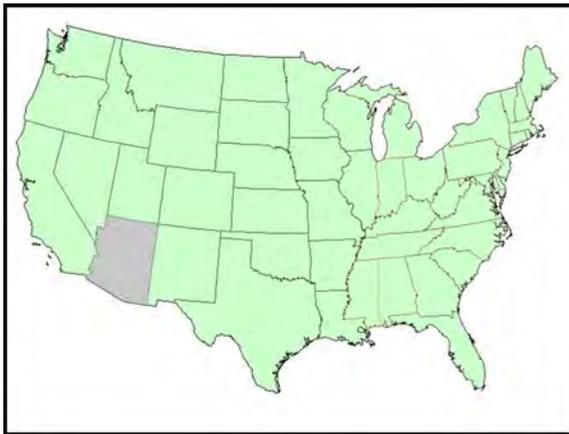
1980



2005



2007



# *thicker multiyear ice being replaced by thinner first year ice*

Winter 2004

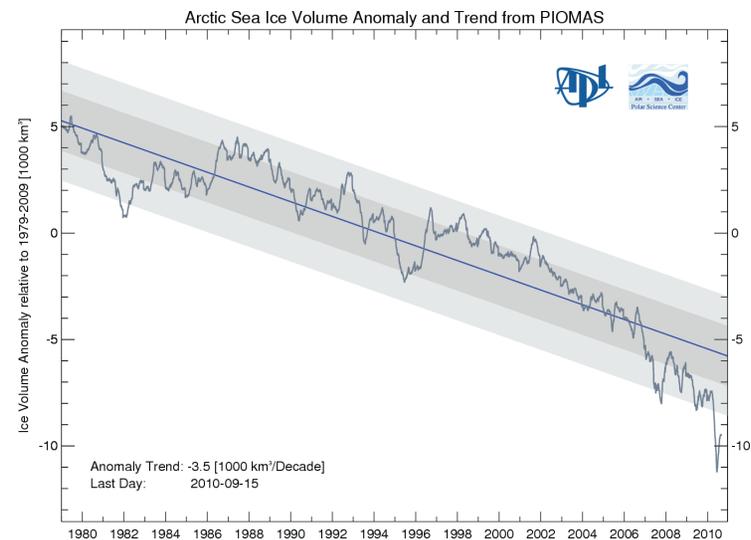


Winter 2008

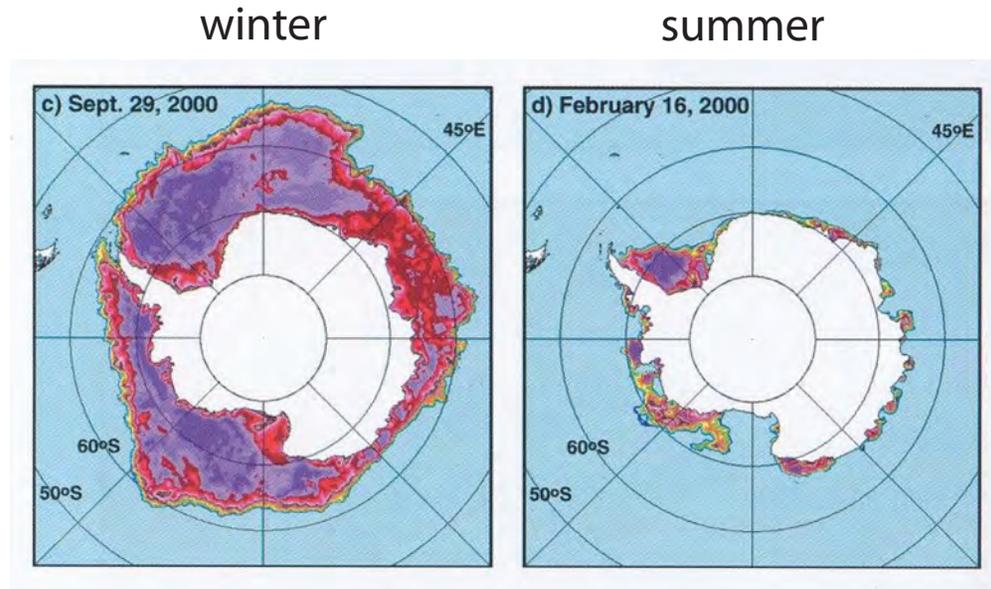


Son Nghiem

*... and sea ice volume is declining*



# *Antarctic sea ice pack is already seasonal*



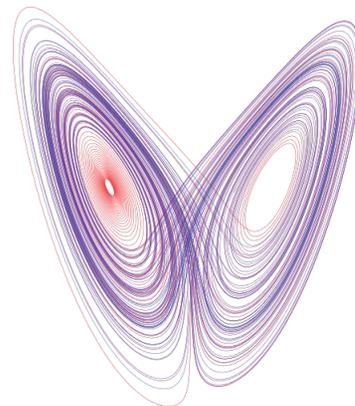
**Is global warming  
Antarctifying  
the Arctic ?**

# ***Can Arctic sea ice rebound?***

... or have we passed a **critical threshold**, a “tipping point” in a transition from an ice-covered to ice-free Arctic Ocean?

unlikely in transition from current perennial sea-ice conditions to seasonally ice-free

may be likely with further warming and sudden loss of wintertime-only ice cover



Lorenz butterfly

“I stand by my previous statements that the Arctic summer sea ice cover is in a **death spiral**. It’s not going to recover.”  
Mark Serreze, Director of the National Snow and Ice Data Center 9/20/10

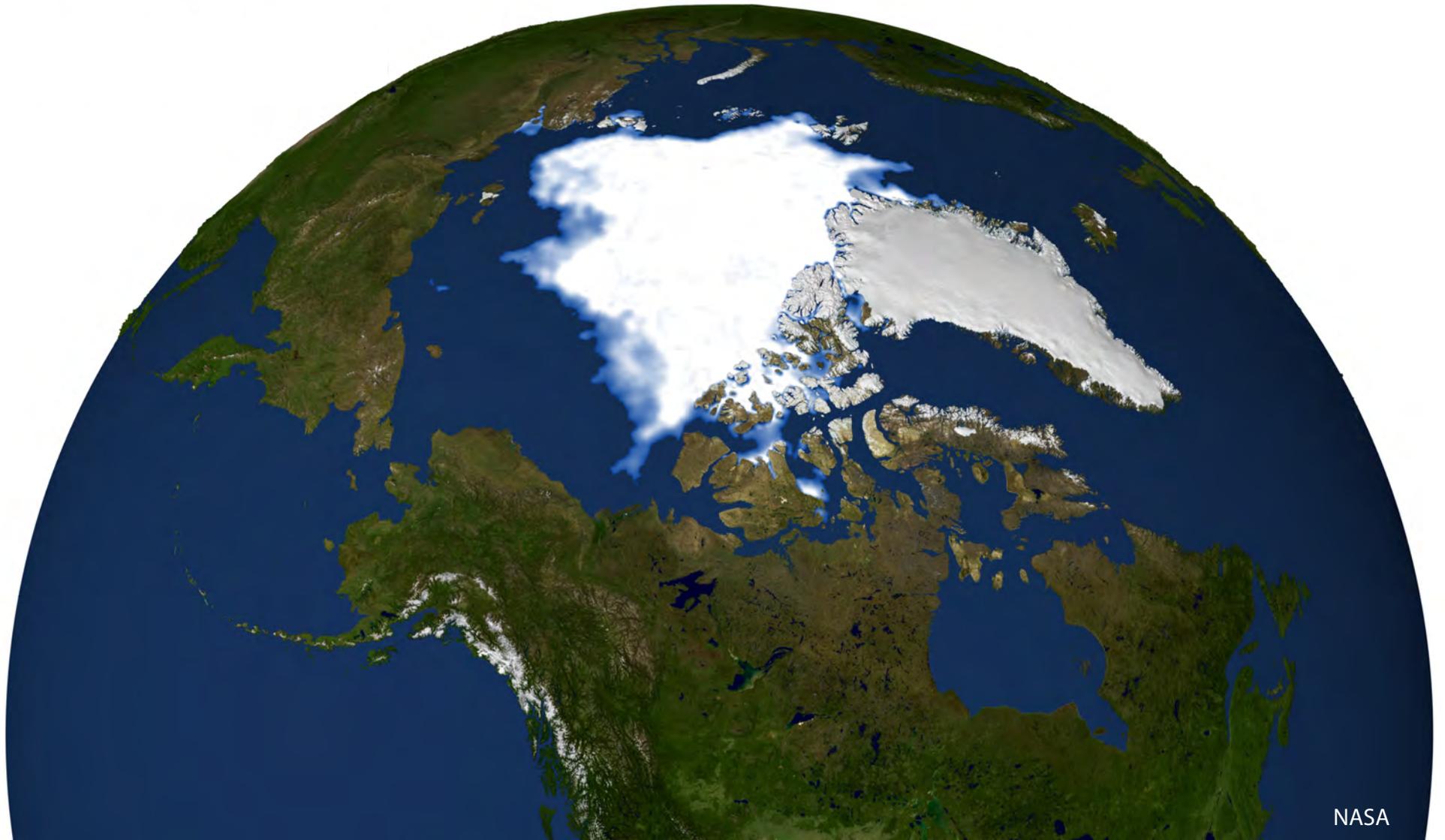
***Eisenman, Wettlaufer 2009***

***Notz 2009***

***Abbot, Silber, Pierrehumbert 2011***

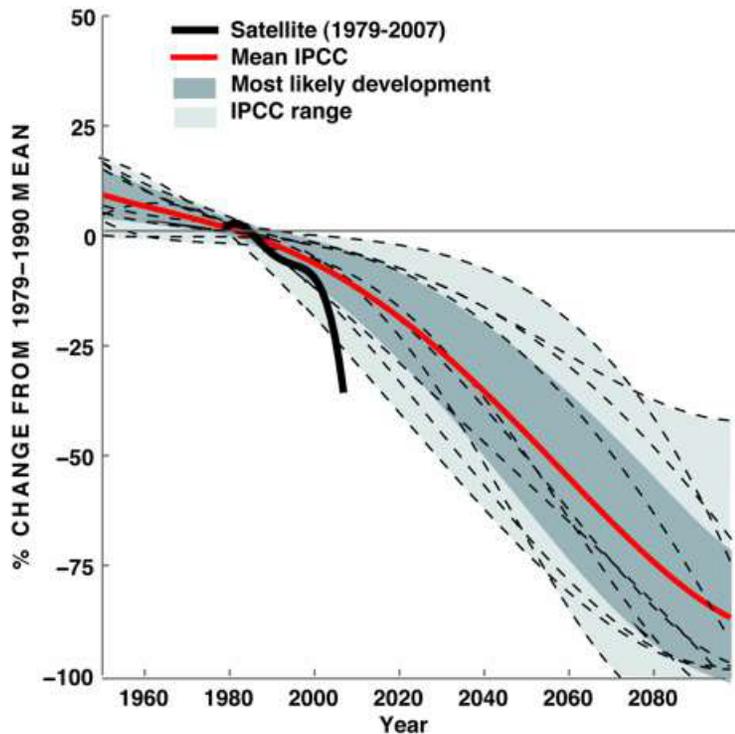
climate change is **amplified** in the polar regions

14 September 2008



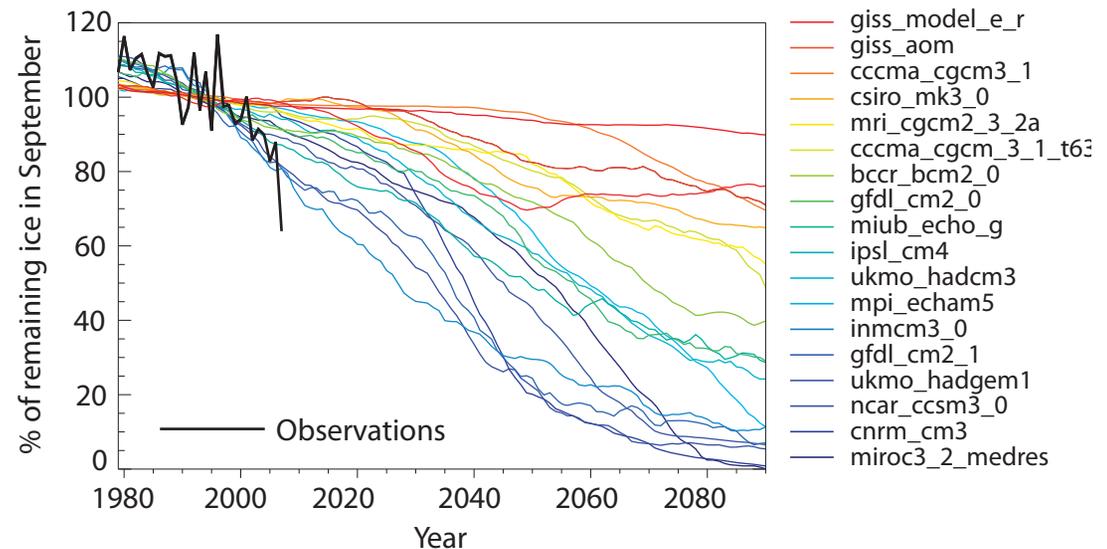
# IPCC (Intergovernmental Panel on Climate Change) projections

global climate models **underestimate** observed decline in summer Arctic sea ice extent



Arctic sea ice loss compared to IPCC models

September 2007



March 2009

Boé, Hall, Qu 2009

# What is essential in a numerical sea ice model for climate studies?

Simulate the climatological mean annual cycle of the ice and snow

Represent the sensitivity to perturbations - must have the key feedbacks

Physics appropriate to the model's spatial scale, parameterizations for sub-scale behaviors

# What are the key ingredients -- or ***governing equations*** that need to be solved on powerful computers?

1. Ice thickness distribution evolution equation (Thorndike et al. 1975)

*dynamics*  
+  
*thermodynamics*

*nonlinear PDE incorporating ice velocity field, ice growth and melting  
mechanical redistribution - ridging and opening*

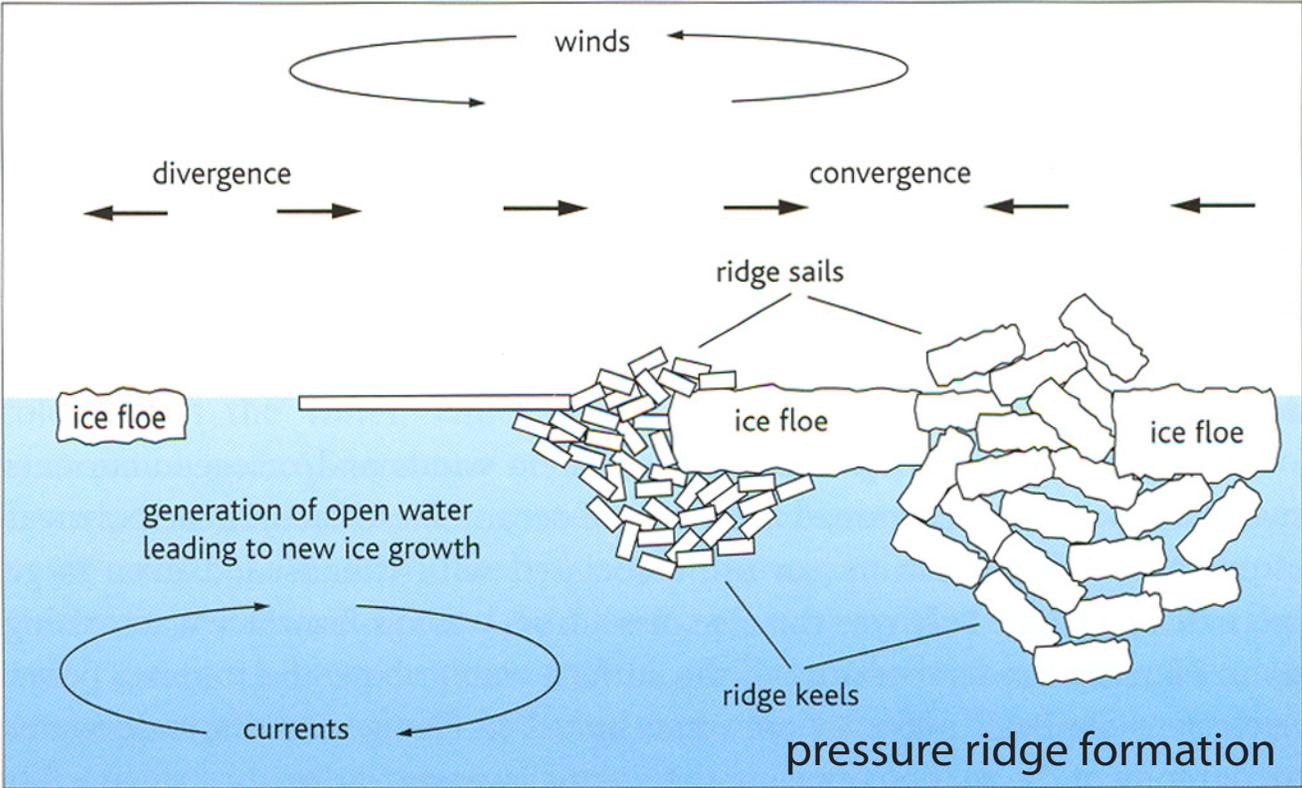
2. Conservation of momentum (Hibler 1979)

*dynamics*

3. Heat equation of sea ice and snow (Maykut and Untersteiner 1971)

*thermodynamics*

# dynamically modifying the ice thickness distribution



thinning

thickening

# measuring ice depth in ridges off Barrow, AK



dynamic sea ice

***What must we understand better about  
sea ice processes & air-ice-ocean interactions  
to improve climate projections?***

**What must be represented more realistically in models?**

# Ice albedo feedback

*Melting*

+

*Absorbed  
sunlight*



+

*Lower  
albedo*

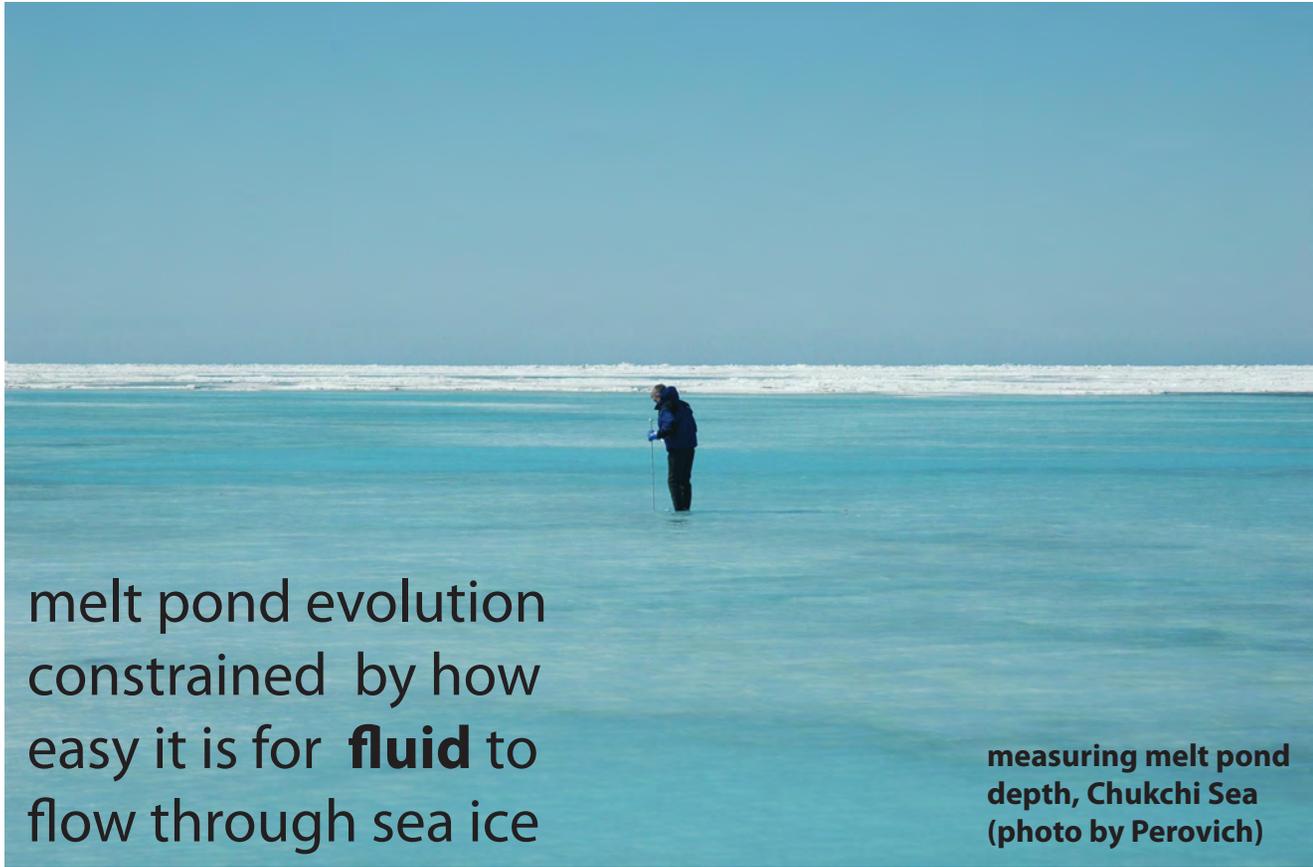
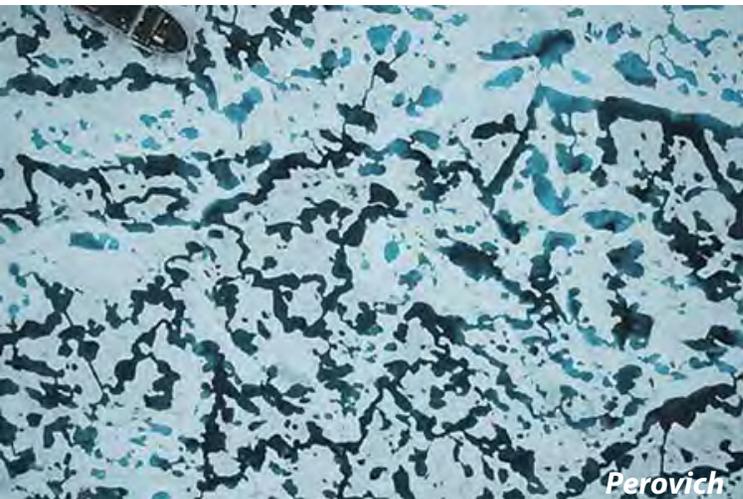
+

## ice-albedo feedback played key role in the 2007 decline:

1. Ice mass balance observations show extraordinary amount of **bottom melting** in the Beaufort Sea in summer of 2007.
2. Calculations indicate that **solar heating of the upper ocean** was the primary source of heat for this observed melting.
3. **Increase in open water fraction** resulted in 500% positive anomaly in solar heat input to the upper ocean, triggering an **ice-albedo feedback**.



sea ice albedo determined by melt ponds



melt pond evolution constrained by how easy it is for **fluid** to flow through sea ice

measuring melt pond depth, Chukchi Sea (photo by Perovich)



**fluid** ↓  
**permeability**  
depends on microstructure





*drainage vortex*

***melt pond evolution depends also on large-scale “pores” in ice cover***

*photos courtesy of C. Polashenski and D. Perovich*





***Does melting sea ice contribute to sea level rise? - not directly***

**glacial ice  
(iceberg, ice shelf)**



**sea ice**

***sea ice is in isostatic balance with the ocean  
when it melts, sea level doesn't change***

***... but indirect effects and feedbacks can influence sea level rise***

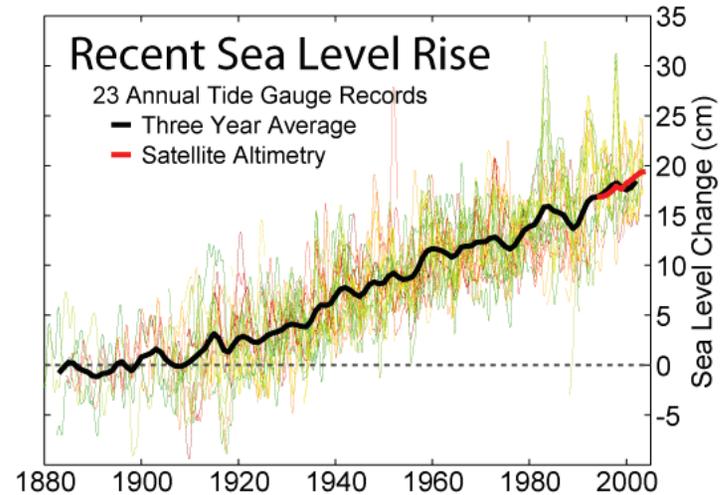
***ice sheet and glacier modeling in climate projections **not mature*****

***no coupling yet of ice sheets and ocean in climate models***

***-- no feedback effects***

(David Holland)

# *As Earth's climate warms, why does sea level rise?*



- ***melting land ice: Antarctica, Greenland, mountain glaciers***

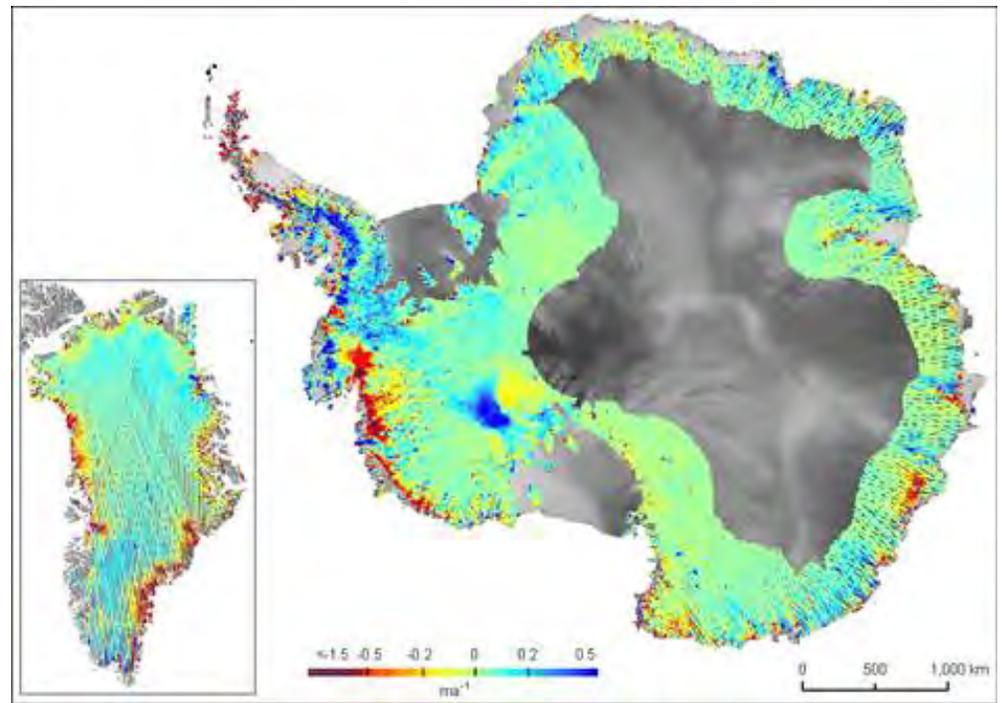
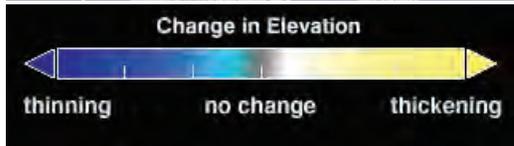
- ***thermal expansion of warming ocean***

*accounts for about 25% of rise in last half of 20th century, rate ~ tripled in 21st century*

- ***continental rebound***

*rise of land masses that were depressed by the huge weight of ice sheets*





***if all land ice melted, contributions to SLR:***

***Antarctica -- 57 m***

***Greenland -- 7 m***

***mountain glaciers -- 0.5 m***

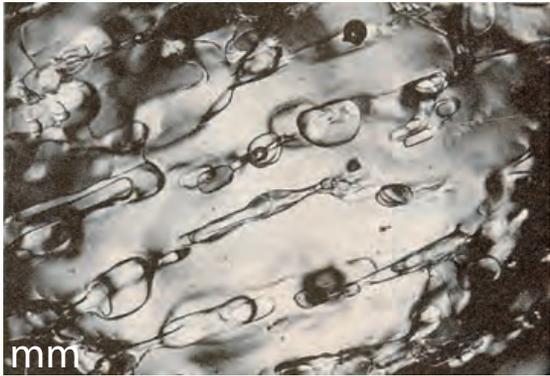


# sea ice displays *multiscale* structure over 10 orders of magnitude

0.1 millimeter



1 meter



brine inclusions



polycrystals



horizontal



dm

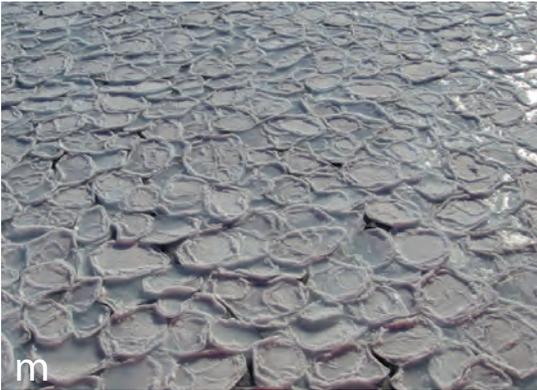


vertical

brine channels



dm



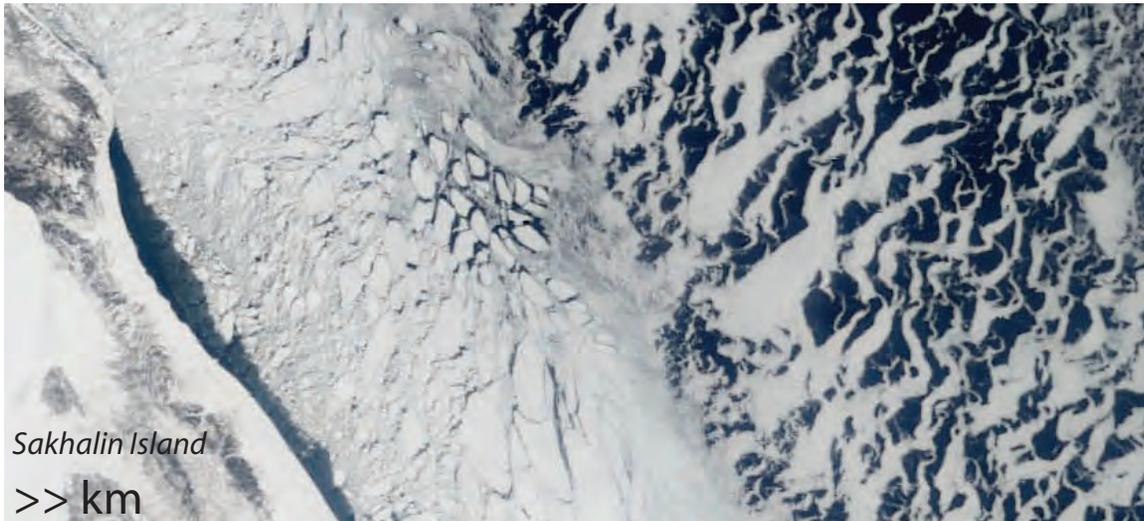
m

pancake ice

1 meter



100 kilometers



***fractals*** have self-similar structure on many length scales



# fractal dimension



$$L = 1$$

$$A = 1$$



$$L = 2$$

$$A = 4$$



$$L = 3$$

$$A = 9$$

....

$$A \sim L^d$$

$$d = 2$$

*Euclidean*



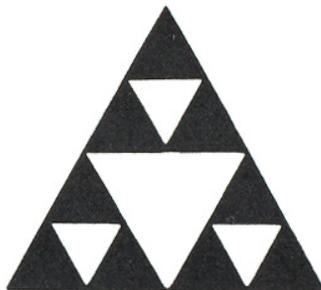
$$L = 1$$

$$A = 1$$



$$L = 2$$

$$A = 3$$



$$L = 4 = 2^2$$

$$A = 9 = 3^2$$

....

$$A \sim L^D$$

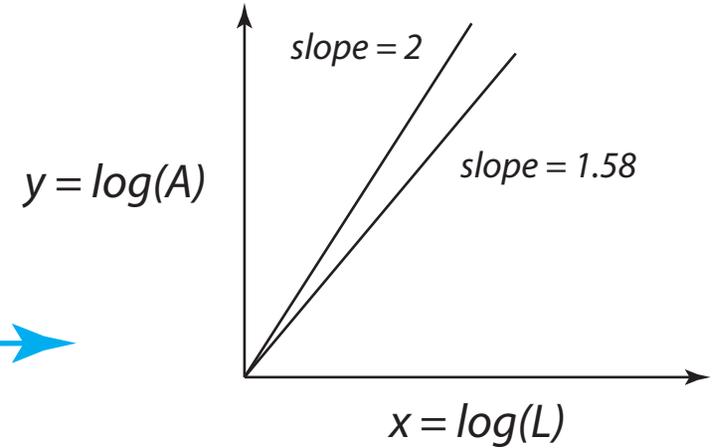
$$D = \frac{\log 3}{\log 2} = 1.58\dots$$

*fractal*

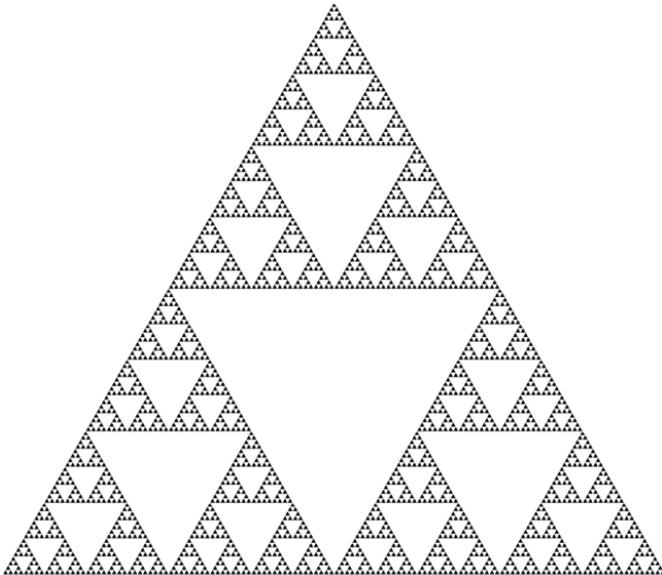
$$A = L^D$$

$$\log(A) = \log(L^D) = D \log(L)$$

$$y = D x$$



$$D = 1.58\dots$$

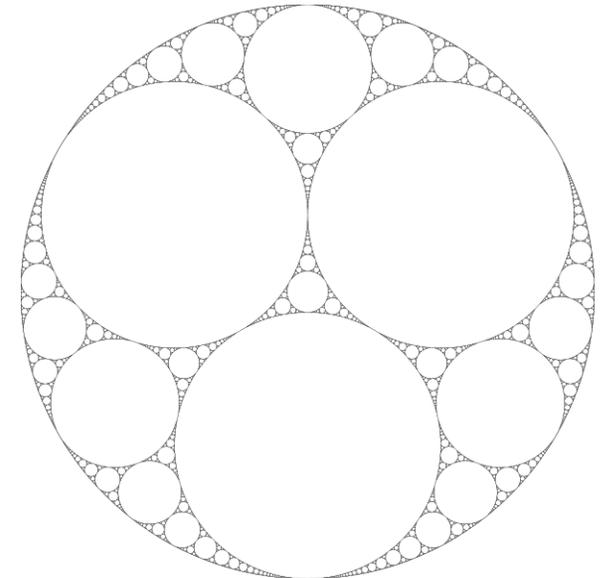


**Sierpinski triangle**

**self-similar structure  
with fractal dimension**

**fractal dimension < 2  
no longer two dimensional  
-- too much removed**

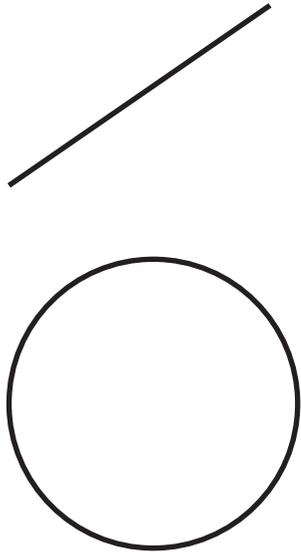
$$D = 1.31\dots$$



**Apollonian gasket**

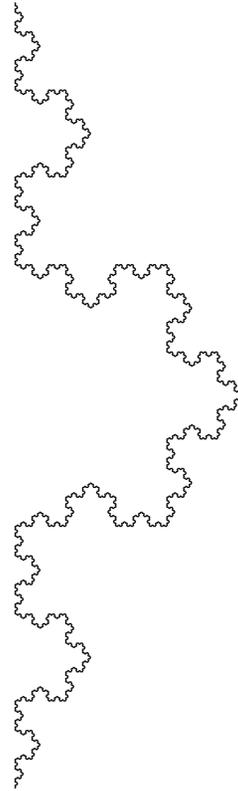
# fractal curves in the plane

they wiggle so much that their dimension is  $>1$



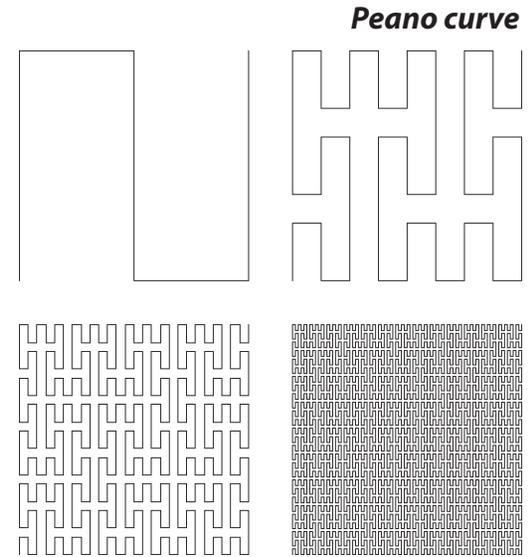
*simple curves*

$D = 1$



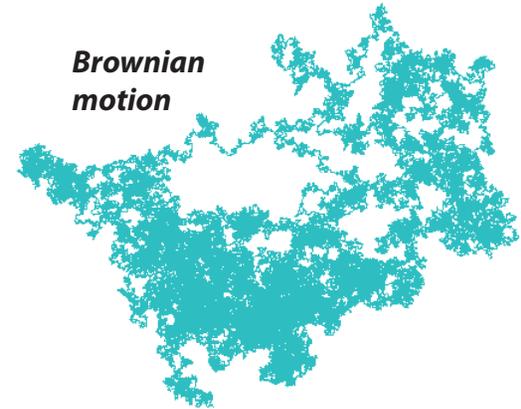
*Koch snowflake*

$D = 1.26$



*Peano curve*

*Brownian motion*



*space filling curves*

$D = 2$

# the sea ice pack is a *fractal*

displaying self-similar structure on many scales

floe size distribution, areas, perimeters, etc. important  
in dynamics (fracture), thermodynamics (melting)

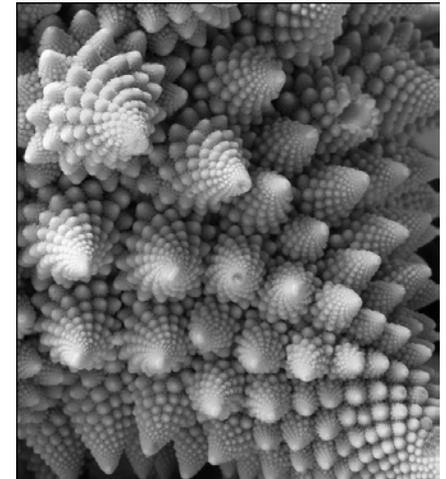
***bigger floes easier to break, smaller floes easier to melt***



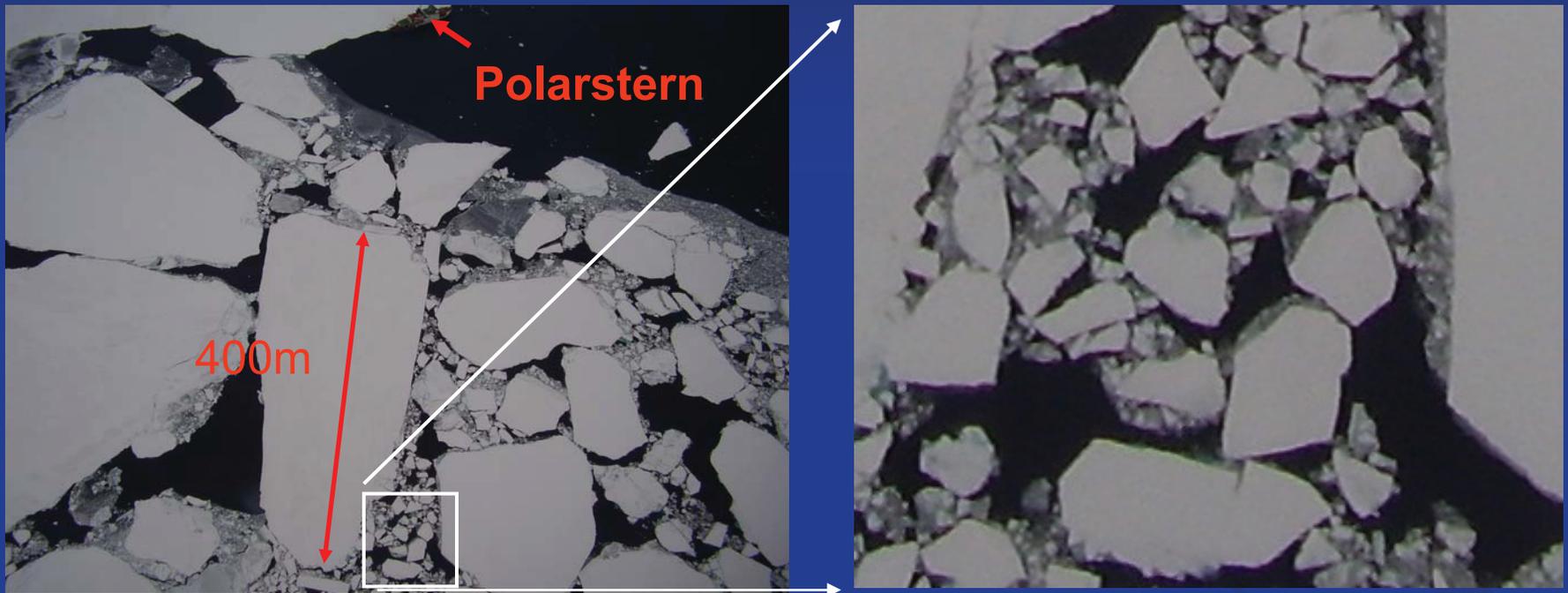
J. Robertson



E. Walker



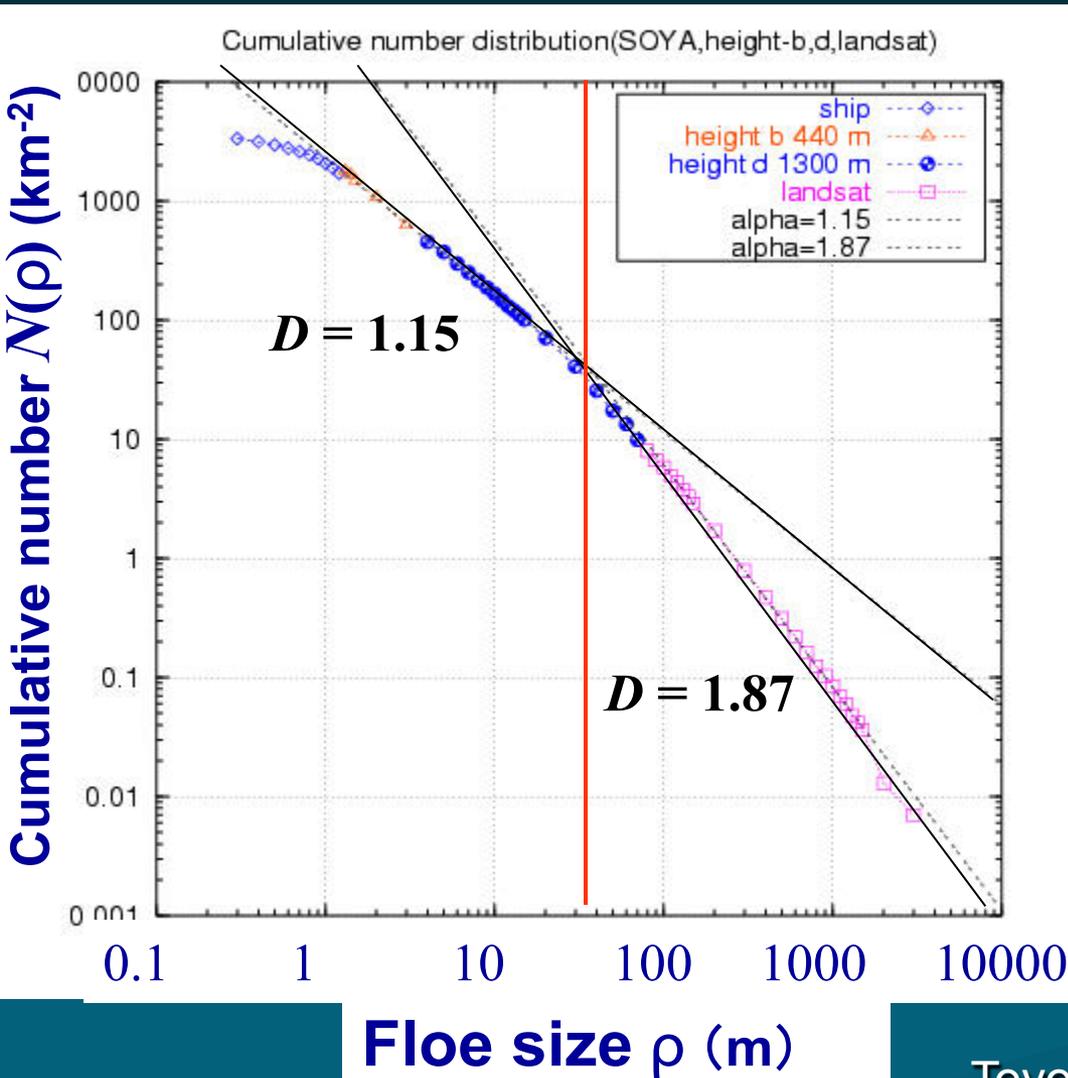
# Self-similarity of sea ice floes



Weddell Sea, Antarctica  
2006

Takenobu Toyota

# Self-similar structure in Okhotsk Sea ice pack



## Two regimes in floe size distribution:

size	fractal dimension
1 ~ 20 m :	$D = 1.15 \pm 0.02$
100 ~ 1500 m :	$D = 1.87 \pm 0.02$

$$N(\rho) \sim \rho^{-D}$$

Number of floes per unit area  
no smaller than  $\rho$

Toyota, et al., *Geophys. Res. Lett.* 2006

(Rothrock and Thorndike, *J. Geophys. Res.* 1984)

***Do melt ponds exhibit interesting multiscale structure?***

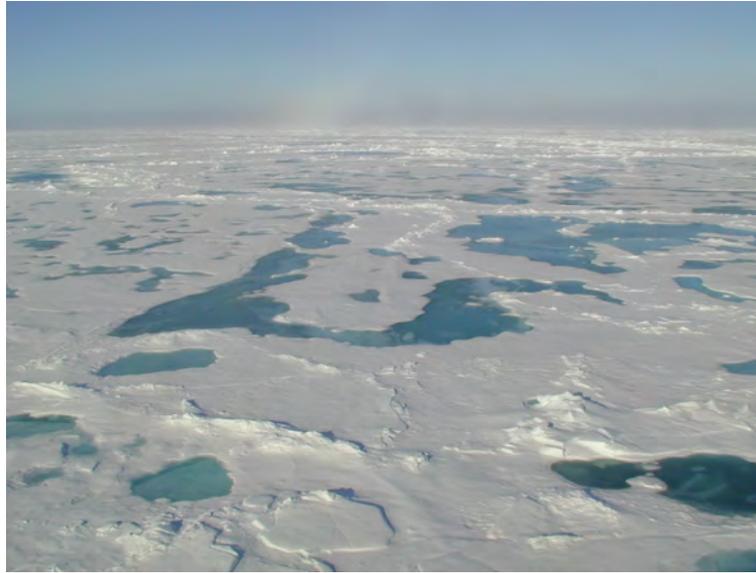
**Are there universal features of the evolution  
not yet uncovered by detailed, mechanistic models?**

***Space filling curves and the evolution of Arctic melt ponds***

Christel Hohenegger, Kyle Steffen, Don Perovich, Ken Golden

# *melt pond connectivity*

ponds disconnected  
ice connected



ponds connected  
ice connected

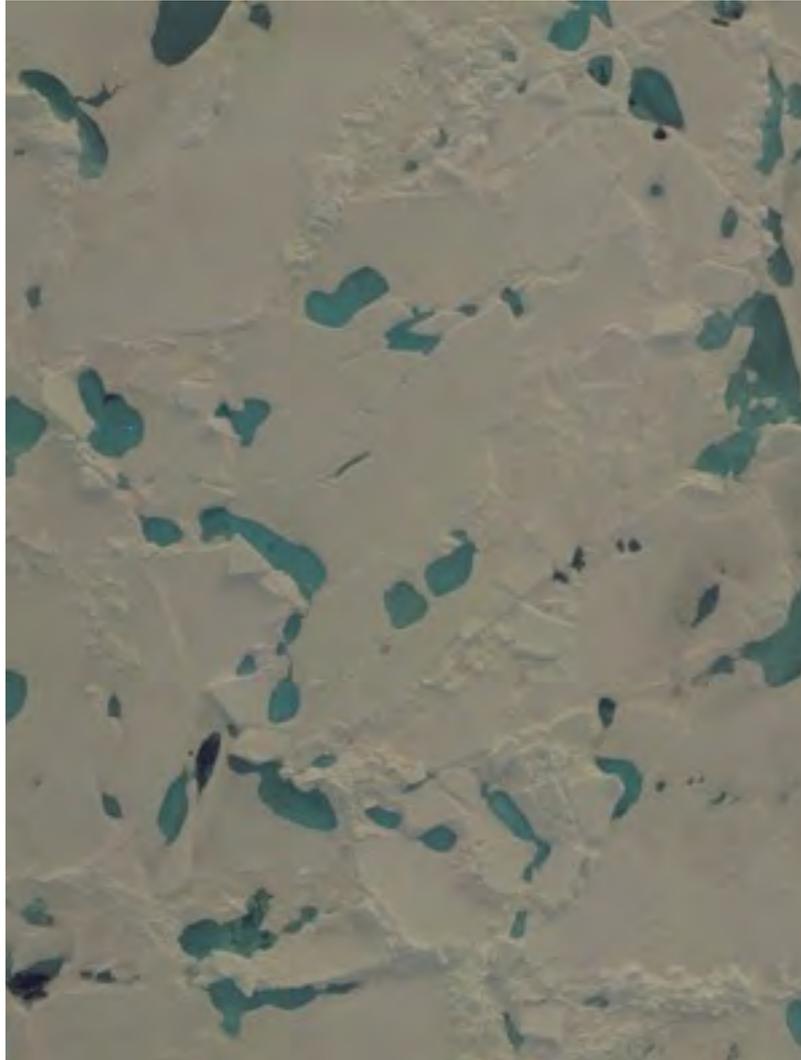


ponds connected  
ice disconnected



**as ponds coalesce and  
grow, the boundaries  
become increasingly  
complex and contorted  
with self-similar structure**

***characterize the geometry  
with the fractal dimension***



July 8 (SHEBA)



July 8 (SHEBA)

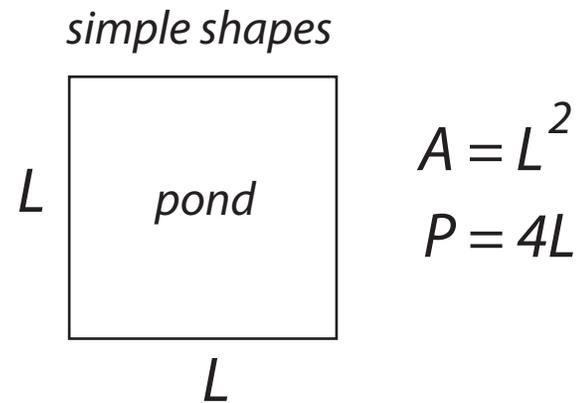


August 18 (HOTRAX)

Surface Heat Budget of the Arctic (SHEBA), 1998  
Healy-Oden Trans-Arctic Expedition (HOTRAX), 2005

use Area - Perimeter relation to investigate melt pond geometry

$$P \sim \sqrt{A}$$



*in general, for fractals with dimension  $D$*

$$P \sim \sqrt{A}^D$$

$$D = 1.26\dots$$

*Sierpinski triangle*

$$D = 2$$

$$P \sim A$$

*plane filling curves*

$$D = 1$$

*simple boundary curves*

## clouds exhibit fractal behavior from 1 to 1000 km

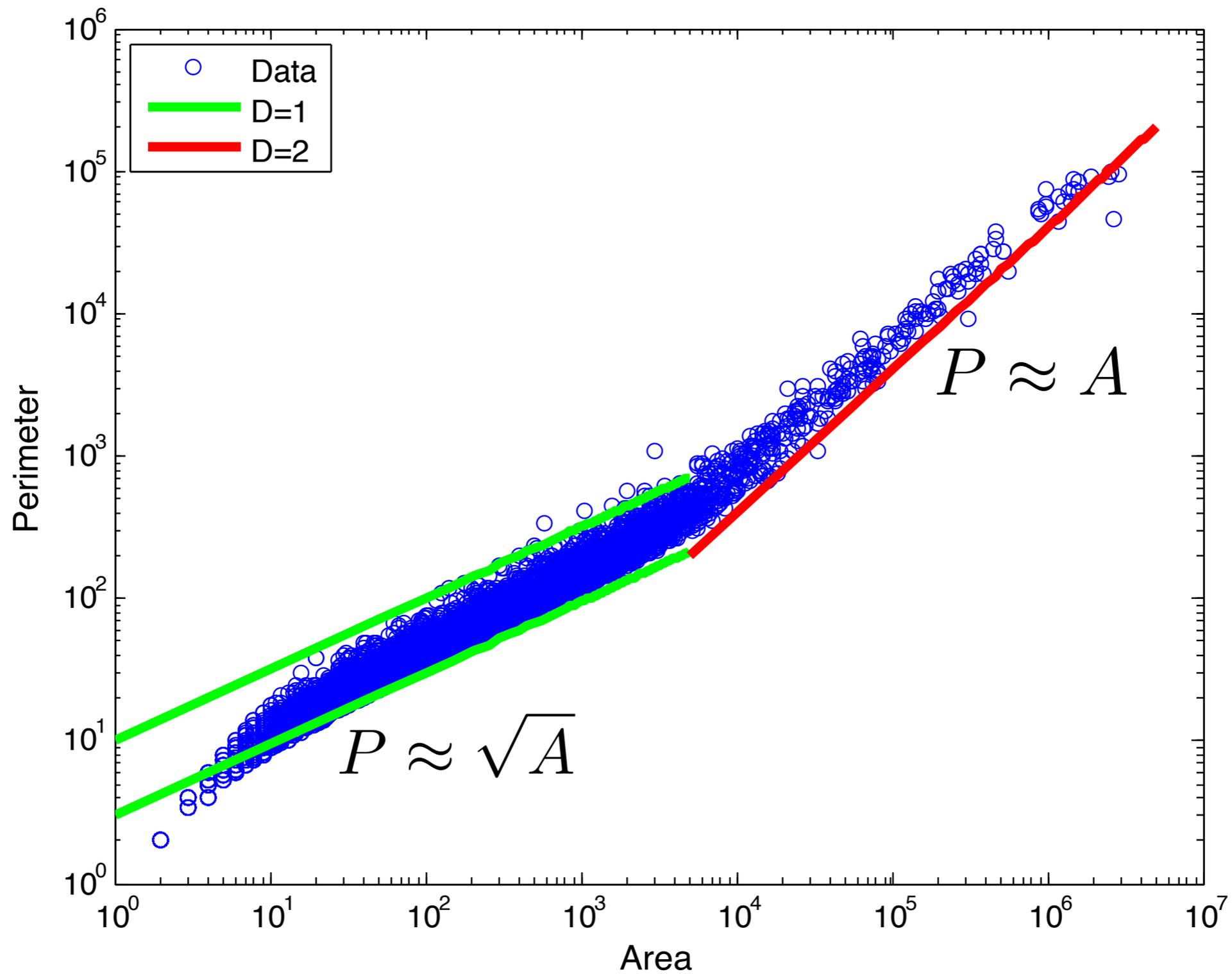
**clouds a critical component of climate models -- significant source of uncertainty  
they trap warmth and reflect sunlight in spring and summer**

*S. Lovejoy, Science, 1982*

use **perimeter-area** relation to find that  
cloud and rain perimeters are fractals

$$D \approx 1.35$$





A-P for melt ponds

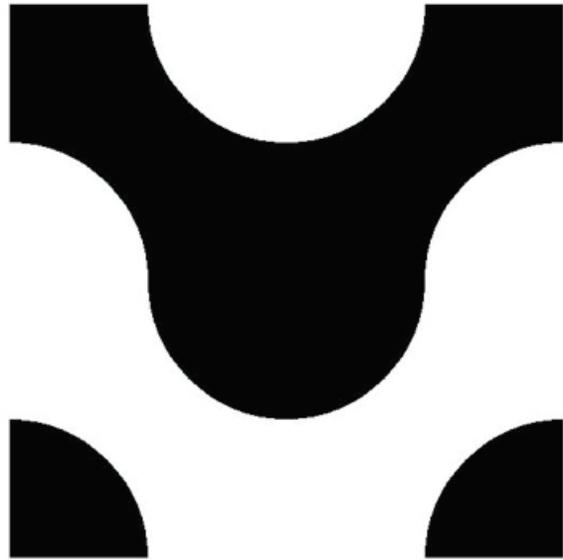
August 14 (HOTRAX): 22 images with fully developed melt ponds



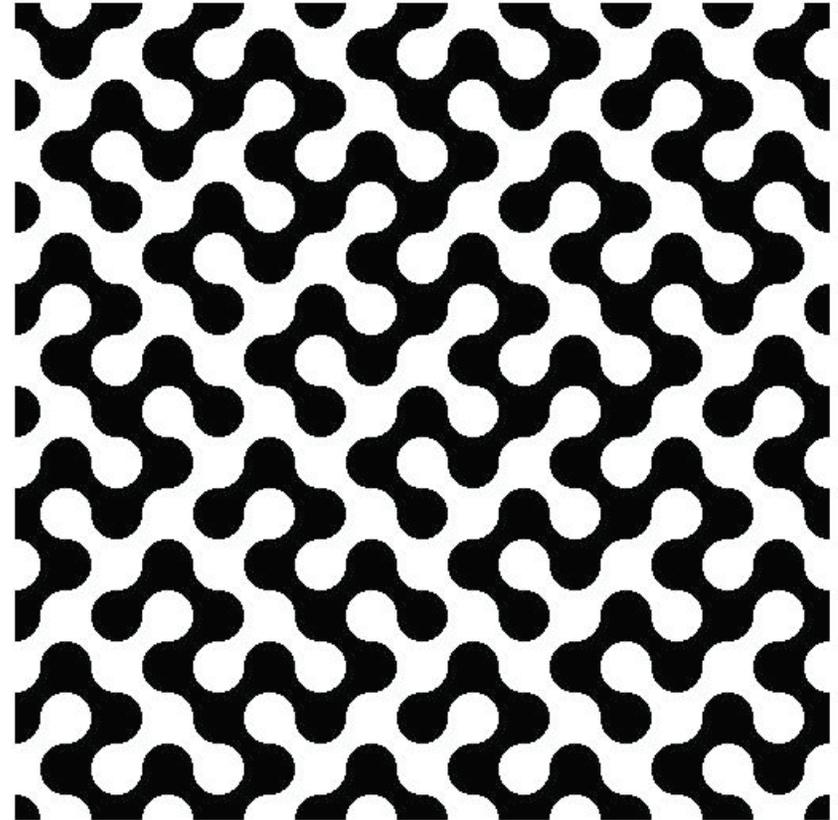
*August 14, 2005*

*Healy-Oden Trans Arctic Expedition (HOTRAX)*

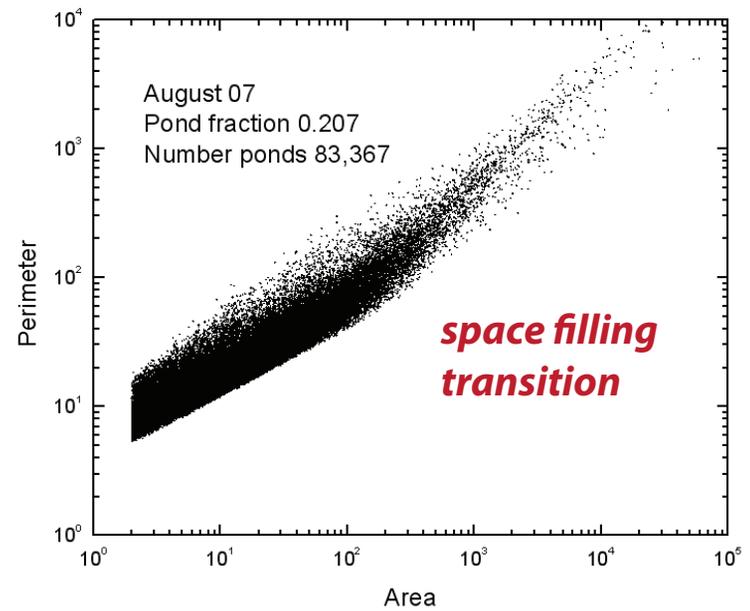
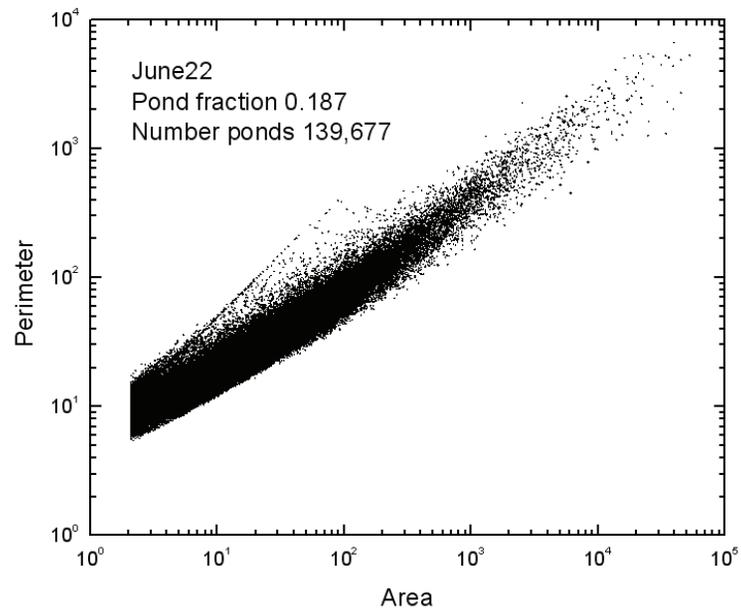
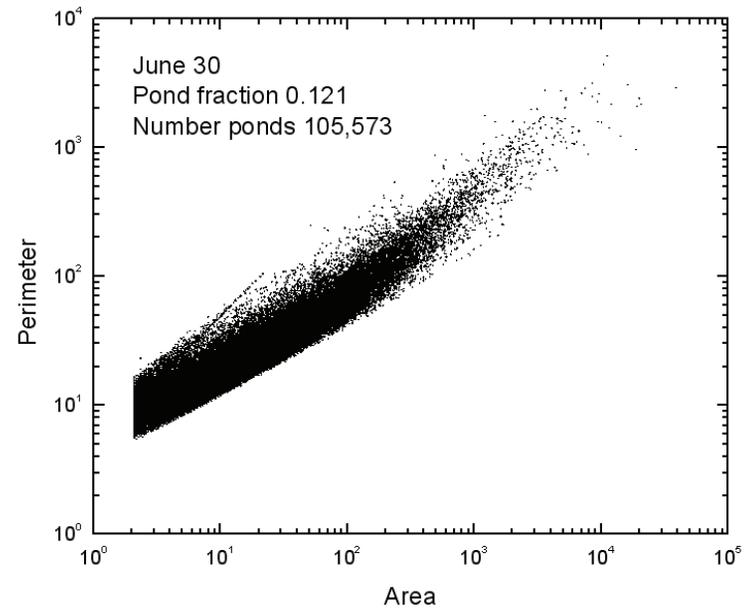
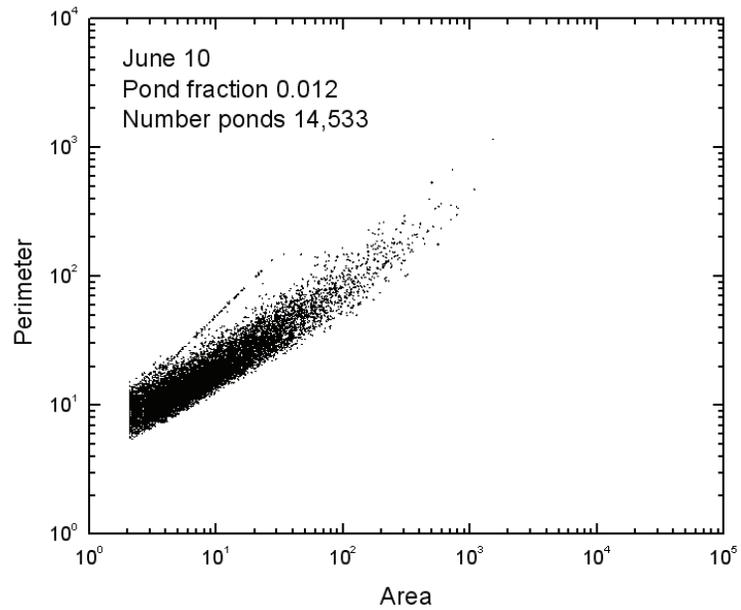
*initial shape*



*4th generation*



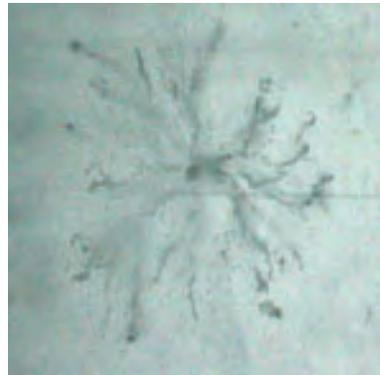
# Time evolution of melt pond structure



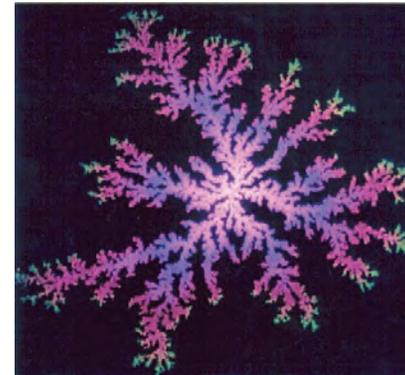
## *fractal microstructures*



electrorheological fluid  
with metal spheres



brine channel  
in sea ice

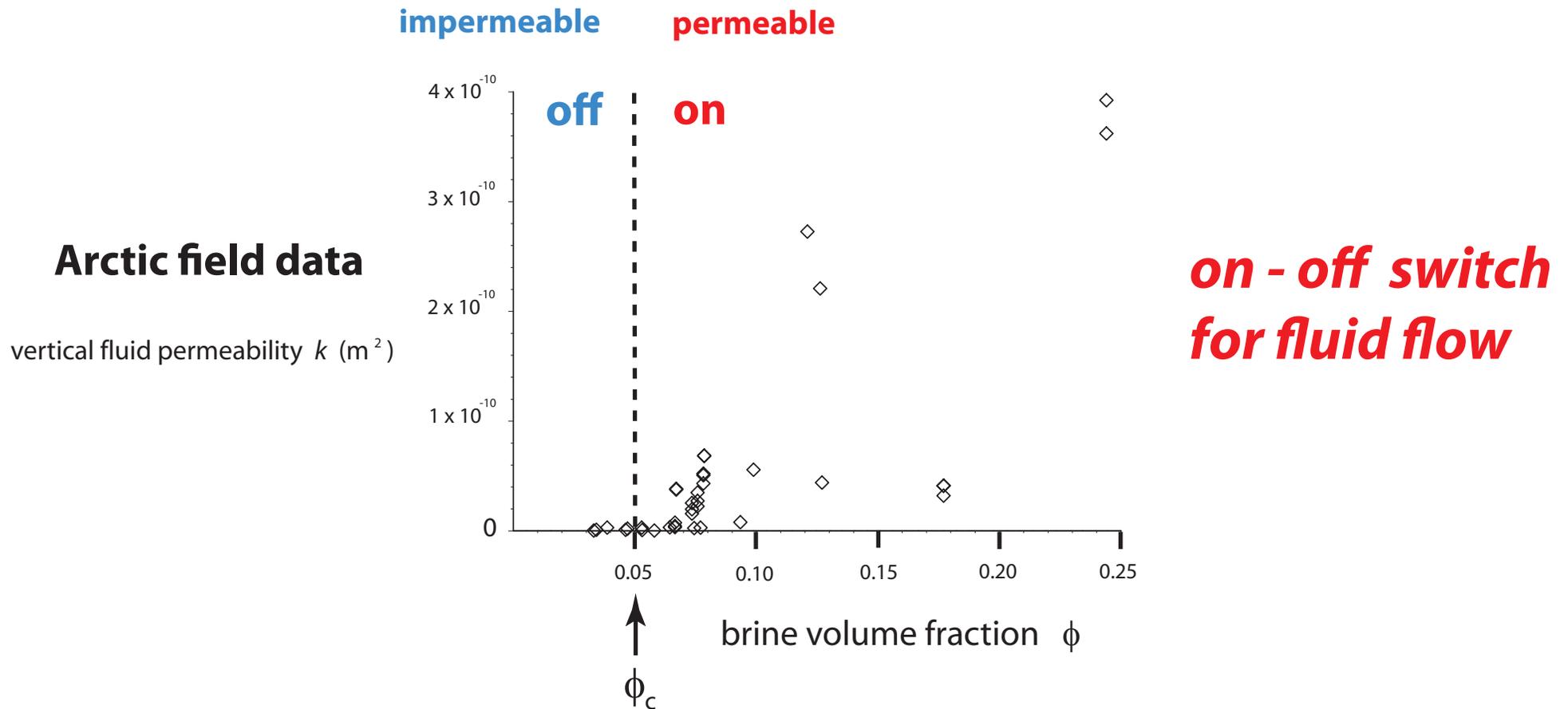


diffusion limited  
aggregation

***sea ice microphysics***

***fluid transport***

# Critical behavior of fluid transport in sea ice



critical brine volume fraction  $\phi_c \approx 5\%$   $\longleftrightarrow$   $T_c \approx -5^\circ \text{C}$ ,  $S \approx 5 \text{ ppt}$

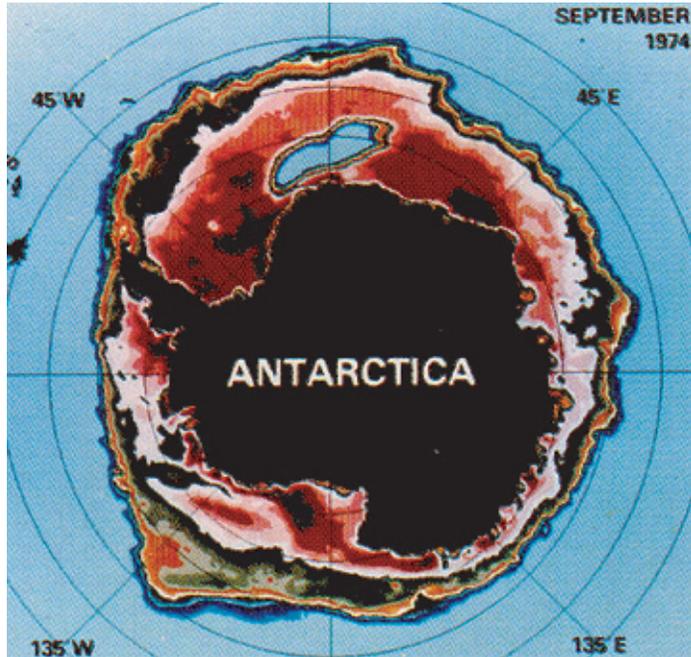
**RULE OF FIVES**

Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

# sea ice permeability constrains:



## Antarctic Zone Flux Experiment (ANZFLUX)

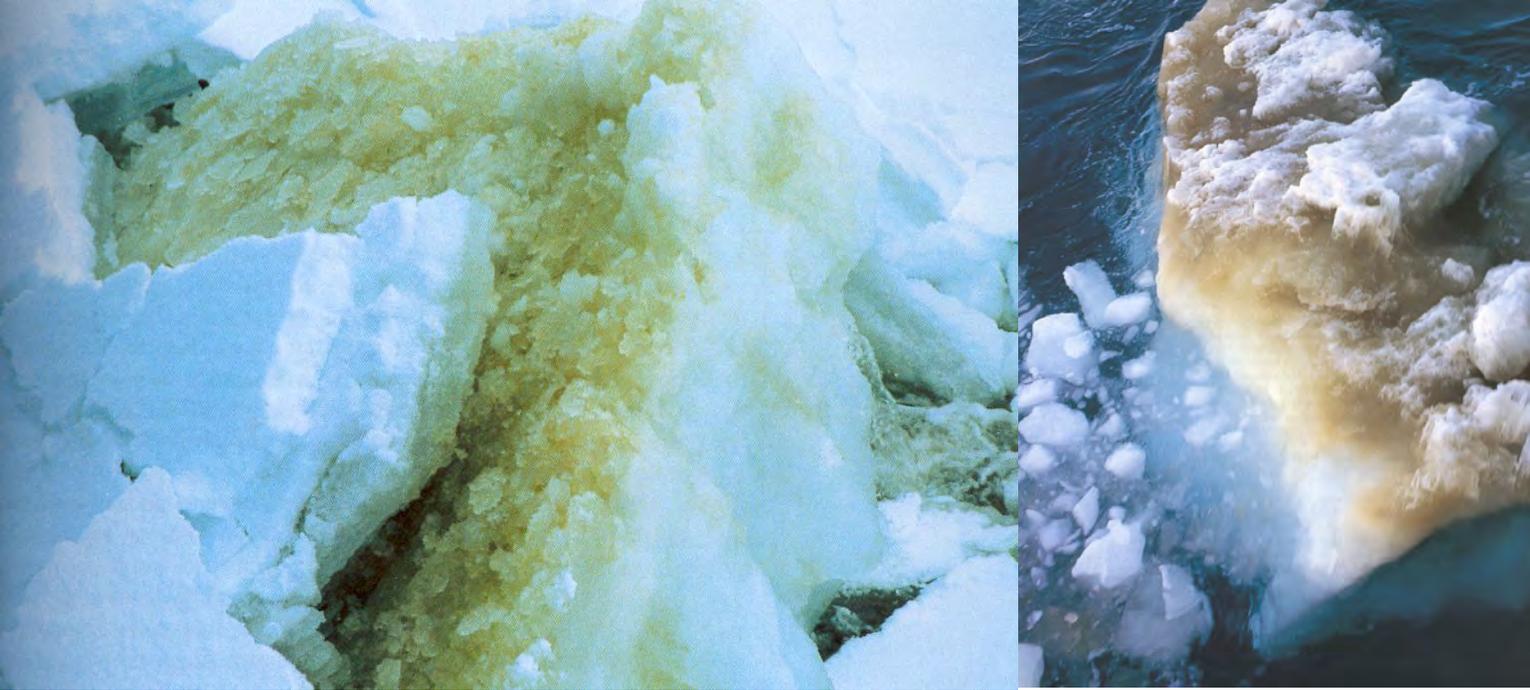
Eastern Weddell Gyre  
July - August, 1994

- **snow-ice production** - freezing of flooded ice surfaces  
*increased precipitation in changing climate can thicken ice pack*  
  
Ackley, Lytle, Golden, Darling, Kuehn, 1995  
Maksym and Jeffries, 2001, ...  
D. C. Powell, T. Markus, A. Stossel, 2005
- evolution of **salinity profiles**  
*currently assumed constant in climate models*
- enhancing **thermal fluxes and exchanges**  
  
Lytle and Ackley, 1996  
Trodahl, *et. al.*, 2000, 2001  
Wang, Zhu, Golden, 2011
- microwave signatures in **remote sensing**  
  
Lytle and Golden, 1995  
Hosseinmostafa, *et. al.*, 1995, ...

# sea ice ecosystem



sea ice algae  
support life in the polar oceans



## sea ice algal communities

D. Thomas 2004



nutrient replenishment  
controlled by ice permeability

biological activity turns on  
or off according to  
***rule of fives***

***Golden, Ackley, Lytle***

***Science 1998***

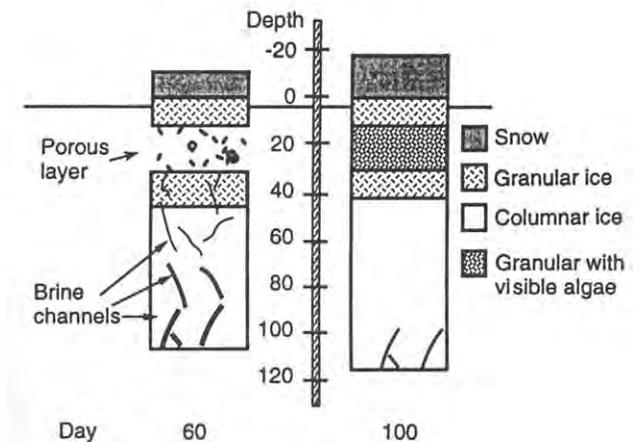
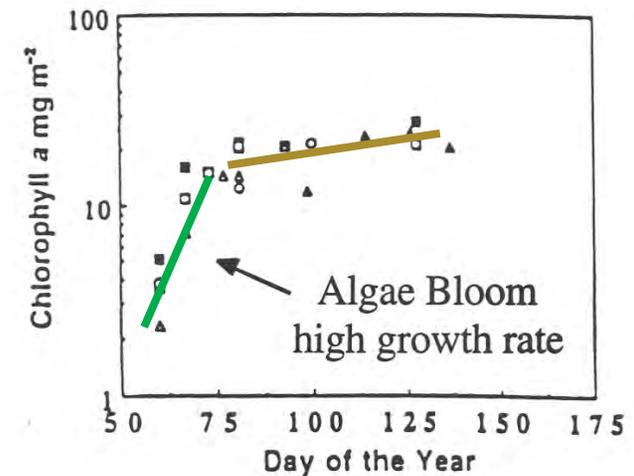
***Fritsen, Lytle, Ackley, Sullivan***

***Science 1994***

# critical behavior of microbial activity

## Convection-fueled algae bloom Ice Station Weddell

(Fritsen, Lytle, Ackley, Sullivan, *Science* 1994)

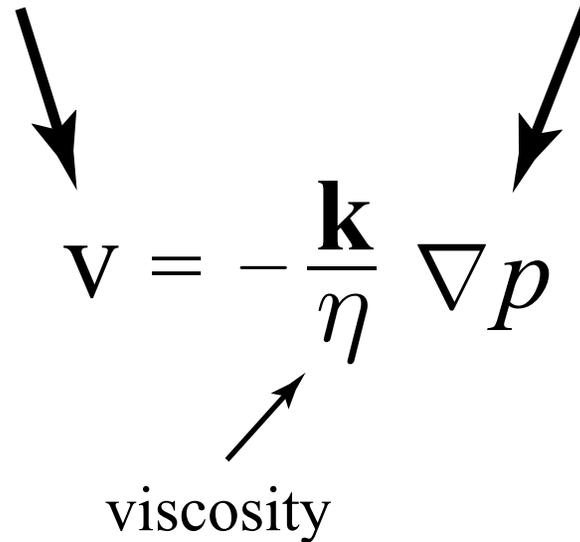


**Golden, Ackley, Lytle *Science* 1998**

# *Darcy's Law* for slow viscous flow in a porous medium

averaged  
fluid velocity

pressure  
gradient



The diagram shows the equation  $\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$  with three arrows pointing to its components: one from 'averaged fluid velocity' to  $\mathbf{v}$ , one from 'pressure gradient' to  $\nabla p$ , and one from 'viscosity' to  $\eta$ .

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

$\mathbf{k}$  = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

# Unified approach to understanding fluid permeability in sea ice:

Thermal evolution of permeability and microstructure in sea ice, K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu, *Geophysical Research Letters*, vol. 34, 2007 (+ cover).

1. Homogenization and Darcy's law

2. Rigorous bounds

3. Percolation theory

4. Hierarchical models

5. Network model

+

X-ray CT imaging and pore analysis provide unprecedented look at temperature evolution of brine phase and its connectivity

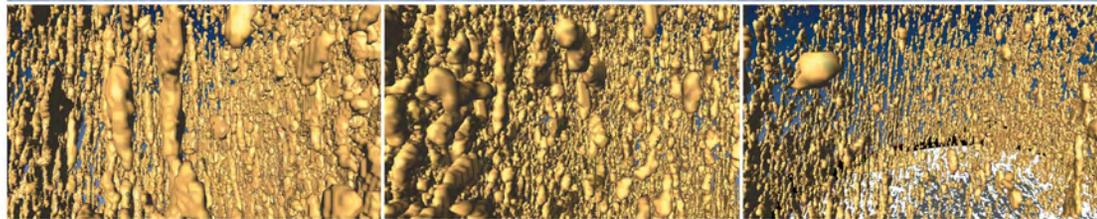
Validated with lab and Arctic field data.

# Geophysical Research Letters

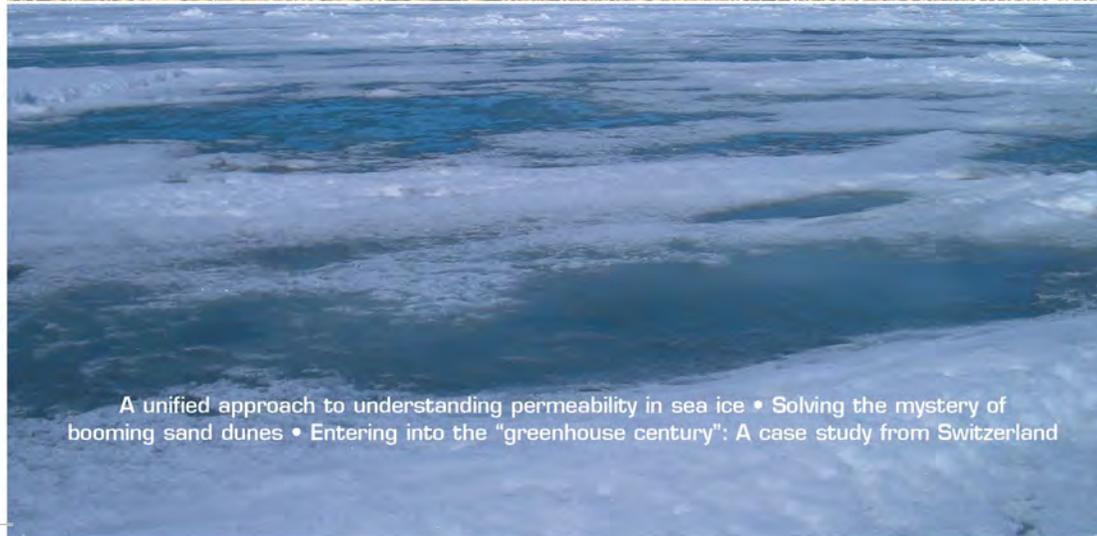
28 AUGUST 2007  
Volume 34 Number 16  
American Geophysical Union



Arctic Ocean near  
Point Barrow, June 2007



brine inclusion tomography



Arctic Ocean near  
Point Barrow, June 2004

A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

micro-scale  
controls

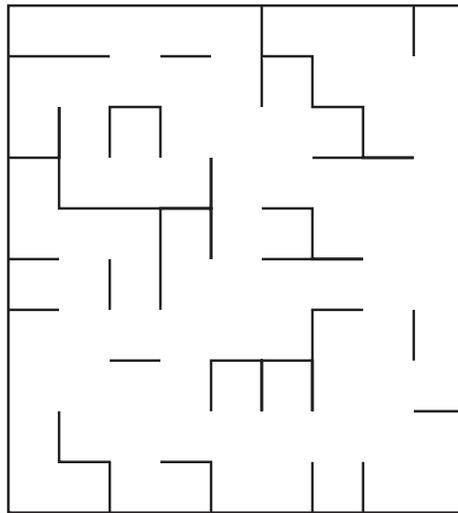
macro-scale  
processes in  
global climate

# Theoretical predictions of ice permeability using percolation and hierarchical models

# percolation theory

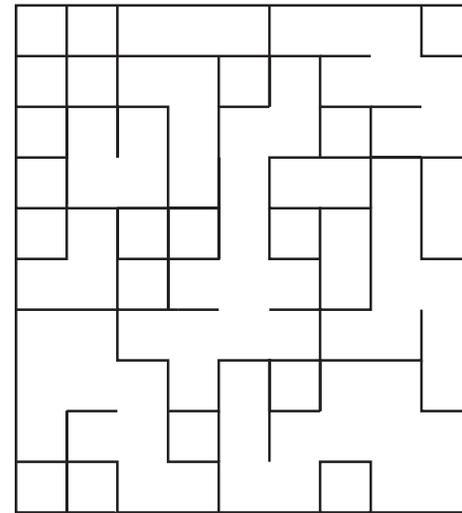
***mathematical theory of connectedness***

impermeable



$$p = 1/3$$

permeable



$$p = 2/3$$

*a bond is **open** with probability  $p$   
**closed** with probability  $1-p$*

**percolation threshold**

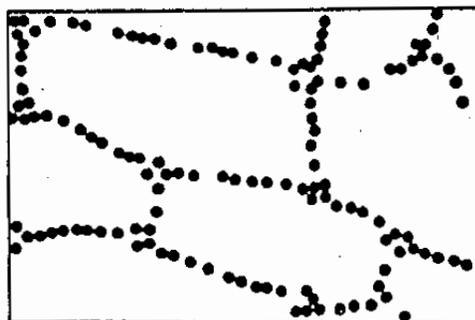
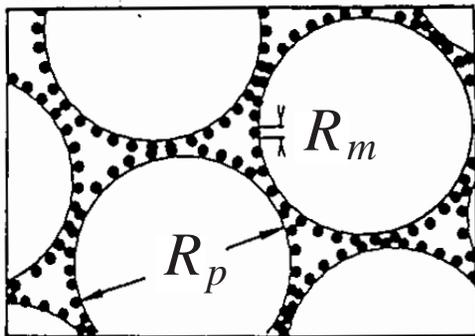
$$p_c = 1/2 \quad \text{for } d = 2$$

***first appearance  
of infinite cluster***

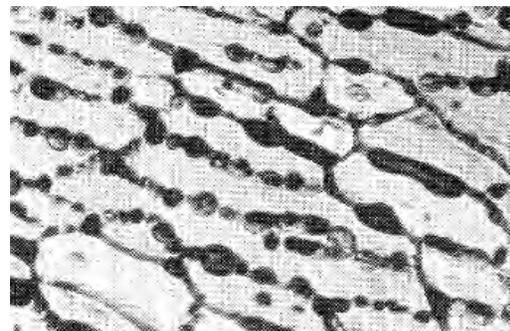
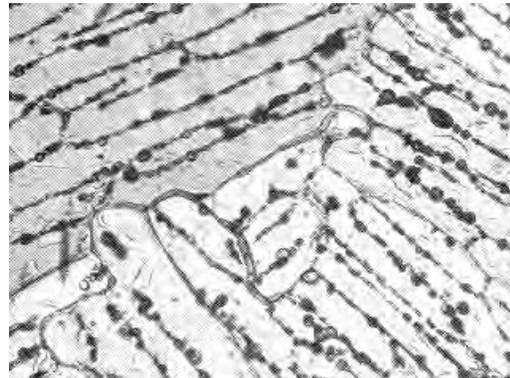
*Continuum* percolation model for stealthy materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5\%$$

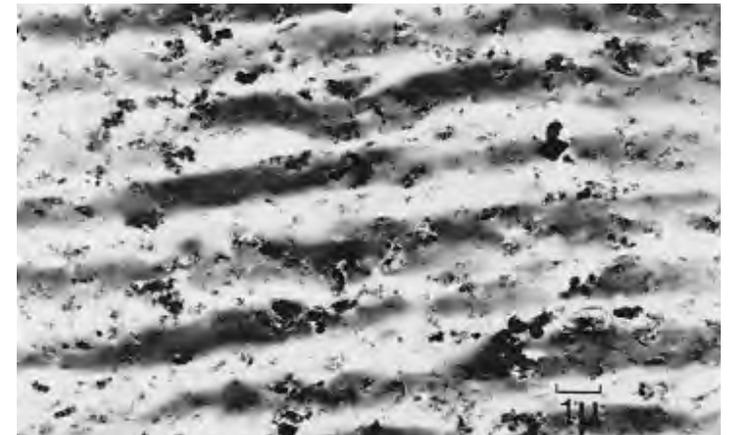
Golden, Ackley, Lytle, *Science*, 1998



compressed powder



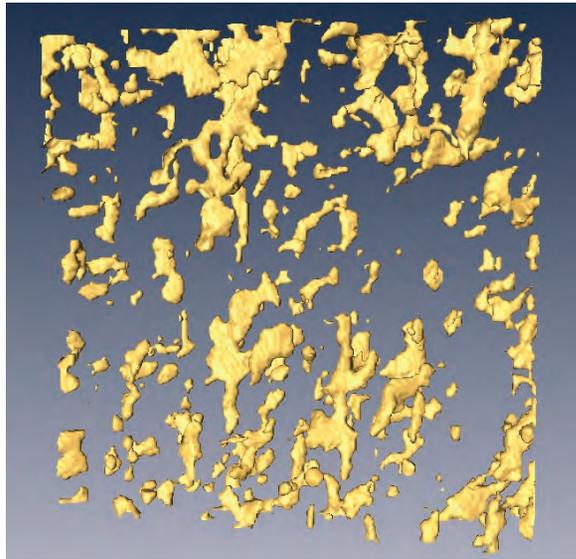
sea ice



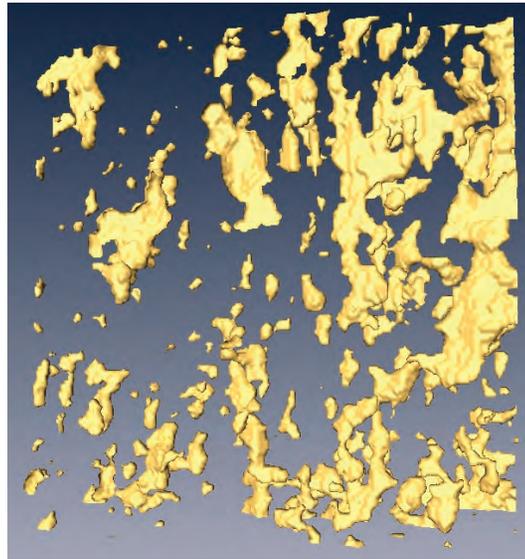
microstructure of radar absorbing composite

# brine connectivity (over cm scale)

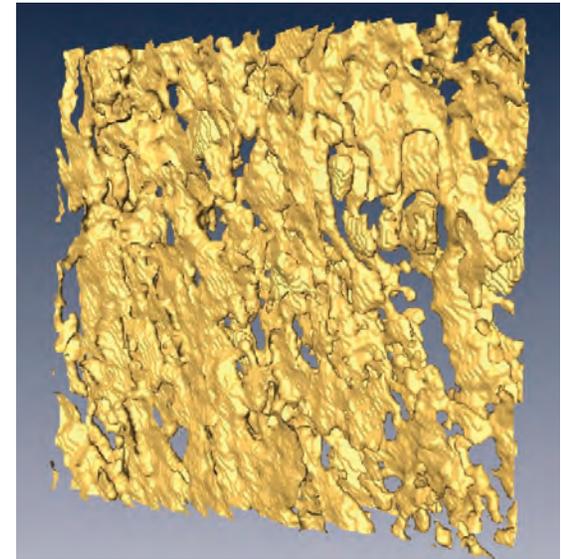
8 x 8 x 2 mm



-15 °C,  $\phi = 0.033$



-6 °C,  $\phi = 0.075$



-3 °C,  $\phi = 0.143$

first direct evidence of conjectured transition in connectivity

3-D images  
pores and throats



3-D graph  
nodes and edges

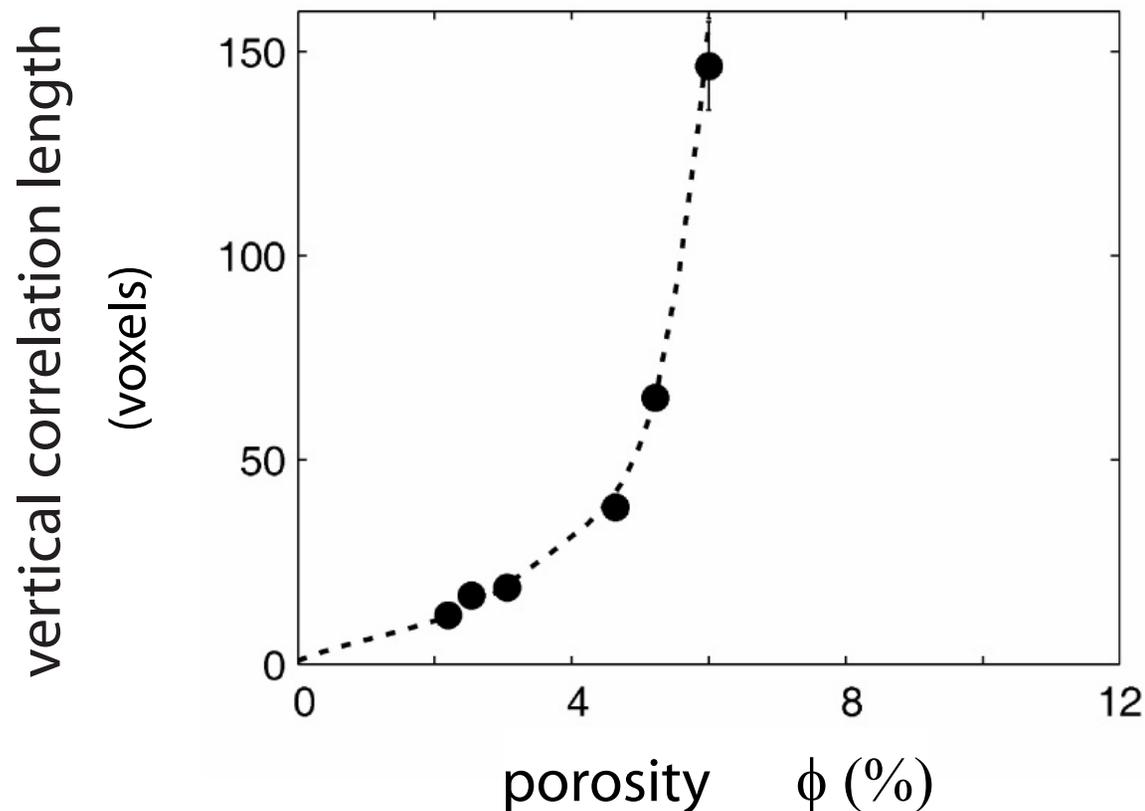
***analyze graph connectivity as function of temperature and sample size***

***use finite size scaling techniques to confirm rule of fives***

The key connectivity functions of percolation theory have been computed **extensively** for many lattice models, but NOT for natural materials.

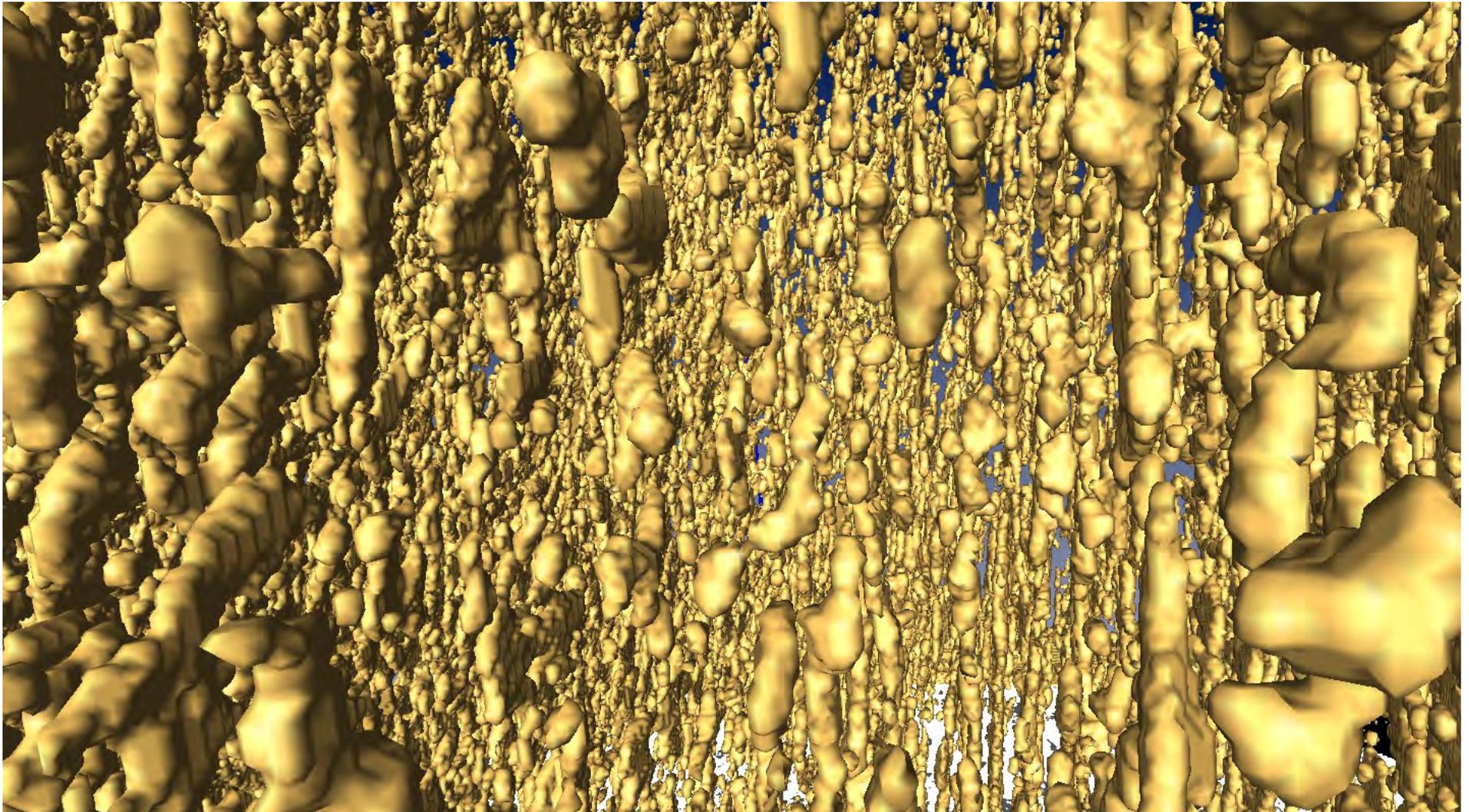
We have calculated them for sea ice single crystals -- a first for percolation investigations -- and estimated anisotropic percolation thresholds.

Pringle, Miner, Eicken, Golden, JGR (Oceans) 2009



divergence of  
correlation length  
for single crystal data

X-ray computed tomography (CT) and medial axis analysis (Prodanovic, et al., 2006) of lab-grown sea-ice single crystals doped with CsCl to improve ice/brine resolution.



$$\phi = 5.7 \% \quad T = -8$$

lattice and continuum percolation theories yield:

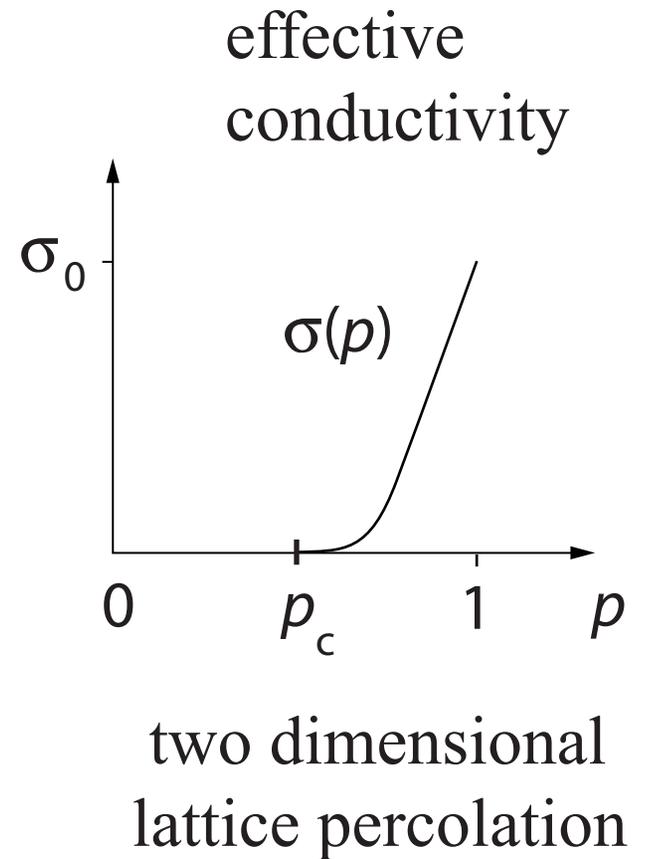
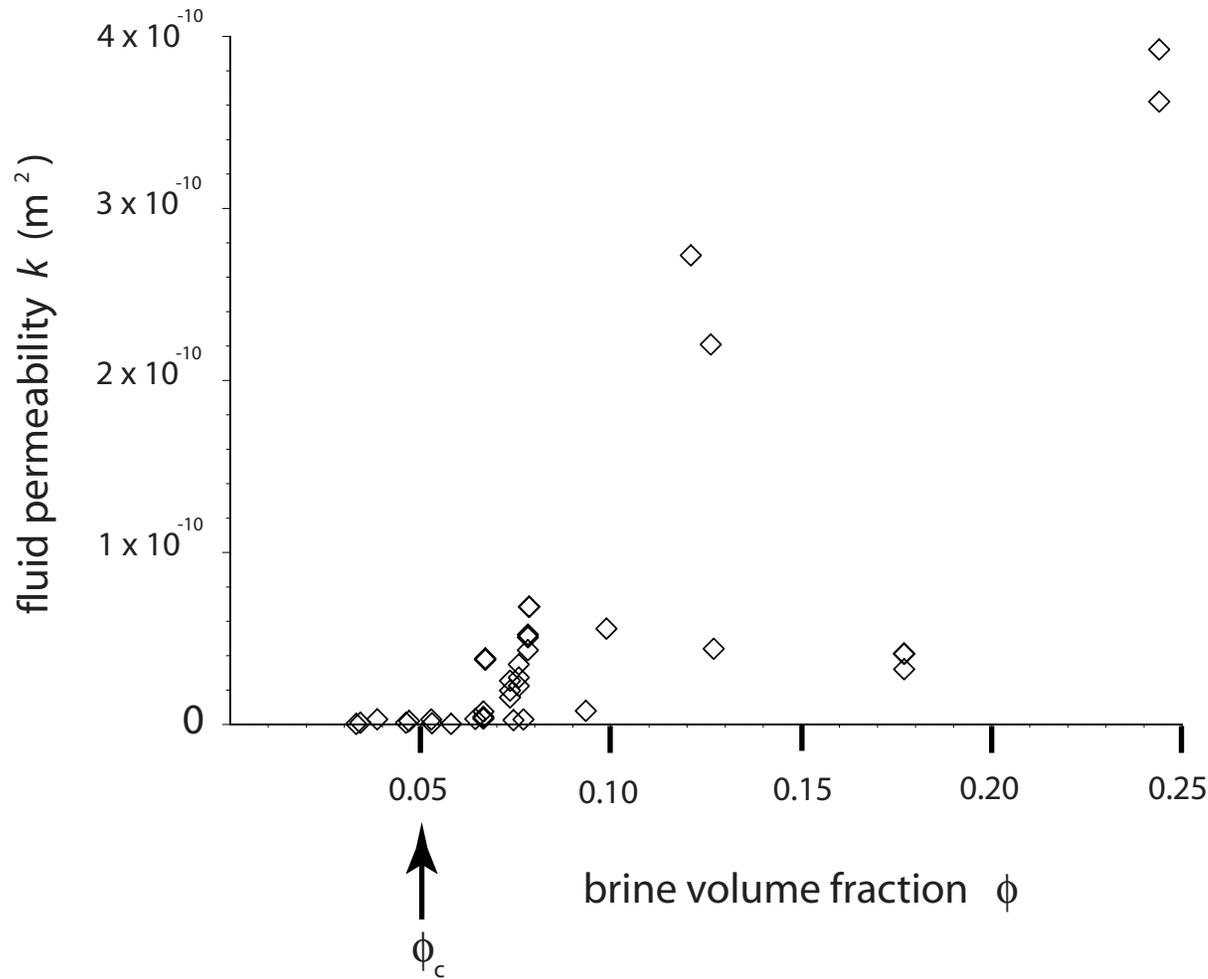
$$k(\phi) = k_0 (\phi - \phi_c)^t$$

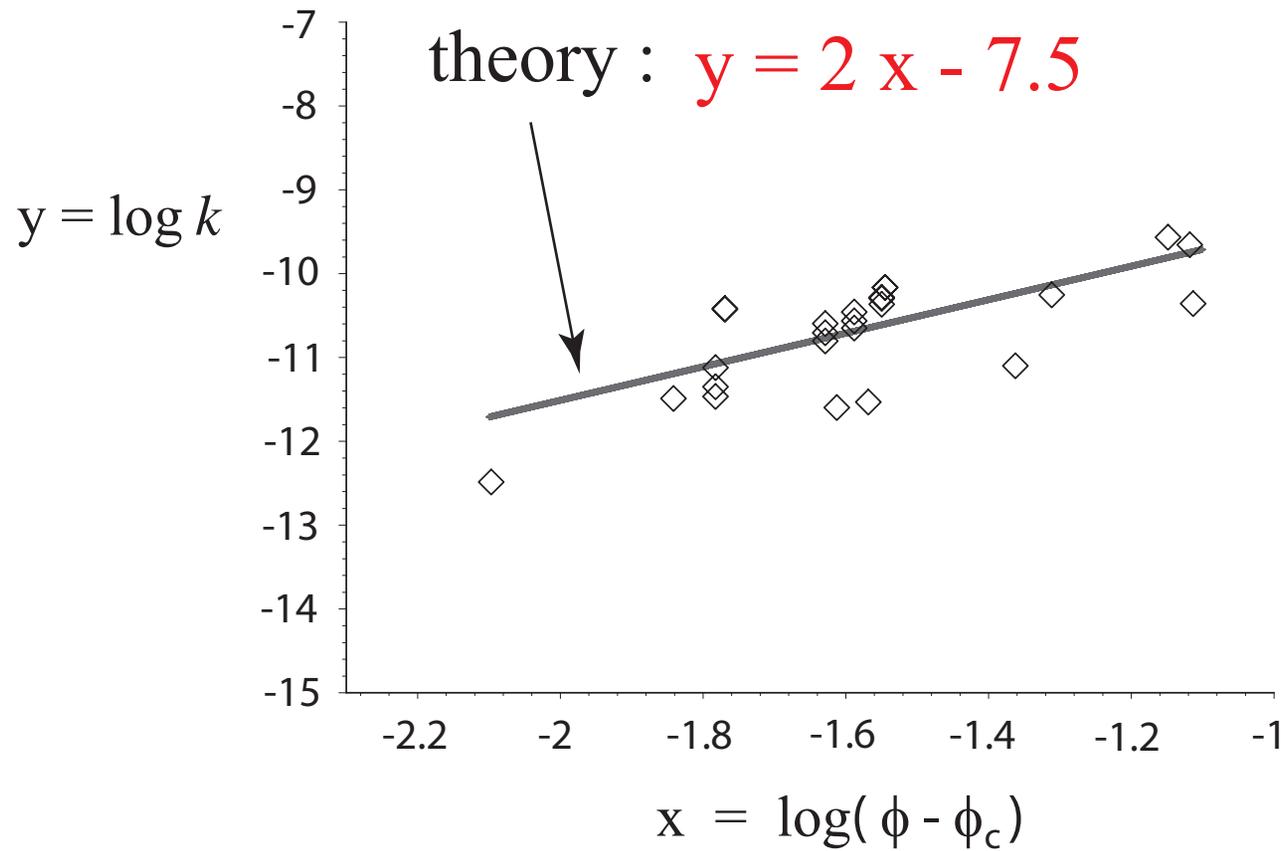
critical  
exponent  
 $t$

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

- exponent is **UNIVERSAL** lattice value  $t \approx 2.0$  from general structure of brine inclusion distribution function (-- other saline ice?)
- **sedimentary rocks** like sandstones also exhibit universality
- **critical path analysis** -- developed for electronic hopping conduction -- yields scaling factor  $k_0$
- no free parameters - microstructural input only

# *in situ* data on vertical fluid permeability of Arctic sea ice

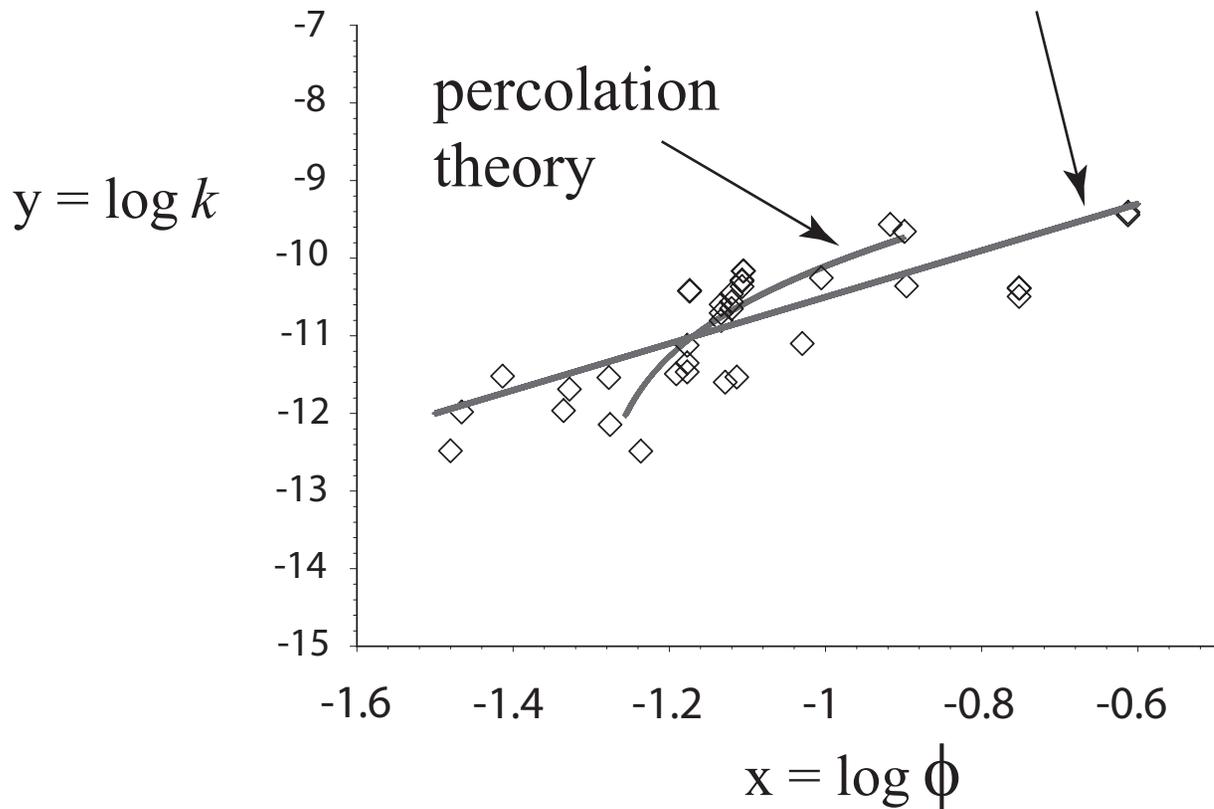




statistical best fit of data:

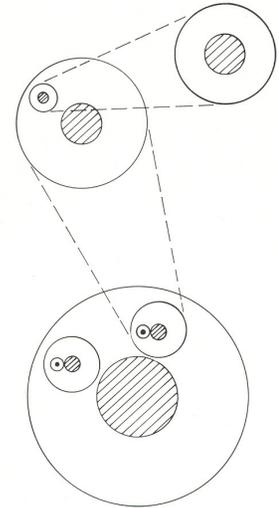
$$y = 2.07x - 7.45$$

hierarchical model :  $y = 3 x - 7.5$



statistical best fit of data:  $y = 3.05 x - 7.50$

**brine-coated  
spherical ice grains**



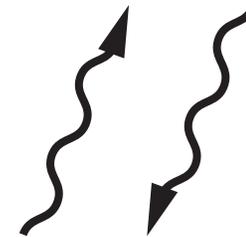
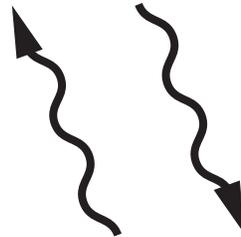
$$k(\phi) = k_0 \phi^3$$

over full range  
of porosities

**self-similar model  
used for porous rocks**

**Sen, Scala, Cohen 1981  
Sheng 1990  
Wong, Koplick, Tomanic 1984**

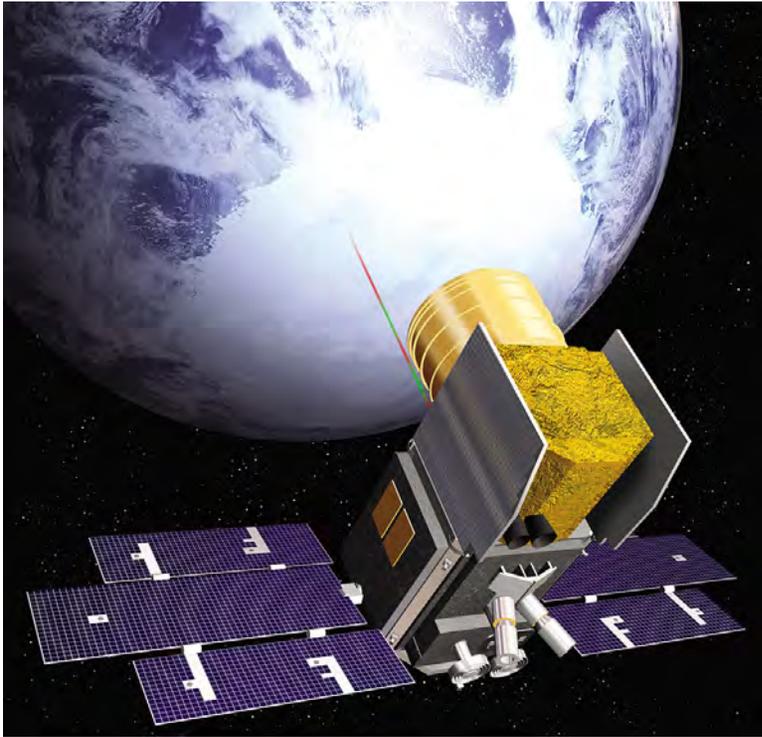
# Remote sensing of sea ice



Recover sea ice properties from electromagnetic (EM) data



INVERSE  
PROBLEM



NASA's Ice, Cloud and Land Elevation Satellite (ICESat)



The Worbot - a low frequency EM induction instrument for measuring sea ice thickness

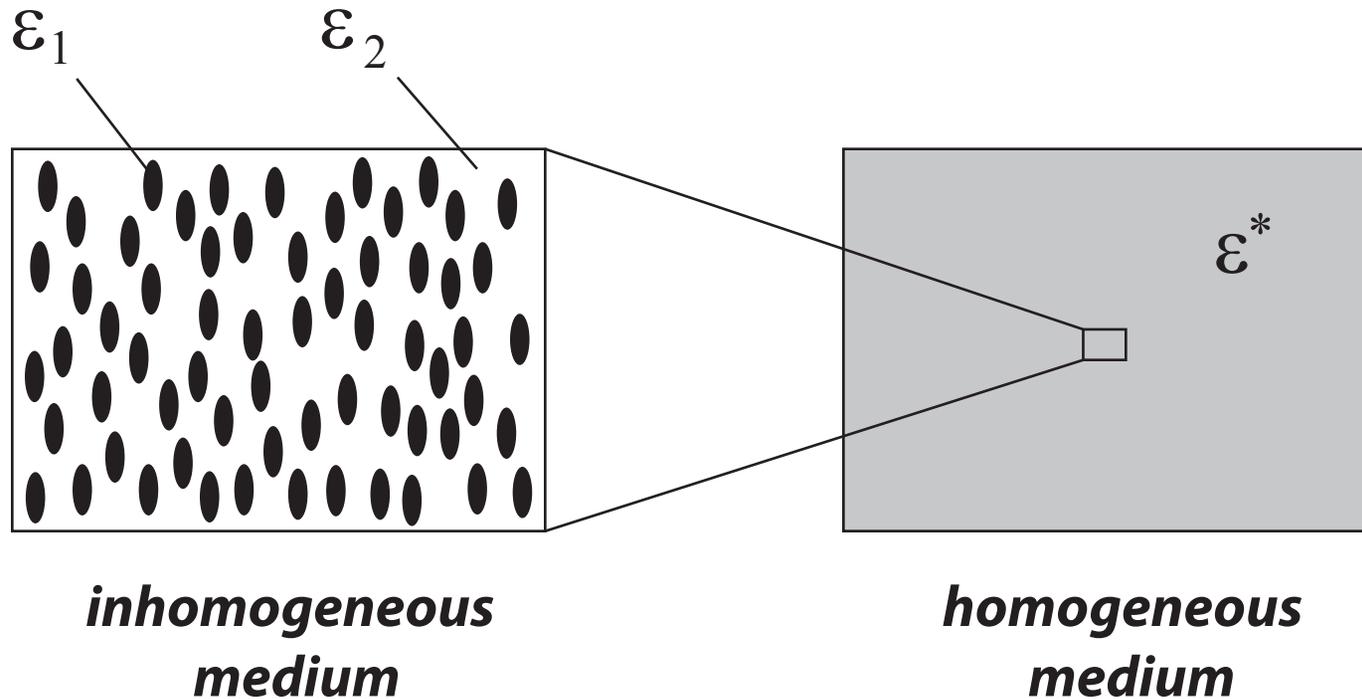
The key parameter in modeling the response of sea ice to an EM field is its

*complex permittivity or dielectric constant*  $\epsilon^*$

which depends strongly on the brine microstructure

*e.g.*, interpretation of EM thickness data depends on knowledge of  $\epsilon^*$

# HOMOGENIZATION



**find the homogeneous medium which  
behaves macroscopically the same as  
the inhomogeneous medium**

*ocean swells propagating through a vast field of pancake ice*

**HOMOGENIZATION:** long wave sees an effective medium, not individual floes



# ***Theory of Effective Electromagnetic Behavior of Composites***

***(analytic continuation method)***

***Forward Homogenization*** Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

***composite geometry  
(spectral measure)***   $\epsilon^*$

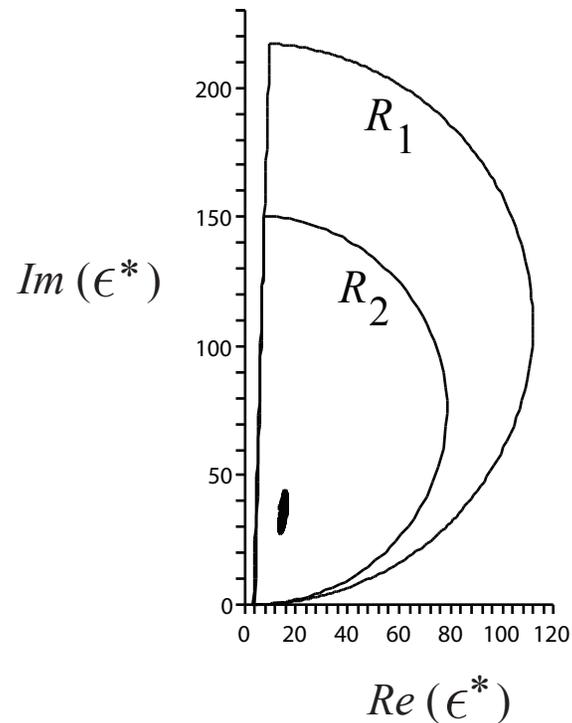
rigorous bounds, approximations, etc.

***Inverse Homogenization*** Cherkaev and Golden (1998), Cherkaev (2001)

$\epsilon^*$   ***composite geometry  
(spectral measure)***

recover brine volume fraction, connectivity, etc.

# forward and inverse bounds for sea ice



forward bounds

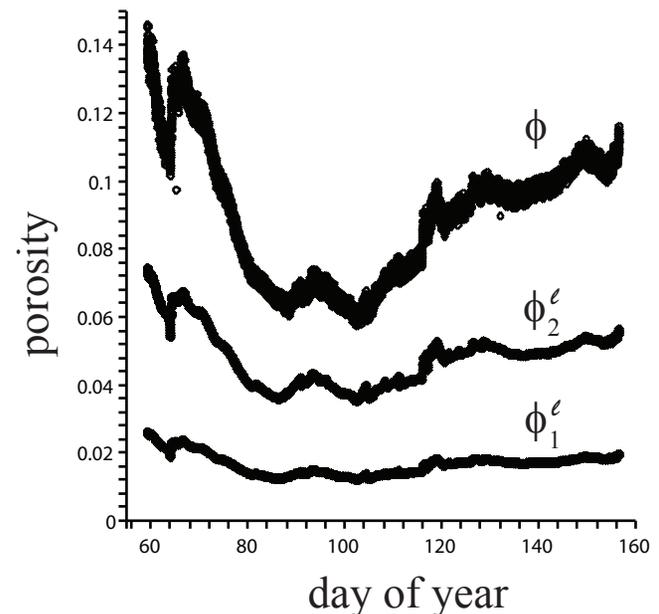
50 MHz capacitance probe  
data taken near Barrow, AK

inverse bounds and  
microstructural recovery

*Gully, Backstrom, Eicken, Golden, Physica B, 2007*

**recovery of inclusion separations**

**Orum, Cherkaev, Golden 2011**

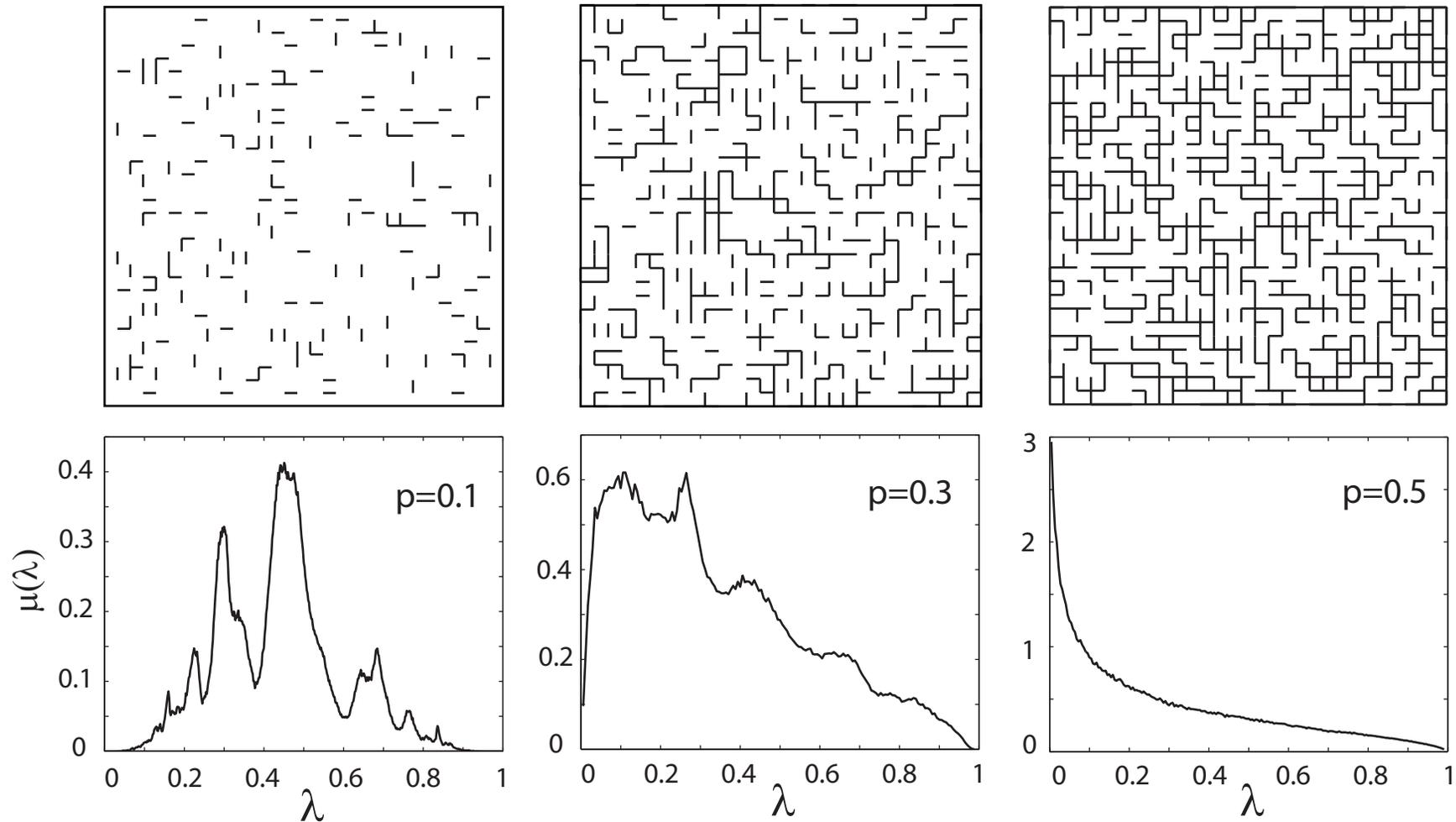


# ***Spectral analysis of multiscale sea ice structures***

***homogenization for brine inclusions, melt ponds, and sea ice pack***

***N. B. Murphy, C. Hohenegger, C. S. Sampson, B. Alali, K. Steffen,  
D. K. Perovich, H. Eicken, and K. M. Golden 2011***

## *spectral measures for 2-d random resistor network*

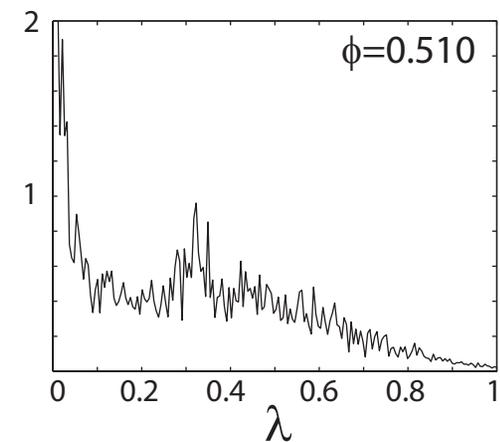
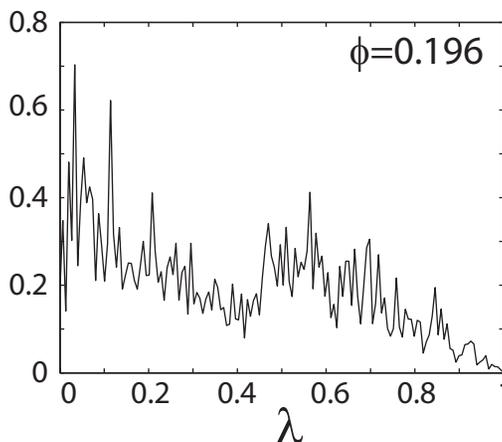
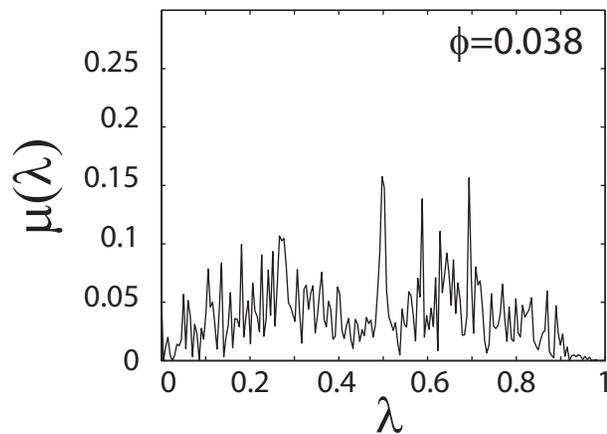
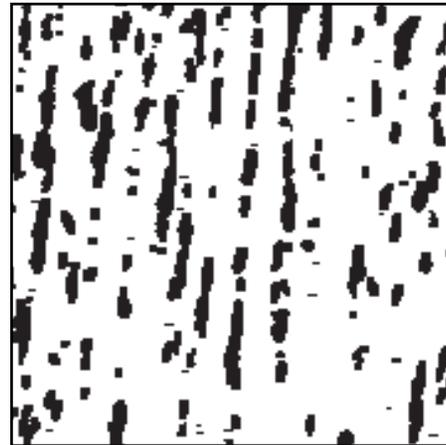


***area under curve =  $p$  = probability of open bond***

***spectral gap closes as percolation threshold is approached***

***random matrix theory calculations of eigenvalue spacing distributions help characterize transitions***

## *spectral measures for brine phase in sea ice*

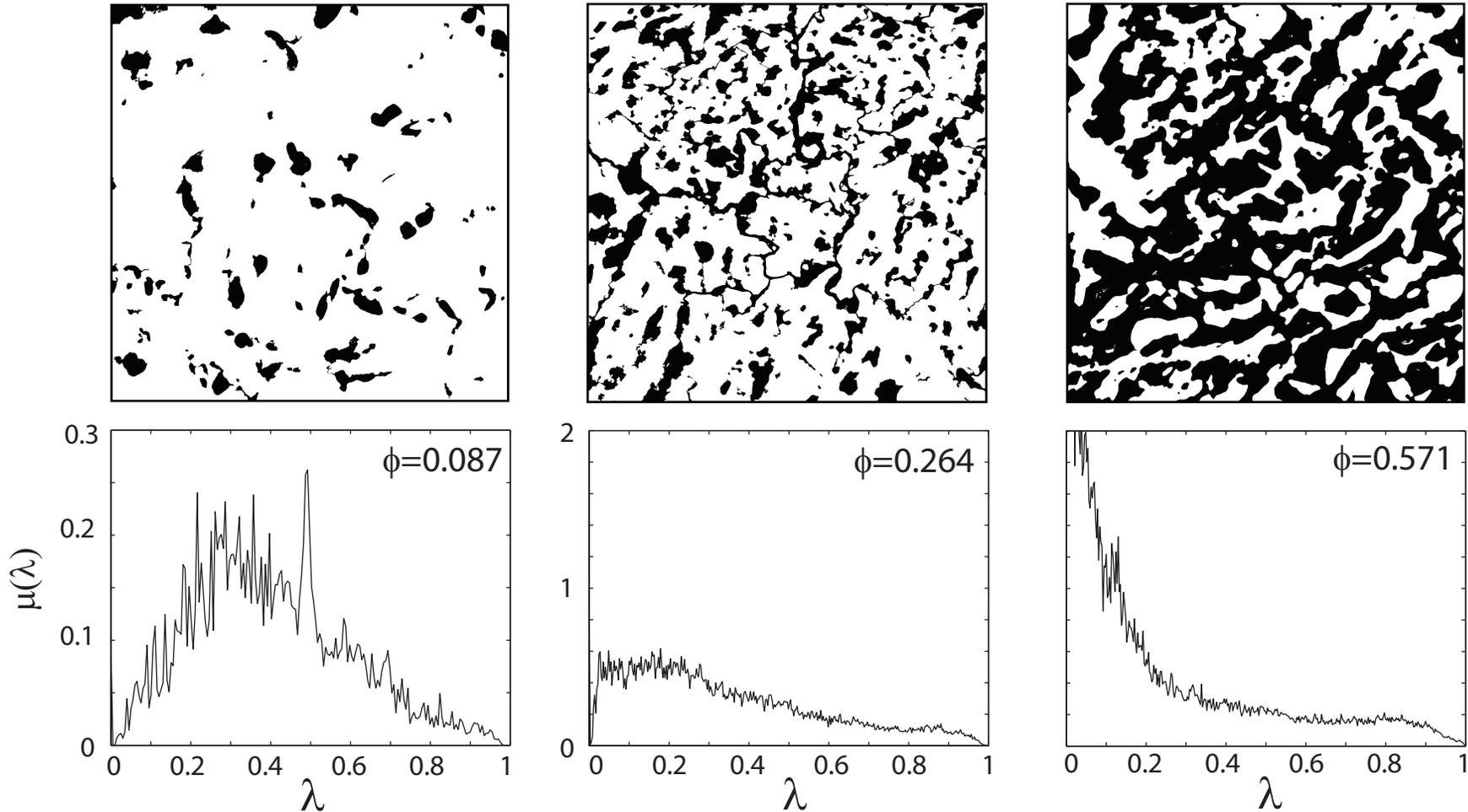


***area under curve =  $\phi$  = brine volume fraction***

***spectral gap closes as percolation threshold is approached***

resonance structure depends on inclusion geometry

## *spectral measures for Arctic melt ponds*

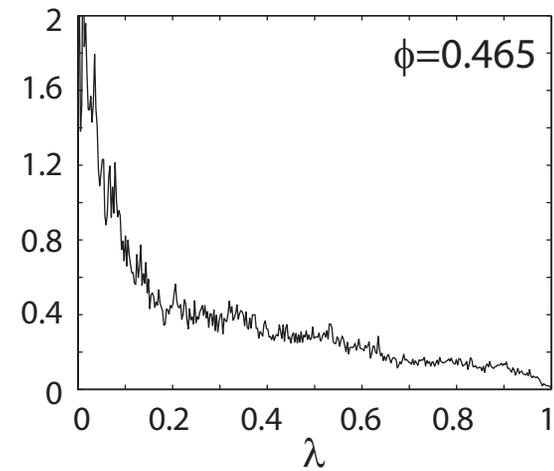
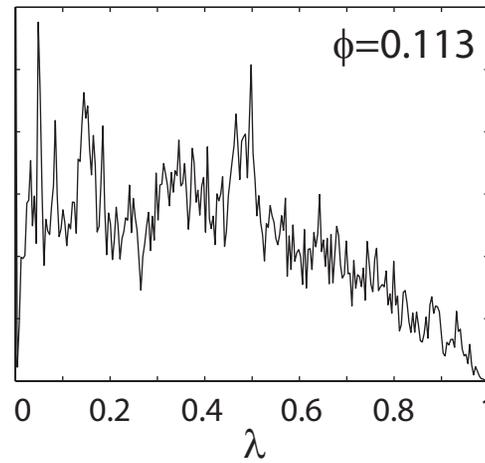
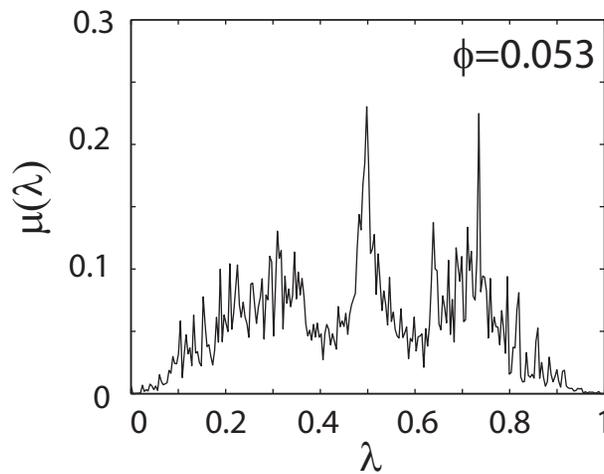
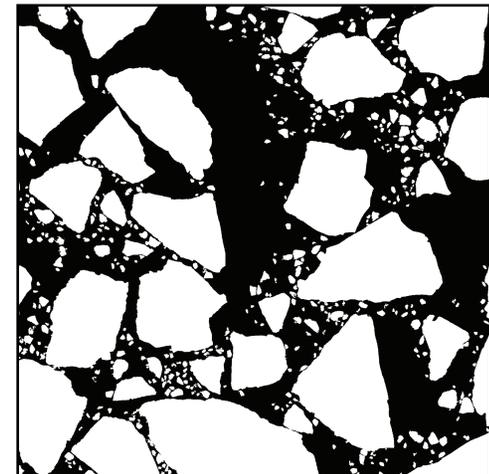
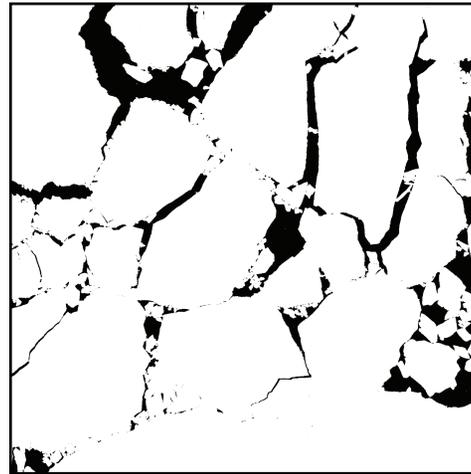
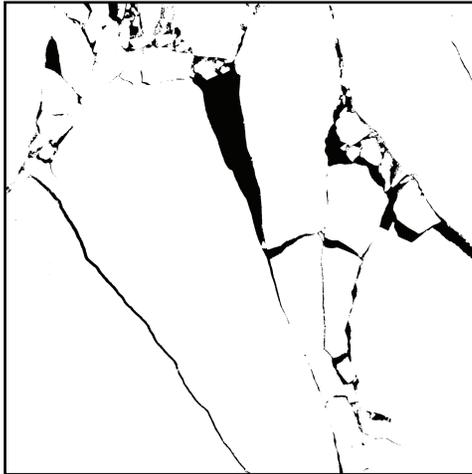


***area under curve =  $\phi$  = melt pond area fraction***

***spectral gap closes as melt ponds become connected***

resonance structure depends on pond geometry

## *spectral measures for Arctic sea ice pack*



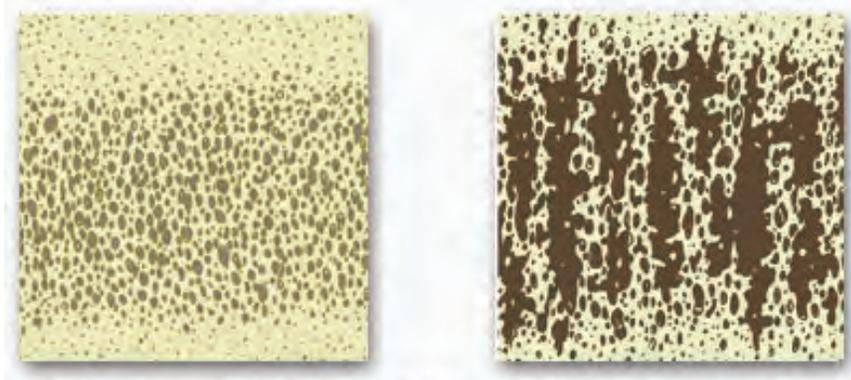
***area under curve =  $\phi$  = open water fraction***

***spectral gap closes as ocean phase becomes connected***

# Inverse Homogenization for Evaluation of Bone Structure

Carlos Bonifasi-Lista, Elena Cherkaev, 2006

bone porosity can be estimated from the effective torsional modulus  
using reconstruction of spectral measure



normal

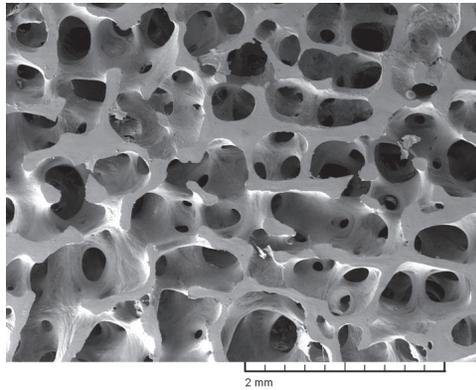
osteoporotic

*Application:*  
*Monitoring osteoporosis*

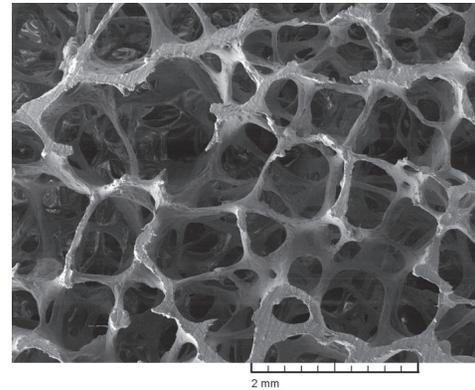
*the math doesn't care if it's sea ice or bone*

# spectral characterization of porous microstructures in bone

(a) young healthy trabecular bone

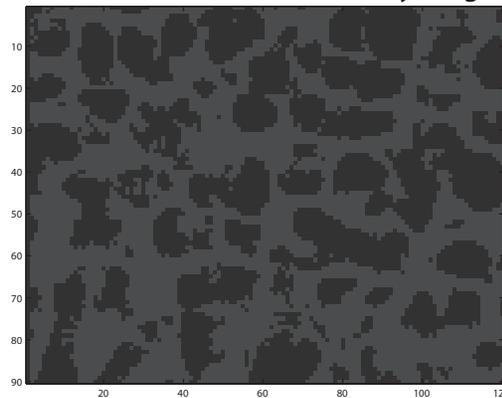


(b) old osteoporotic trabecular bone

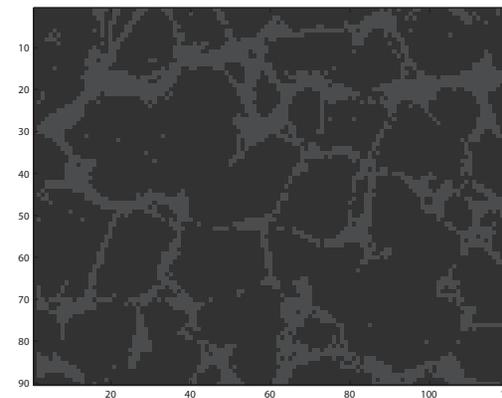


P. Hansma

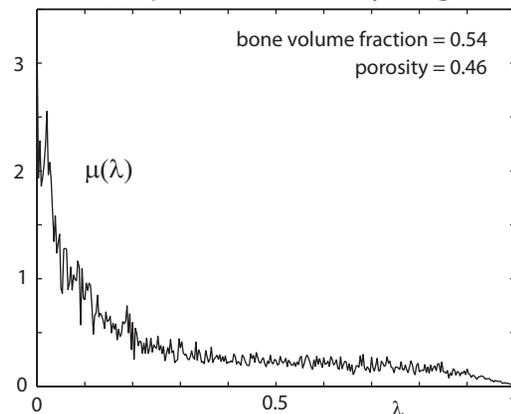
(c) 90 x 120 discretization - young



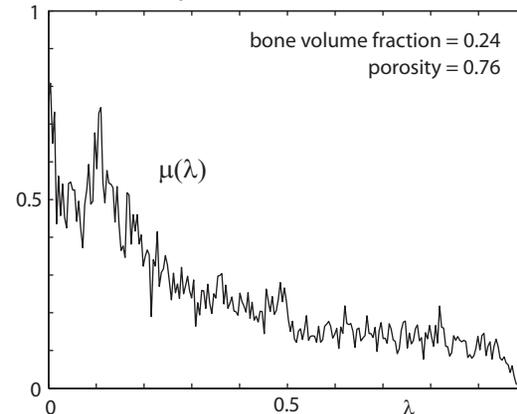
(d) 90 x 120 discretization - old



(e) spectral measure - young



(f) spectral measure - old

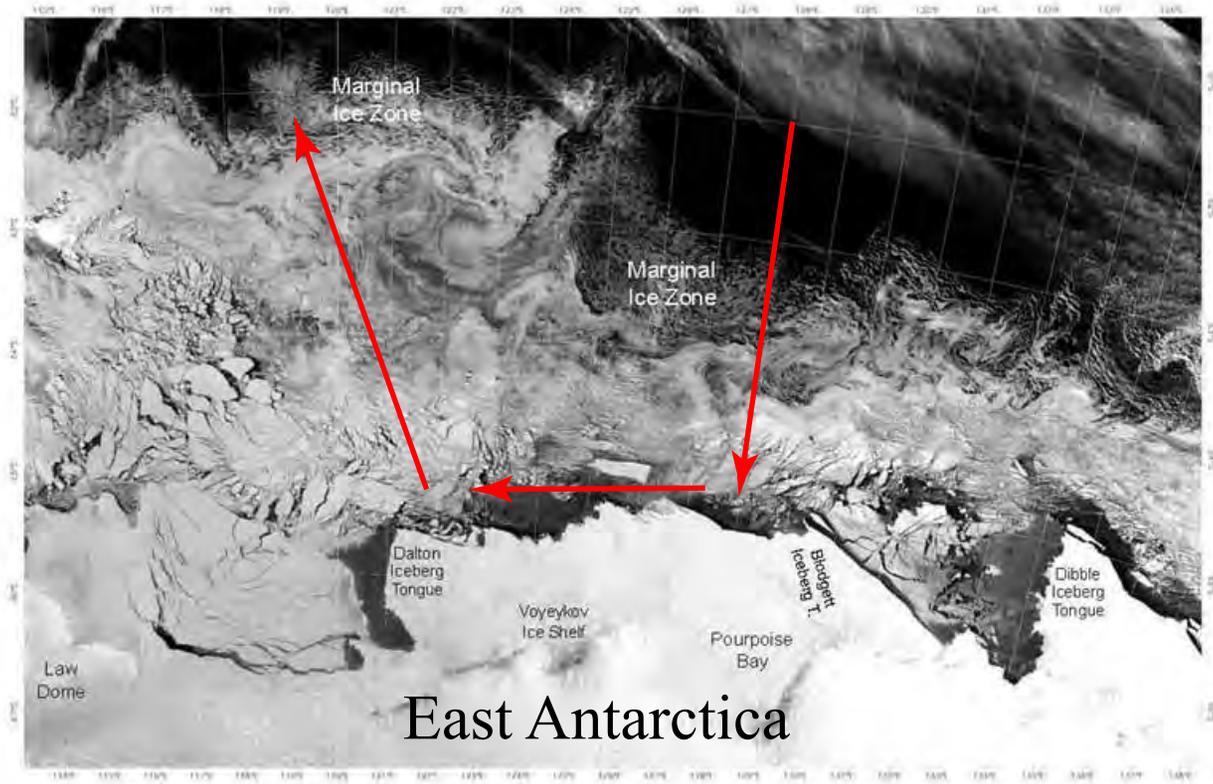


in sea ice, porosity changes significantly  
with small changes in temperature

***if bone behaved similarly, you could get  
osteoporosis from a fever!***

# Sea Ice Physics and Ecosystem eXperiment (SIPEX)

4 September - 17 October 2007



Australian Government  
Department of the Environment  
and Water Resources  
Australian Antarctic Division

**International Polar Year (IPY) 2007-2008**



ANTARCTIC CLIMATE  
& ECOSYSTEMS  
COOPERATIVE RESEARCH CENTRE

# Measuring electrical and fluid transport during SIPEX

with Adam Gully

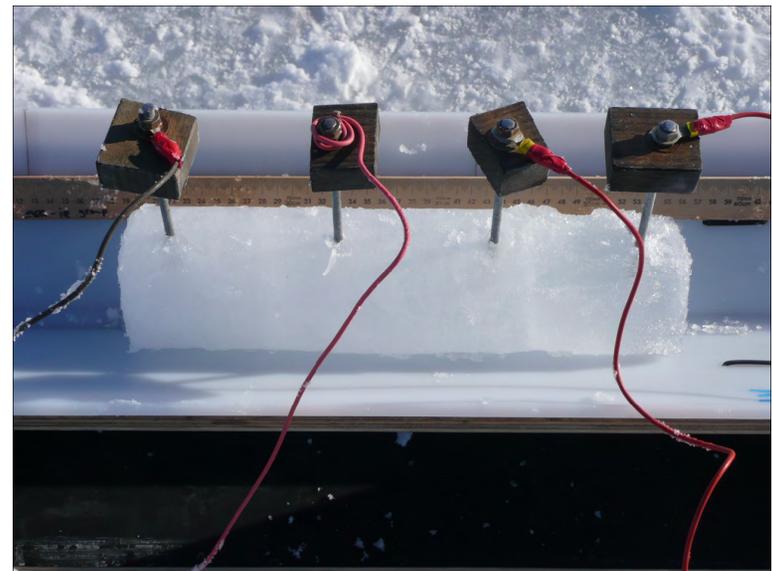
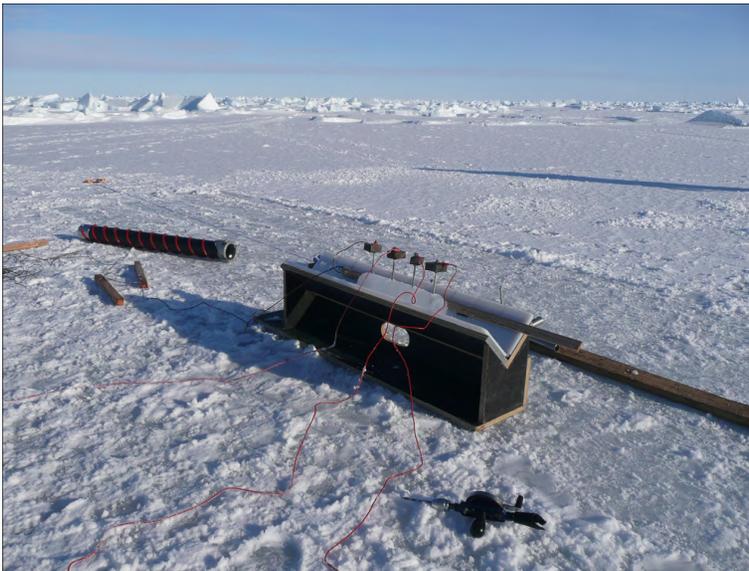
**electrical properties** --- help monitor **fluid** transport processes  
--- important in **thickness** measurements, e.g. EMI

1. **Direct measurements of DC vertical conductivity** from extracted sea ice cores --- first experiments of this type
2. **Surface impedance tomography** with Wenner array --- invert DC resistance data to reconstruct profile of electrical properties
3. **Fluid permeability** --- first measurements in Antarctic pack ice
4. **Tracer studies** --- pour fluids through inverted sea ice blocks

## electrical measurements



## Wenner array



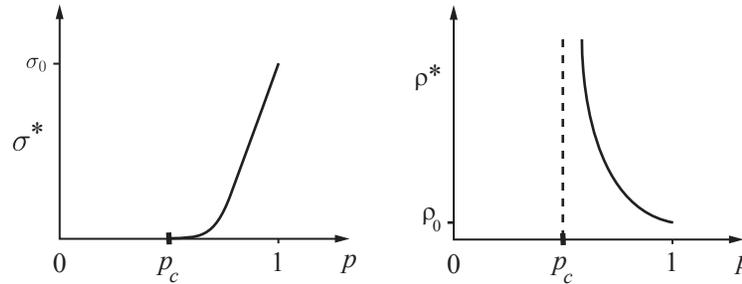
## vertical conductivity

Zhu, Golden, Gully, Sampson *Physica B* 2010

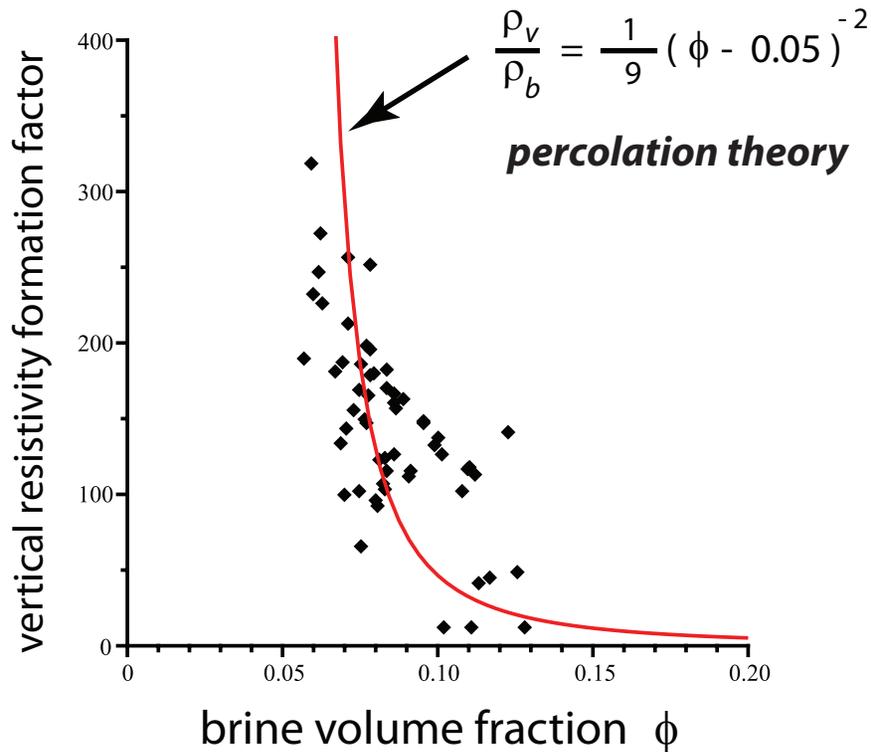
Sampson, Golden, Gully, Worby *Deep Sea Research* 2010

# critical behavior of electrical transport in sea ice

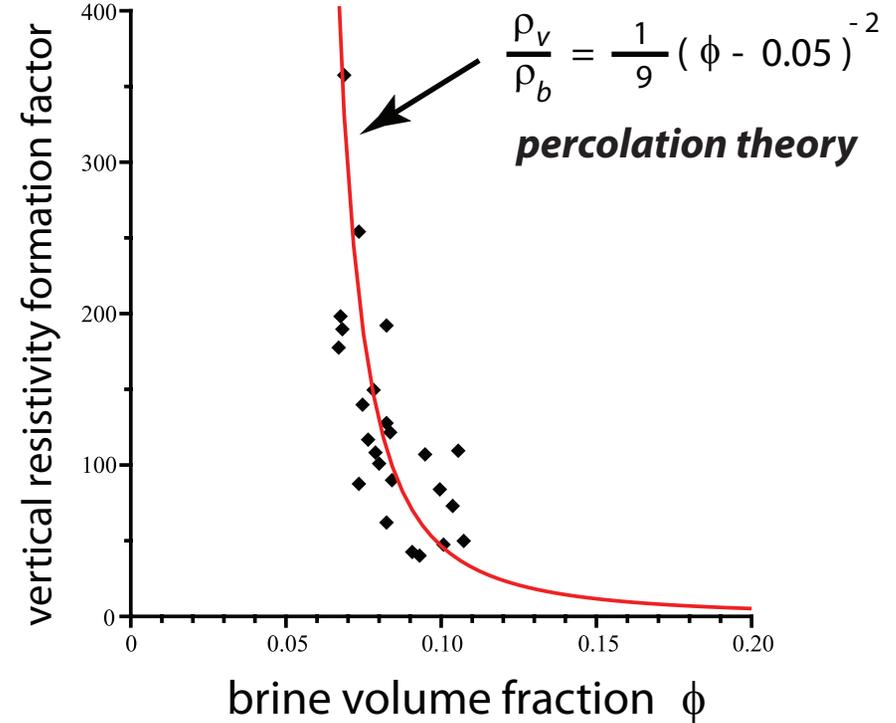
## electrical signature of the on-off switch for fluid flow



### Arctic



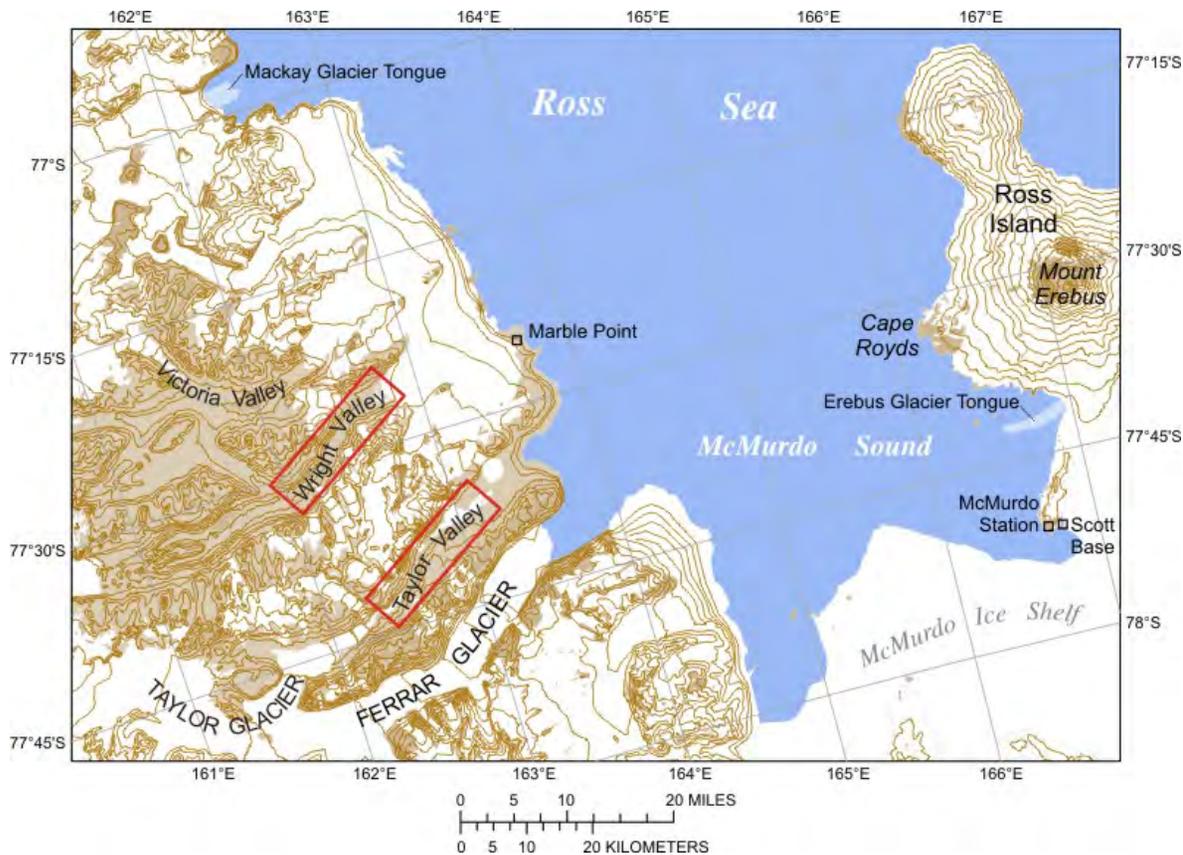
### Antarctic



# 2010 Antarctic expedition to a remote NZ field camp in McMurdo Sound

Ken Golden, Joyce Lin, Cindy Furse (ECE), David Lubbers (ECE)

measure fluid, electromagnetic, and microstructural properties of sea ice  
collaborate with our NZ colleagues in cross-borehole tomography

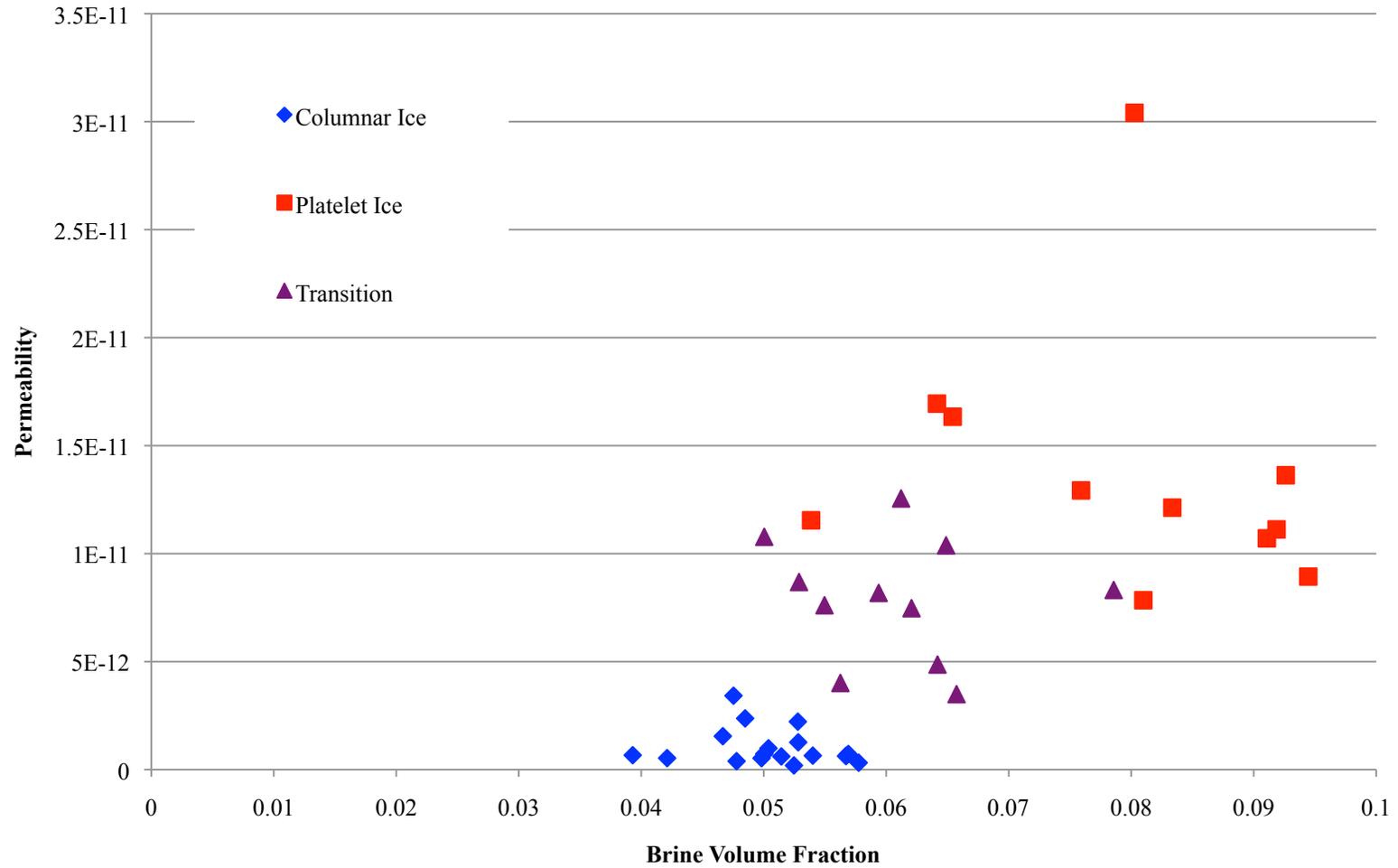


## ***New Zealand's Scott Base***

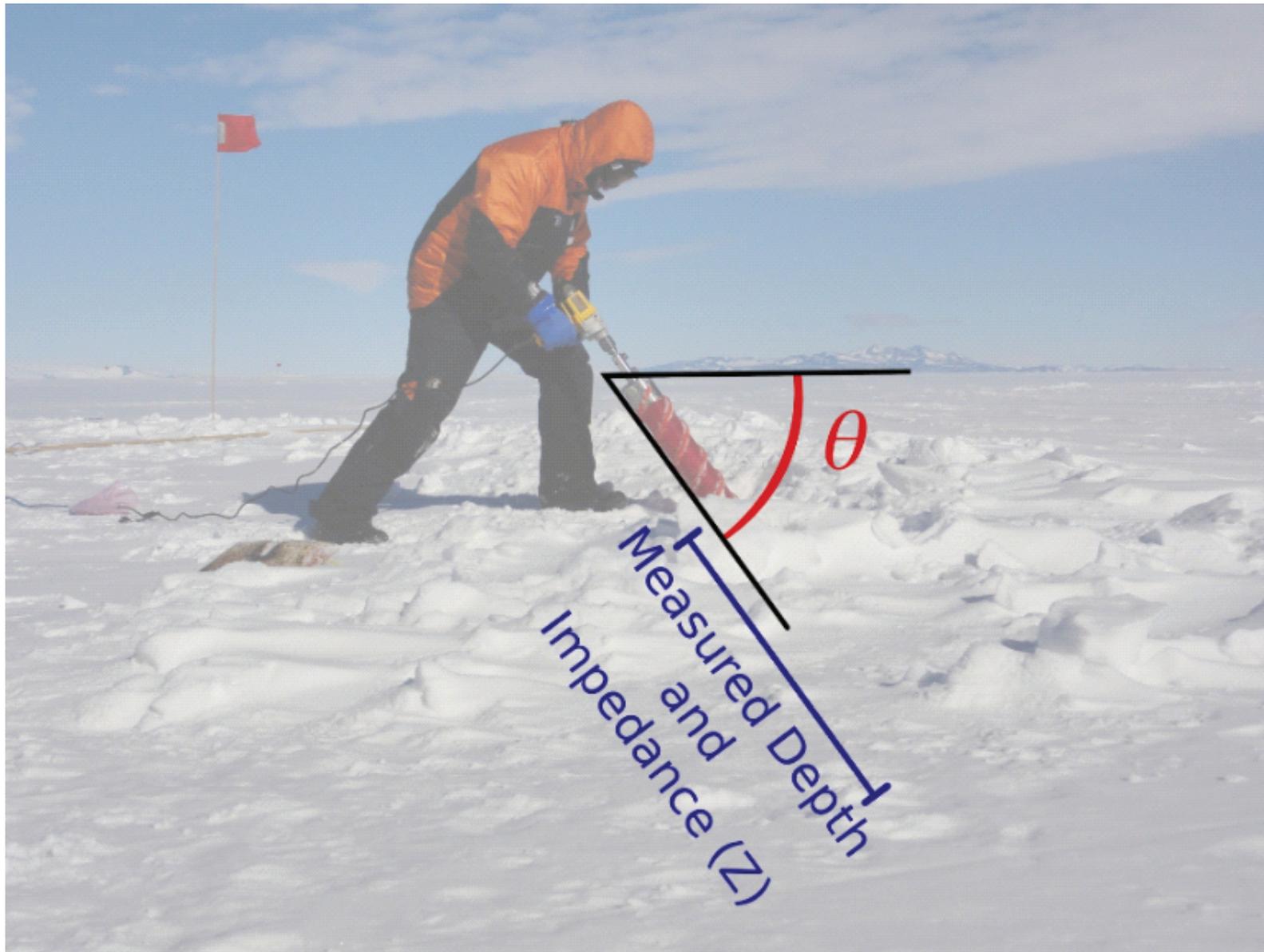




# Vertical Permeability



***“crooked core” method to measure components of anisotropic conductivity tensor***





## *preparing for an Antarctic expedition*

(can't run to Home Depot if something breaks or is forgotten!)



Math Department parking lot



Scott Base, Antarctica

***“two is one, one is none”***

# Conclusions

1. Polar ice is melting. *Critical behavior* a natural part of system.
2. Melt ponds exhibit a space filling transition as they grow and coalesce.
3. *Brine flow* through sea ice is a key to geophysics and biology of polar regions, and displays critical behavior.
4. Developed comprehensive theory of fluid permeability and electrical conductivity, using models from statistical physics, and measured these properties in the Arctic and Antarctic.
5. Developed idea of *spectral upscaling* for sea ice structures.
6. Sea ice processes such as melt pond evolution, snow-ice formation, nutrient flux can be modeled more realistically, and monitored electromagnetically.

*our permeability results currently being used to improve how climate models treat evolution of salinity profiles and melt ponds  
e.g. Los Alamos CICE -> NCAR CCSM*

# ***THANK YOU***

## **National Science Foundation**

Division of Mathematical Sciences

Arctic Natural Sciences

Office of Polar Programs

CMG Program

(Collaboration in Mathematical Geosciences)

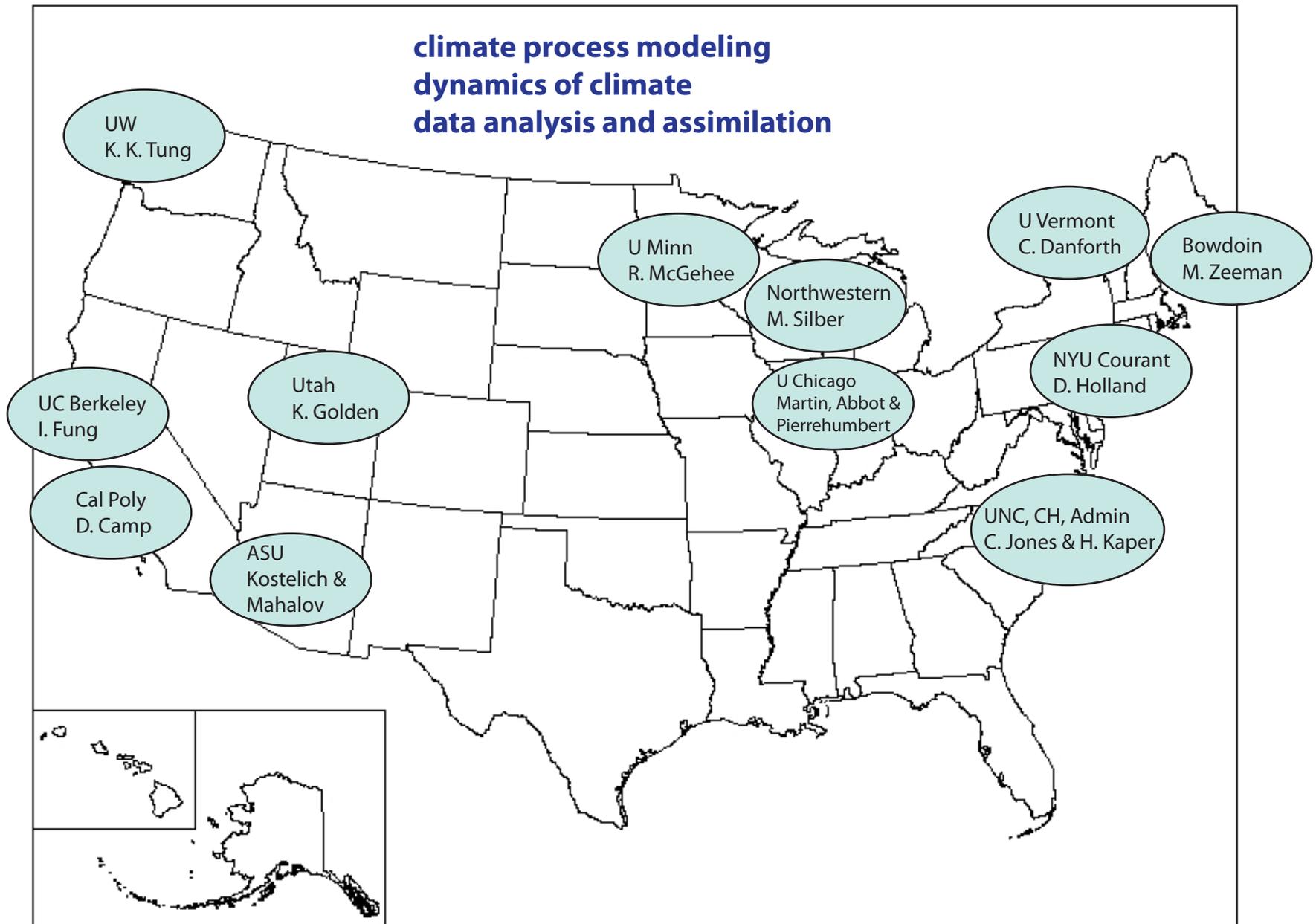
VIGRE Program

REU Program



***Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999***

# Mathematics and Climate Change Research Network



**NSF DMS 2010-2015, Lorenz postdocs, grad, undergrad, polar expeditions**

**Jones, Golden, Kaper, Zeeman**

# Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and  
the Mathematics of  
Transport in Sea Ice

page 562

Mathematics and the  
Internet: A Source of  
Enormous Confusion  
and Great Potential

page 586



*Real analysis in polar coordinates (see page 613)*

Mathematics Awareness Month - April 2009

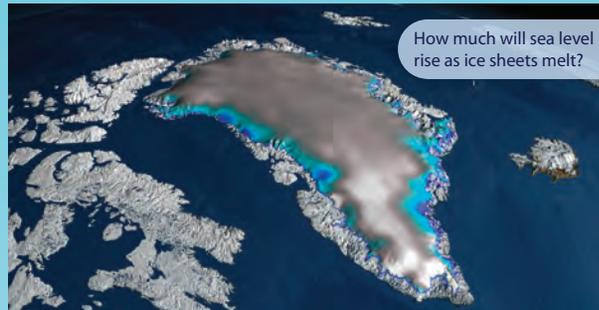
# Mathematics and Climate

Find out how math and science are used to address questions of climate change:

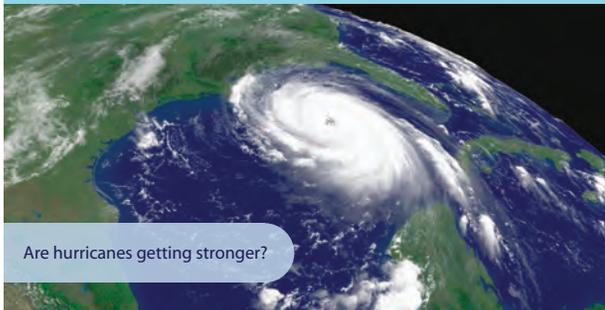
How long will the summer Arctic sea ice pack survive?



How much will sea level rise as ice sheets melt?



Are hurricanes getting stronger?



How do human activities impact global warming?



How is climate monitored on a global scale?



How can we improve our understanding of climate change and what can we do about it?



$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \mathbf{F} + \frac{\mu}{\rho} \nabla^2 \mathbf{u}$$
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

[www.mathaware.org](http://www.mathaware.org)

Committee  
Chair: Kenneth Golden (University of Utah)

Kerry Emanuel (MIT)  
Margot Gerritsen (Stanford)  
Jon Huntsman, Jr. (Governor of Utah)  
Mary Lou Zeeman (Bowdoin)

Inez Fung (UC Berkeley)  
David Holland (NYU)  
David Neelin (UCLA)  
Jay Zwally (NASA)

# Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship Aurora Australis without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 186km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

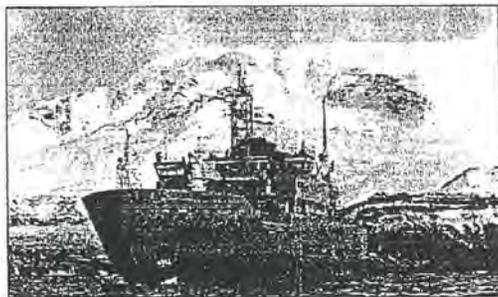
The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the Aurora Australis in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The Aurora Australis is owned by P&O Australia and chartered by the Antarctic Div-

ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

## Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire,

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

### THE CANBERRA TIMES

Thursday 23 July 1998

Page 4

## Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker Aurora Australis in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected Aurora Australis would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The Aurora, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room ..."

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats ..."

Sydney Morning Herald  
23 July, 1998

#### ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the Aurora Australis yesterday. The 54 people on board were forced on deck in the

