Nine

I $f: \mathbb{R}^n \to \mathbb{R}$

II graphs

III level curves and contour maps

IV level surfaces

I $f : \mathbb{R}^n \to \mathbb{R}$

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R} \quad , \quad f(x,y) = \frac{3x-y}{xy}$$

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$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$, $f(x,y) = \dfrac{3x-y}{xy}$

$$f(2,5) = \frac{3\cdot 2-5}{2\cdot 5} = \frac{6-5}{10} = \frac{1}{10}$$

$g : \mathbb{R}^2 \longrightarrow \mathbb{R}$, $g(x,y) = \cos x \sin y - \dfrac{x+y}{\pi}$

$$g\left(0,\frac{\pi}{2}\right) =$$

$h : \mathbb{R}^3 \longrightarrow \mathbb{R}$, $h(x,y,z) = x^2 z + 2yz - 4$

$$h(2,-1,3) =$$

$$f:\mathbb{R}^2\longrightarrow\mathbb{R}\quad,\quad f(x,y)=\frac{3x-y}{xy}$$

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$$g\left(0,\frac{\pi}{2}\right)=\cos 0\,\sin\frac{\pi}{2}-\frac{0+\frac{\pi}{2}}{\pi}=1-\frac{1}{2}=\frac{1}{2}$$

$$h:\mathbb{R}^3\longrightarrow\mathbb{R}\quad,\quad h(x,y,z)=x^2z+2yz-4$$

$$h(2,-1,3)=2^2\cdot 3+2\cdot(-1)\cdot 3-4=12-6-4=2$$

The **domain** of $f : \mathbb{R}^n \to \mathbb{R}$ are those $p \in \mathbb{R}^n$ for which $f(p)$ makes sense.

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Domain of $g(x,y) = \frac{x}{y}$

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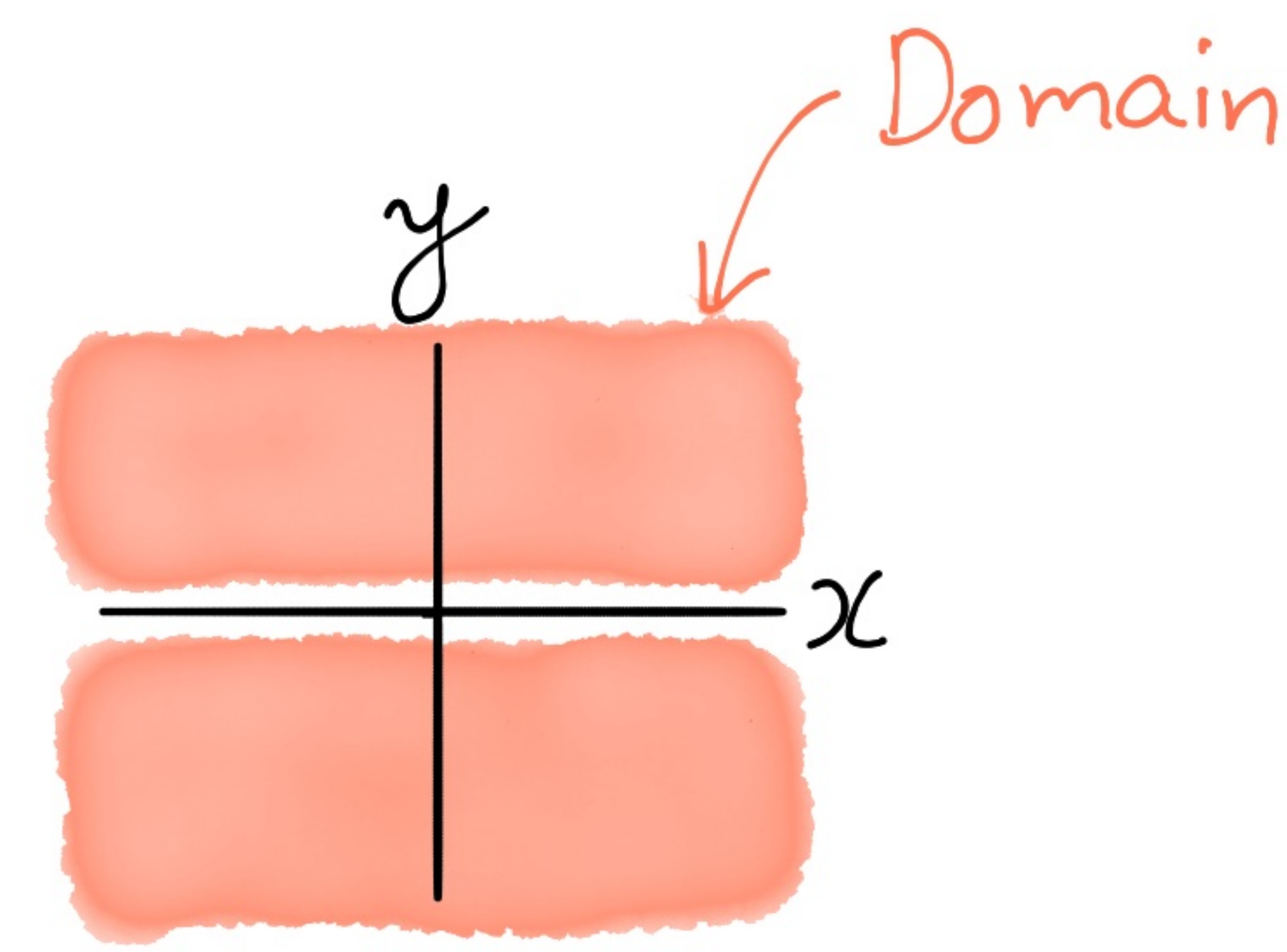
Domain of $g(x,y) = \dfrac{x}{y}$

Any point $(x,y) \in \mathbb{R}^2$ for which $y \neq 0$.



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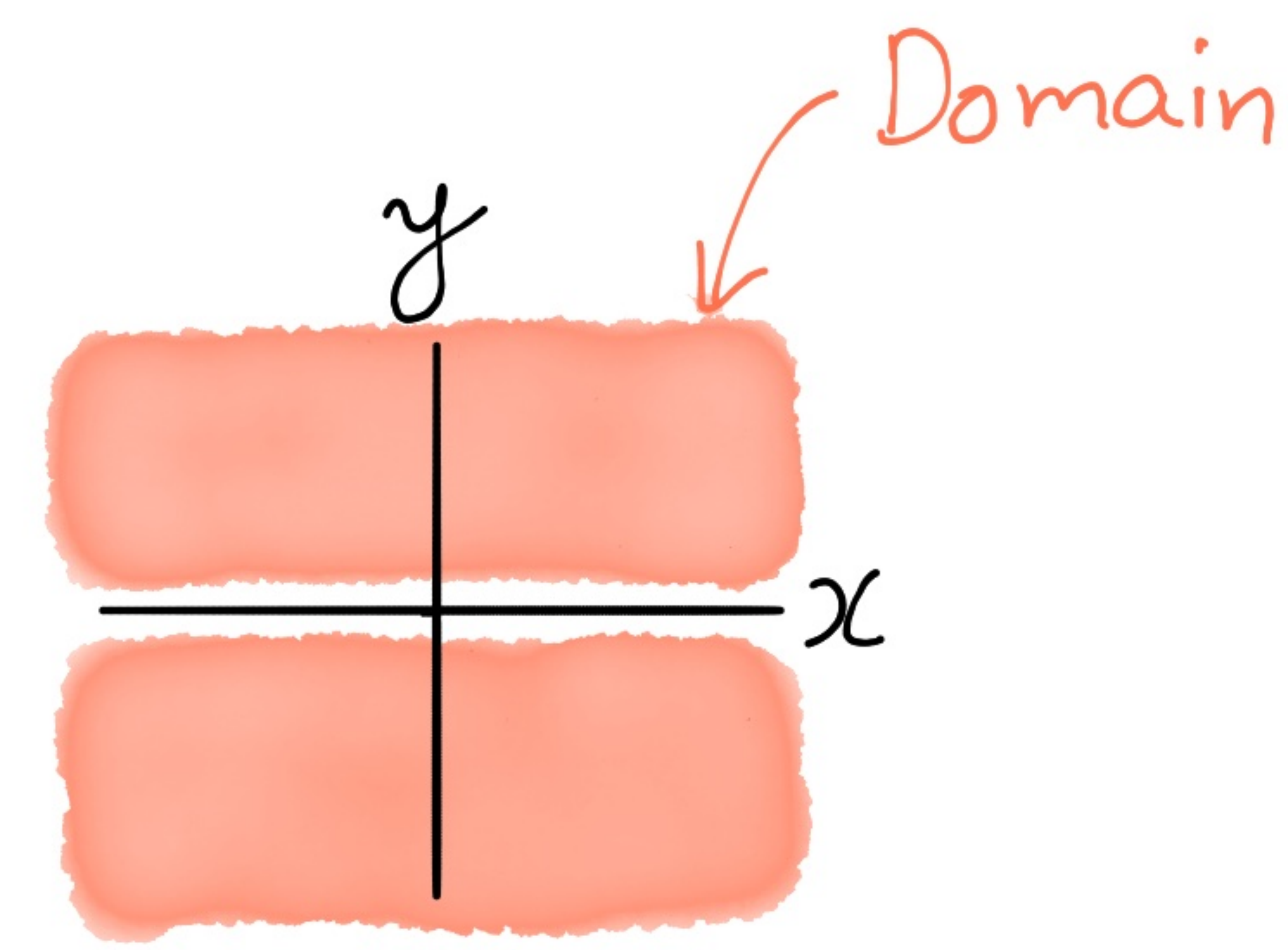
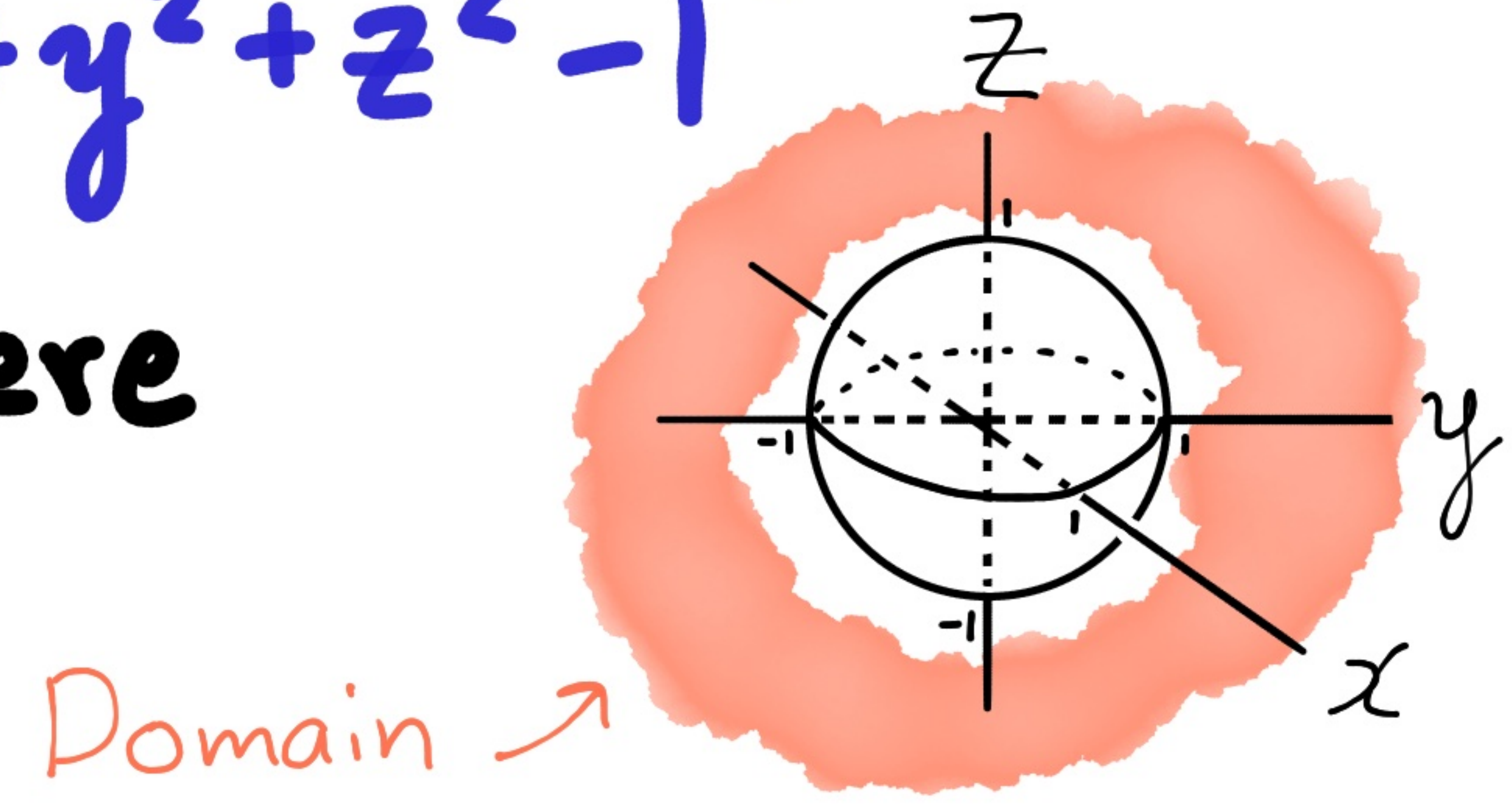


Domain of $f(x,y,z) = \sqrt{x^2+y^2+z^2-1}$

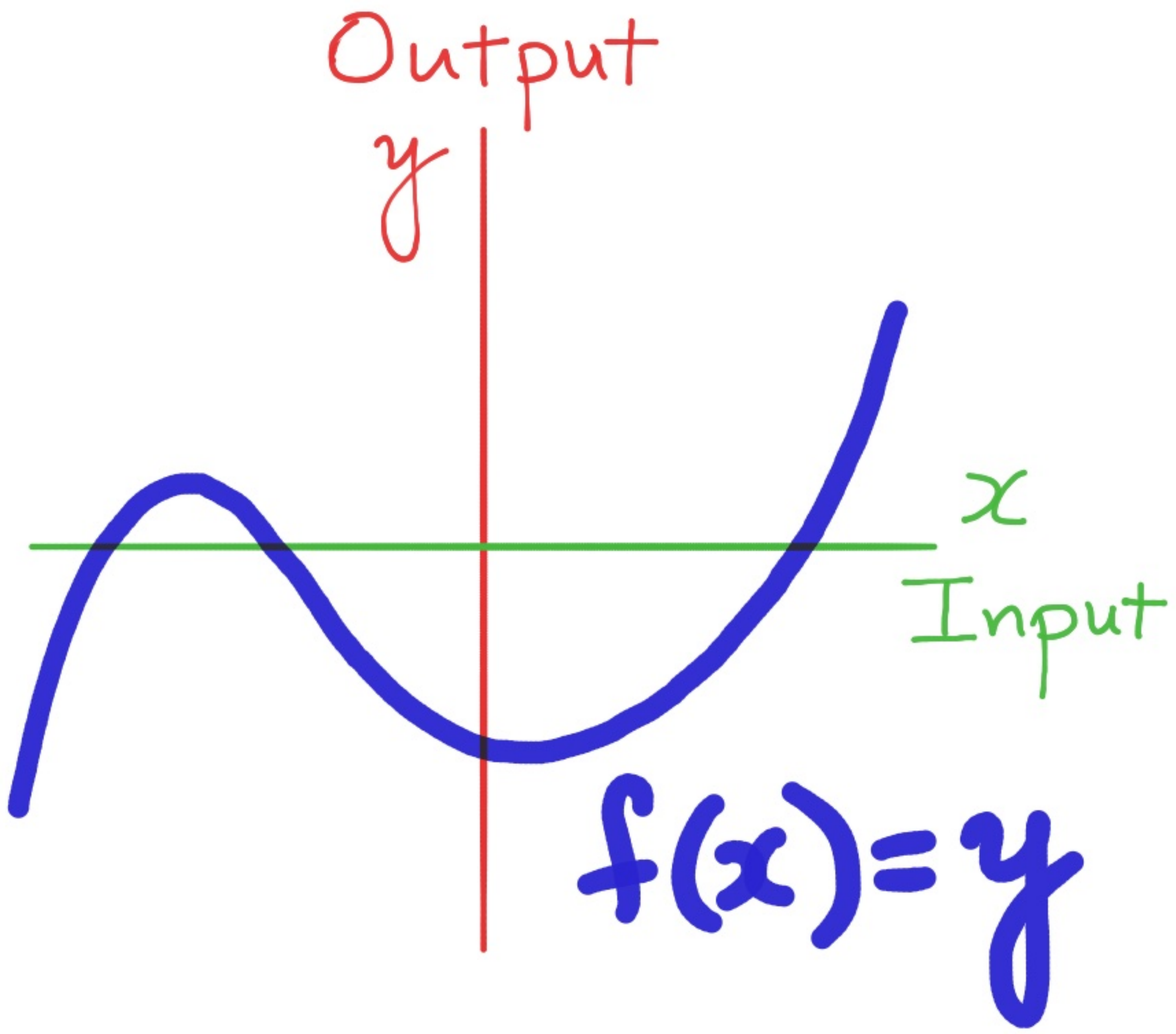
Any point $(x,y,z) \in \mathbb{R}^3$ where

$x^2+y^2+z^2 \geq 1$, $\|(x,y,z)\| \geq 1$.

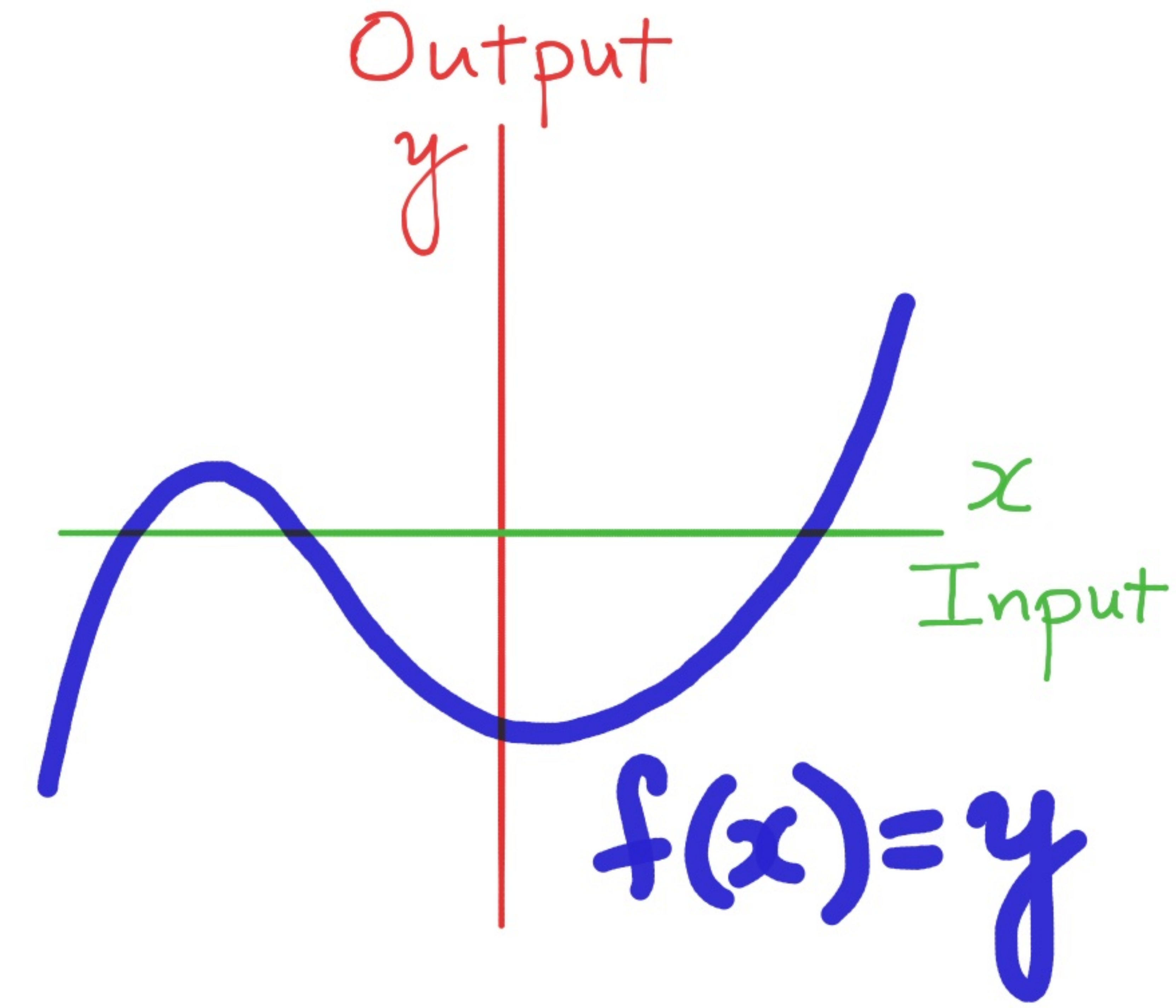


II Graphs

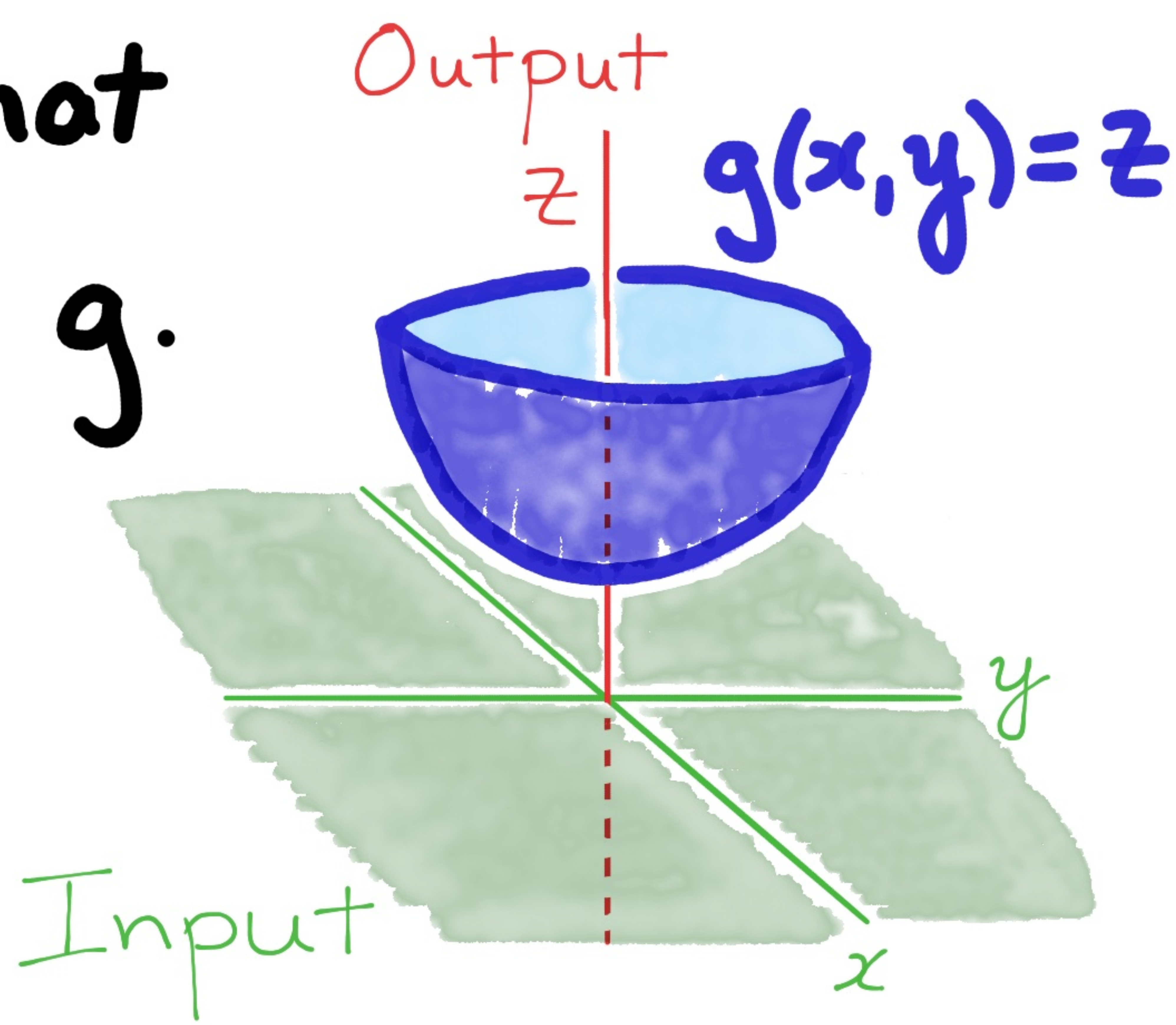
Let $f:\mathbb{R}\to\mathbb{R}$. The set of all $(x,y)\in\mathbb{R}^2$ such that $f(x)=y$ is the graph of f.



Let $f: \mathbb{R} \to \mathbb{R}$. The set of all $(x,y) \in \mathbb{R}^2$ such that $f(x)=y$ is the **graph** of f.

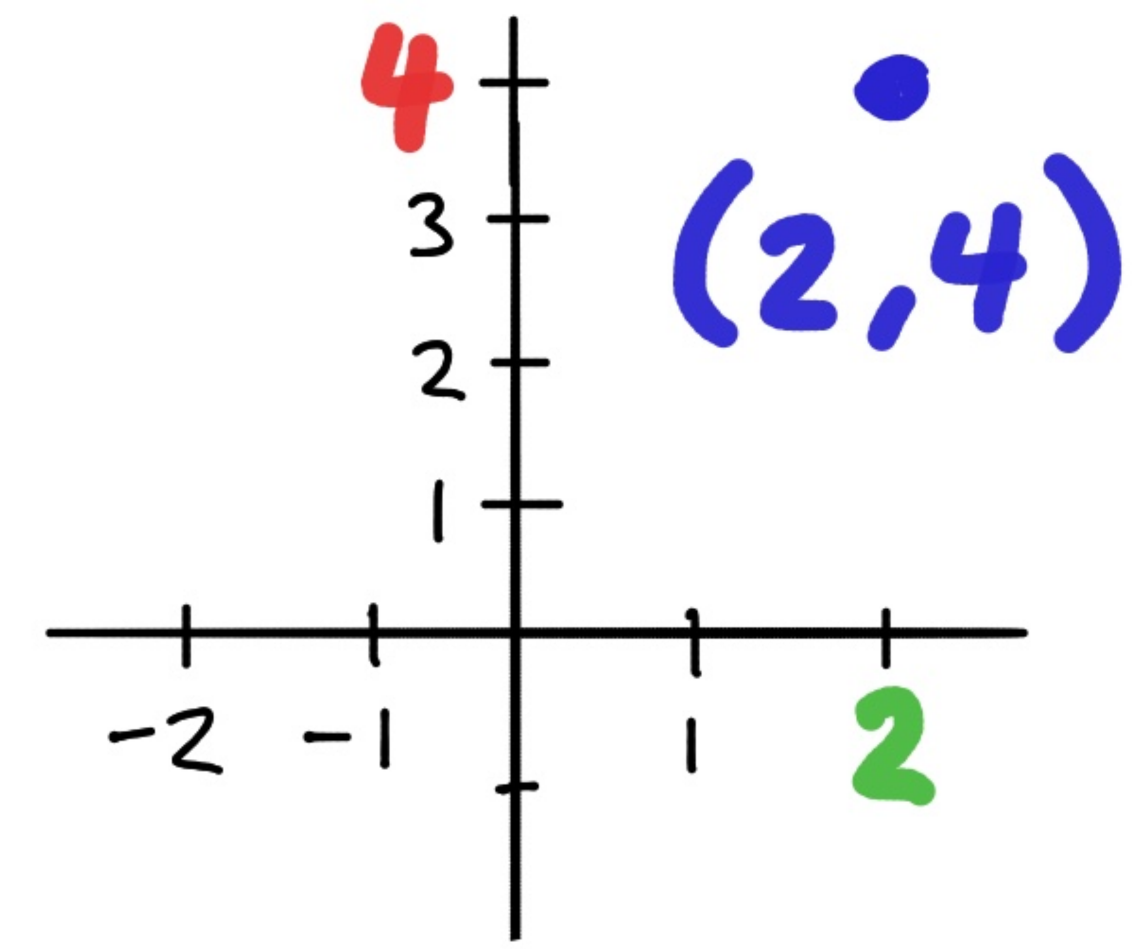


Let $g: \mathbb{R}^2 \to \mathbb{R}$. The set of all $(x,y,z) \in \mathbb{R}^3$ such that $g(x,y)=z$ is the **graph** of g.



Let $f: \mathbb{R} \to \mathbb{R}$ be $f(x) = x^2$.

$x = 2$ gives the point $(2, 4)$ in the graph of f.



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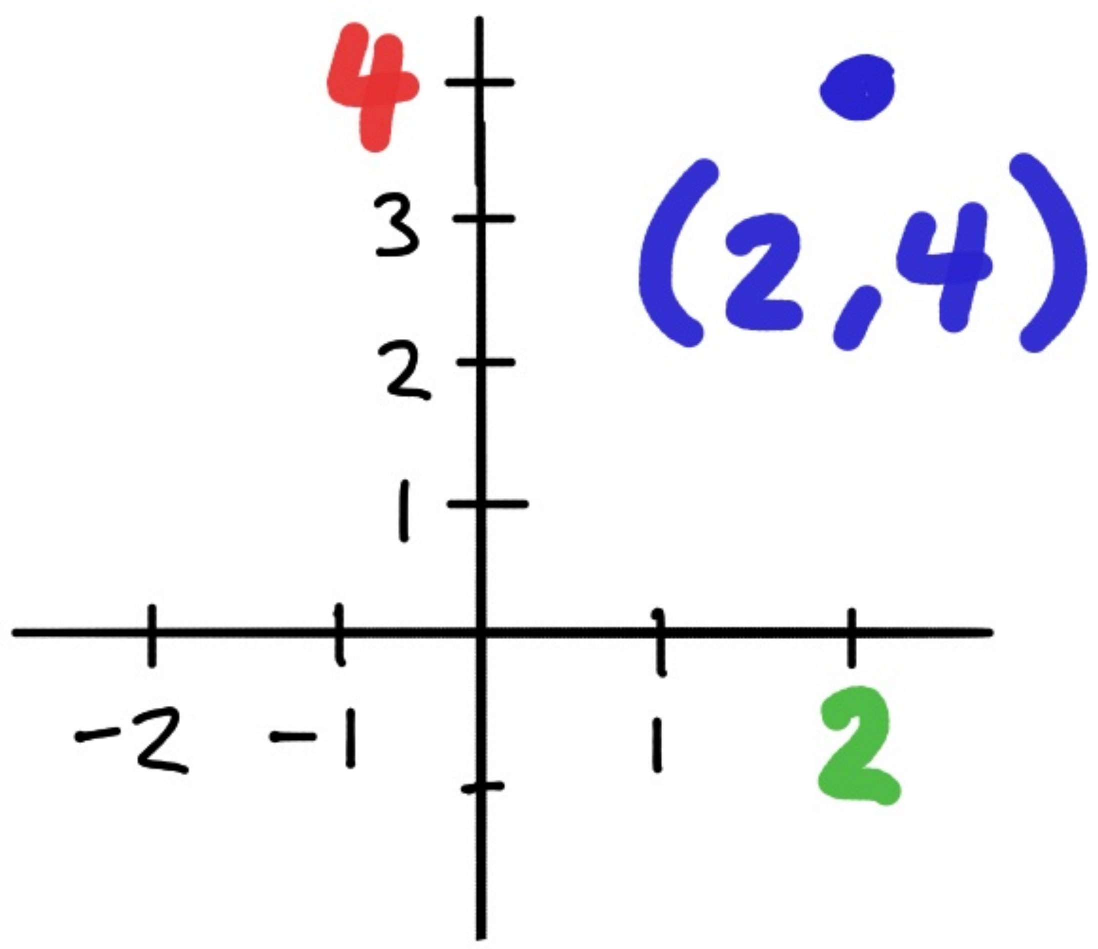
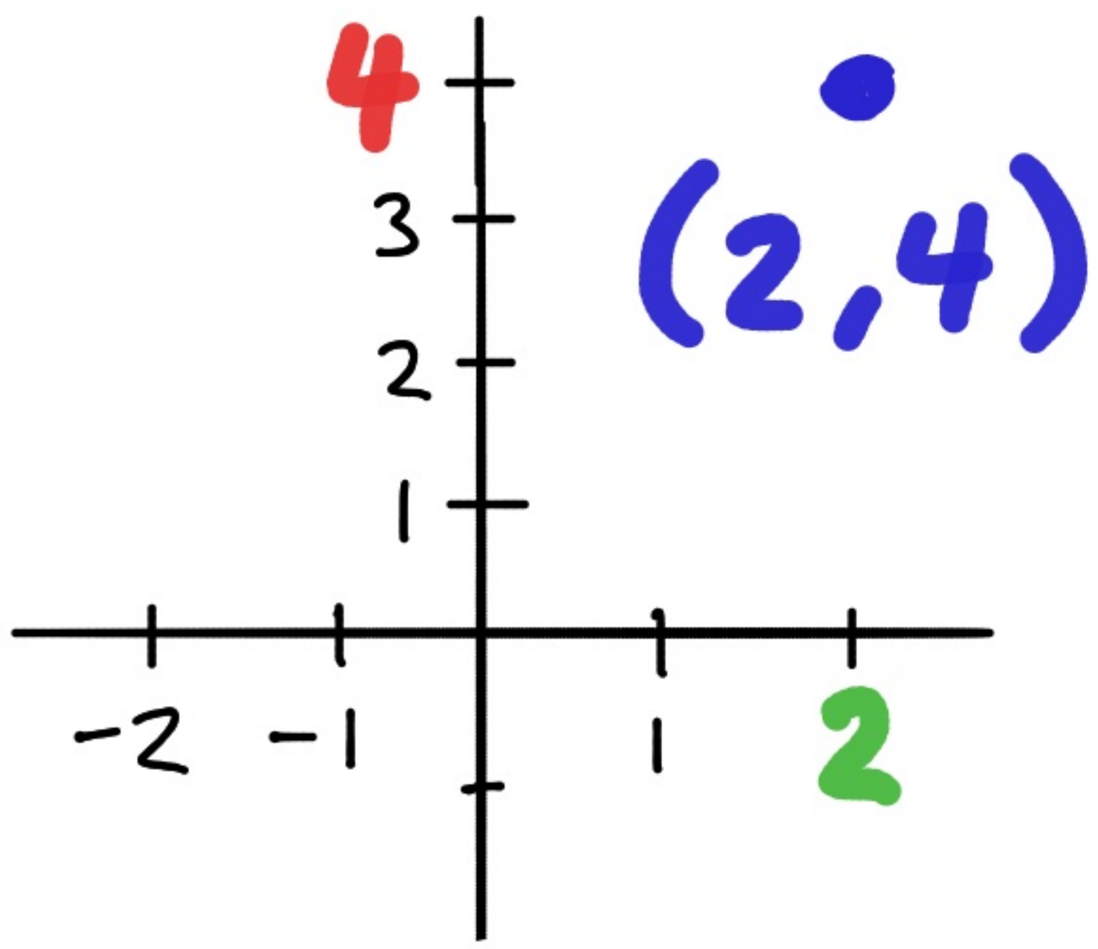


Let $g: \mathbb{R}^2 \to \mathbb{R}$ be $g(x,y) = -xy$. Which point in the graph of g corresponds to the input $(2,-1) \in \mathbb{R}^2$?

Let $f:\mathbb{R}\to\mathbb{R}$ be $f(x)=x^2$.

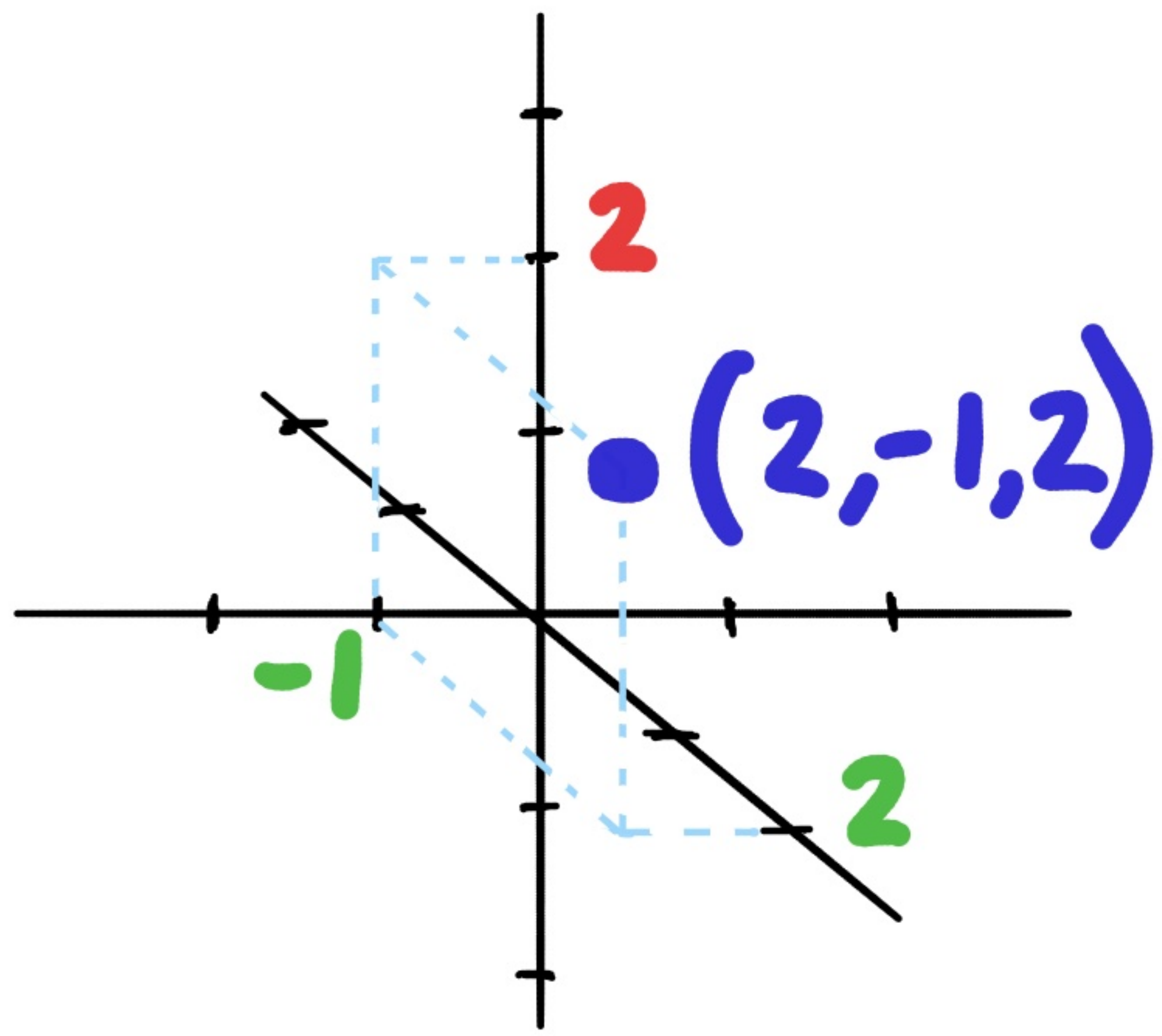
$x=2$ gives the point $(2,4)$ in the graph of f.



Let $g:\mathbb{R}^2\to\mathbb{R}$ be $g(x,y)=-xy$.

Which point in the graph of g corresponds to the input $(2,-1)\in\mathbb{R}^2$?

$$(2,-1,-2(-1))$$



Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

Points in graph: $(x, y, x^2 + y^2)$

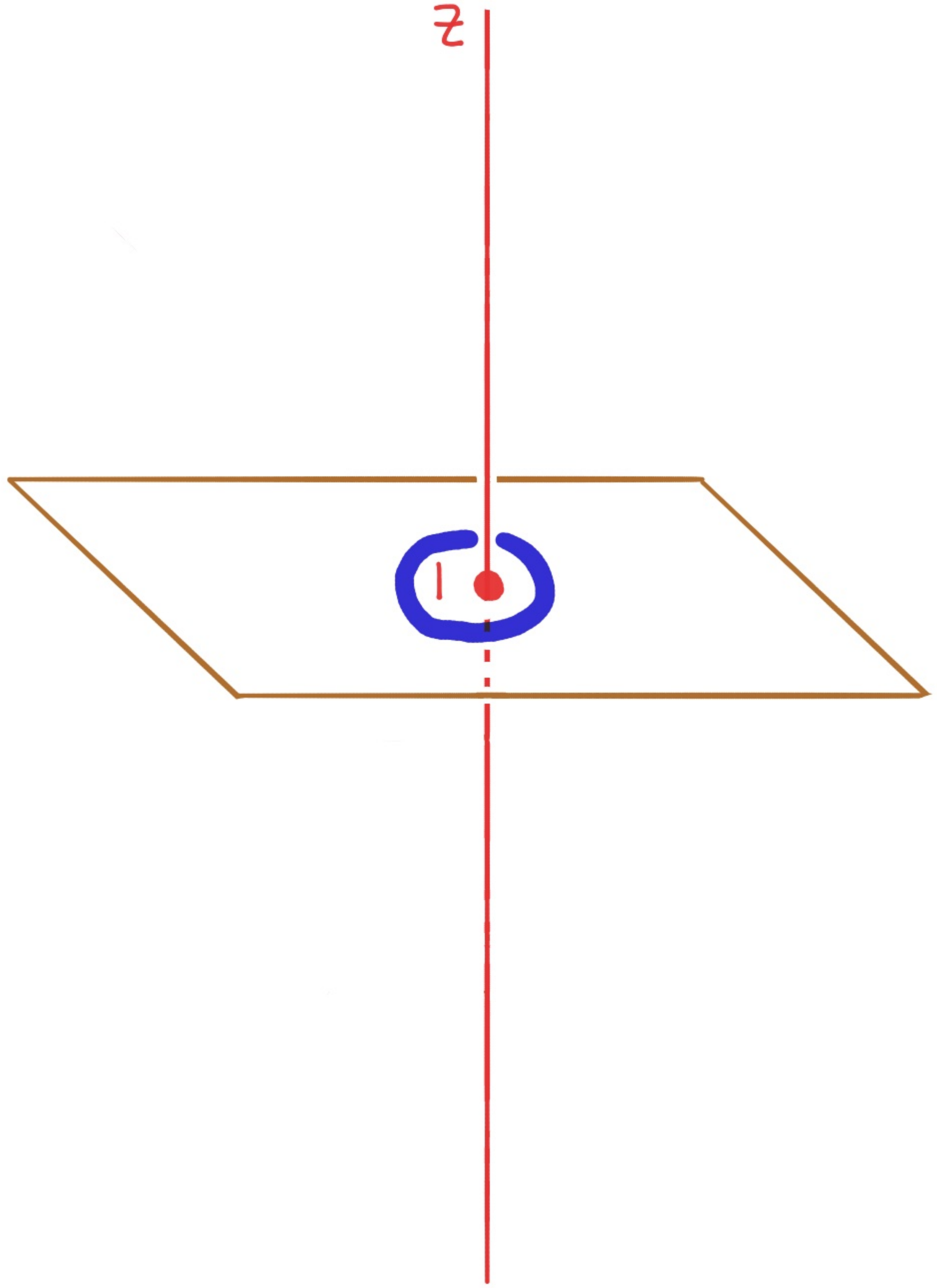
$(1, 1, 2)$ $(2, 0, 4)$

$(0, -1, 1)$ $(-3, 2, 13)$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

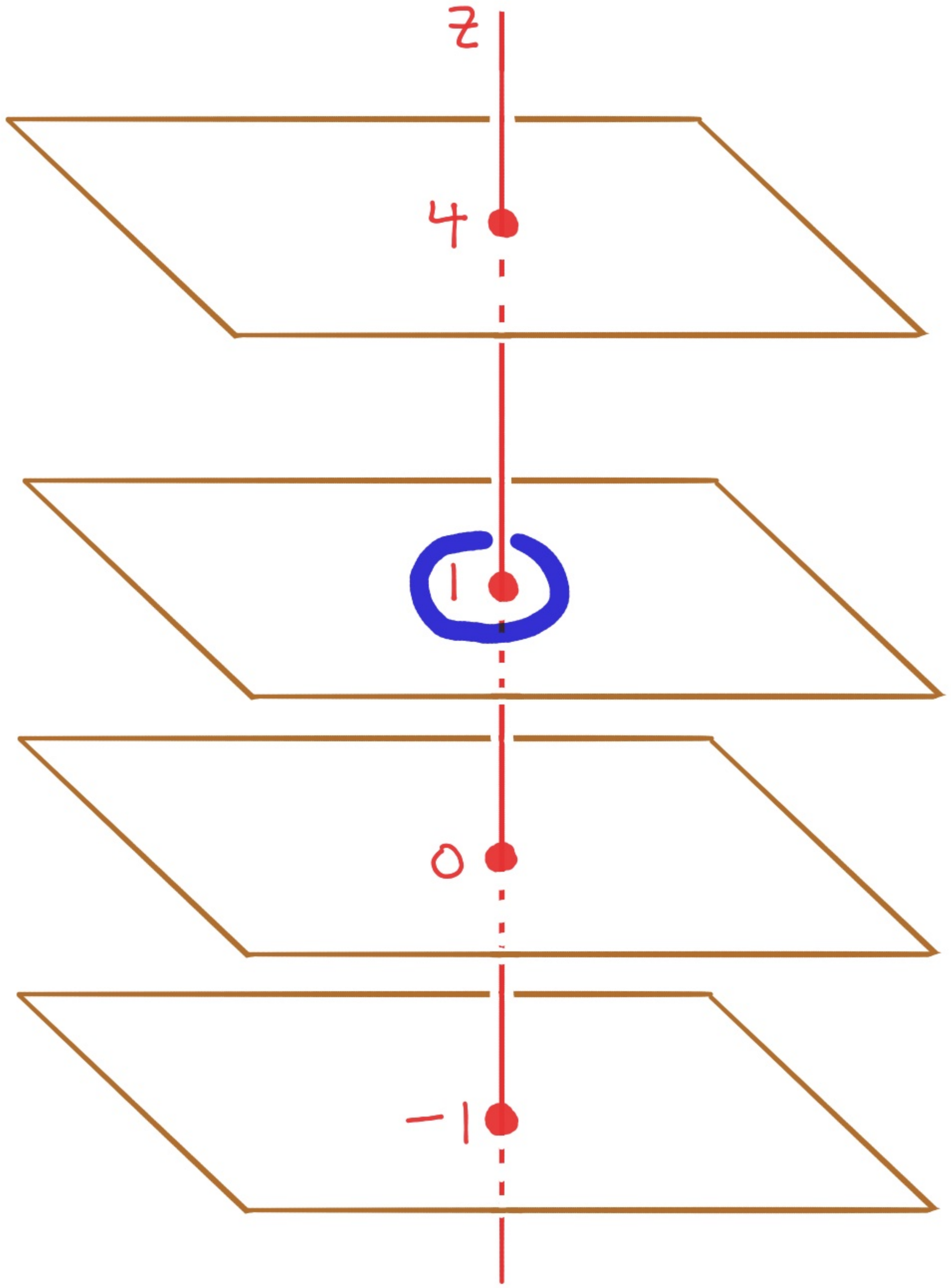
Graph z = constant cross sections



$z = 1$, $x^2 + y^2 = 1$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$



Graph $z = \text{constant}$ cross sections

$z = 4, \quad x^2 + y^2 = 4$

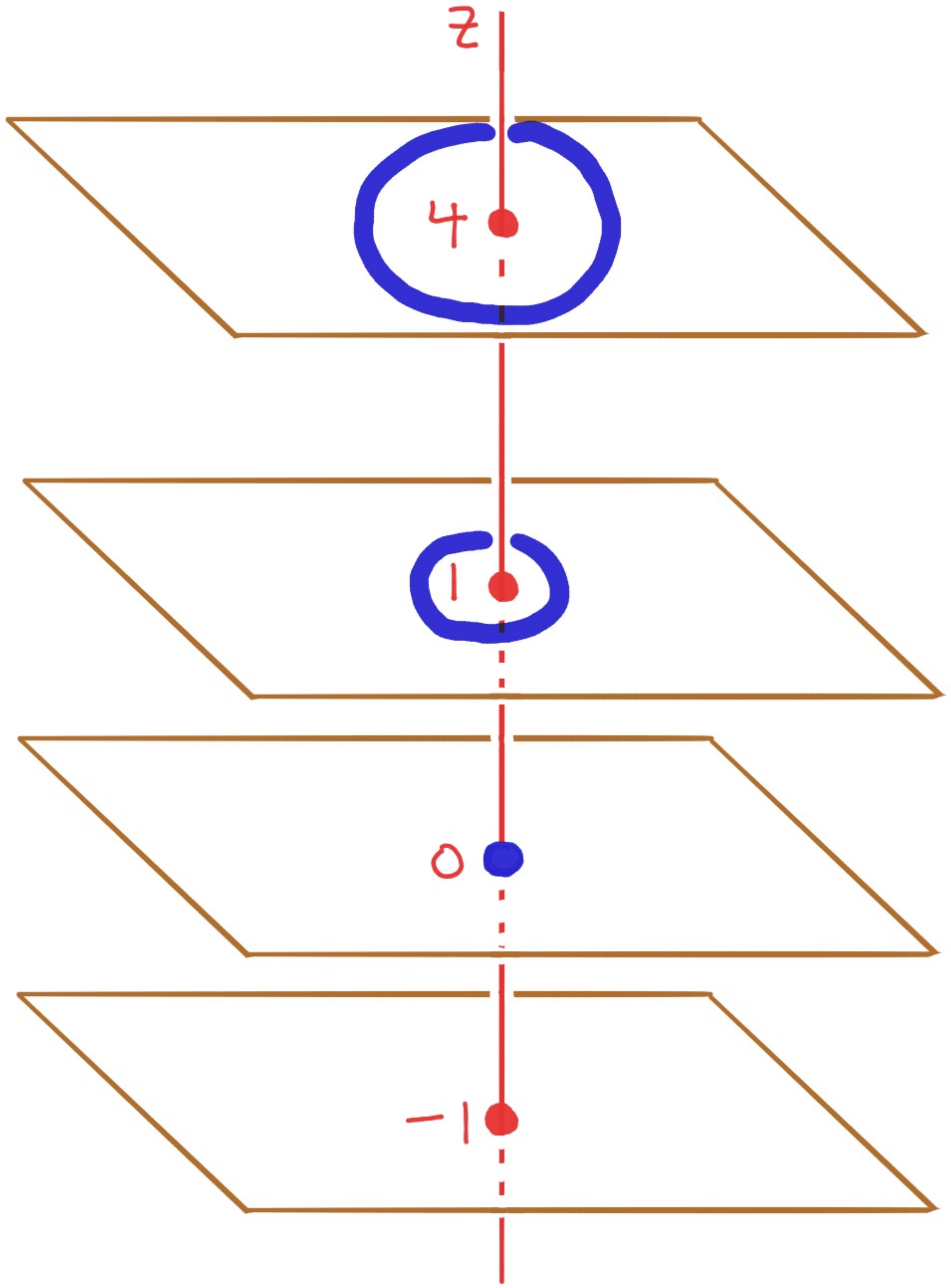
$z = 1, \quad x^2 + y^2 = 1$

$z = 0, \quad x^2 + y^2 = 0$

$z = -1, \quad x^2 + y^2 = -1$

Graph of $g(x,y) = x^2+y^2$

$x^2+y^2=z$



Graph z = constant cross sections

$z=4, \quad x^2+y^2=4$

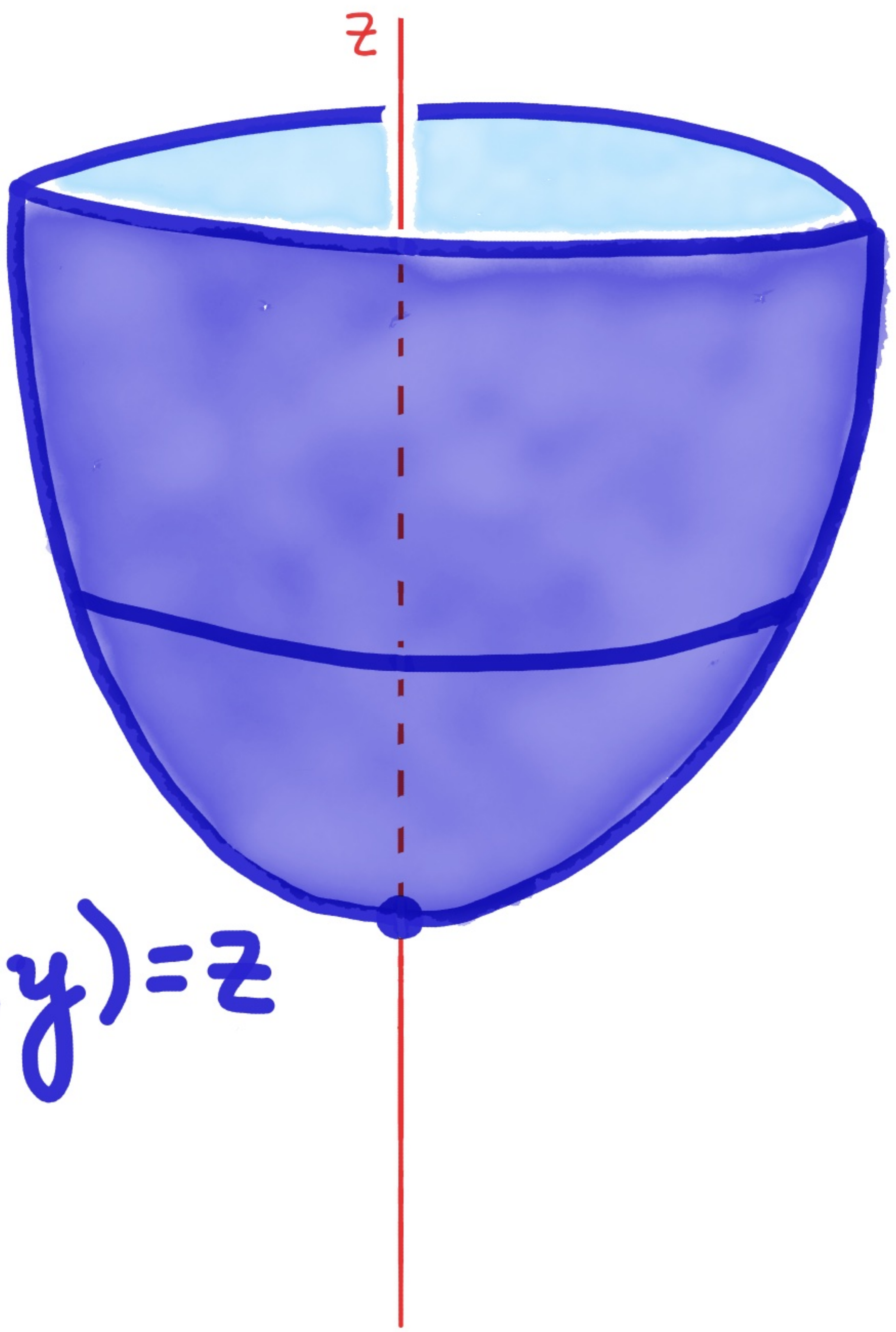
$z=1, \quad x^2+y^2=1$

$z=0, \quad x^2+y^2=0$

$z=-1, \quad x^2+y^2=-1$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$



$g(x,y) = z$

Graph $z = \text{constant}$ cross sections

$z = 4$, $x^2 + y^2 = 4$

$z = 1$, $x^2 + y^2 = 1$

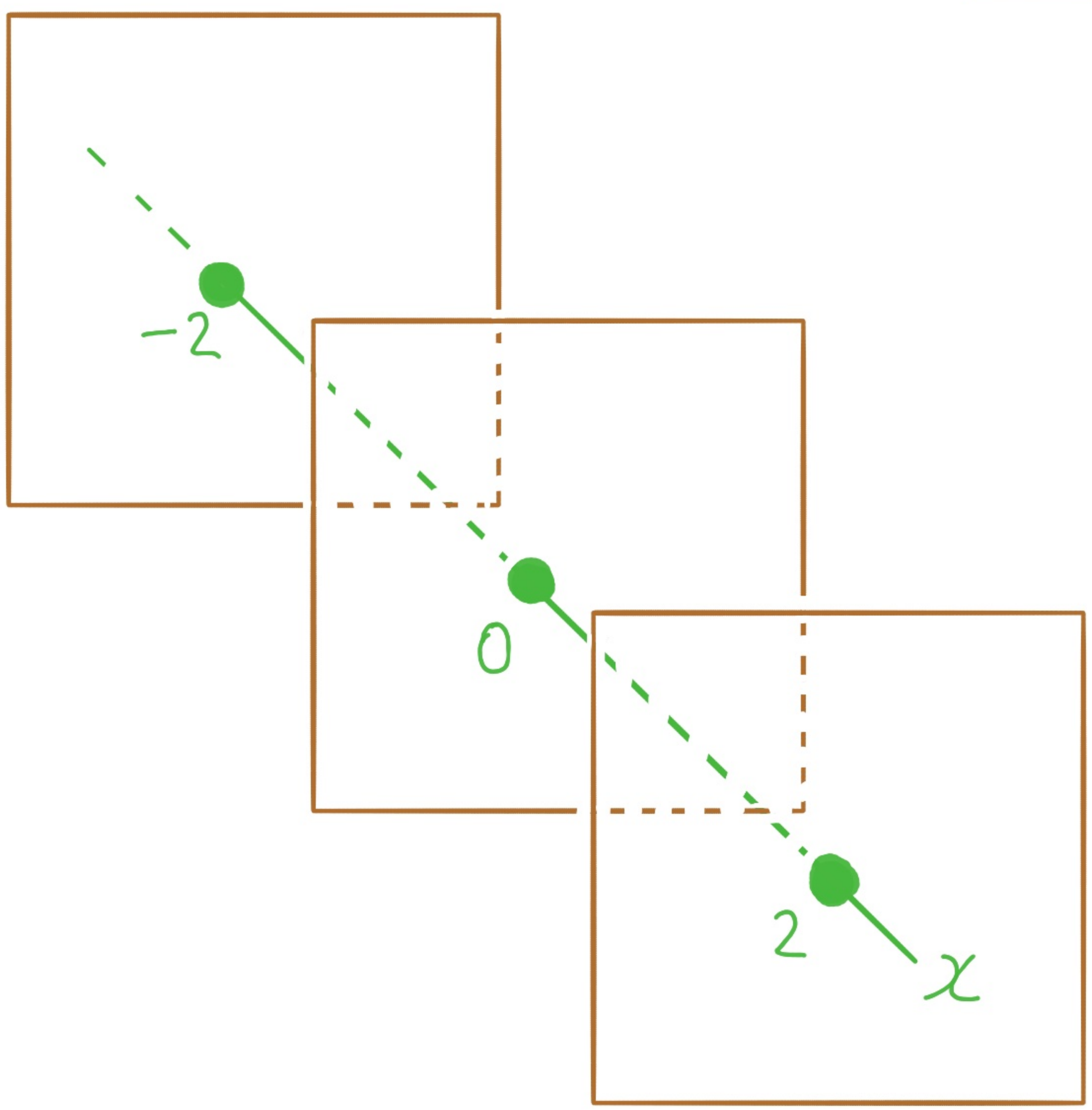
$z = 0$, $x^2 + y^2 = 0$

$z = -1$, $x^2 + y^2 = -1$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

Graph $x = $ constant cross sections



$x = -2, \quad 4 + y^2 = z$

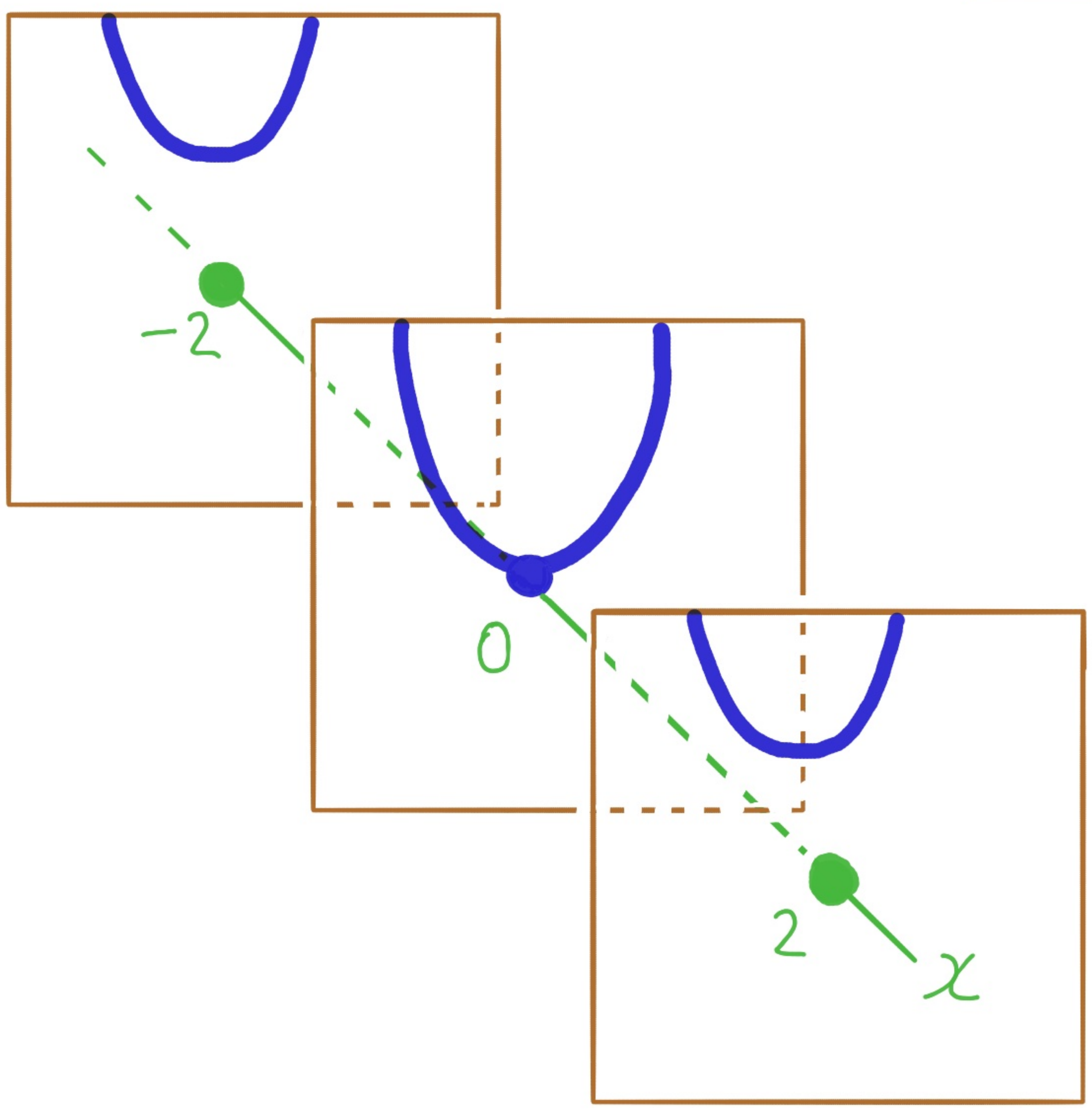
$x = 0, \quad y^2 = z$

$x = 2, \quad 4 + y^2 = z$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

Graph $x = \text{constant}$ cross sections



$x = -2, \quad 4 + y^2 = z$

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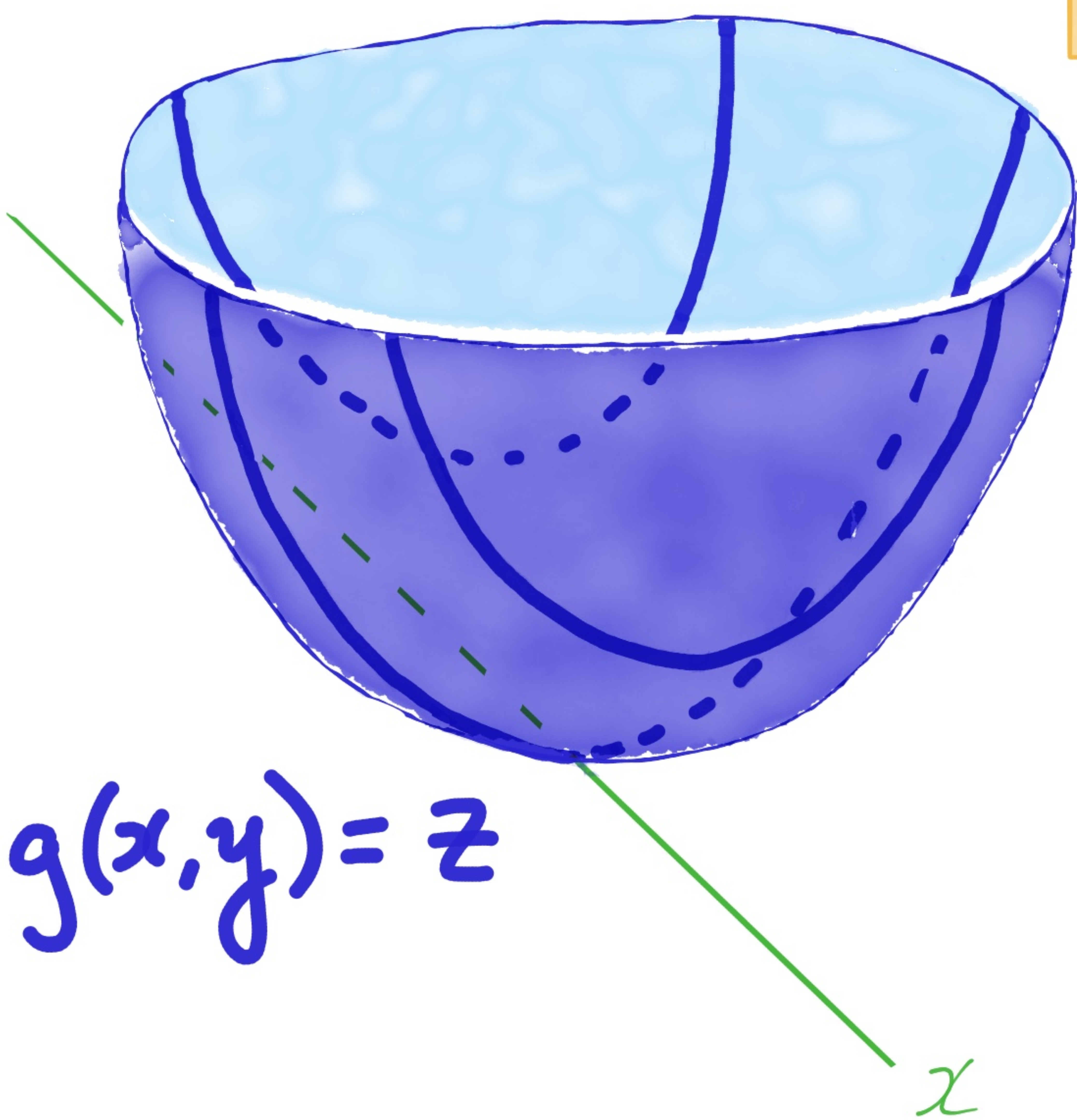
Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$



$g(x,y) = z$

Graph $x = \text{constant}$ cross sections

$x = -2, \quad 4 + y^2 = z$

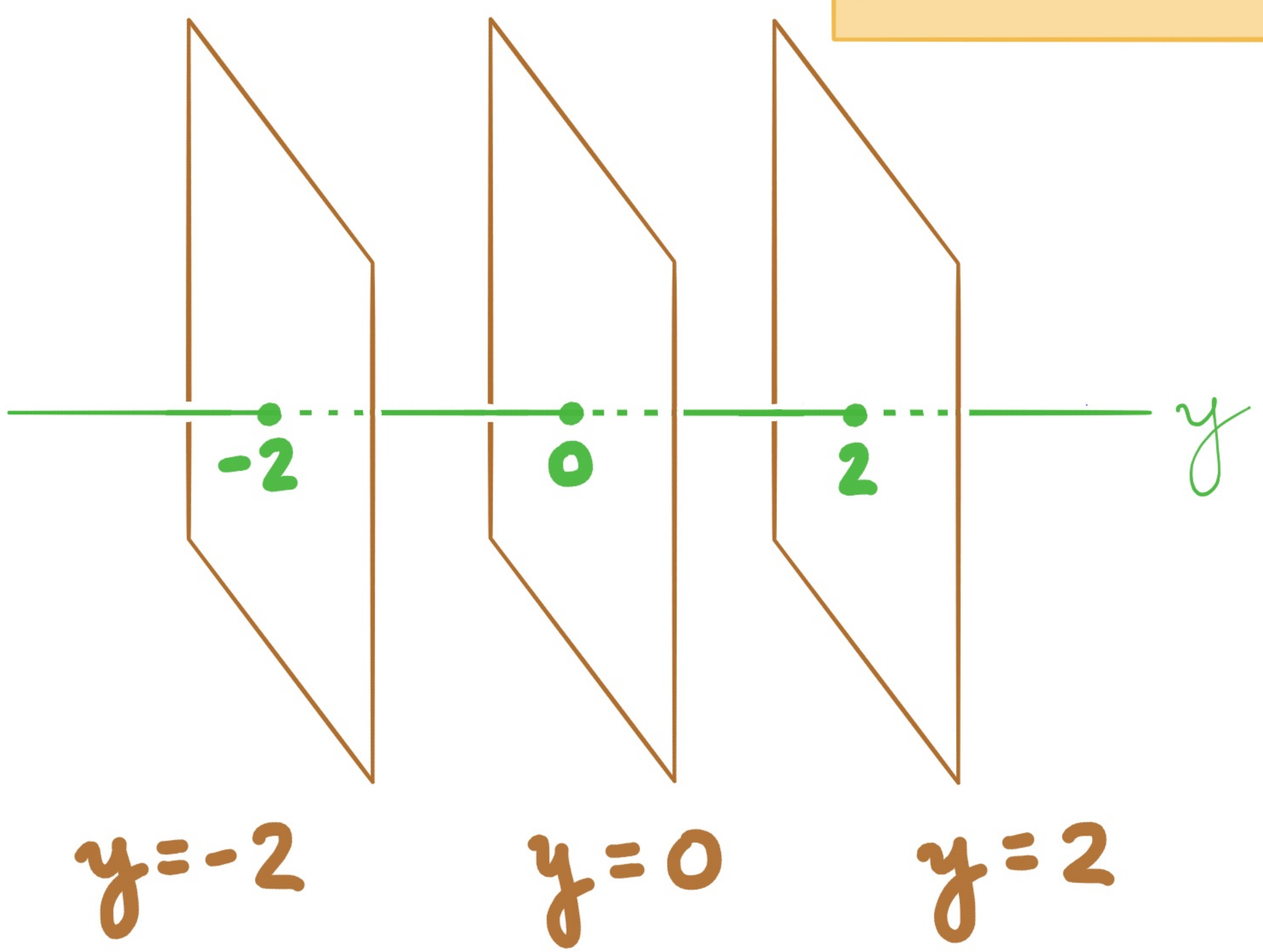
$x = 0, \quad y^2 = z$

$x = 2, \quad 4 + y^2 = z$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

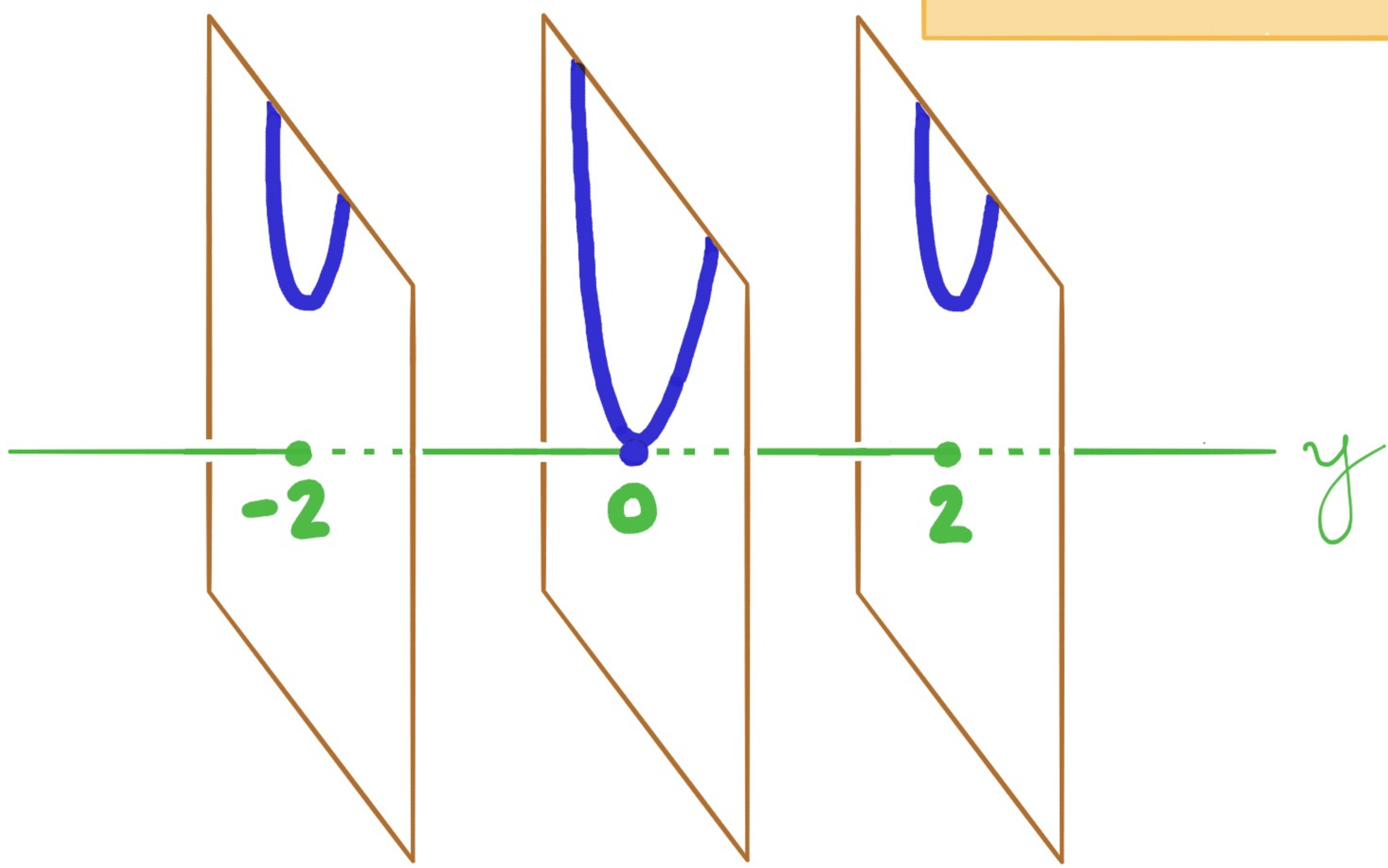
Graph $y = \text{constant}$ cross sections



Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

Graph $y = $ constant cross sections



$y = -2$ $\quad$ $x^2 + 4 = z$

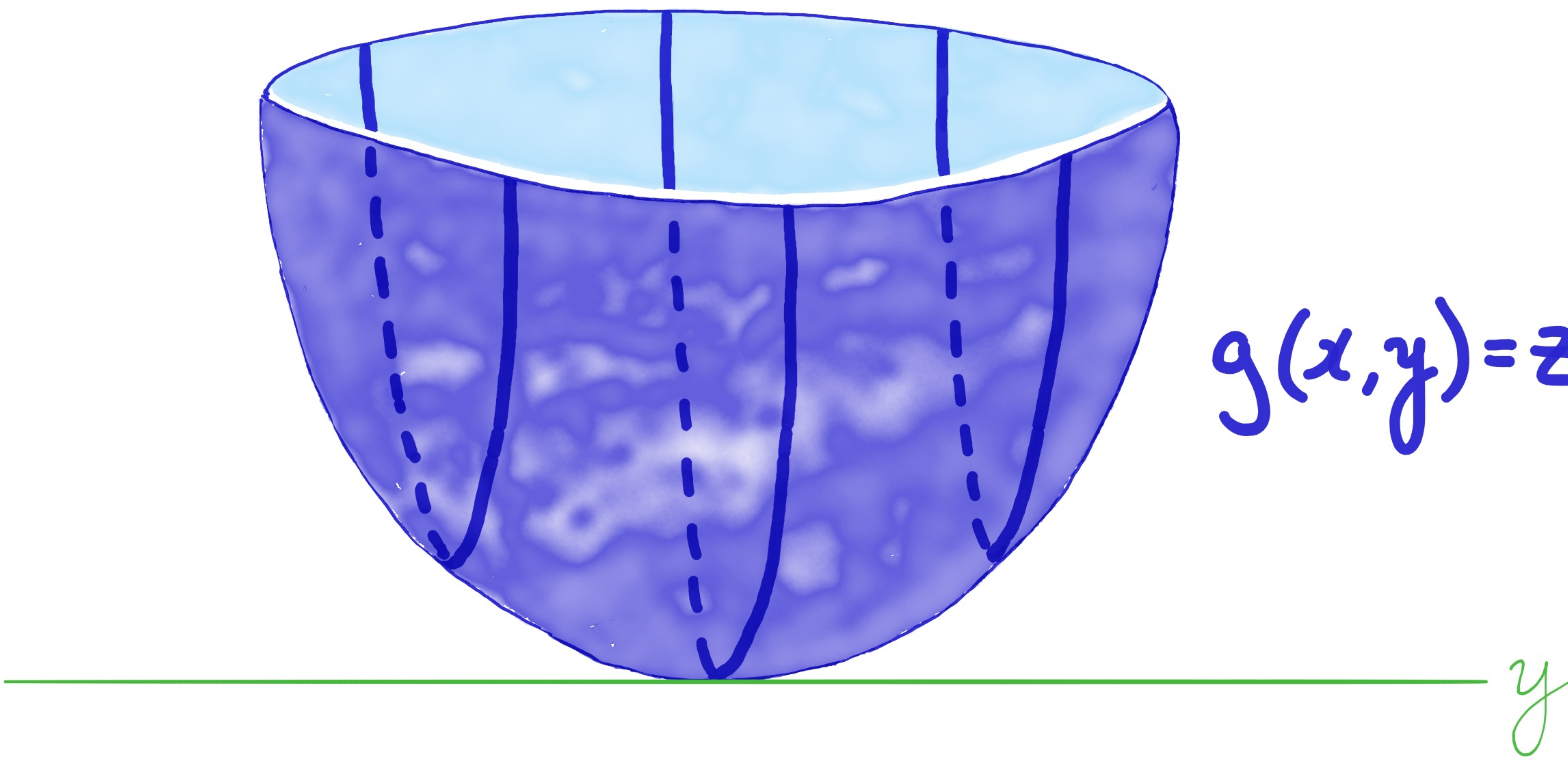
$y = 0$ $\quad$ $x^2 = z$

$y = 2$ $\quad$ $x^2 + 4 = z$

Graph of $g(x,y) = x^2 + y^2$

$x^2 + y^2 = z$

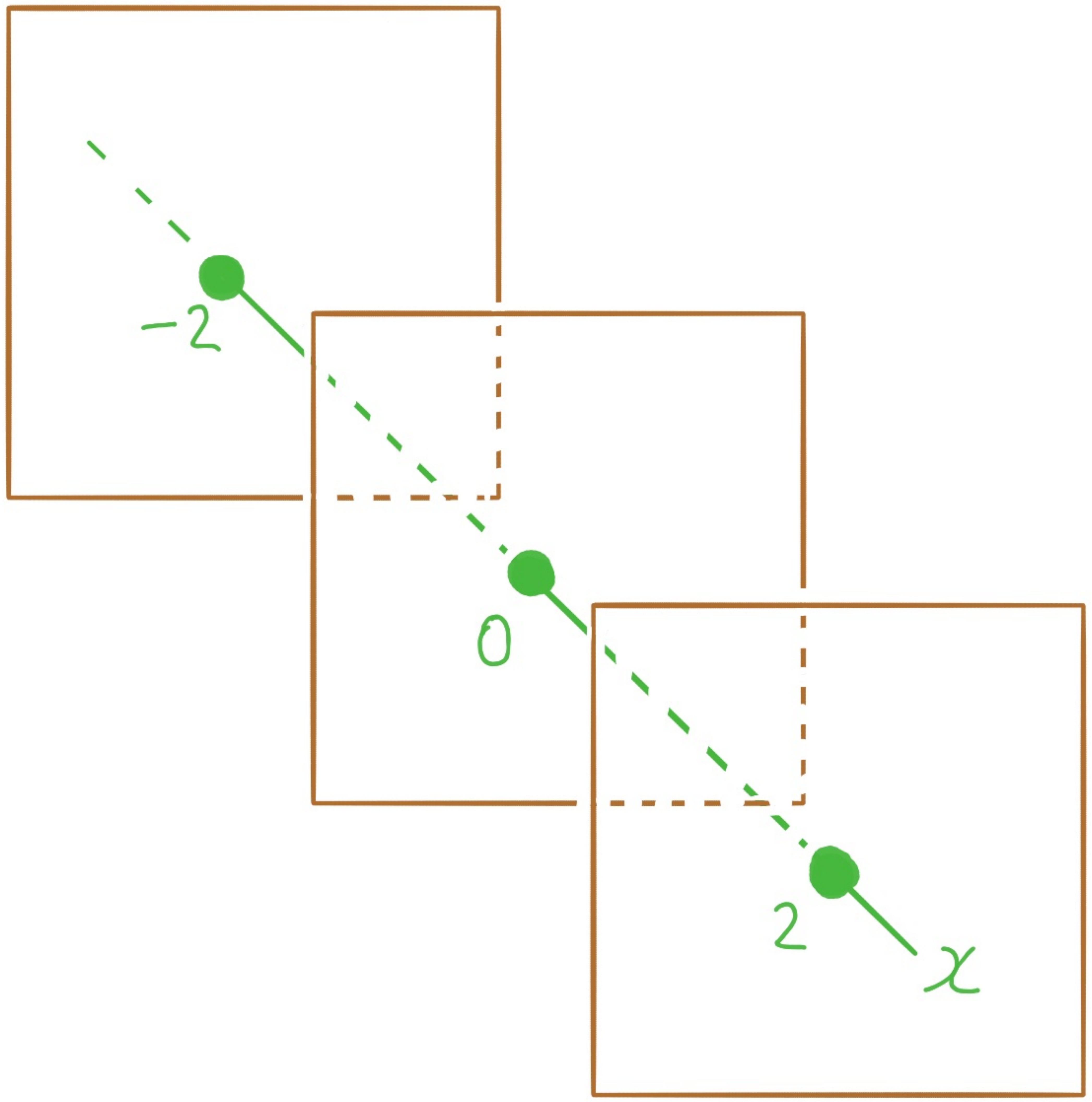
Graph y = constant cross sections



Graph of $f(x,y)=xy$

$xy=z$

Graph $x=$constant cross sections



$x=-2$,

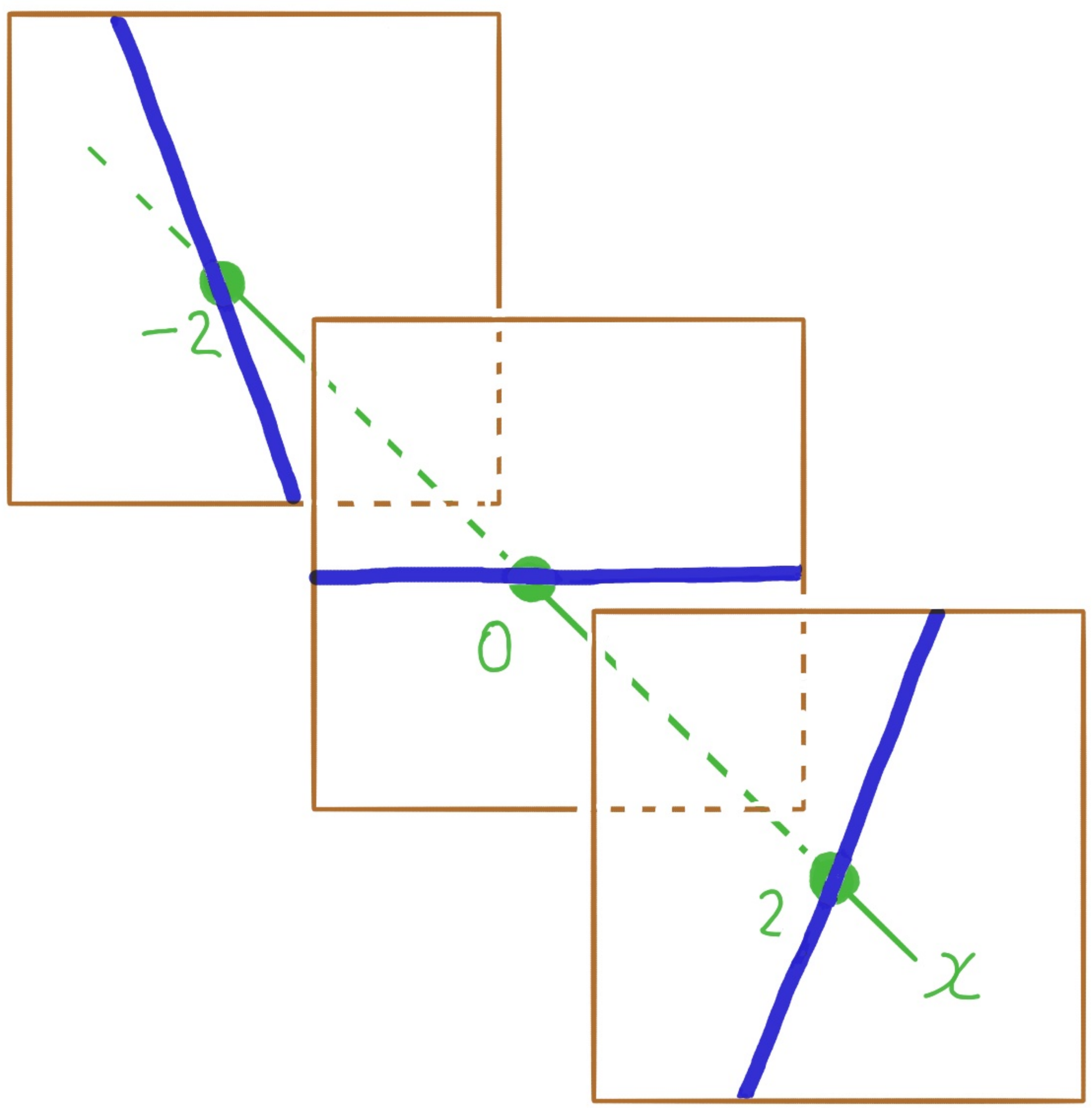
$x=0$,

$x=2$,

Graph of $f(x,y)=xy$

$xy=z$

Graph $x=$constant cross sections



$x=-2, \quad -2y=z$

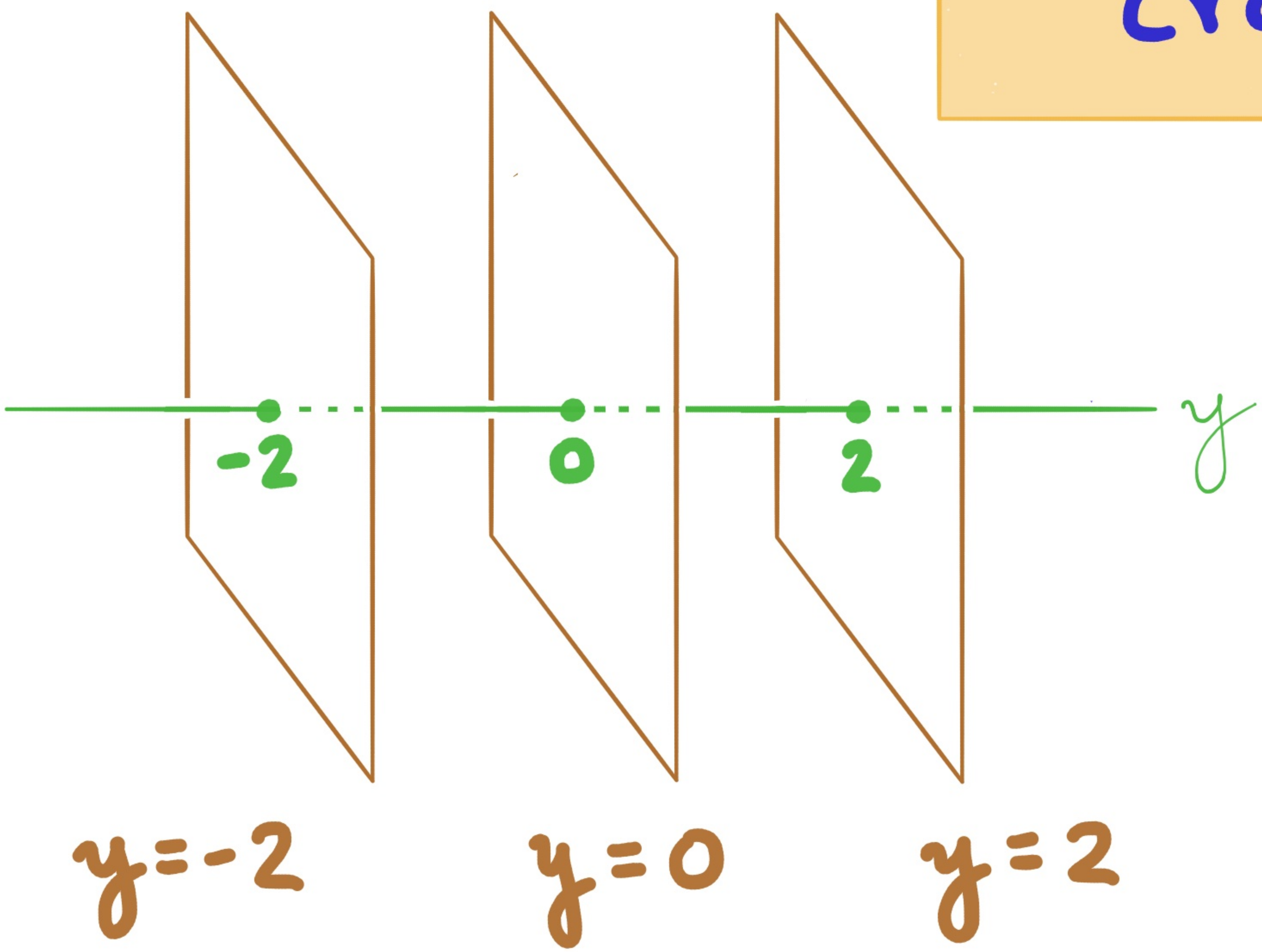
$x=0, \quad 0=z$

$x=2, \quad 2y=z$

Graph of $f(x,y)=xy$

$xy=z$

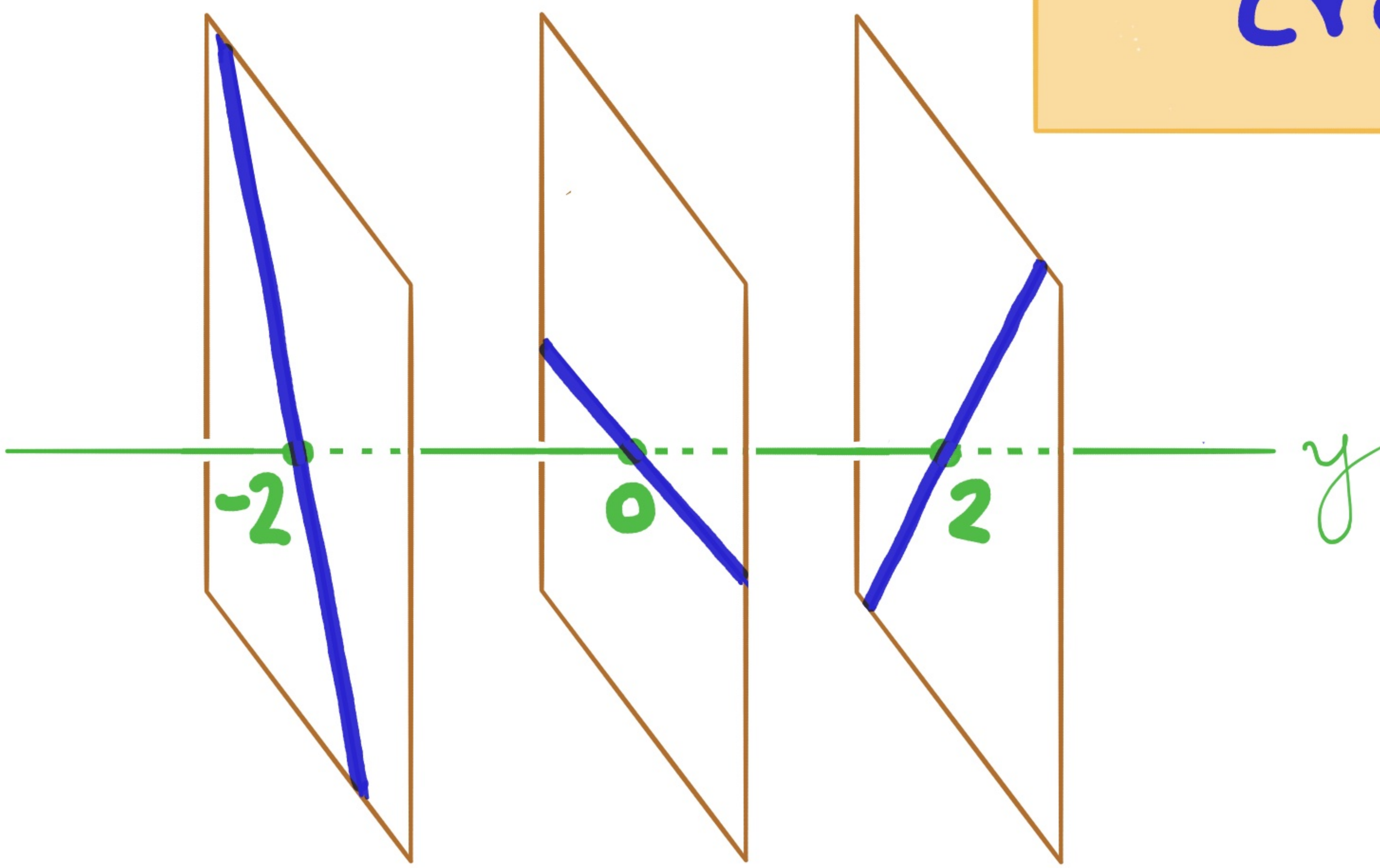
Graph $y=$constant cross sections



Graph of $f(x,y)=xy$

$xy=z$

Graph $y=\text{constant}$ cross sections



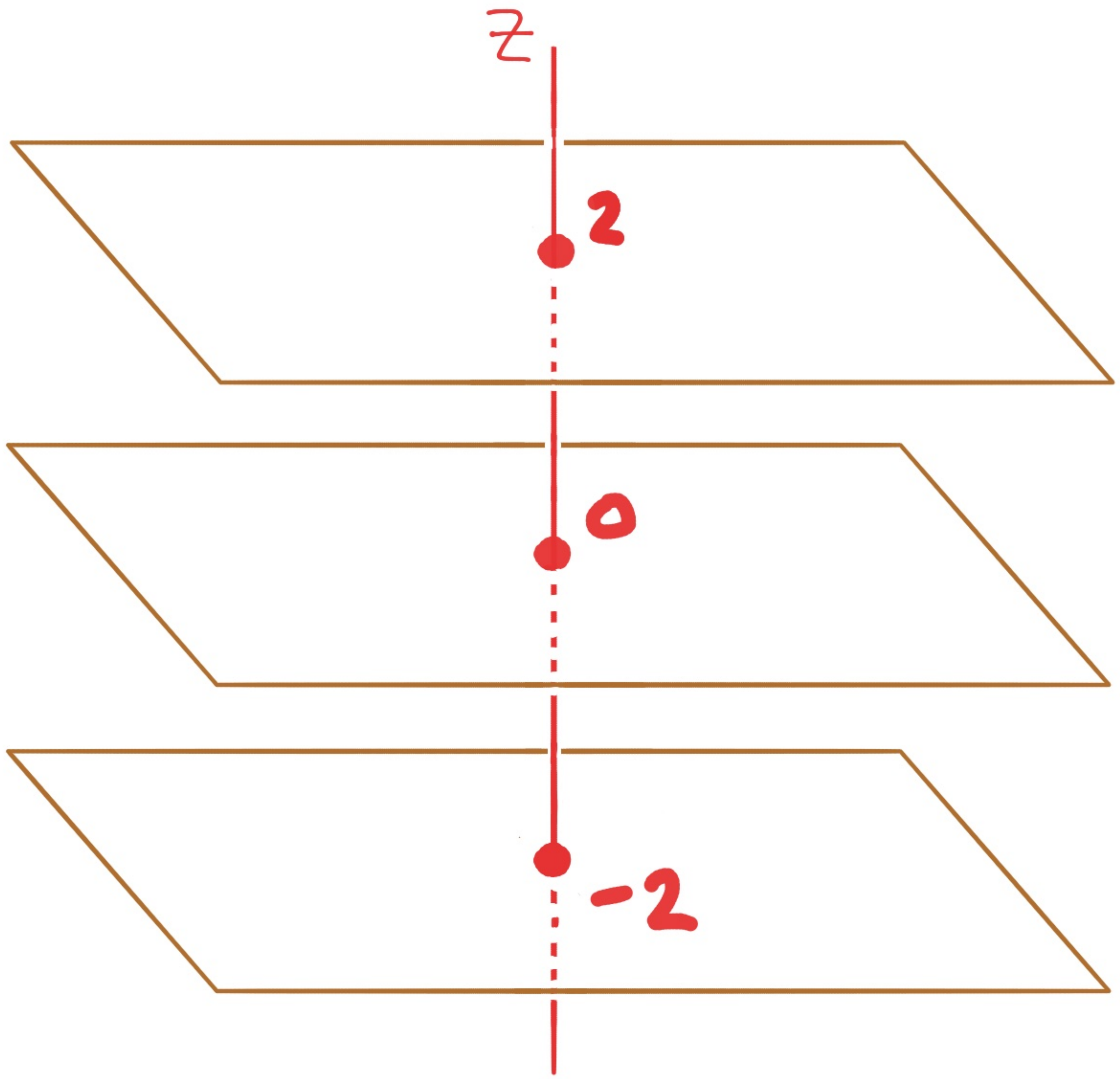
$y=-2$ $y=0$ $y=2$

$-2x=z$ $0=z$ $2x=z$

Graph of $f(x,y) = xy$

$xy = z$

Graph $z = \text{constant}$ cross sections



$z = 2,$

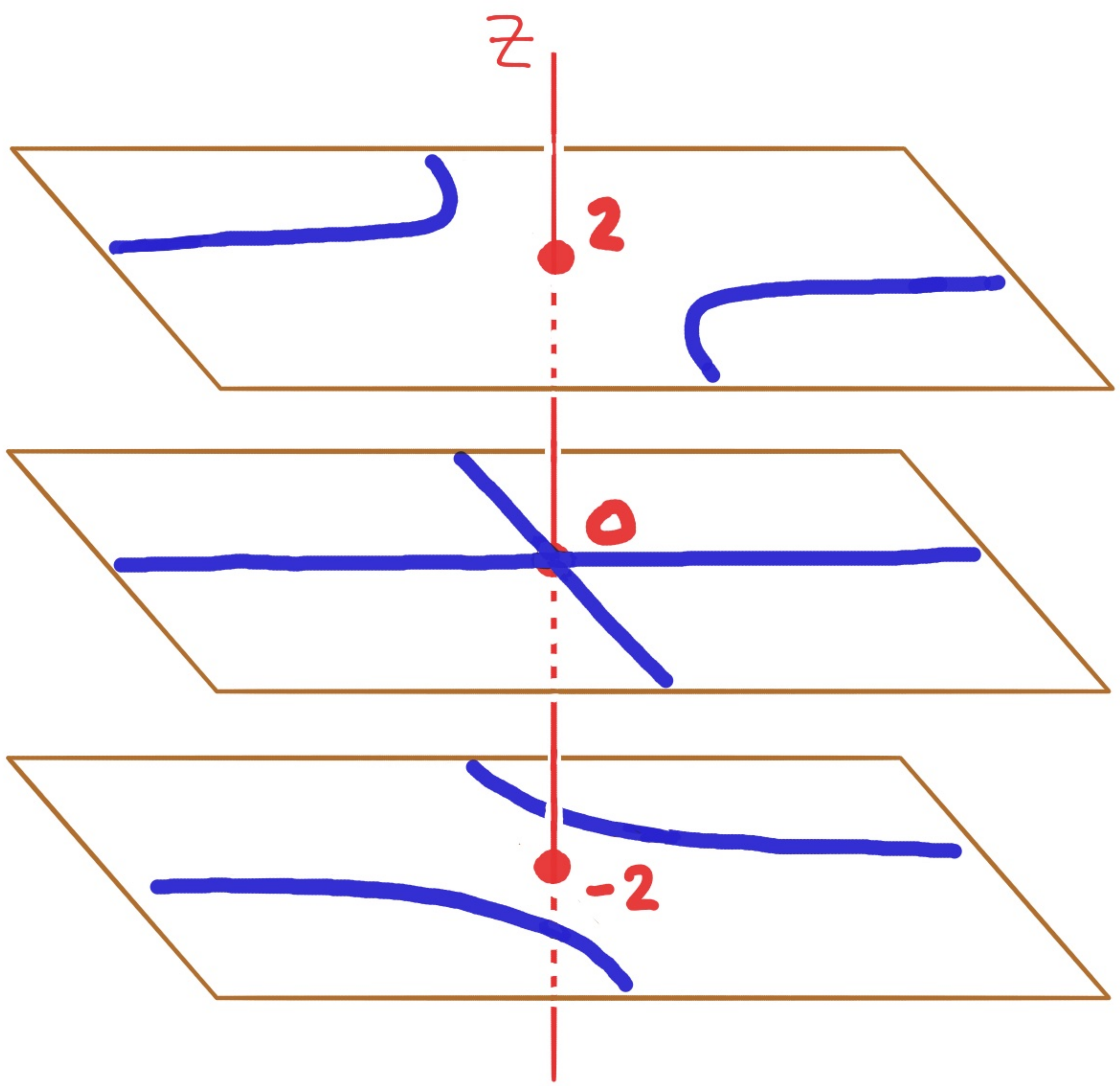
$z = 0,$

$z = -2,$

Graph of $f(x,y) = xy$

$xy = z$

Graph $z = \text{constant}$ cross sections

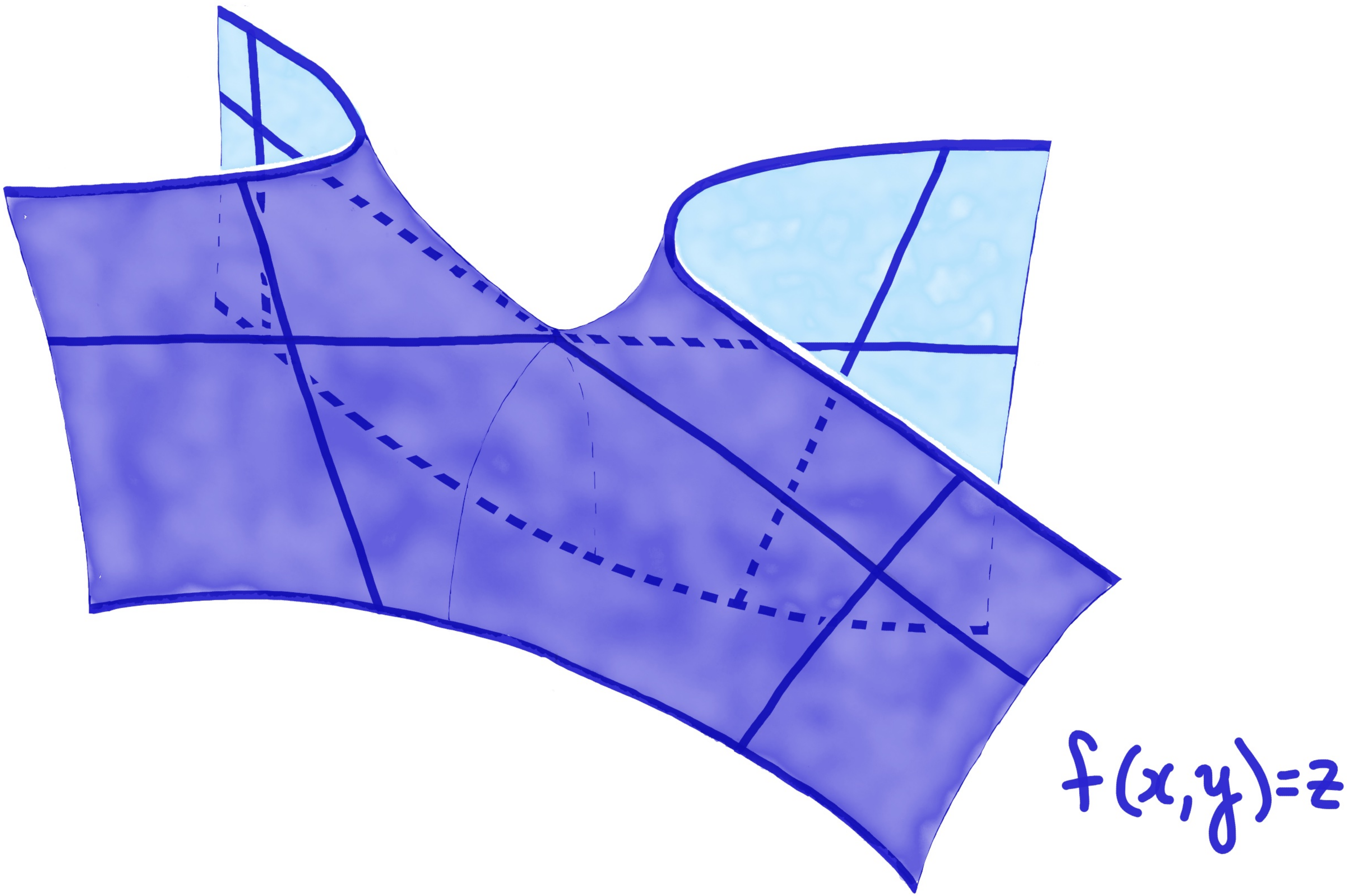


$z = 2, \quad xy = 2$

$z = 0, \quad xy = 0$

$z = -2, \quad xy = -2$

Graph of $f(x,y)=xy$



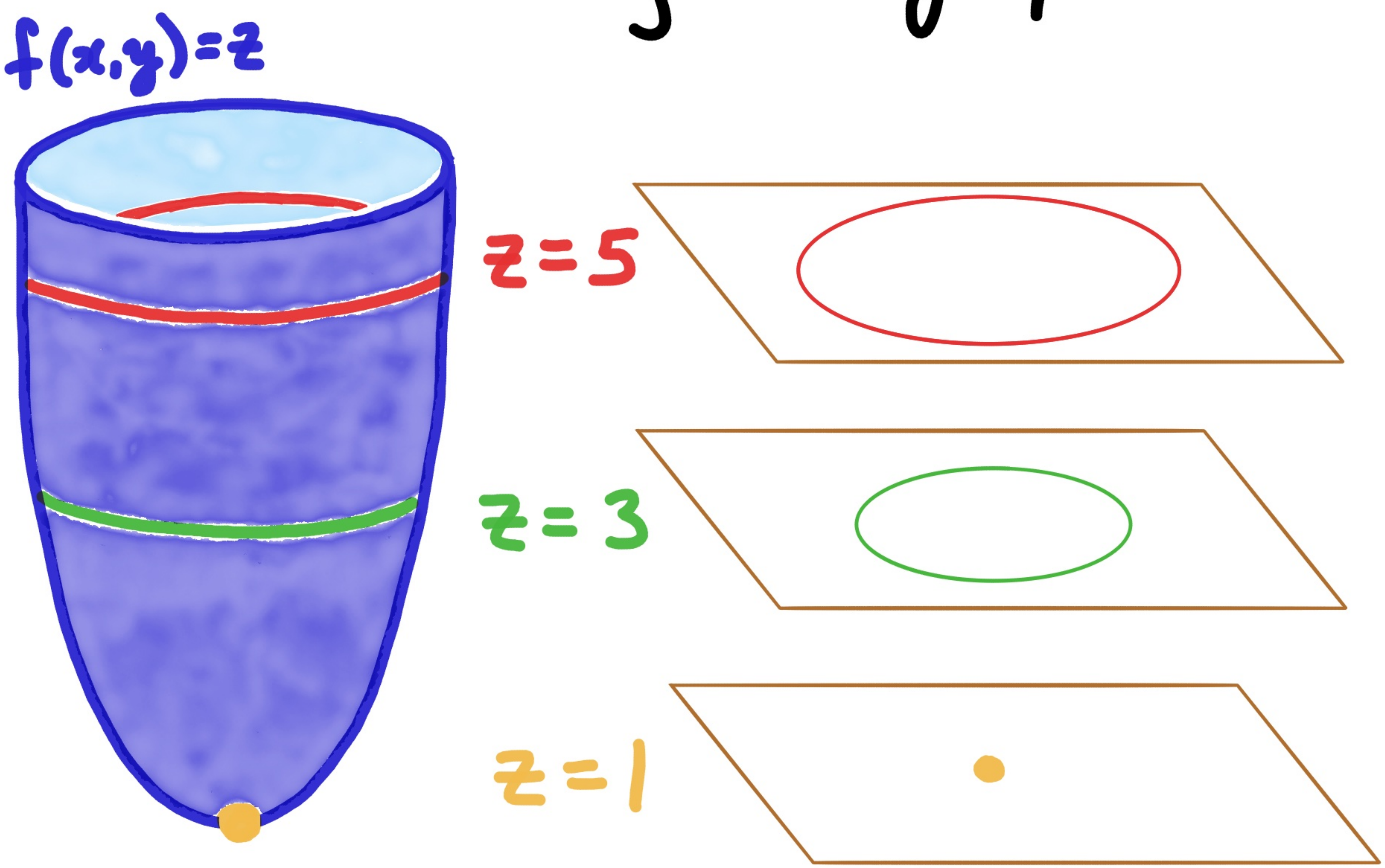
III Level curves and contour maps

Level curves and contour maps

Superimpose all the z = constant cross sections of $f(x,y)=z$ onto a single xy-plane.

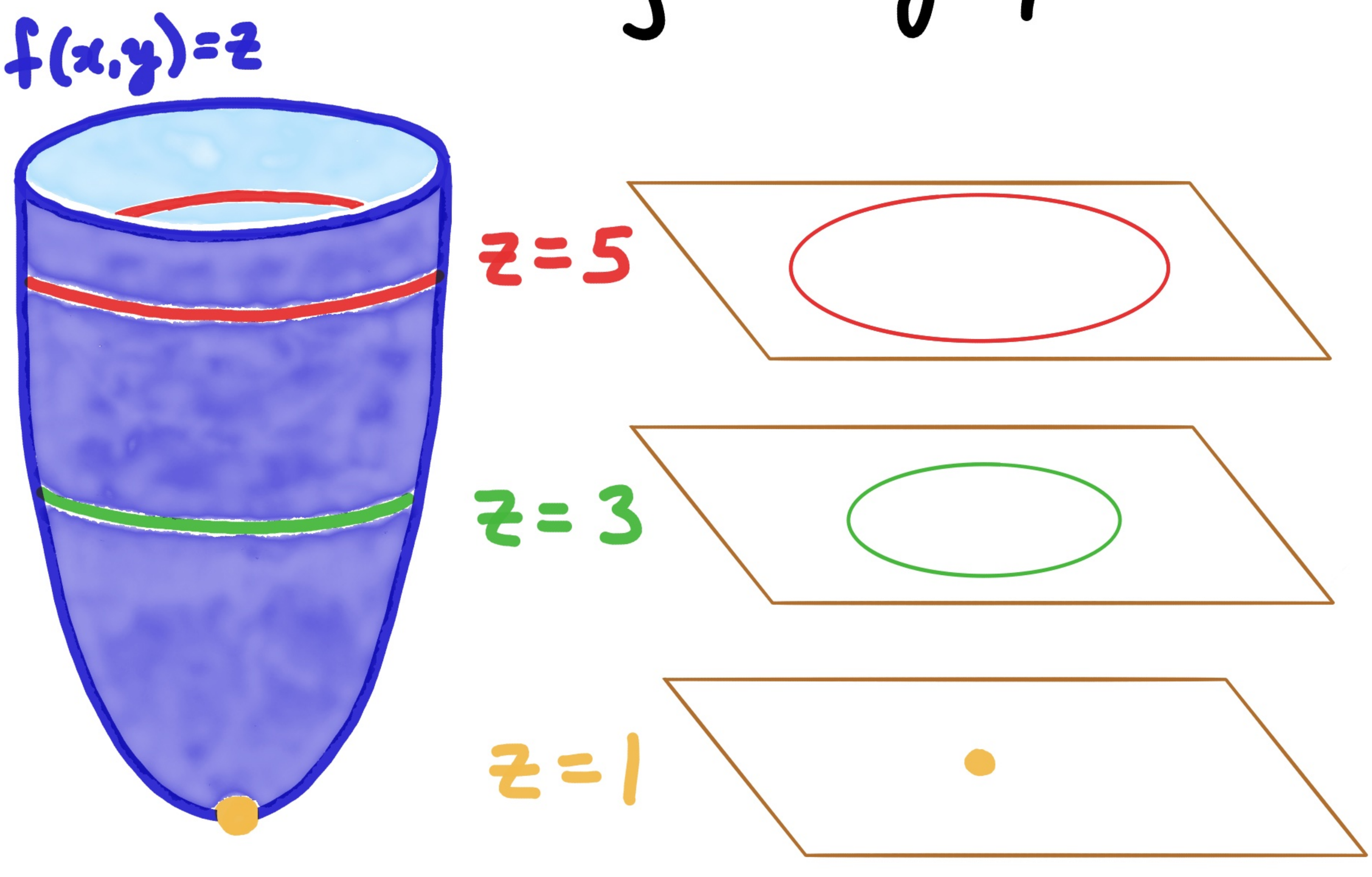
Level curves and contour maps

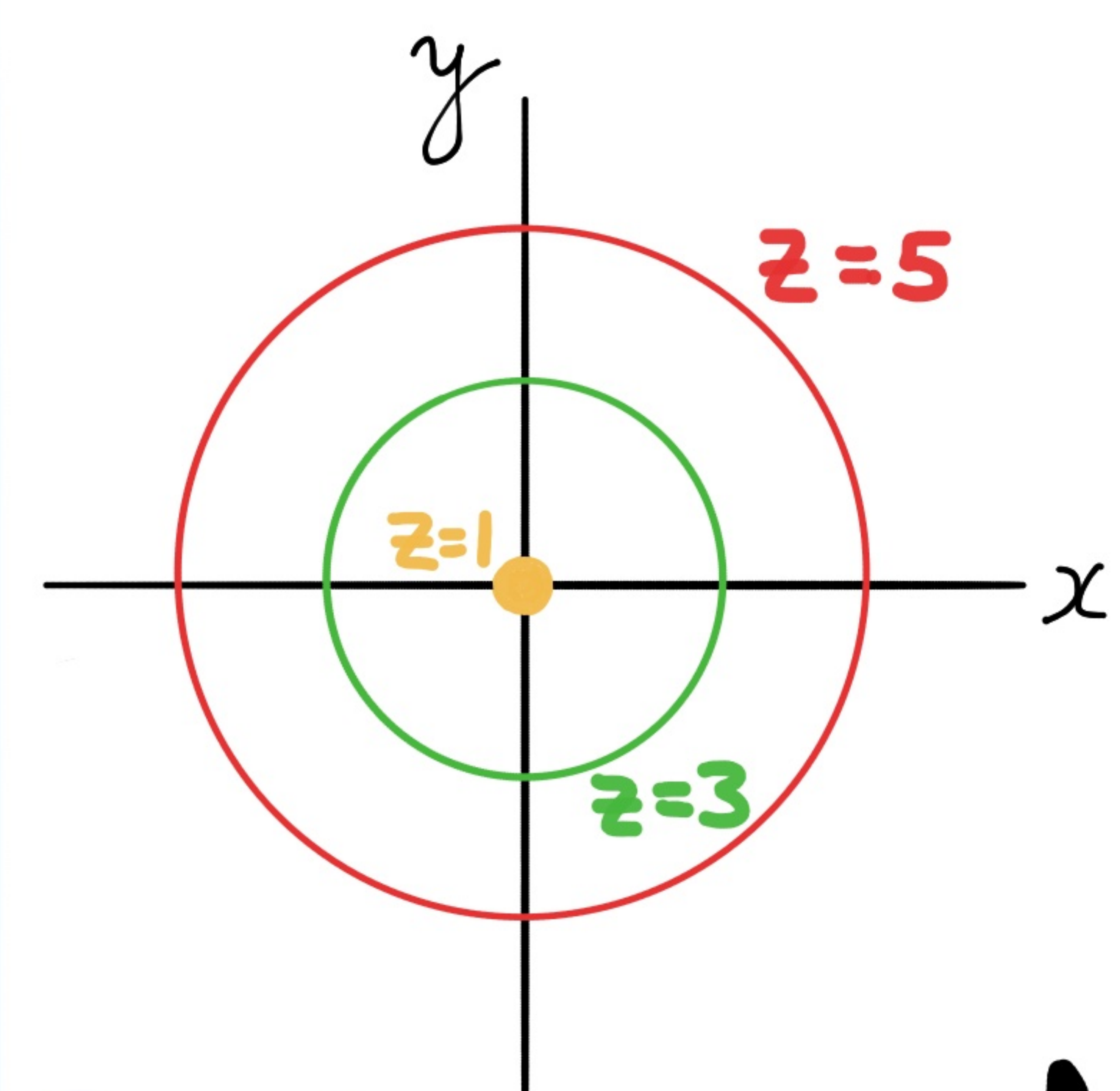
Superimpose all the $z = \text{constant}$ cross sections of $f(x,y) = z$ onto a single xy-plane.



Level curves and contour maps

Superimpose all the z=constant cross sections of $f(x,y)=z$ onto a single xy-plane.



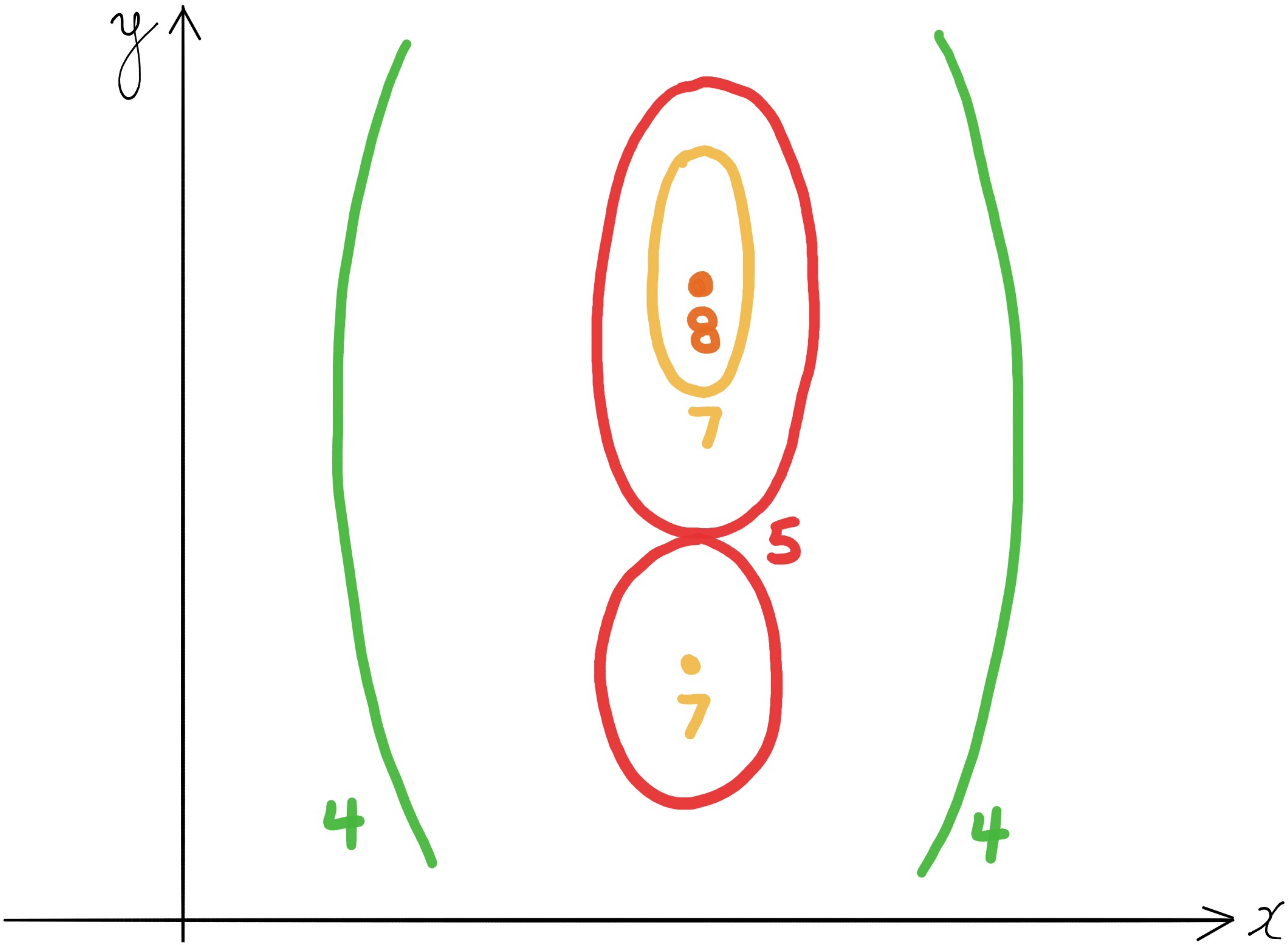


Contour map of $f(x,y)=z$ with 3 level curves.

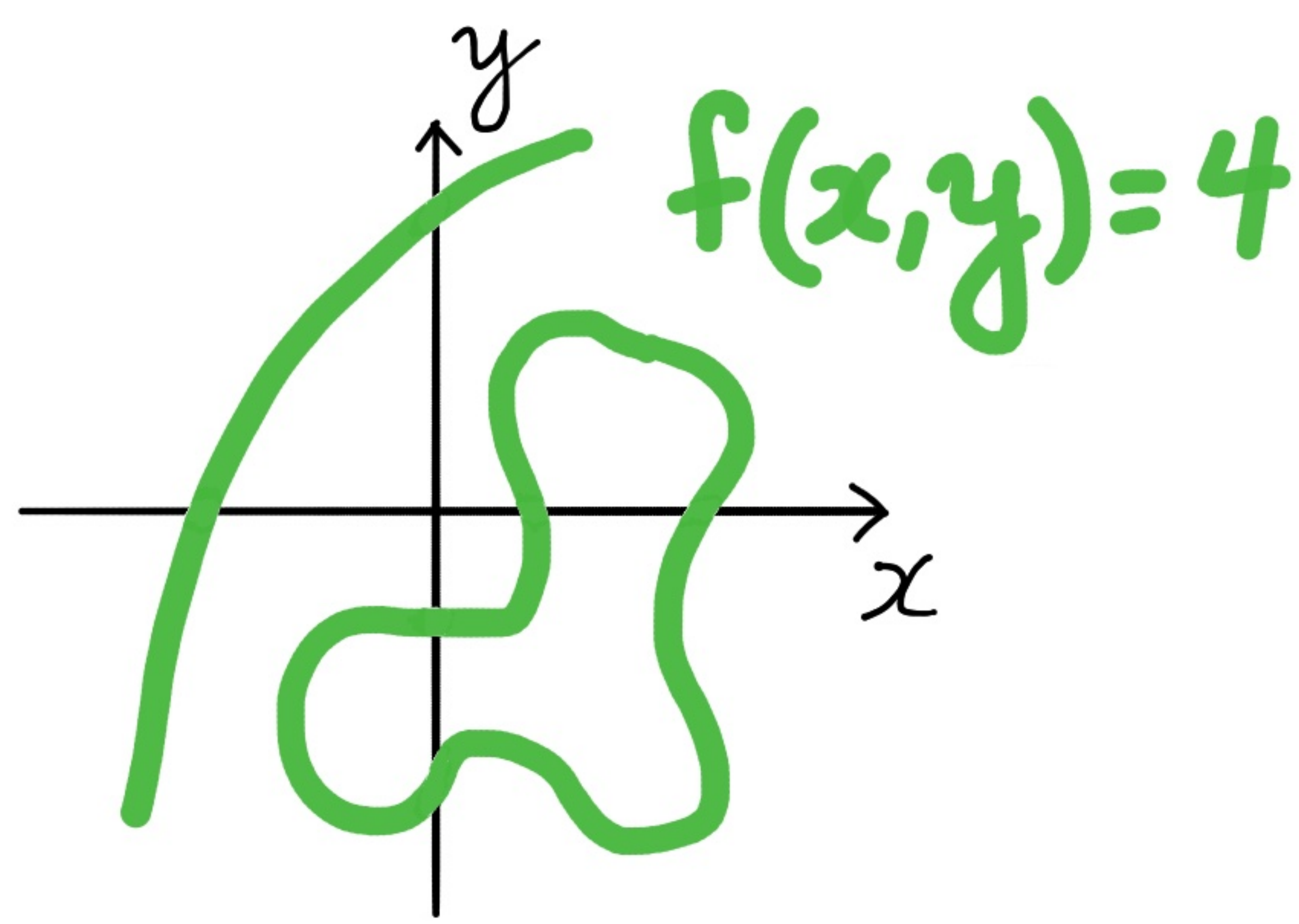
z
8
7
5
4
g(x,y)=z
y
x


z
8
7
5
4
g(x,y)=z
y
x
4
5
7
7
8
5
4

Level curves and contour map for $g(x, y) = z$



A **level curve** for $f(x,y)=z$ is a sketch in $\mathbb{R}^2$ of the solutions of $f(x,y)=\text{constant}.$



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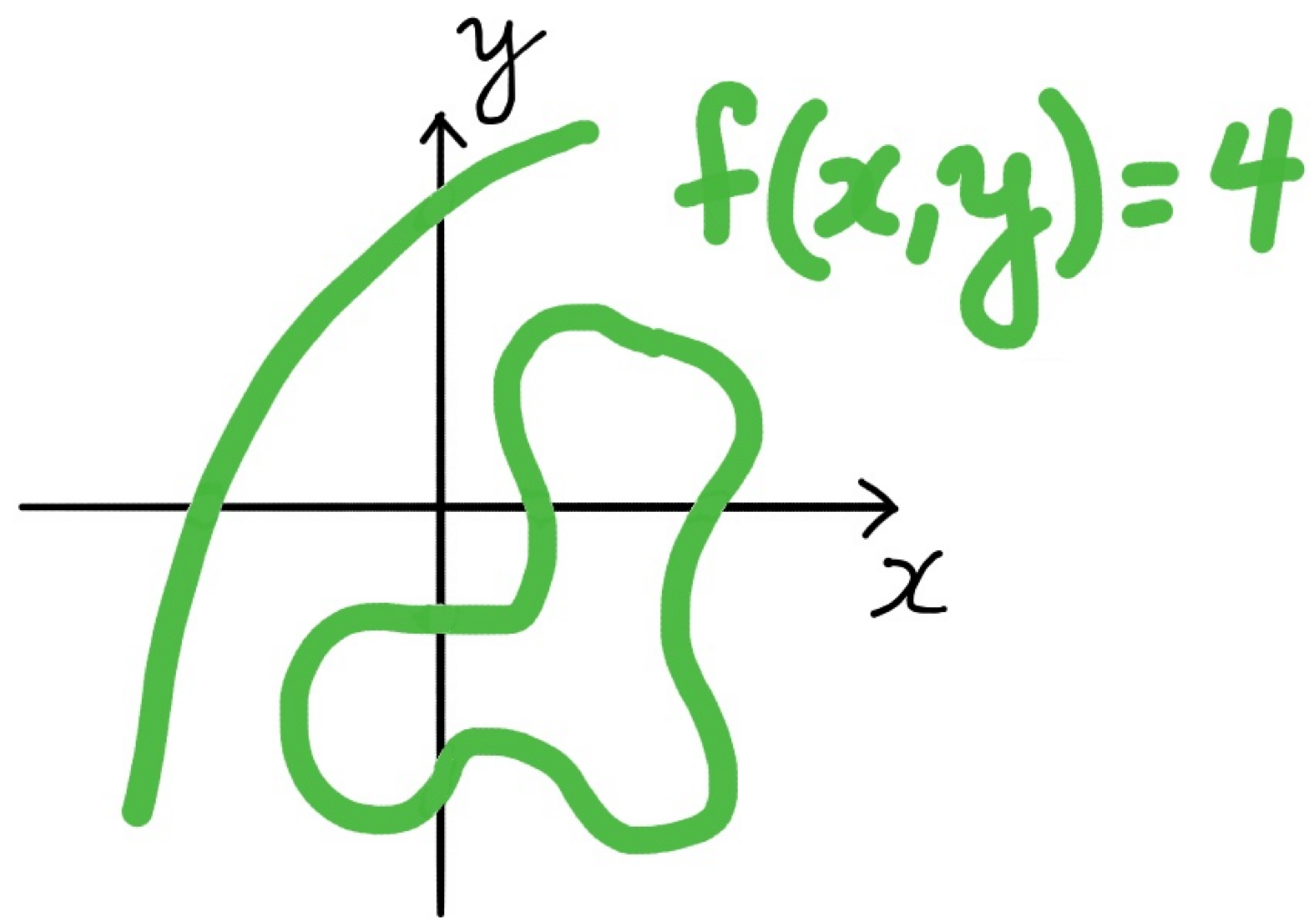
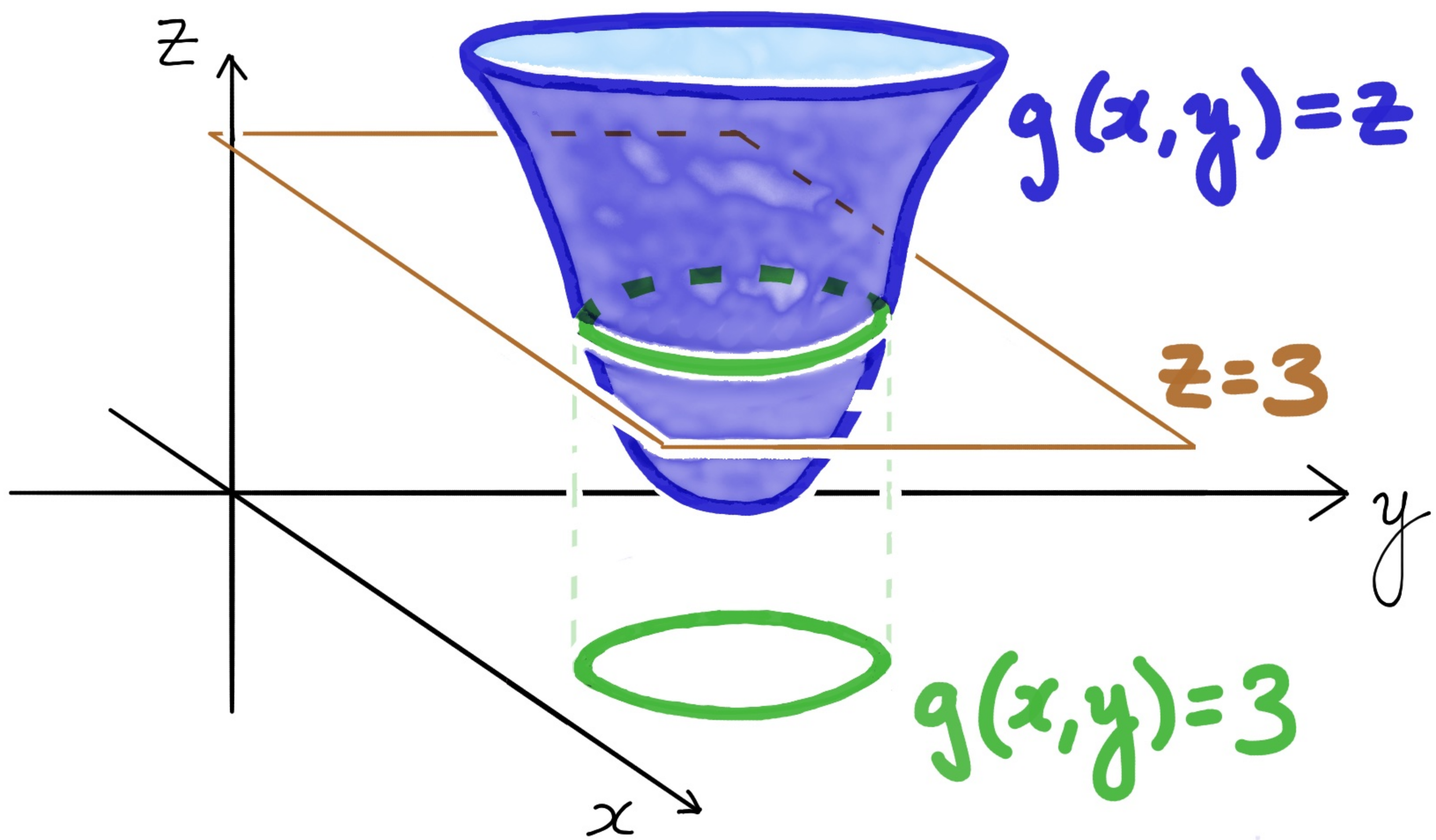
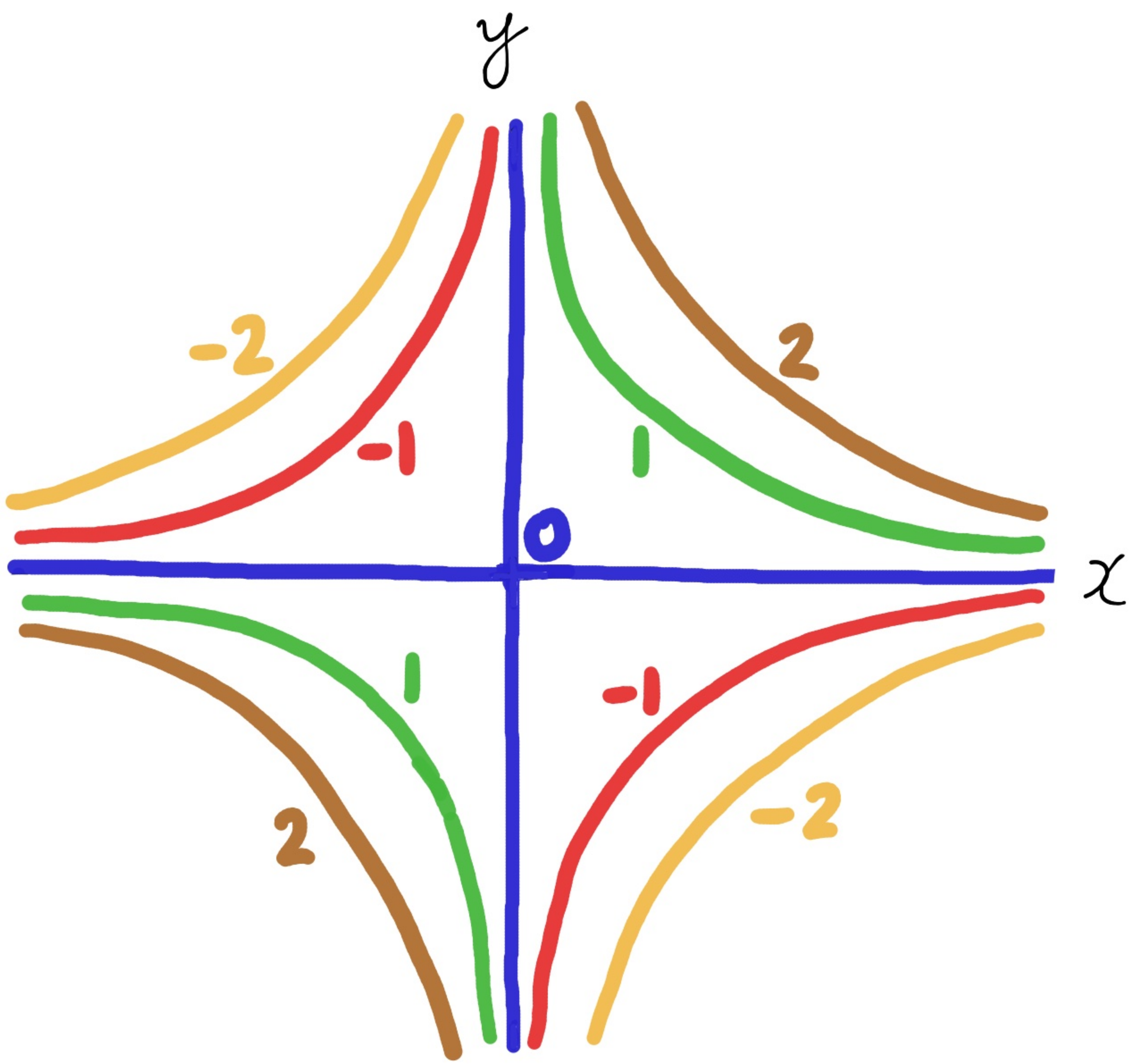




5 level curves for $f(x,y) = xy$.



$xy = 2$

$xy = 1$

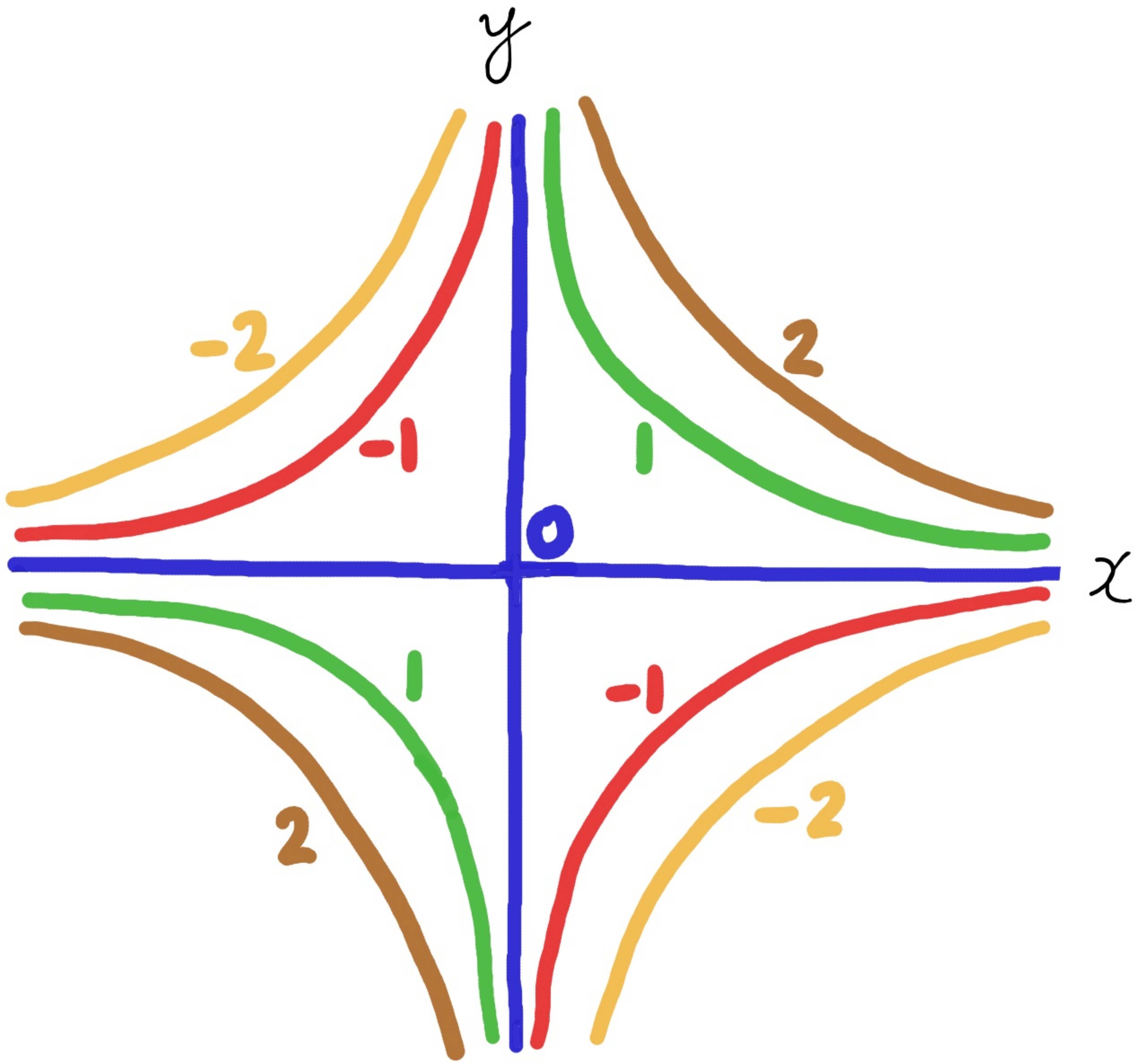
$xy = 0$

$xy = -1$

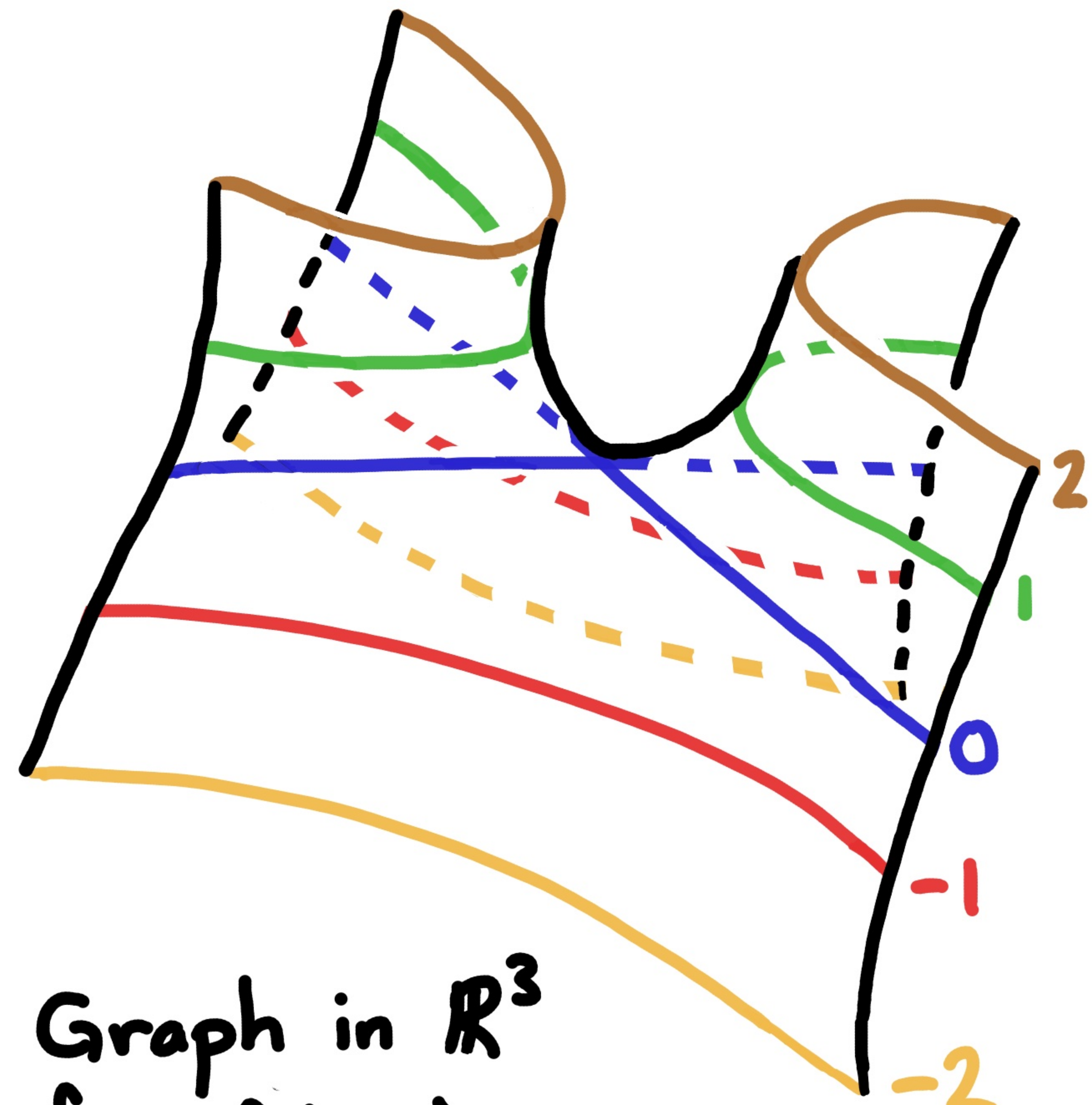
$xy = -2$

Contour map in $\mathbb{R}^2$ for $f(x,y) = xy$.

5 level curves for $f(x,y)=xy$.



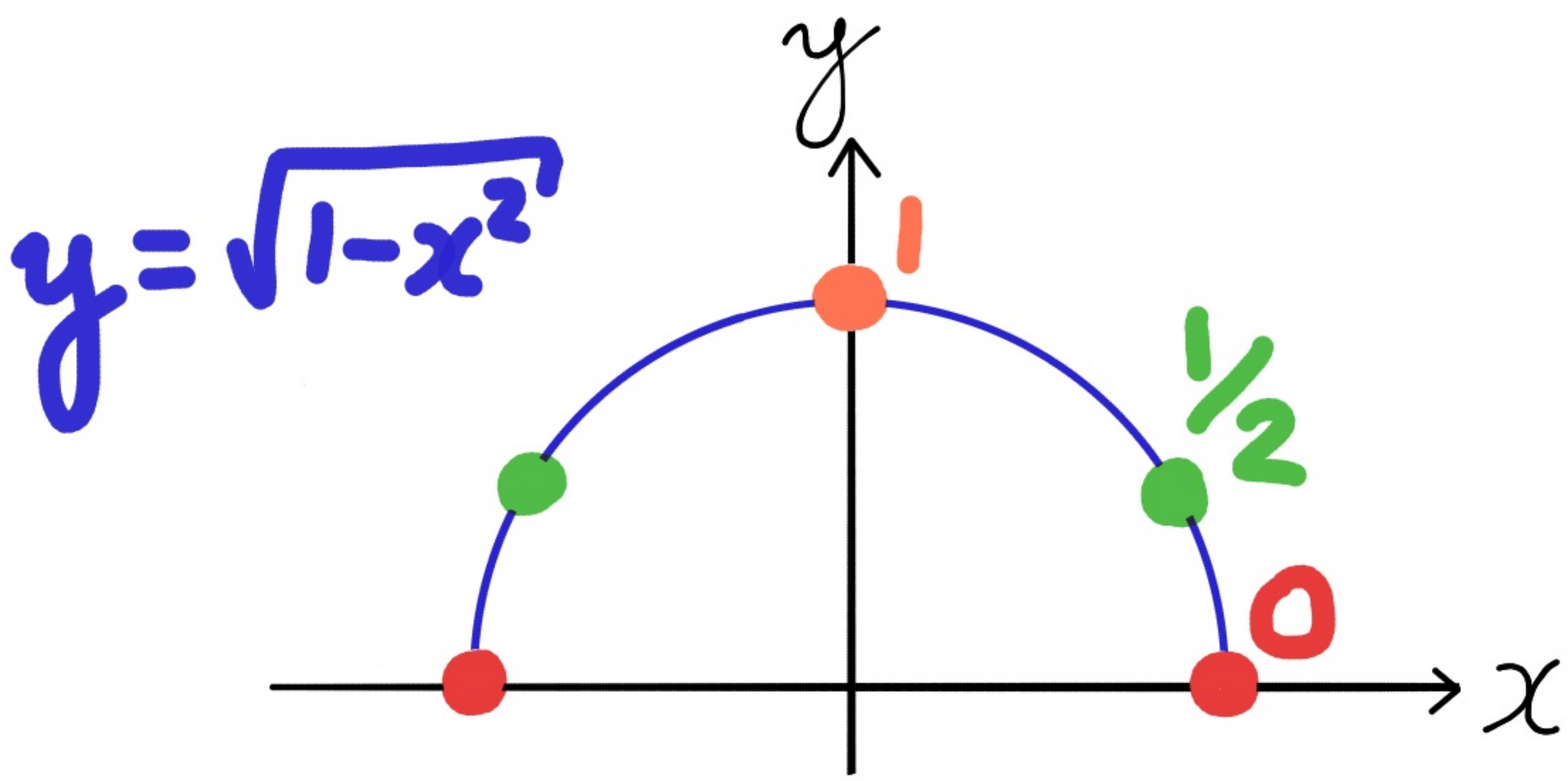
Contour map in $\mathbb{R}^2$ for $f(x,y)=xy$.



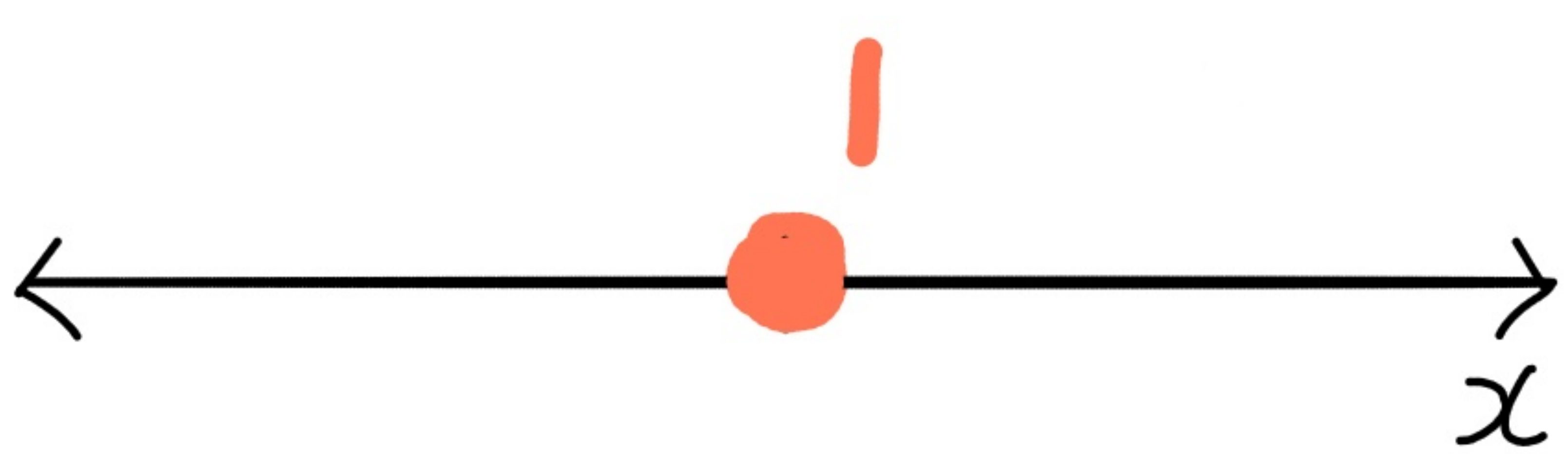
Graph in $\mathbb{R}^3$ for $f(x,y)=xy$.

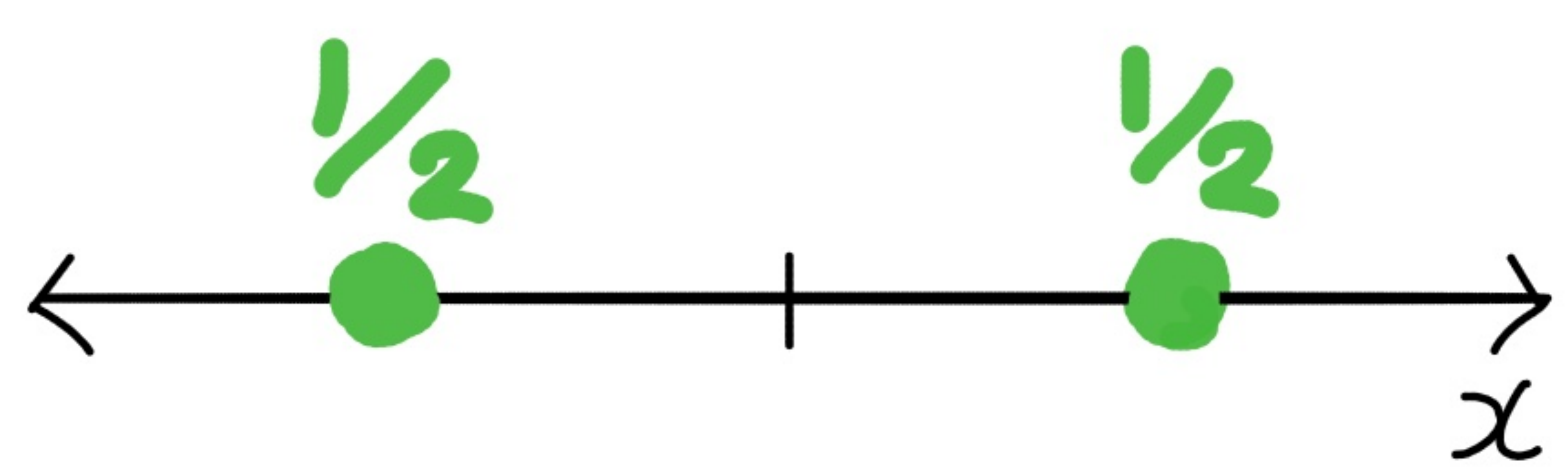
IV Level surfaces

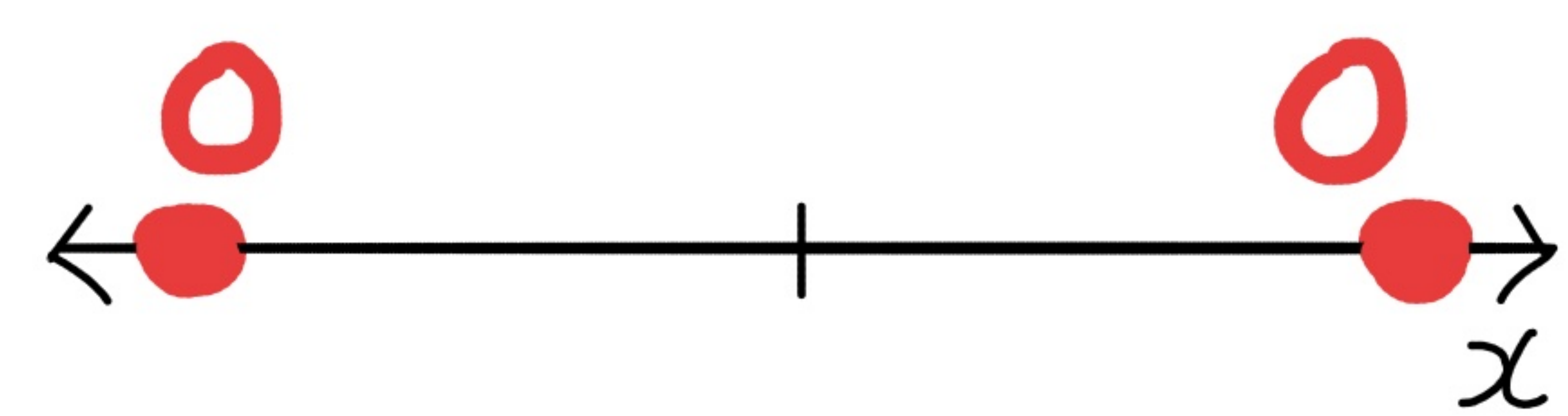
Level points in $\mathbb{R}$ for $f(x) = \sqrt{1-x^2}$.



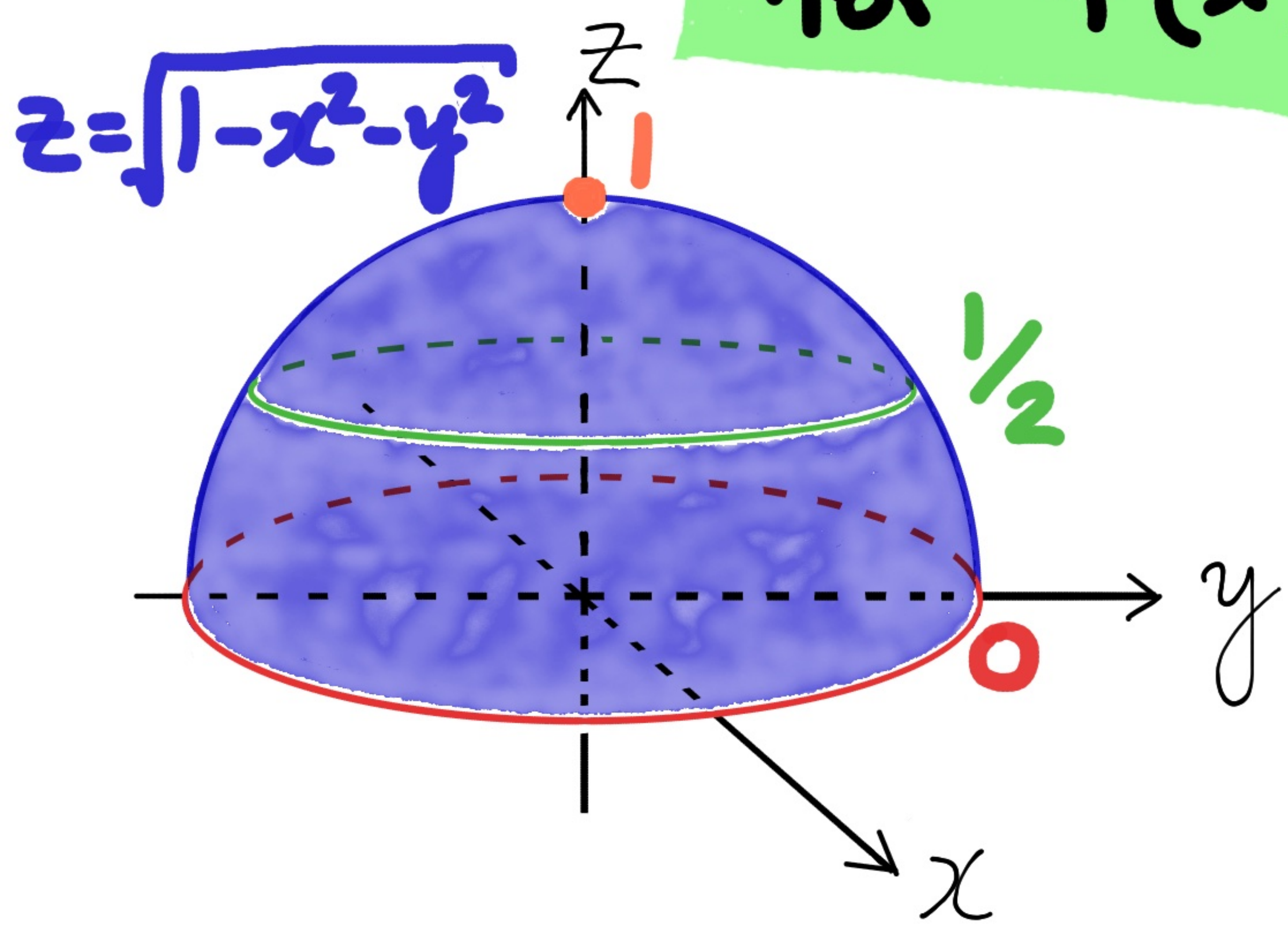
$$y = \sqrt{1-x^2}$$
$$y^2 = 1 - x^2$$
$$x^2 + y^2 = 1$$



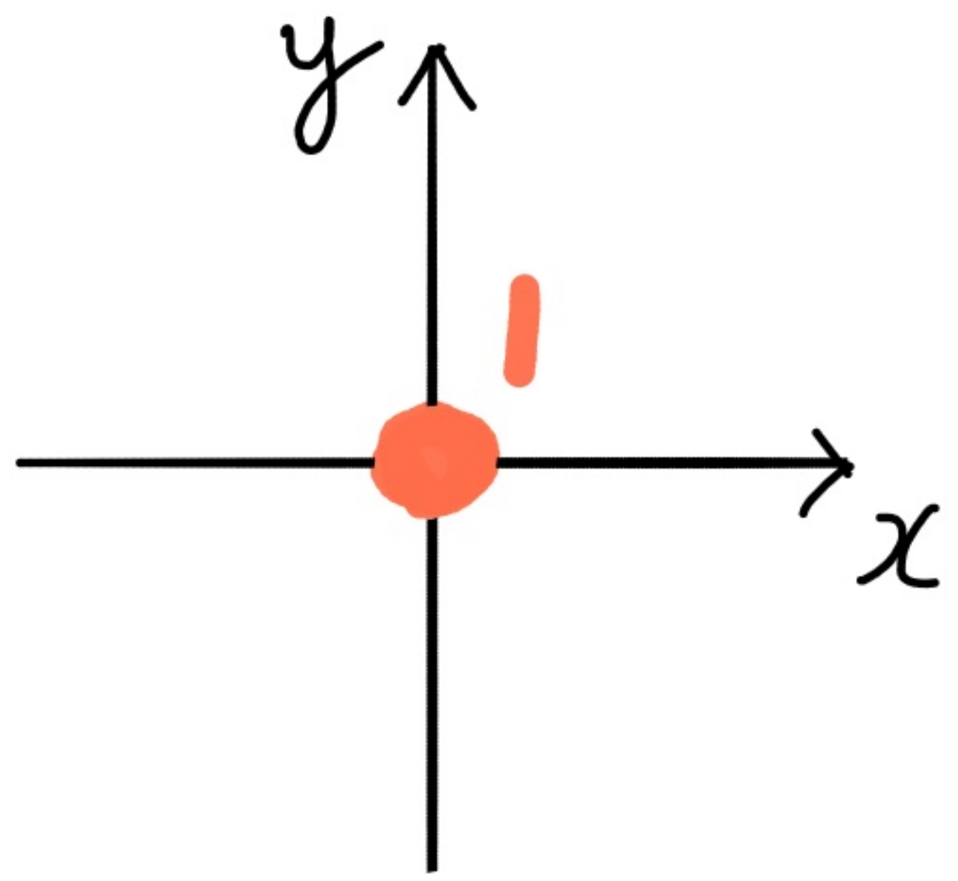


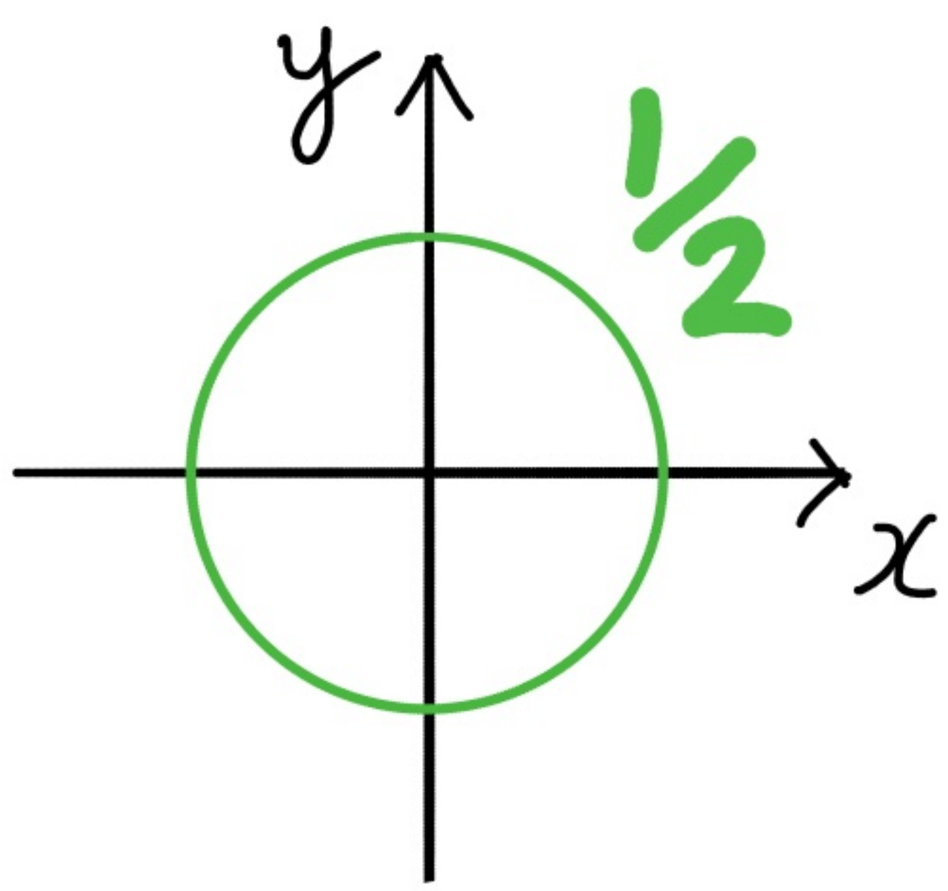


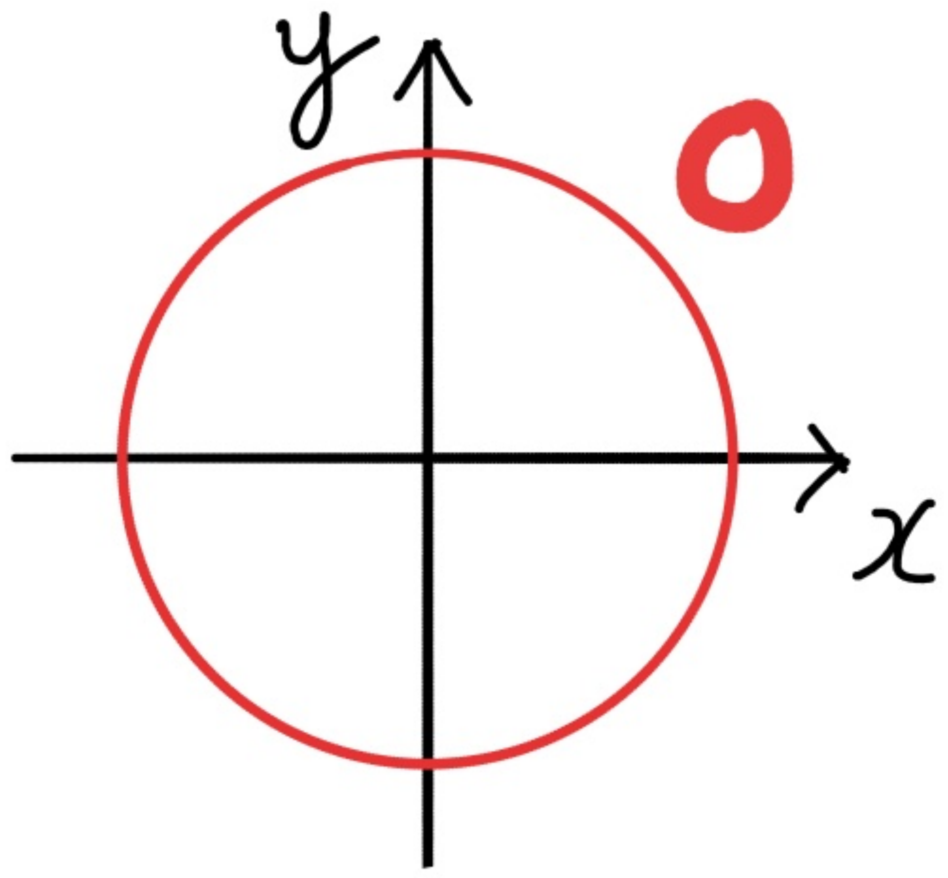
Level curves in $\mathbb{R}^2$ for $f(x,y)=\sqrt{1-x^2-y^2}$.



$$z=\sqrt{1-x^2-y^2}$$
$$z^2=1-x^2-y^2$$
$$x^2+y^2+z^2=1$$





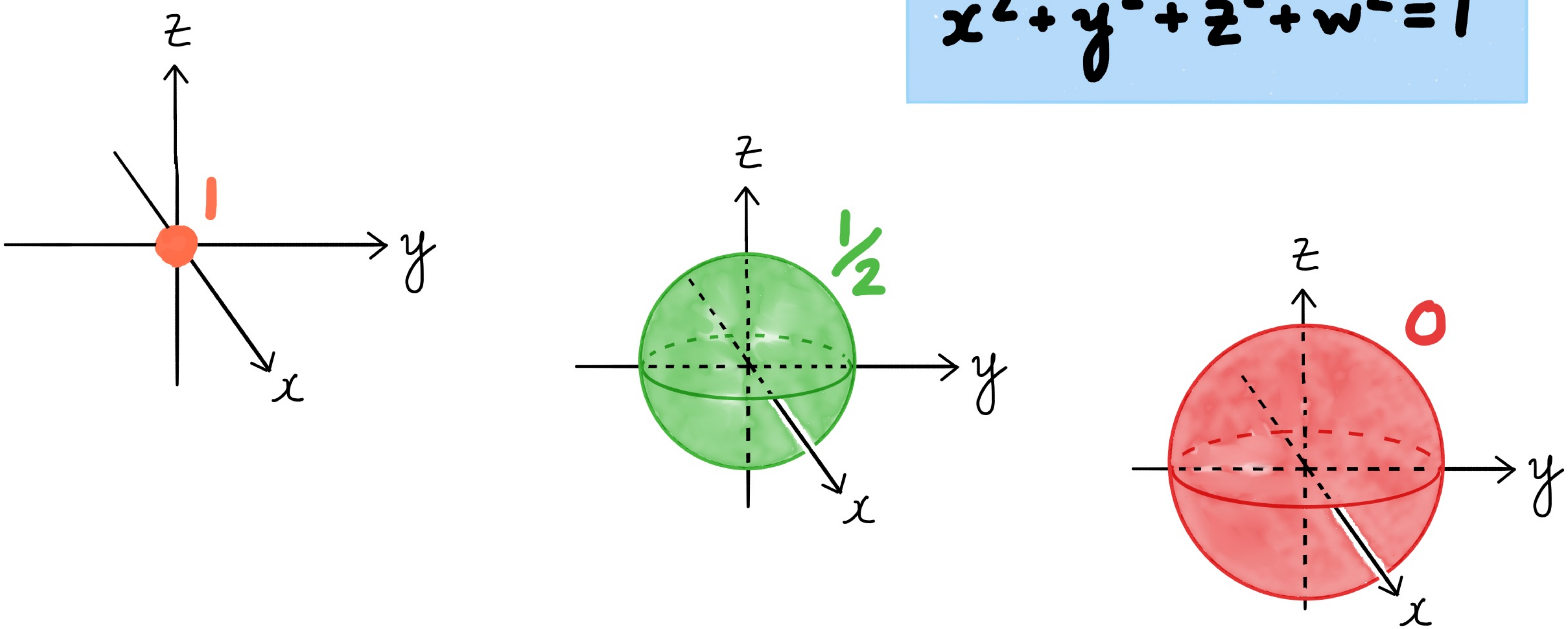


Level surfaces in $\mathbb{R}^3$ for $f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$

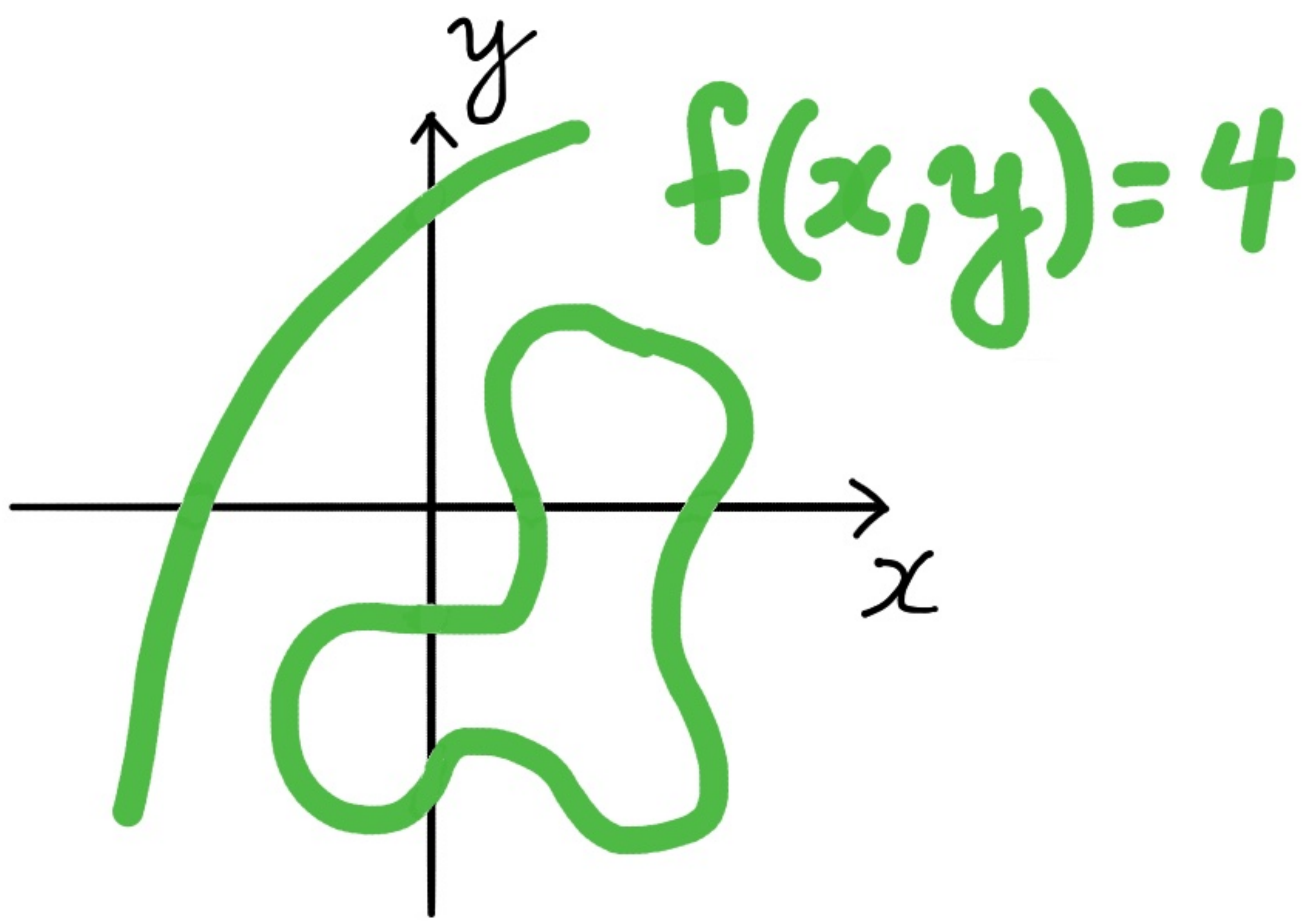
$$w=\sqrt{1-x^2-y^2-z^2}$$

? ?

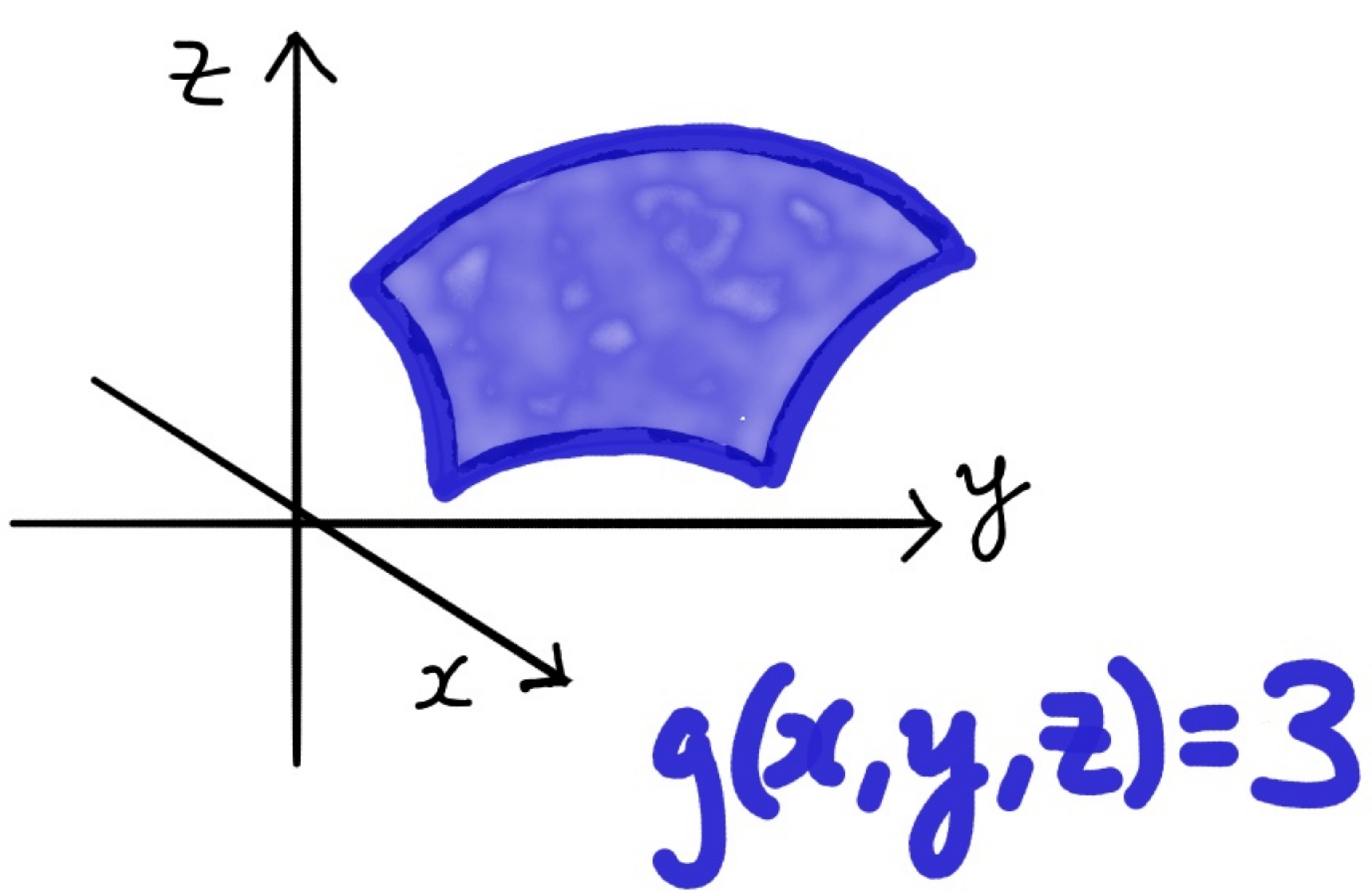
$$w=\sqrt{1-x^2-y^2-z^2}$$
$$w^2=1-x^2-y^2-z^2$$
$$x^2+y^2+z^2+w^2=1$$



A **level curve** for $f(x,y)=z$ is a sketch in $\mathbb{R}^2$ of the solutions of $f(x,y)=\text{constant}$.



A **level surface** for $g(x,y,z)=w$ is a sketch in $\mathbb{R}^3$ of the solutions of $g(x,y,z)=\text{constant}$.



Examples:

① Planes are level surfaces in $\mathbb{R}^3$.

$$f(x,y,z) = 3(x-2) + 4(y-1) + 8(z-5).$$

$f(x,y,z)=0$ is the plane containing $(2,1,5)$ and normal to $(3,4,8)$.

② Spheres are level surfaces in $\mathbb{R}^3$.

$$g(x,y,z) = (x-1)^2 + (y-3)^2 + (z-7)^2 - 4^2.$$

$g(x,y,z)=0$ is the sphere with center $(1,3,7)$ and radius 4.