

Seven

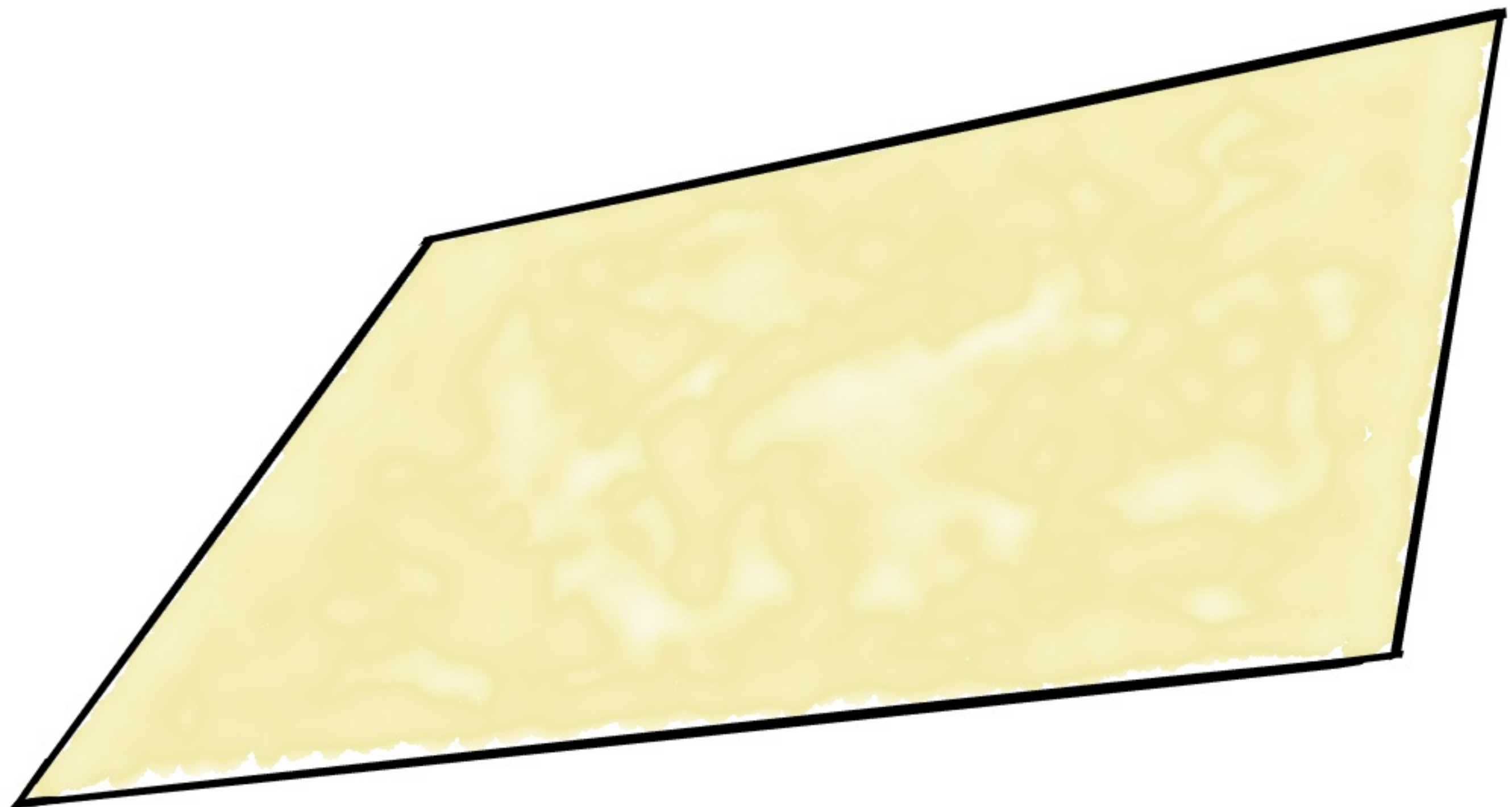
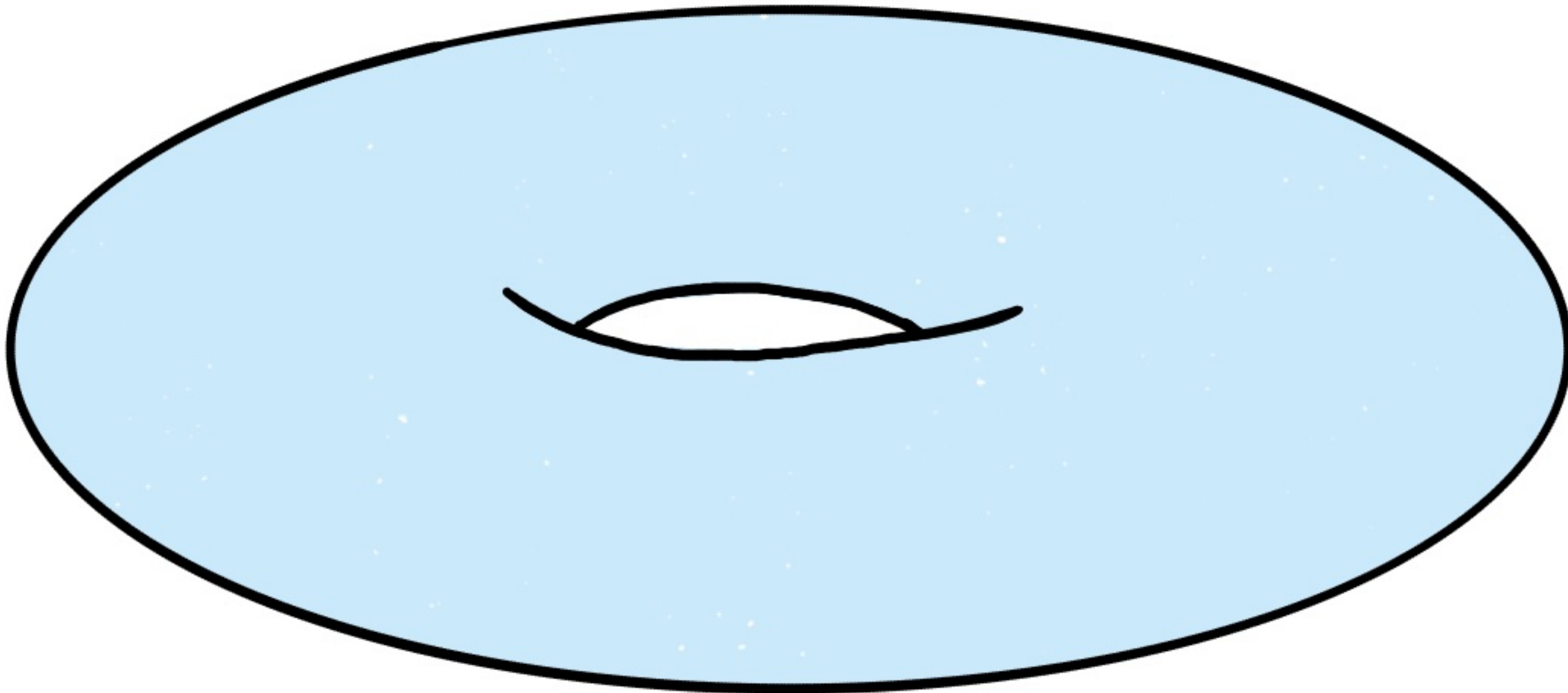
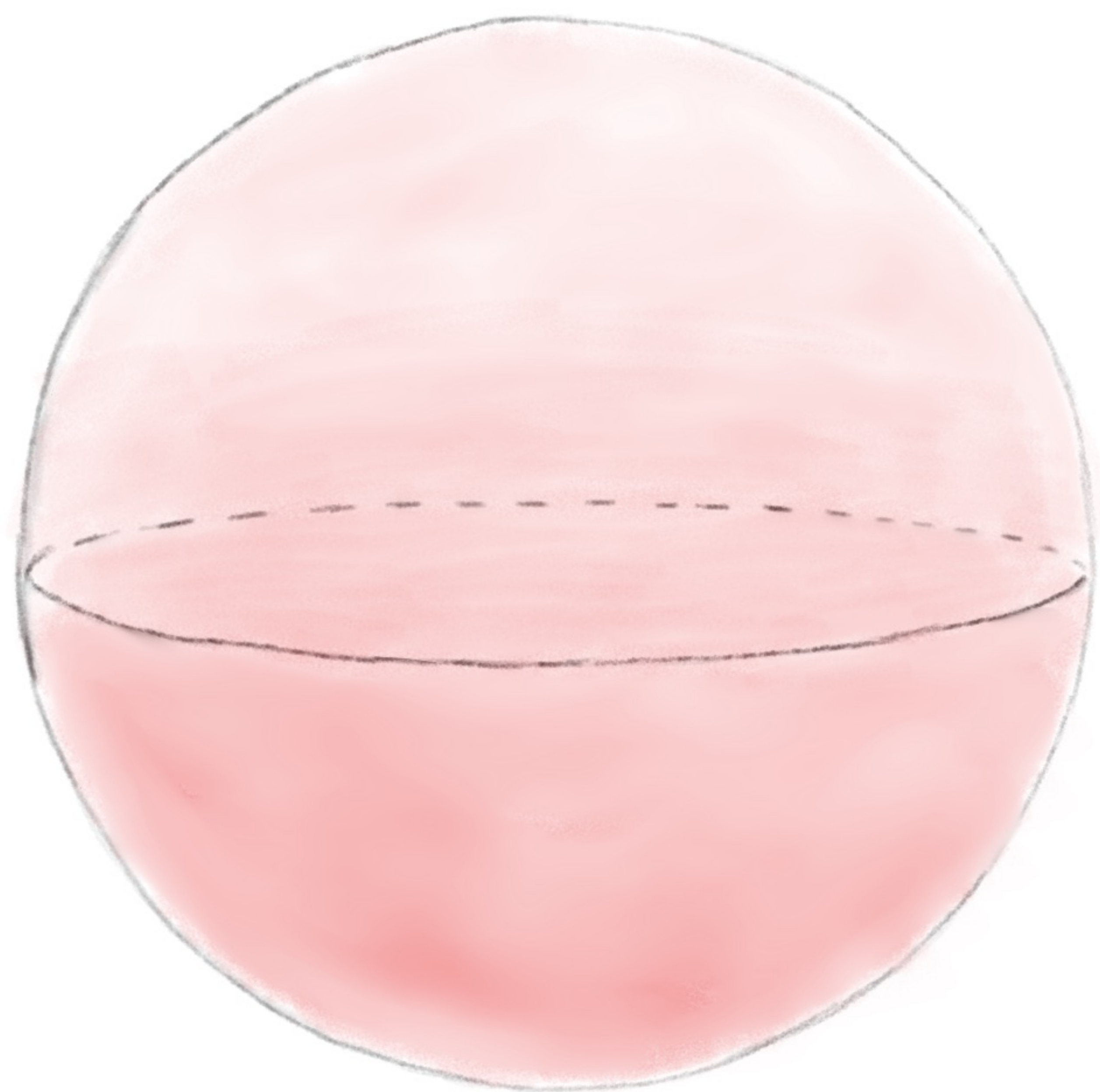
① Surfaces

② Curvature of surfaces

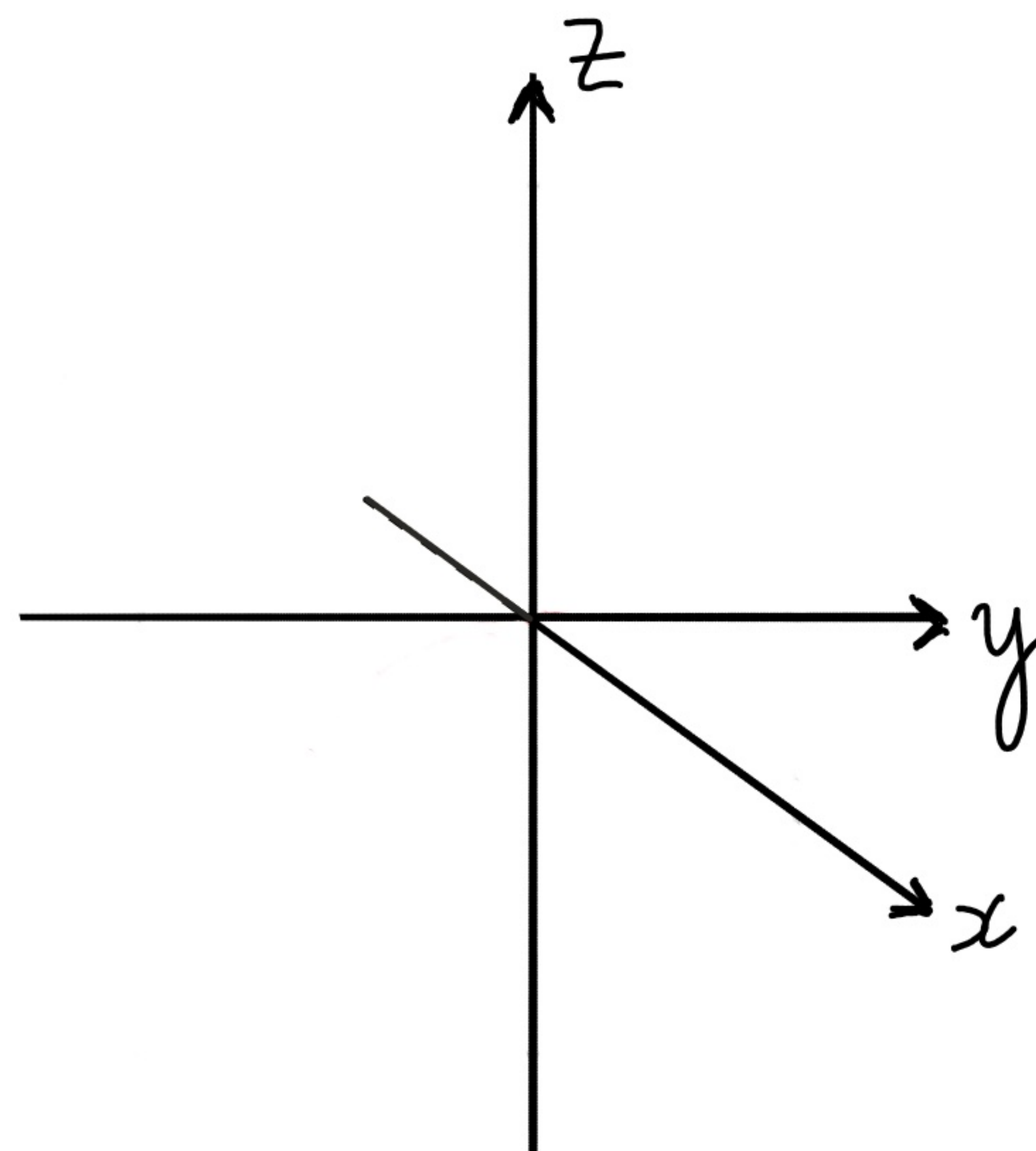
③ Equation of a sphere

① Surfaces

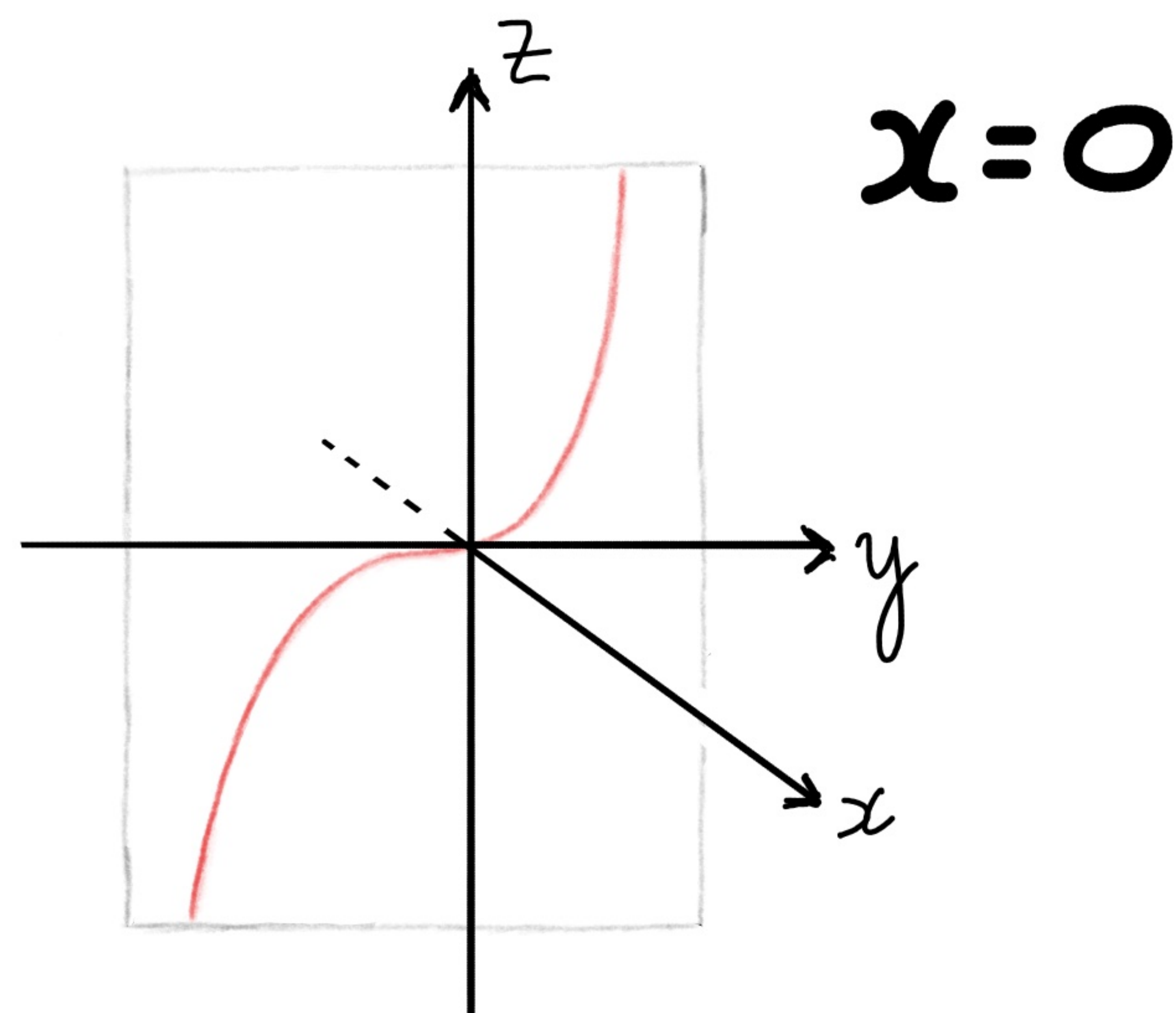
Surfaces



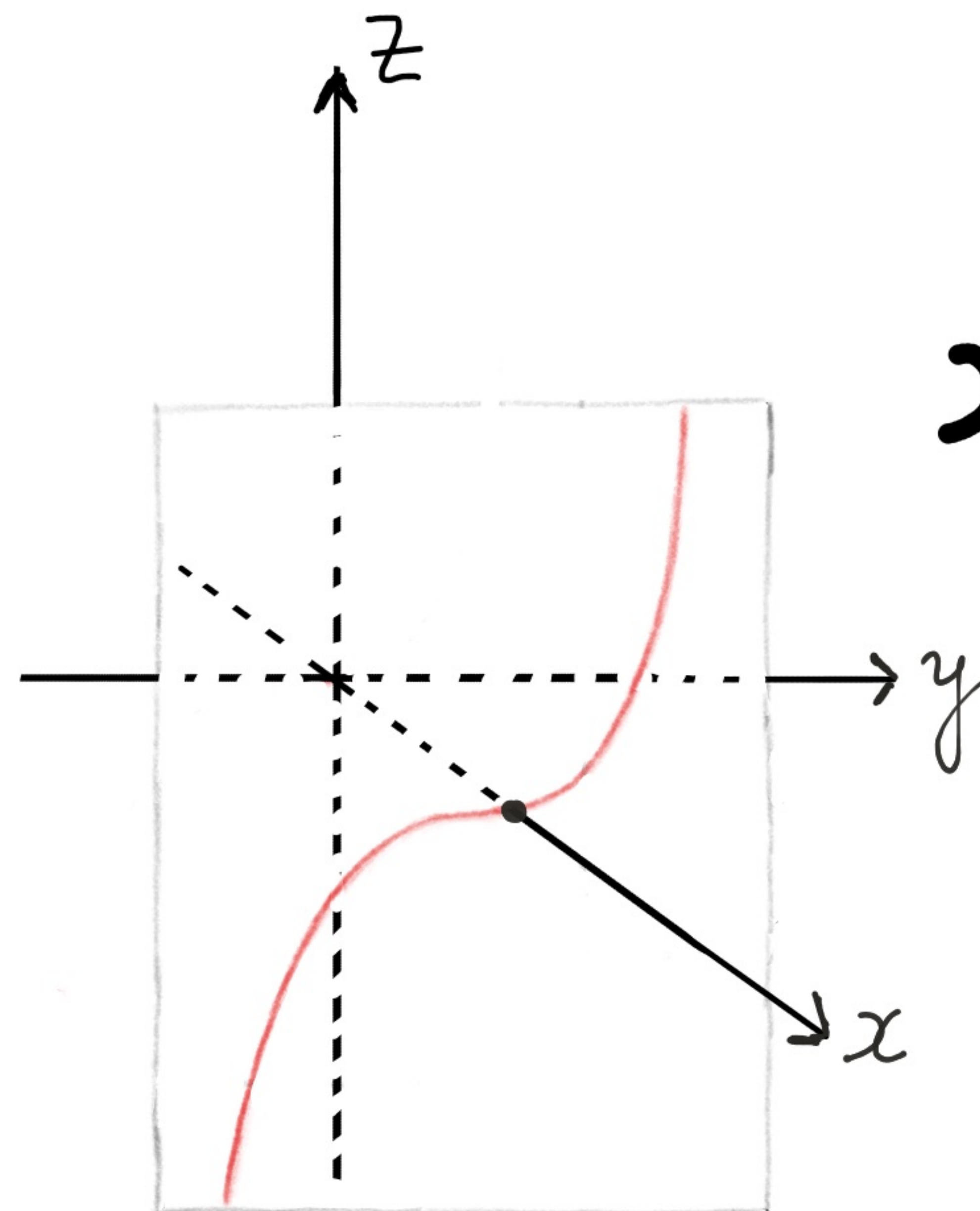
$$y^3 = z$$



$$y^3 = z$$



$$y^3 = z$$



$$x = 1$$

$$y^3 = z$$

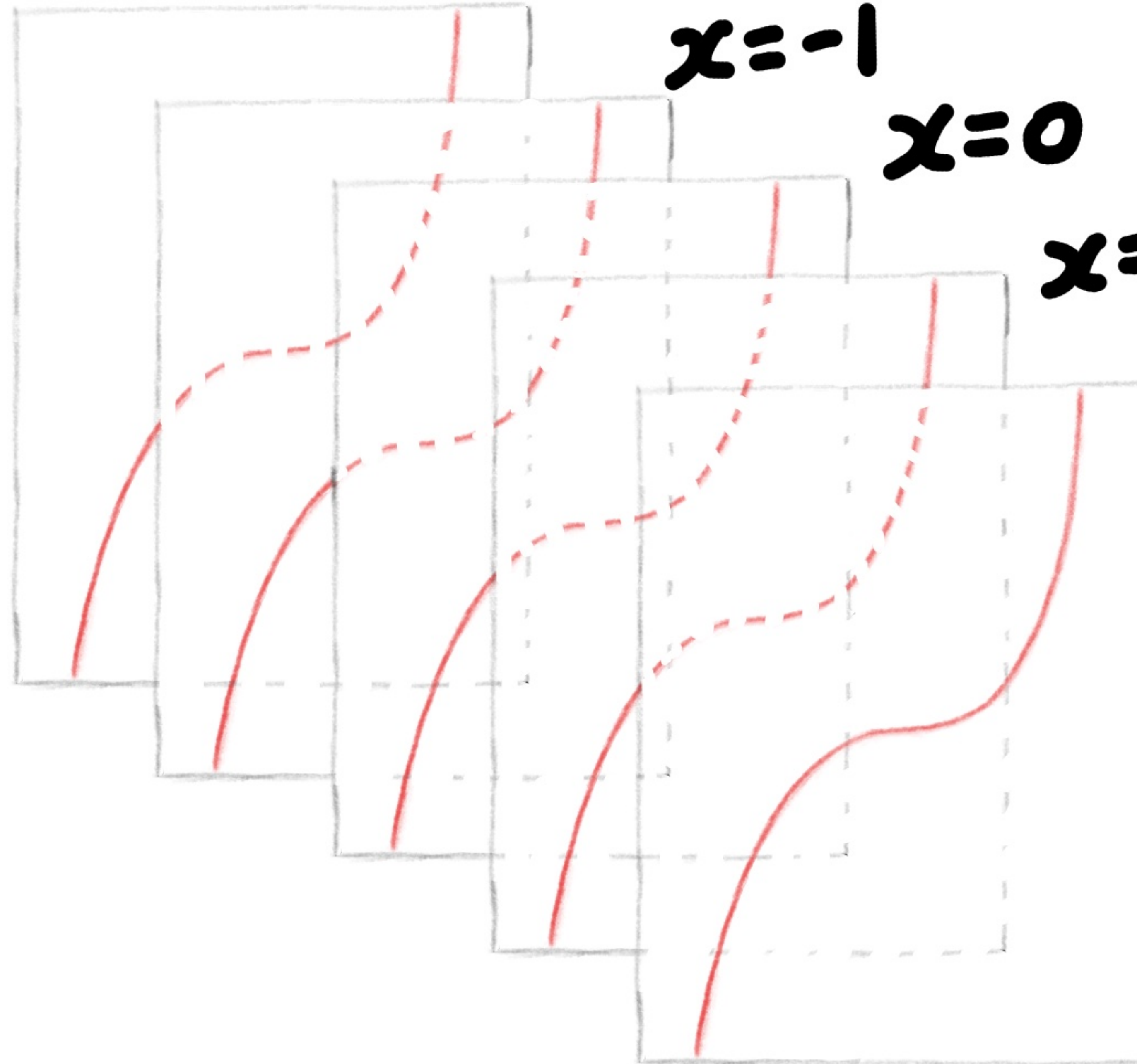
$$x = -2$$

$$x = -1$$

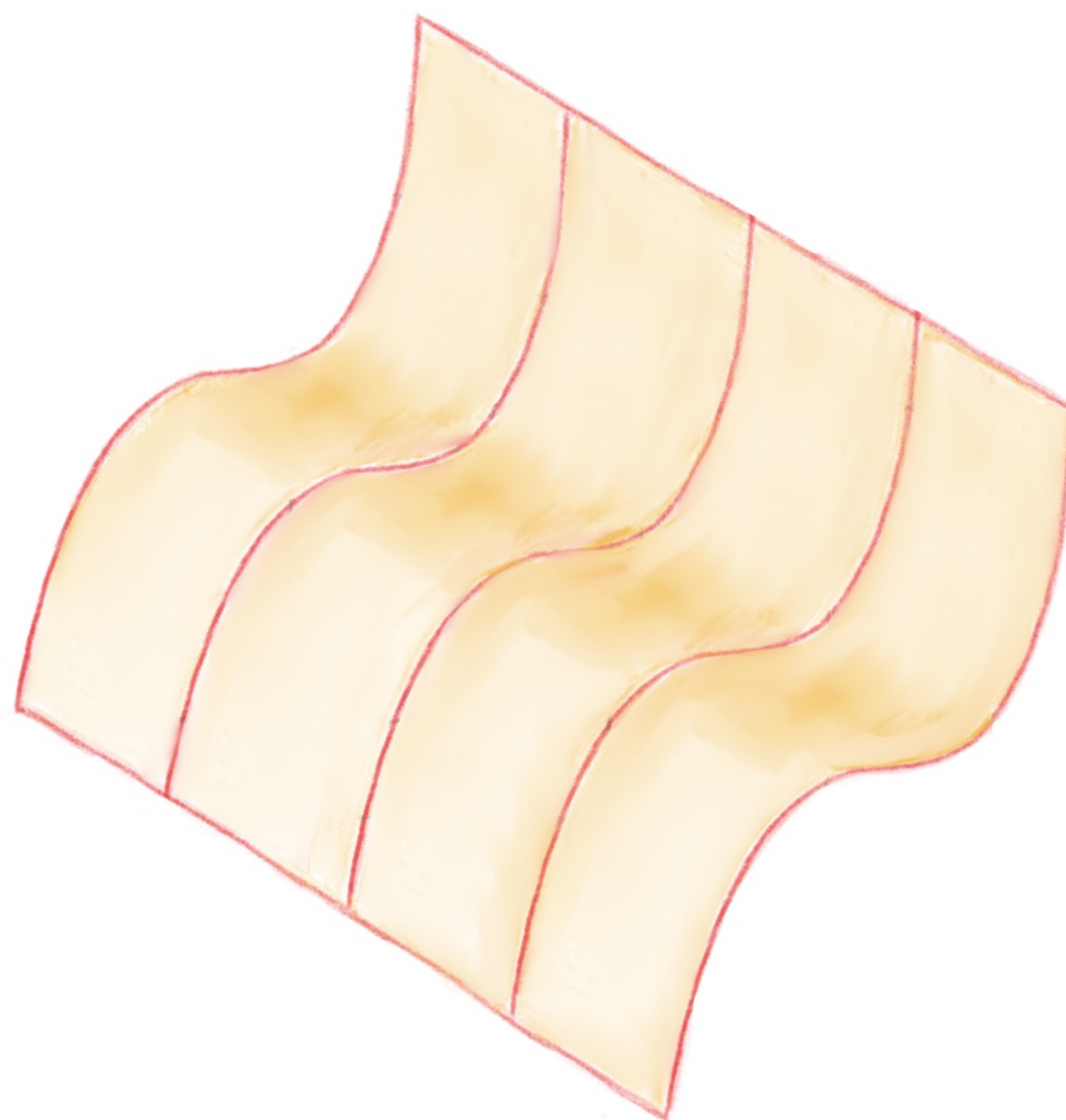
$$x = 0$$

$$x = 1$$

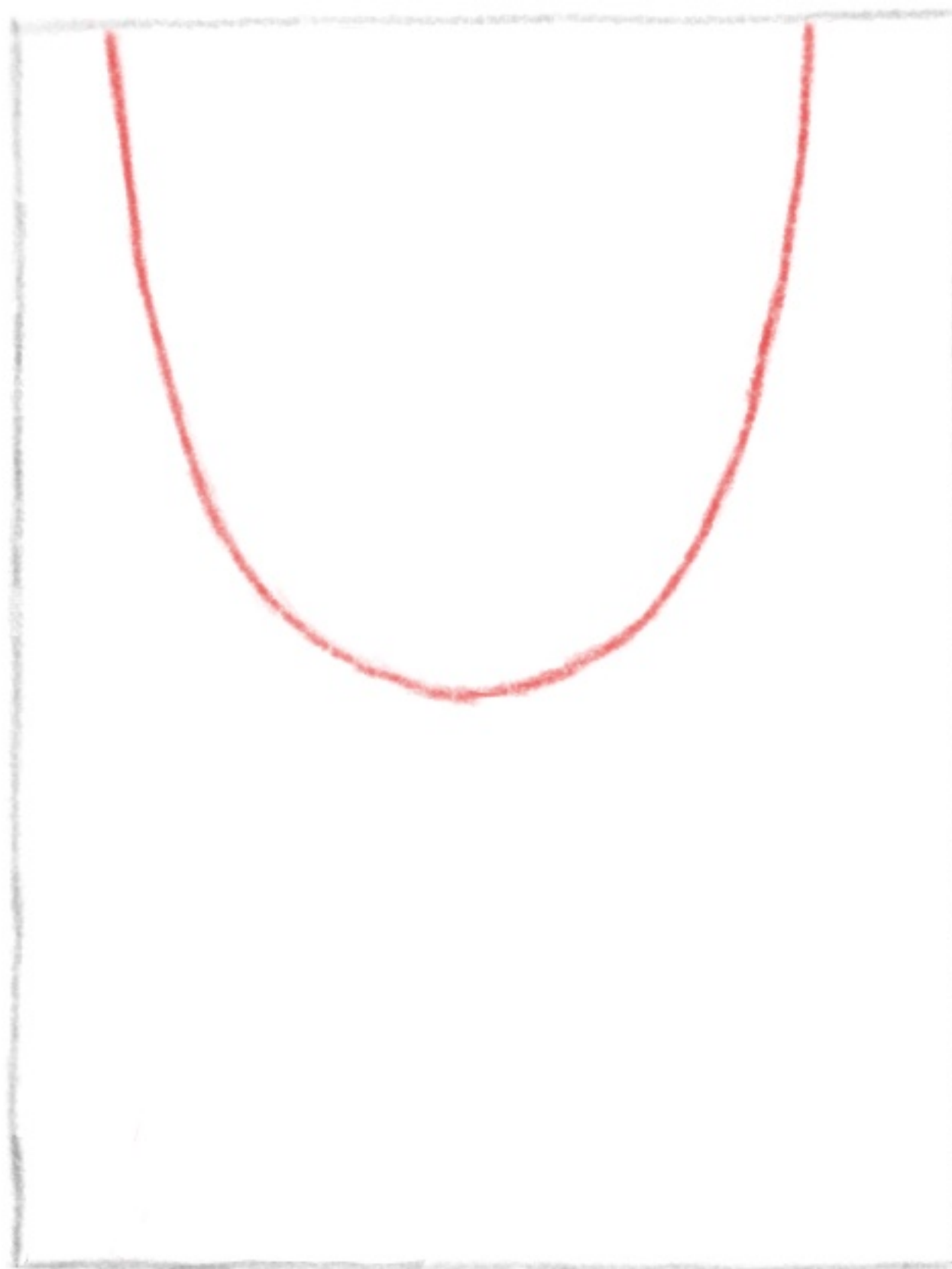
$$x = 2$$



$$y^3 = z$$



$$y^2 = z$$



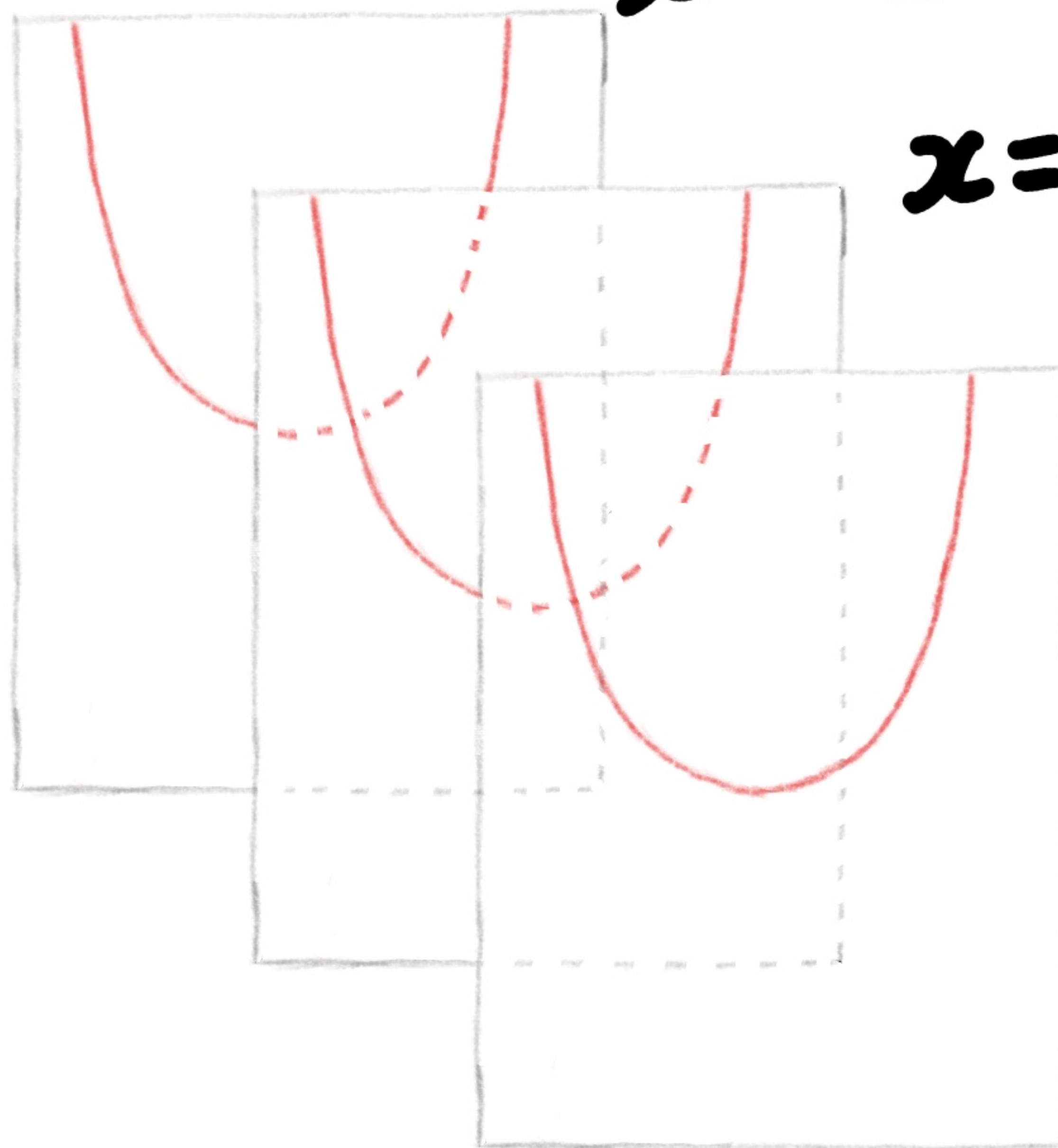
$$x=0$$

$$y^2 = z$$

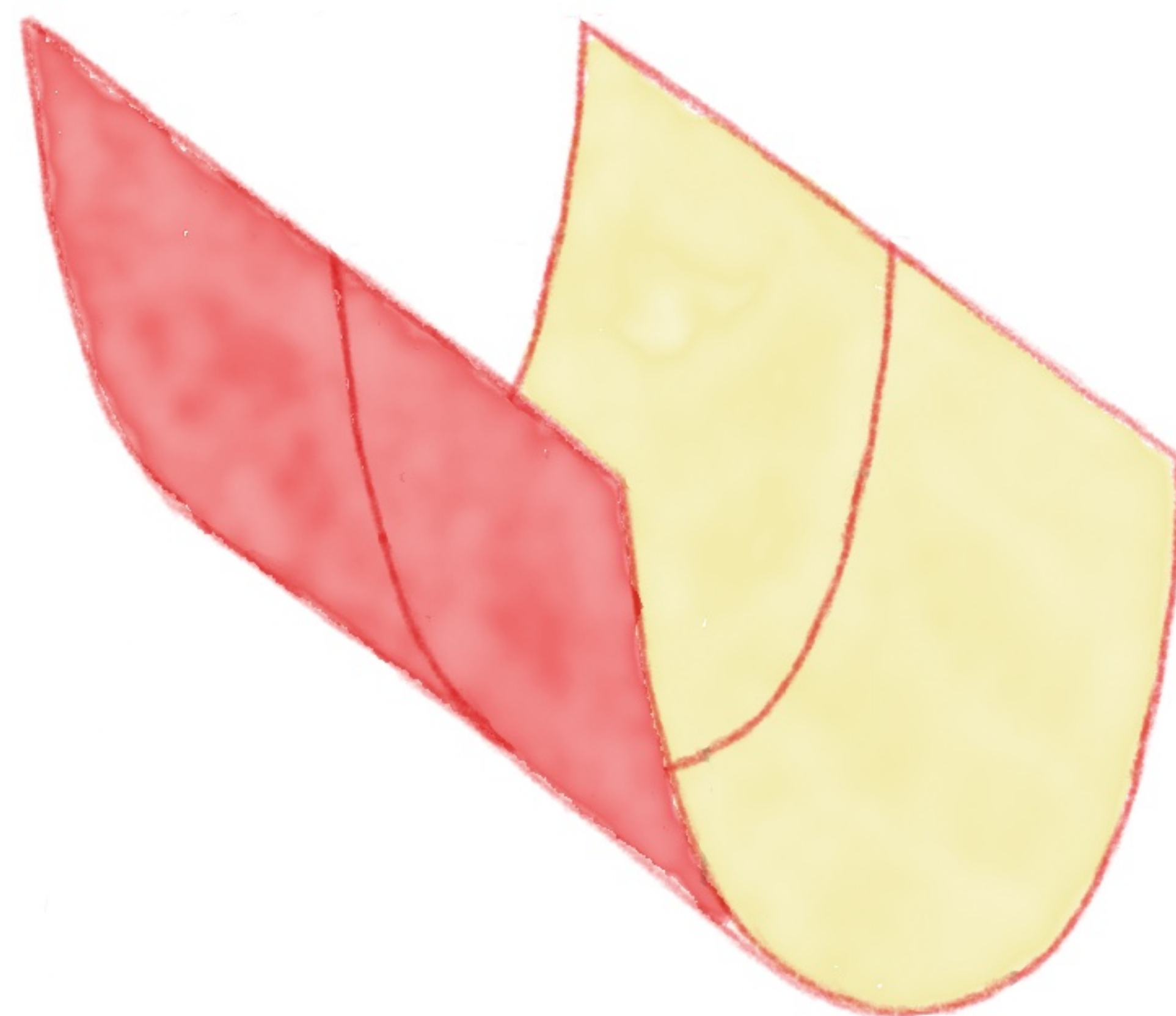
$$x = -3$$

$$x = 0$$

$$x = 3$$



$$y^2 = z$$



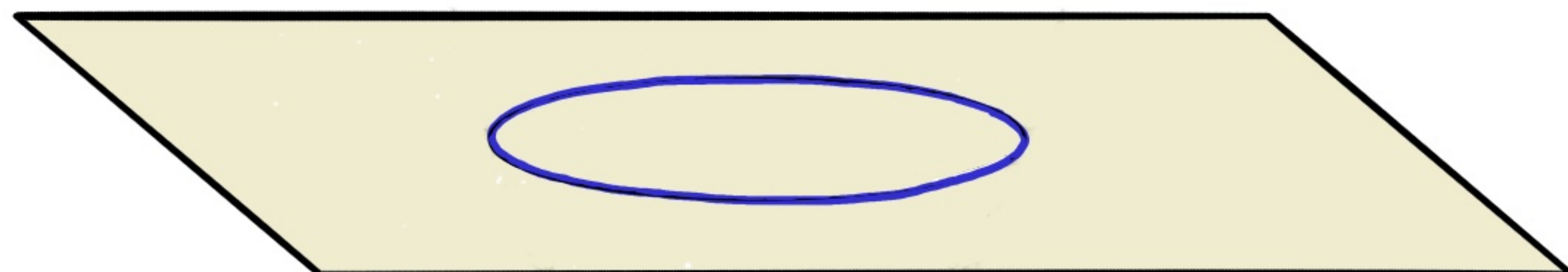
$$3x^2 + y^2 = z$$

$$3x^2 + y^2 = z$$



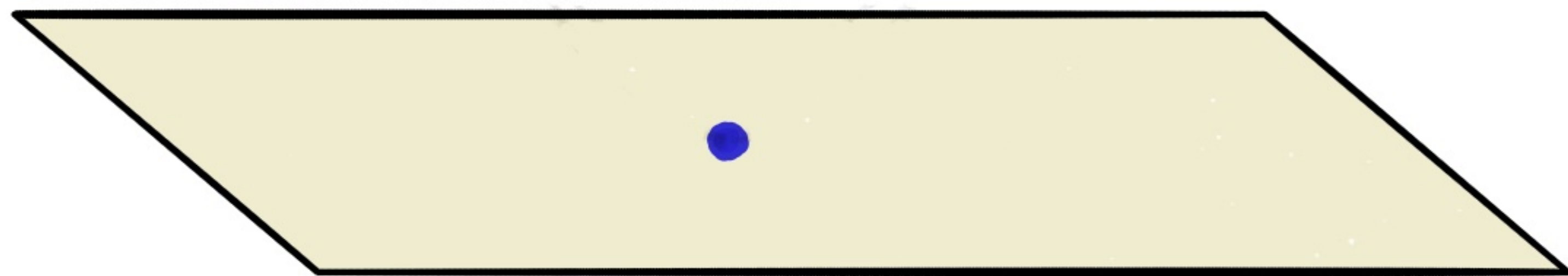
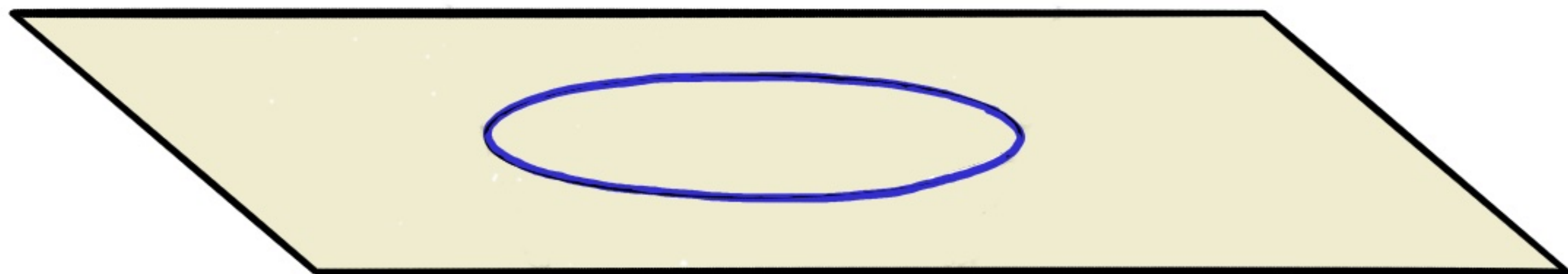
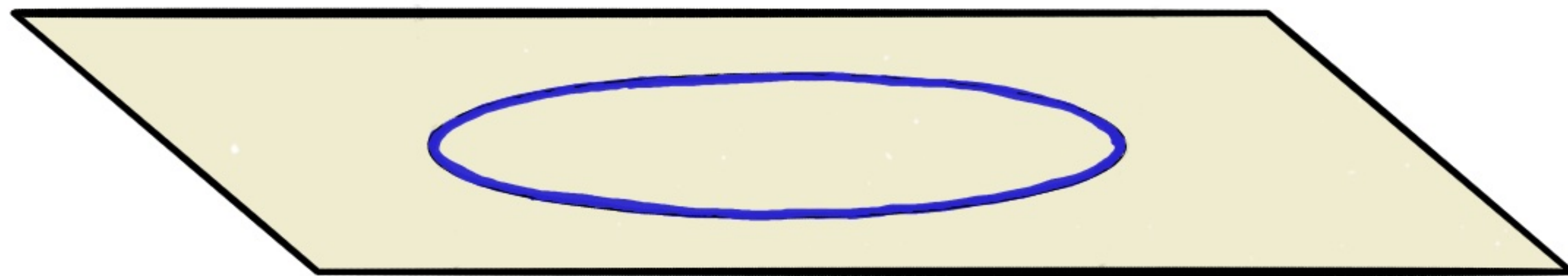
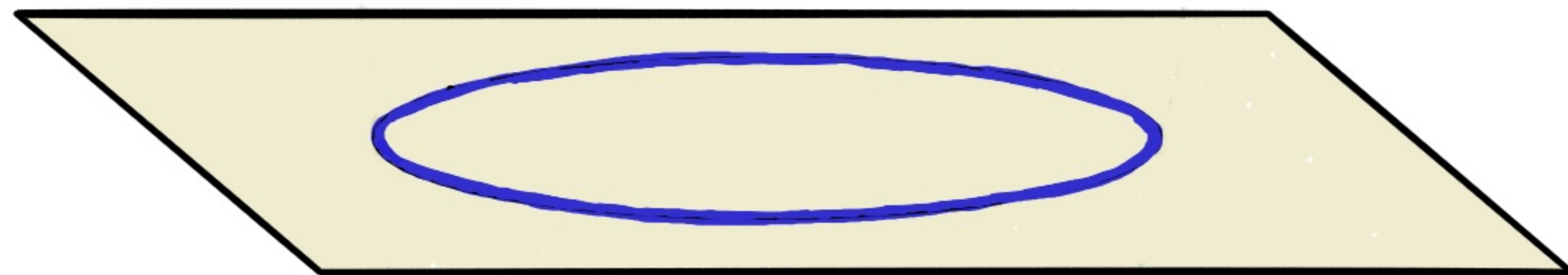
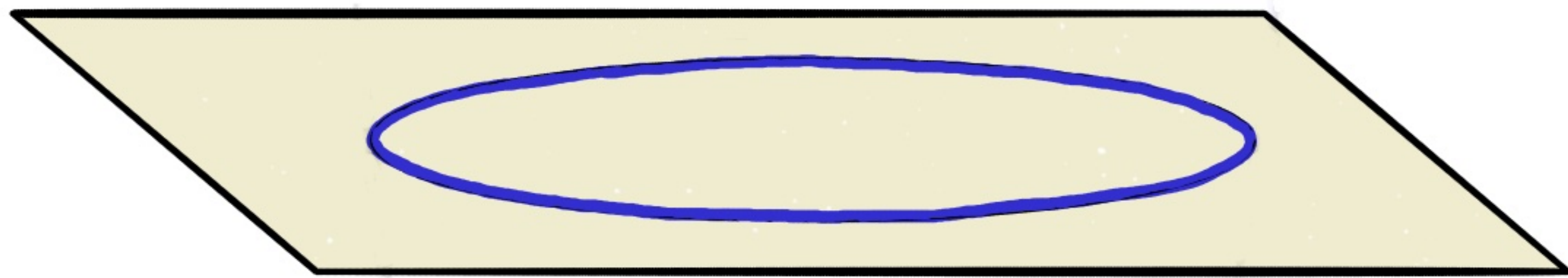
$$z=1$$

$$3x^2 + y^2 = z$$



$$z=1, \quad 3x^2 + y^2 = 1$$

$$3x^2 + y^2 = z$$



$$z=4, \quad 3x^2 + y^2 = 4$$

$$z=3, \quad 3x^2 + y^2 = 3$$

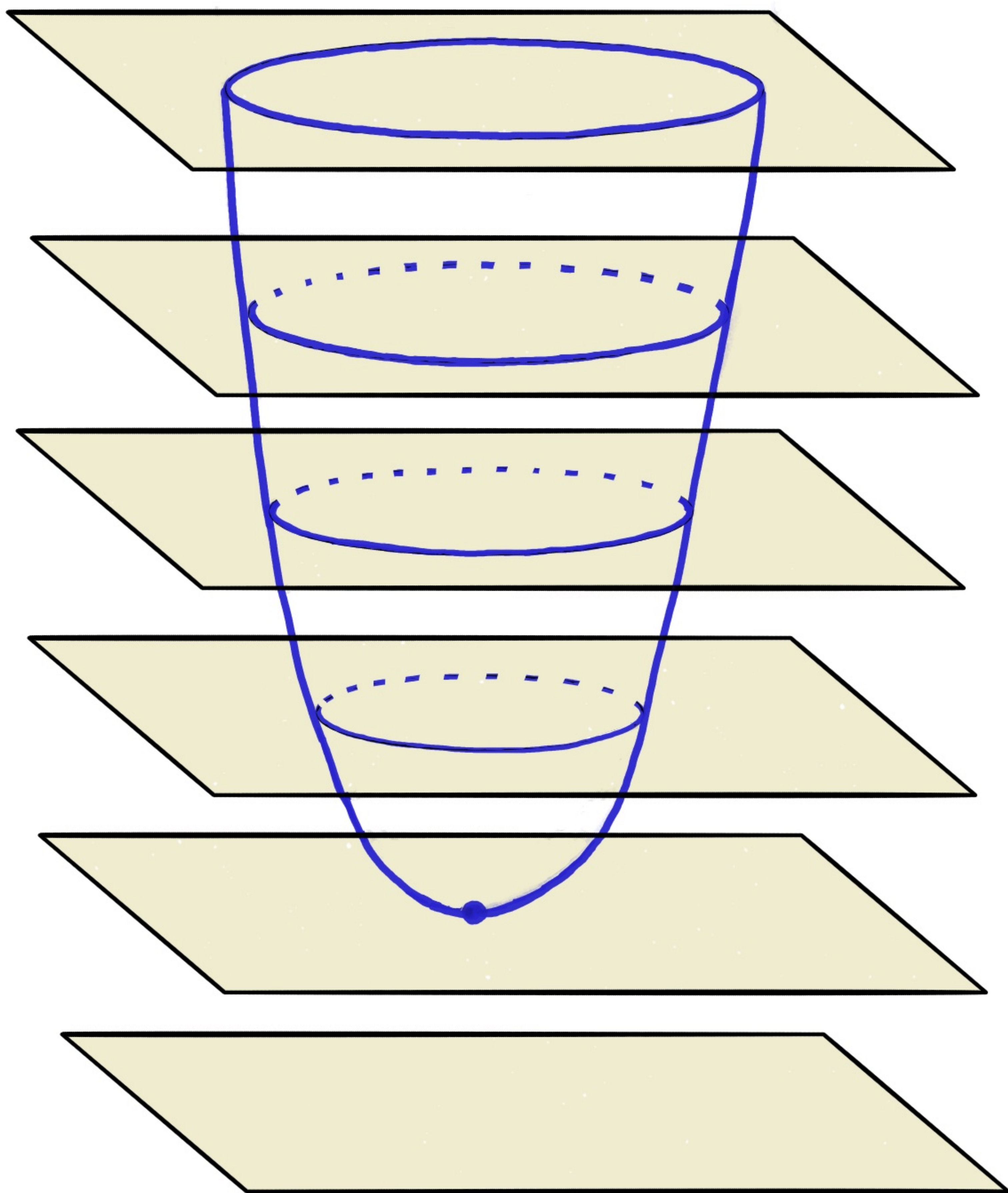
$$z=2, \quad 3x^2 + y^2 = 2$$

$$z=1, \quad 3x^2 + y^2 = 1$$

$$z=0, \quad 3x^2 + y^2 = 0$$

$$z=-1, \quad 3x^2 + y^2 = -1$$

$$3x^2 + y^2 = z$$



$$z=4, \quad 3x^2 + y^2 = 4$$

$$z=3, \quad 3x^2 + y^2 = 3$$

$$z=2, \quad 3x^2 + y^2 = 2$$

$$z=1, \quad 3x^2 + y^2 = 1$$

$$z=0, \quad 3x^2 + y^2 = 0$$

$$z=-1, \quad 3x^2 + y^2 = -1$$

$$3x^2 + y^2 = z$$

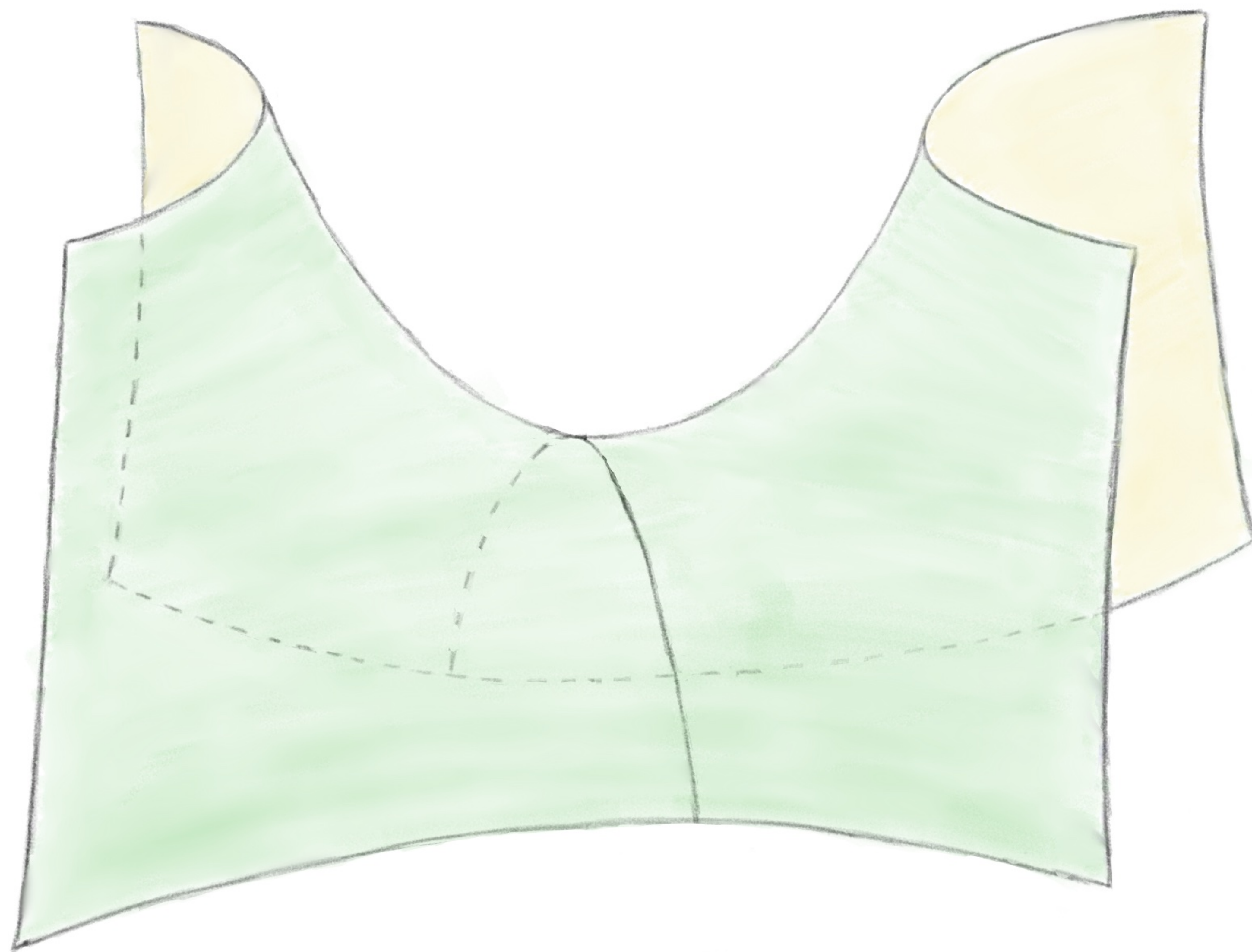




$$3x^2 + y^2 = z$$

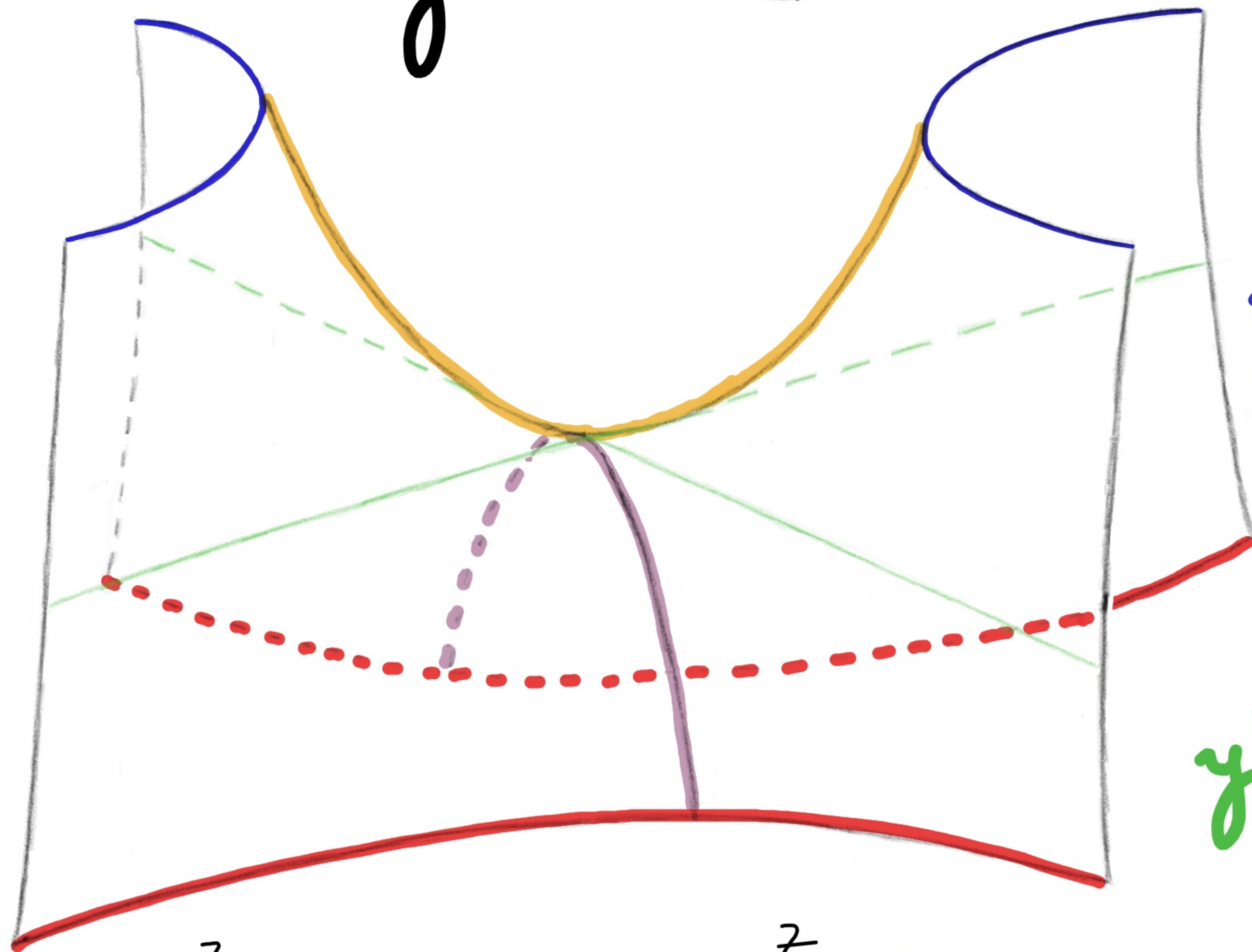
$$-3x^2 - y^2 = z$$





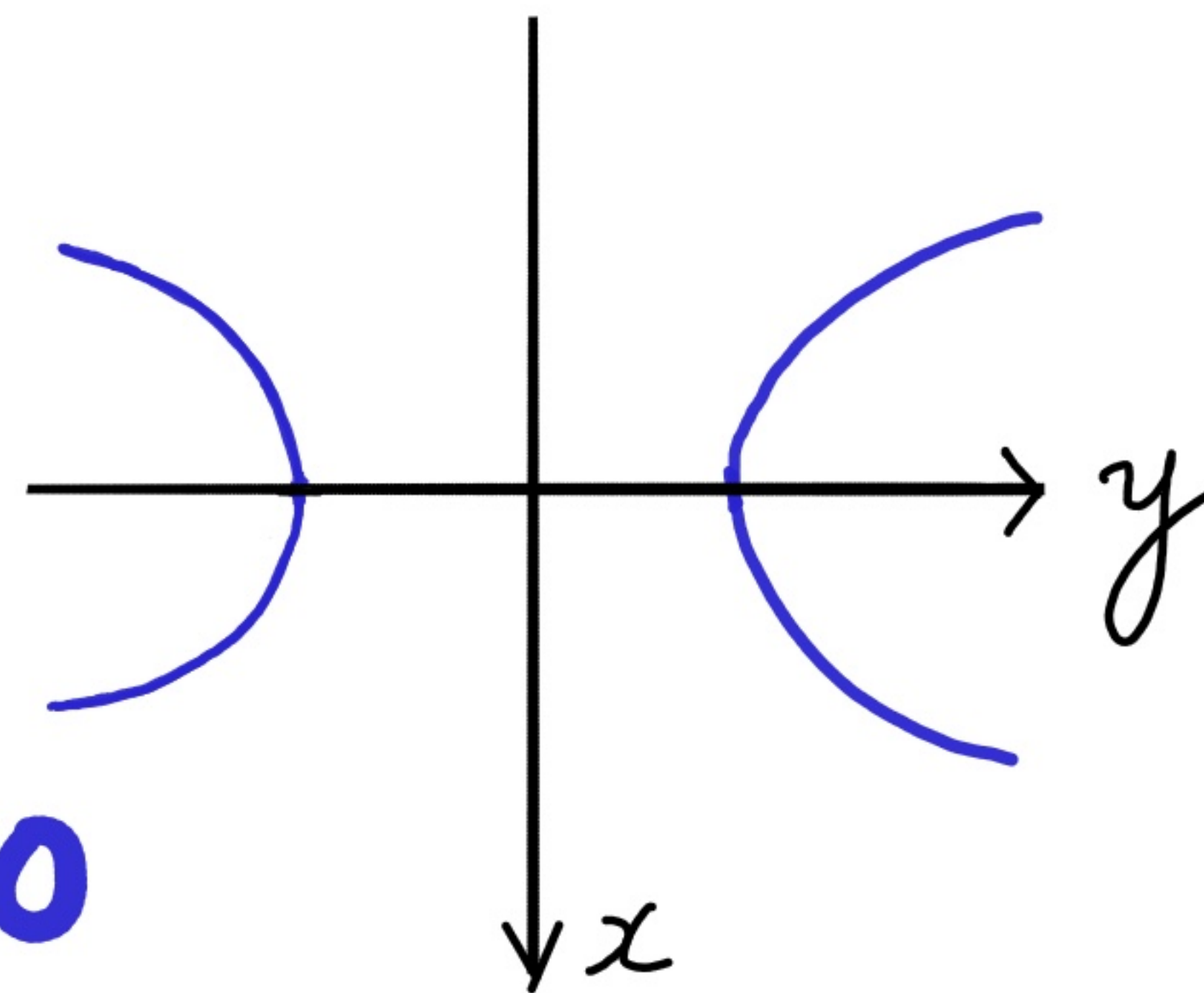
$$y^2 - x^2 = z$$

$$y^2 - x^2 = z$$



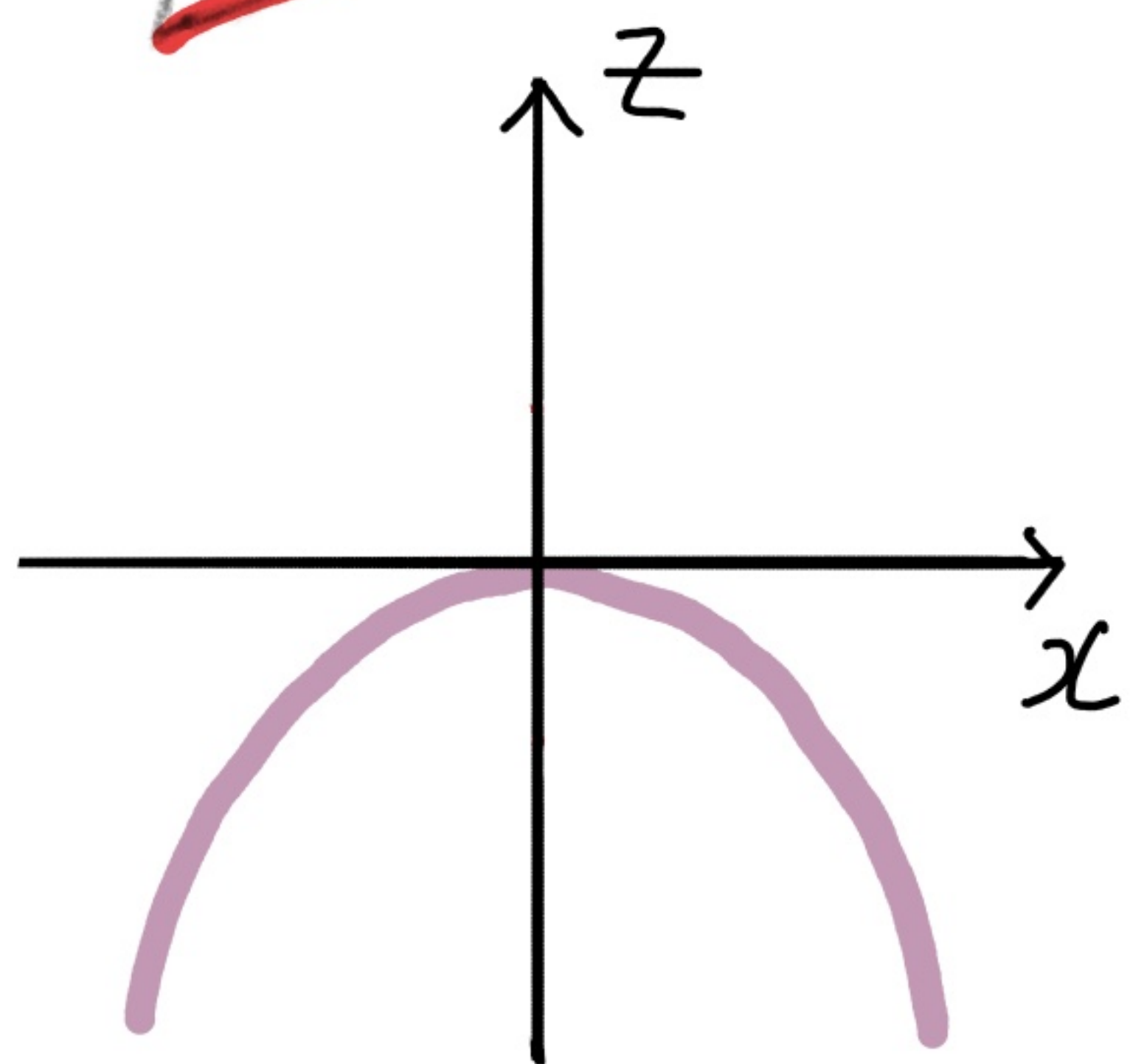
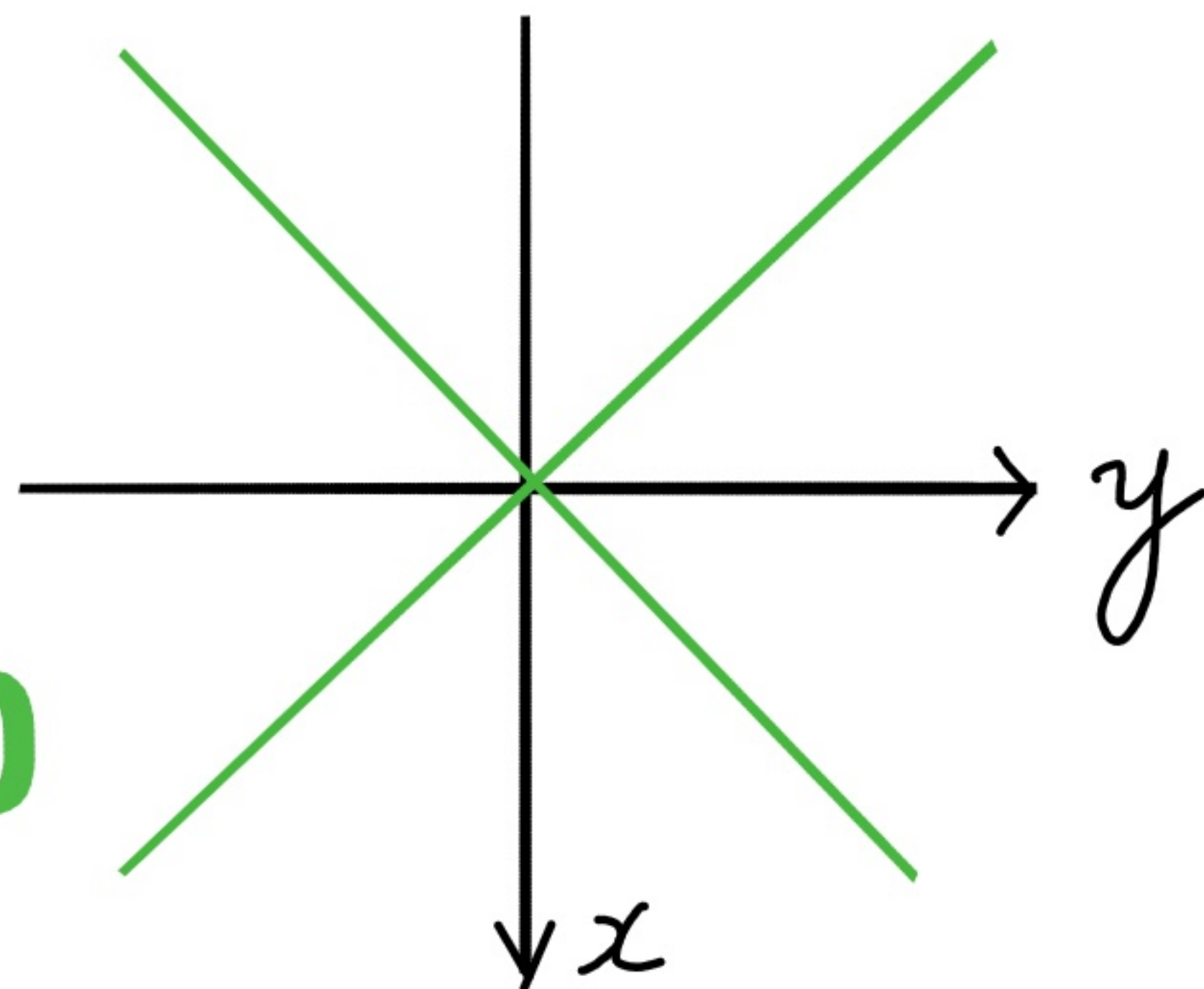
$$z=10$$

$$y^2 - x^2 = 10$$

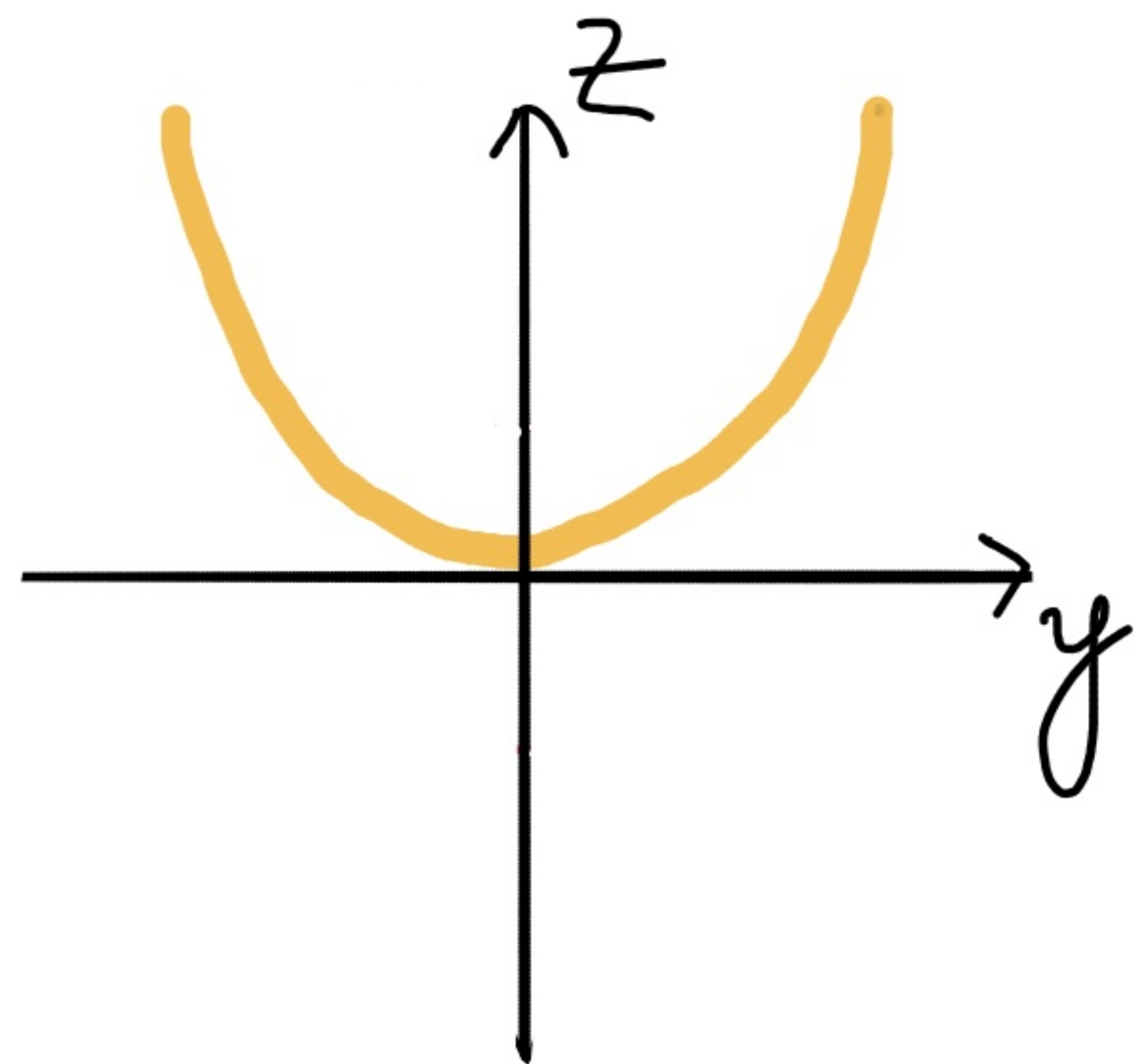


$$z=0$$

$$y^2 - x^2 = 0$$



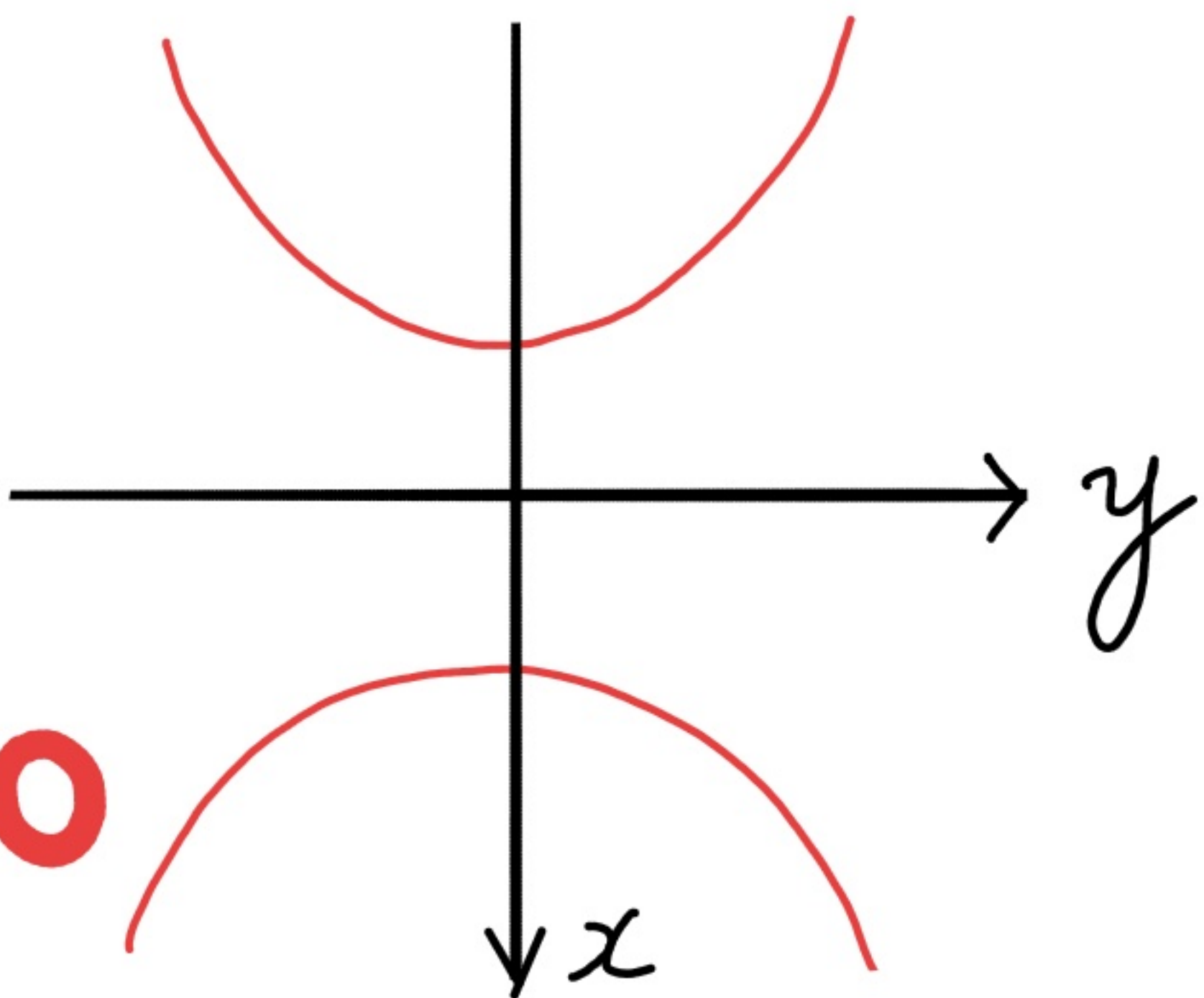
$$y=0, -x^2=z$$



$$x=0, y^2=z$$

$$z=-10$$

$$y^2 - x^2 = -10$$

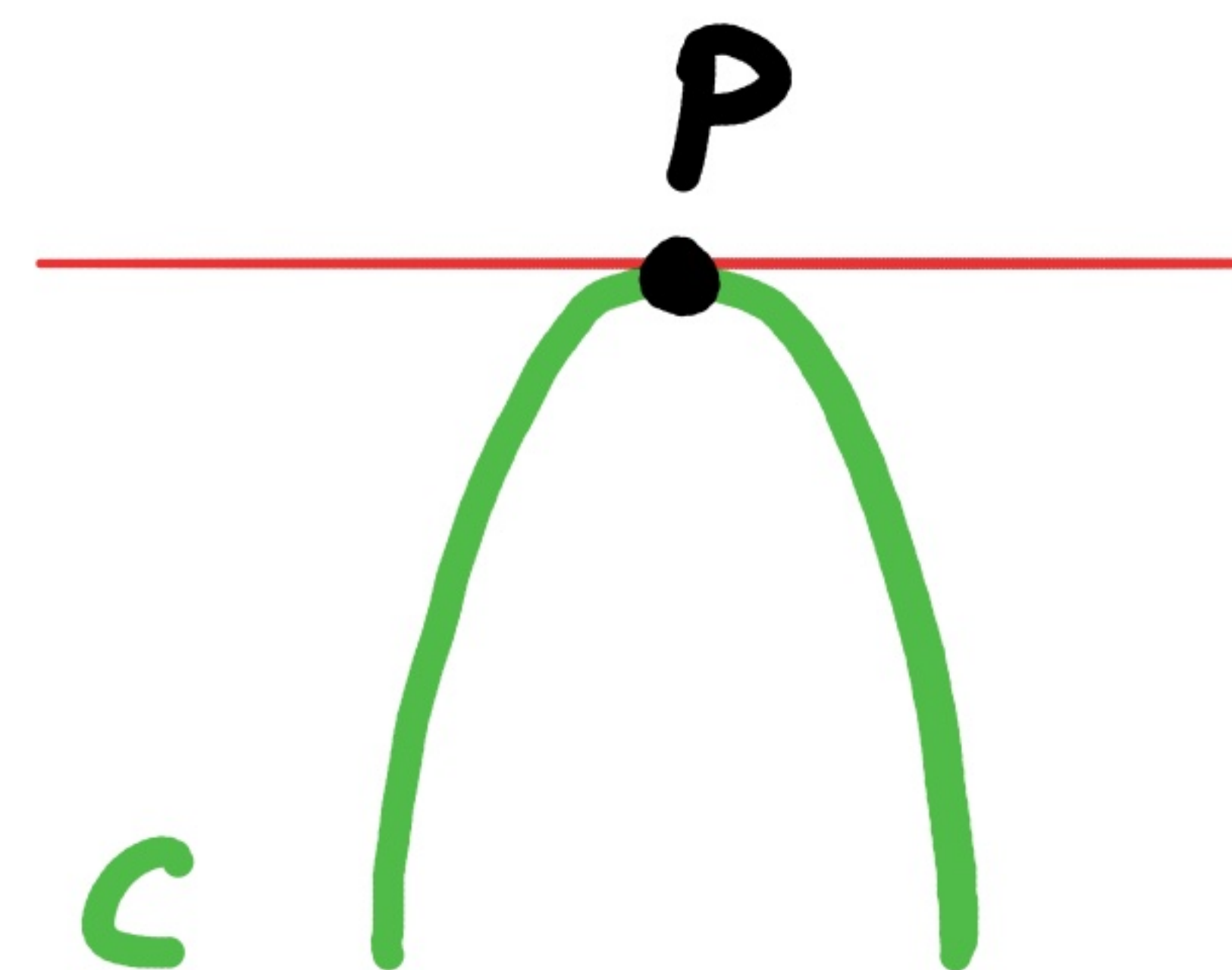
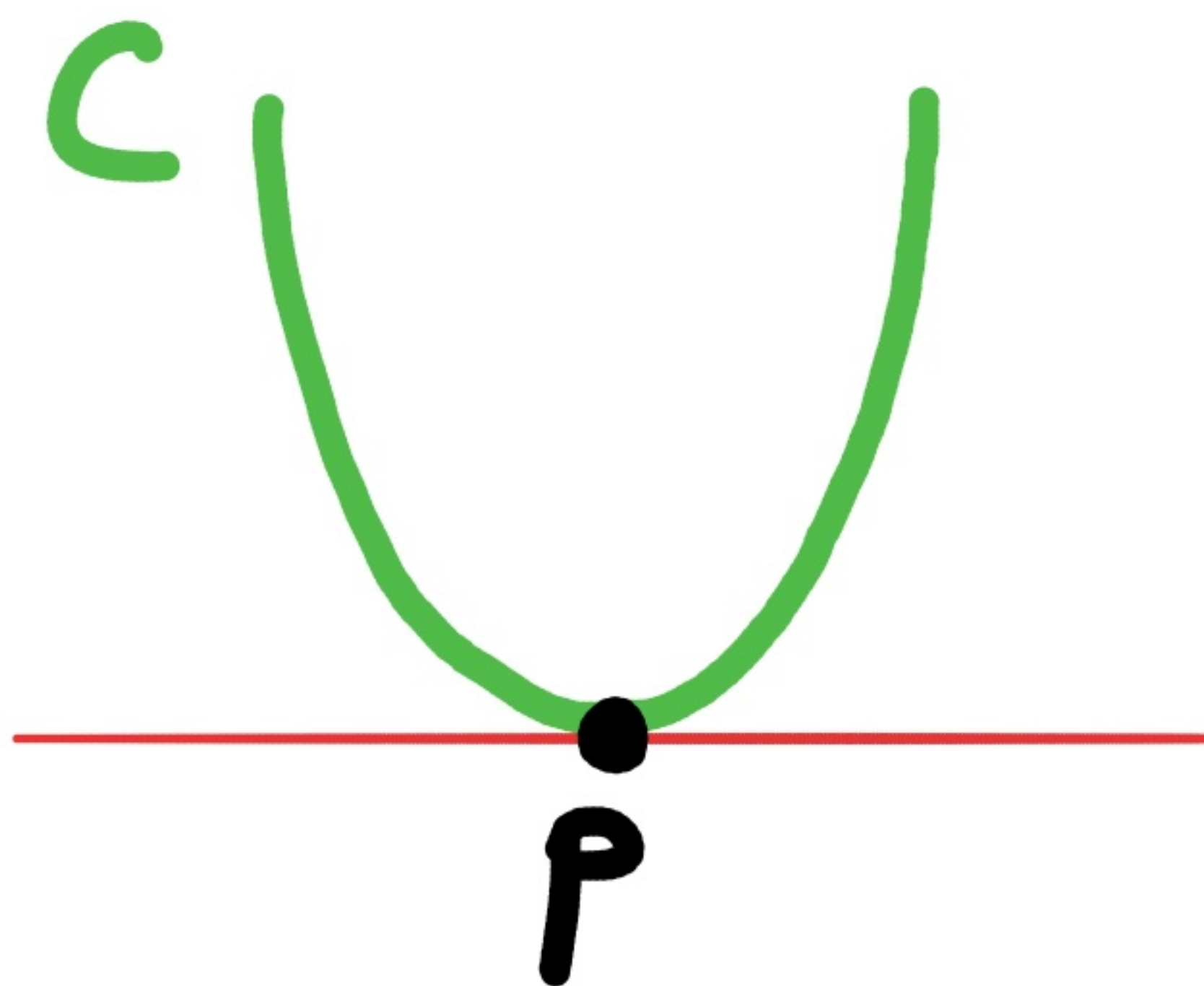


② Curvature for surfaces

Signed curvature for curves

Let p be a point on a curve C . If C is tangent to a horizontal line at p , then "signed curvature", $\hat{K}(p)$, is as follows:

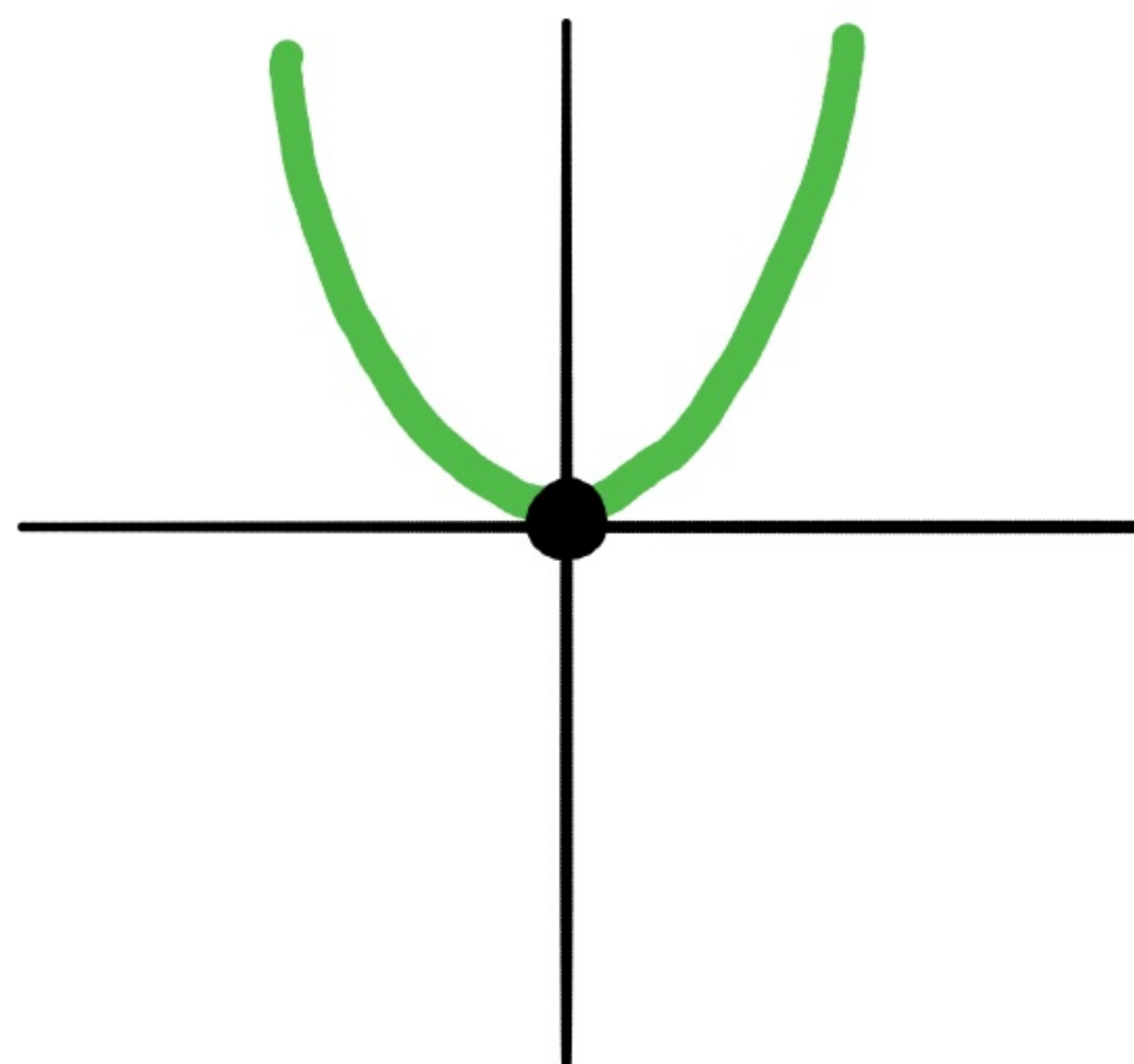
$$\hat{K}(p) = K(p)$$



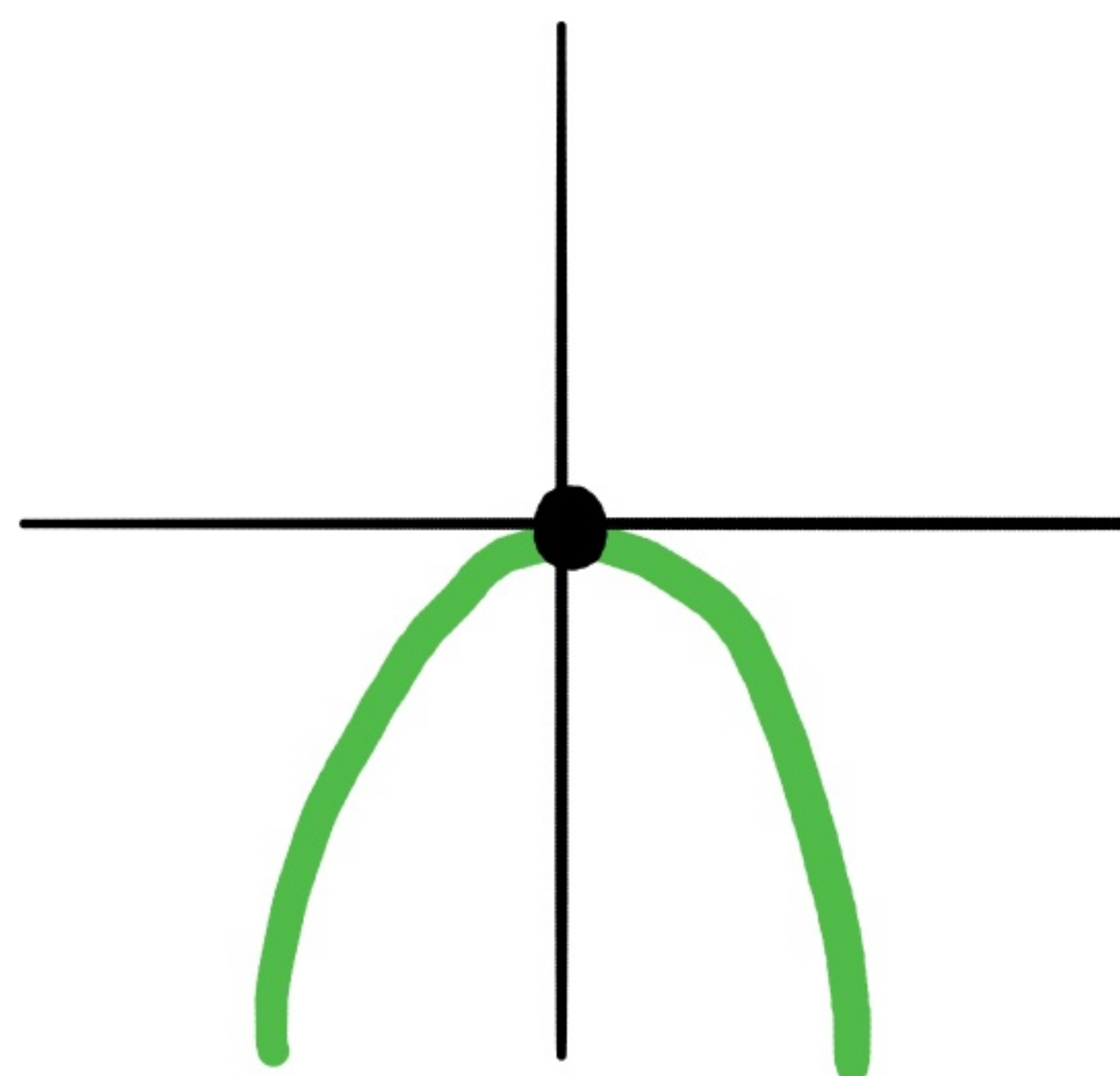
$$\hat{K}(p) = -K(p)$$

Examples:

$$y = x^2$$

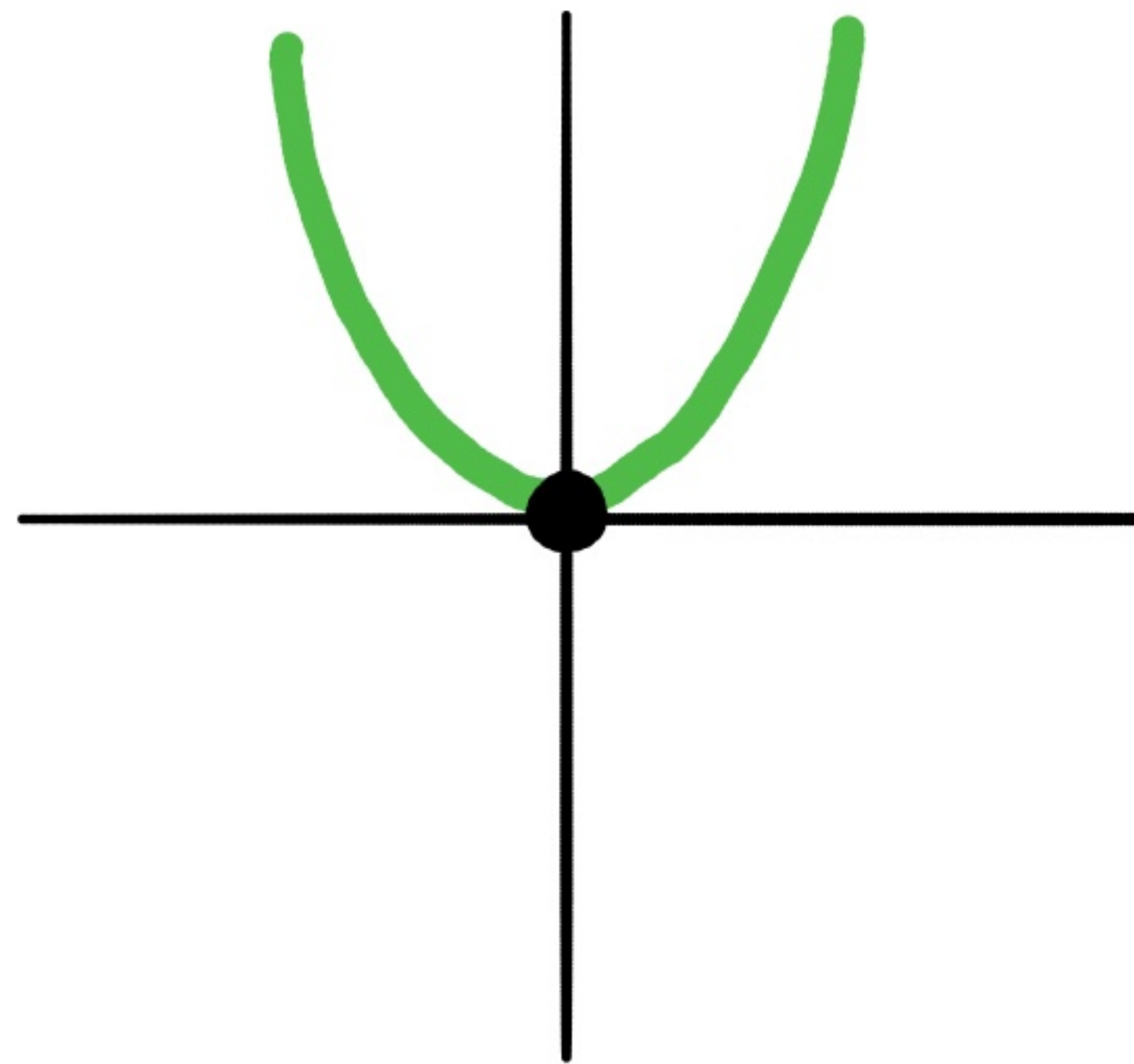


$$y = -x^2$$



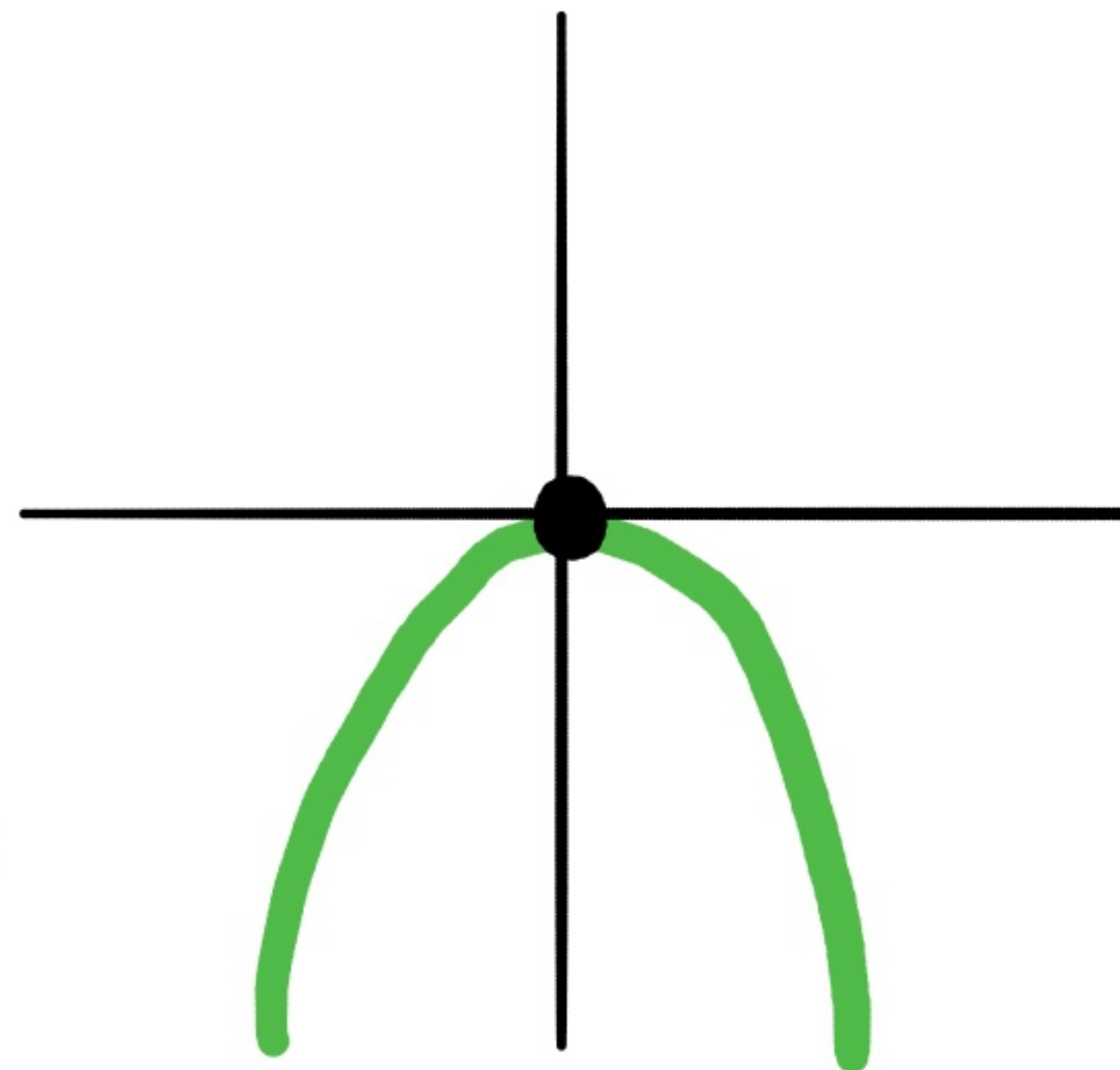
Examples:

$$y = x^2$$



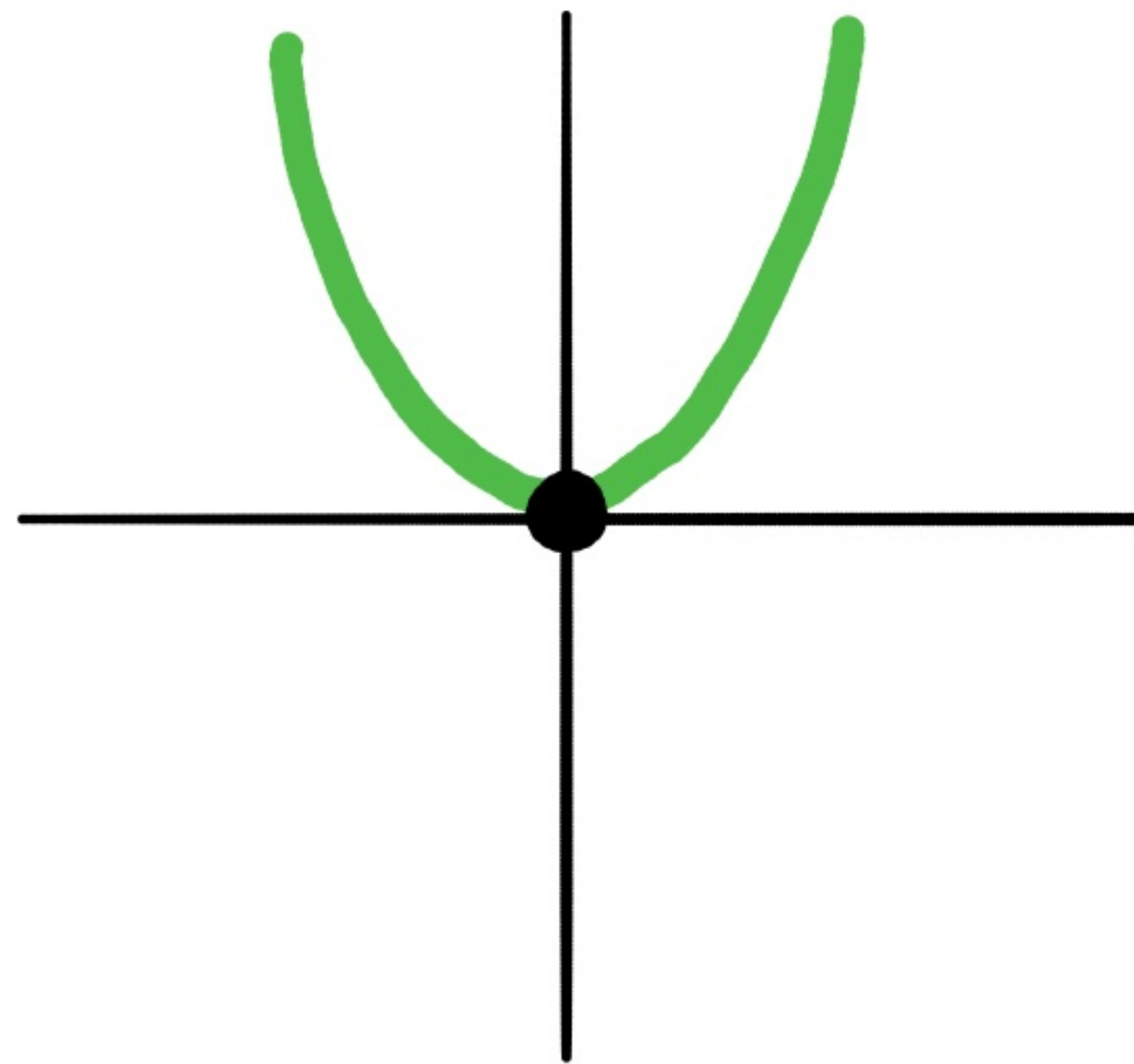
$$K(\vec{0}) = |y''| = |2| = 2$$

$$y = -x^2$$



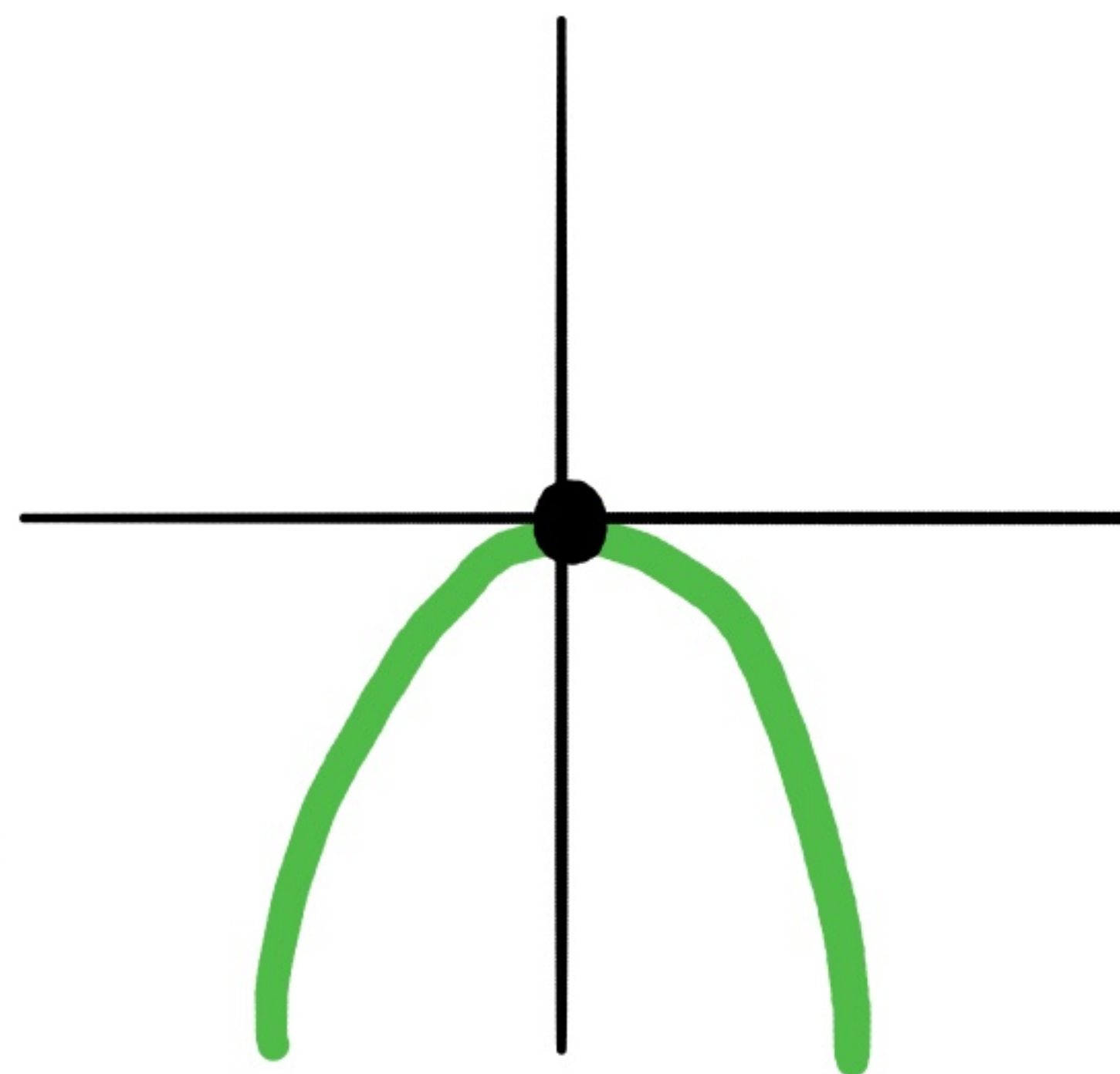
Examples:

$$y = x^2$$



$$K(\vec{0}) = |y''| = |2| = 2$$

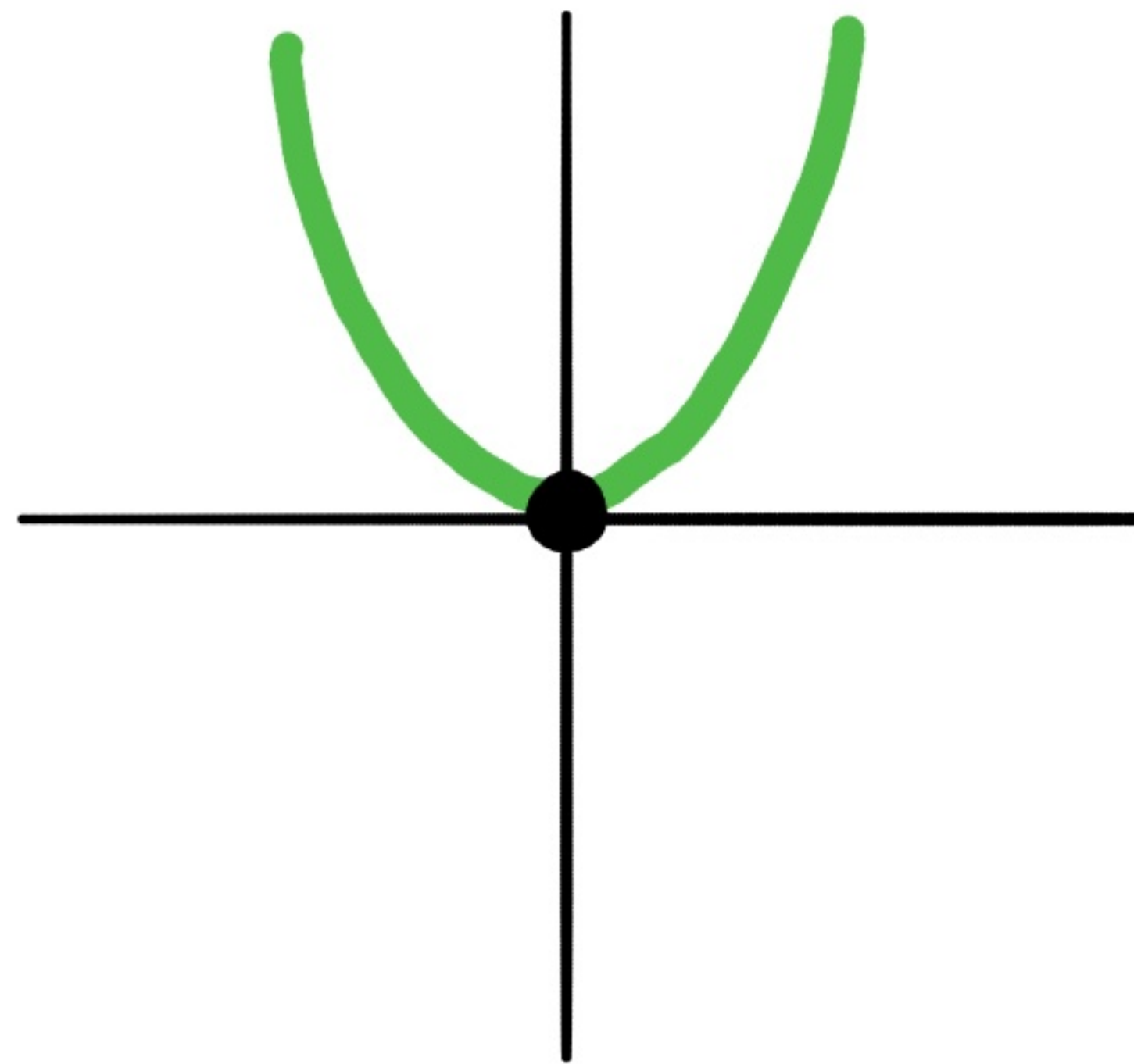
$$y = -x^2$$



$$K(\vec{0}) = |y''| = |-2| = 2$$

Examples:

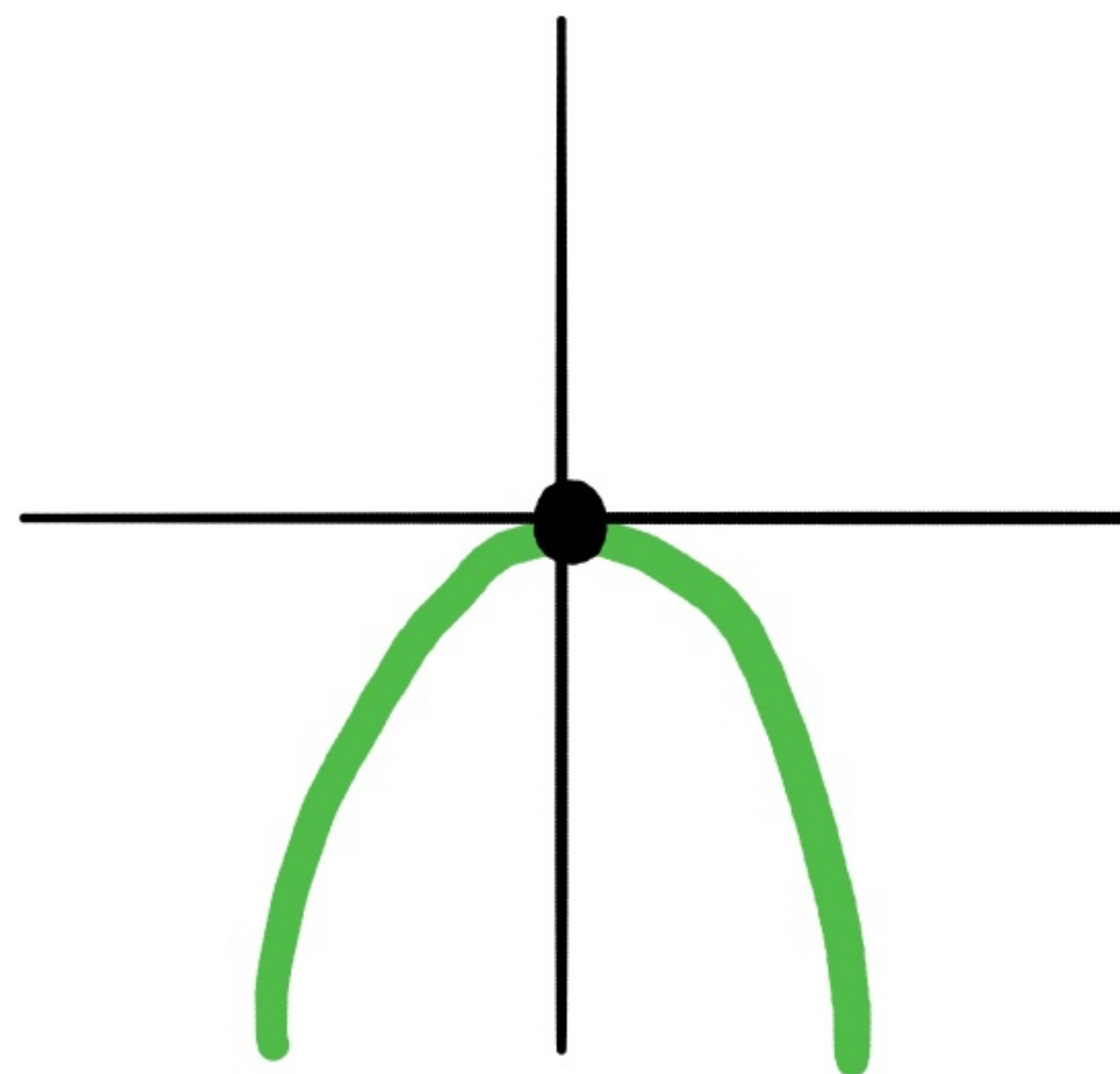
$$y = x^2$$



$$K(\vec{0}) = |y''| = |2| = 2$$

$$\hat{K}(\vec{0}) = 2$$

$$y = -x^2$$



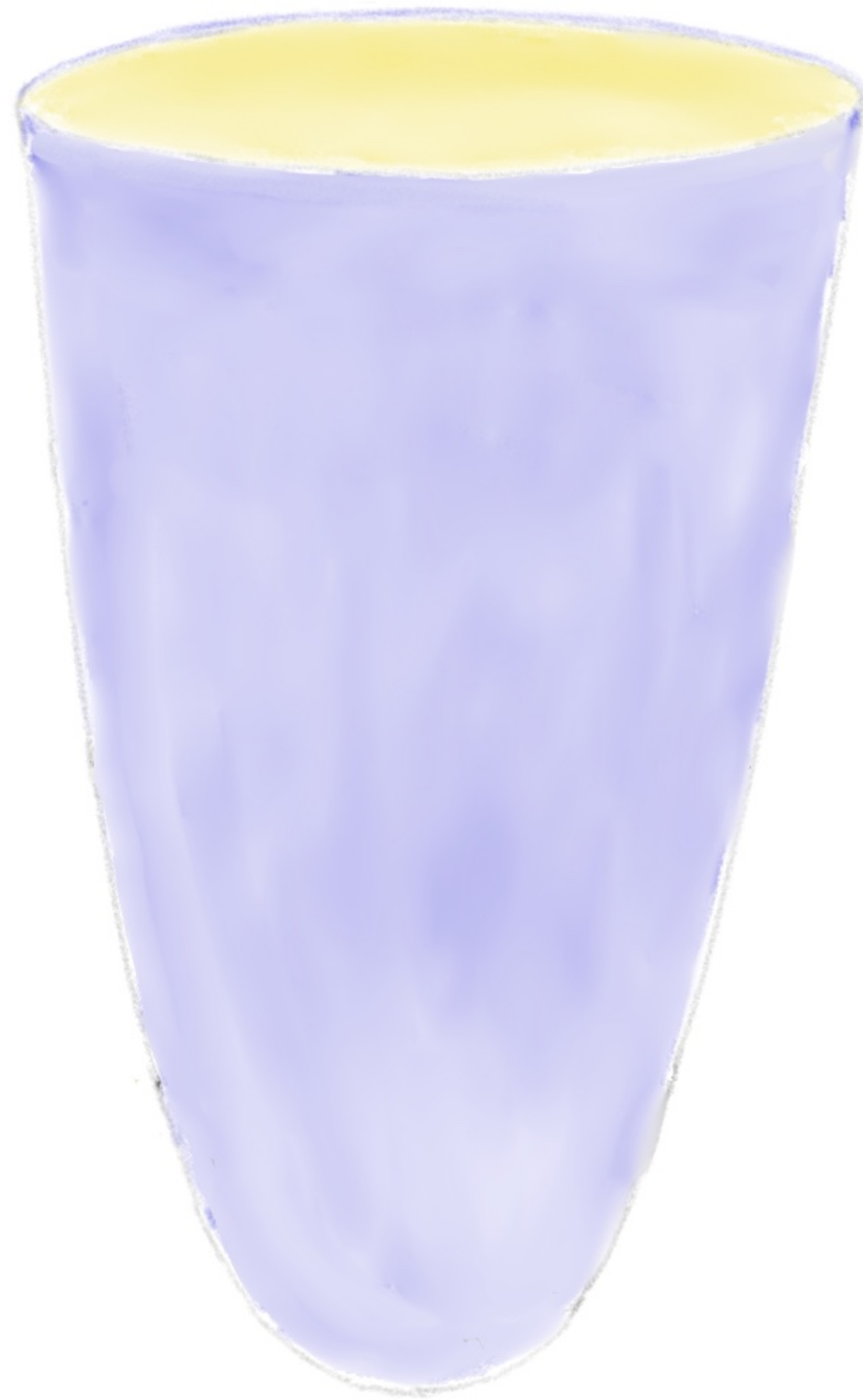
$$K(\vec{0}) = |y''| = |-2| = 2$$

$$\hat{K}(\vec{0}) = -2$$

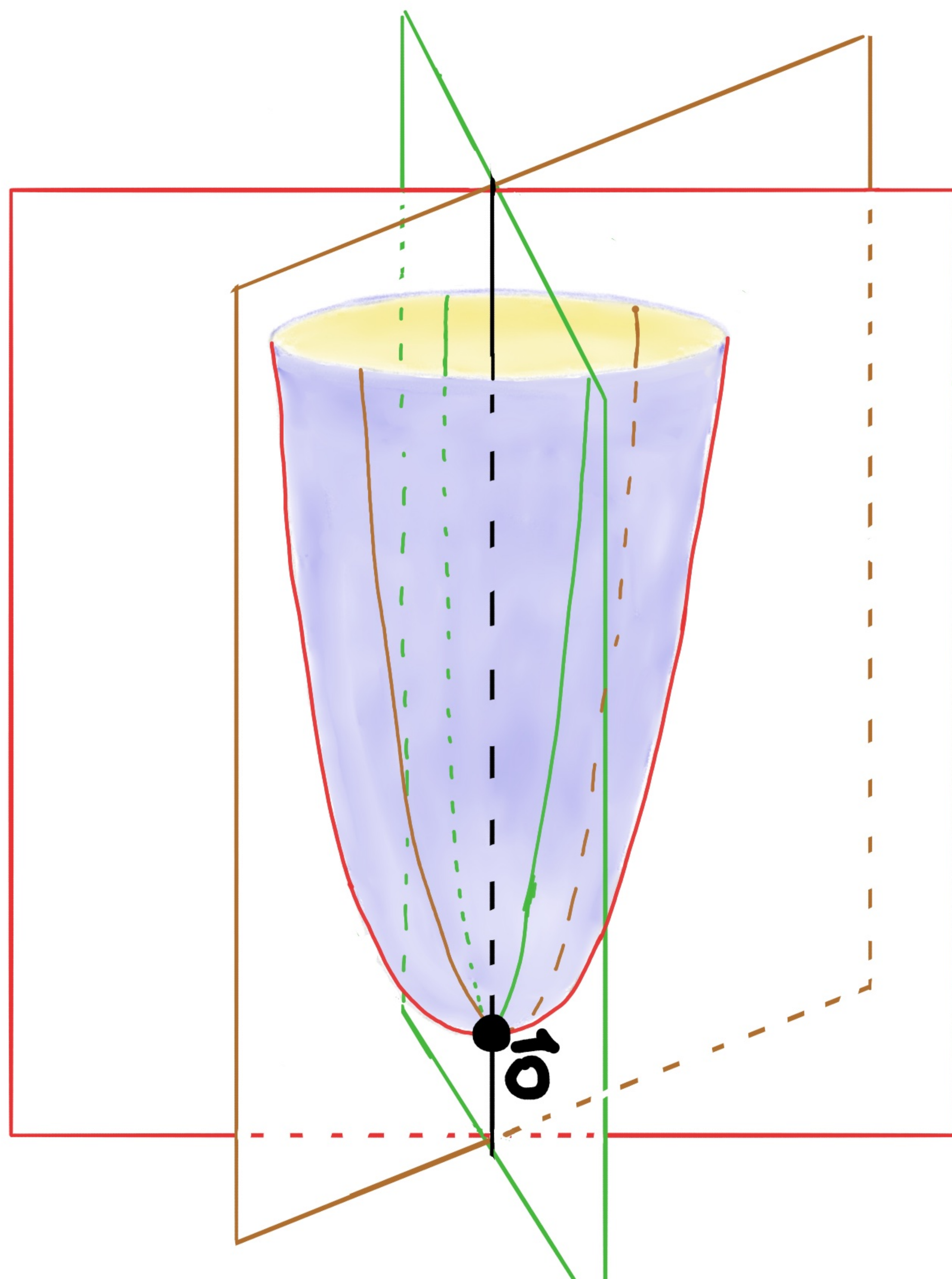
curvature of a surface at a point p

- 1.) Take all vertical cross sections through p .
- 2.) Find signed curvature at p of each resulting curve.
- 3.) Multiply min and max of the signed curvatures from 2.).

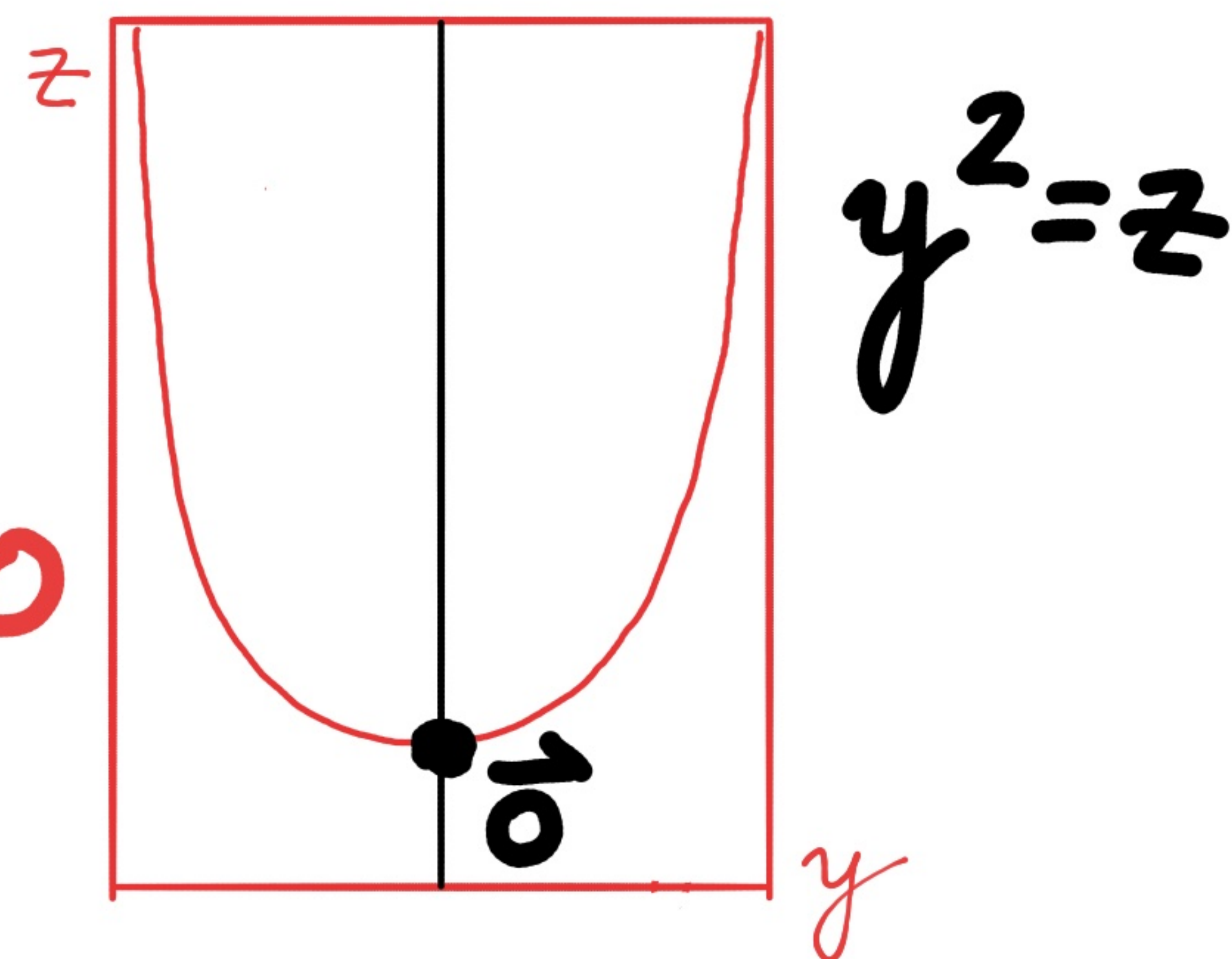
$$3x^2 + y^2 = z$$



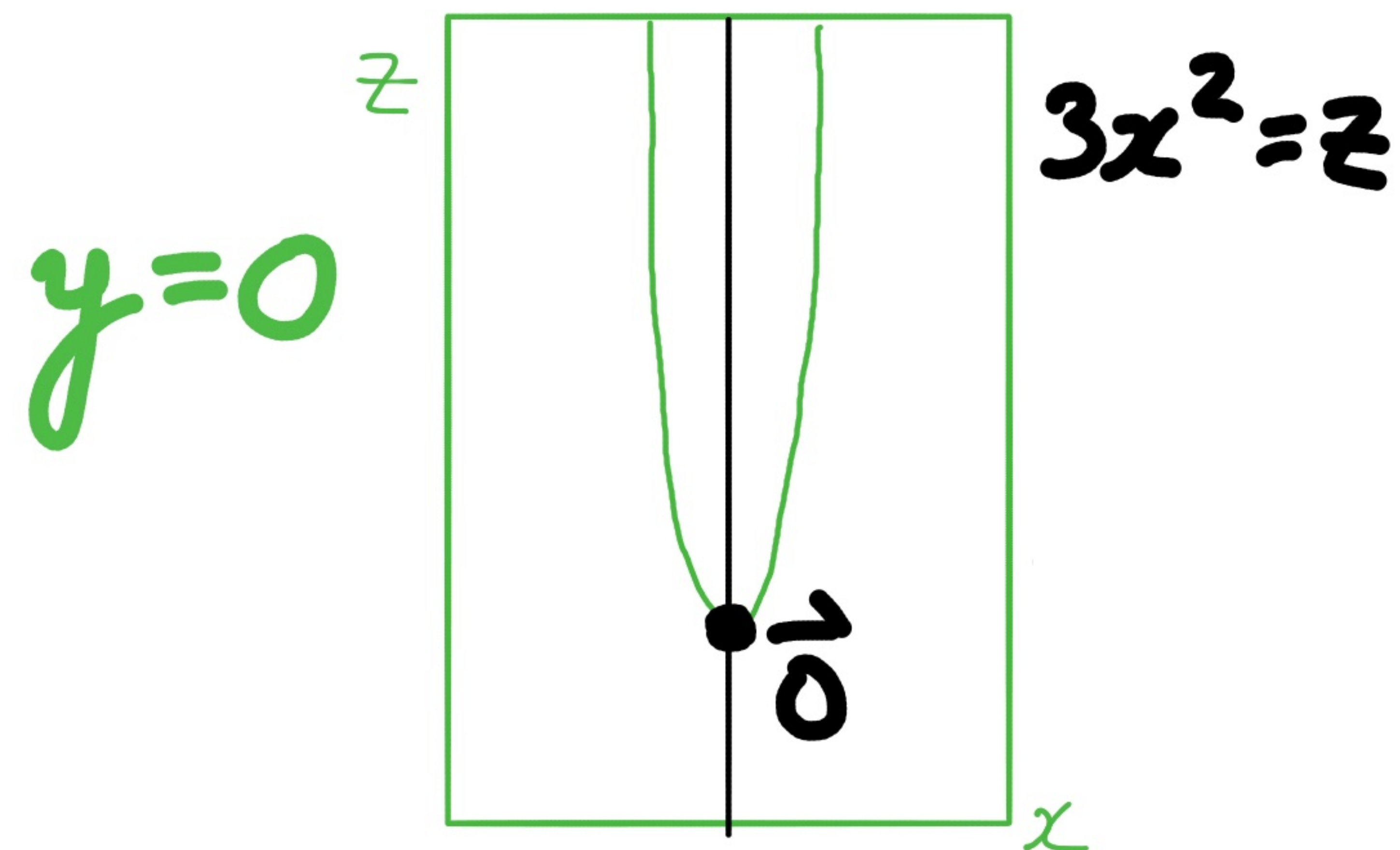
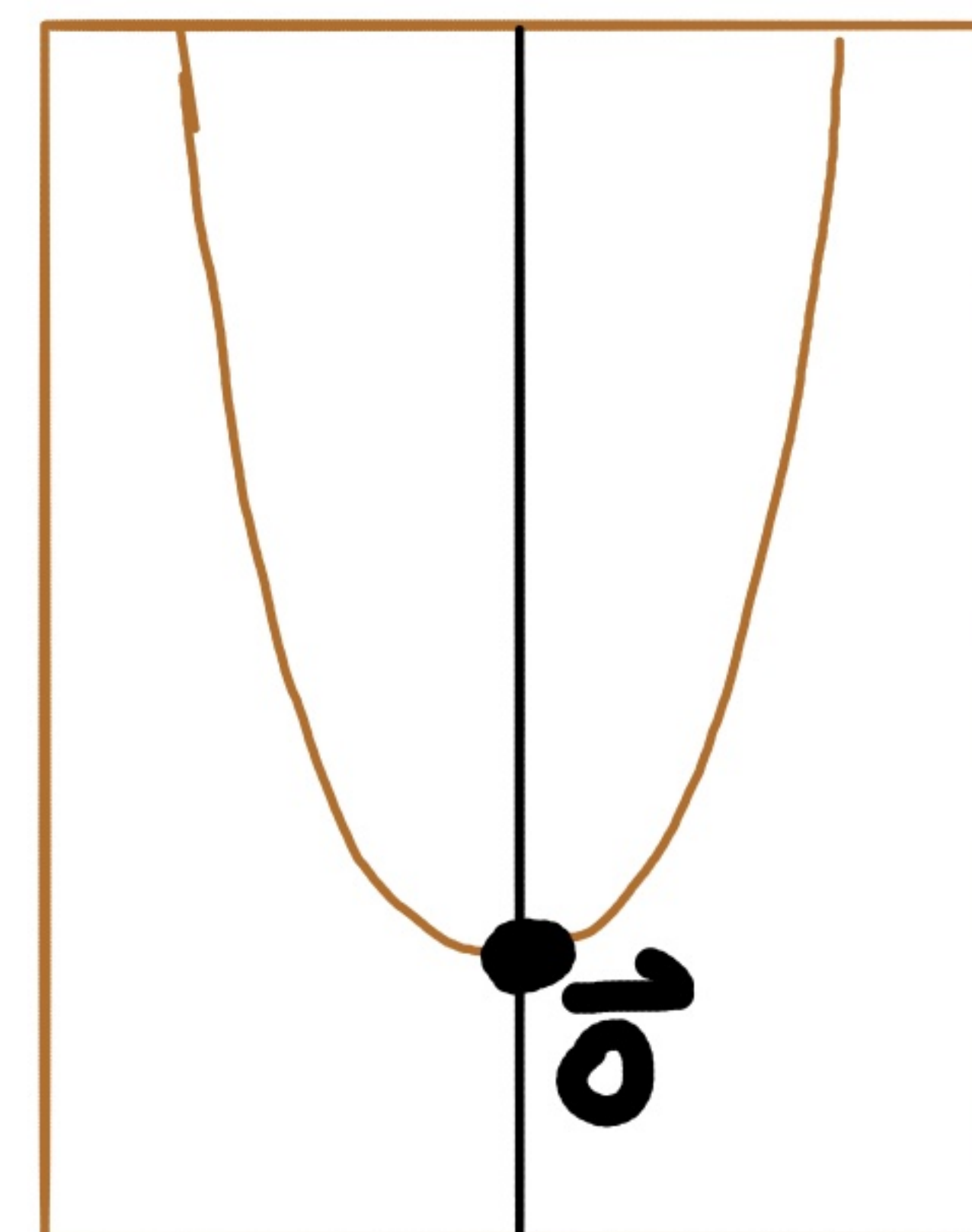
$$3x^2 + y^2 = z$$



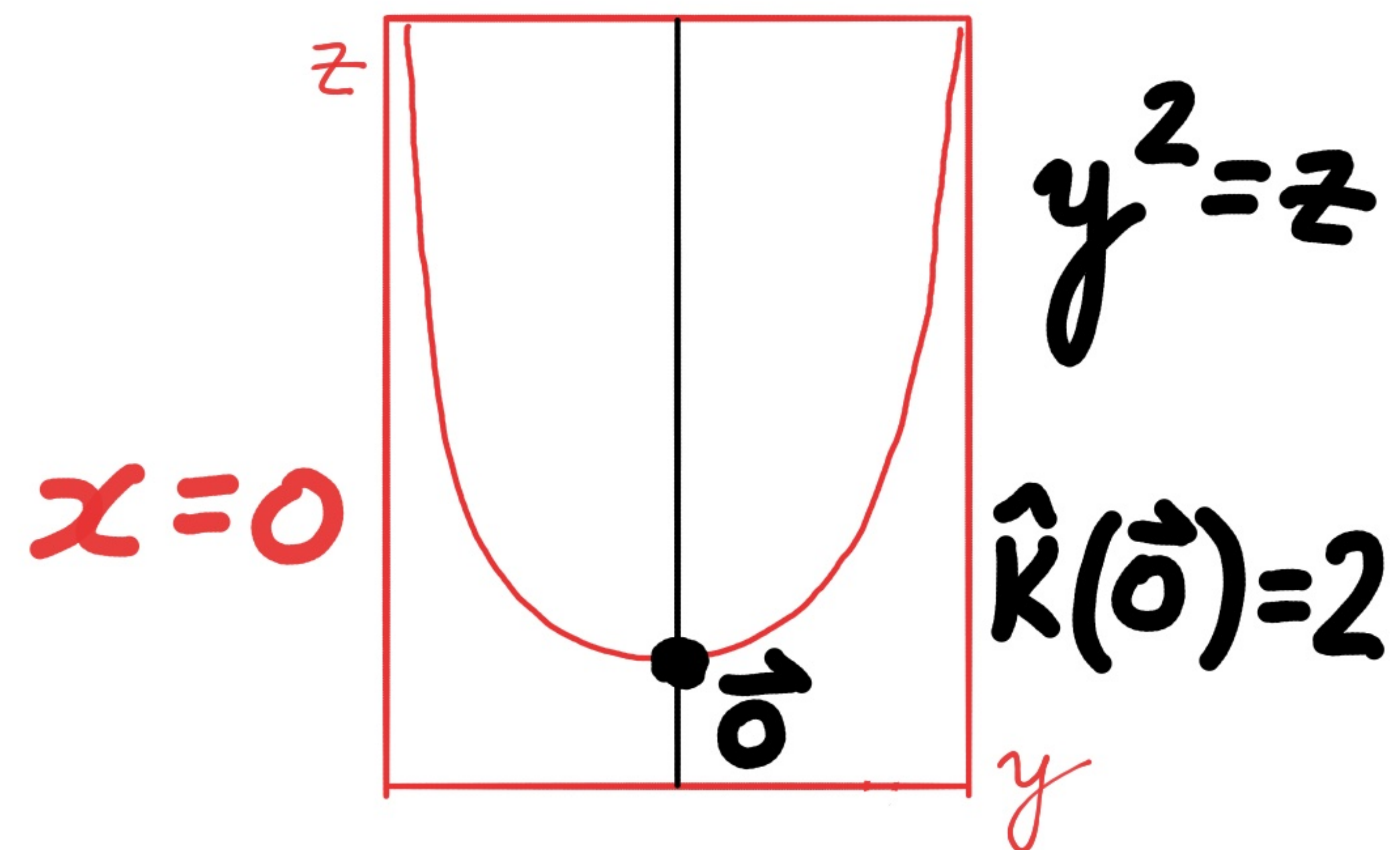
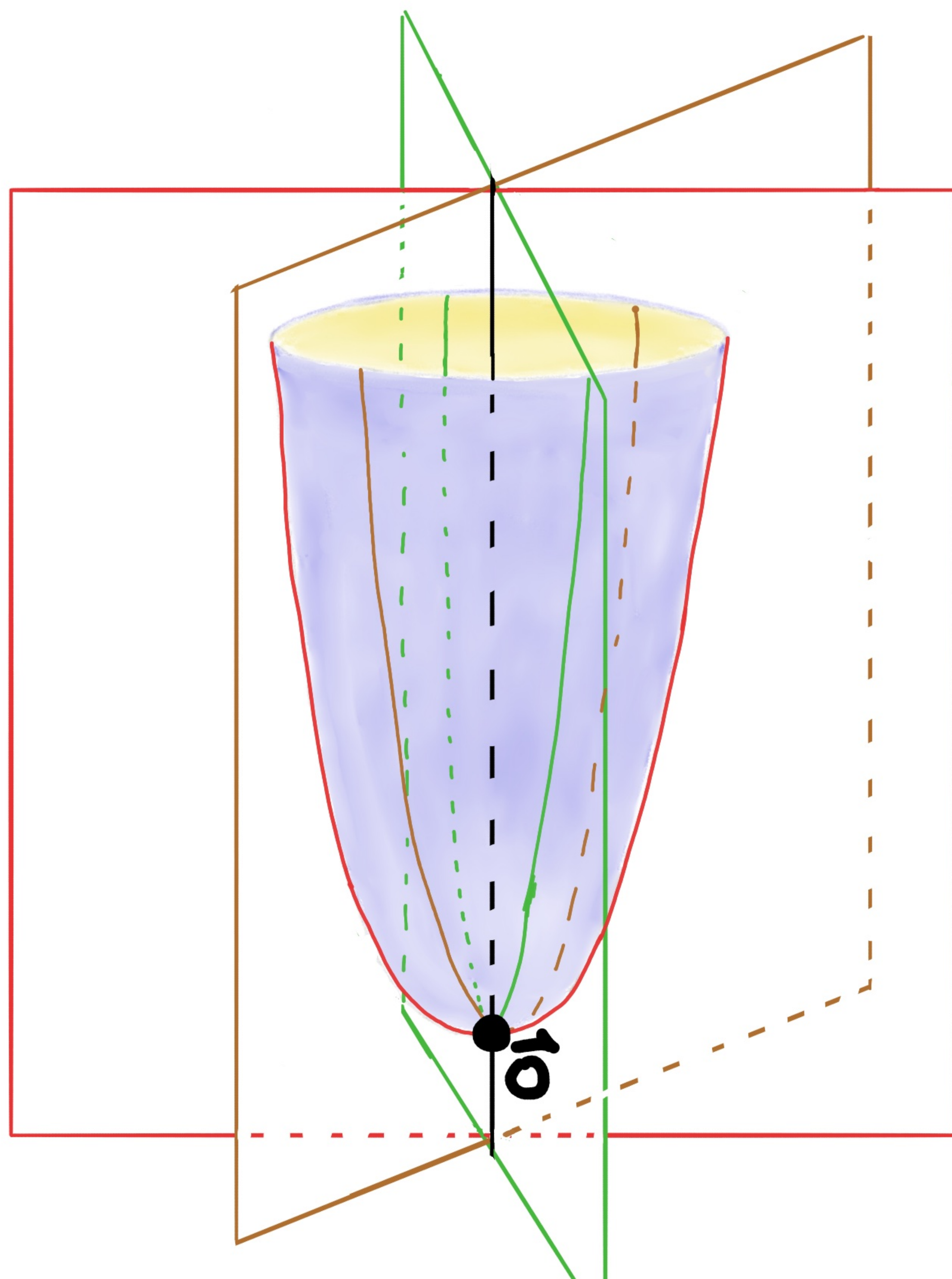
$$x=0$$



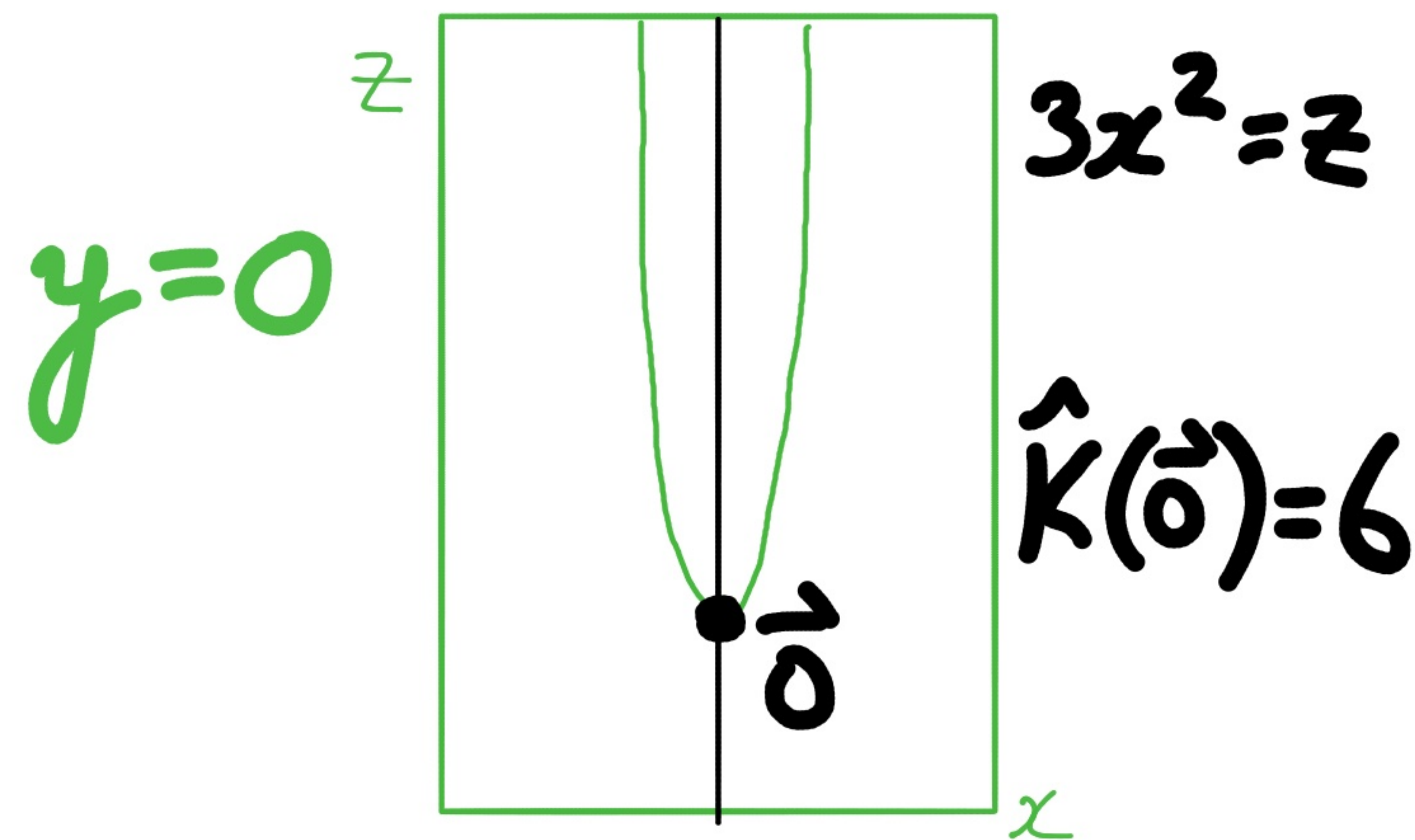
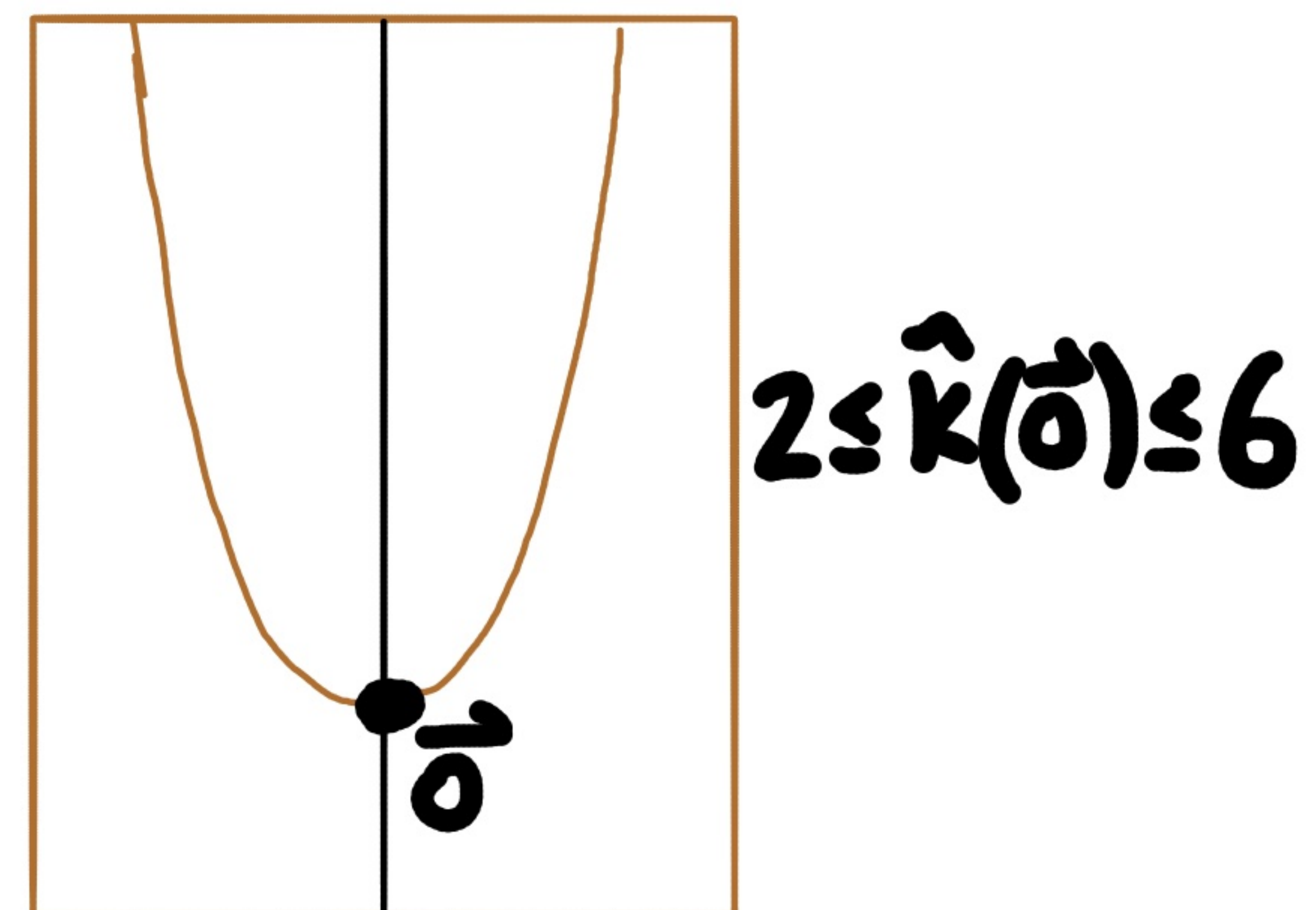
In
between



$$3x^2 + y^2 = z$$

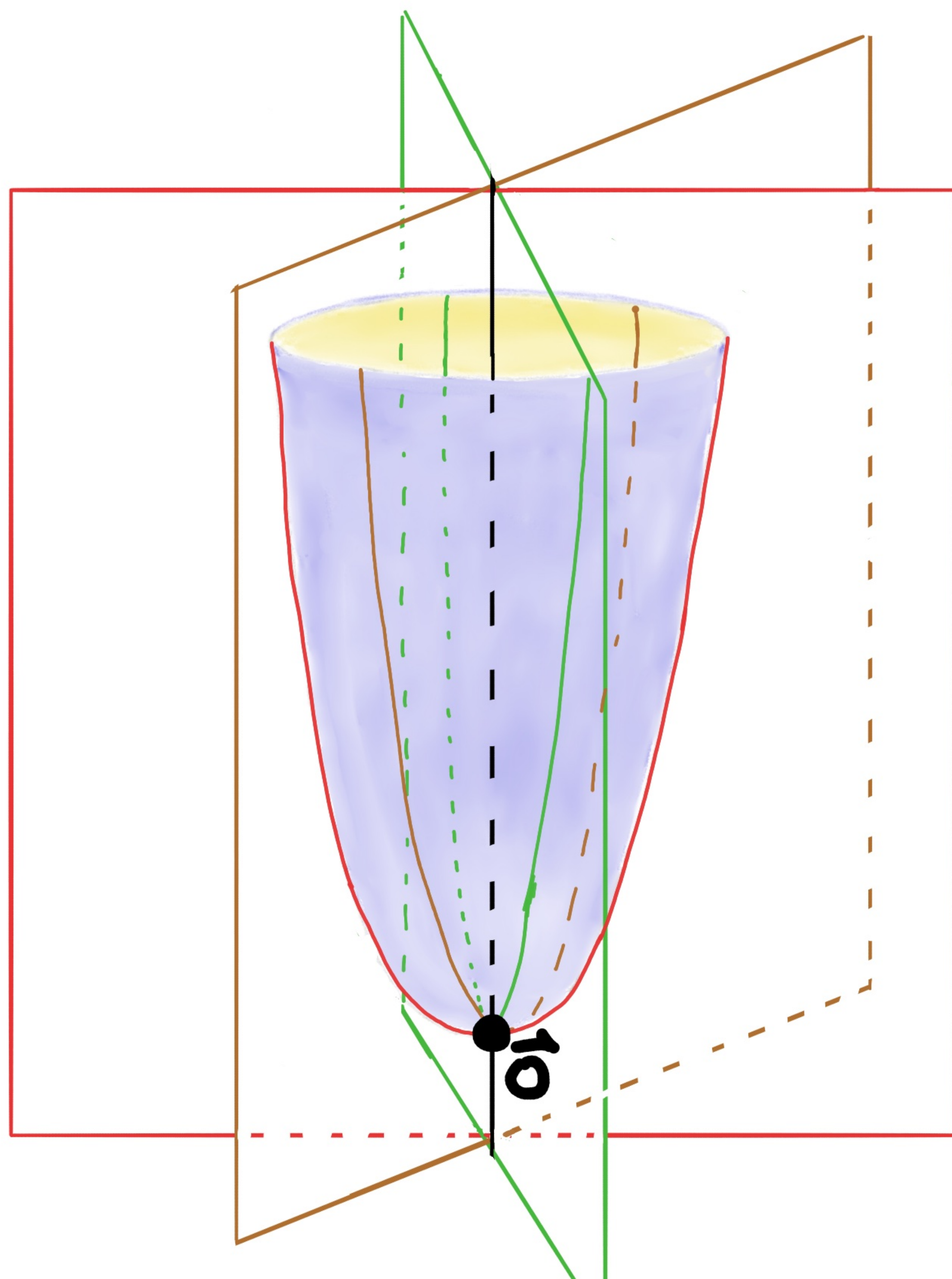


In
between

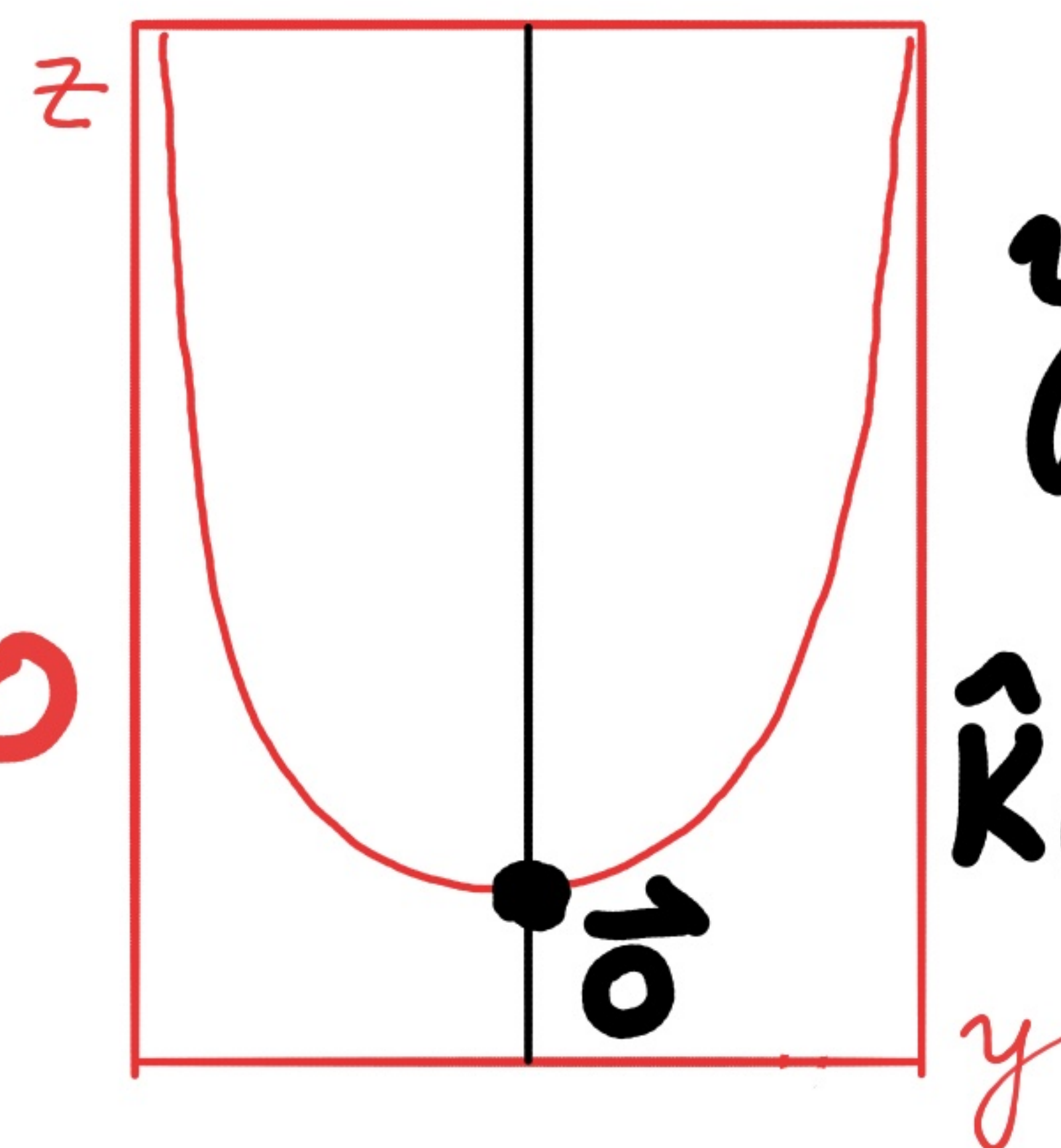


$$3x^2 + y^2 = z$$

Curvature
 $2 \cdot 6 = 12$



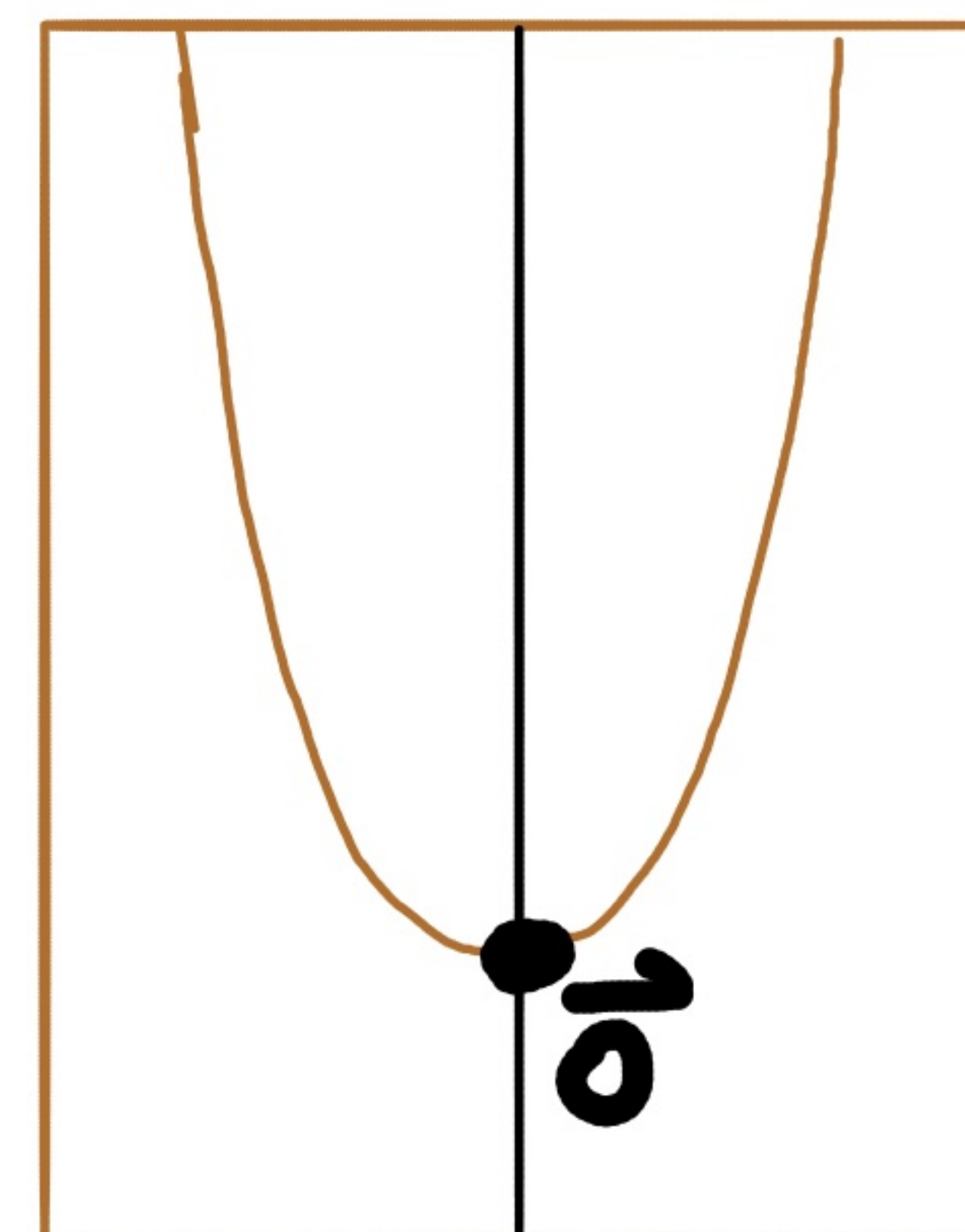
$$x=0$$



$$y^2 = z$$

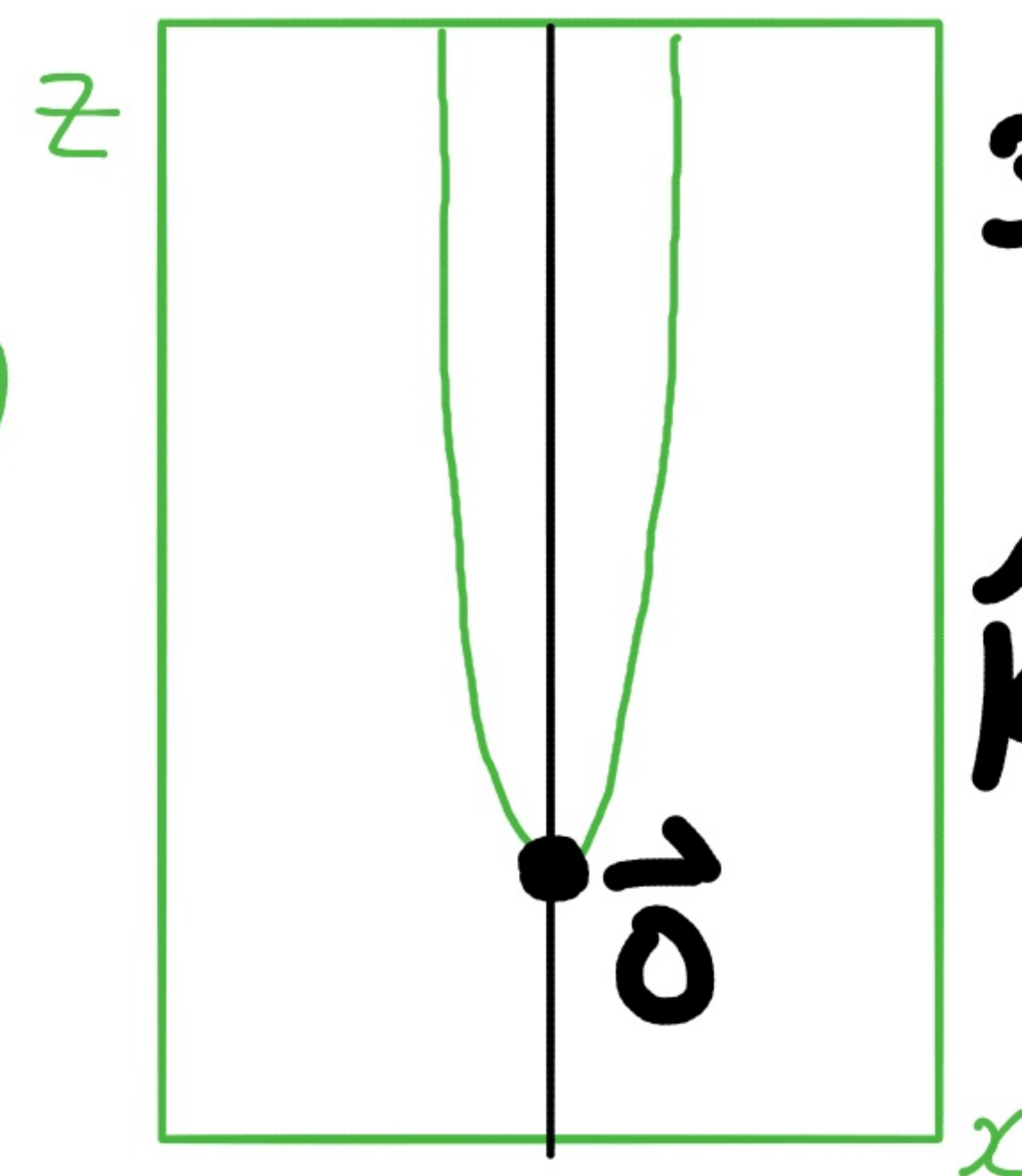
$$\hat{k}(\vec{o}) = 2$$

In
between



$$2 \leq \hat{k}(\vec{o}) \leq 6$$

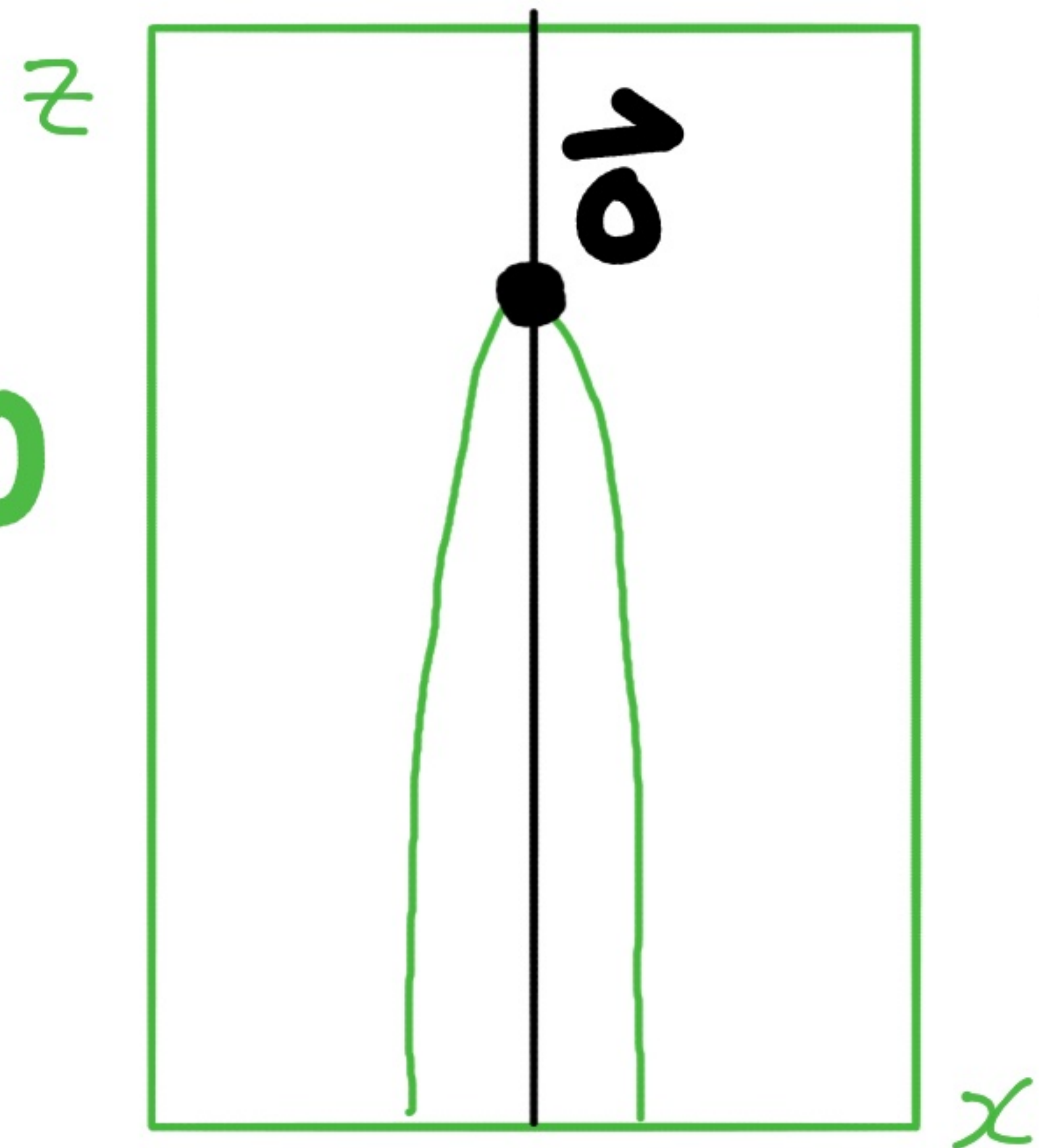
$$y=0$$



$$3x^2 = z$$

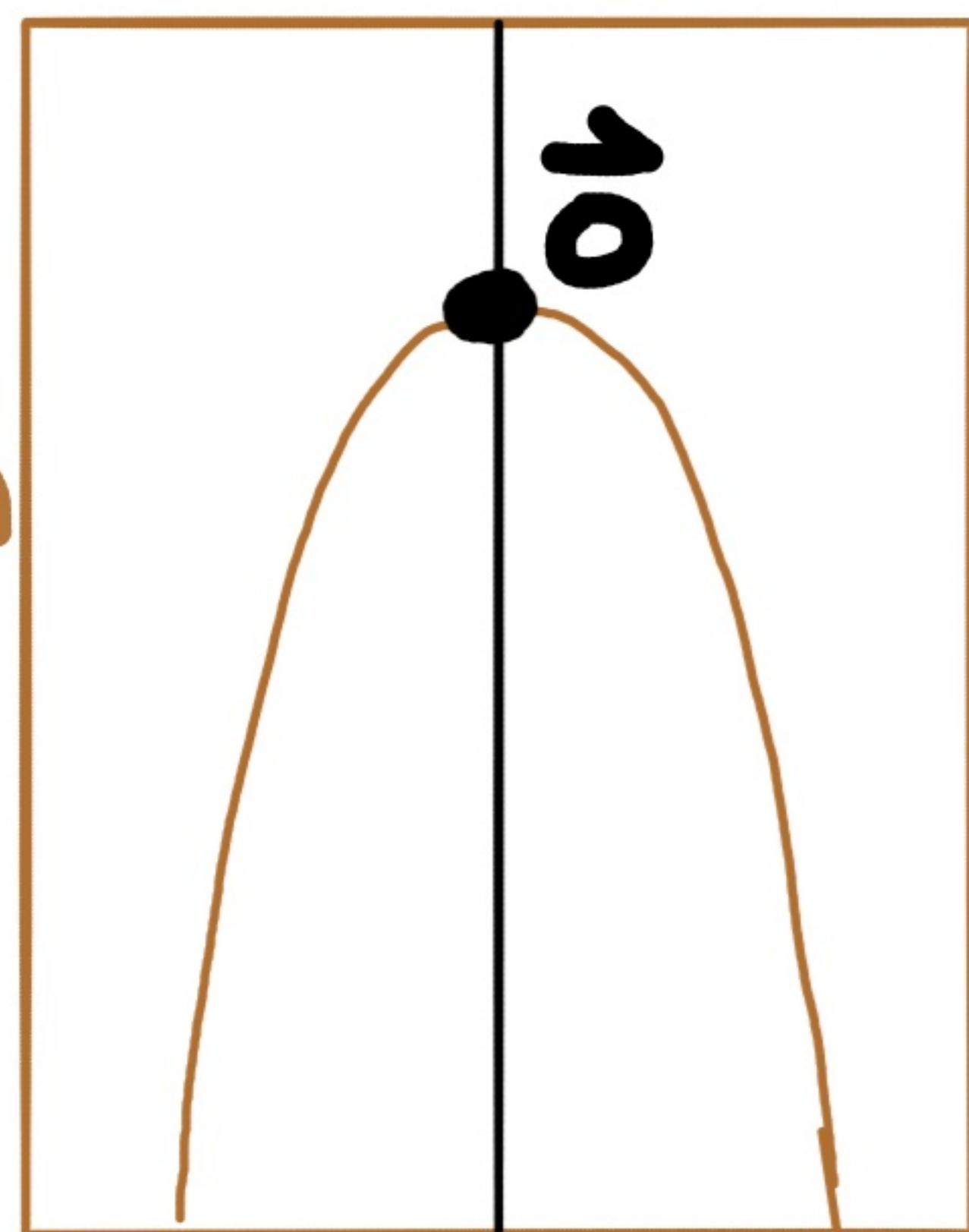
$$\hat{k}(\vec{o}) = 6$$

$$y=0$$

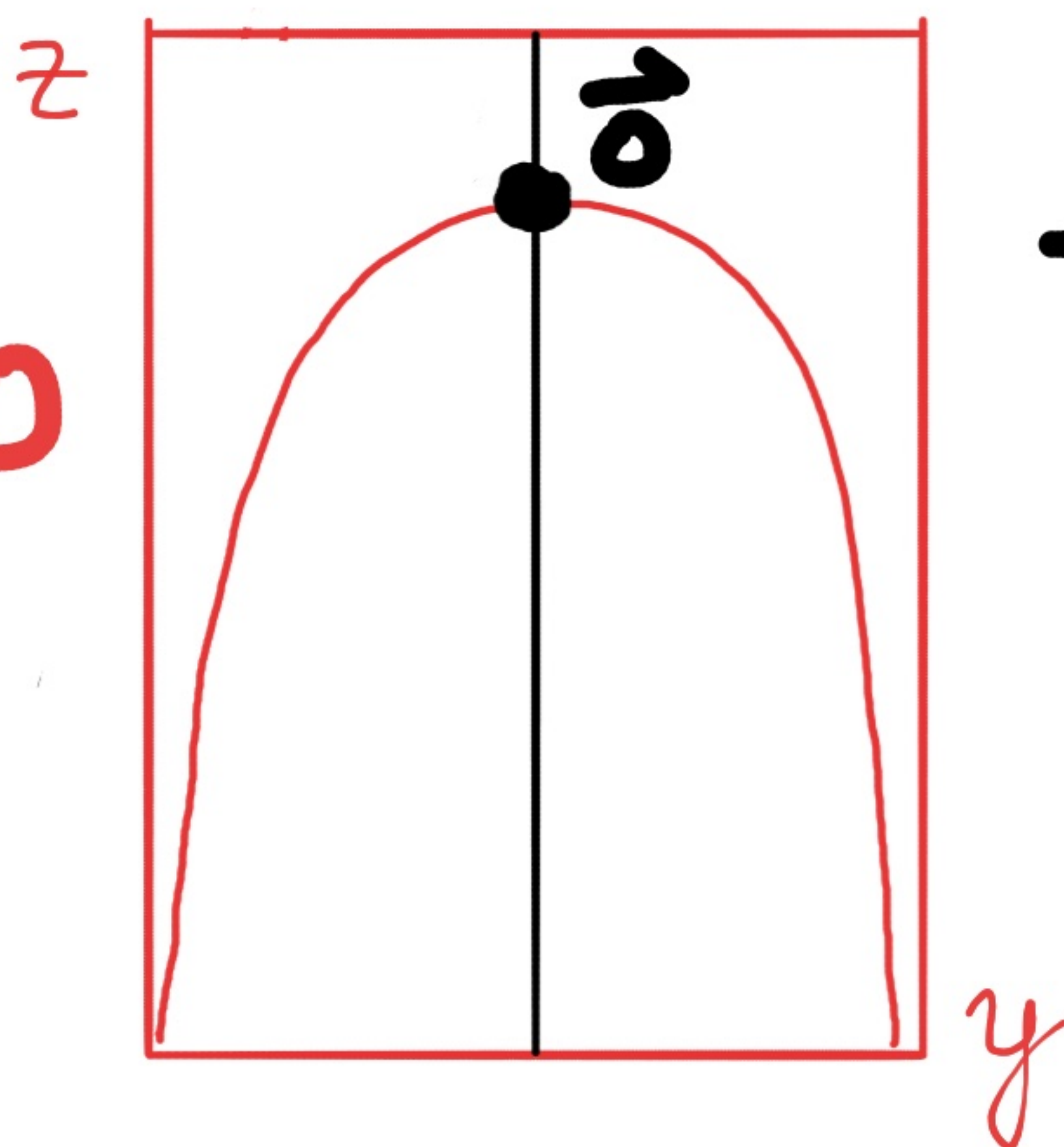


$$-3x^2 = z$$

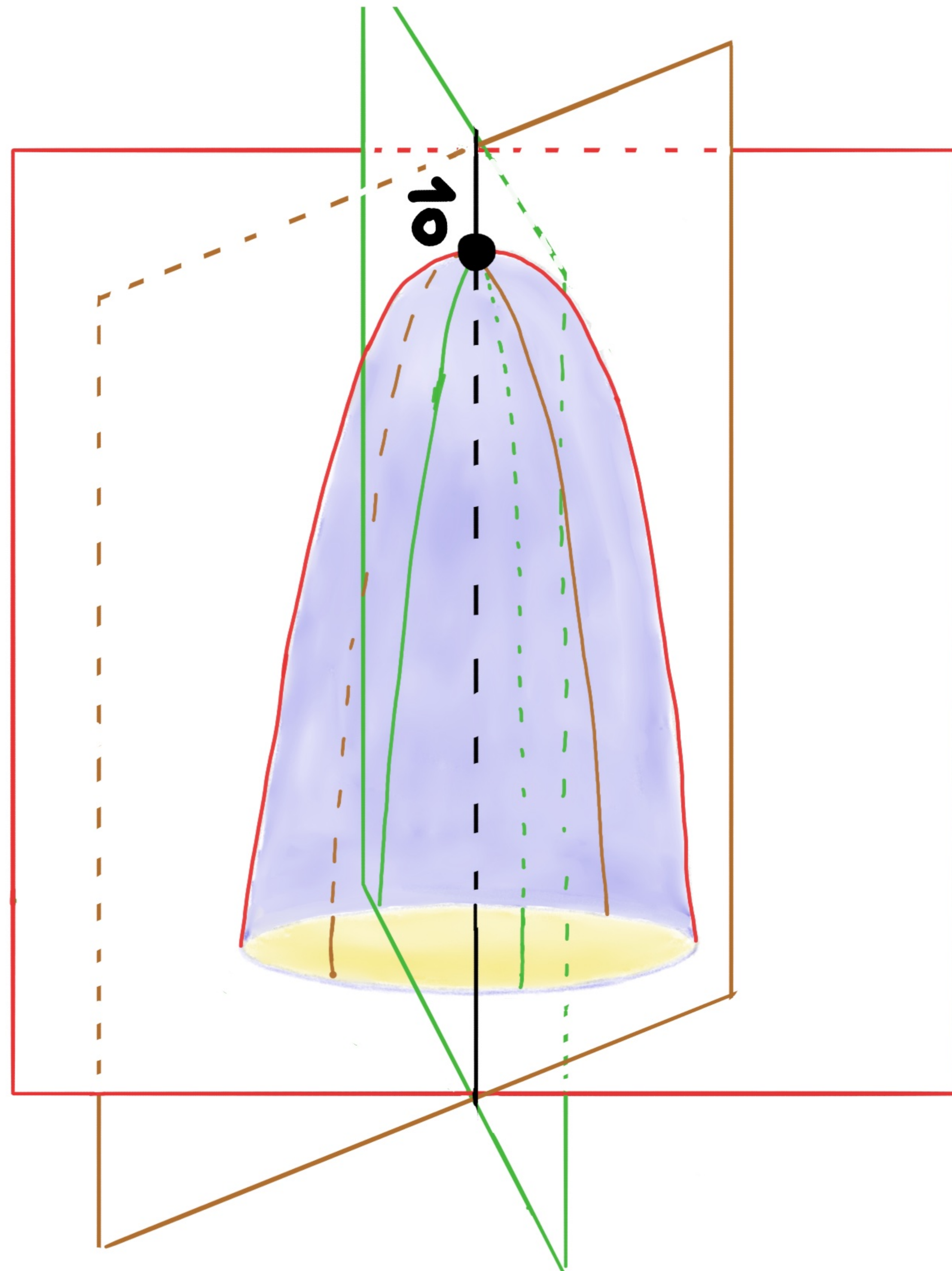
In
between



$$x=0$$

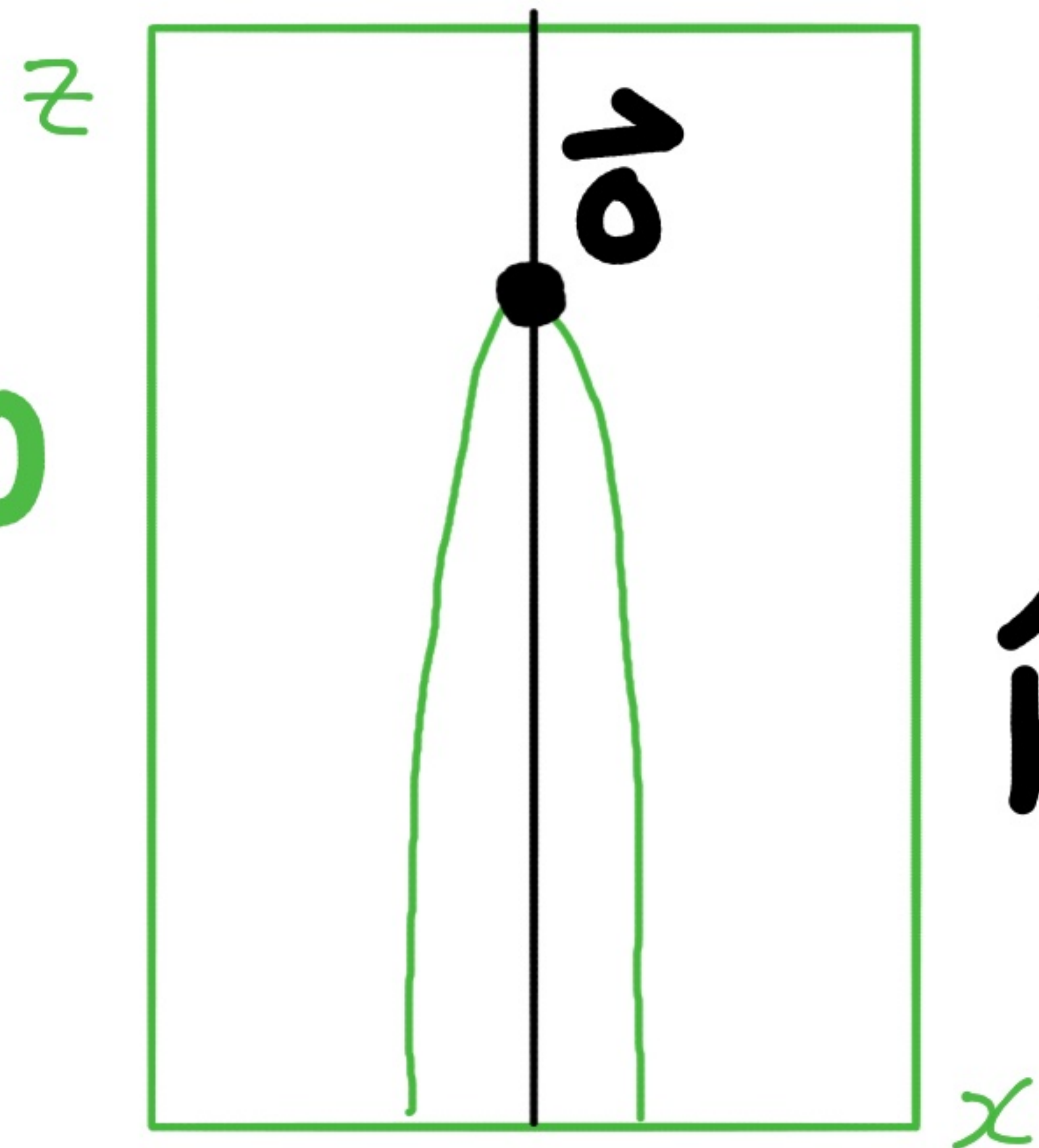


$$-y^2 = z$$



$$-3x^2 - y^2 = z$$

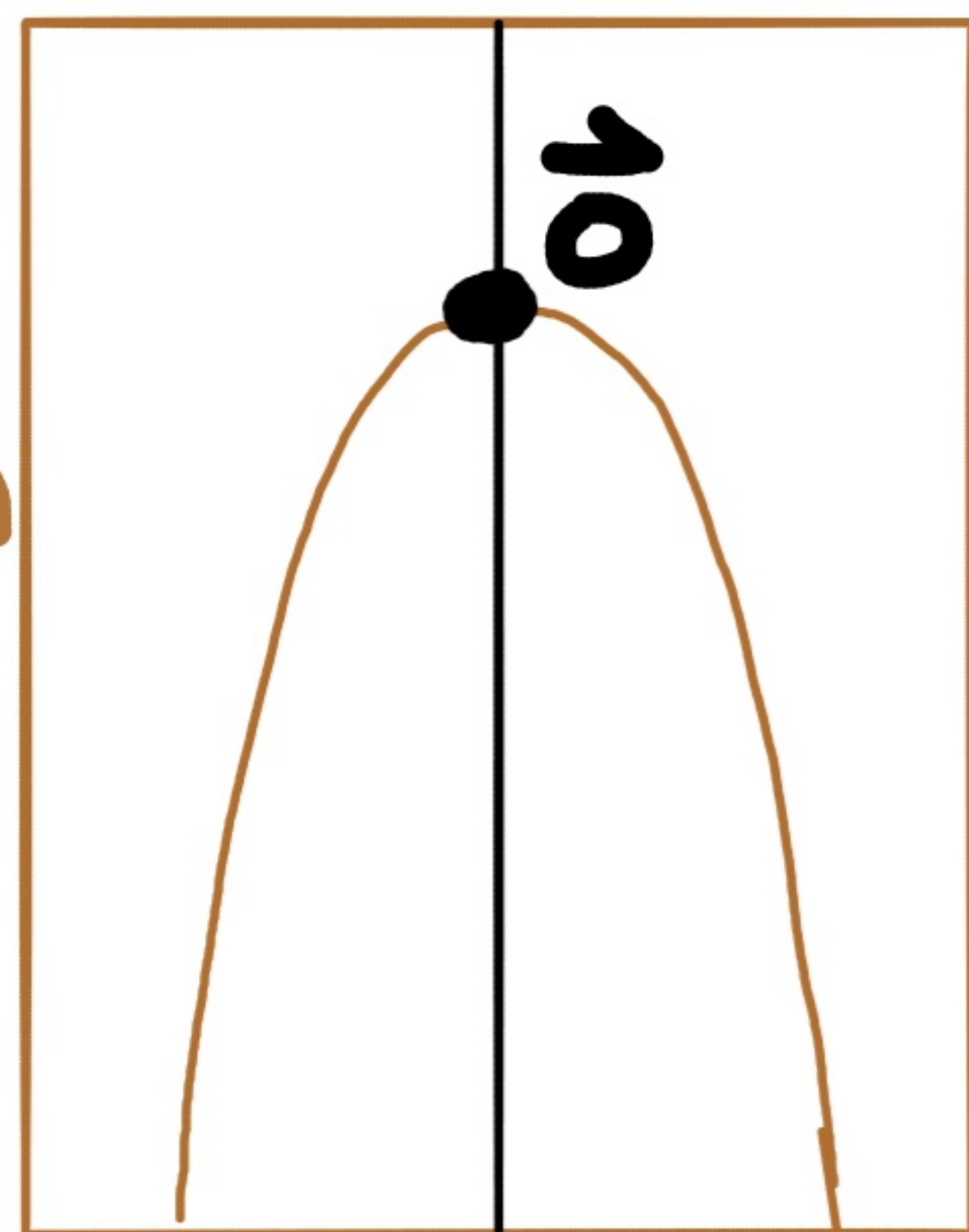
$$y=0$$



$$-3x^2 = z$$

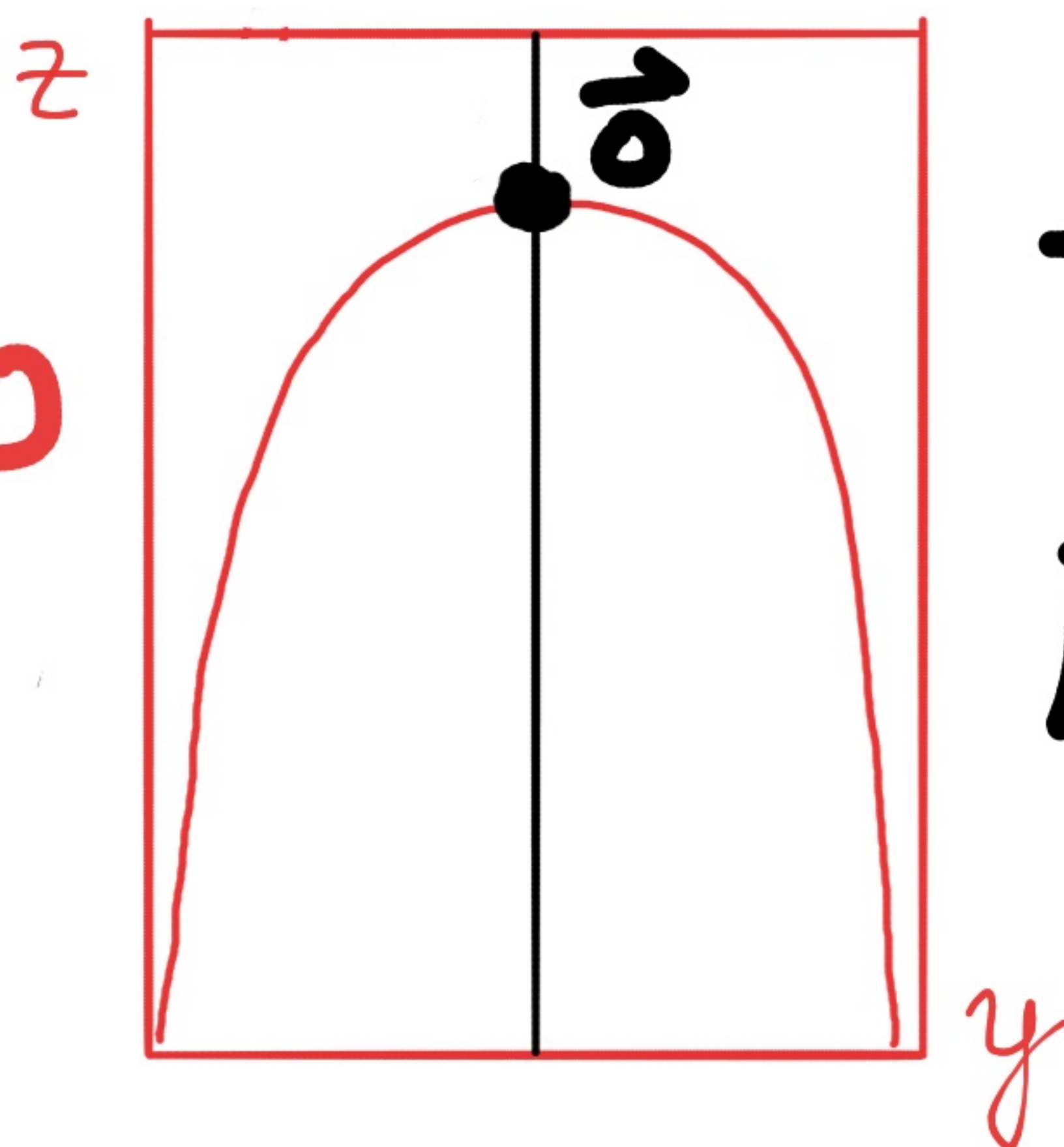
$$\hat{K}(\vec{0}) = -6$$

In
between



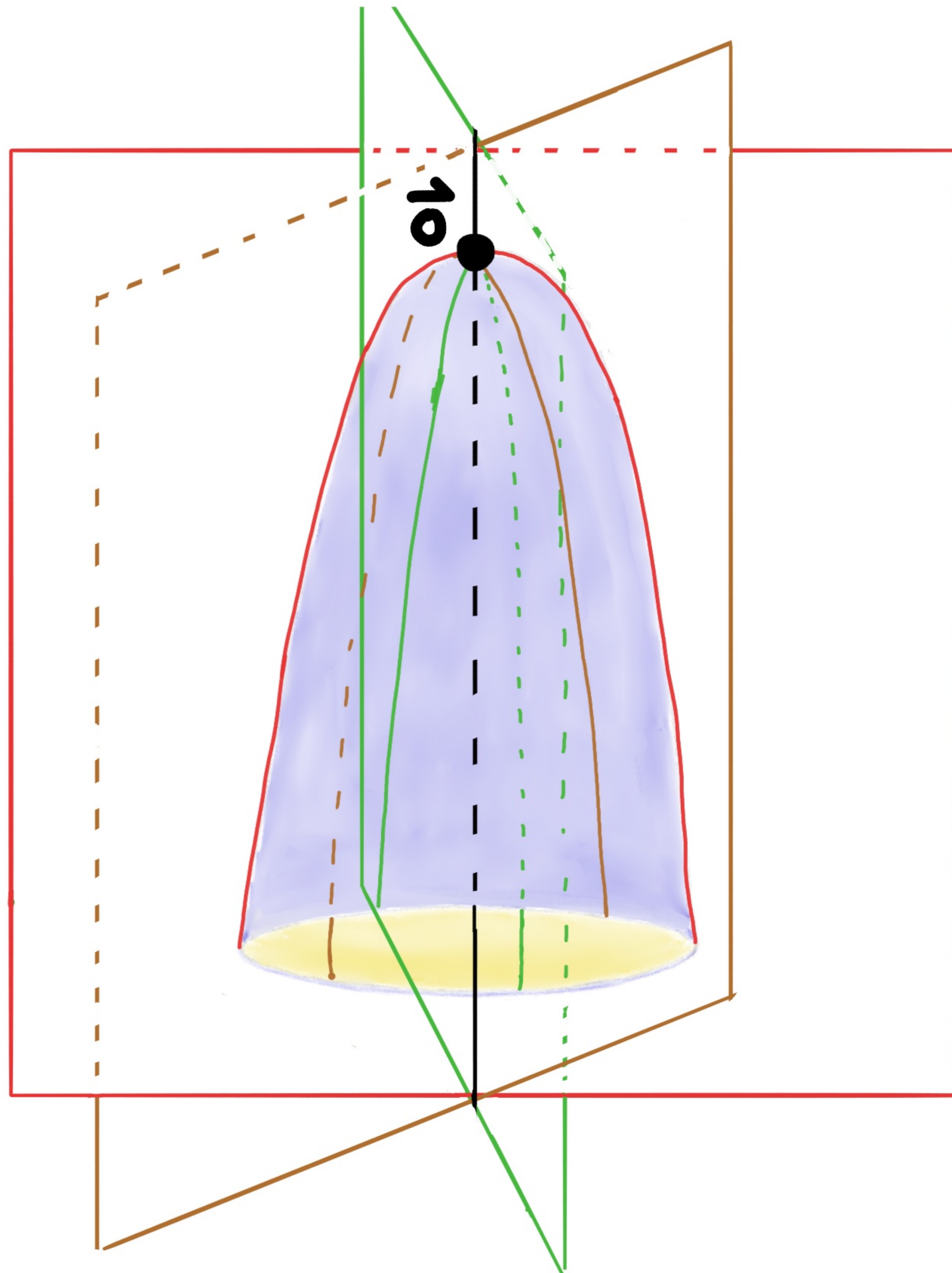
$$-6 \leq \hat{K}(\vec{0}) \leq -2$$

$$x=0$$



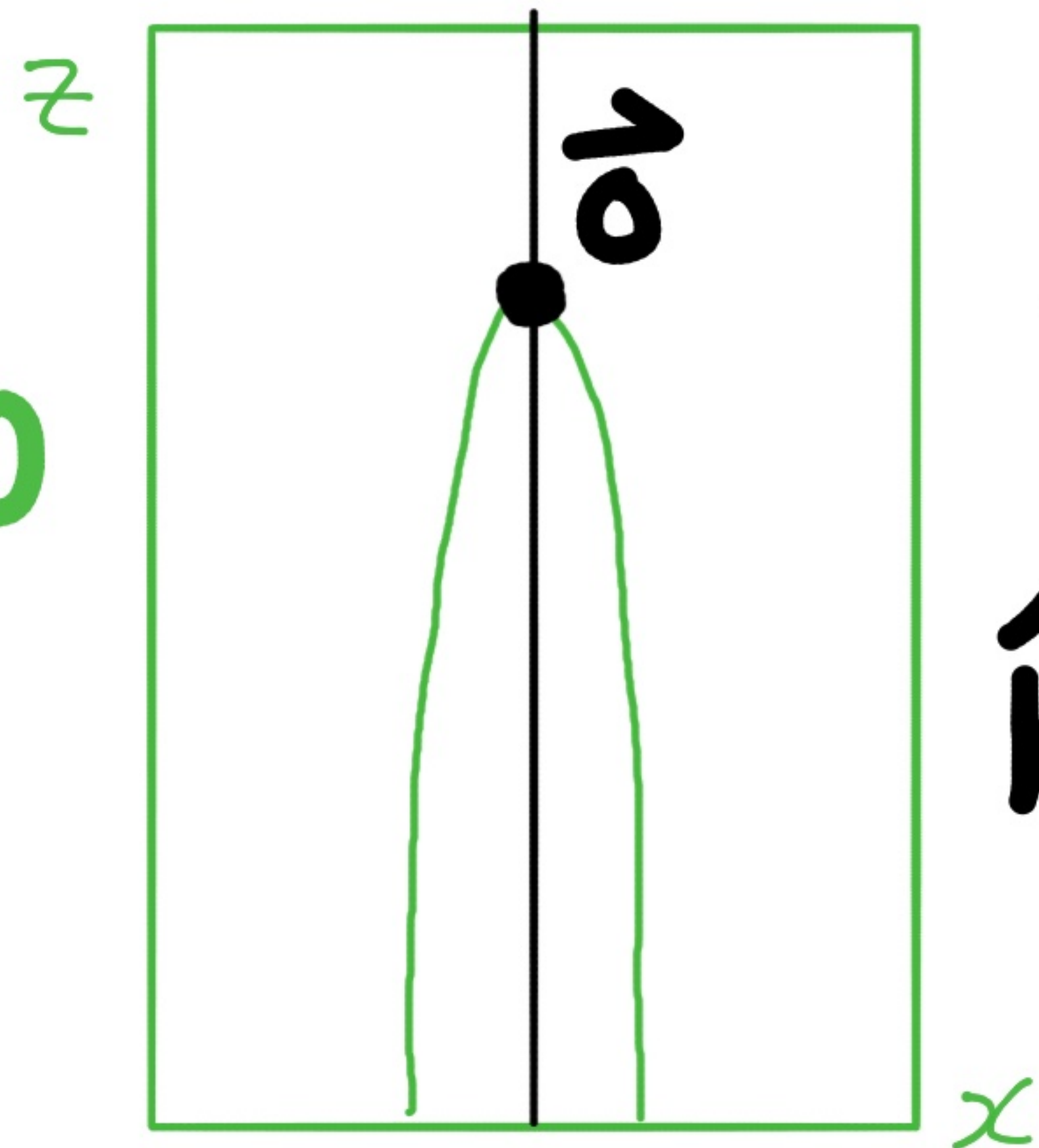
$$-y^2 = z$$

$$\hat{K}(\vec{0}) = -2$$



$$-3x^2 - y^2 = z$$

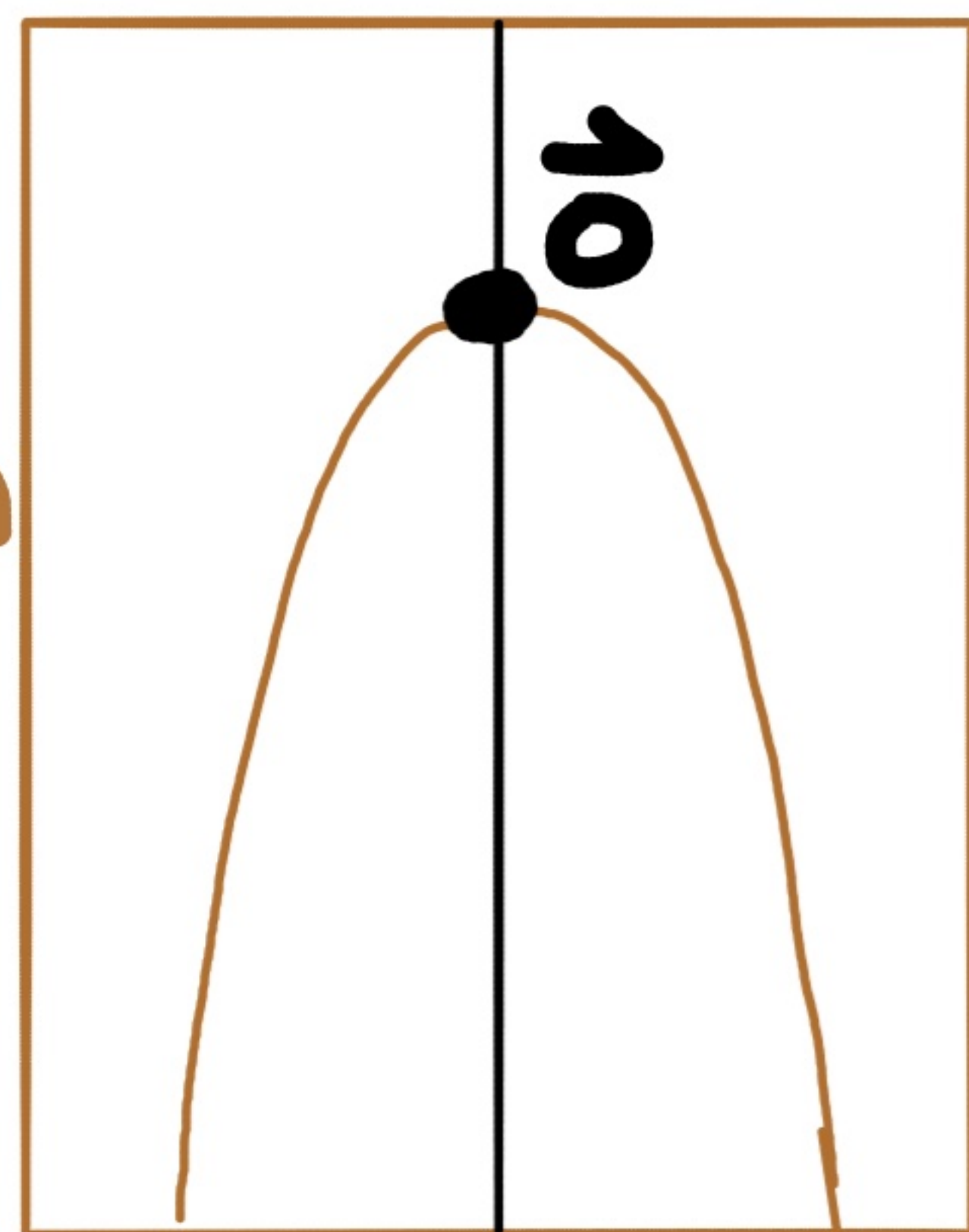
$$y=0$$



$$-3x^2 = z$$

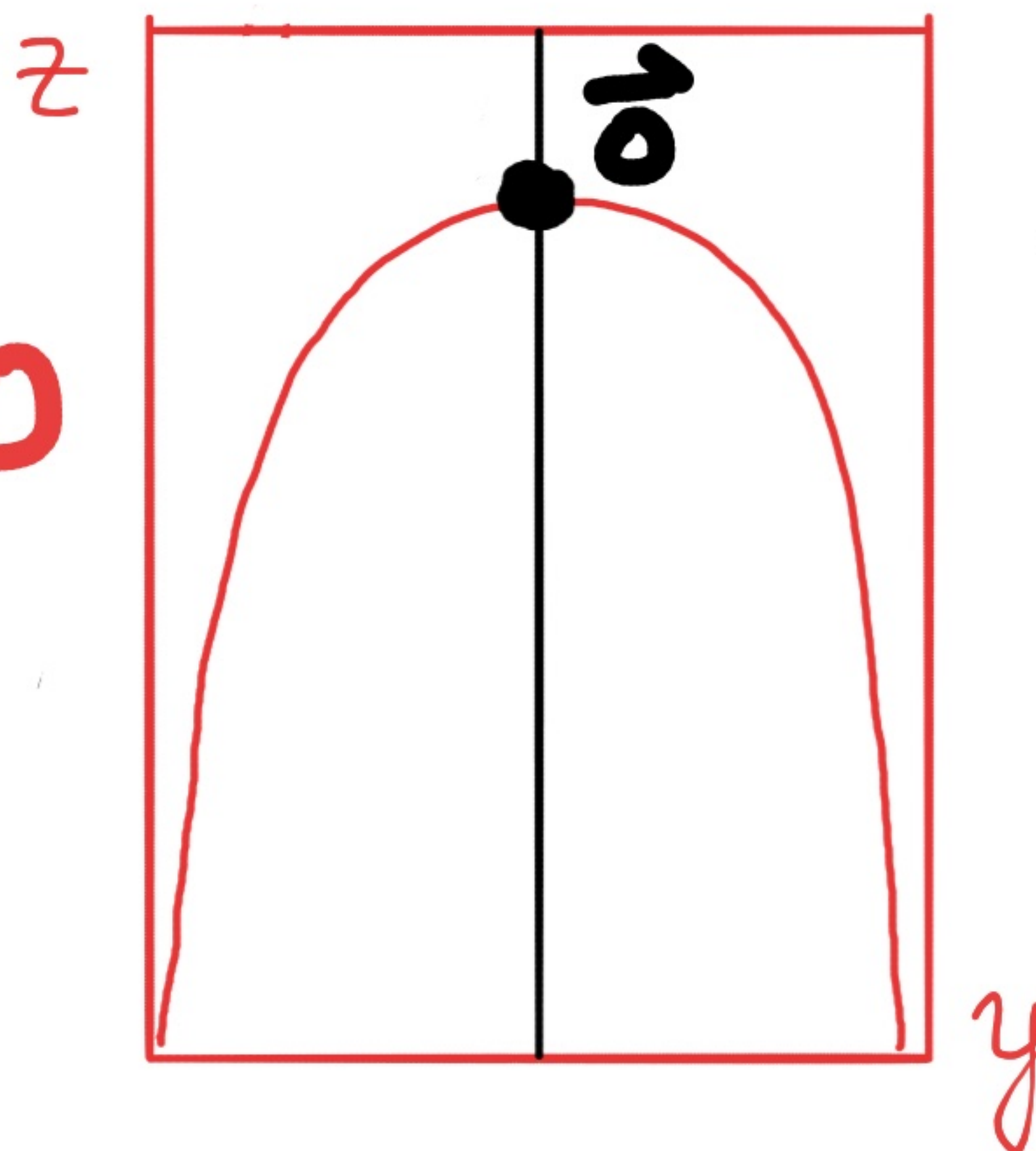
$$\hat{K}(\vec{0}) = -6$$

In
between



$$-6 \leq \hat{K}(\vec{0}) \leq -2$$

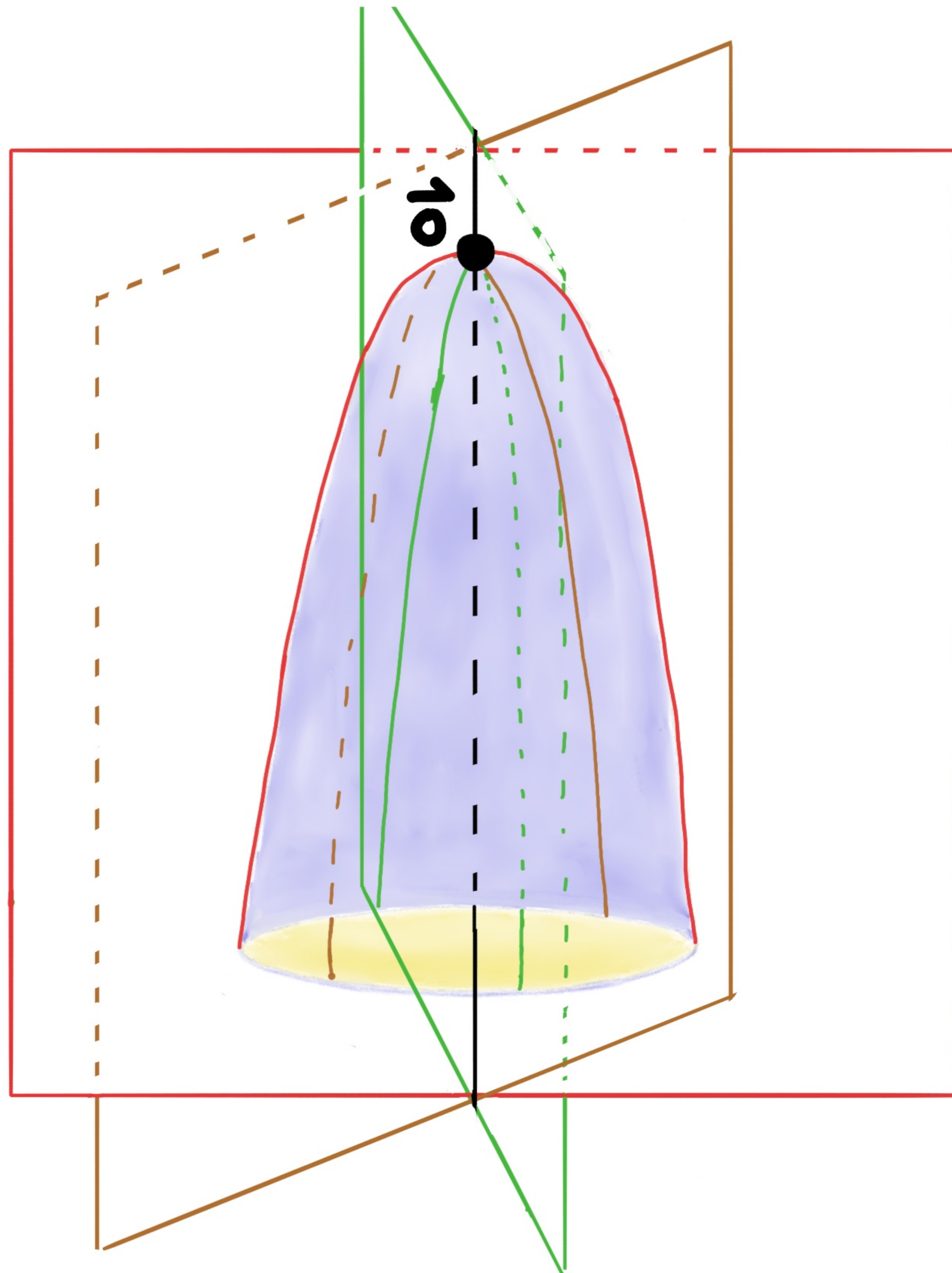
$$x=0$$



$$-y^2 = z$$

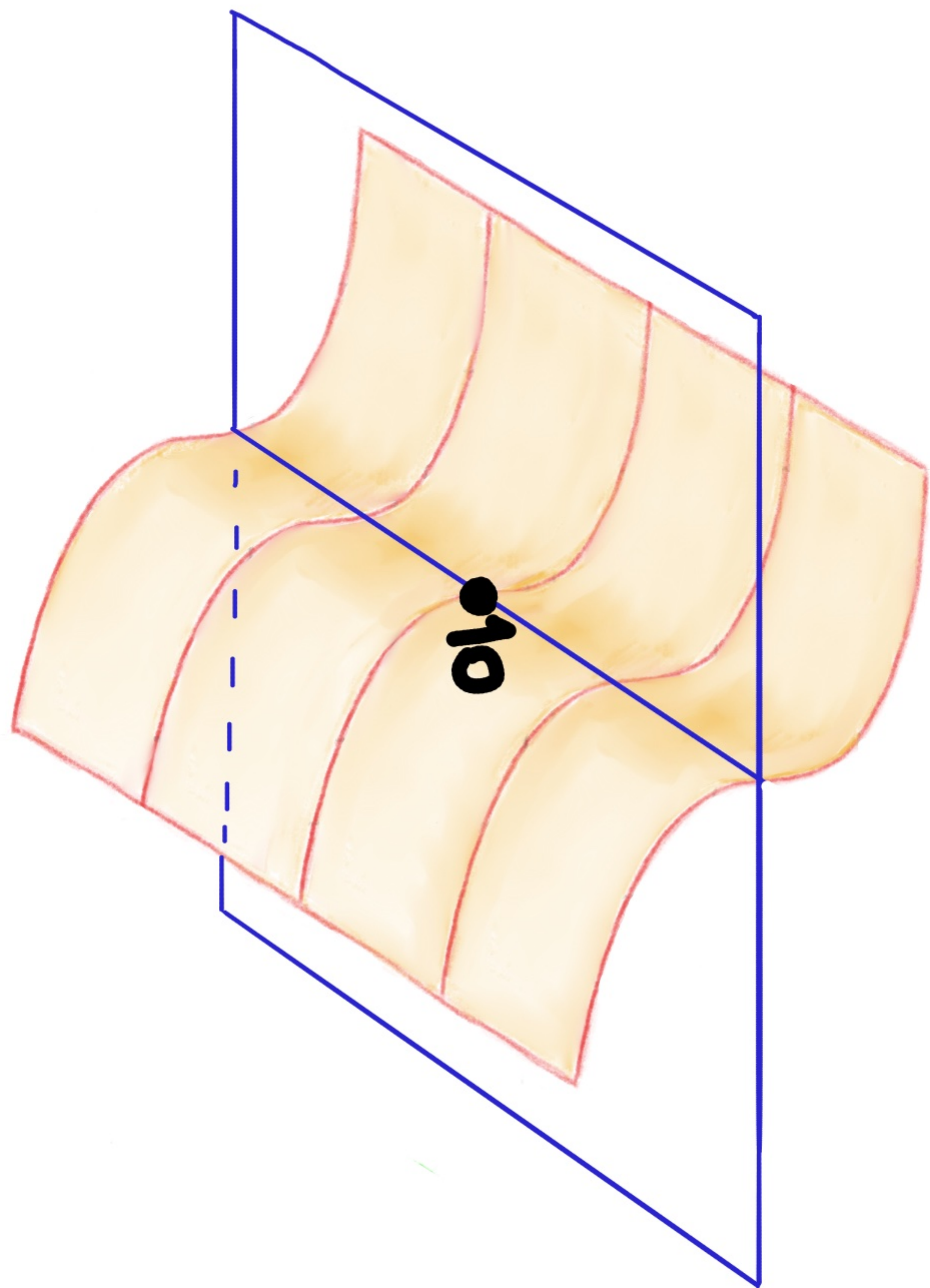
$$\hat{K}(\vec{0}) = -2$$

curvature
 $(-2)(-6) = 12$

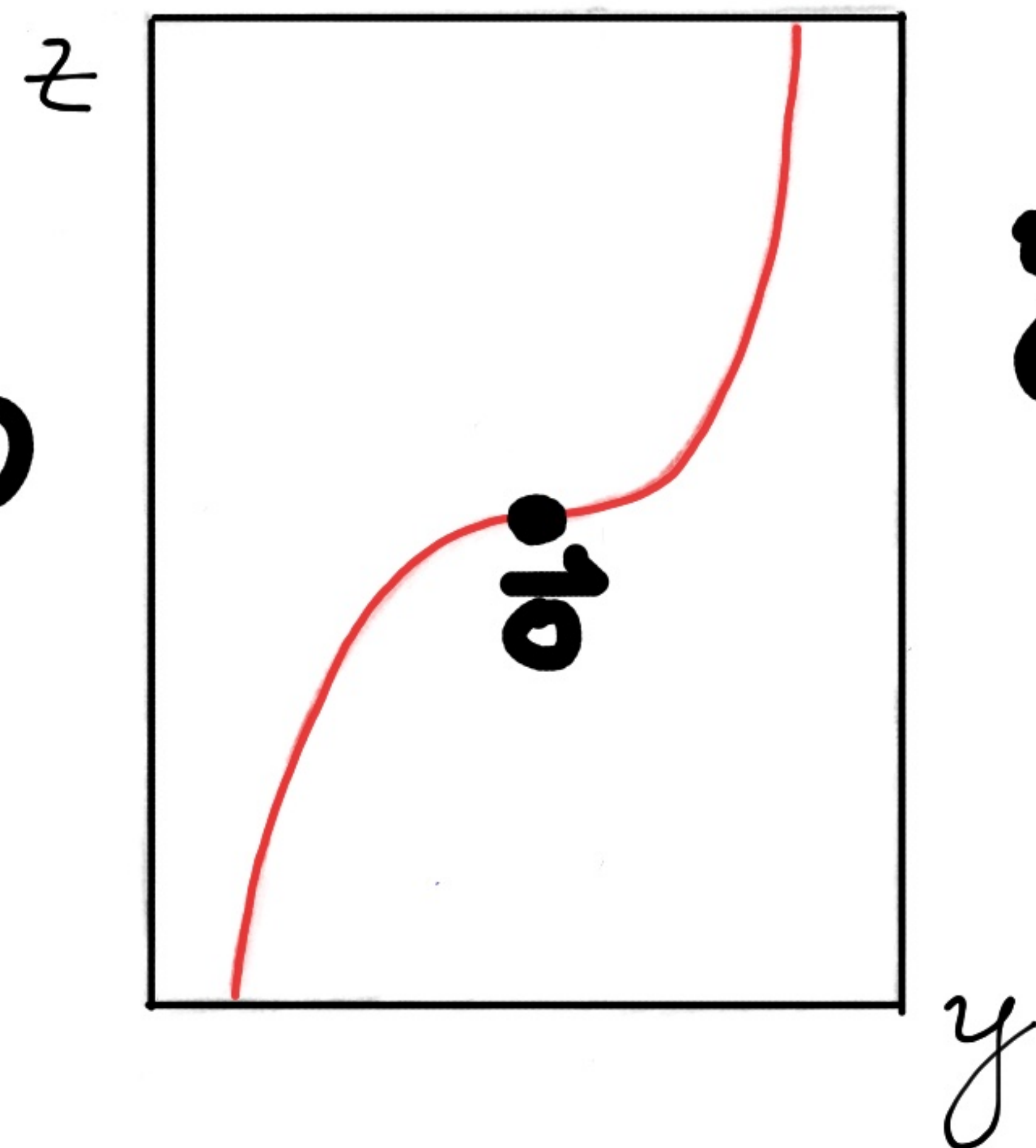


$$-3x^2 - y^2 = z$$

$$y^3 = z$$

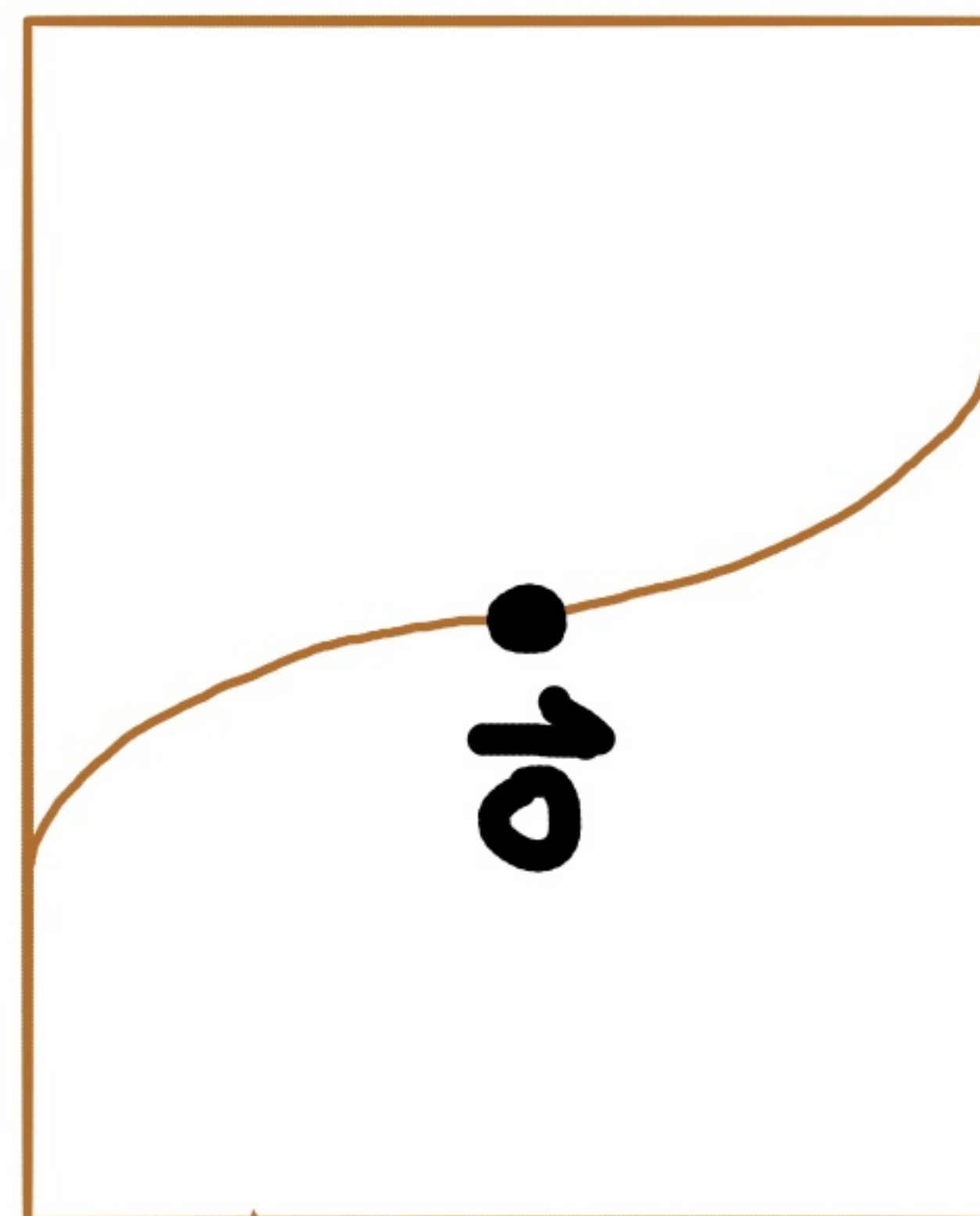


$$x=0$$

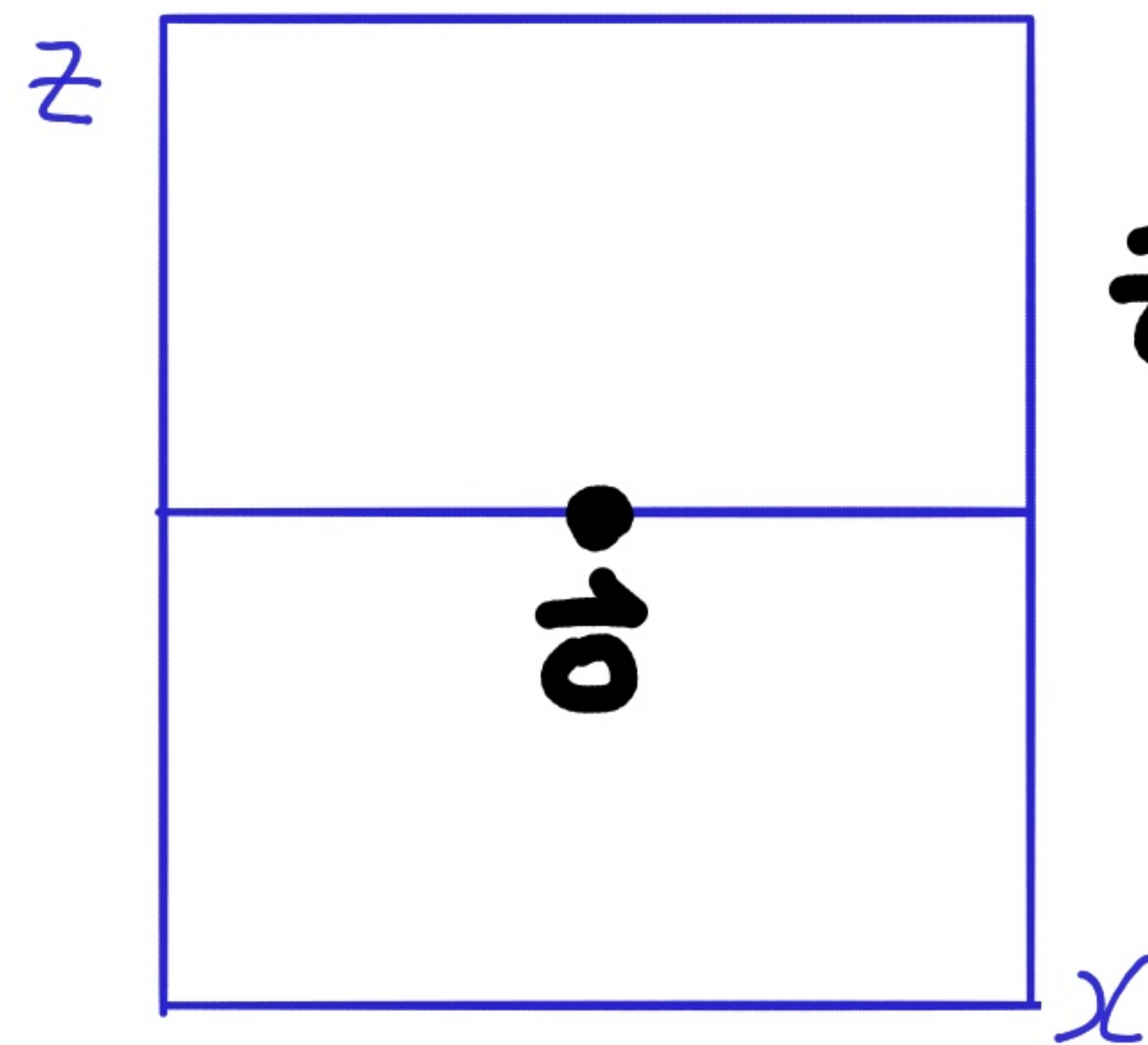


$$y^3 = z$$

In
between

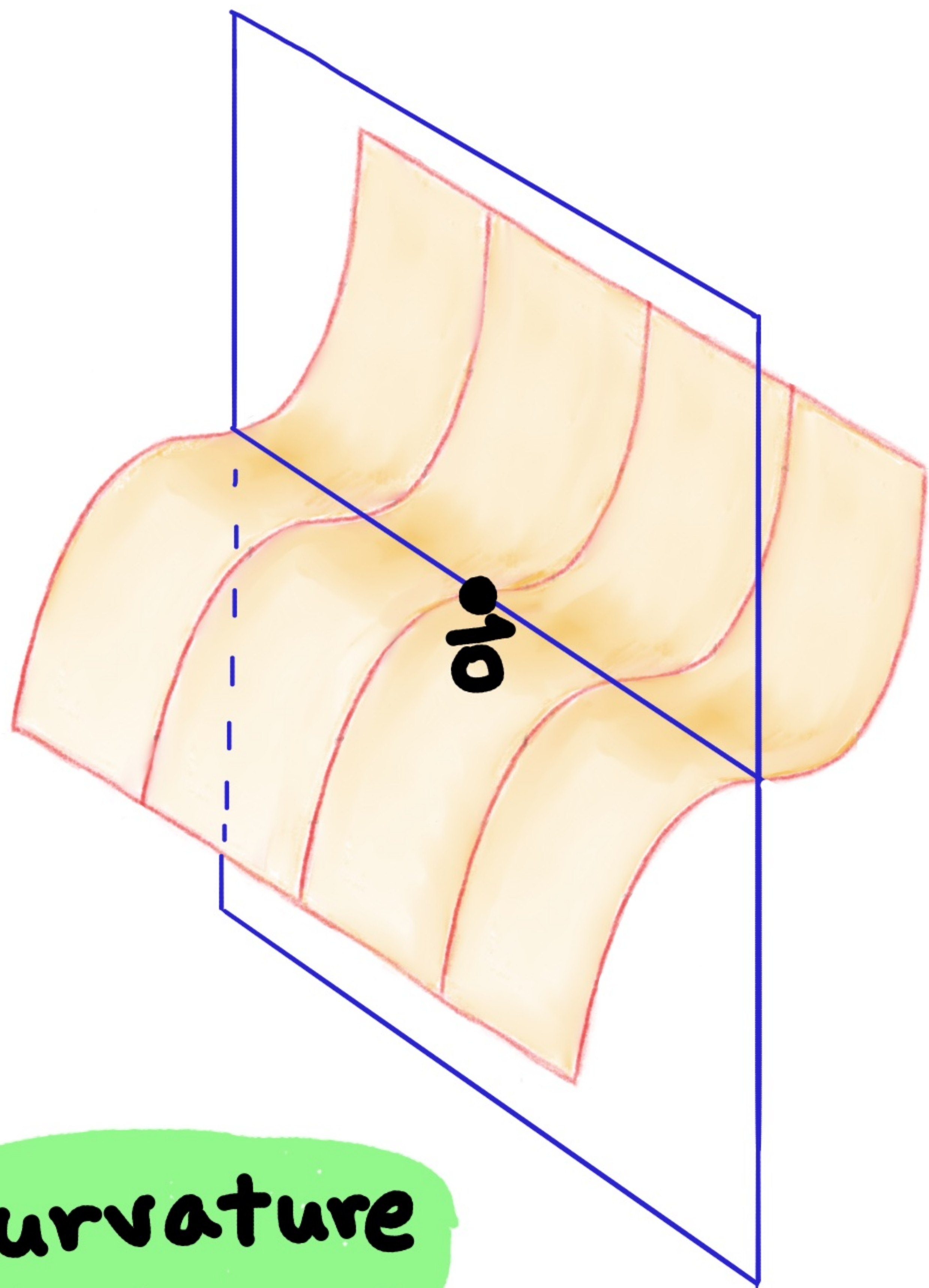


$$y=0$$



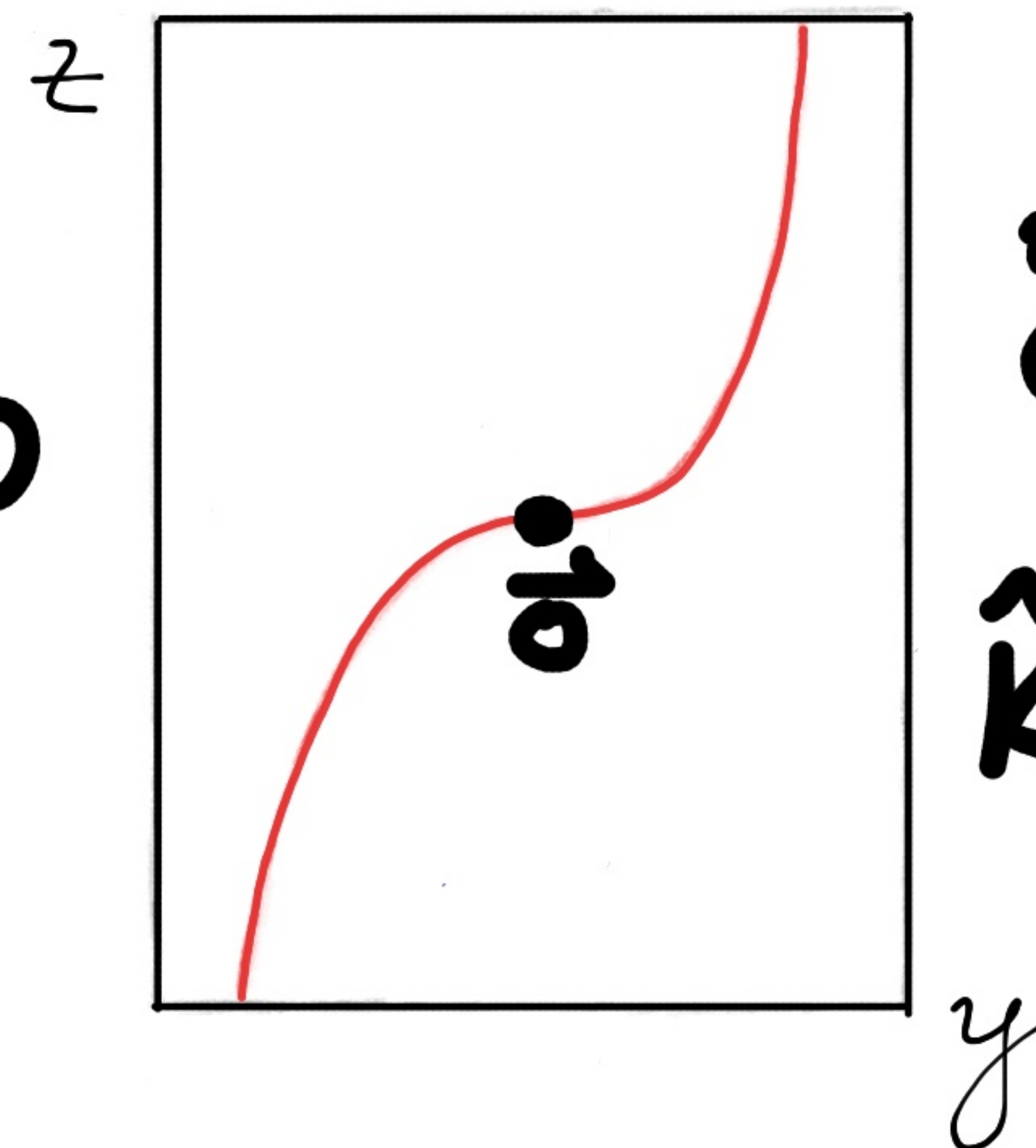
$$z=0$$

$$y^3 = z$$



curvature
 $0 \cdot 0 = 0$

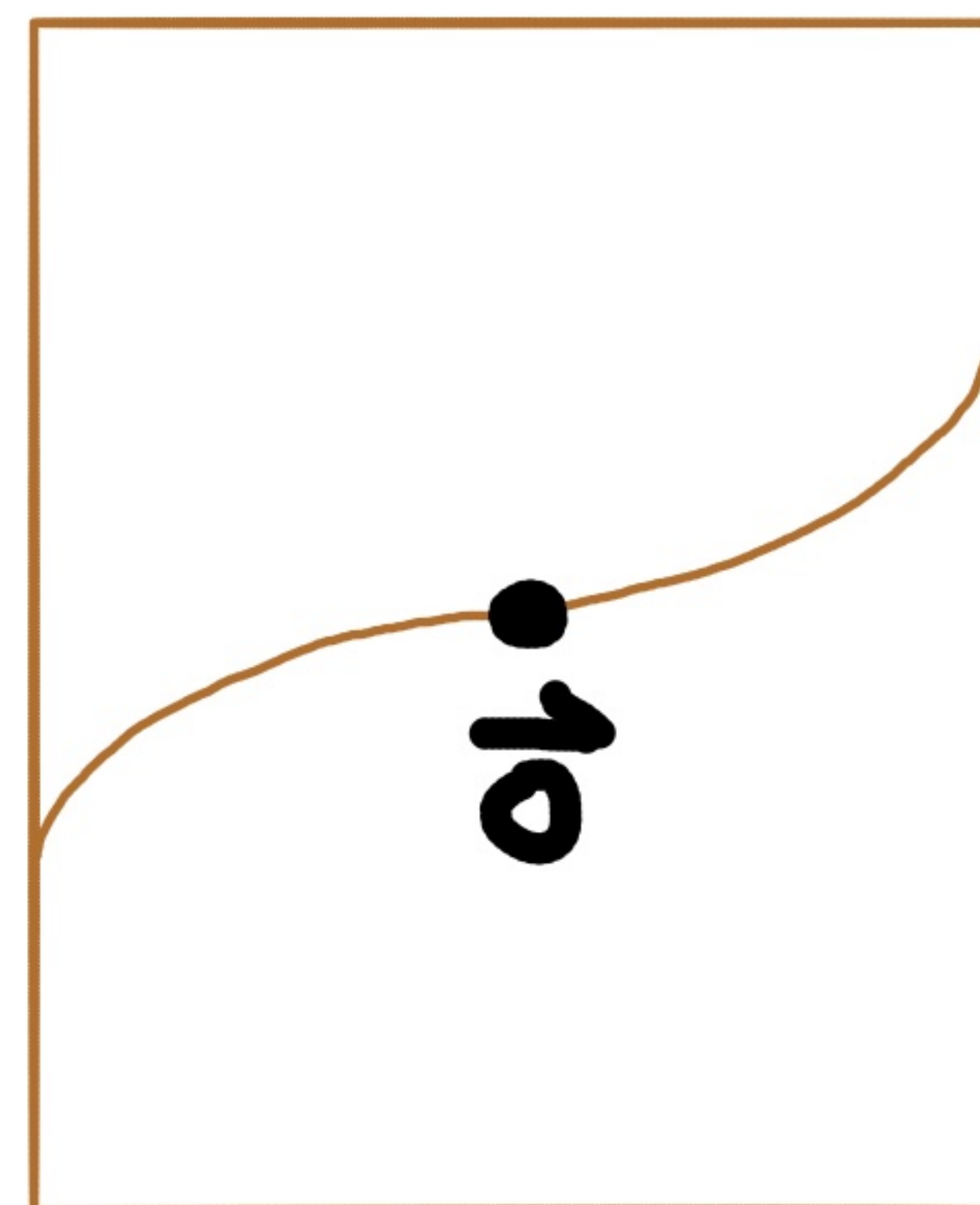
$$x=0$$



$$y^3 = z$$

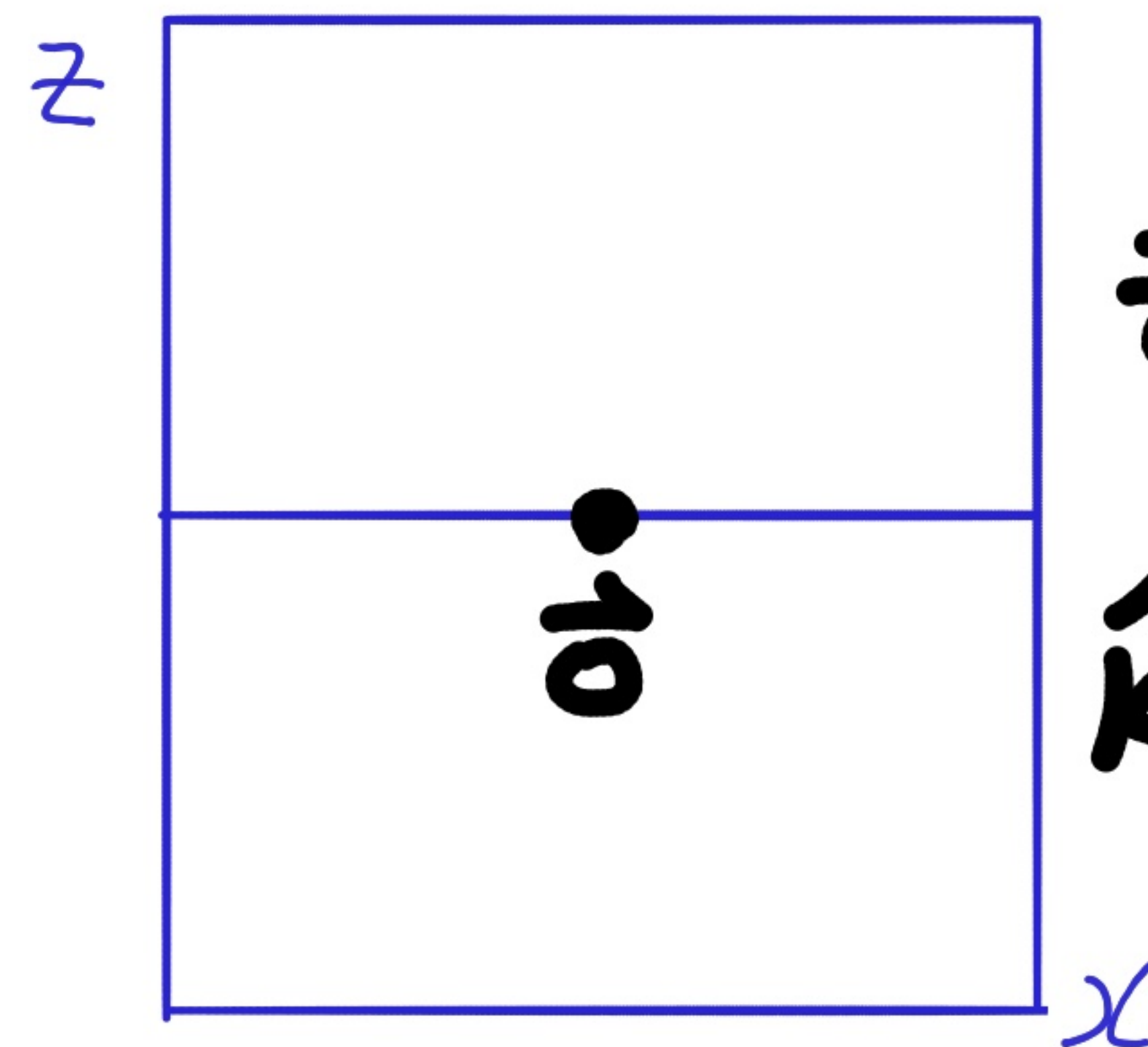
$$\hat{k}(\vec{0}) = 0$$

In
between



$$0 \leq \hat{k}(\vec{0}) \leq 0$$

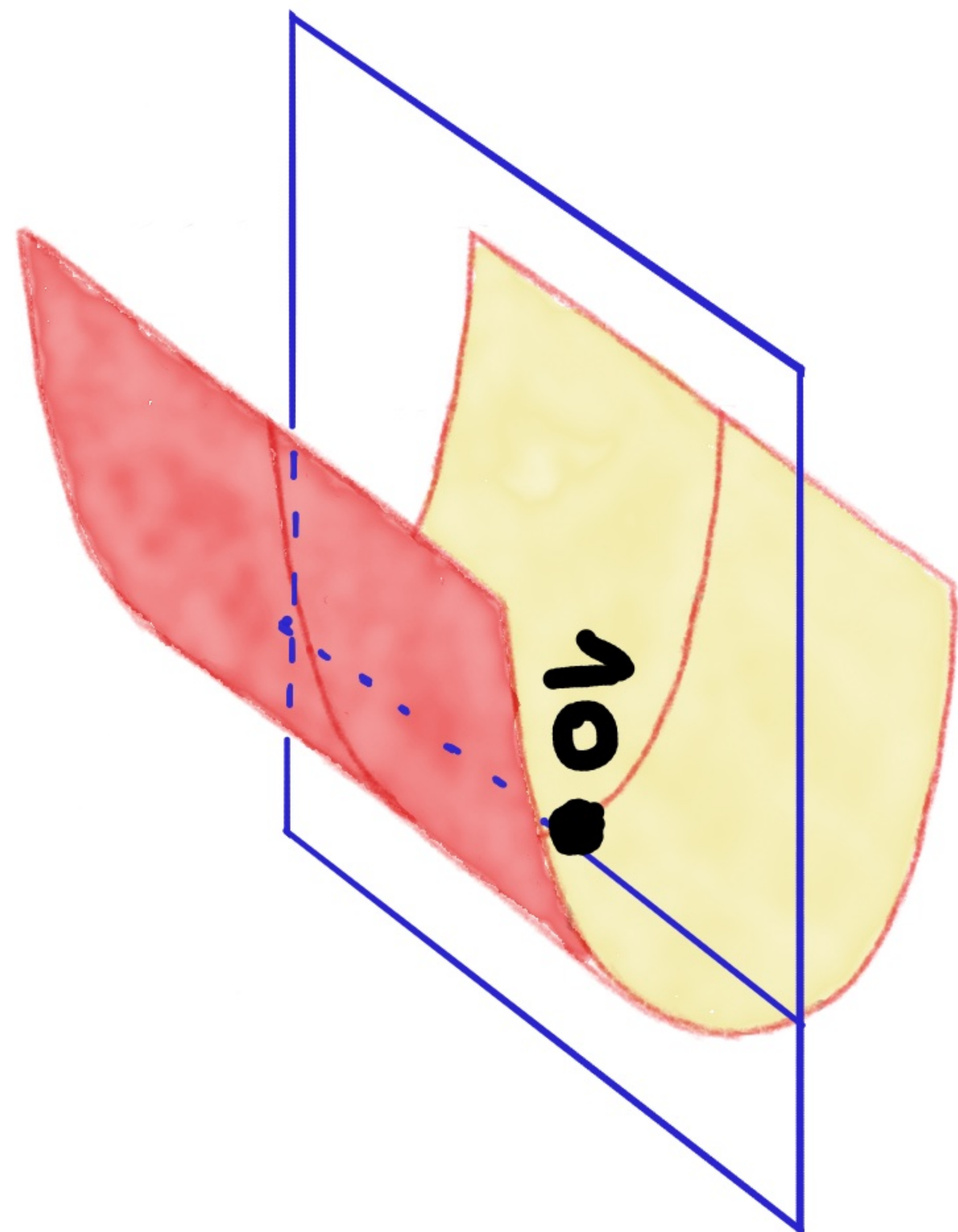
$$y=0$$



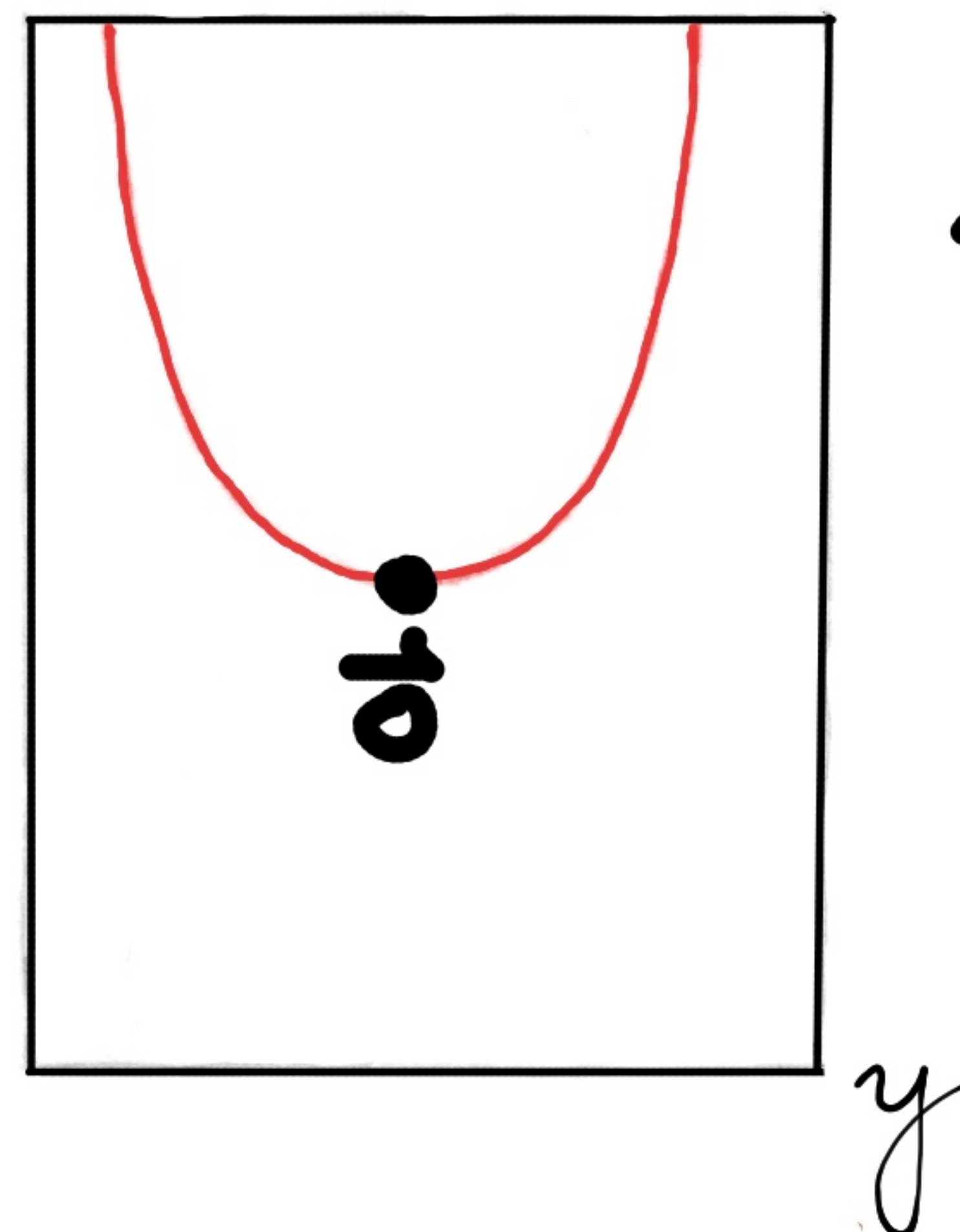
$$z=0$$

$$\hat{k}(\vec{0}) = 0$$

$$y^2 = z$$

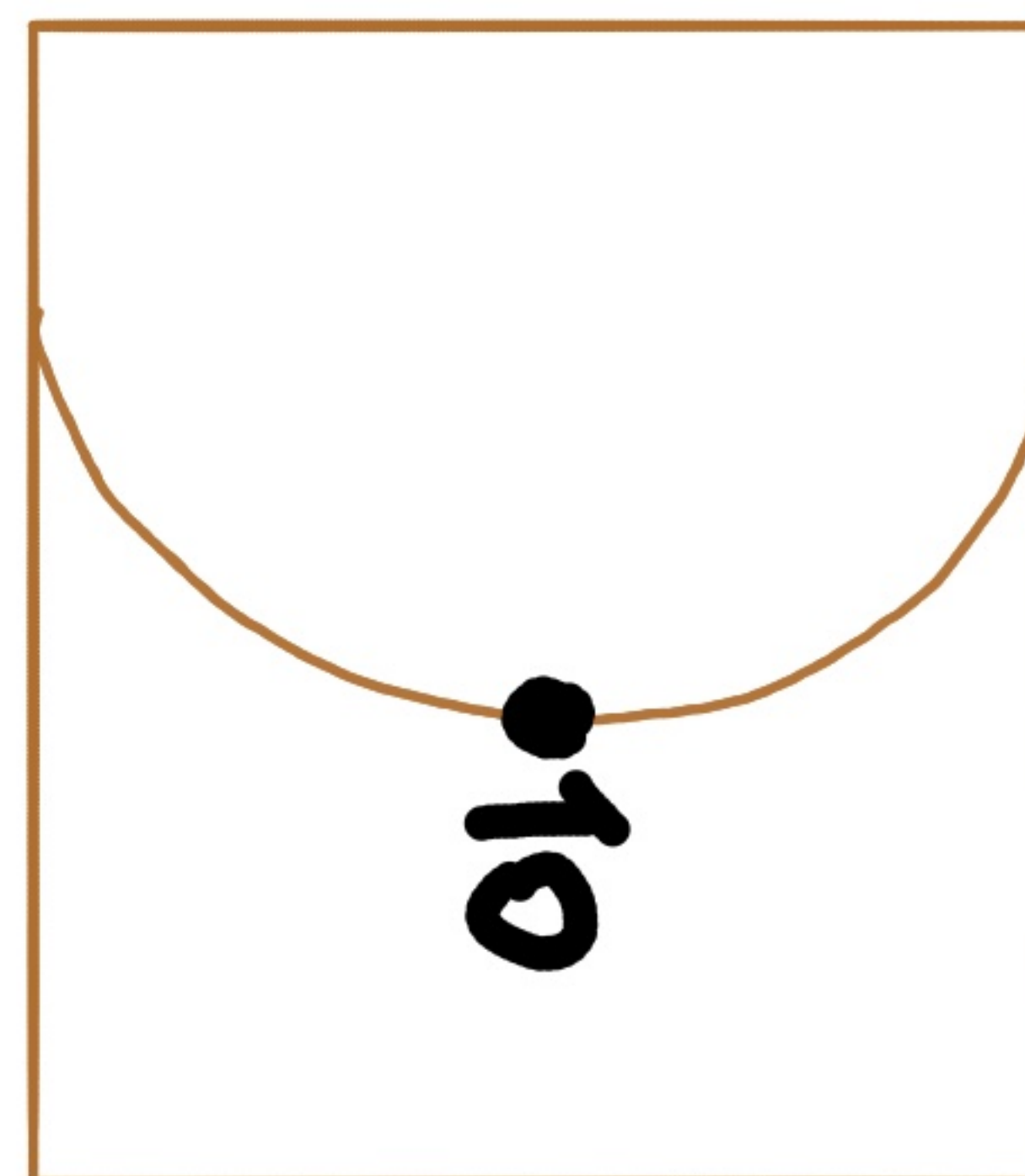


$$x=0$$

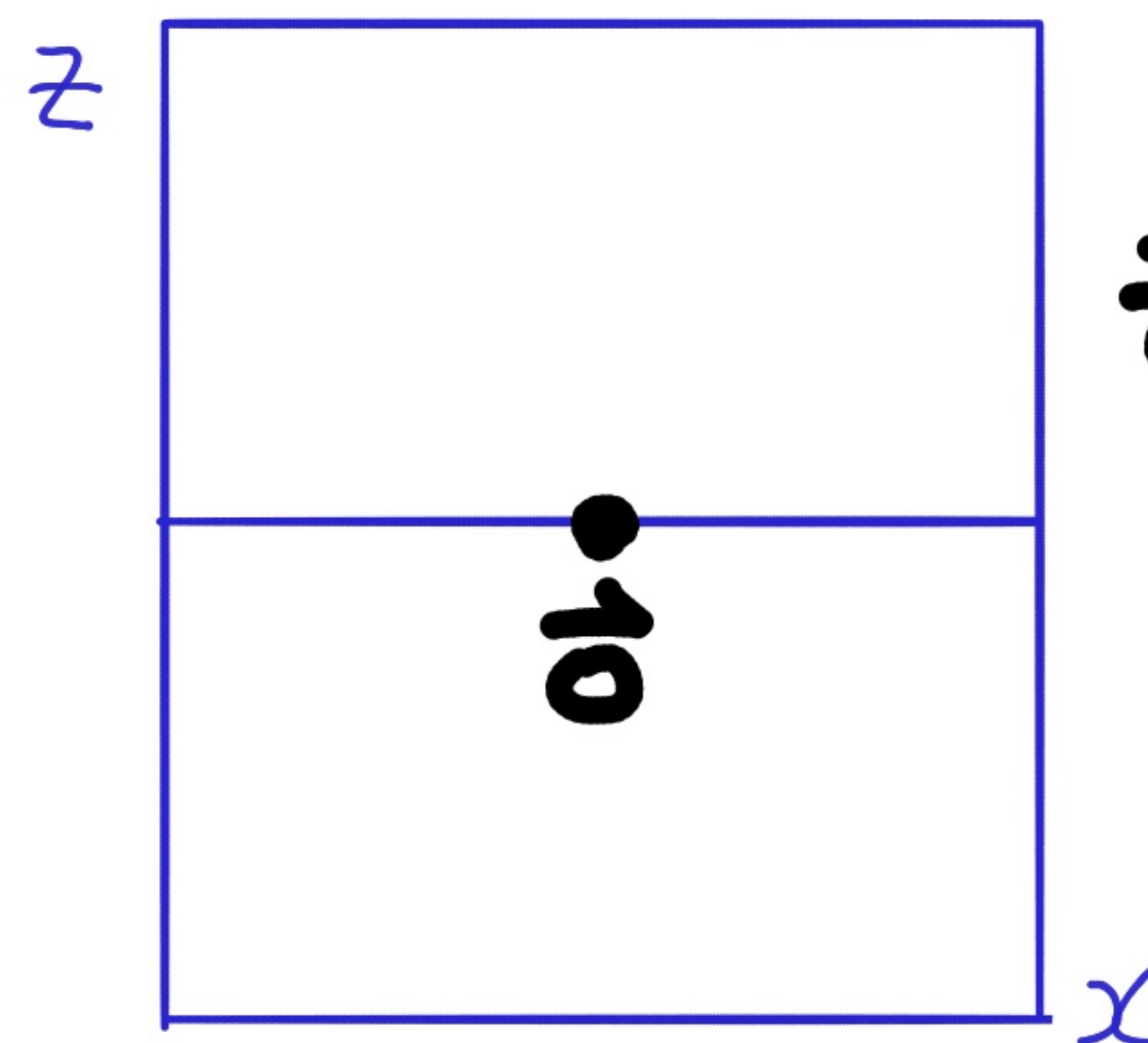


$$y^2 = z$$

In
between

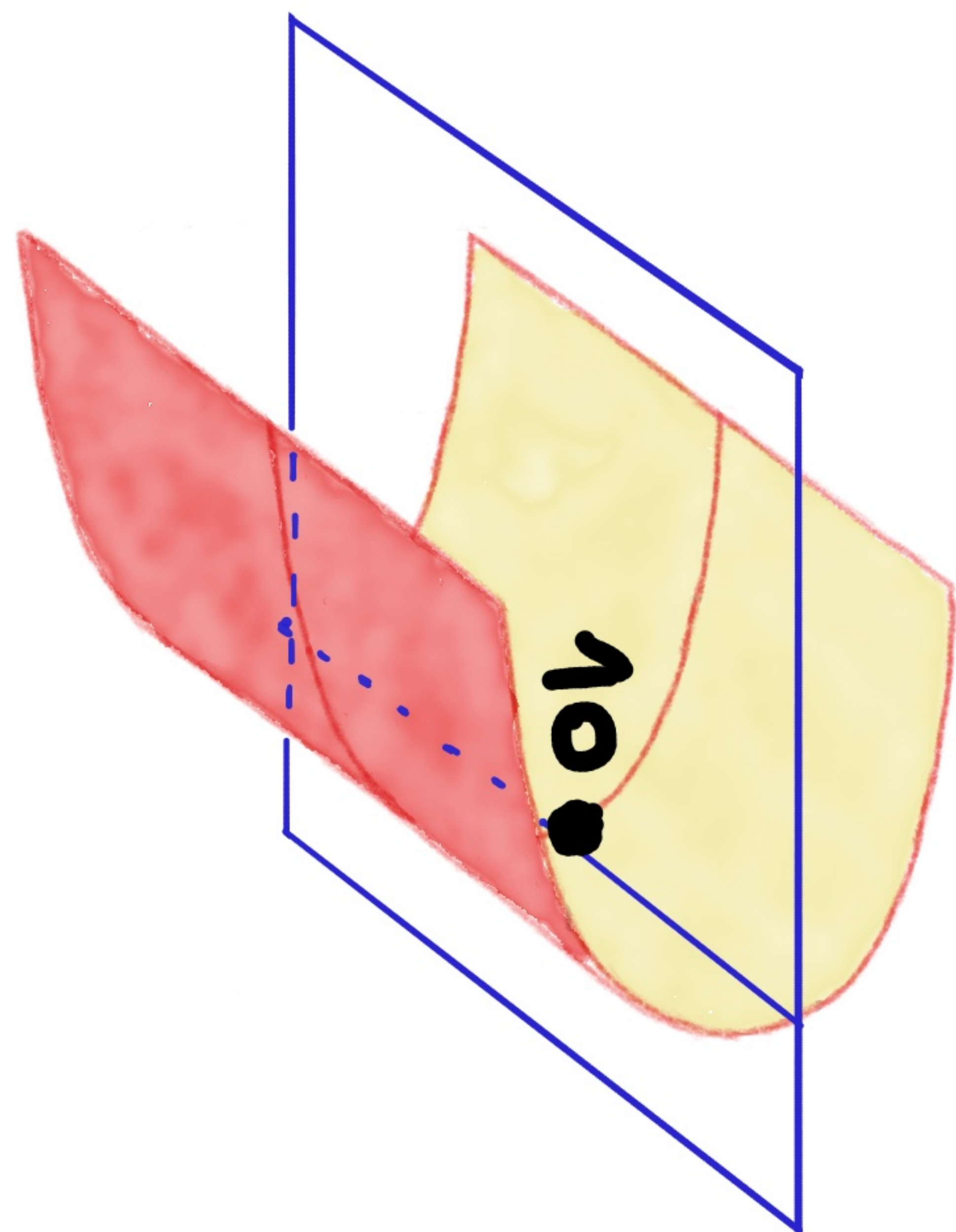


$$y=0$$



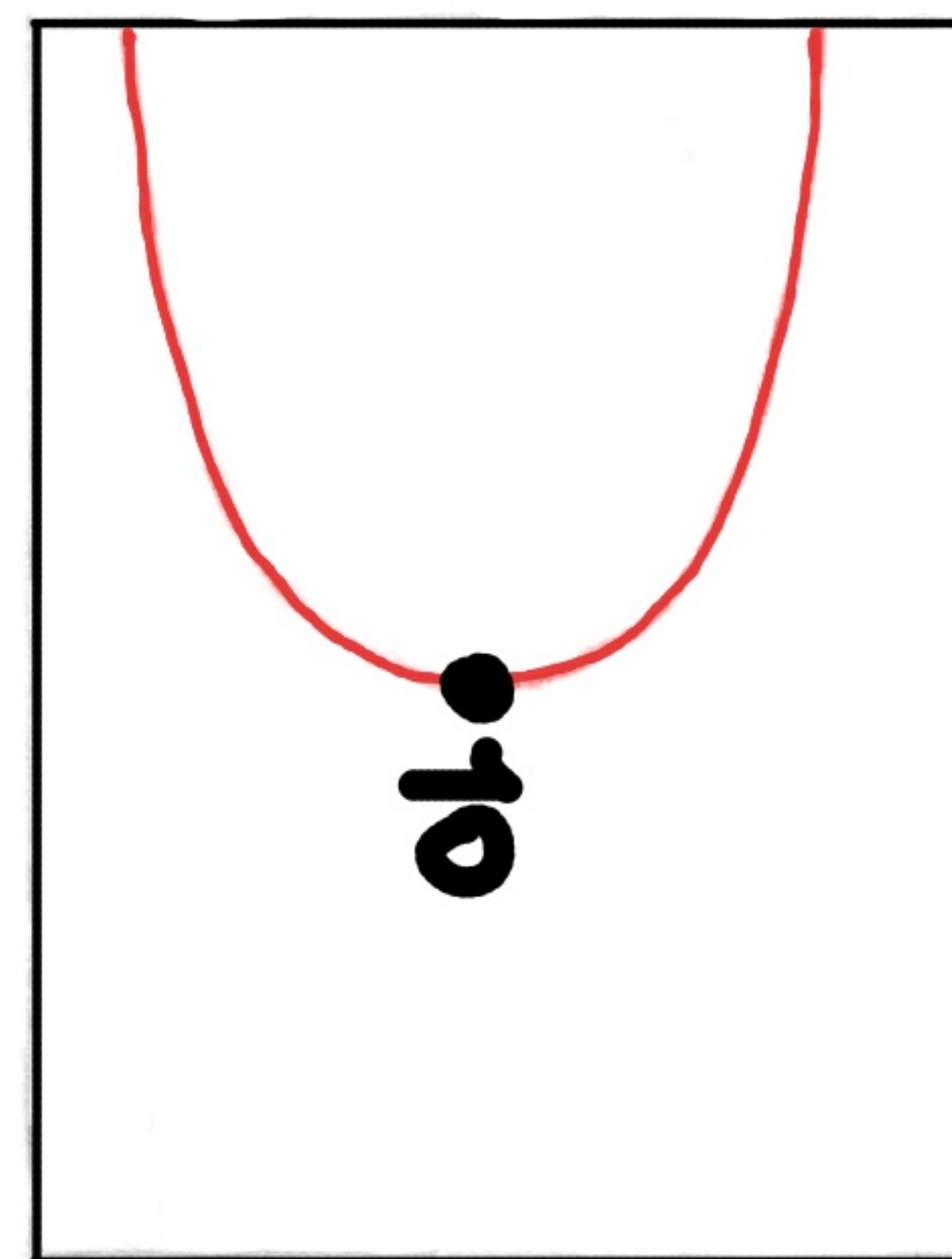
$$z=0$$

$$y^2 = z$$



curvature
 $0 \cdot 2 = 0$

$$x=0$$

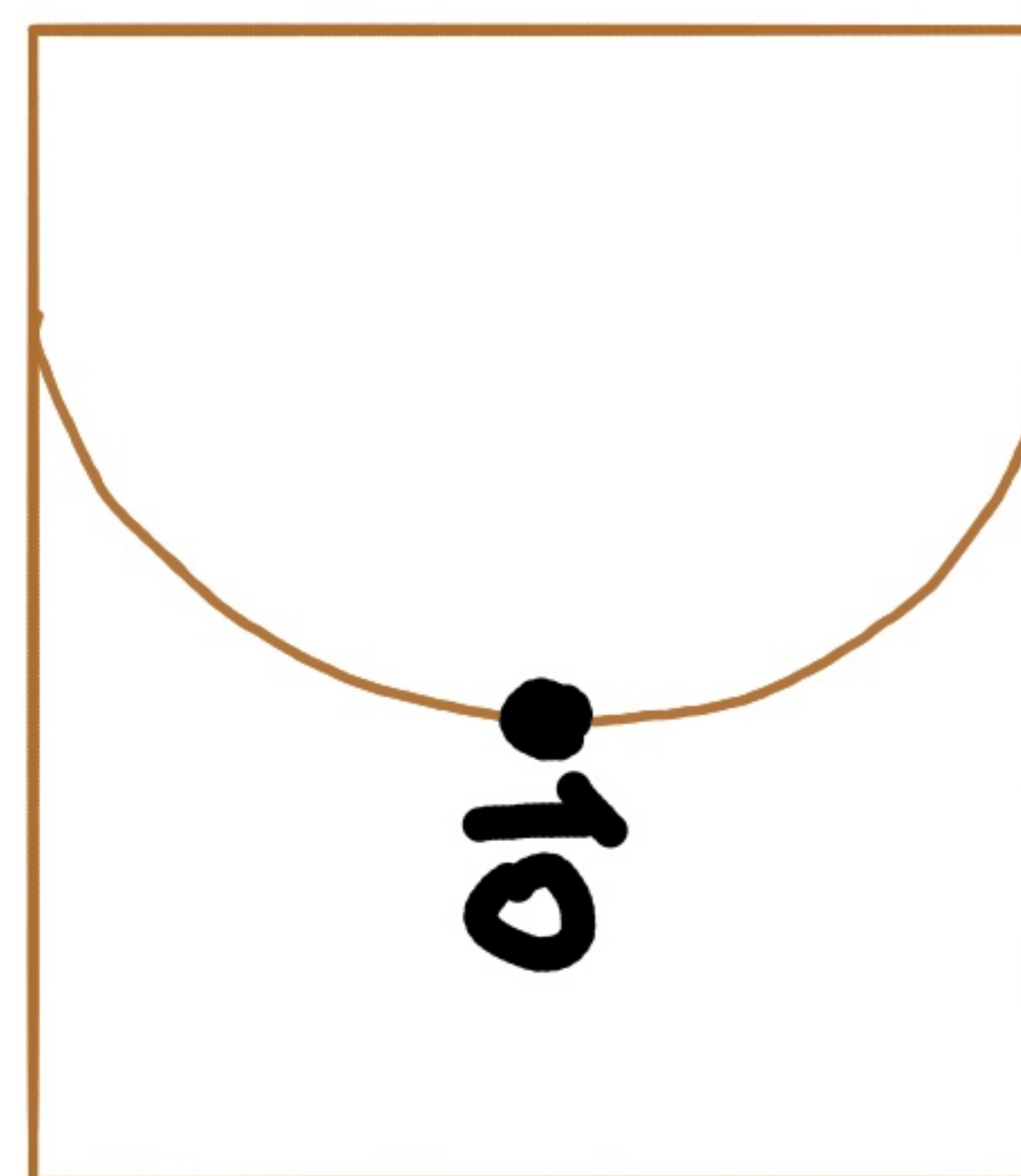
 z


$$y^2 = z$$

$$\hat{k}(\vec{0}) = 2$$

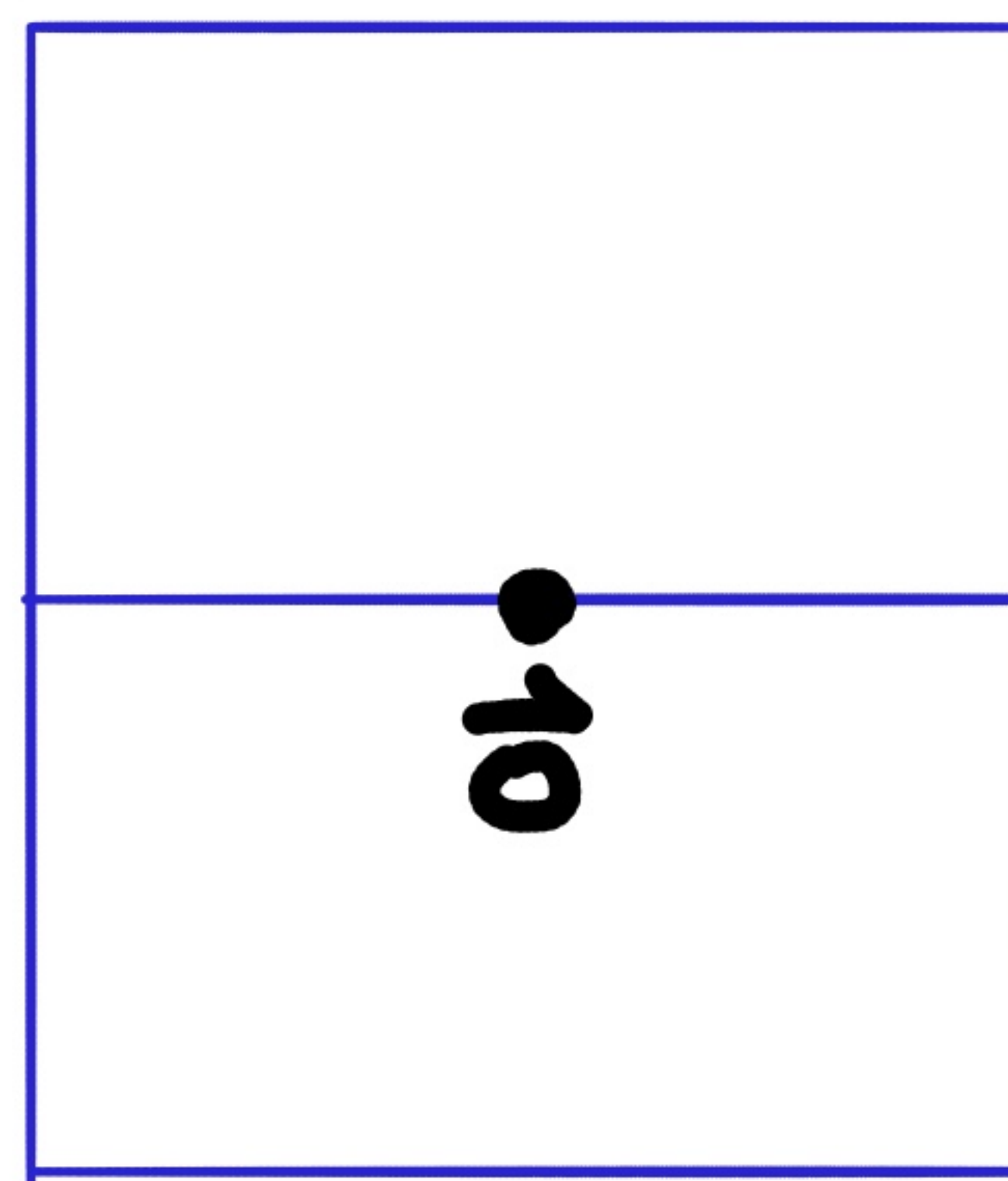
 y

In
between



$$0 \leq \hat{k}(\vec{0}) \leq 2$$

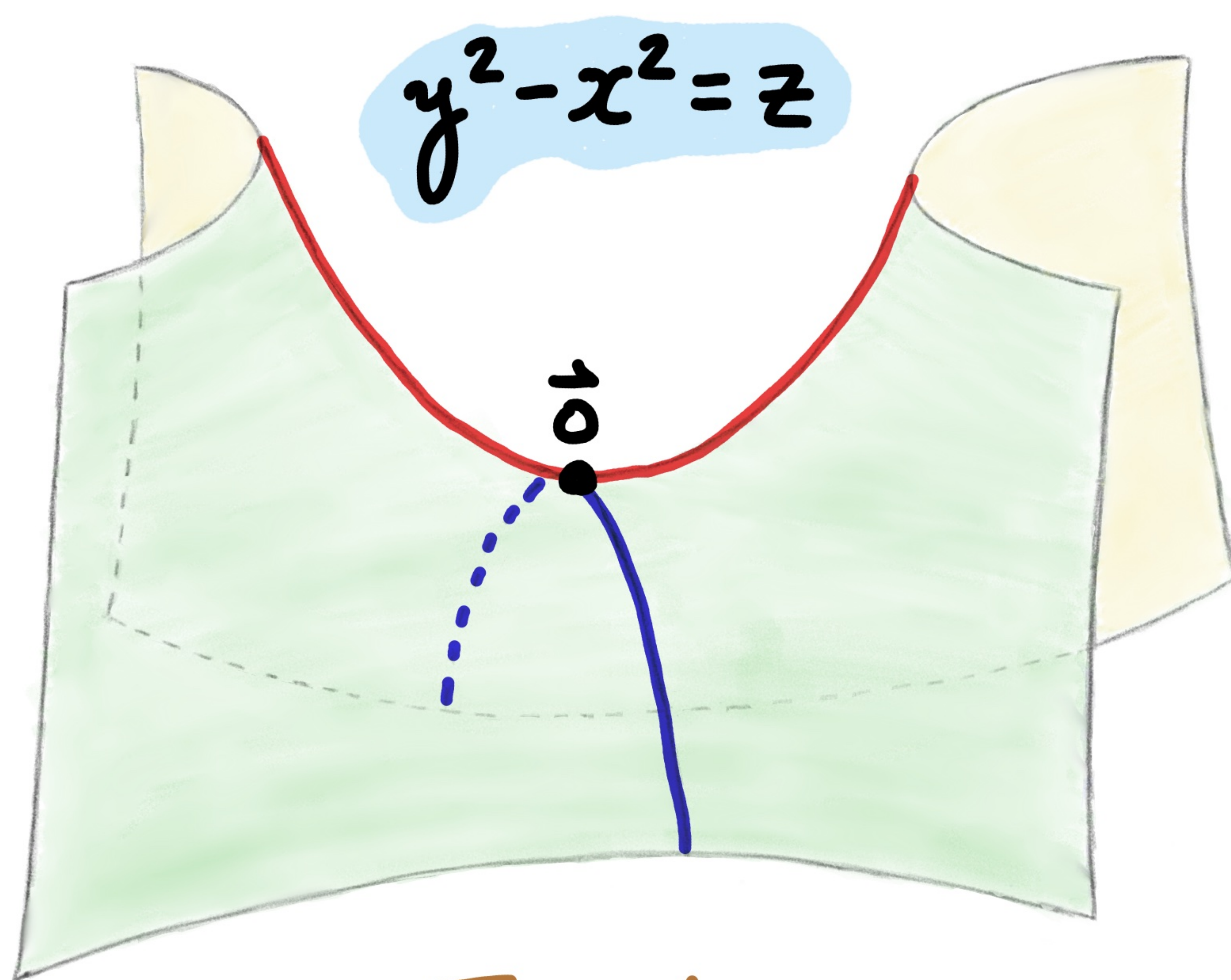
$$y=0$$

 z


$$z=0$$

$$\hat{k}(\vec{0}) = 0$$

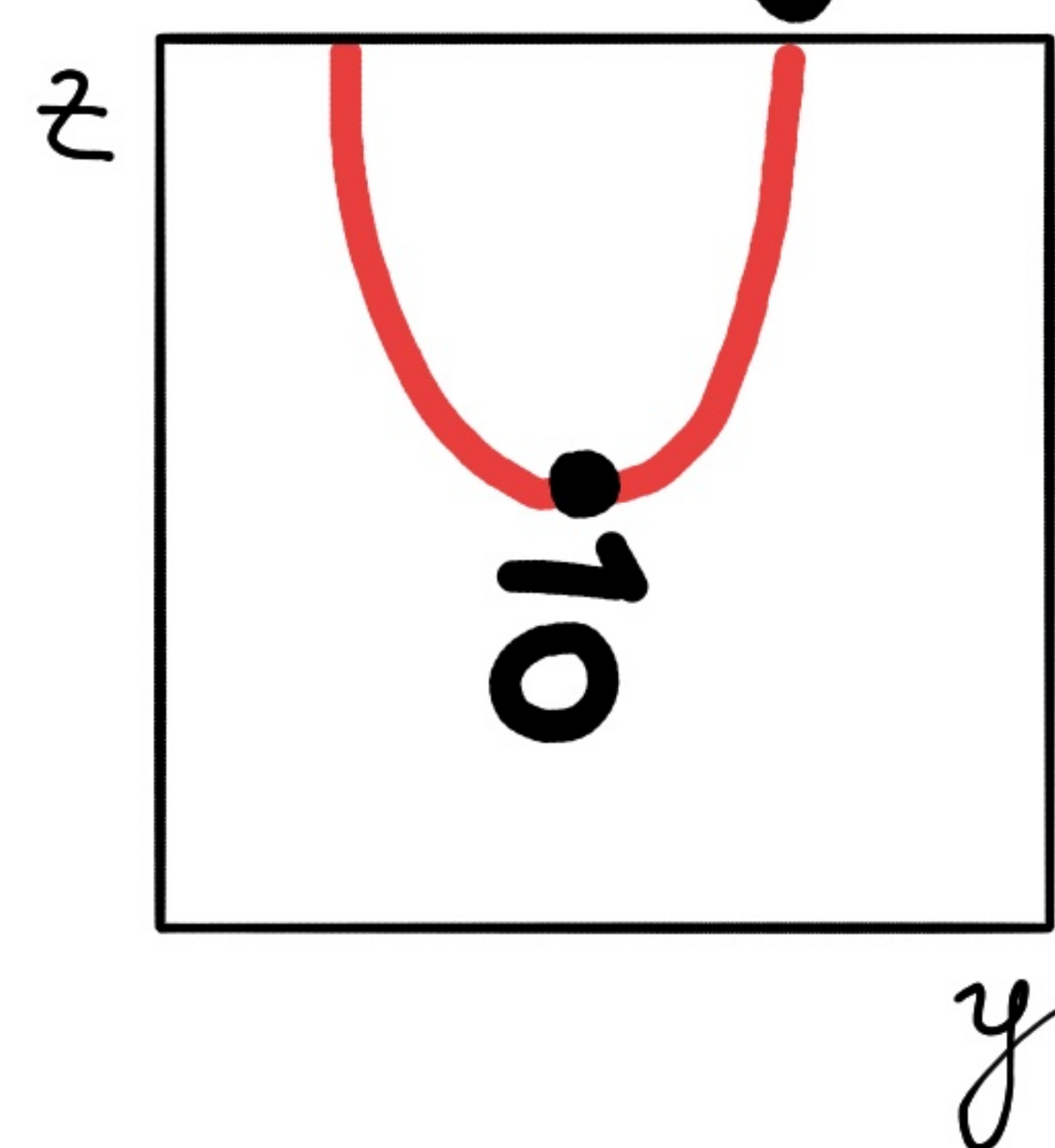
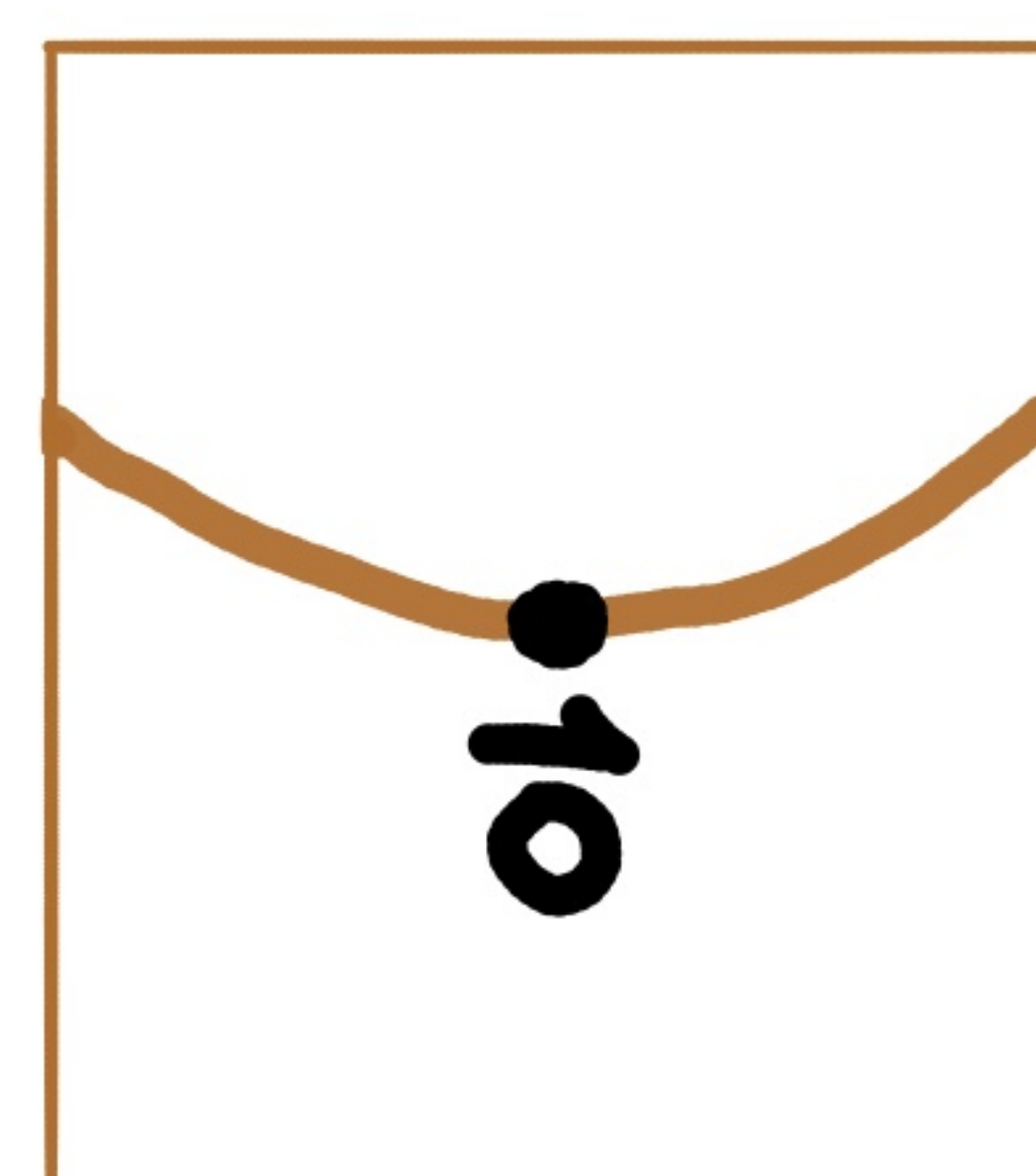
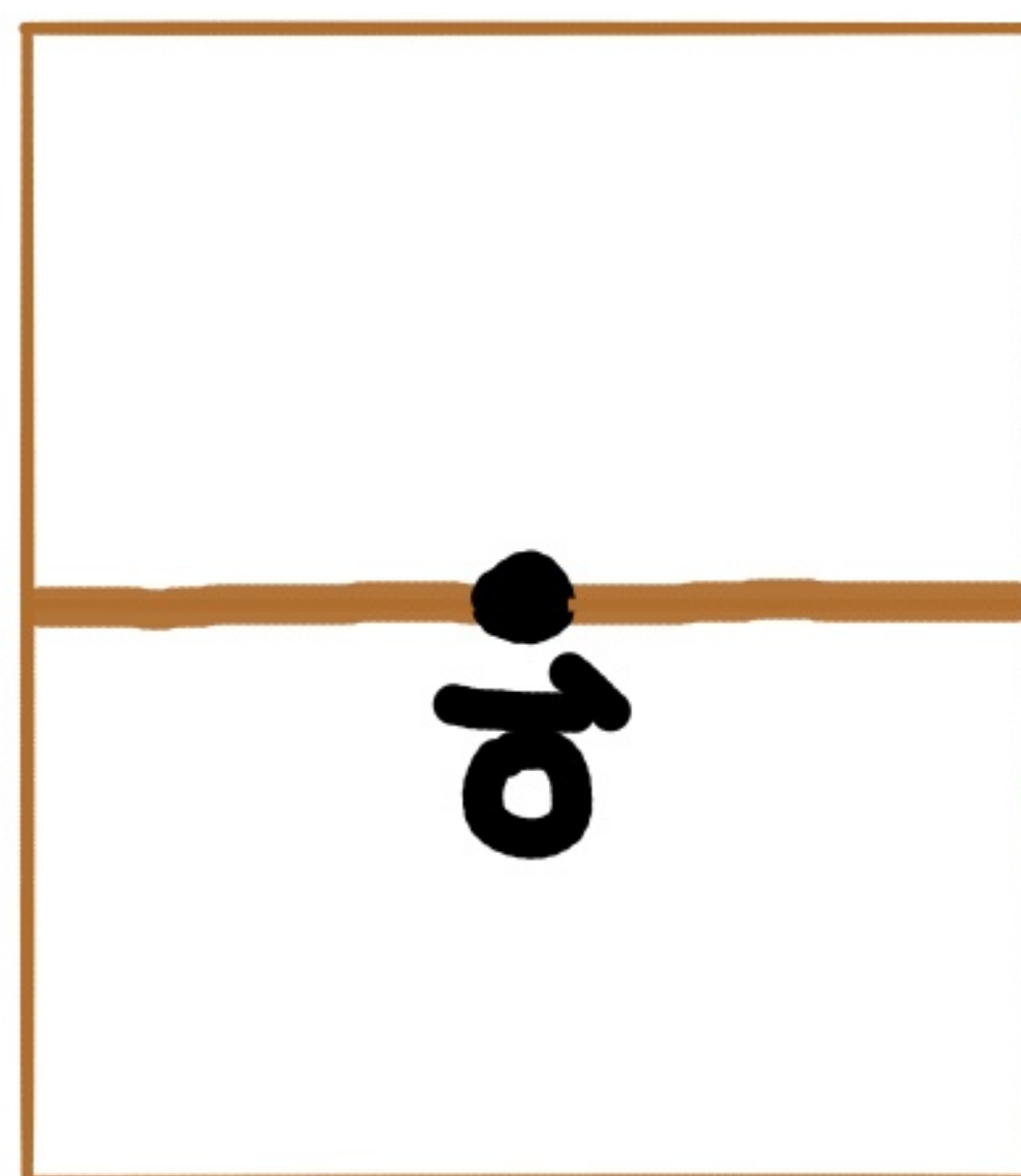
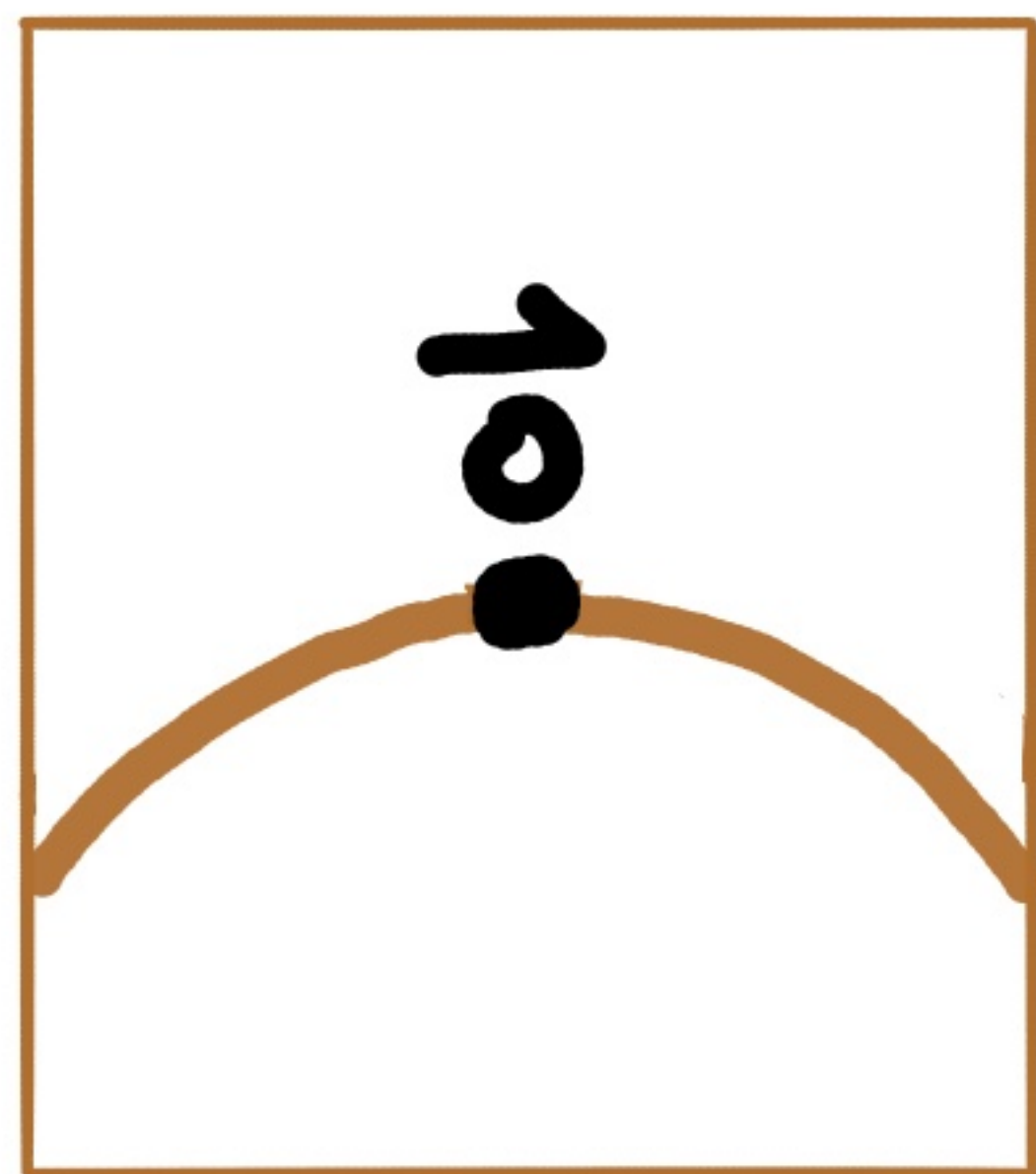
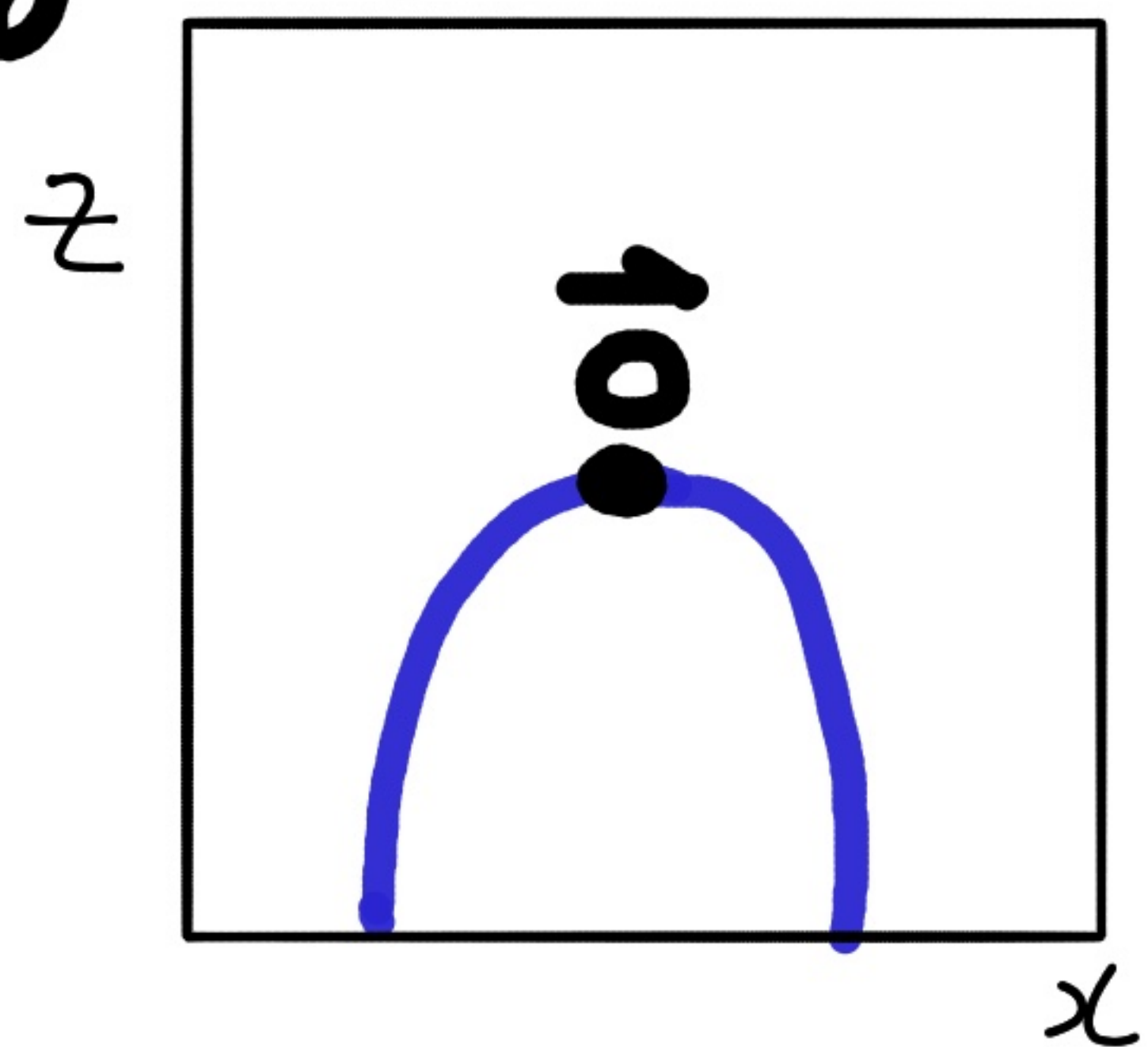
 x

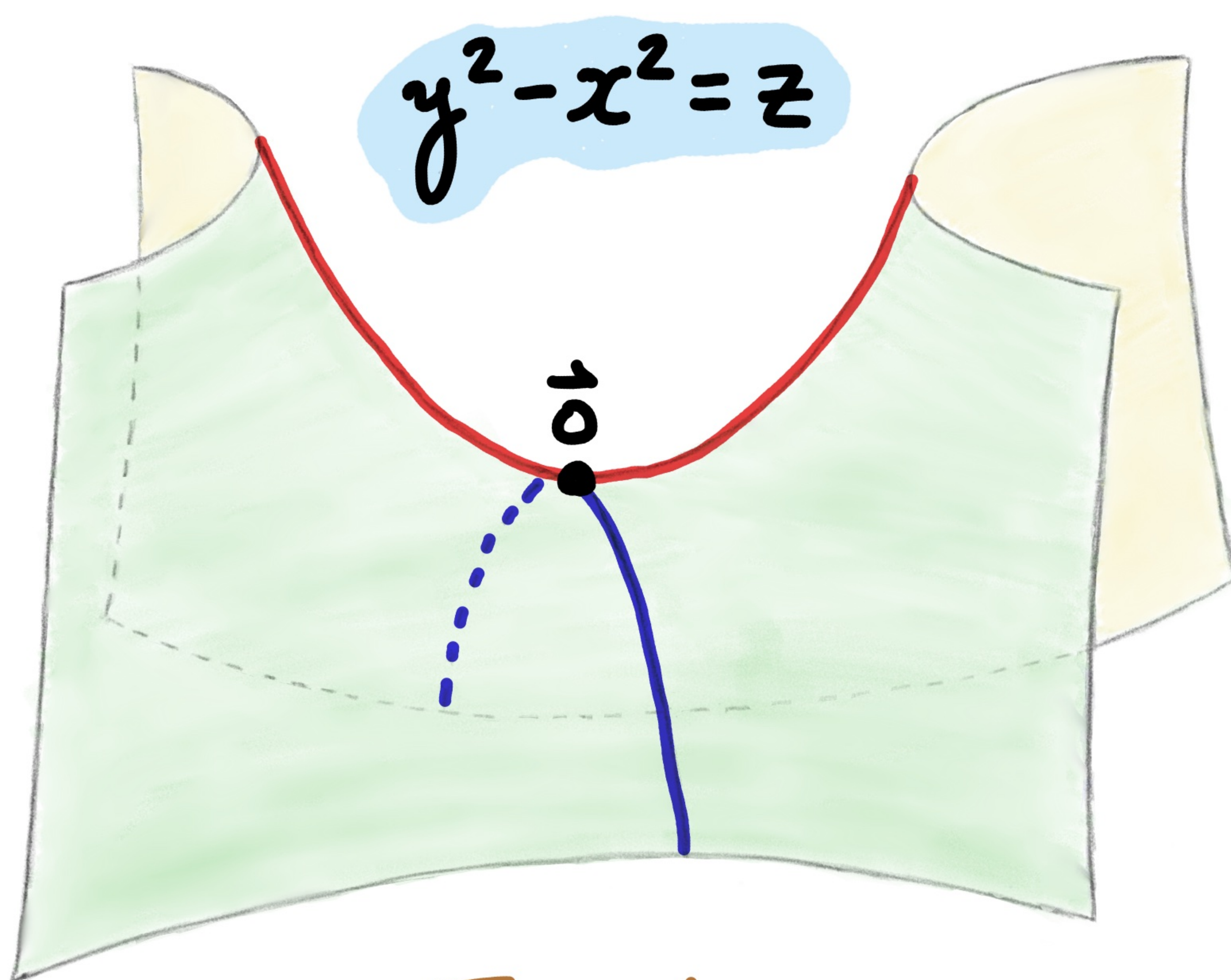


$y=0, -x^2=z$

In between

$x=0, y^2=z$

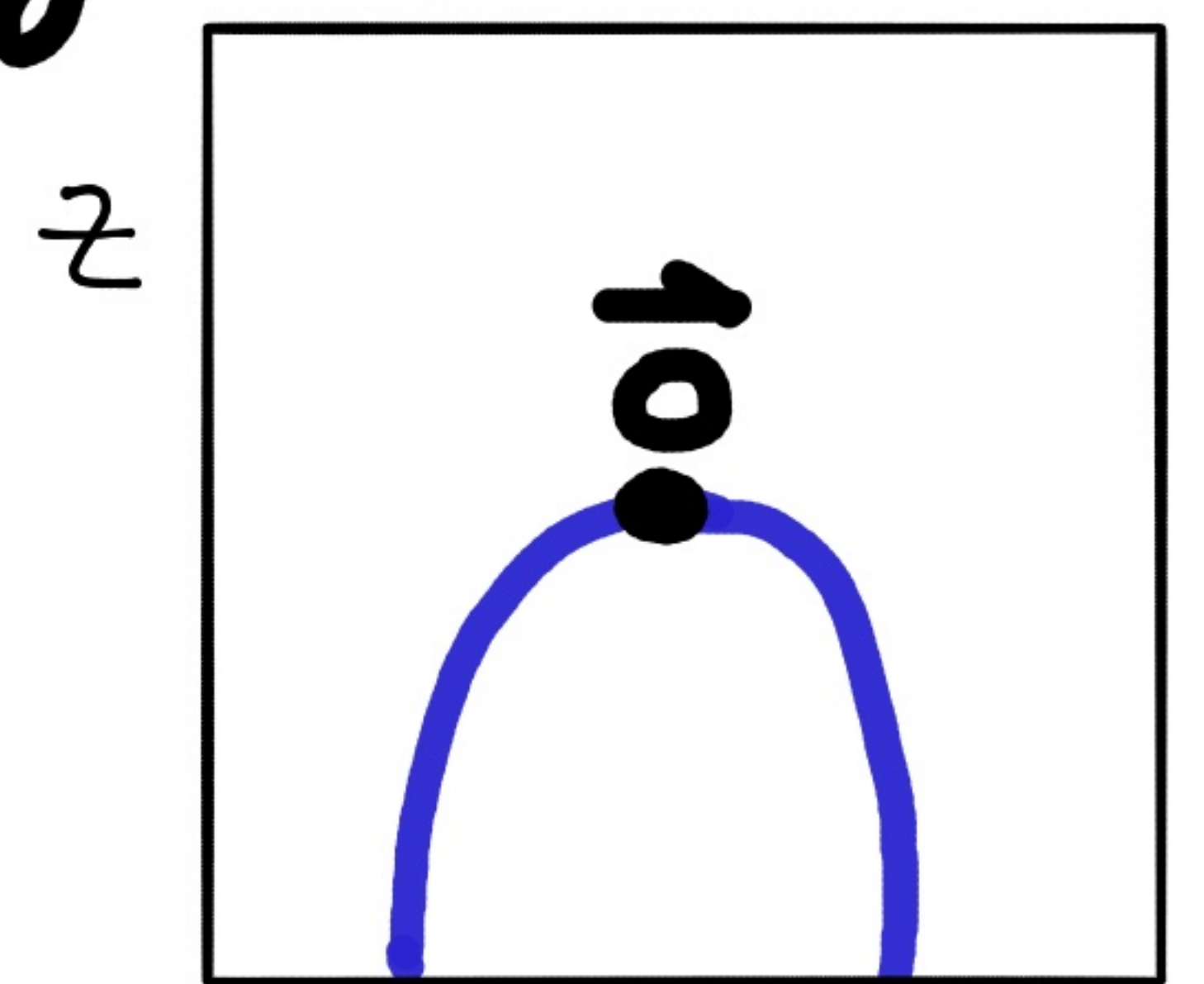




$$y^2 - x^2 = z$$

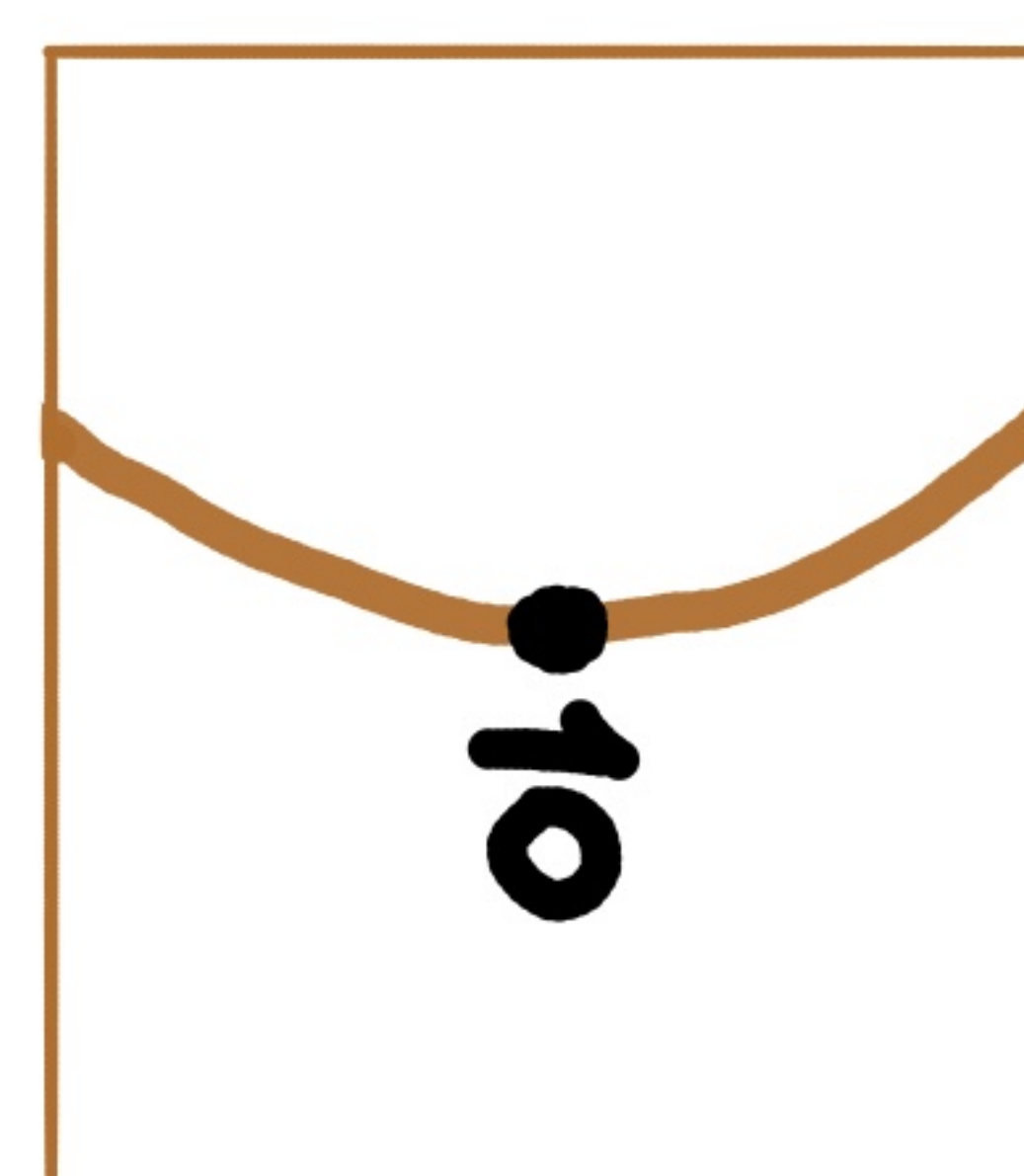
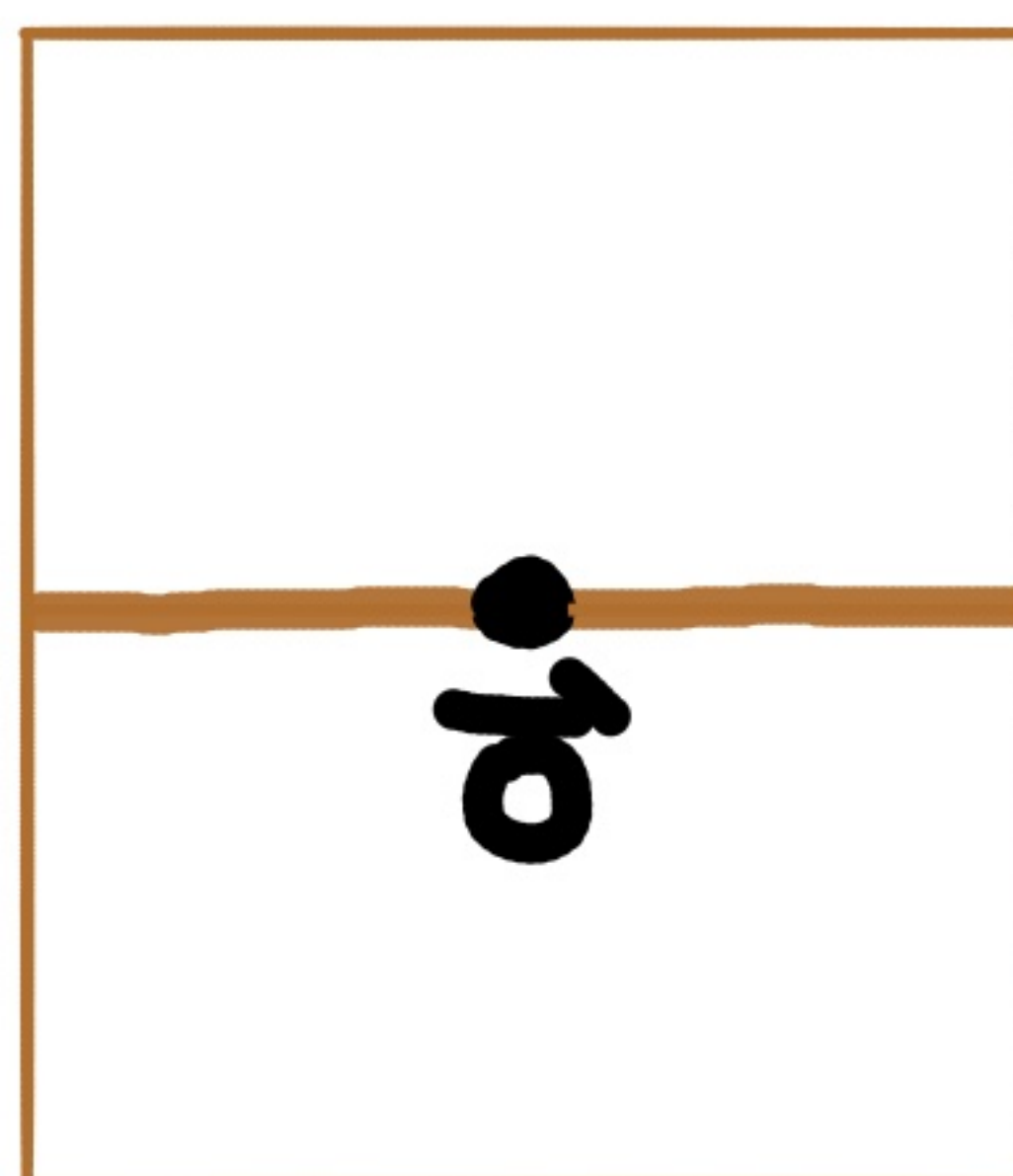
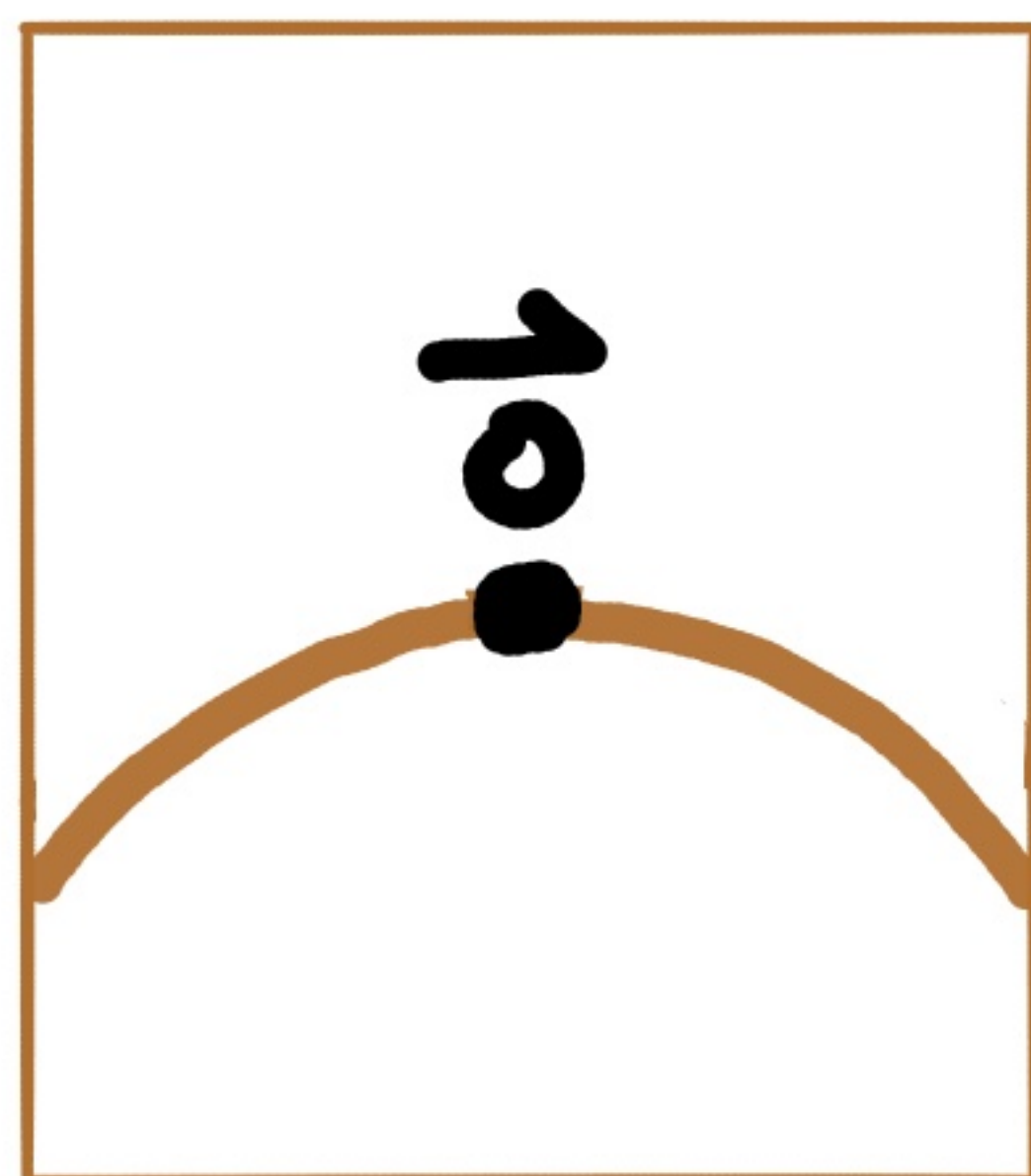
curvature
 $-2 \cdot 2 = -4$

$$y=0, -x^2=z$$



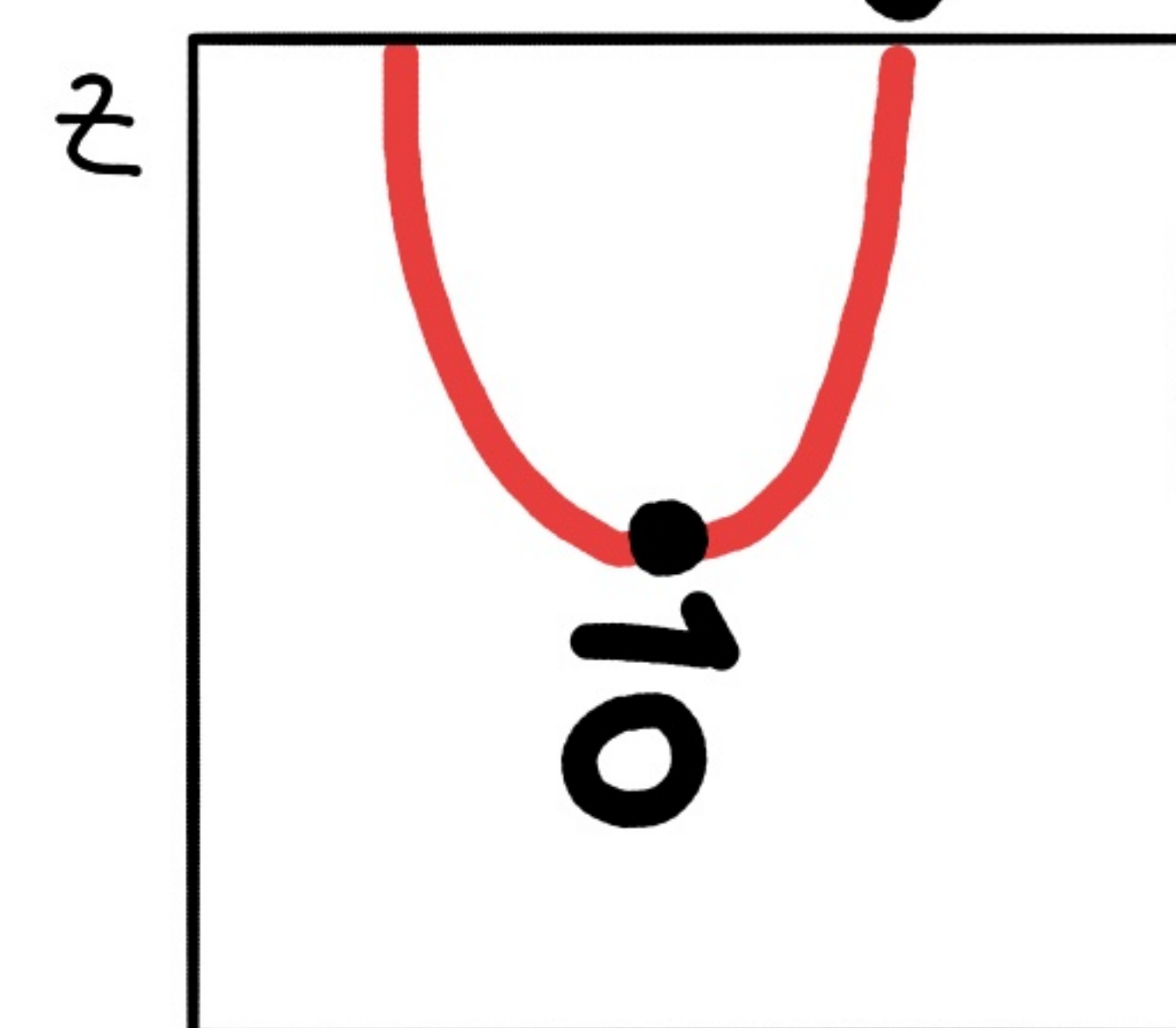
$$\hat{k}(\vec{o}) = -2$$

In between



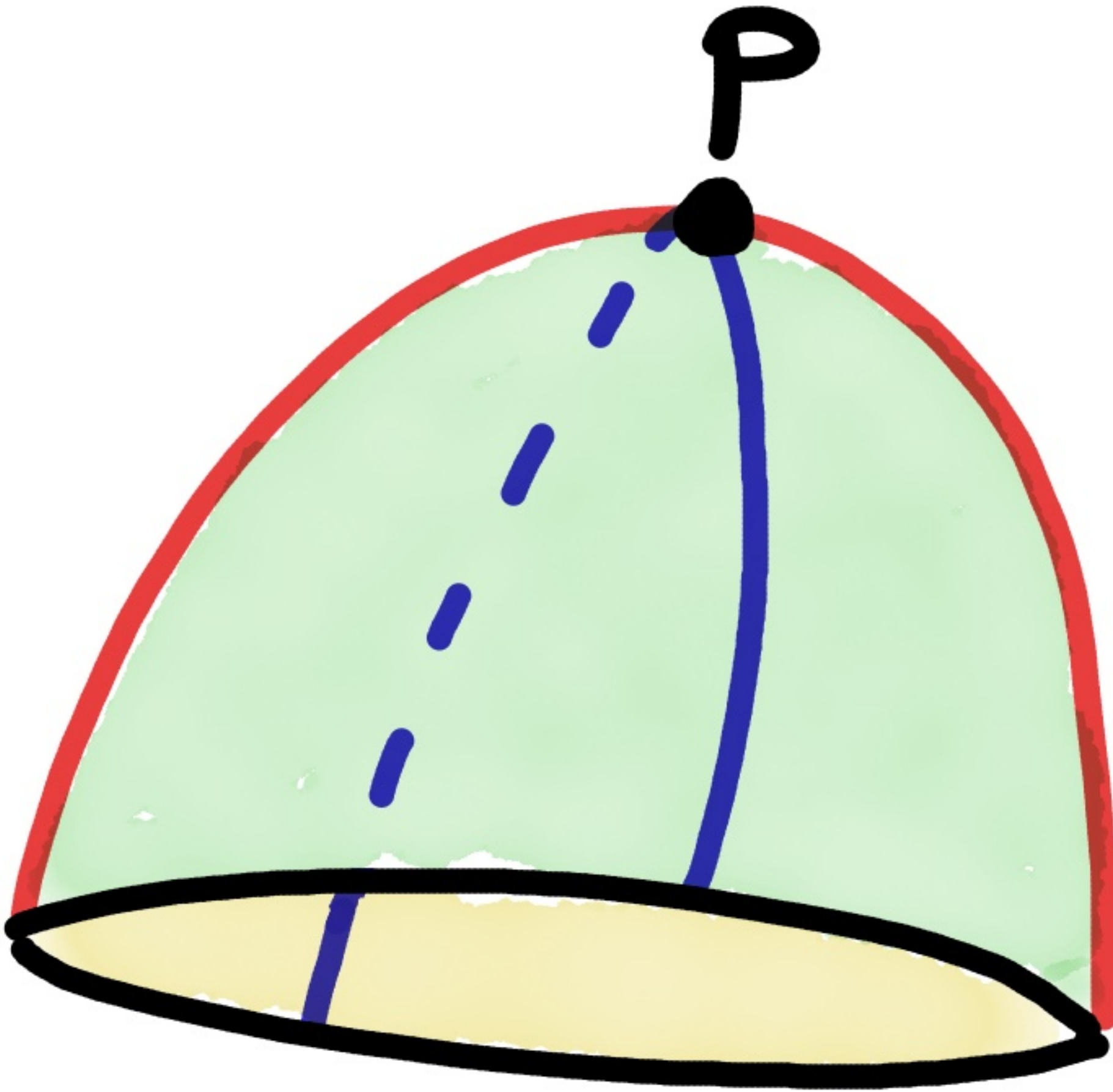
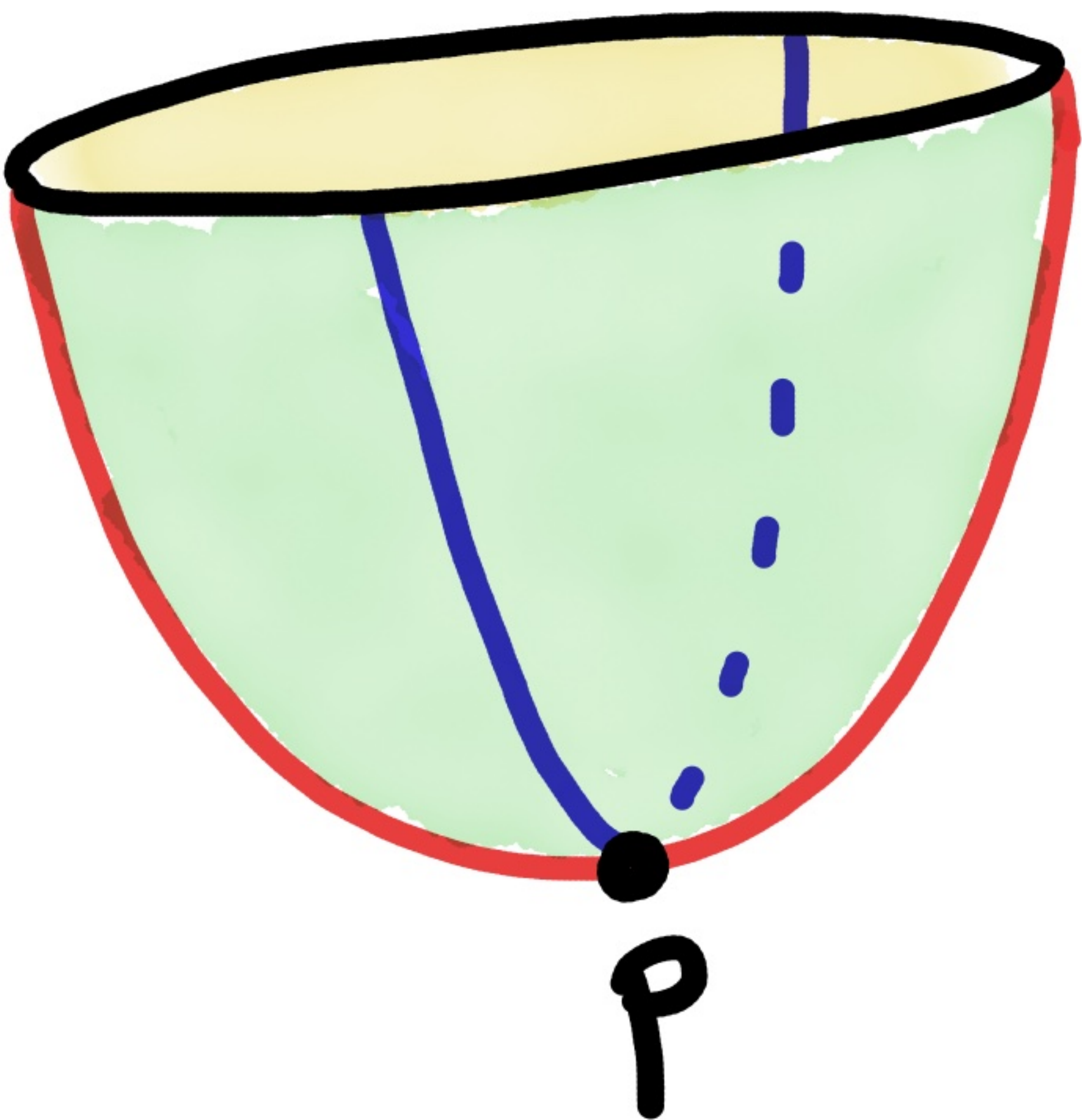
$$-2 \leq \hat{k}(\vec{o}) \leq 2$$

$$x=0, y^2=z$$

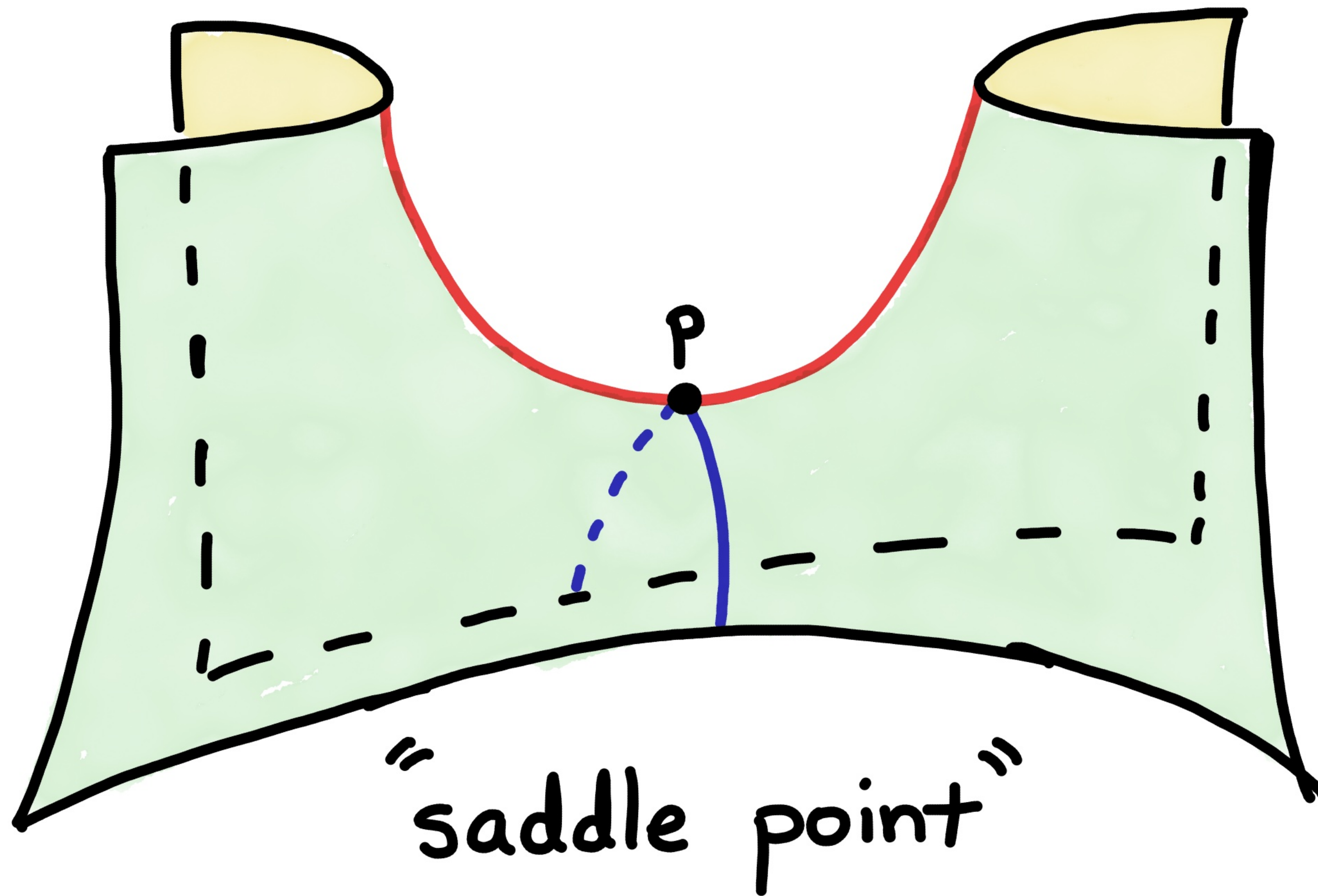


$$\hat{k}(\vec{o}) = 2$$

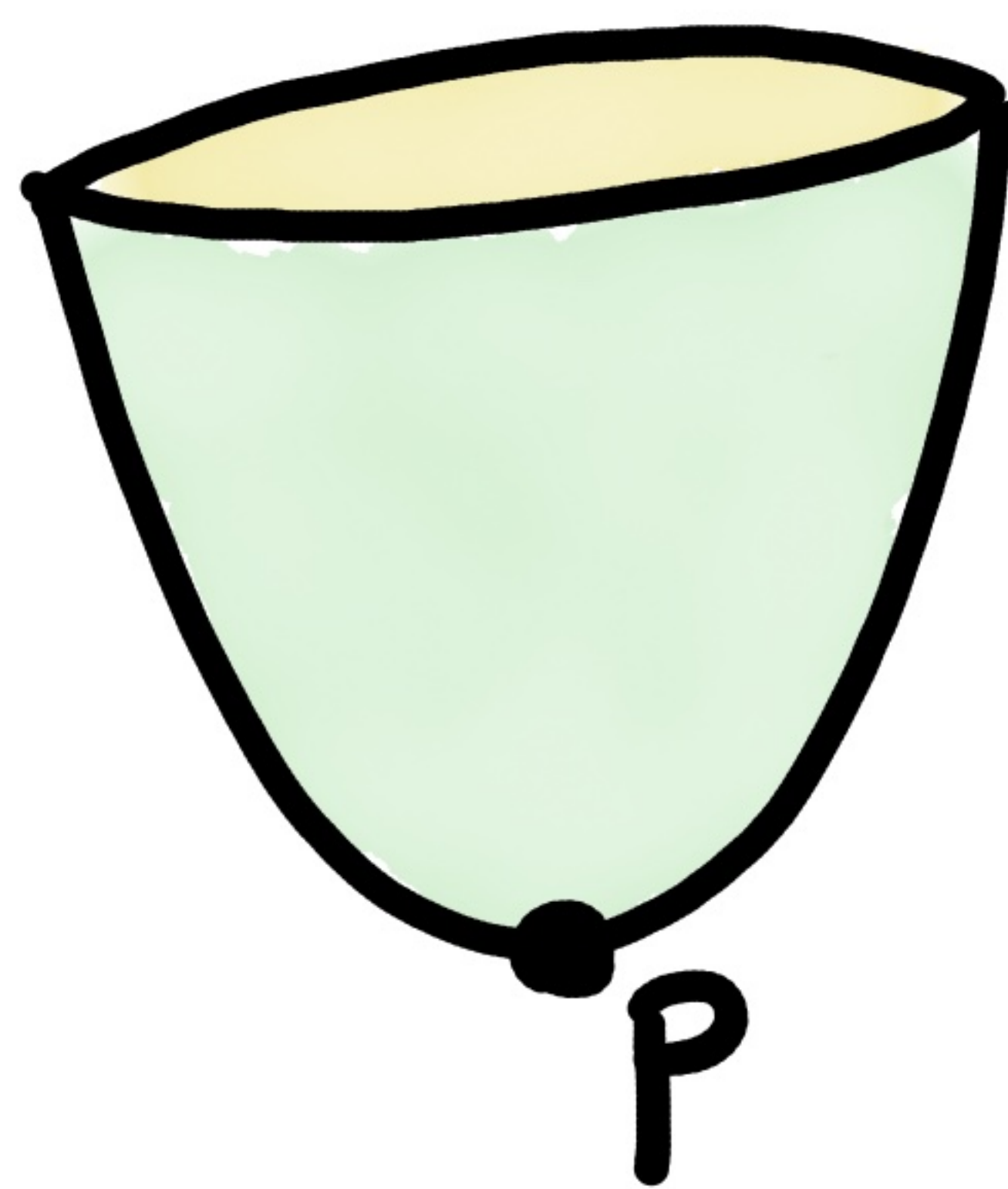
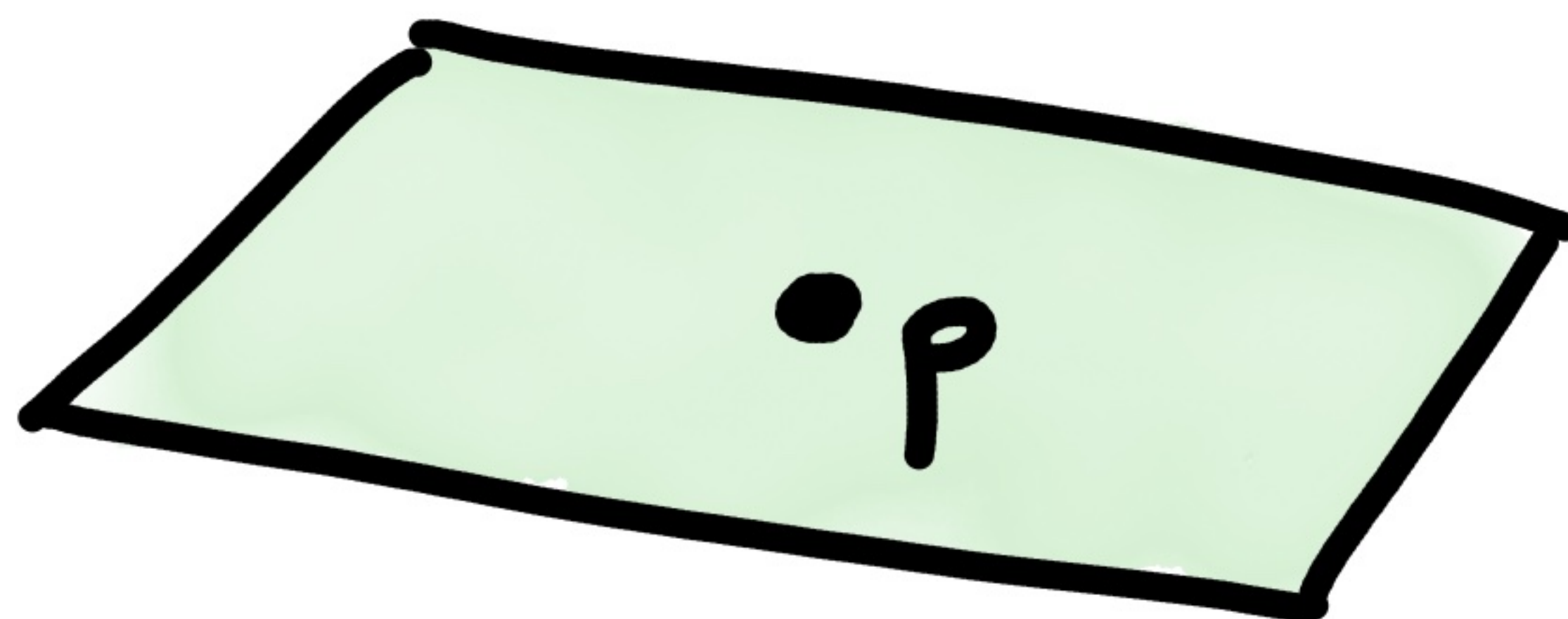
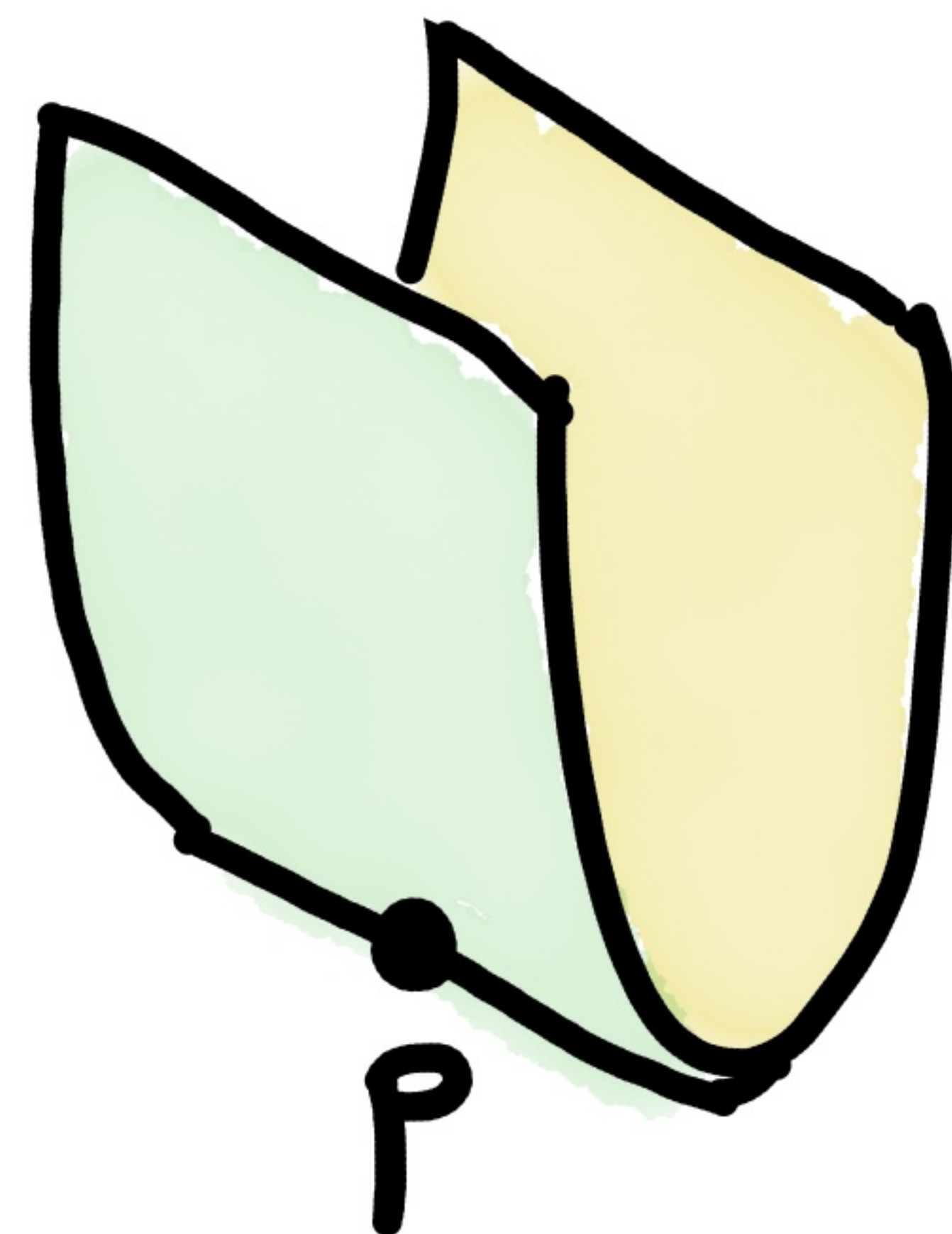
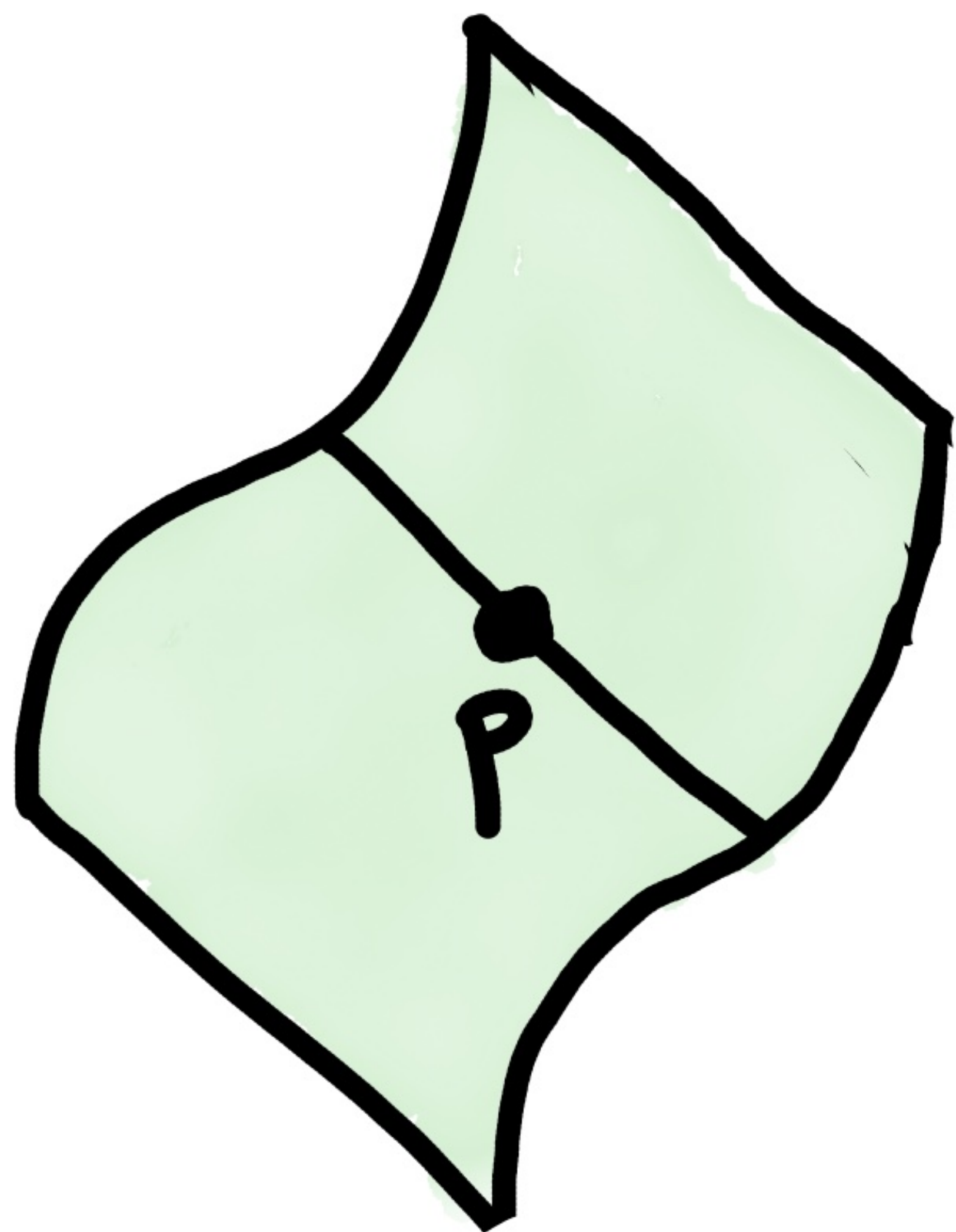
positive curvature



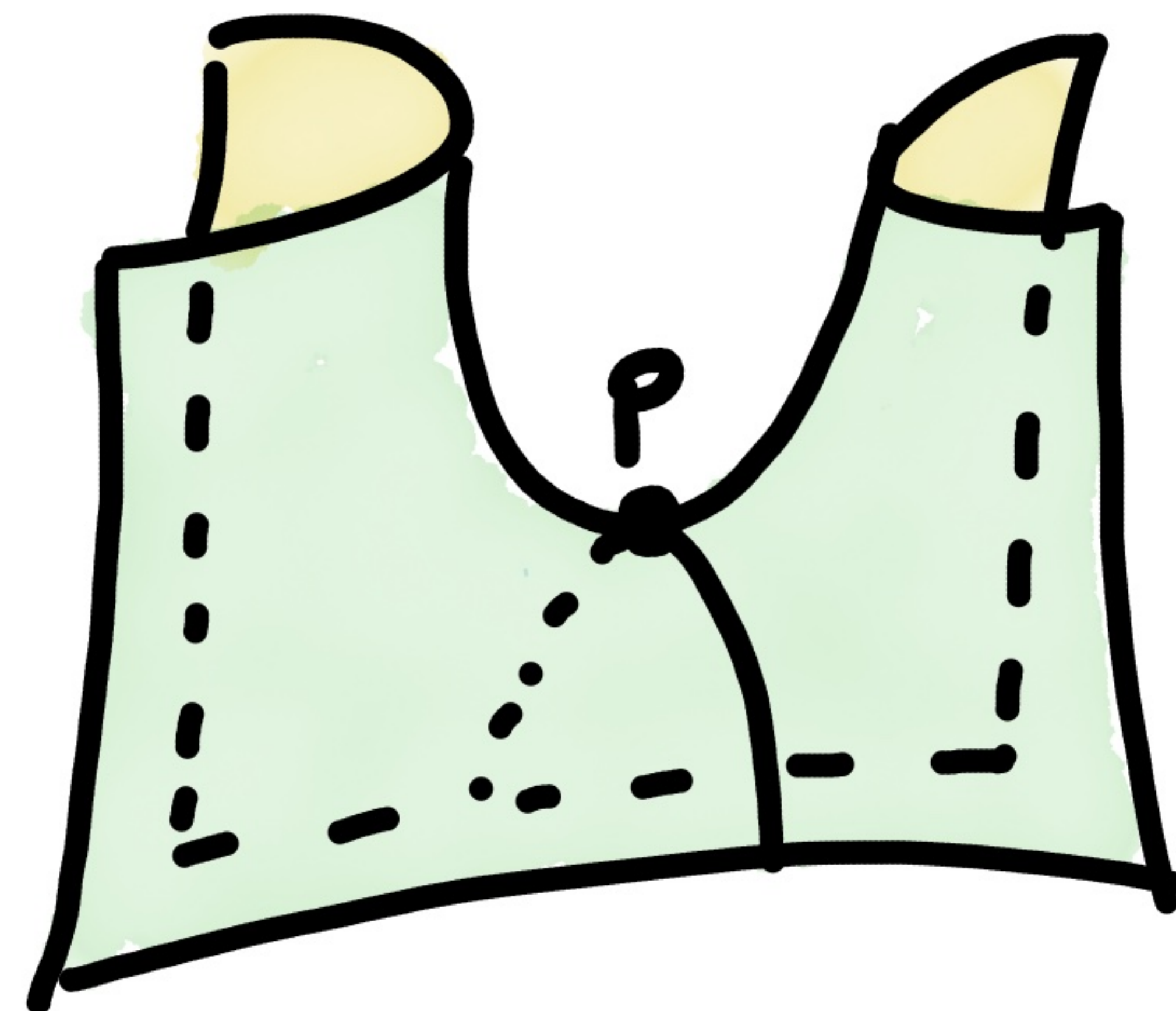
negative curvature



zero curvature



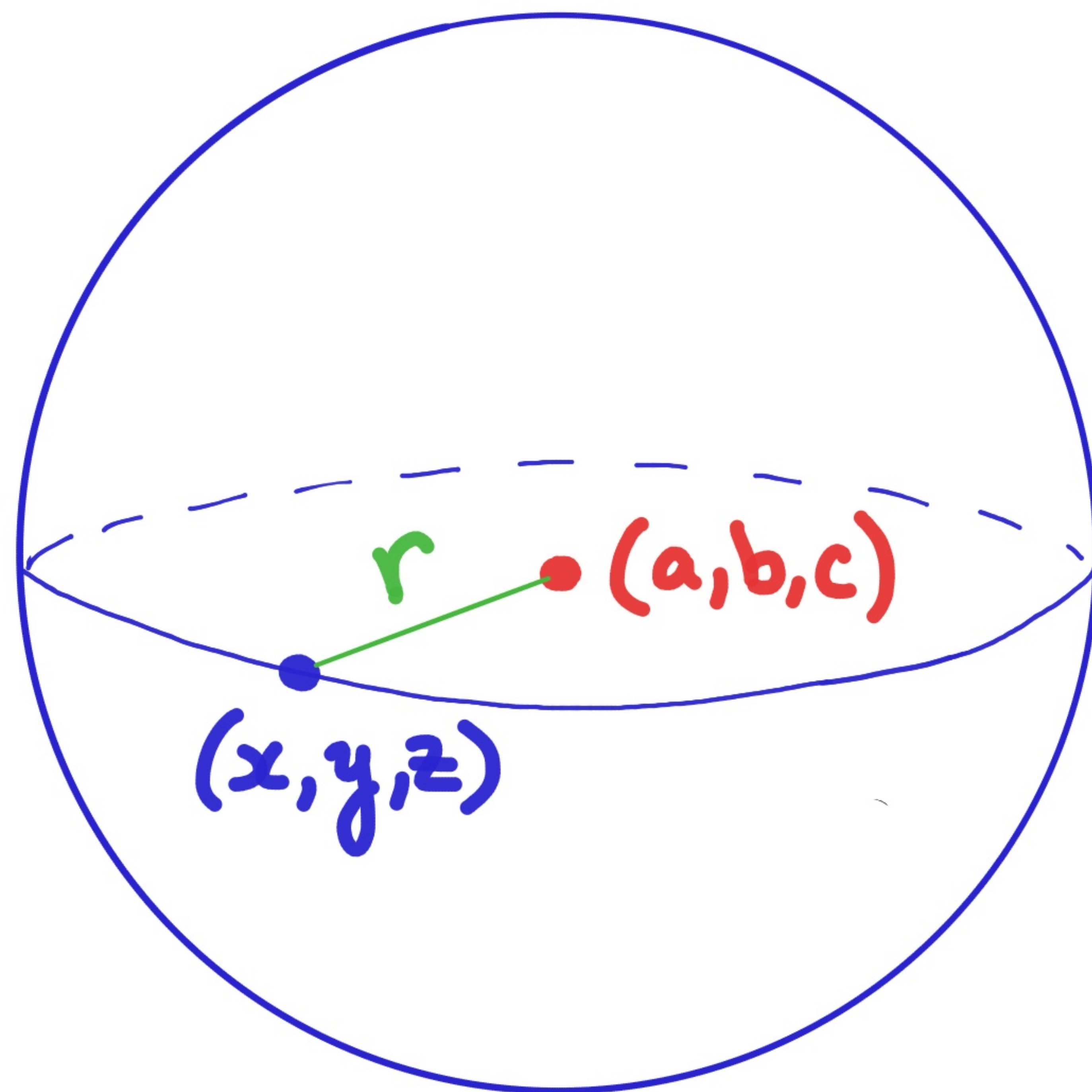
$$x^4 + y^4 = z$$



$$y^4 - x^4 = z$$

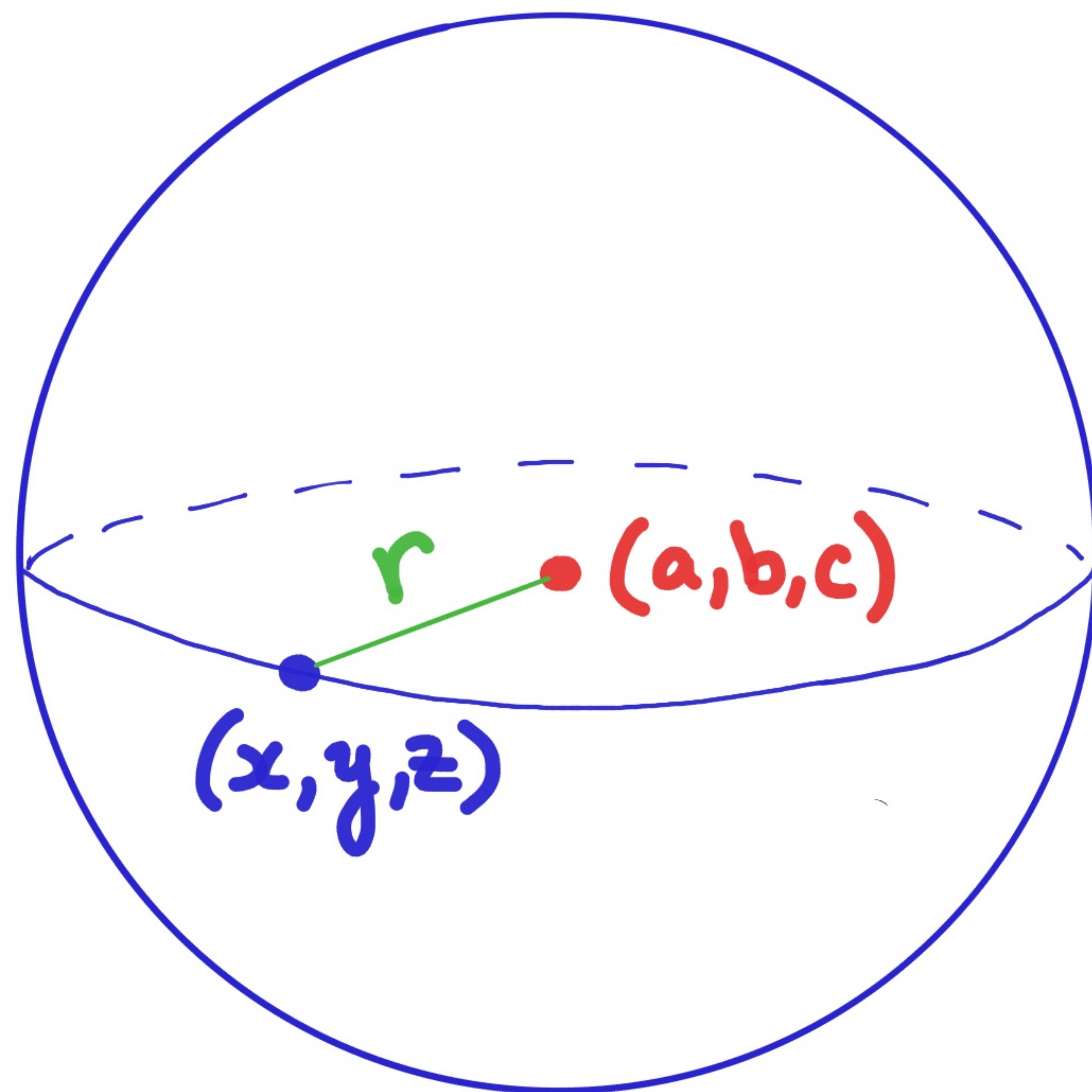
③ Equation of a sphere

points $(x,y,z) \in \mathbb{R}^3$
on a sphere of
radius r centered
at $(a,b,c) \in \mathbb{R}^3$



points $(x,y,z) \in \mathbb{R}^3$
on a sphere of
radius r centered
at $(a,b,c) \in \mathbb{R}^3$

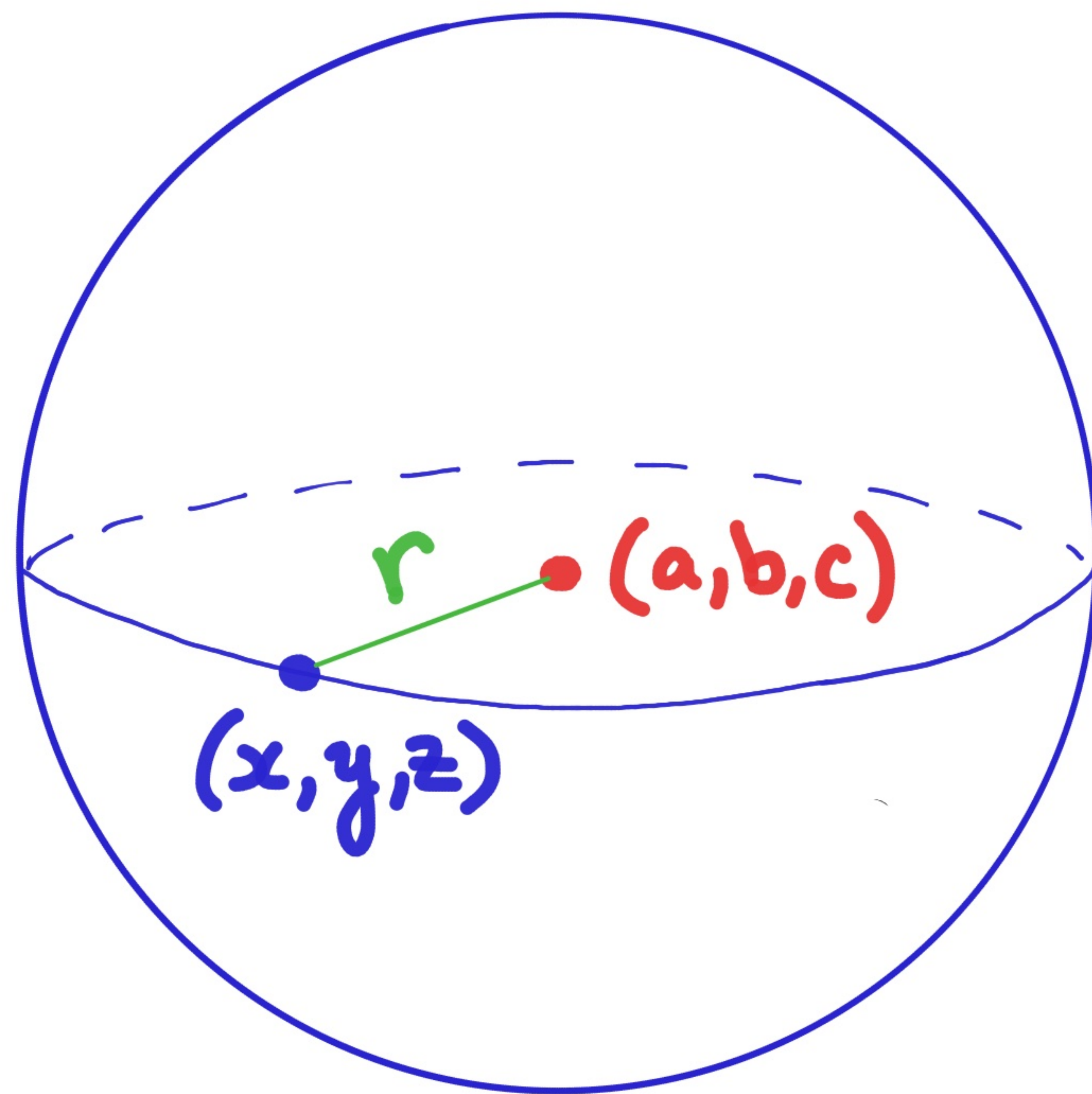
points $(x,y,z) \in \mathbb{R}^3$
that are distance
 r from $(a,b,c) \in \mathbb{R}^3$



points $(x,y,z) \in \mathbb{R}^3$
on a sphere of
radius r centered
at $(a,b,c) \in \mathbb{R}^3$

points $(x,y,z) \in \mathbb{R}^3$
that are distance
 r from $(a,b,c) \in \mathbb{R}^3$

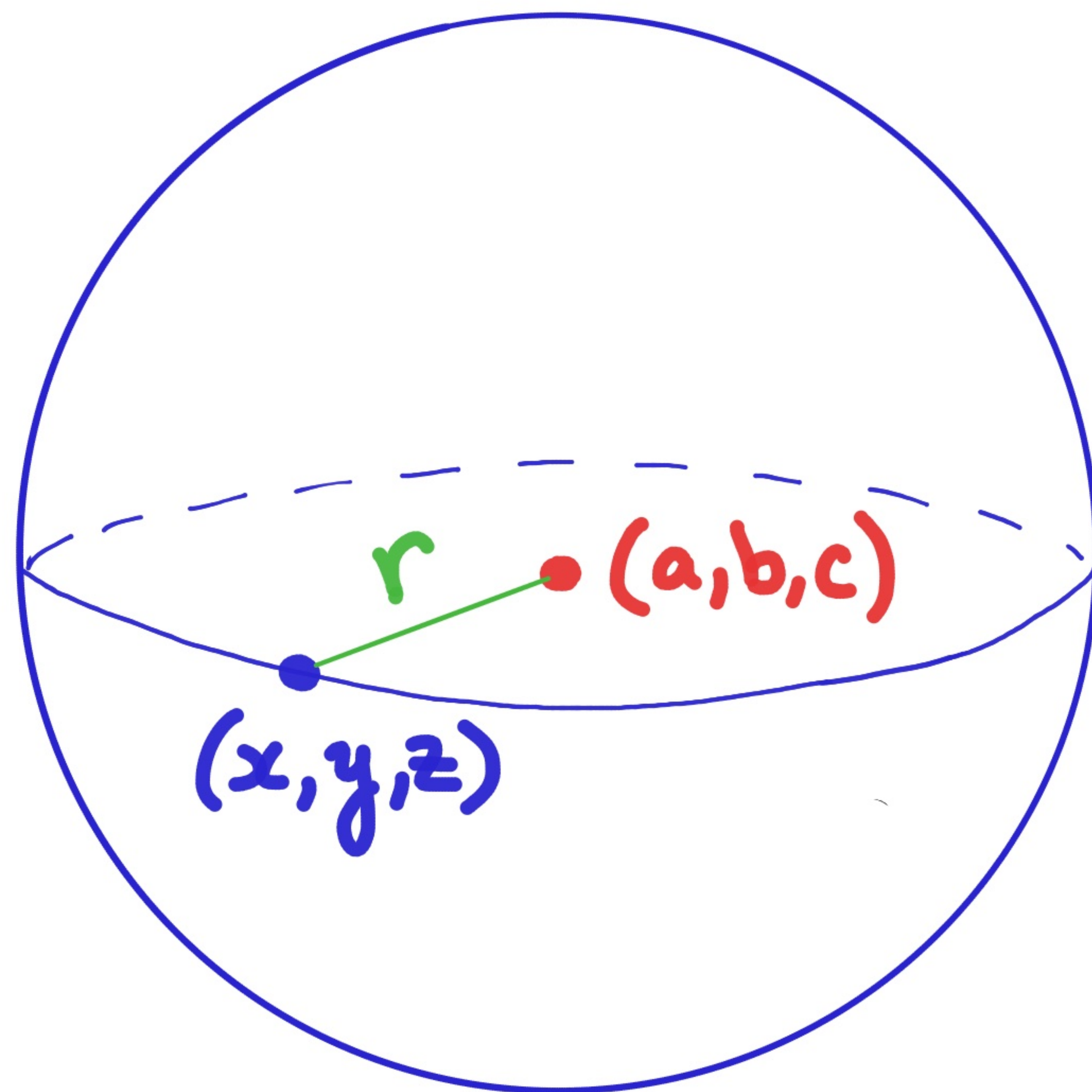
points $(x,y,z) \in \mathbb{R}^3$ with
 $\|(x,y,z) - (a,b,c)\| = r$



points $(x,y,z) \in \mathbb{R}^3$
on a sphere of
radius r centered
at $(a,b,c) \in \mathbb{R}^3$

points $(x,y,z) \in \mathbb{R}^3$
that are distance
 r from $(a,b,c) \in \mathbb{R}^3$

points $(x,y,z) \in \mathbb{R}^3$ with
 $\|(x,y,z) - (a,b,c)\| = r$

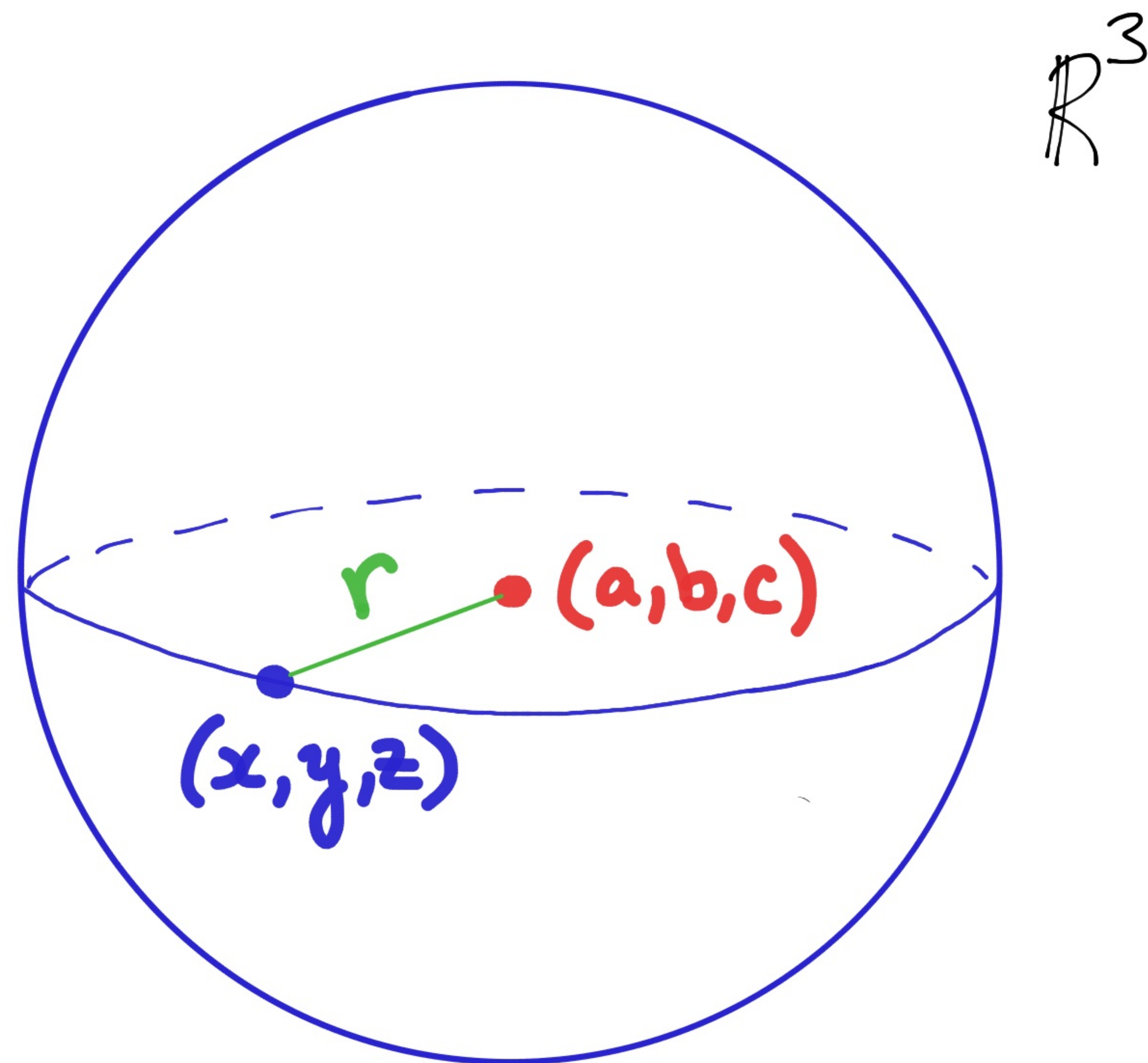


That is, $\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$

points $(x,y,z) \in \mathbb{R}^3$
on a sphere of
radius r centered
at $(a,b,c) \in \mathbb{R}^3$

points $(x,y,z) \in \mathbb{R}^3$
that are distance
 r from $(a,b,c) \in \mathbb{R}^3$

points $(x,y,z) \in \mathbb{R}^3$ with
 $\|(x,y,z) - (a,b,c)\| = r$



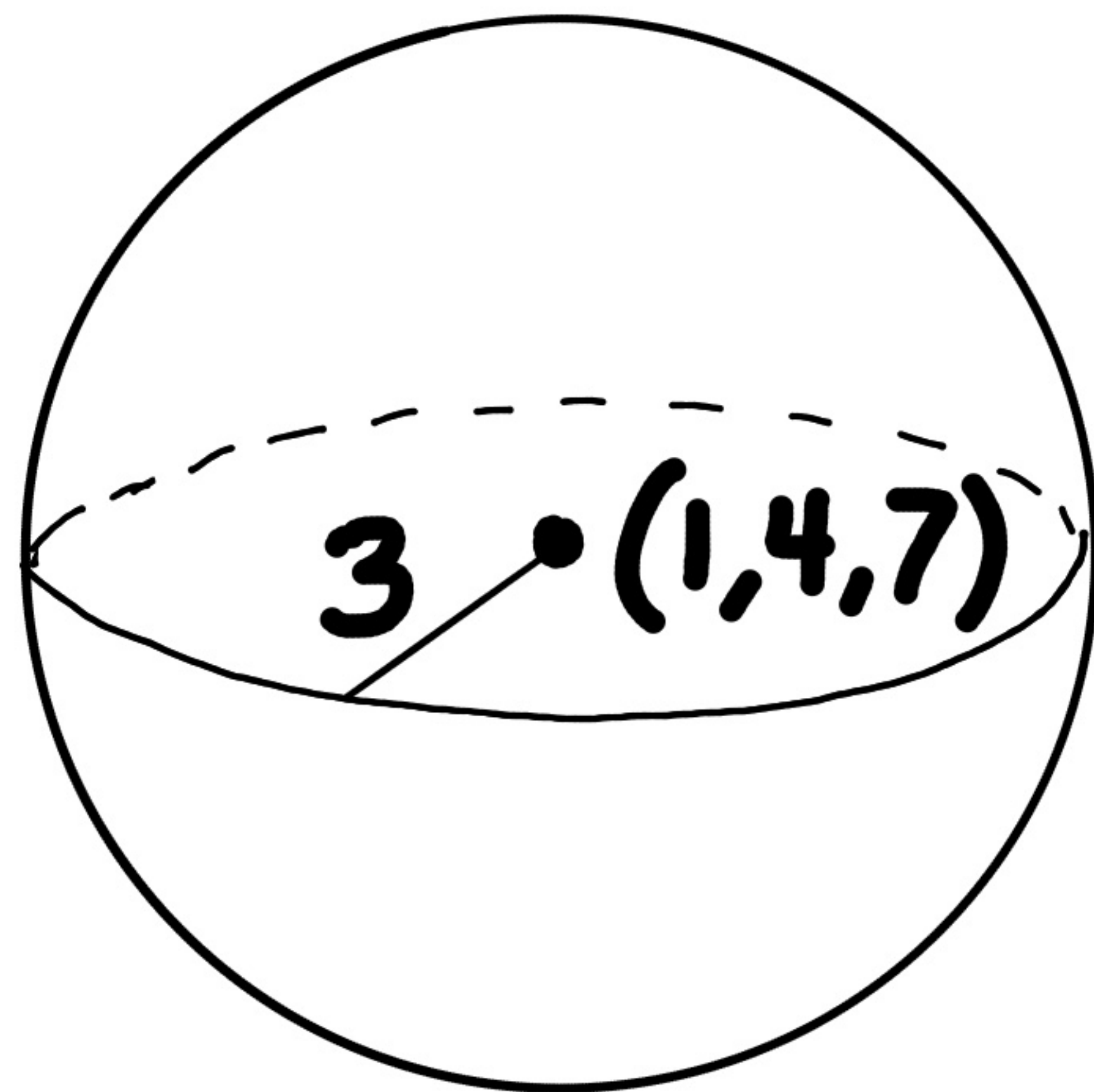
That is, $\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$

That is, $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

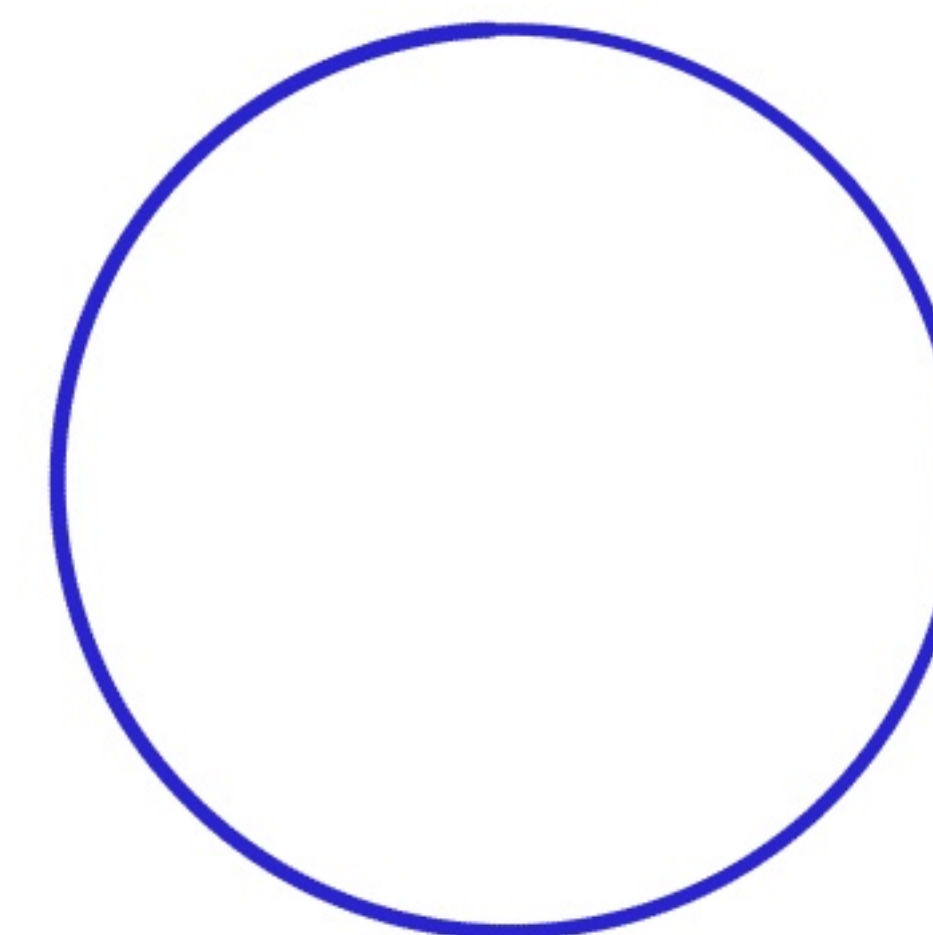
Example:

The equation of the sphere of radius 3 centered at $(1, 4, 7) \in \mathbb{R}^3$ is

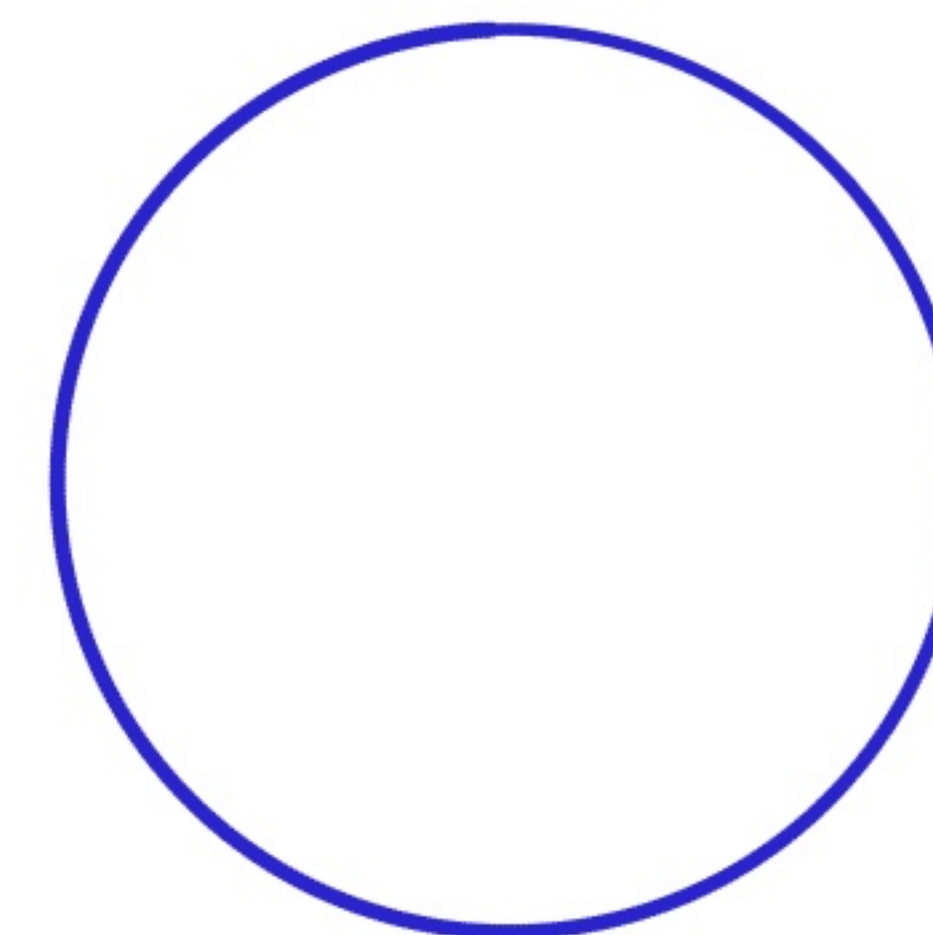
$$(x-1)^2 + (y-4)^2 + (z-7)^2 = 3^2$$



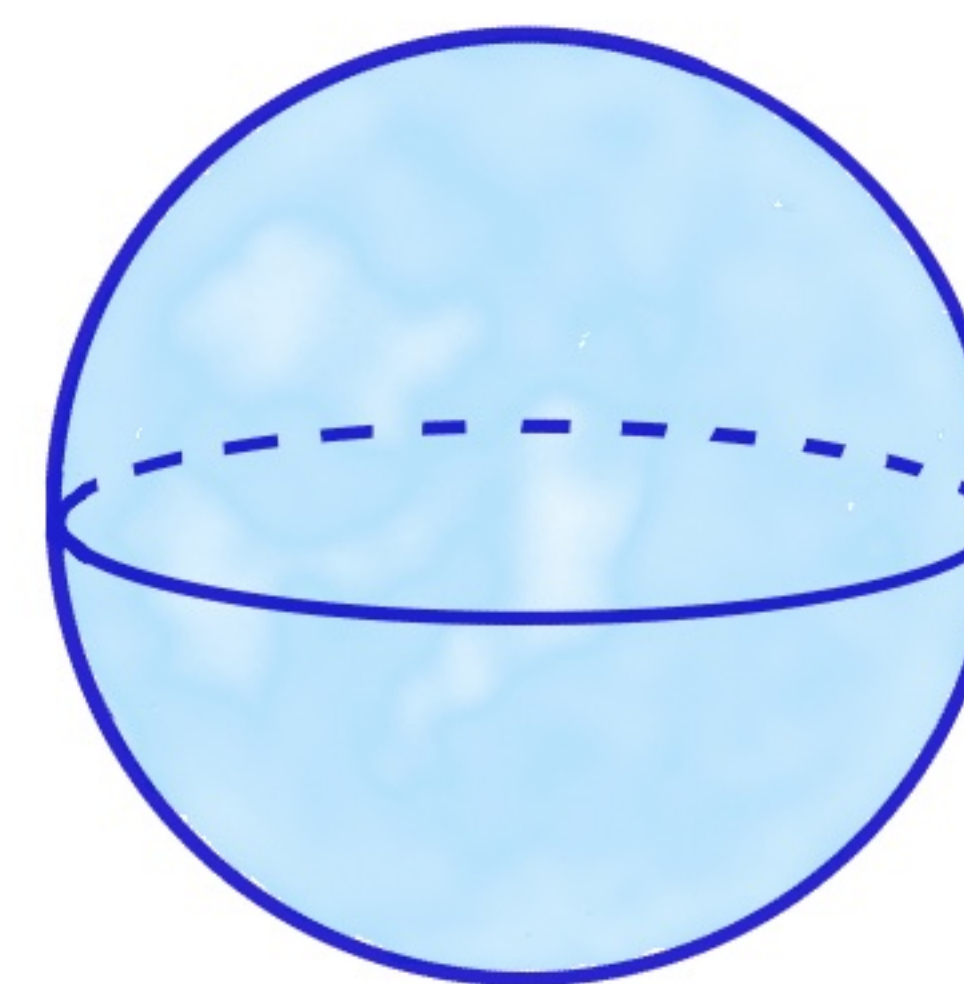
$$\underline{x^2 + y^2 = 1 \text{ in } \mathbb{R}^2:}$$



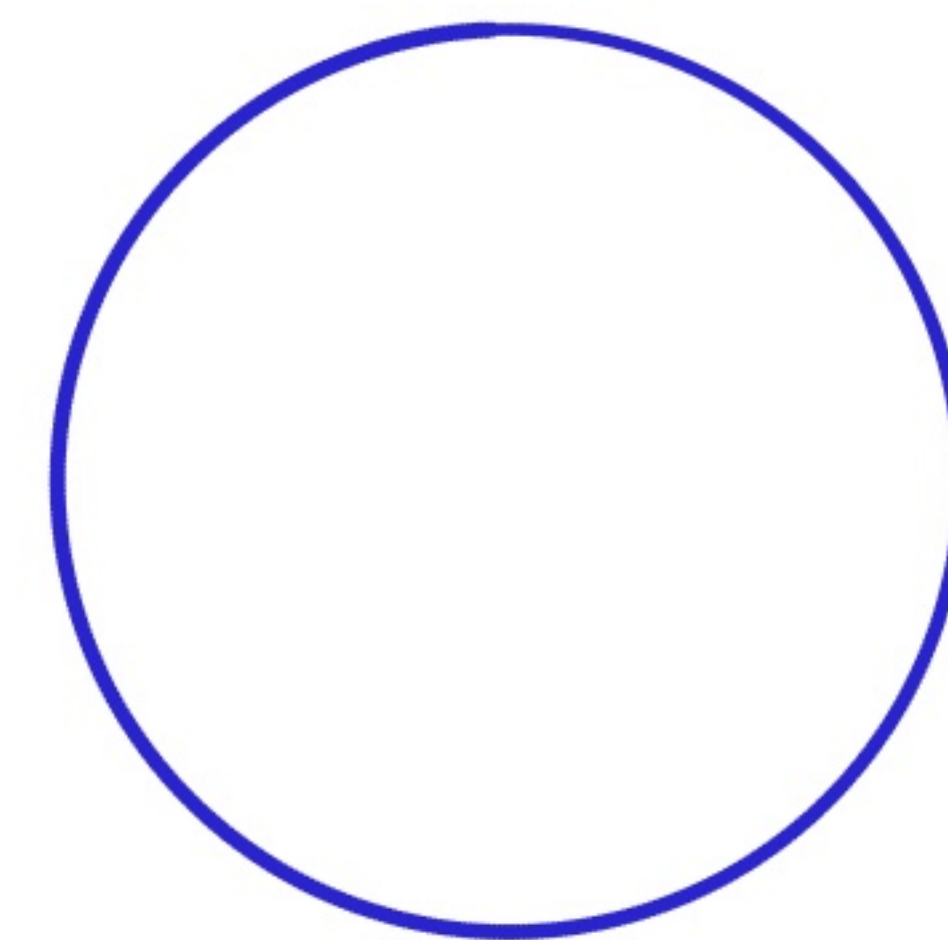
$$\underline{x^2 + y^2 = 1 \text{ in } \mathbb{R}^2:}$$



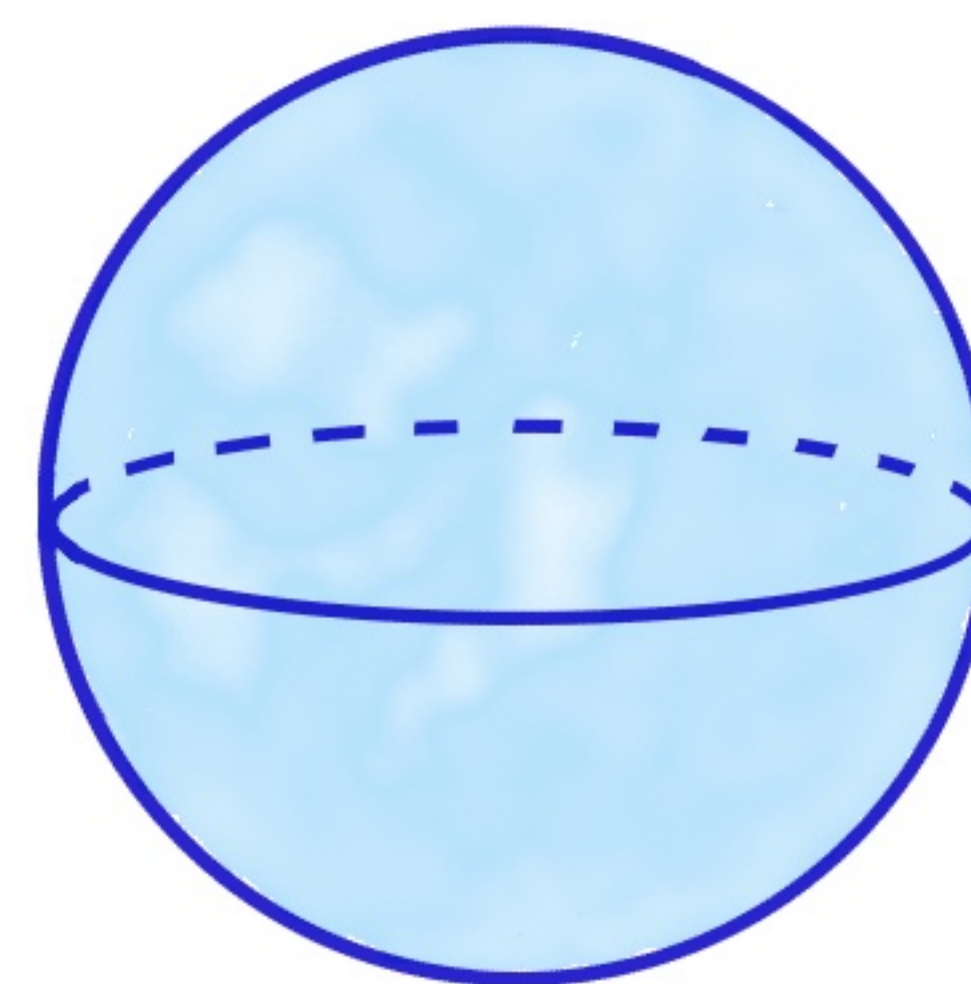
$$\underline{x^2 + y^2 + z^2 = 1 \text{ in } \mathbb{R}^3:}$$



$$\underline{x^2 + y^2 = 1 \text{ in } \mathbb{R}^2:}$$



$$\underline{x^2 + y^2 + z^2 = 1 \text{ in } \mathbb{R}^3:}$$



$$\underline{x^2 + y^2 + z^2 + w^2 = 1 \text{ in } \mathbb{R}^4:}$$

