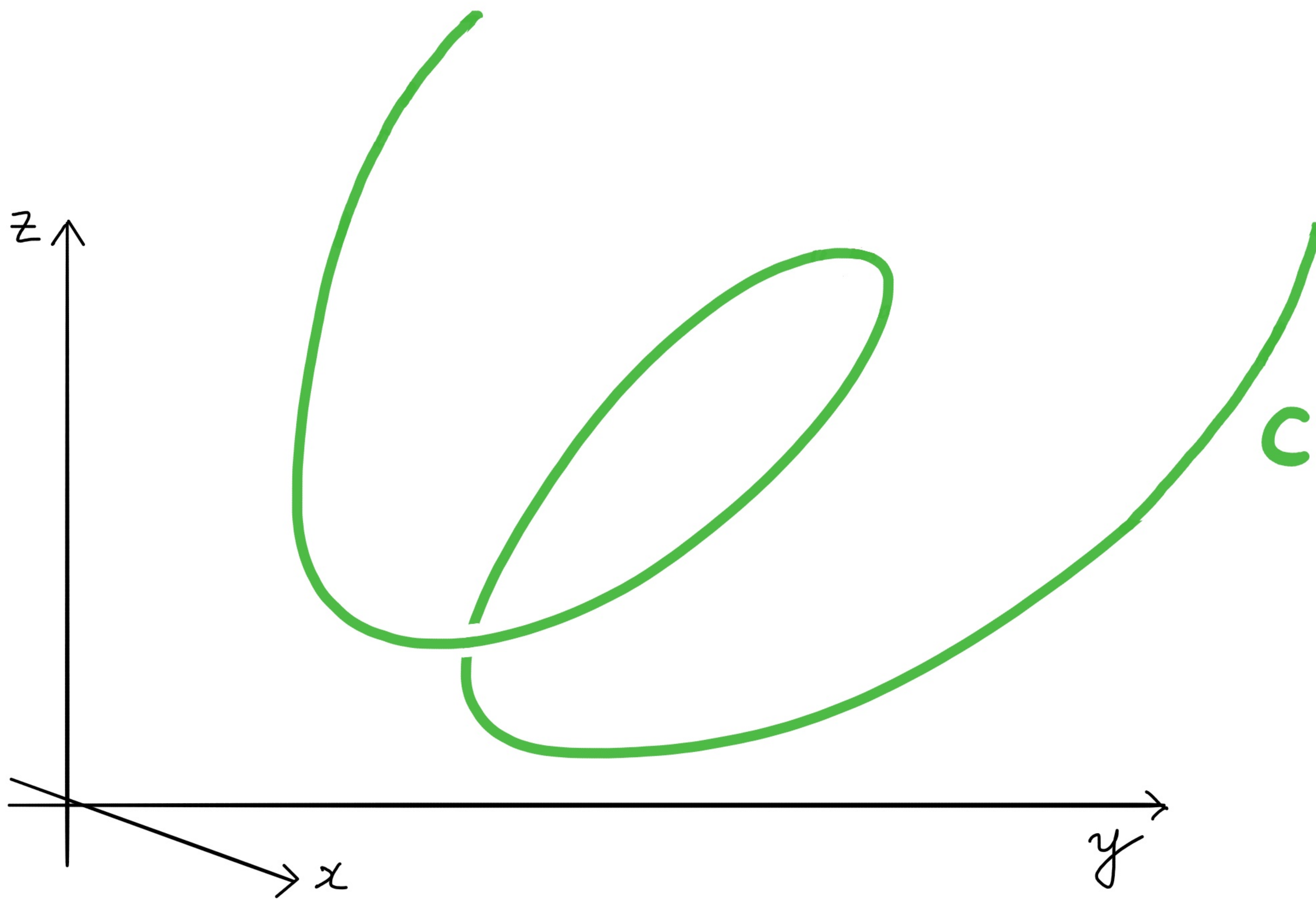


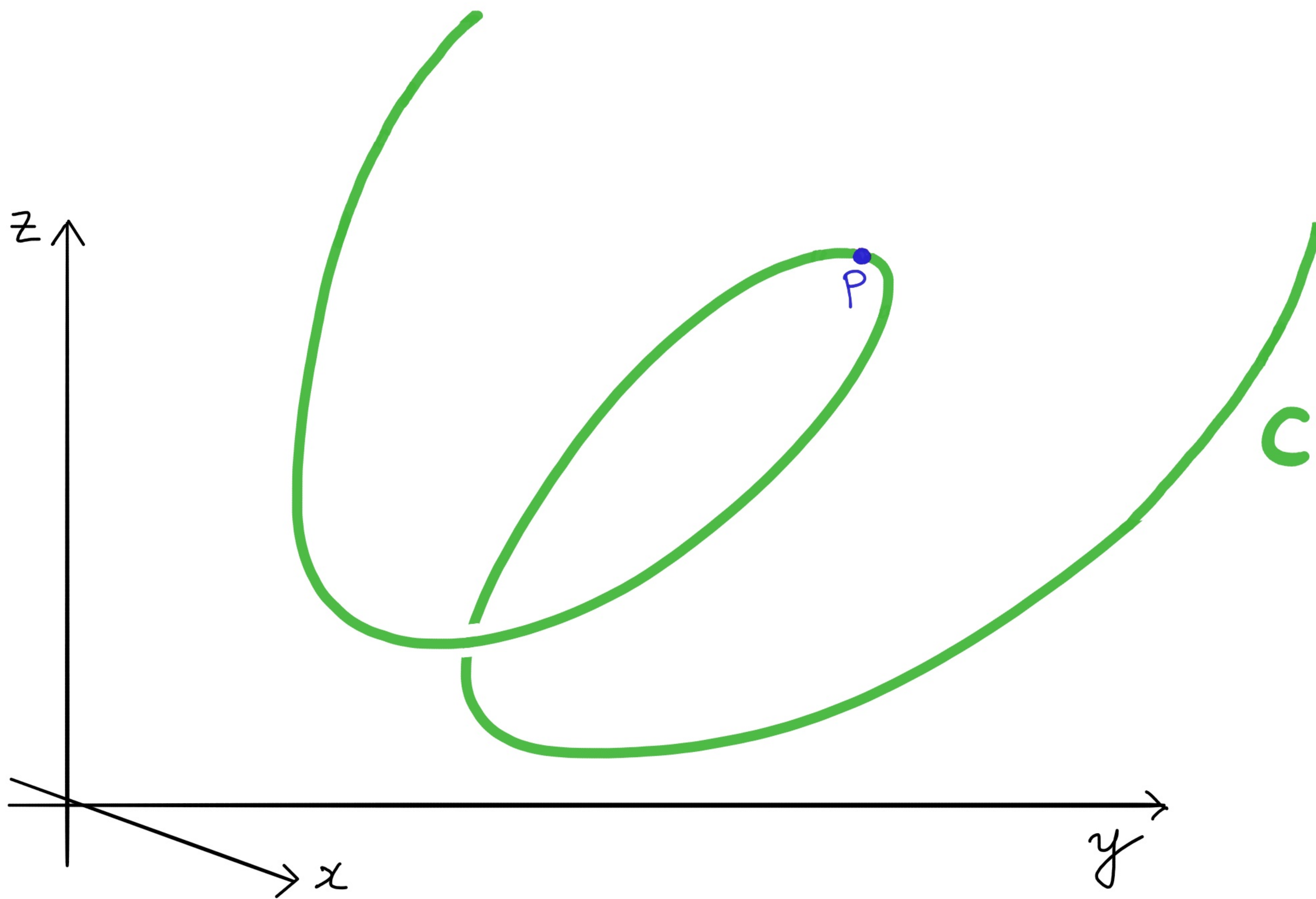
Five

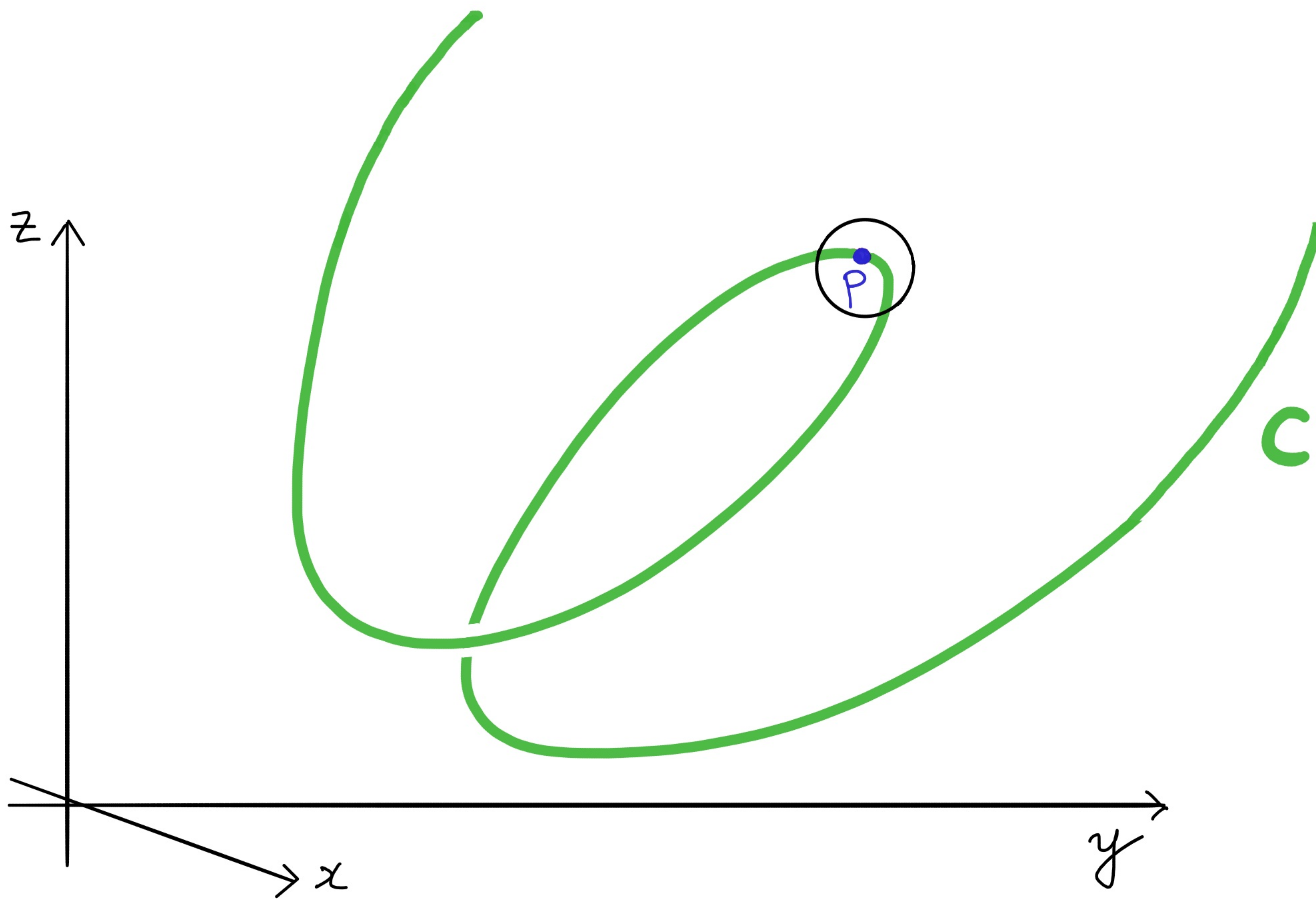
① Tangent line and
tangent vectors

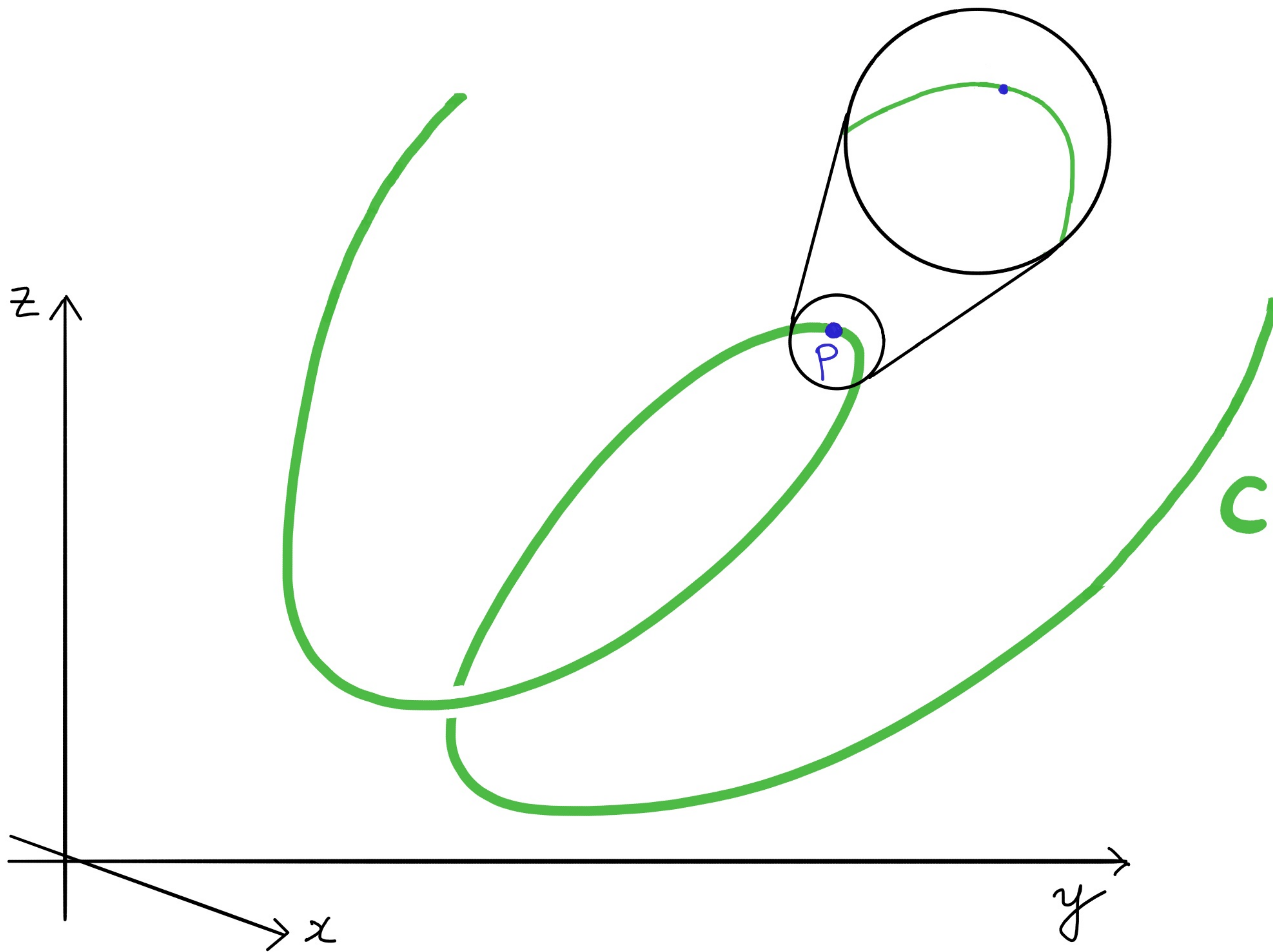
② Equations of tangent lines

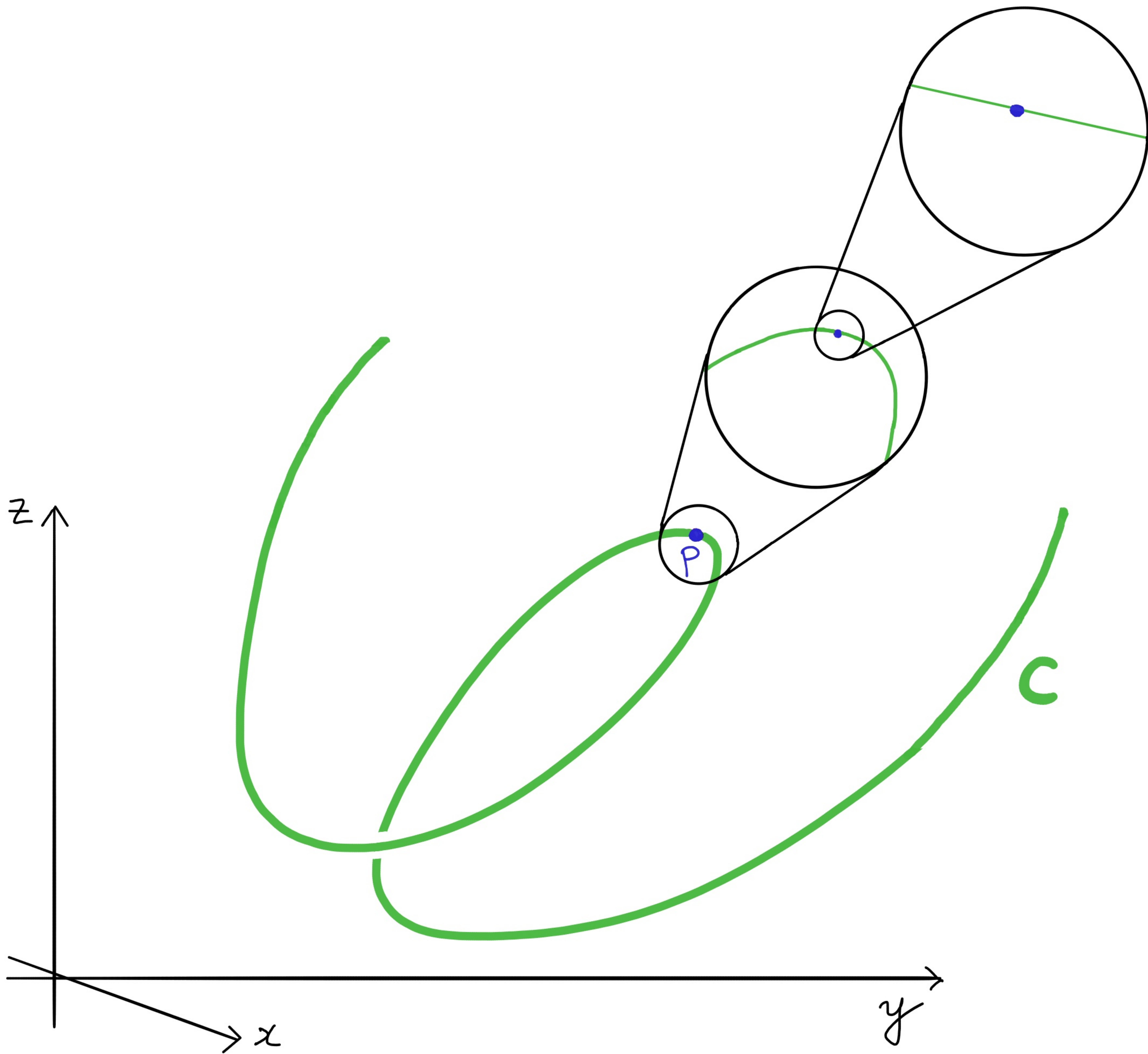
① Tangent lines
and tangent vectors

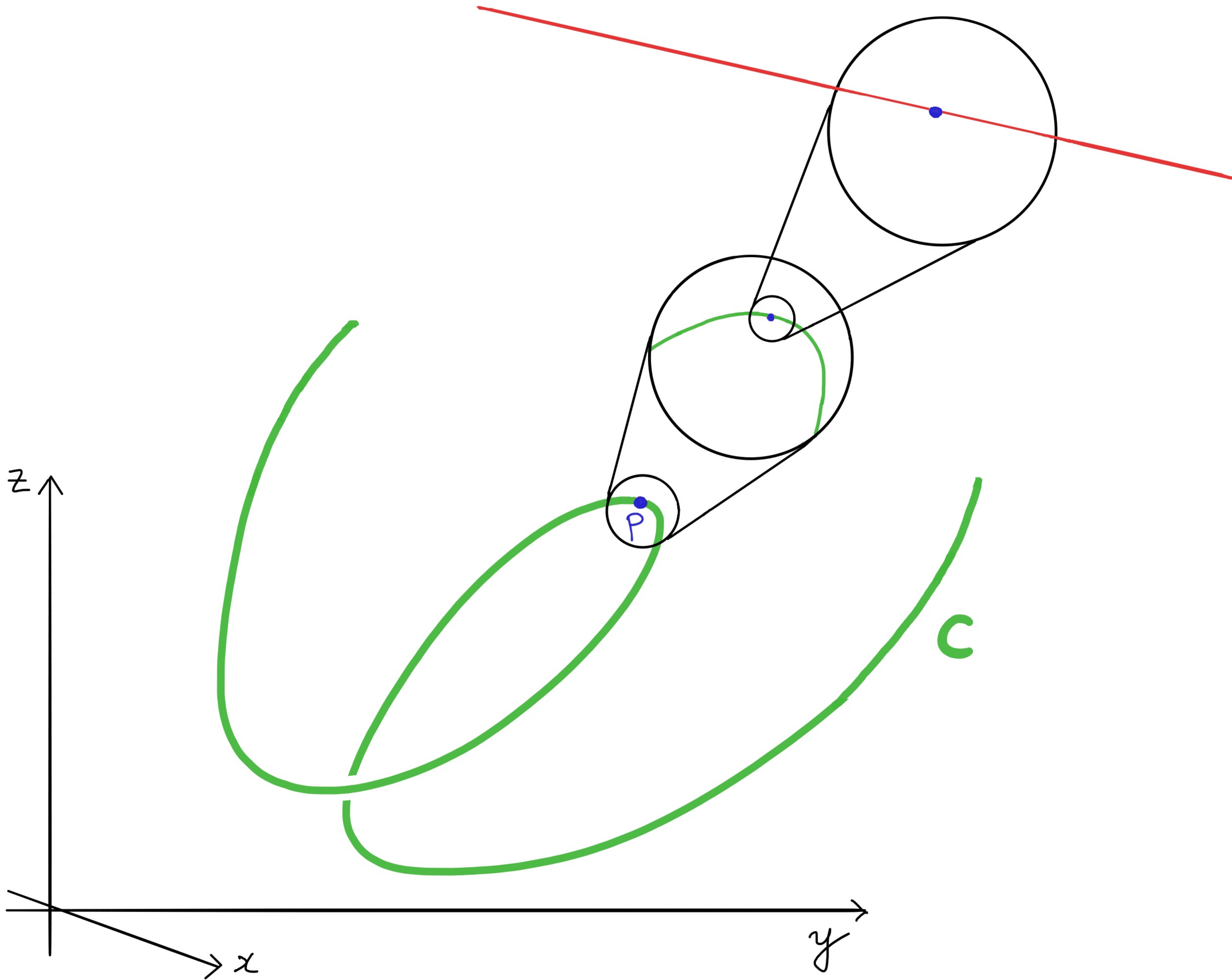


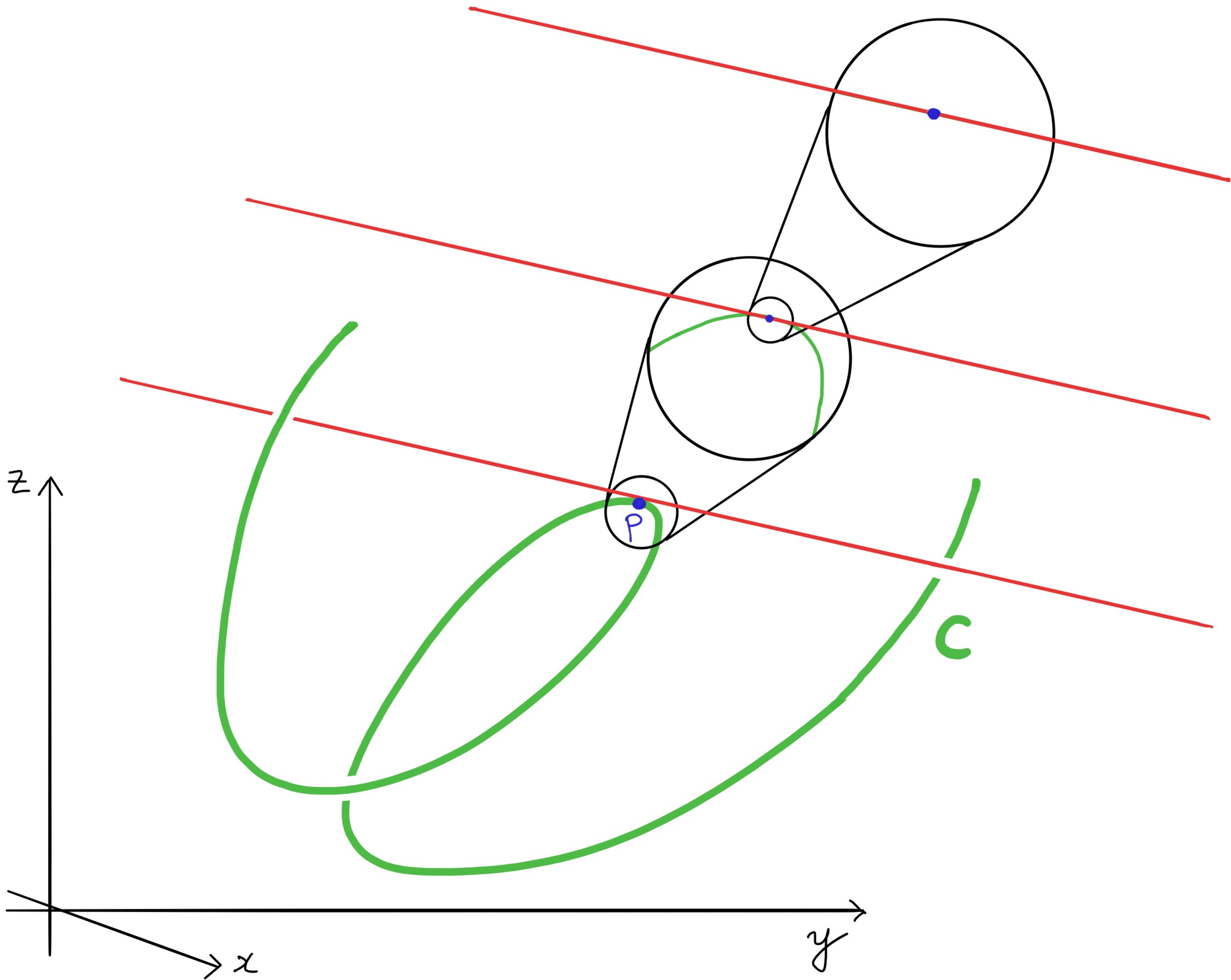


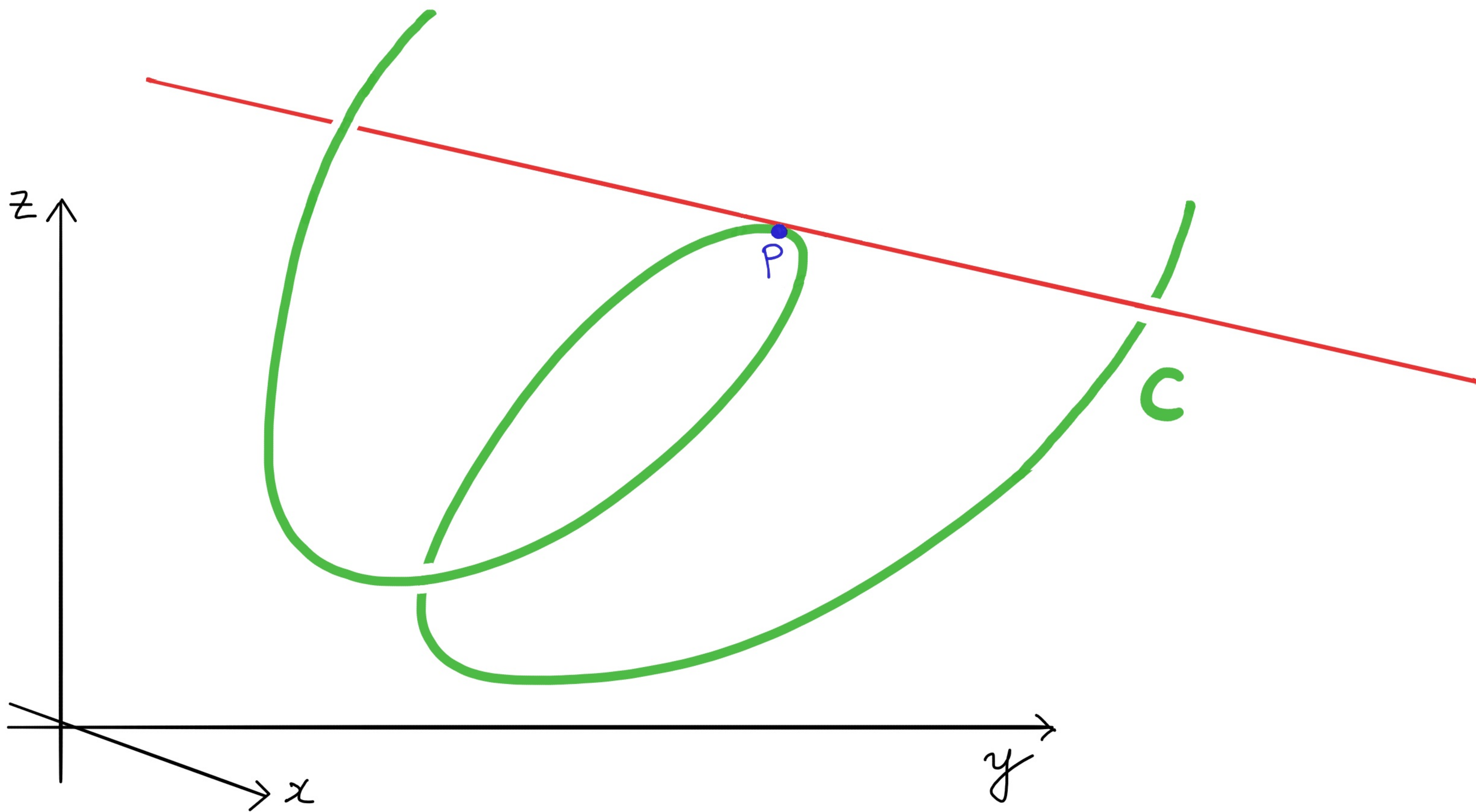




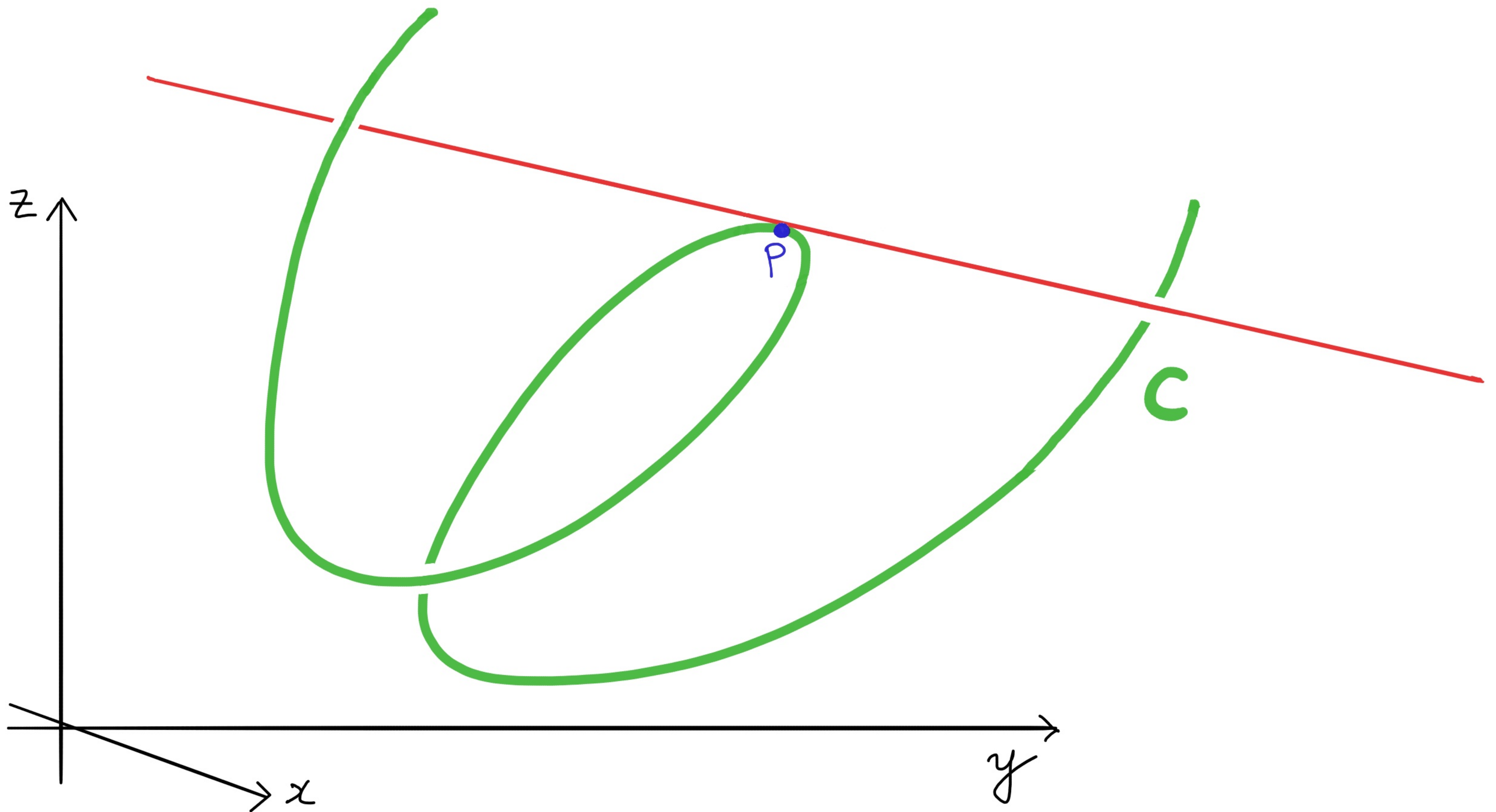


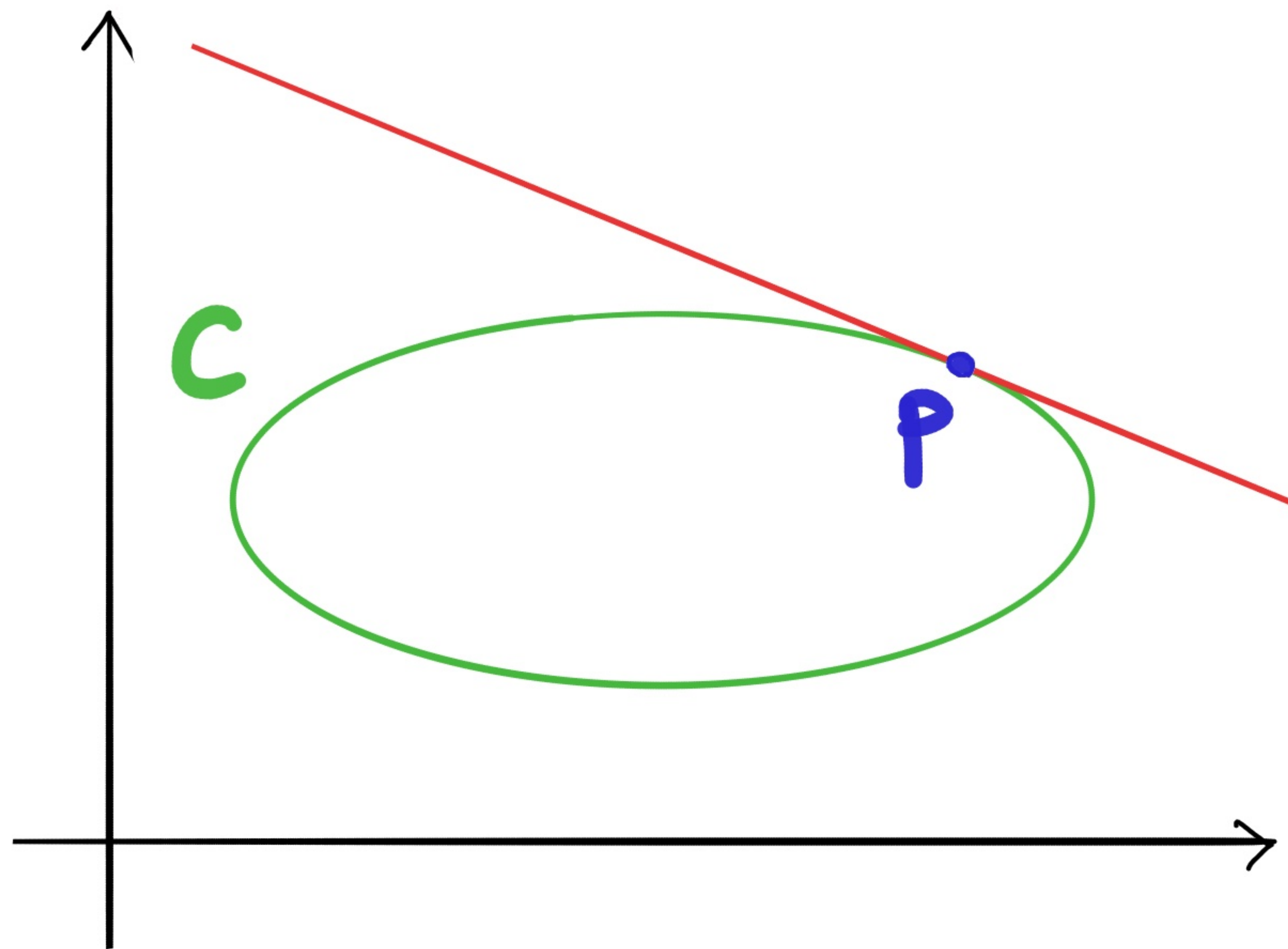




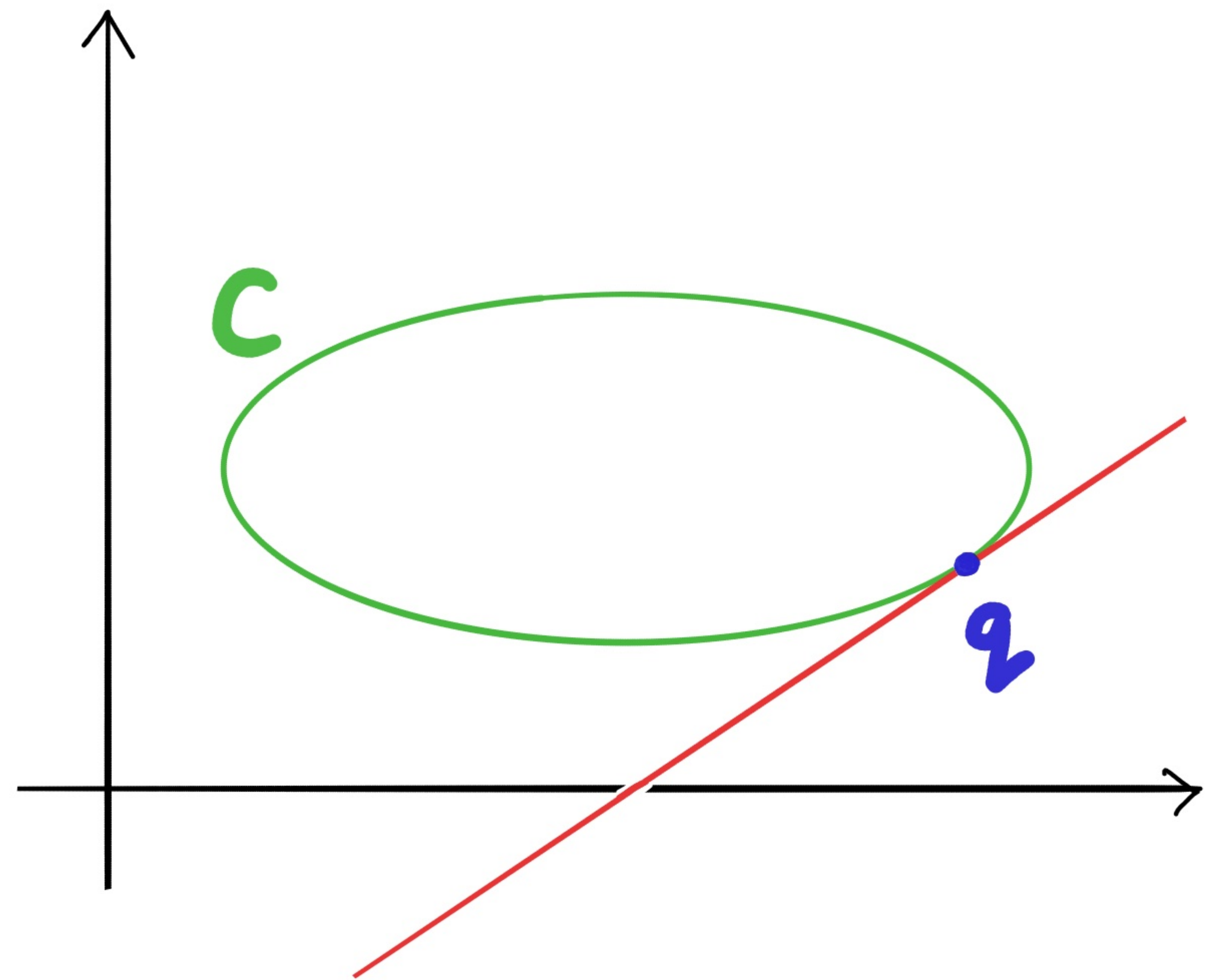


The **tangent line** of C at P is the best approximation of C near P by a line.



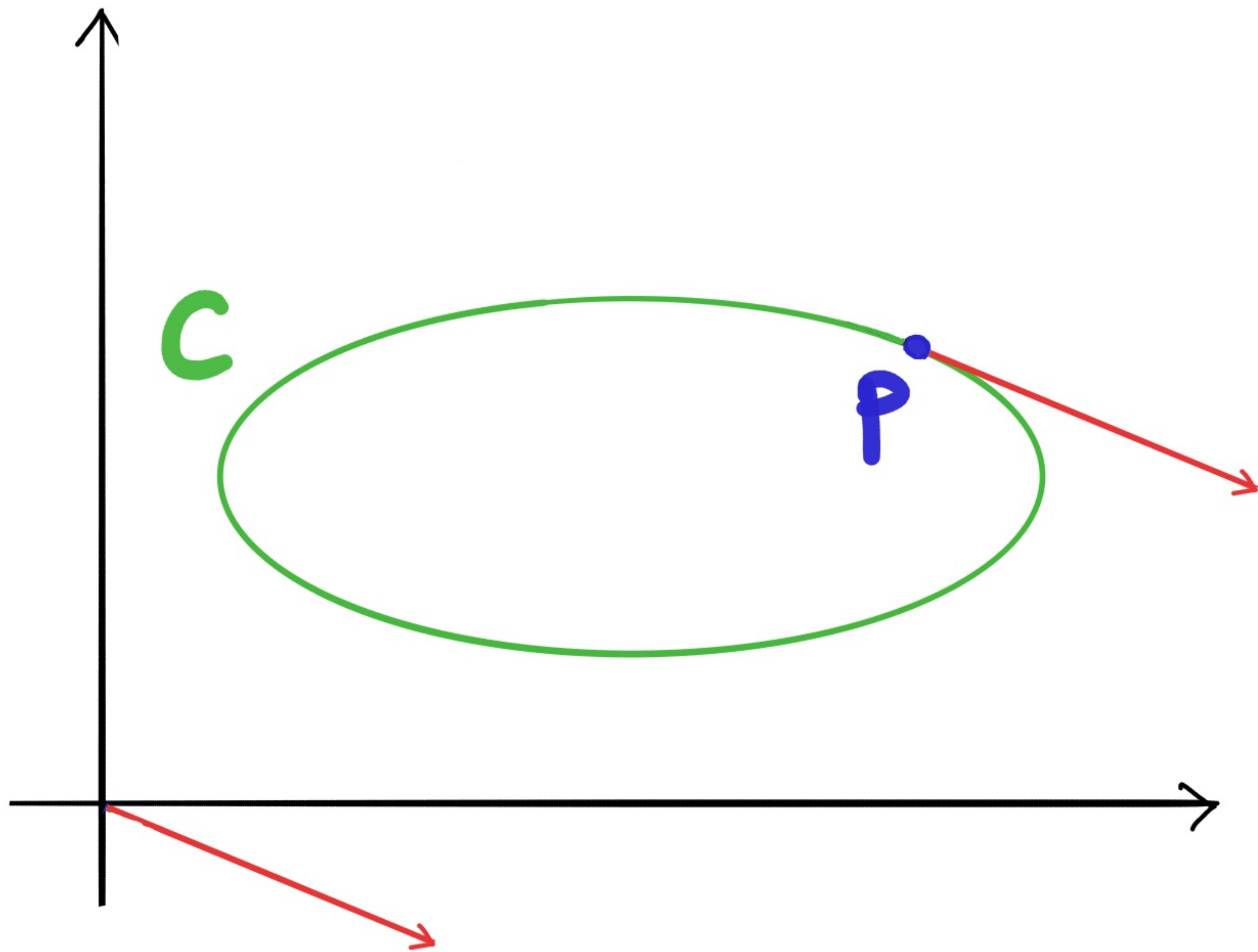


Tangent line
of C at p .

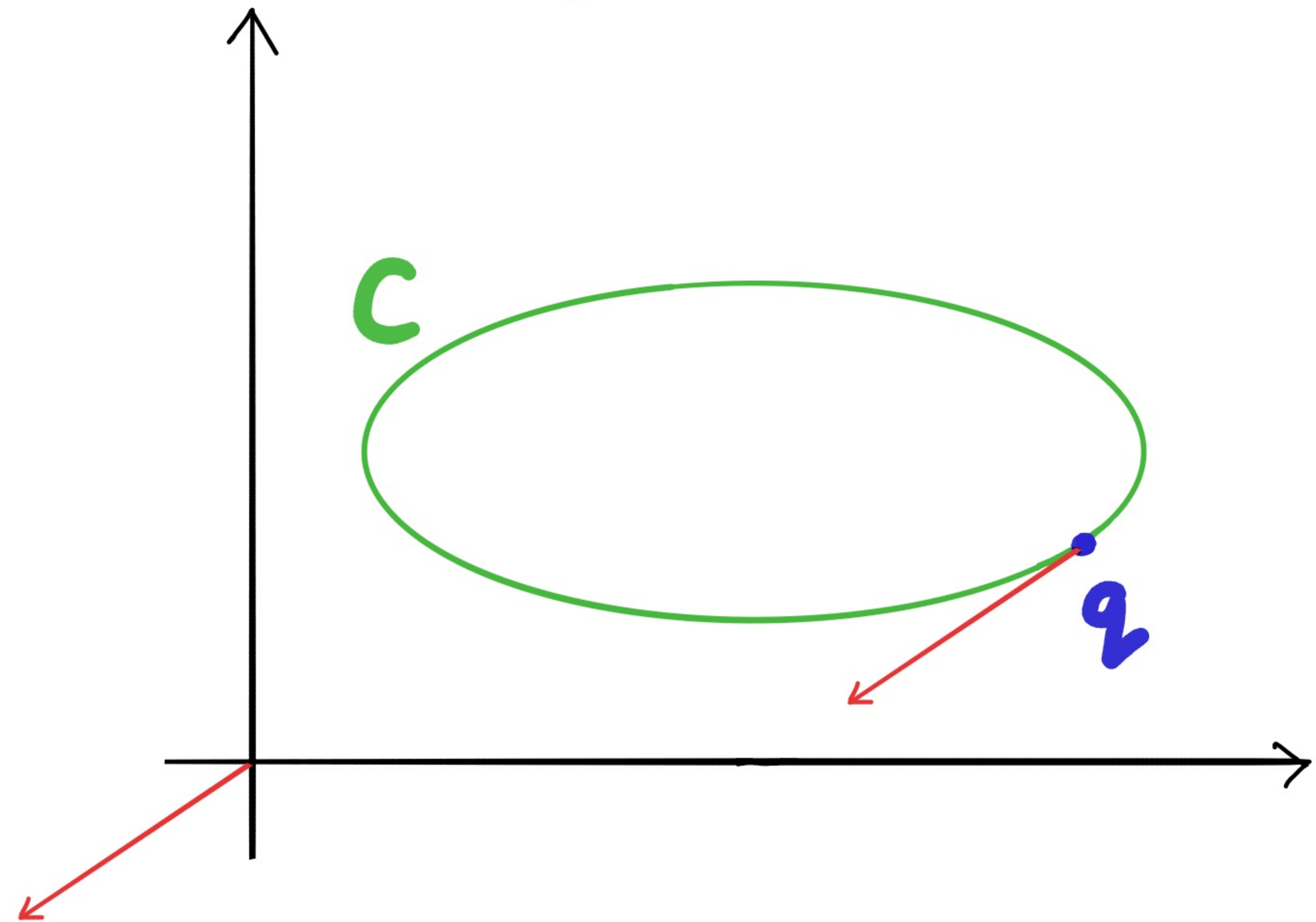


Tangent line
of C at q .

A **tangent vector** of C at p is a vector parallel to the tangent line.

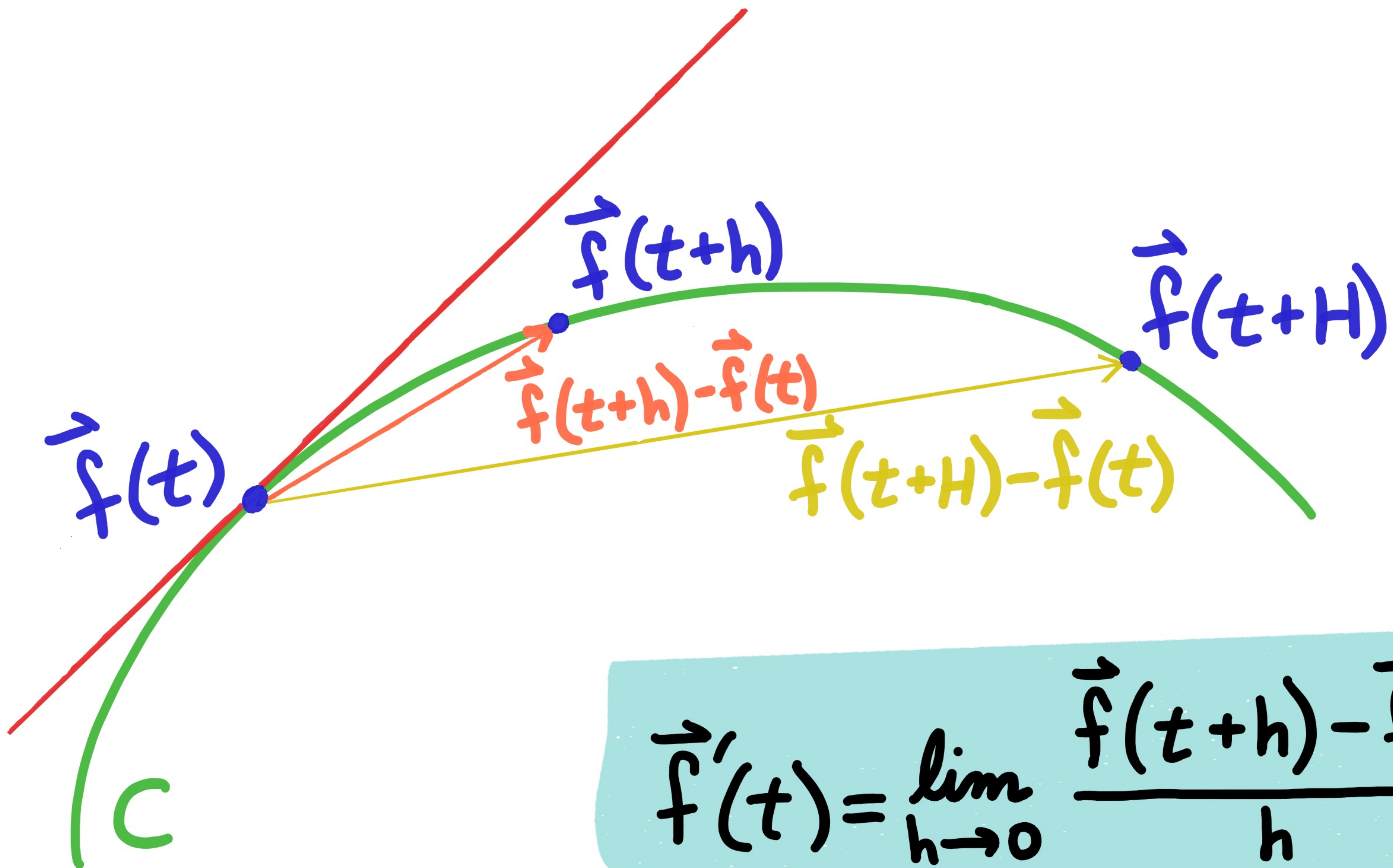


Tangent vector
of C at p .



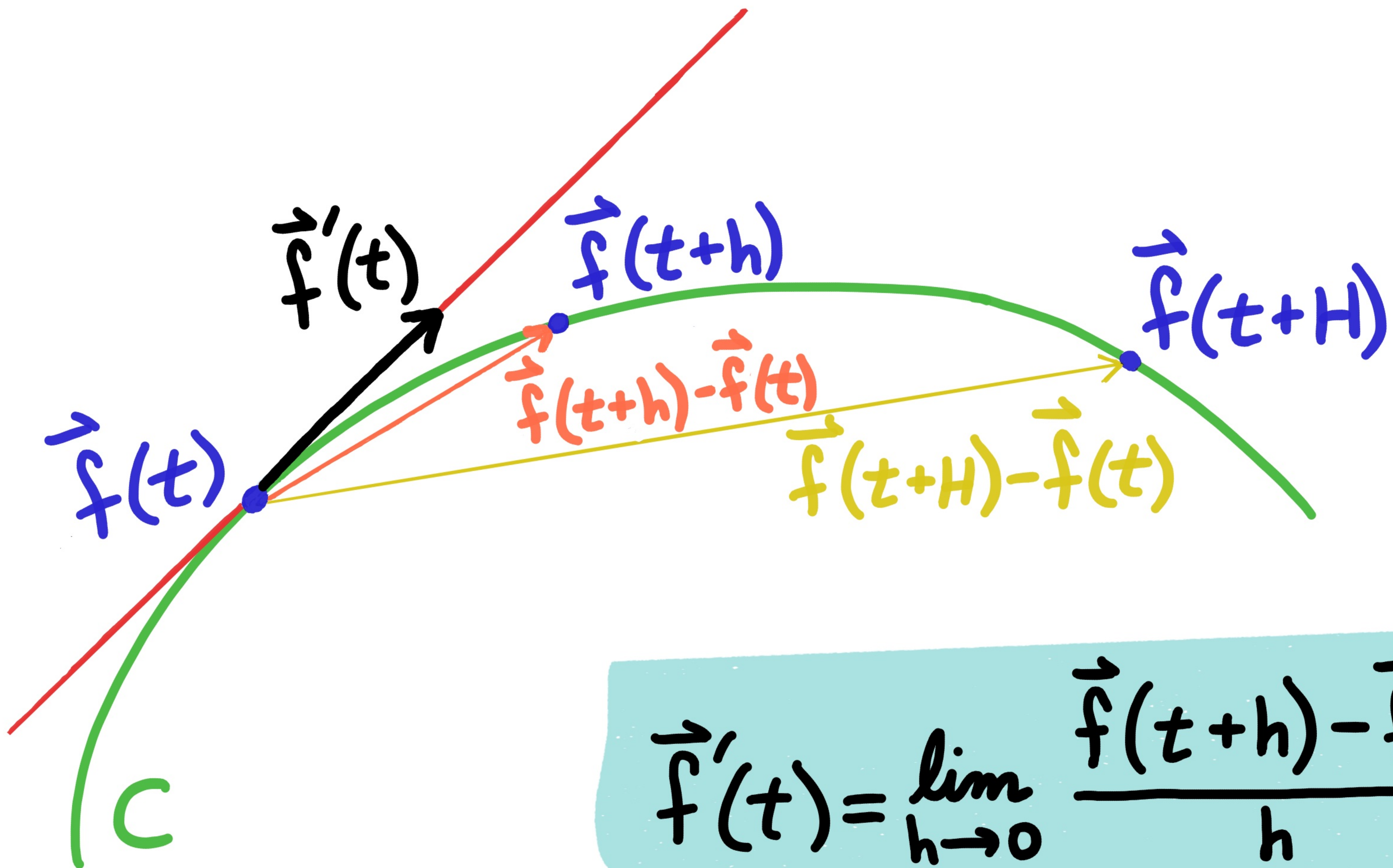
Tangent vector
of C at q .

$\vec{f}'(t)$ is a tangent vector at $\vec{f}(t)$

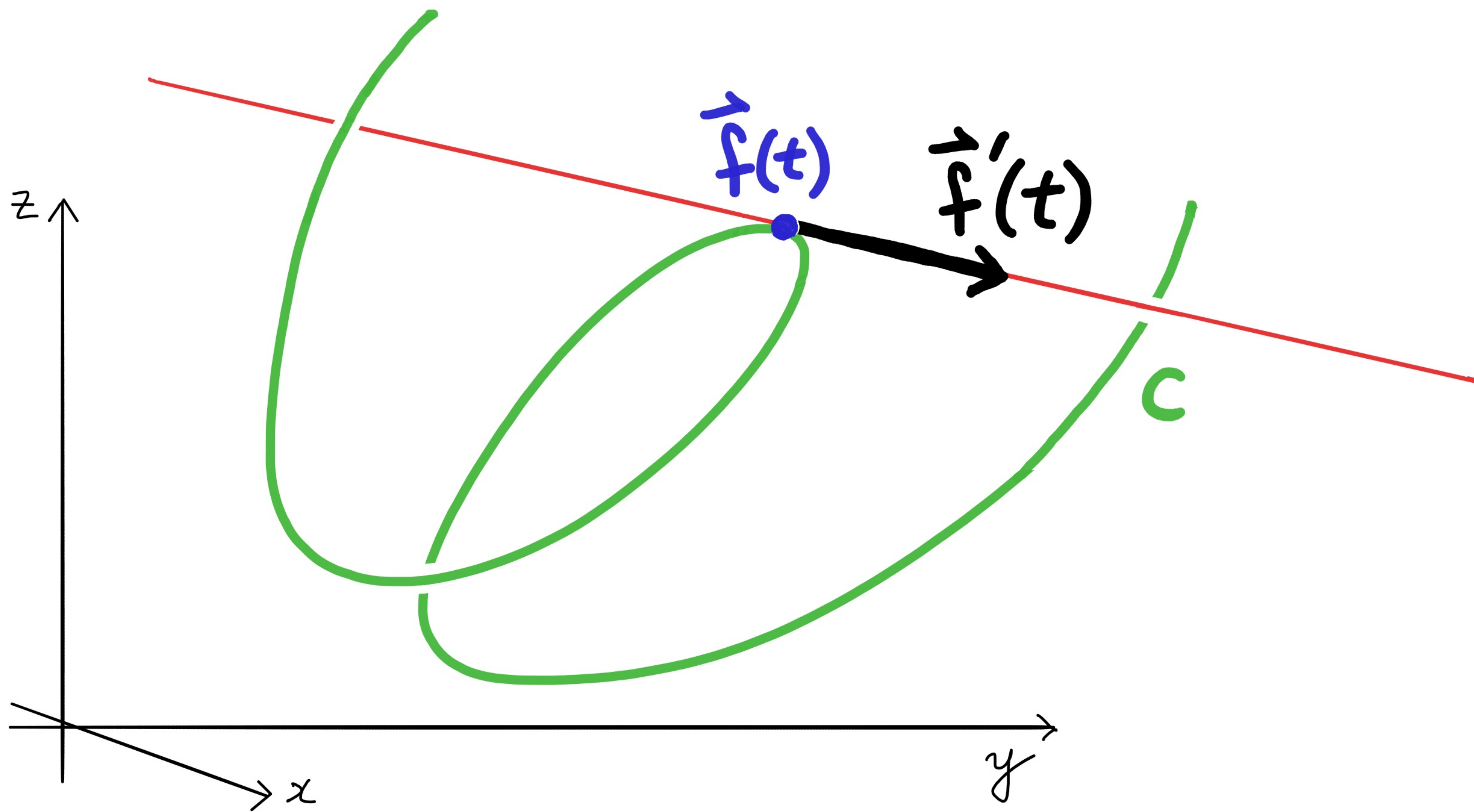


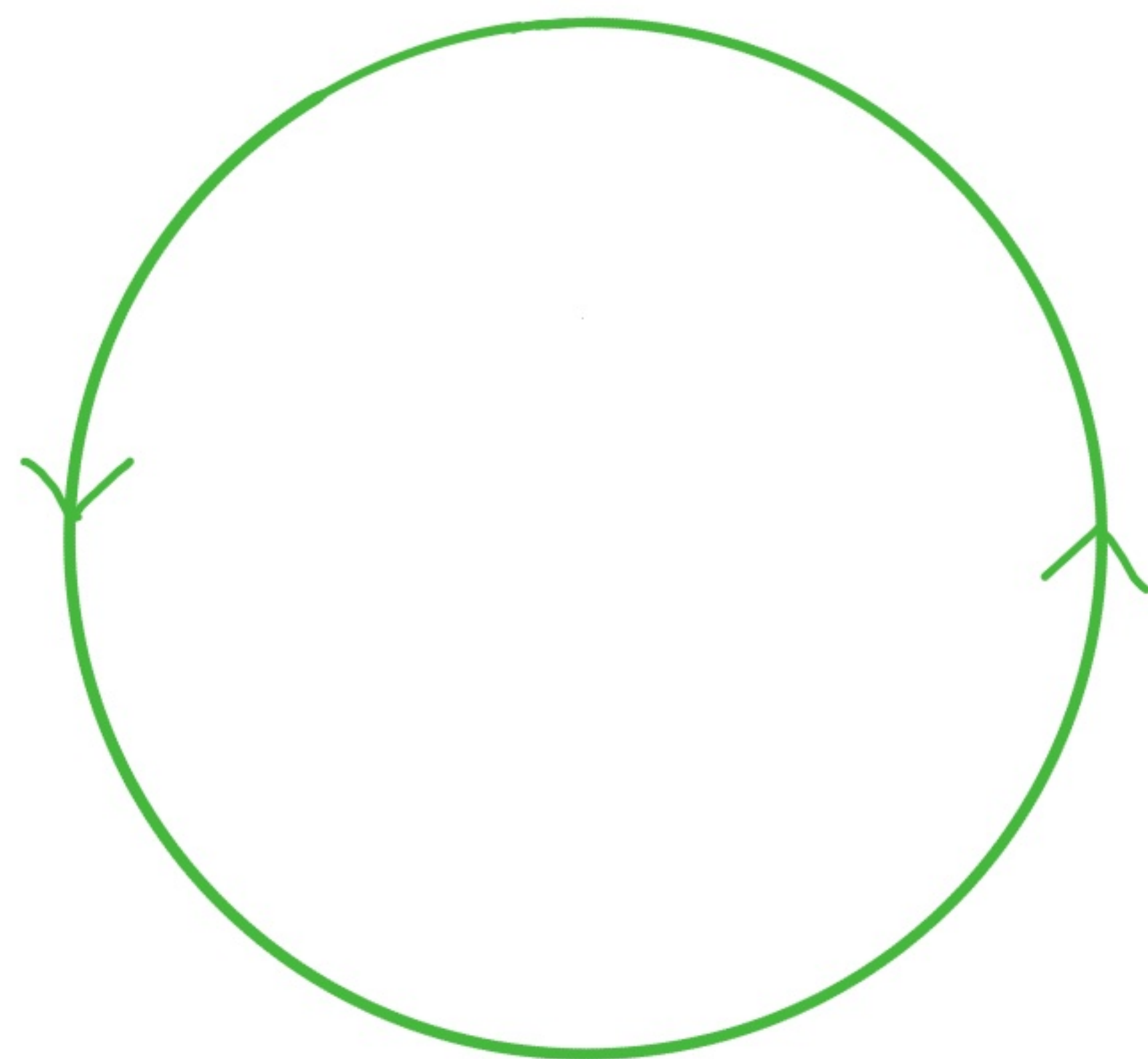
$$\vec{f}'(t) = \lim_{h \rightarrow 0} \frac{\vec{f}(t+h) - \vec{f}(t)}{h}$$

$\vec{f}'(t)$ is a tangent vector at $\vec{f}(t)$

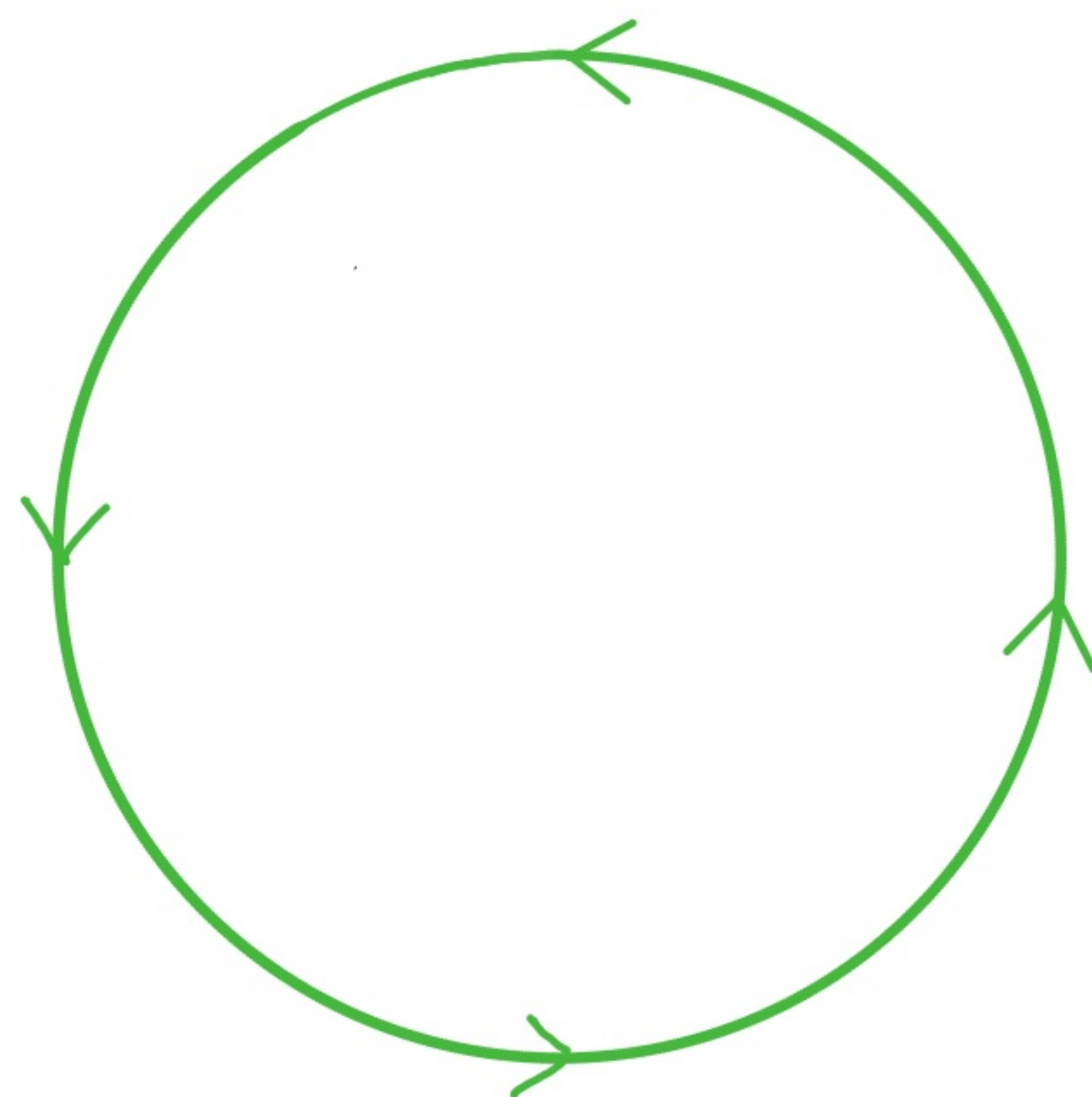


$$\vec{f}'(t) = \lim_{h \rightarrow 0} \frac{\vec{f}(t+h) - \vec{f}(t)}{h}$$

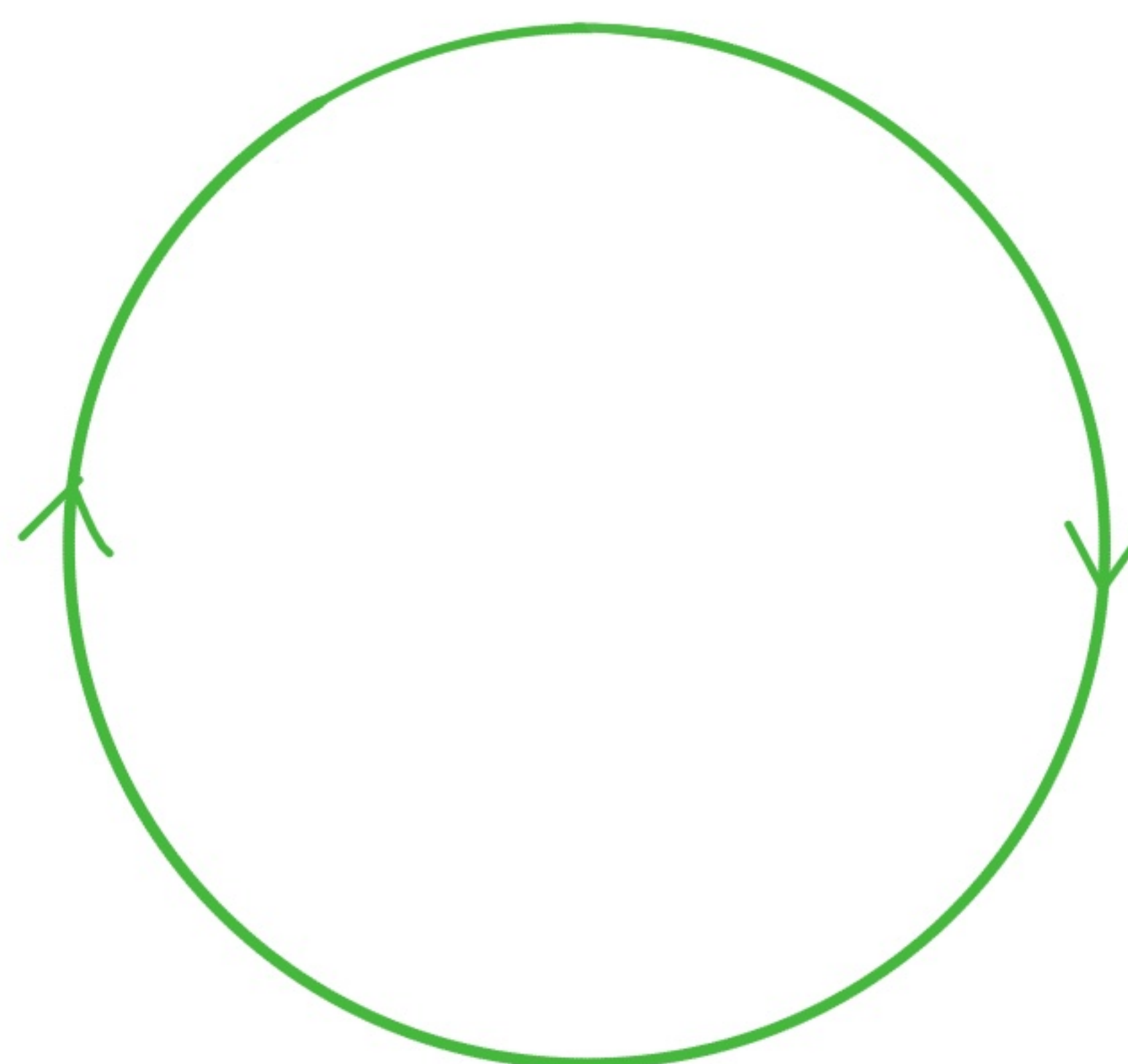




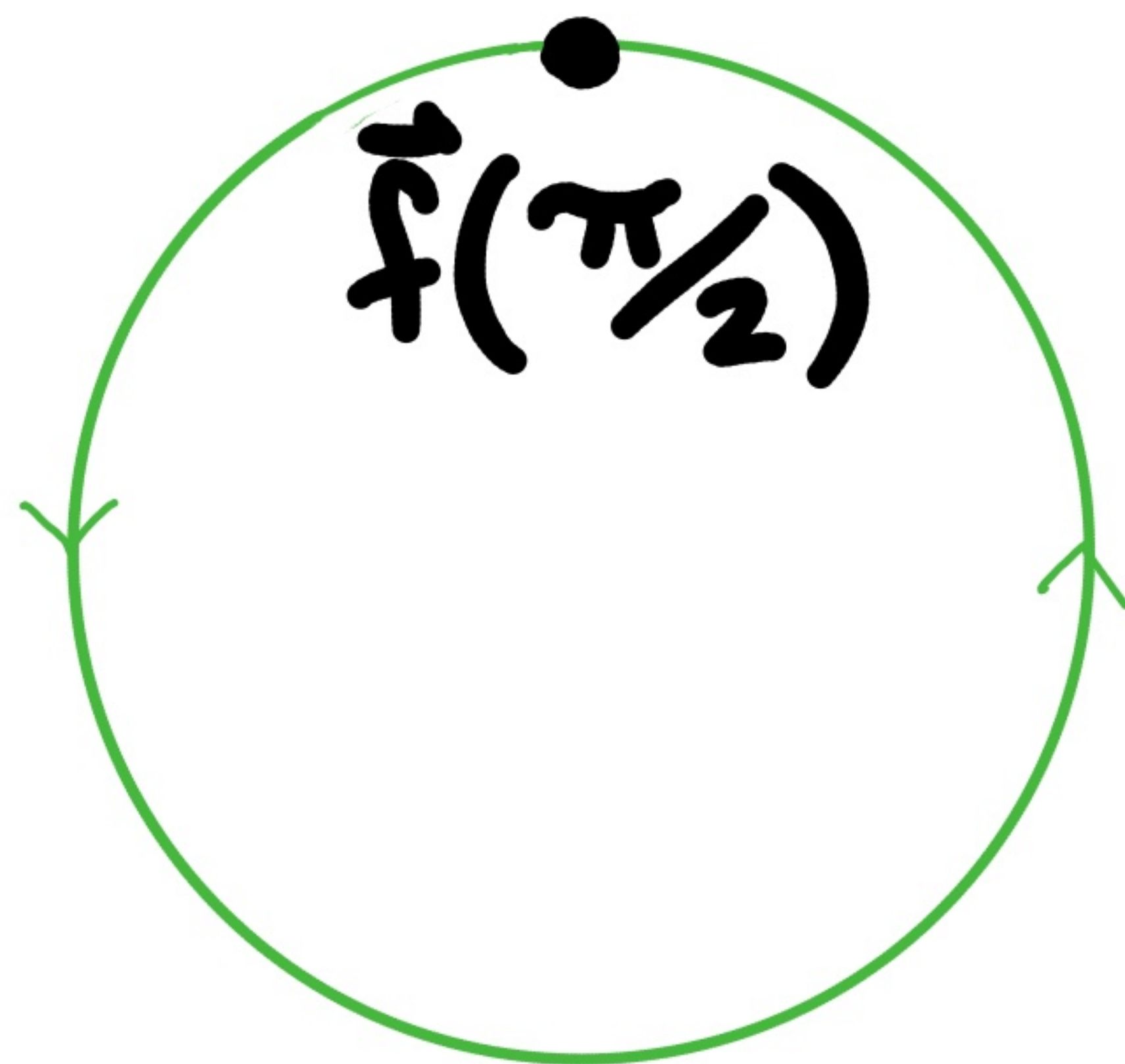
$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$



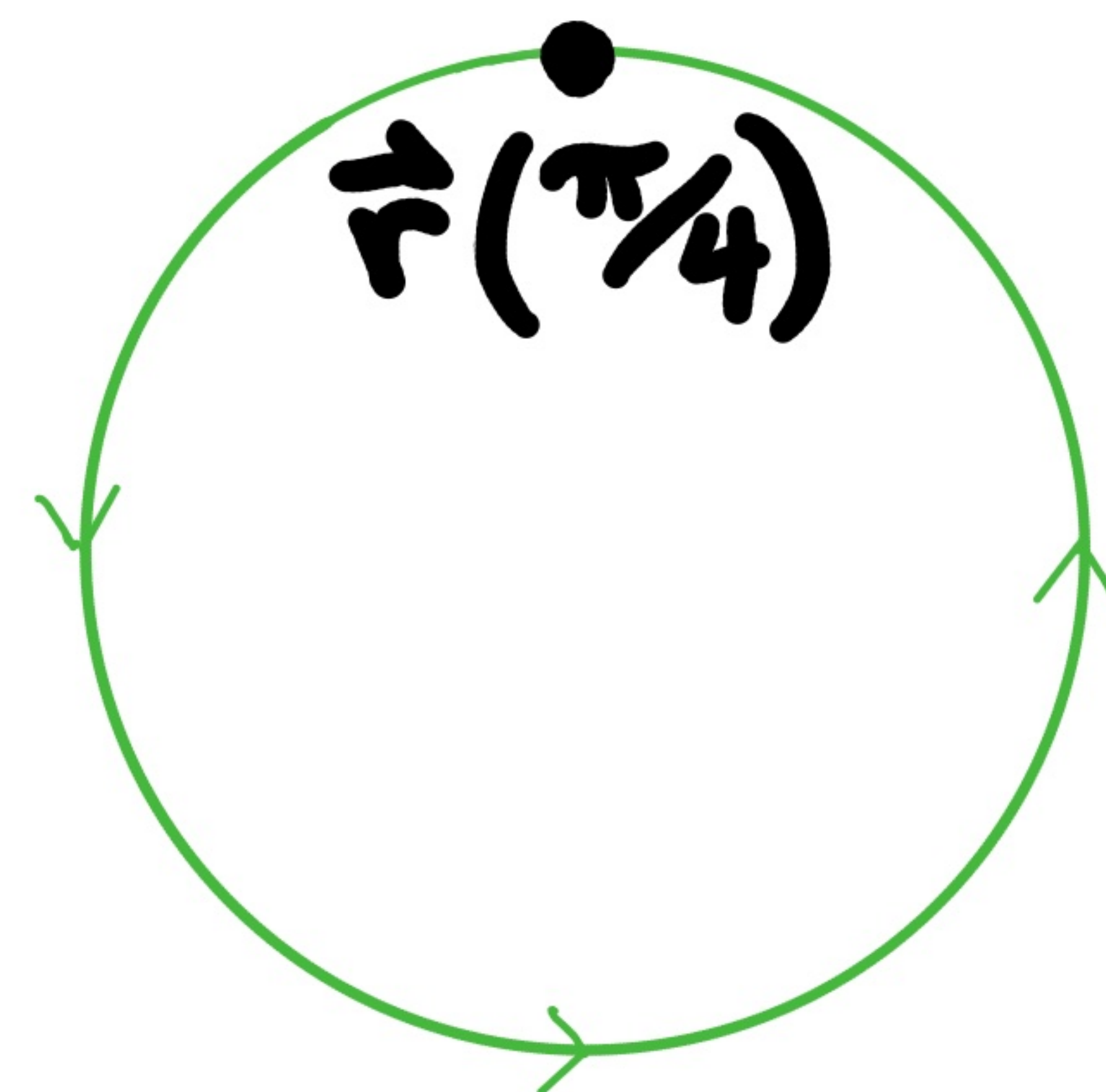
$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$



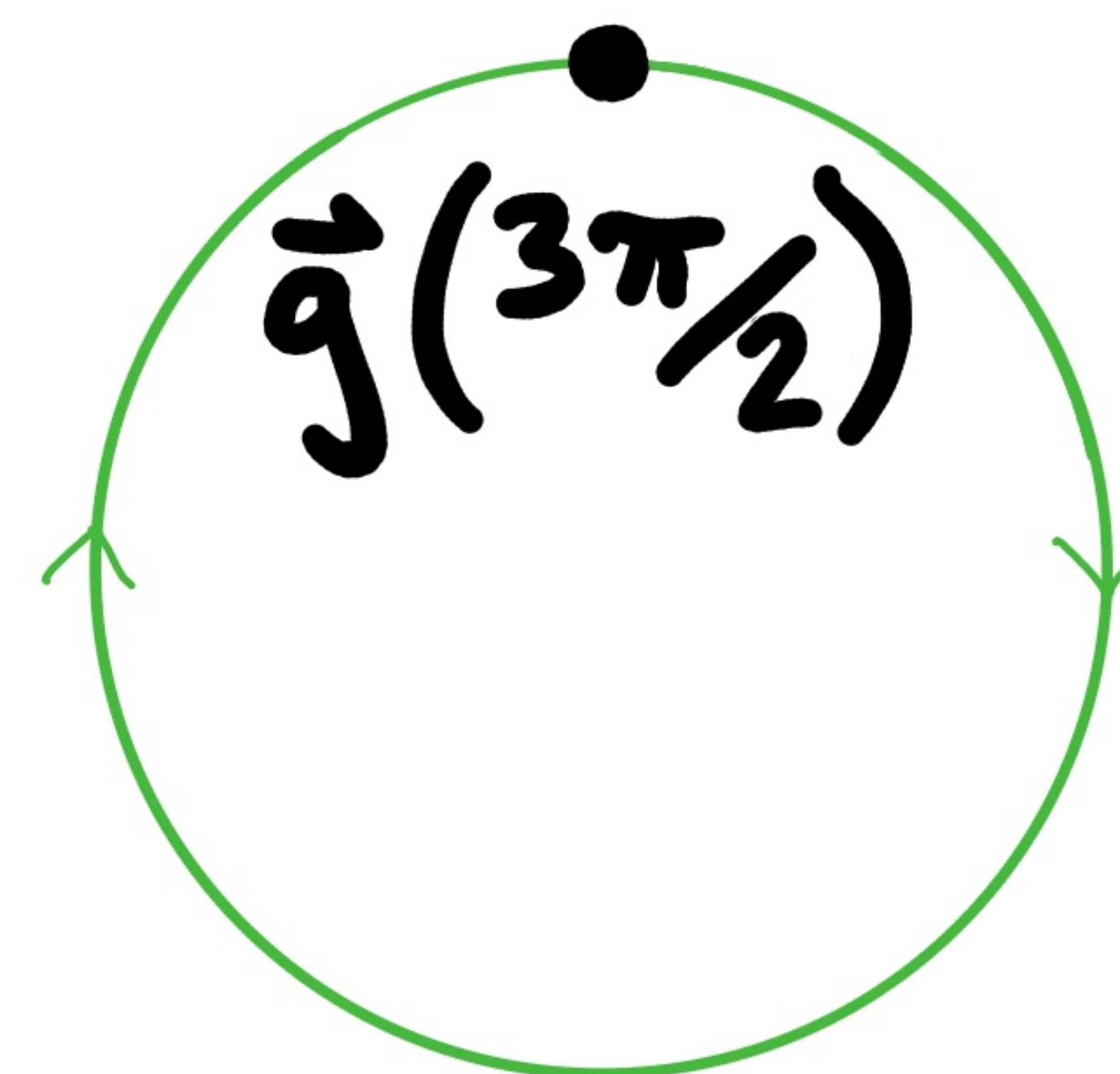
$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$



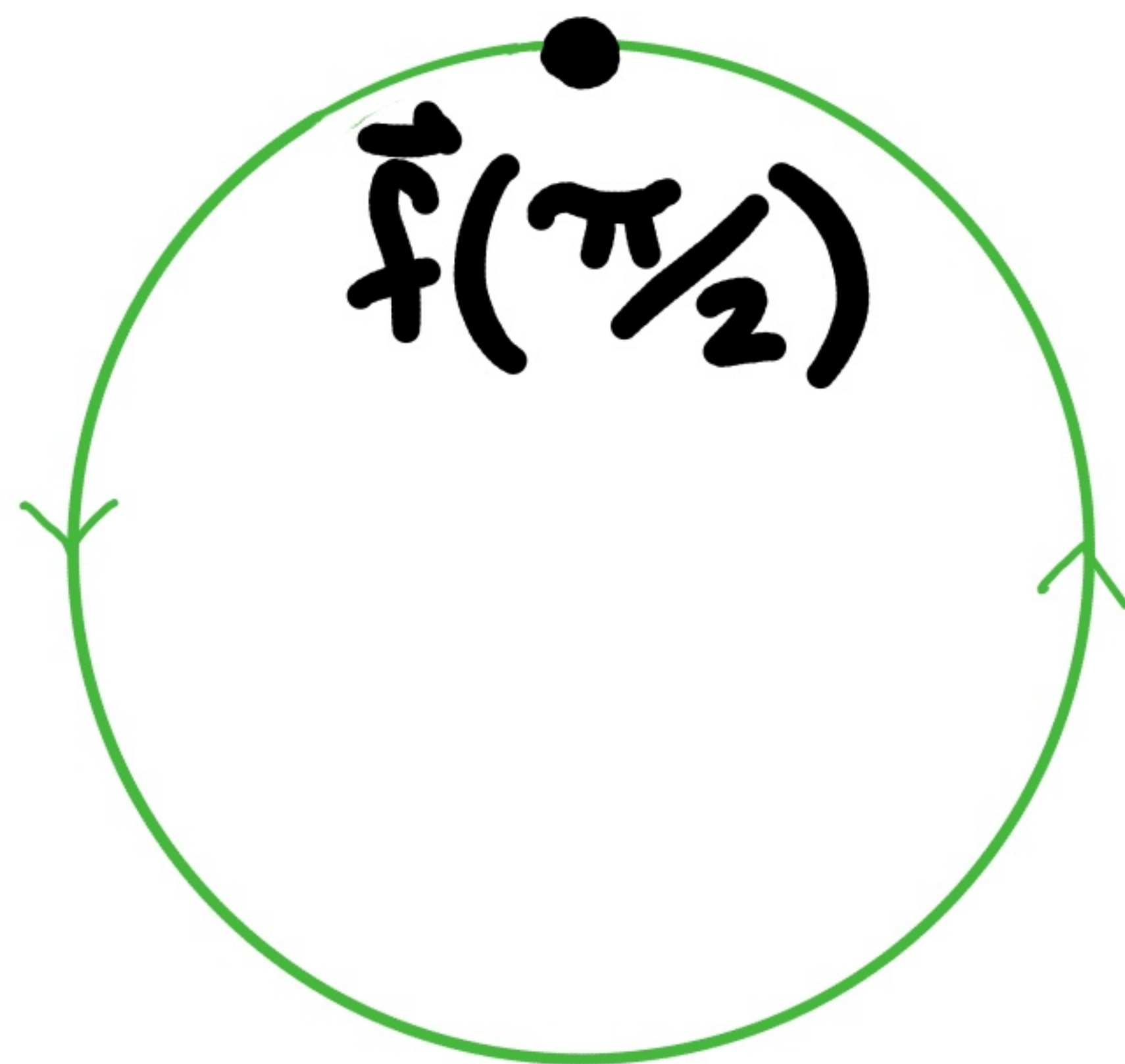
$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$



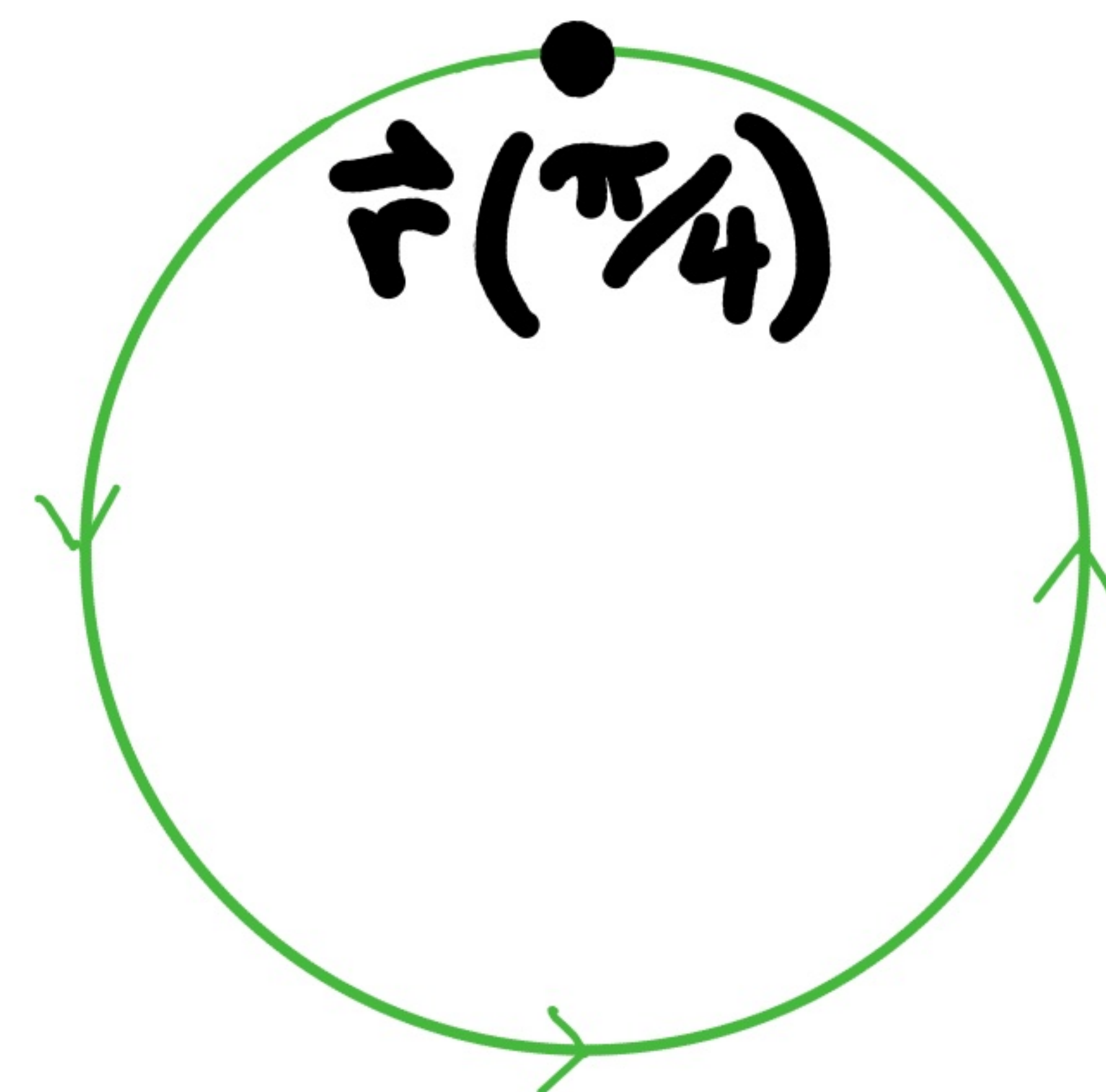
$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$



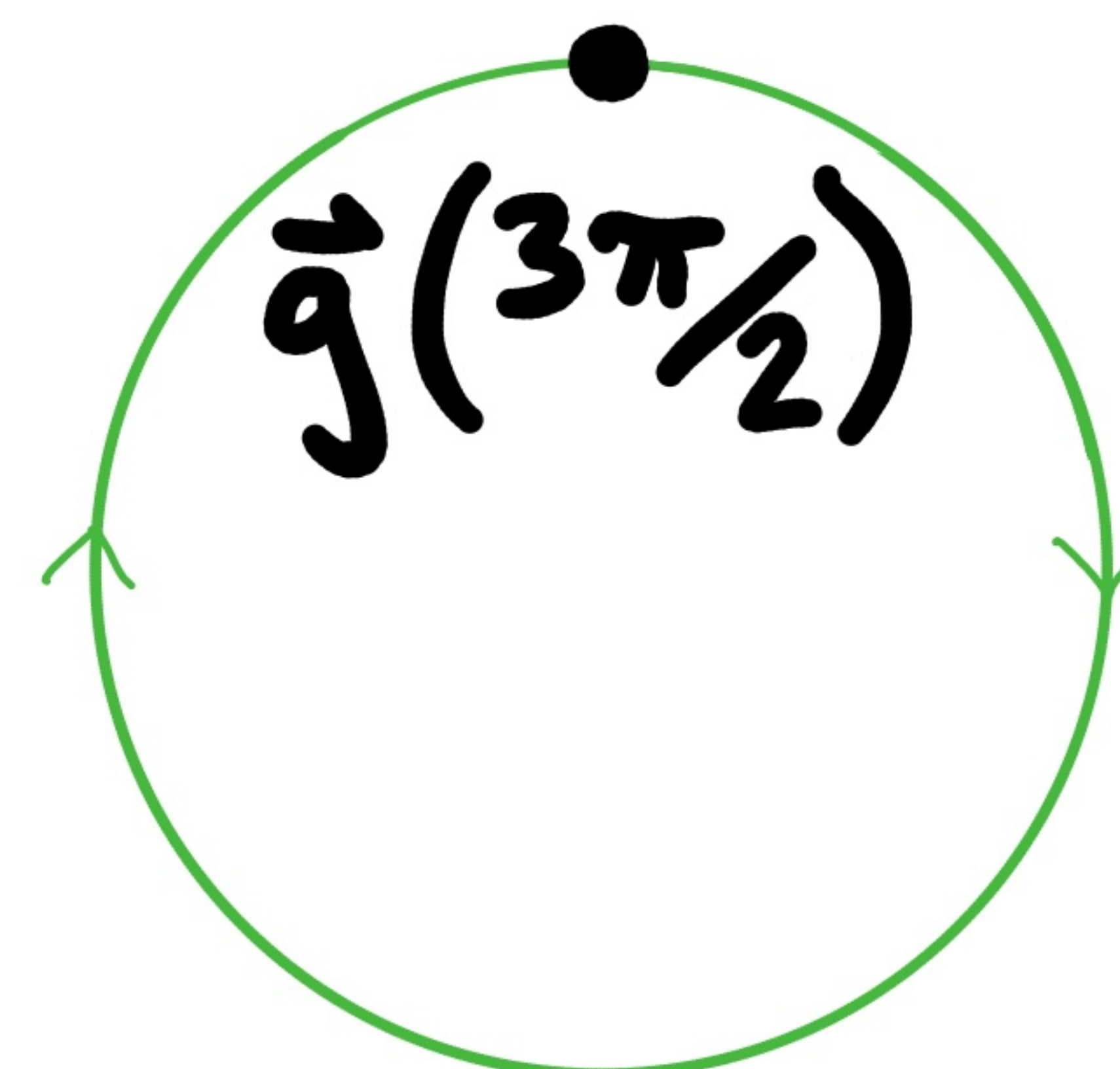
$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$



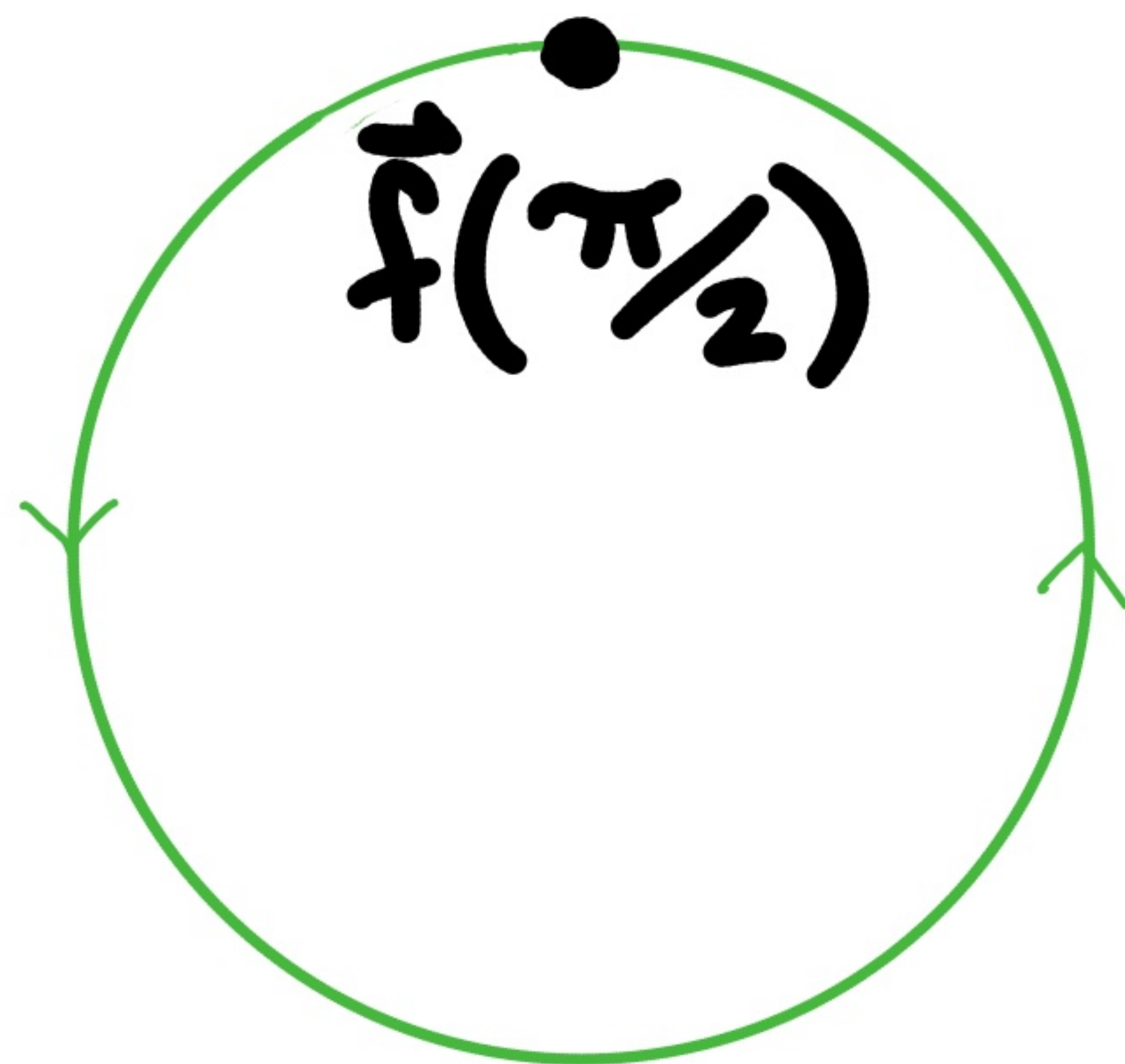
$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$
$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$



$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

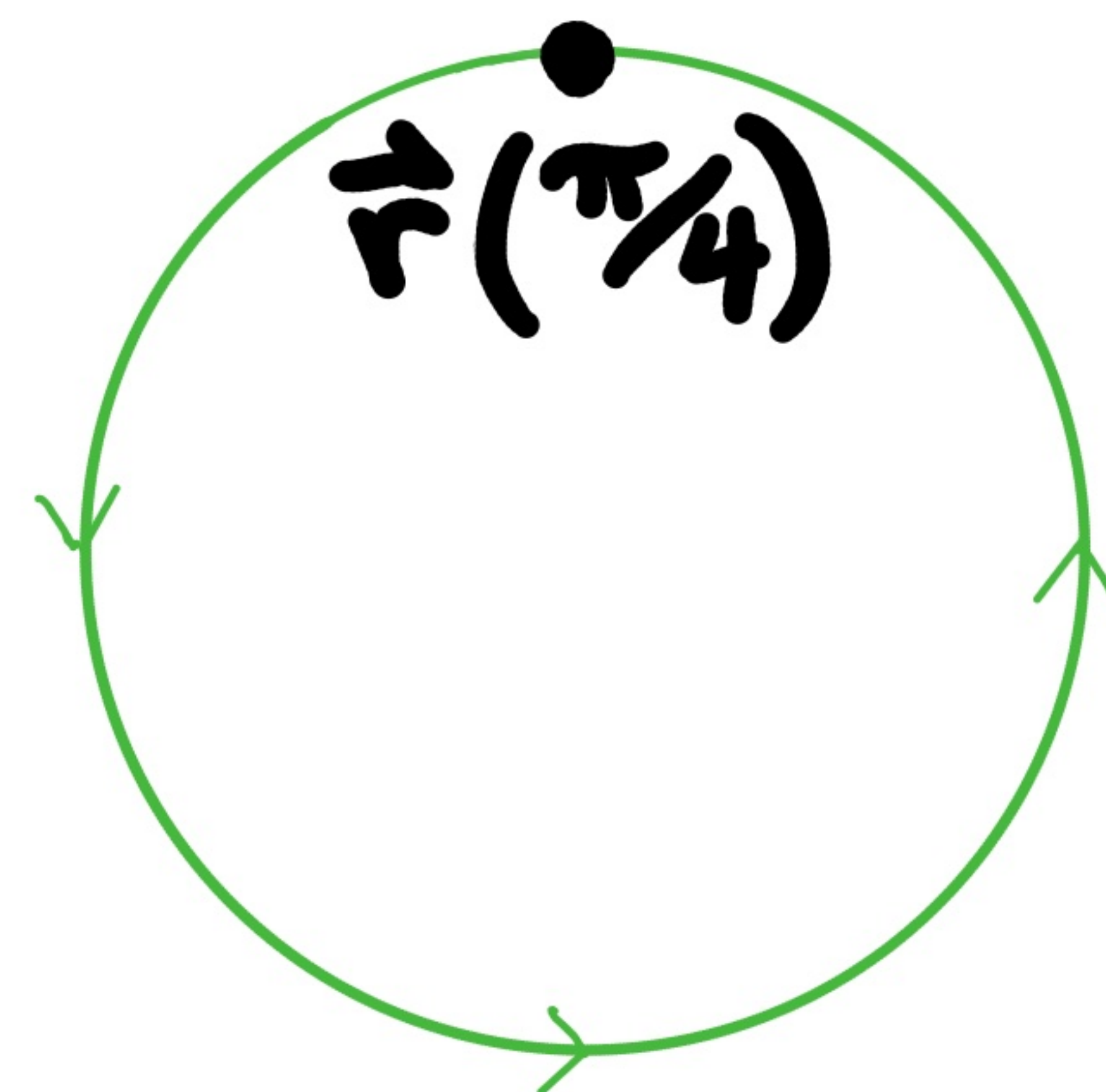


$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$



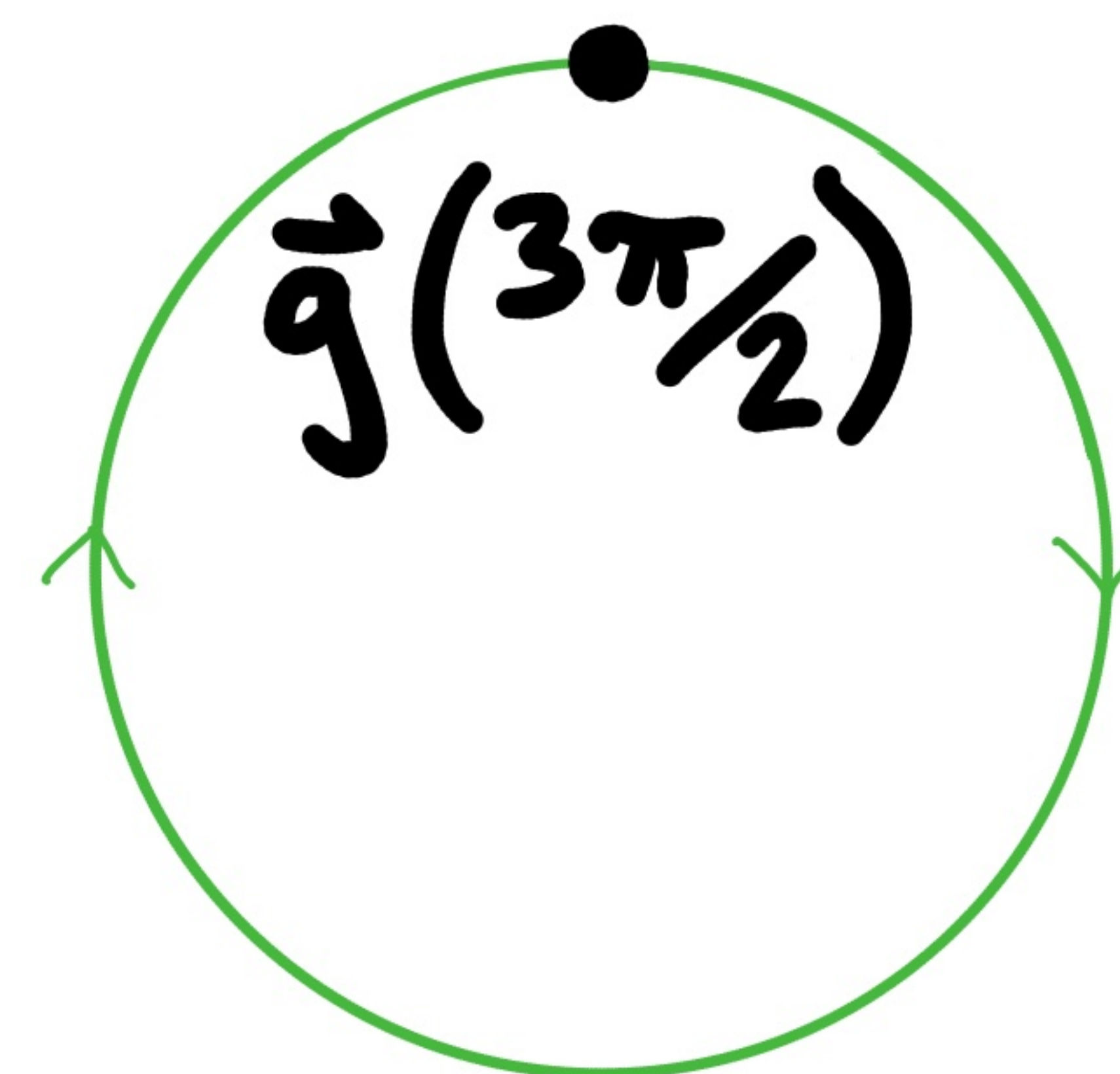
$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$



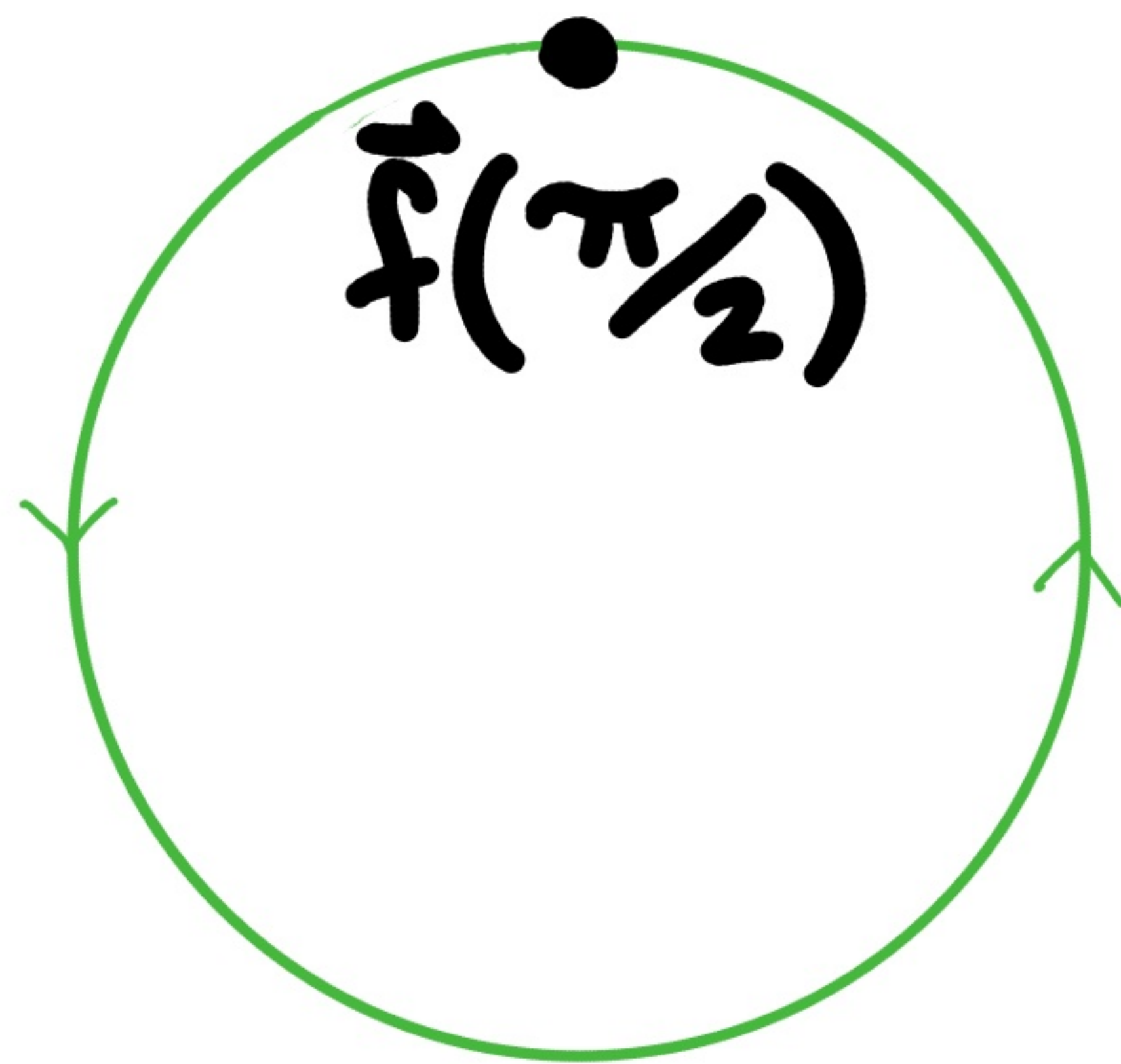
$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) =$$



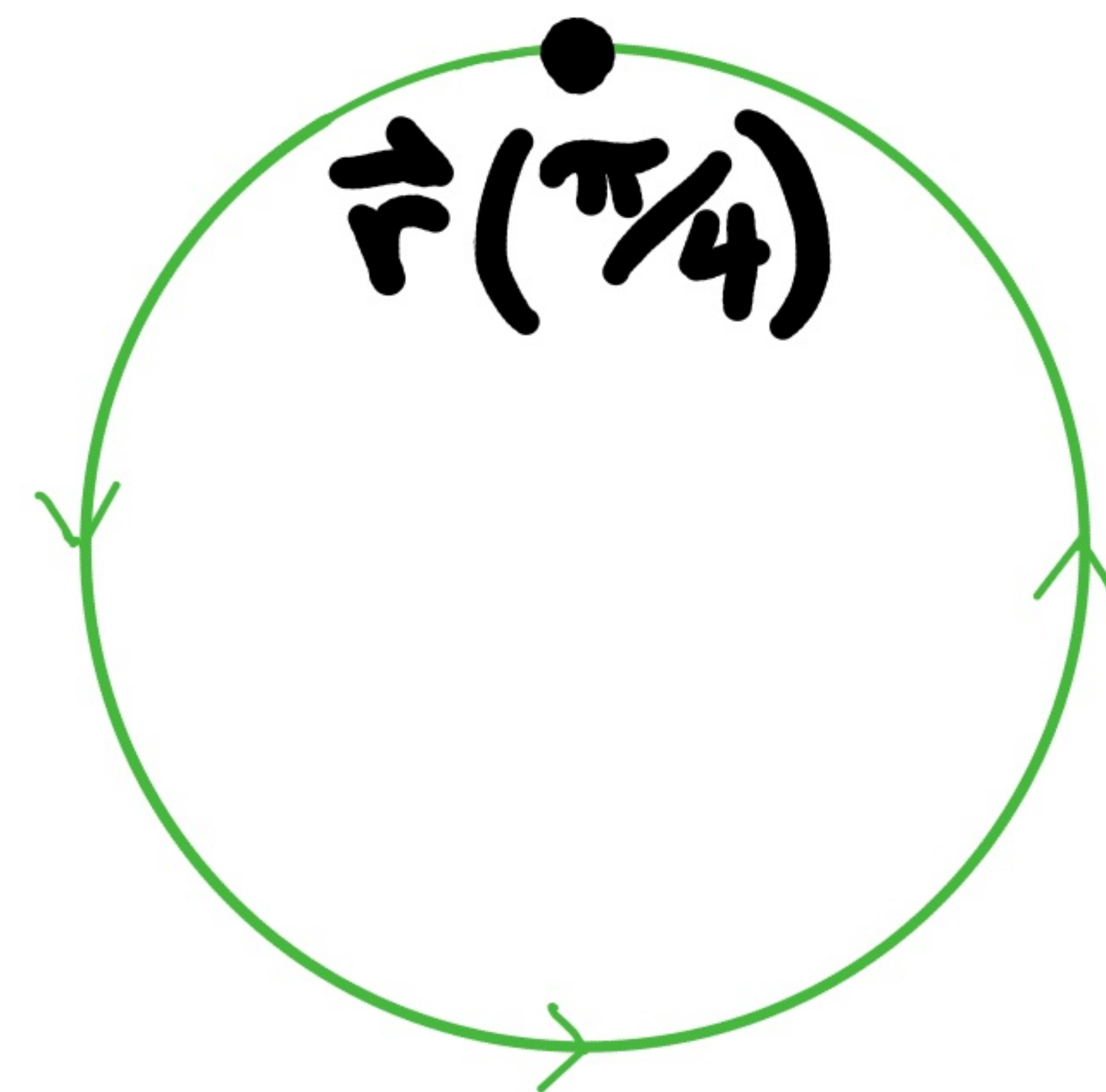
$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

$$\vec{g}'(t) =$$



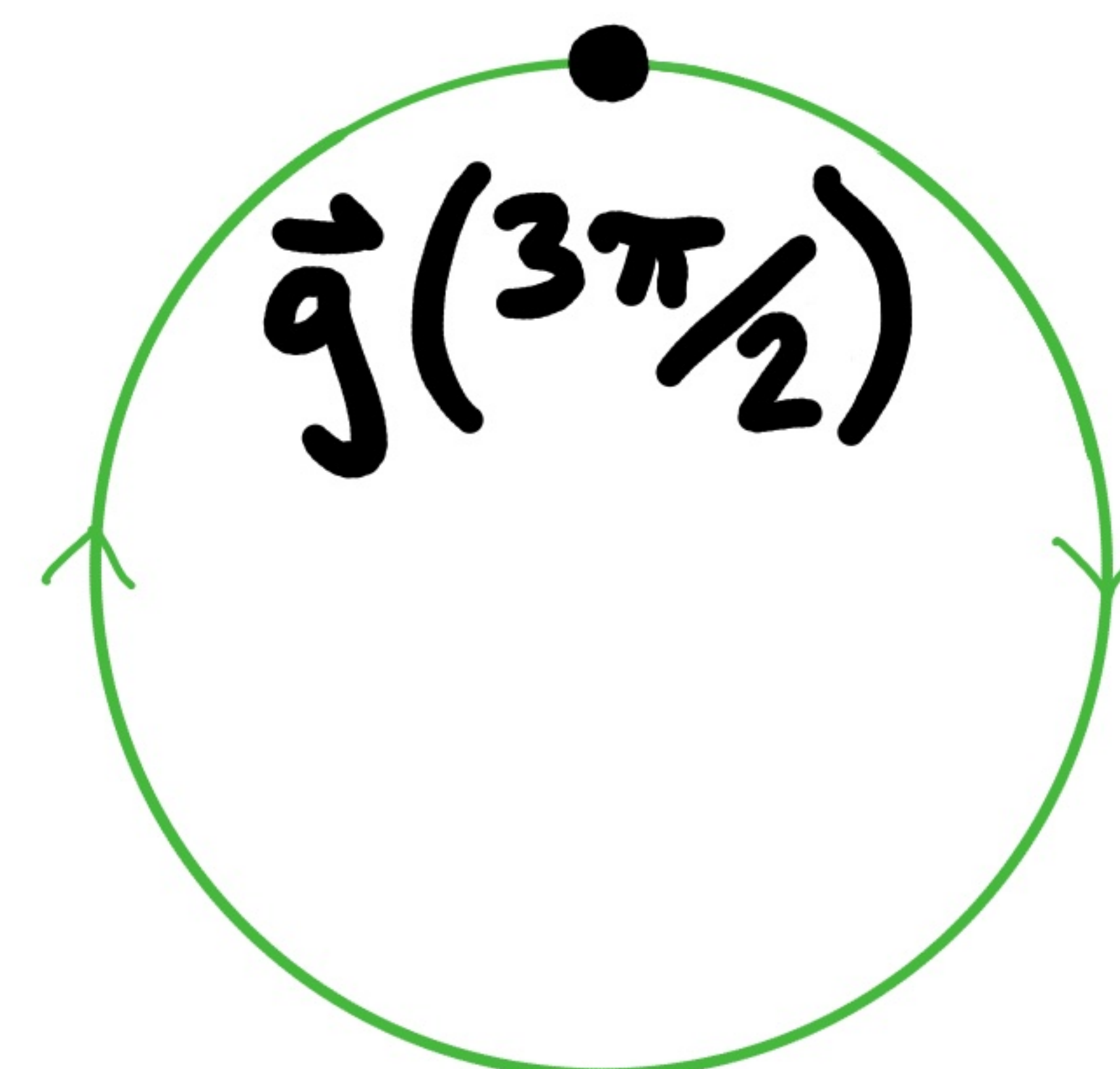
$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$



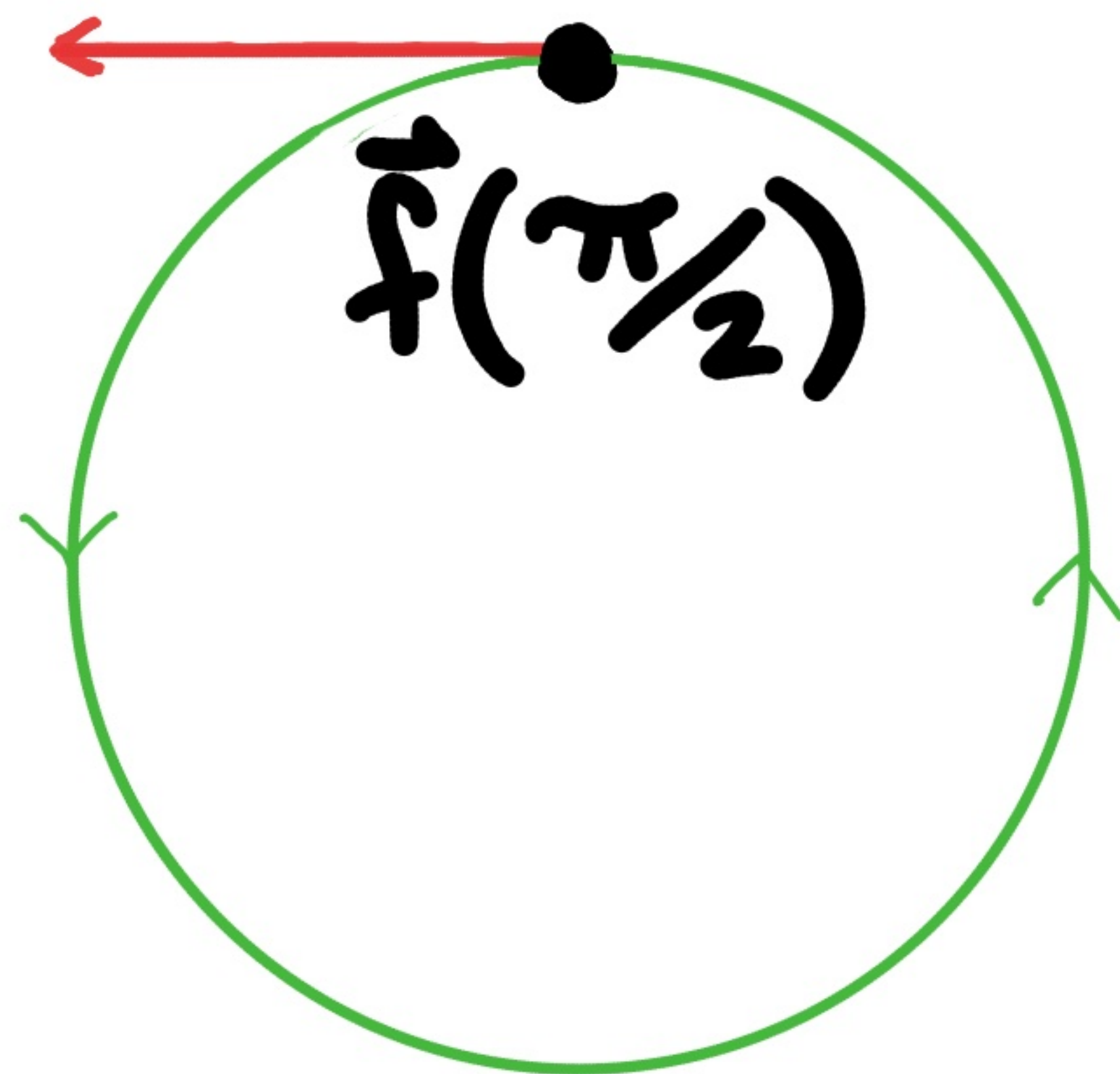
$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) = \langle -2\sin 2t, 2\cos 2t \rangle$$



$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

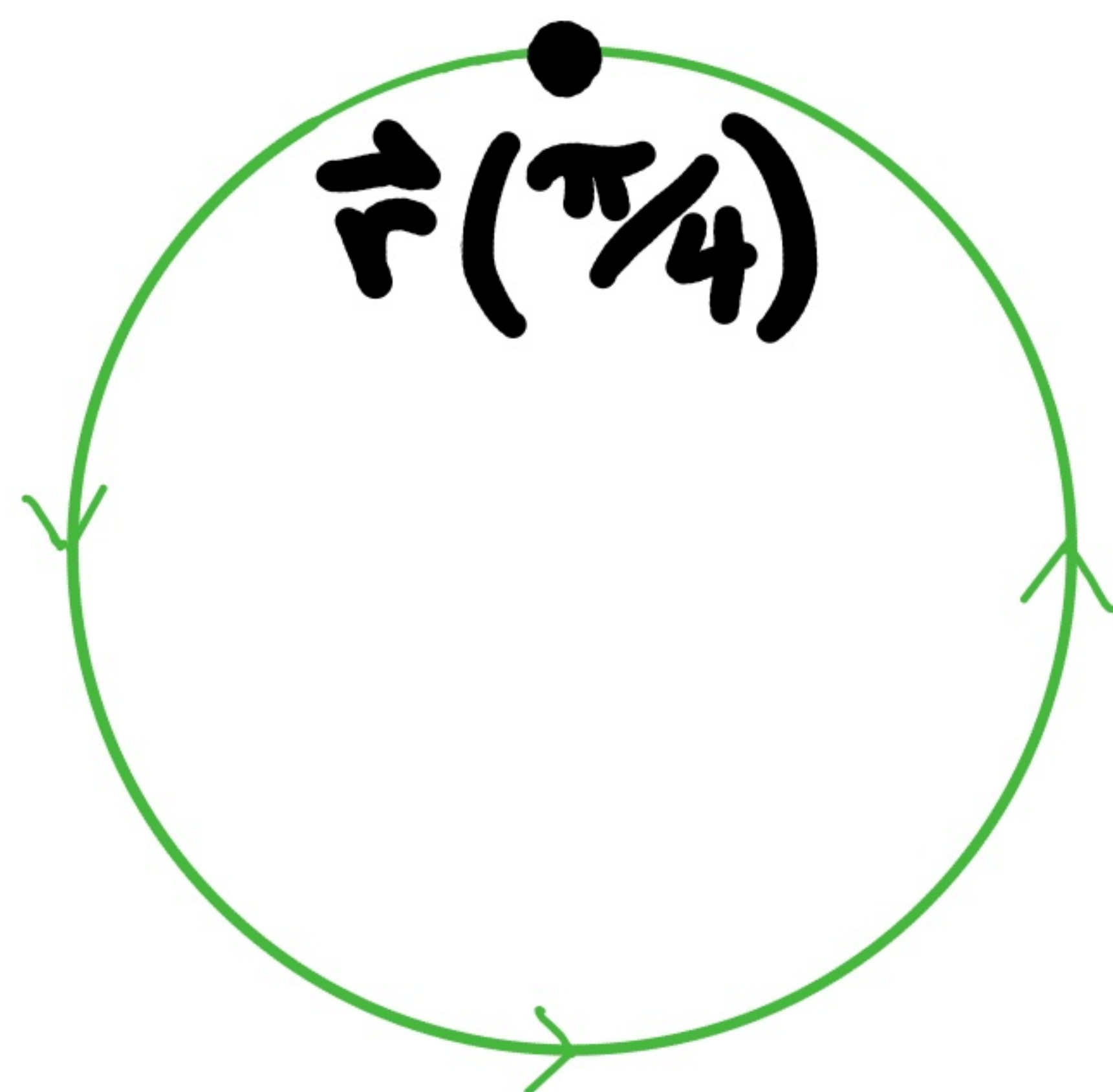
$$\vec{g}'(t) = \langle \sin(-t), -\cos(-t) \rangle$$



$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

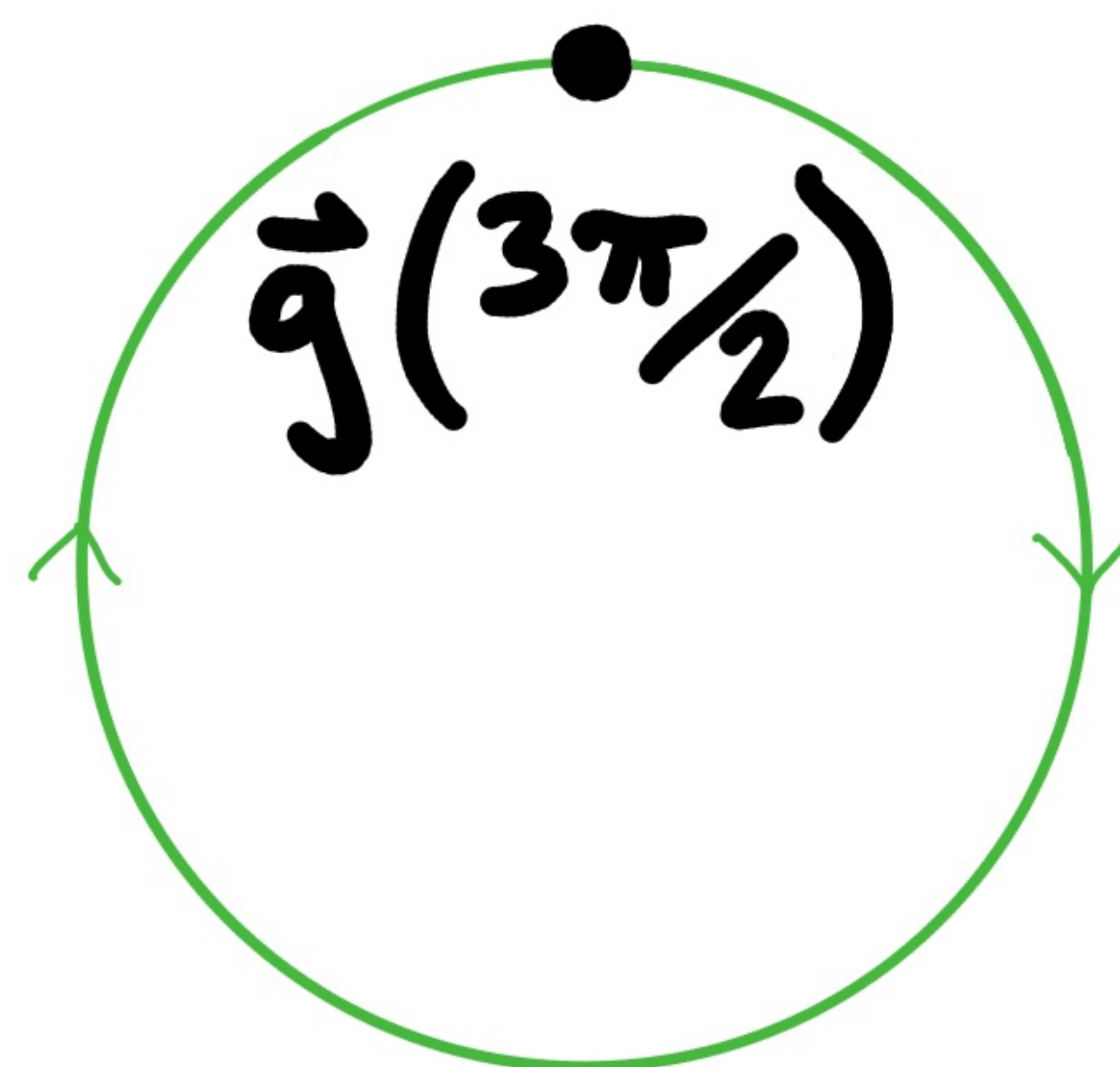
$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$

$$\vec{f}'(\pi/2) = \langle -1, 0 \rangle$$



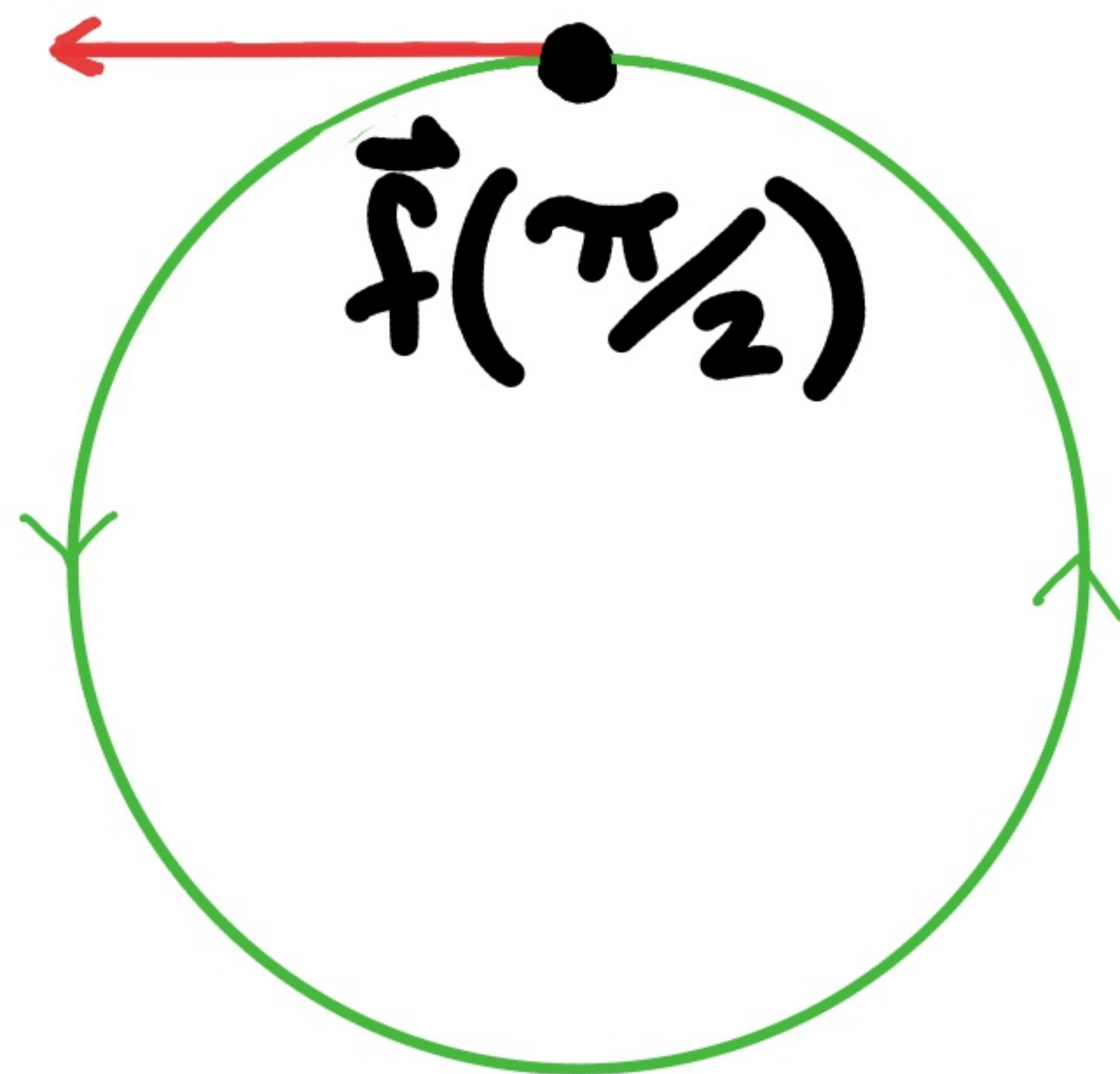
$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) = \langle -2\sin 2t, 2\cos 2t \rangle$$



$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

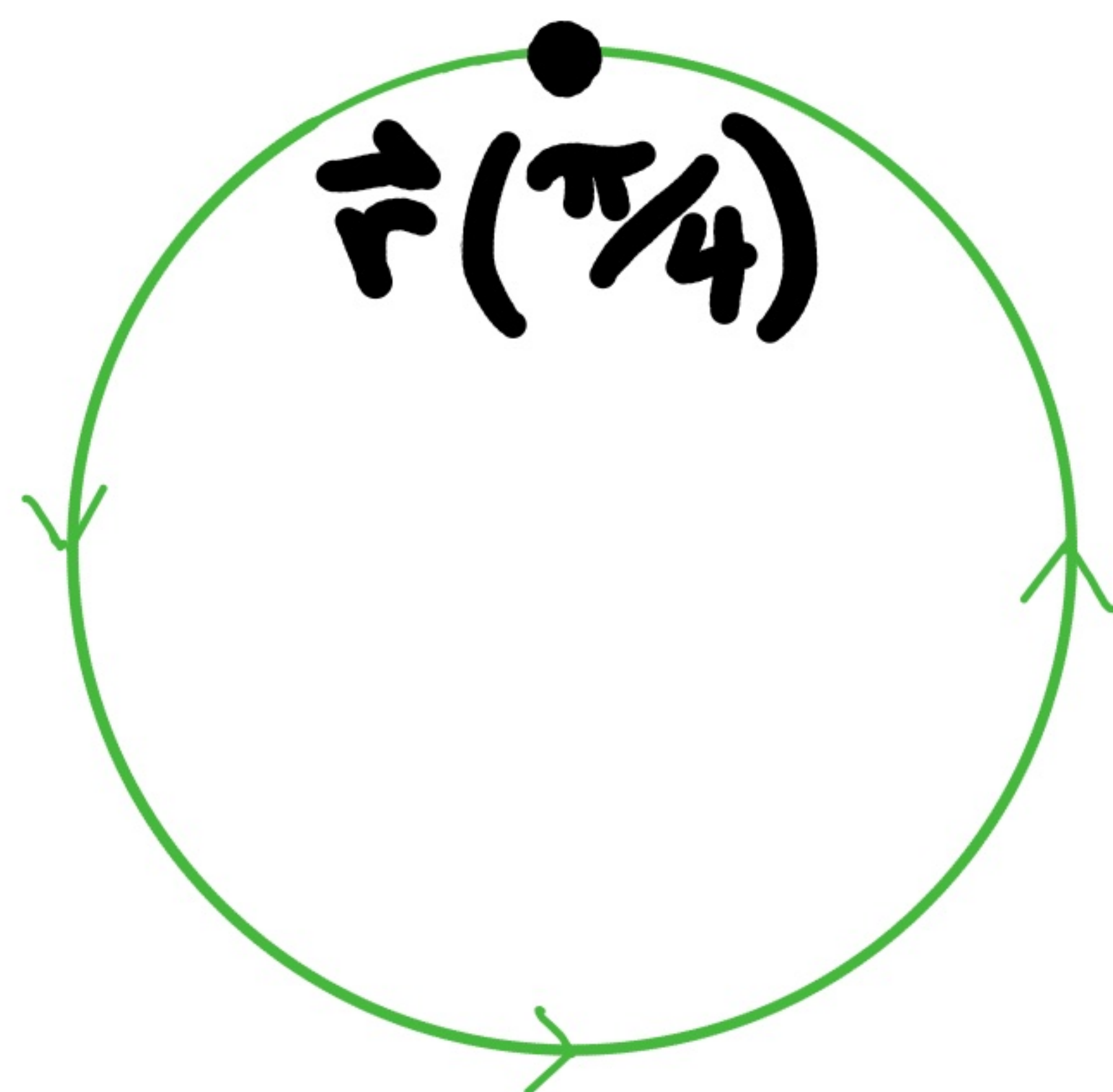
$$\vec{g}'(t) = \langle \sin(-t), -\cos(-t) \rangle$$



$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$

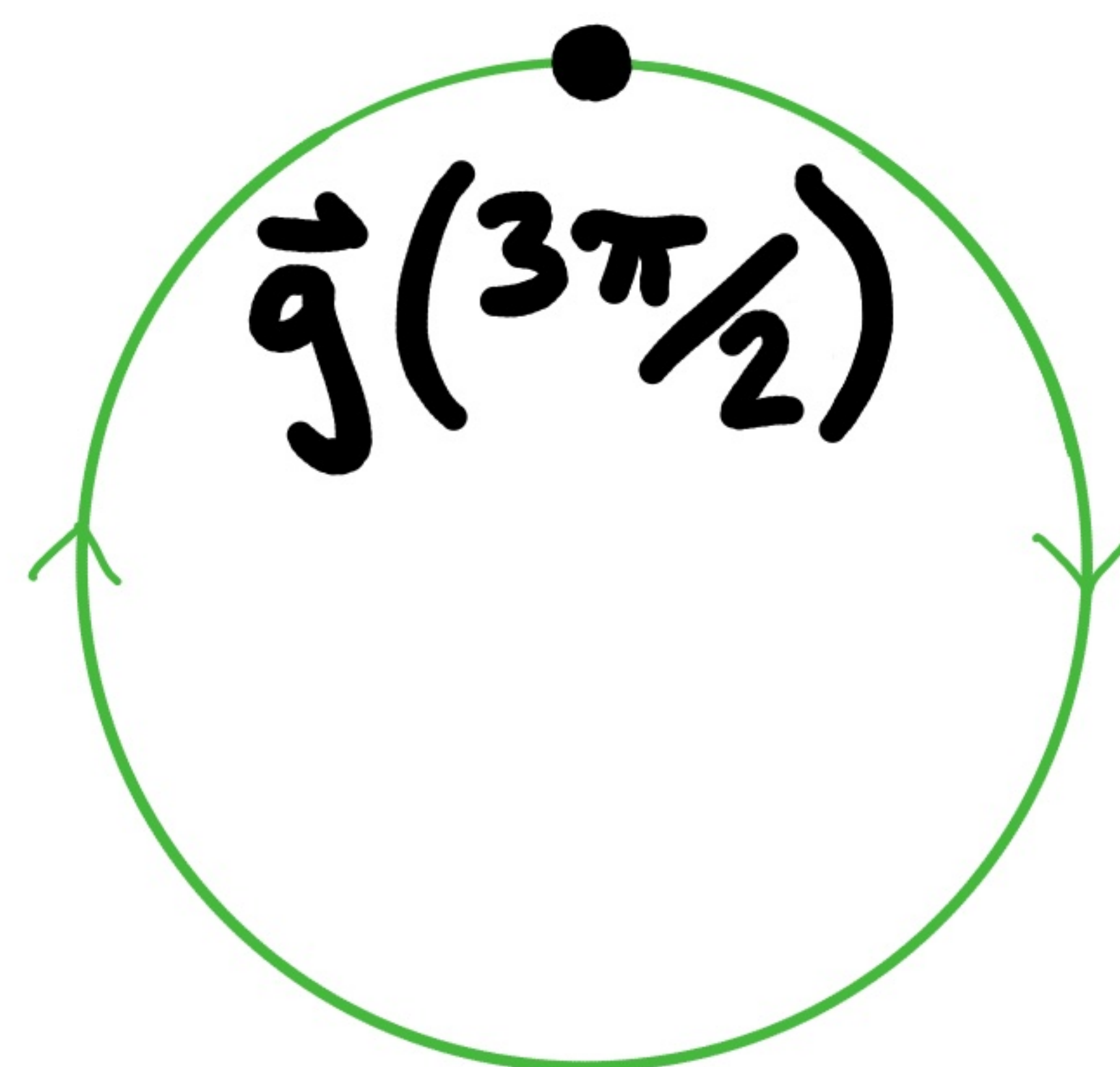
$$\vec{f}'(\pi/2) = \langle -1, 0 \rangle$$



$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) = \langle -2\sin 2t, 2\cos 2t \rangle$$

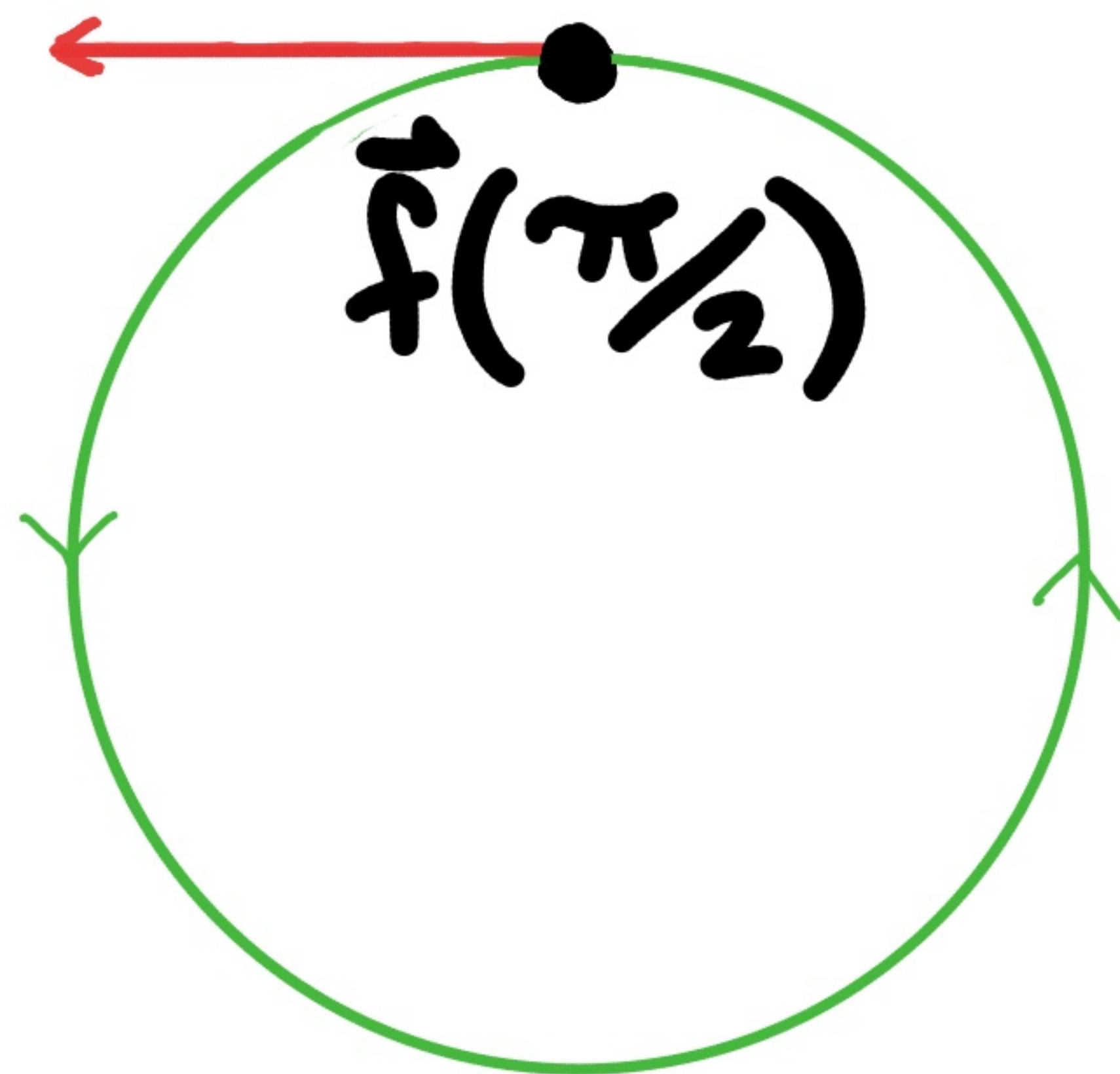
$$\vec{r}'(\pi/4) =$$



$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

$$\vec{g}'(t) = \langle \sin(-t), -\cos(-t) \rangle$$

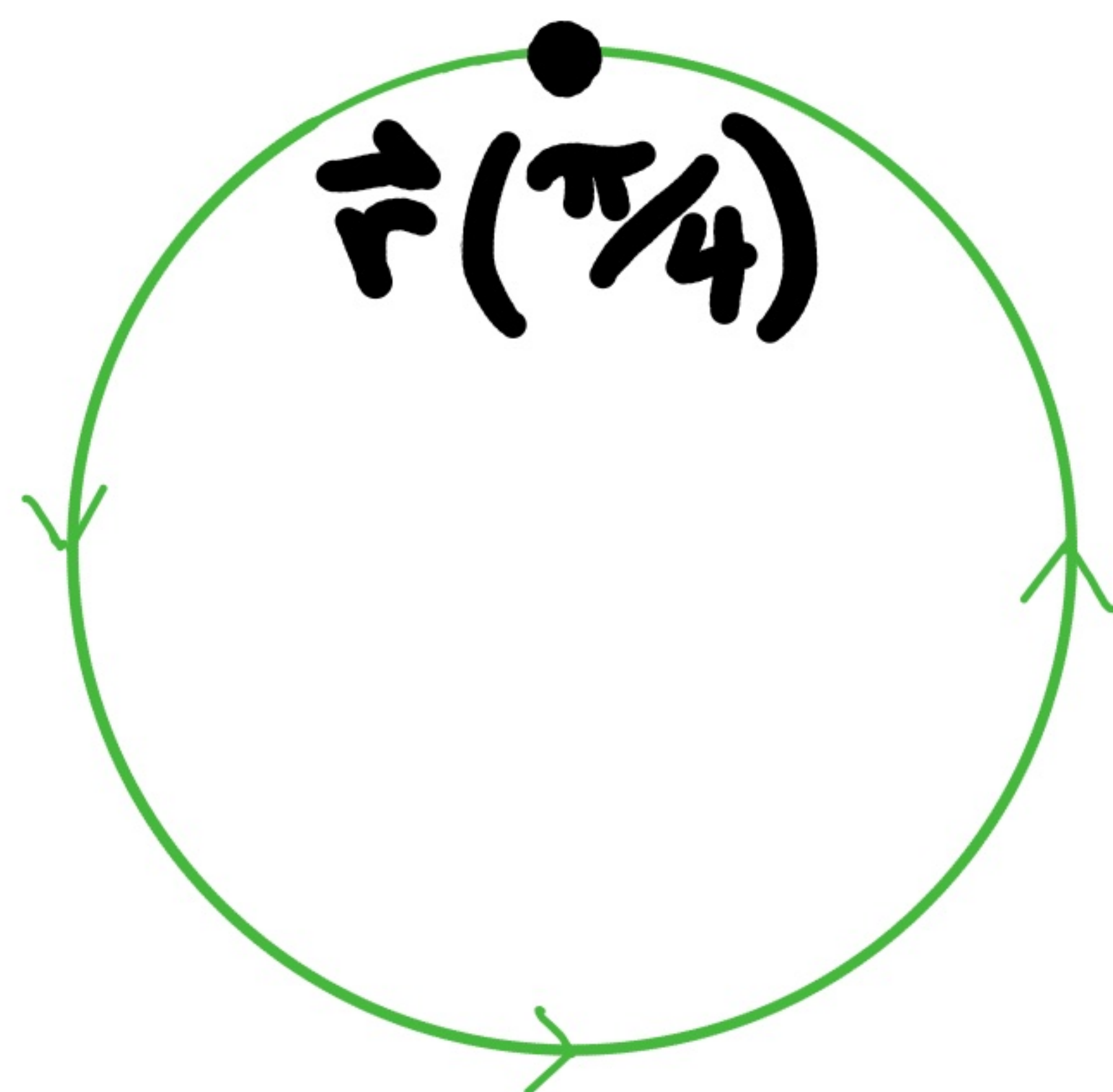
$$\vec{g}'\left(\frac{3\pi}{2}\right) =$$



$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$

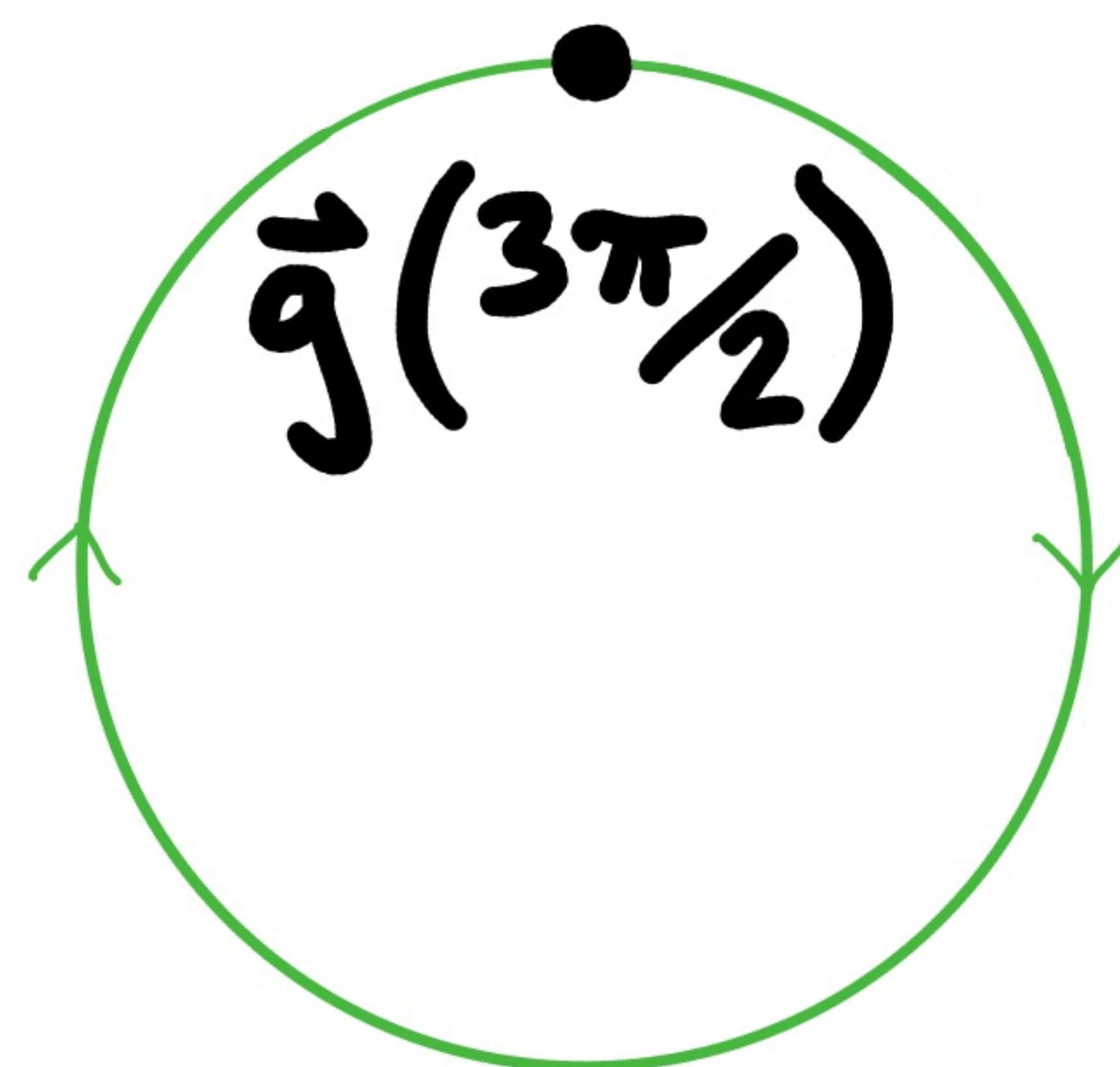
$$\vec{f}'(\pi/2) = \langle -1, 0 \rangle$$



$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) = \langle -2\sin 2t, 2\cos 2t \rangle$$

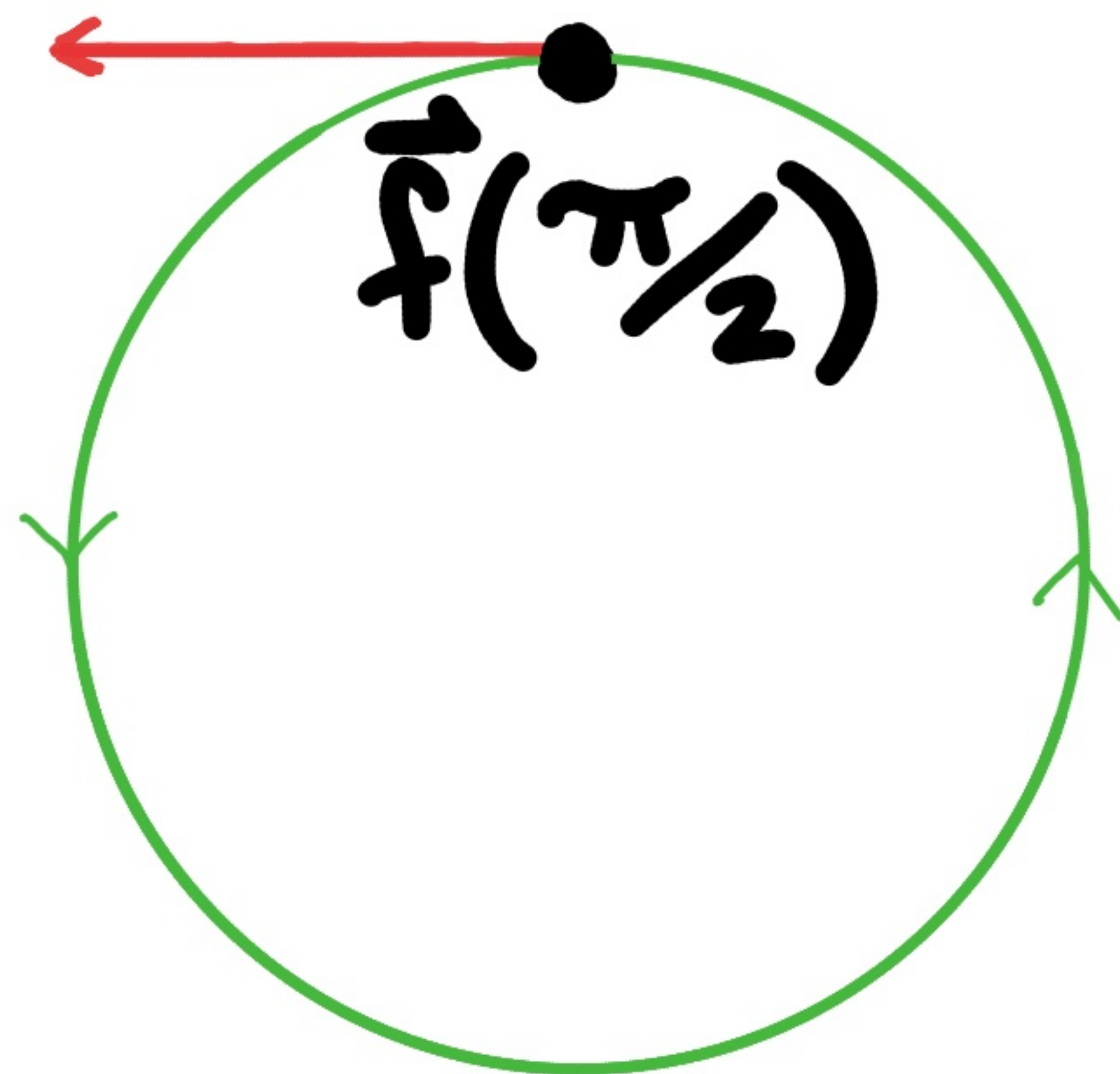
$$\vec{r}'(\pi/4) = \langle -2, 0 \rangle$$



$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

$$\vec{g}'(t) = \langle \sin(-t), -\cos(-t) \rangle$$

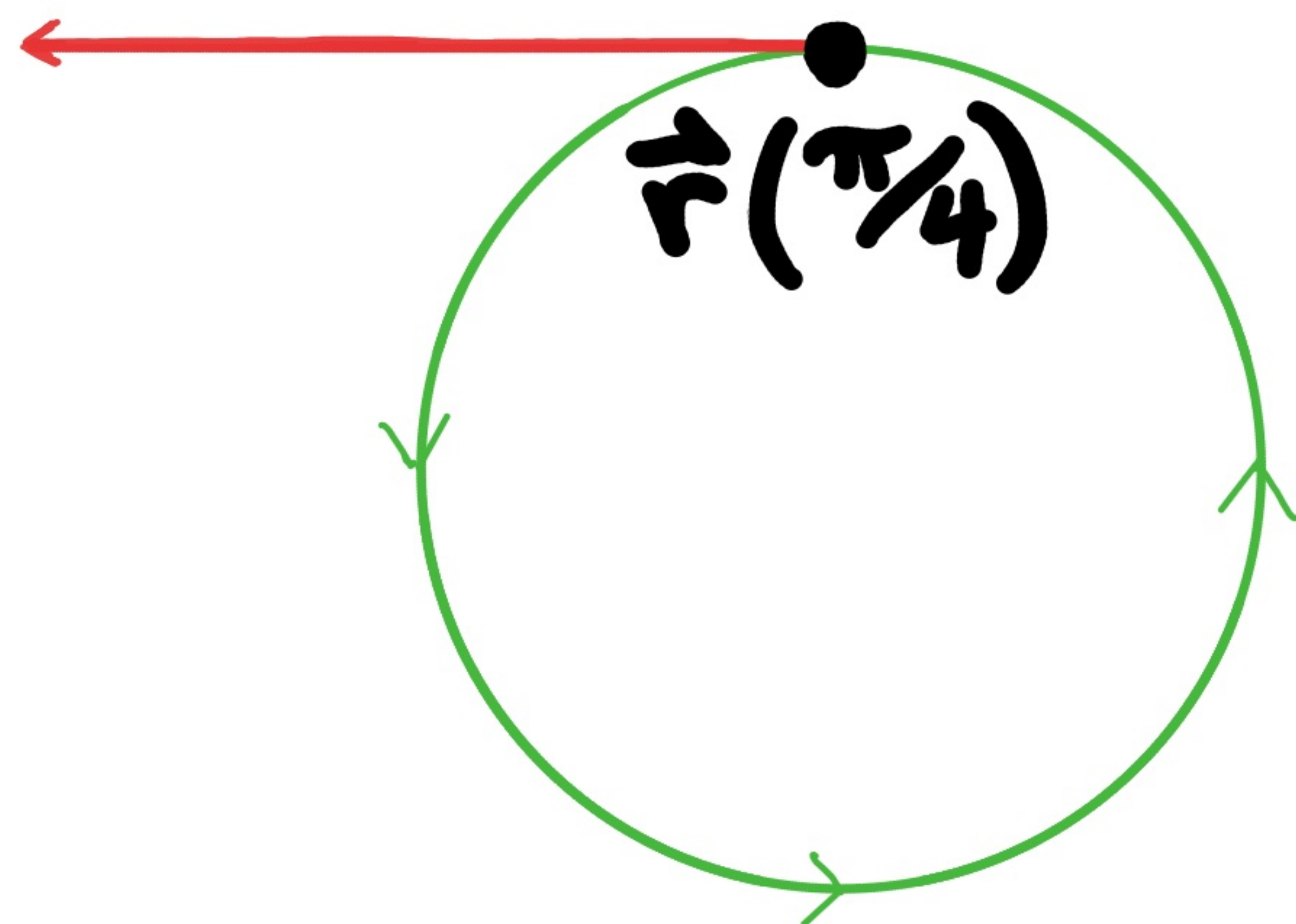
$$\vec{g}'(\frac{3\pi}{2}) = \langle 1, 0 \rangle$$



$$\vec{f}(t) = \langle \cos t, \sin t \rangle$$

$$\vec{f}'(t) = \langle -\sin t, \cos t \rangle$$

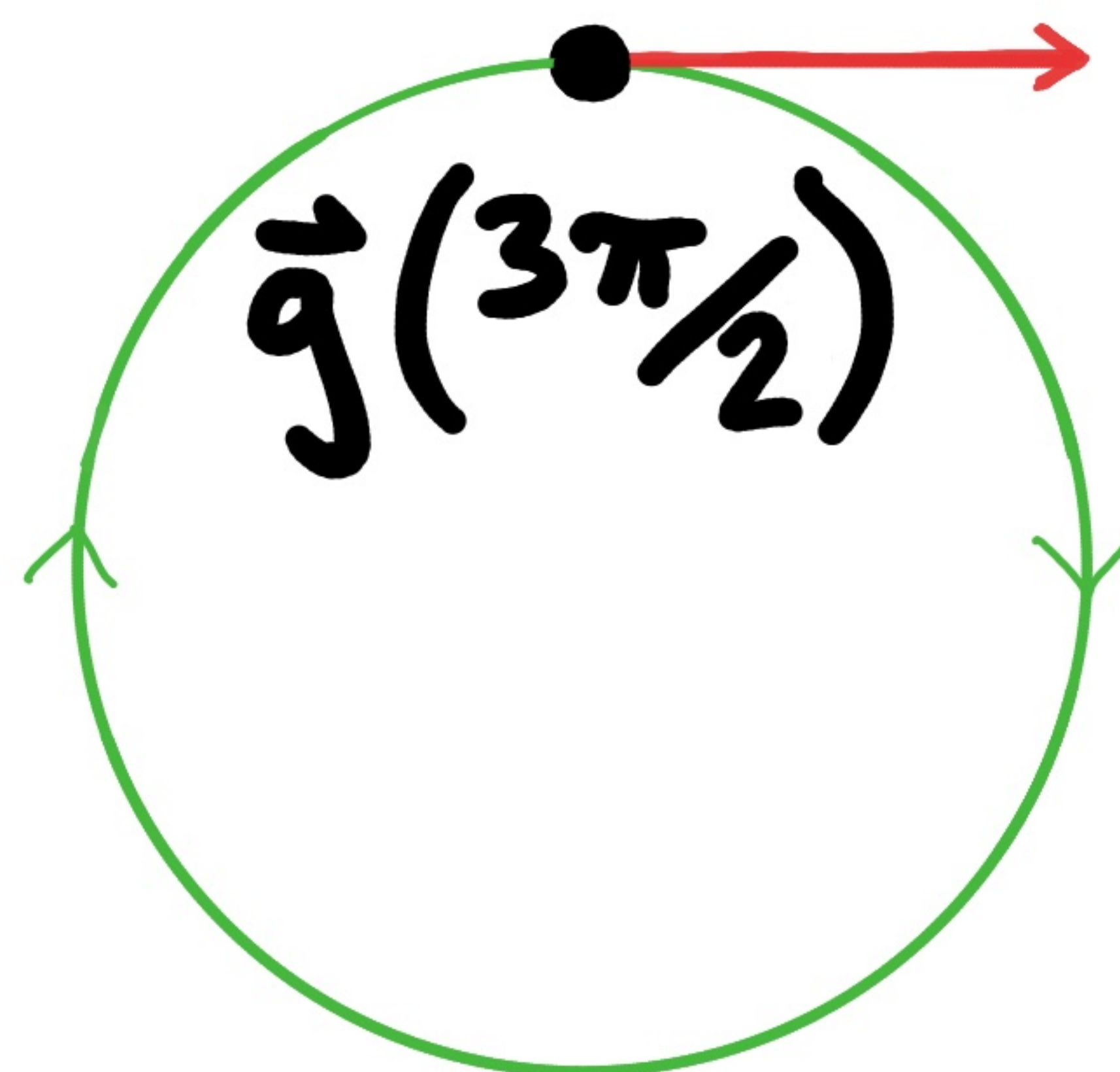
$$\vec{f}'(\pi/2) = \langle -1, 0 \rangle$$



$$\vec{r}(t) = \langle \cos 2t, \sin 2t \rangle$$

$$\vec{r}'(t) = \langle -2\sin 2t, 2\cos 2t \rangle$$

$$\vec{r}'(\pi/4) = \langle -2, 0 \rangle$$



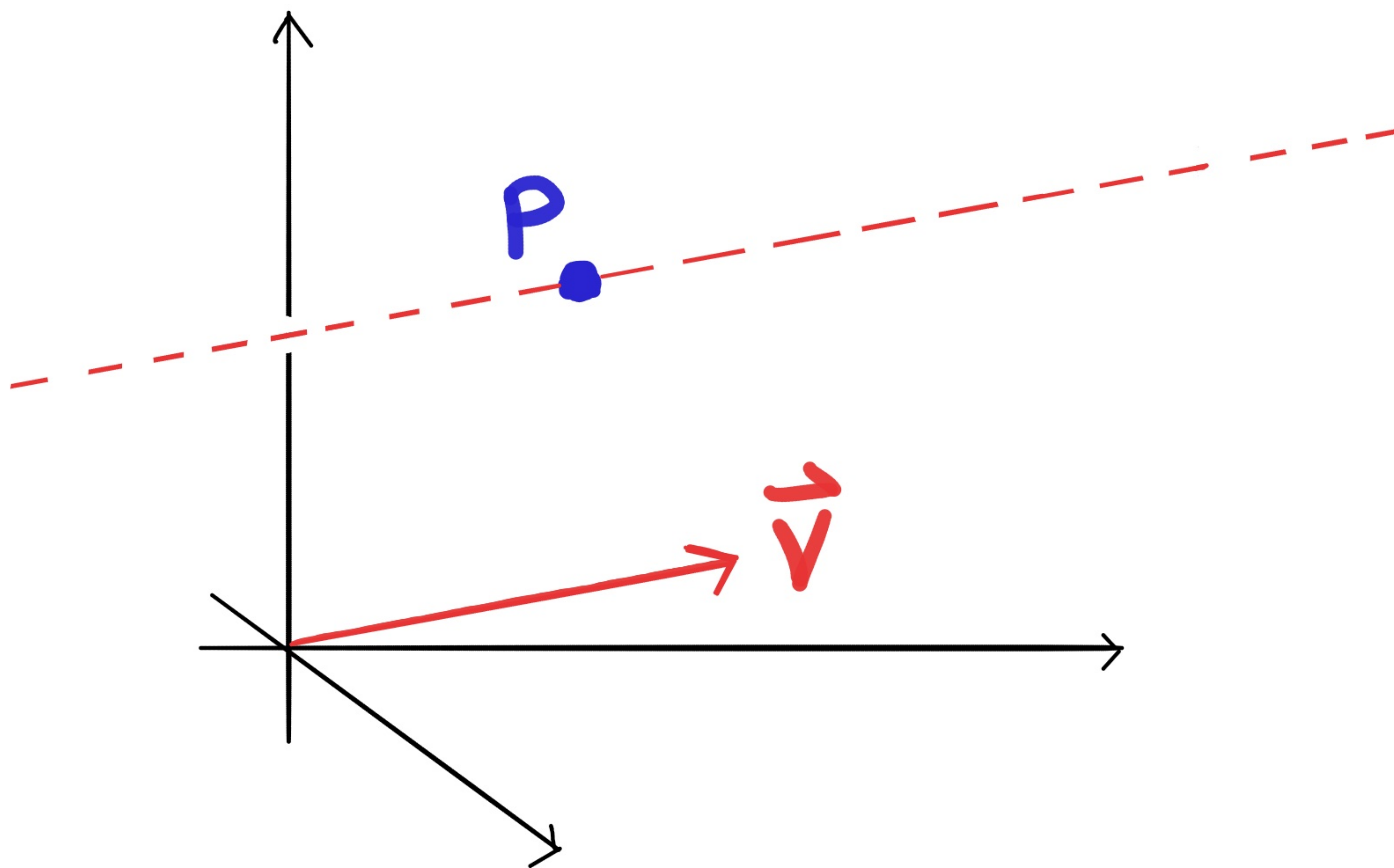
$$\vec{g}(t) = \langle \cos(-t), \sin(-t) \rangle$$

$$\vec{g}'(t) = \langle \sin(-t), -\cos(-t) \rangle$$

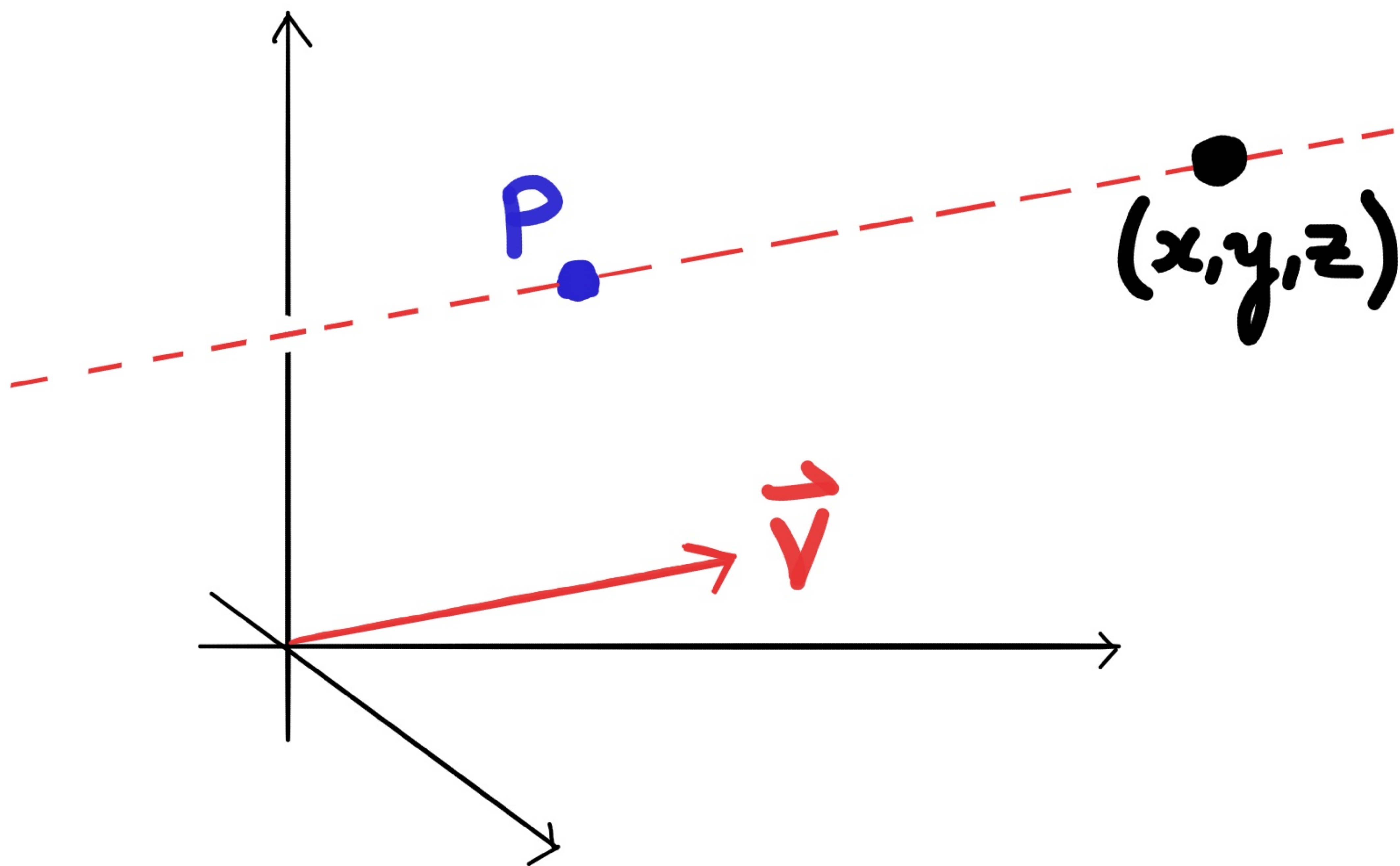
$$\vec{g}'(\frac{3\pi}{2}) = \langle 1, 0 \rangle$$

② Equation of
a tangent line

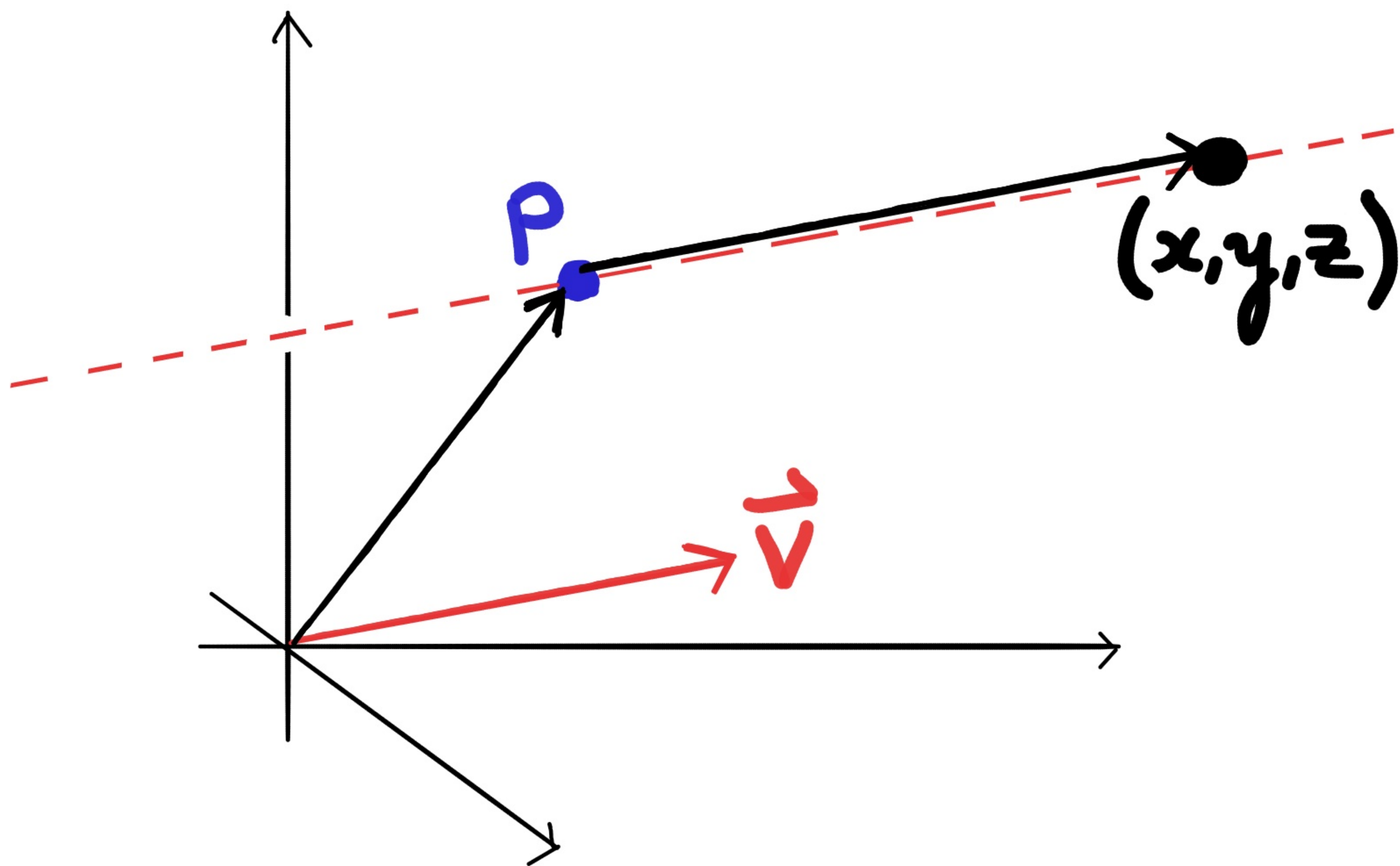
Line containing a point P ,
parallel to a vector $\vec{v}$



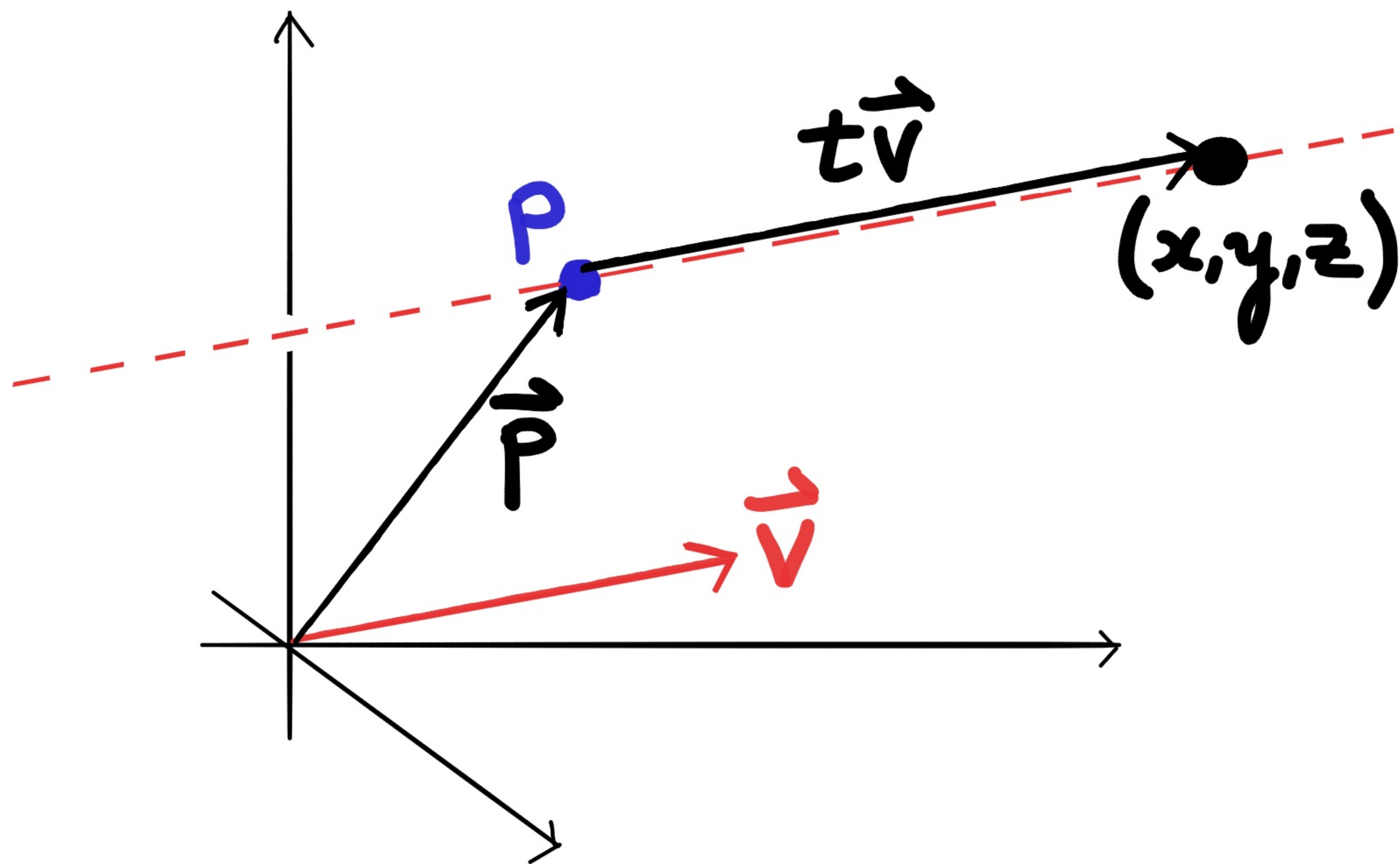
Line containing a point P ,
parallel to a vector $\vec{v}$



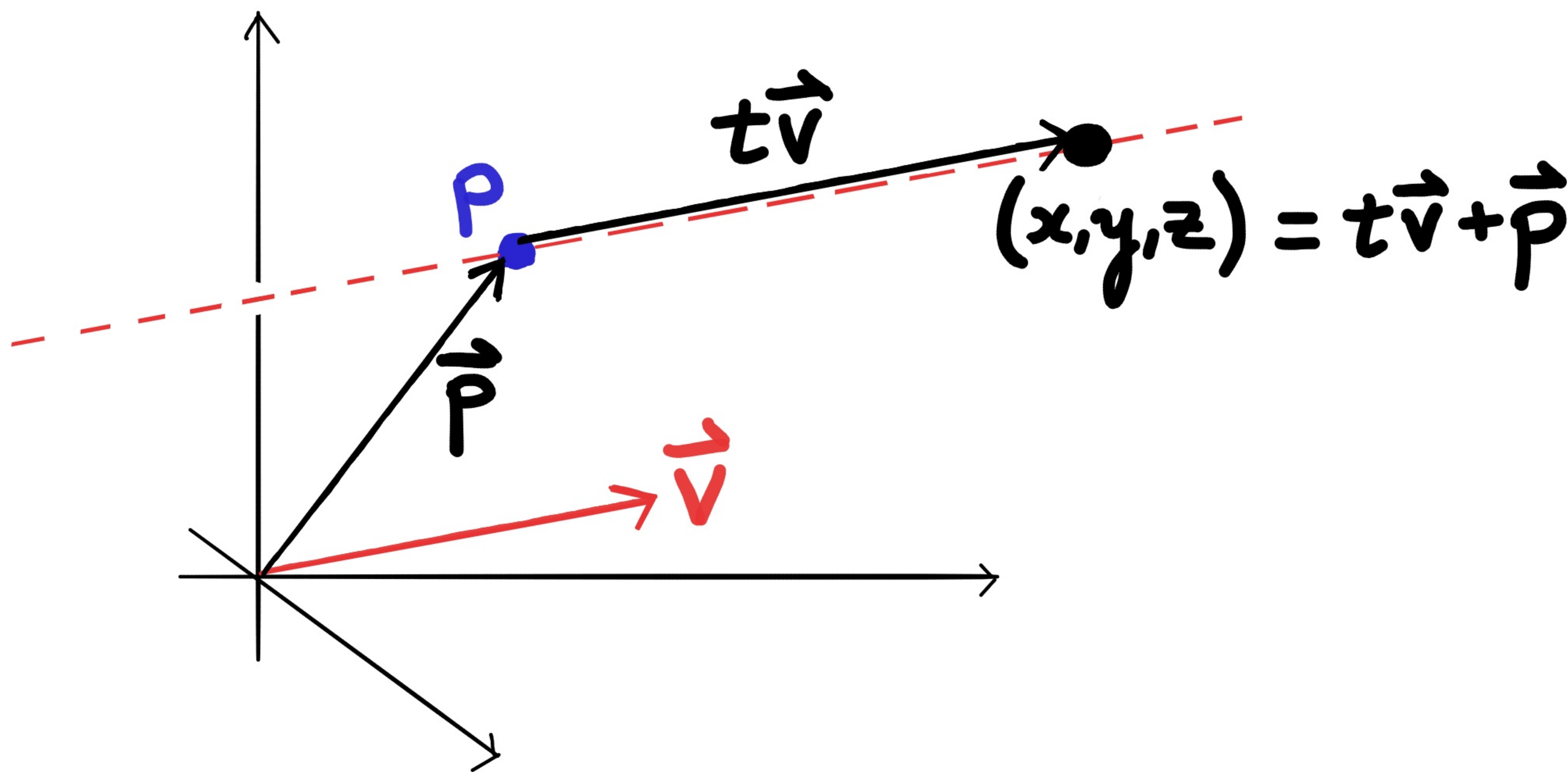
Line containing a point P ,
parallel to a vector $\vec{v}$



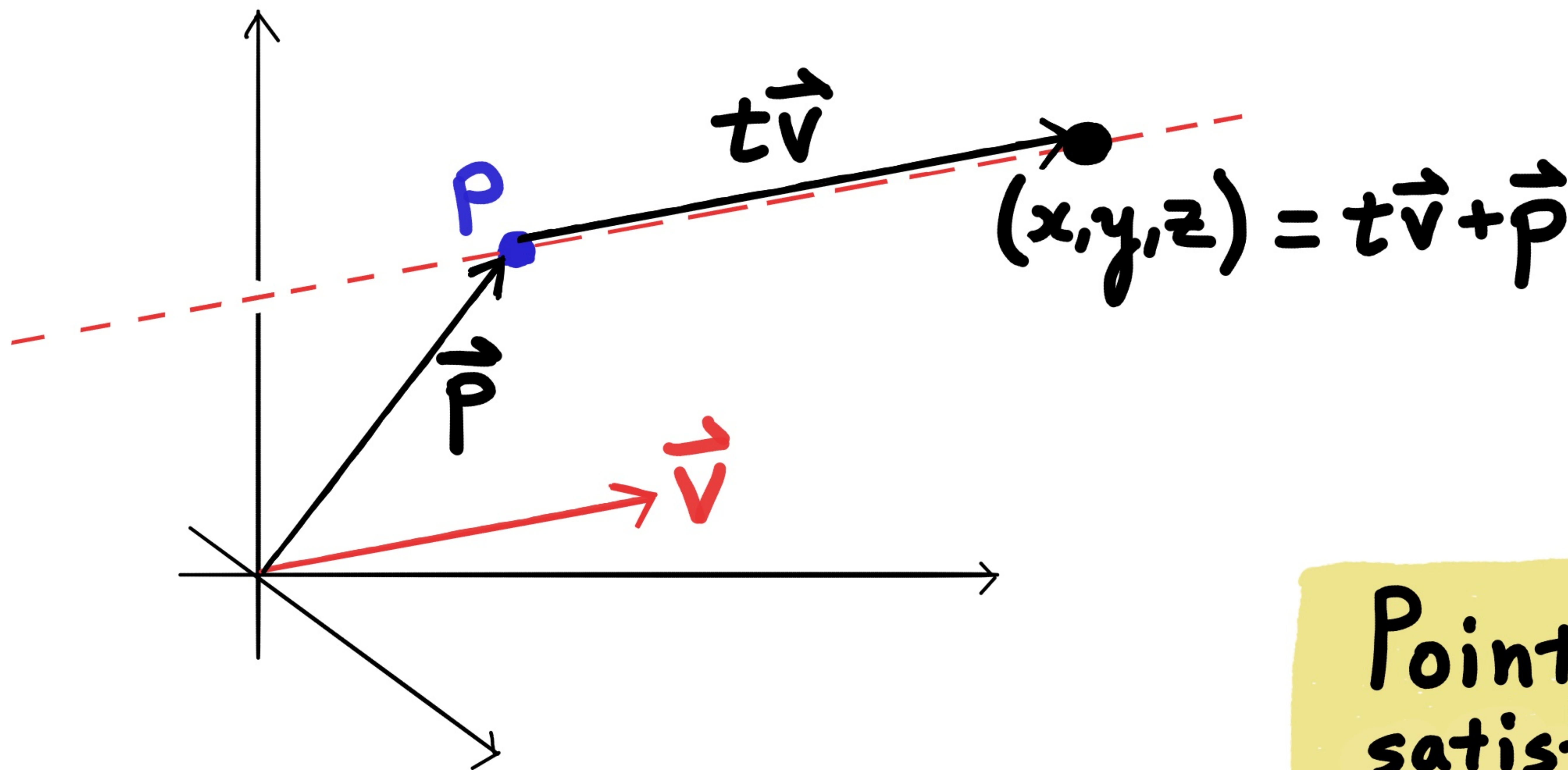
Line containing a point P ,
parallel to a vector $\vec{v}$



Line containing a point P ,
parallel to a vector $\vec{v}$



Line containing a point P ,
parallel to a vector $\vec{v}$

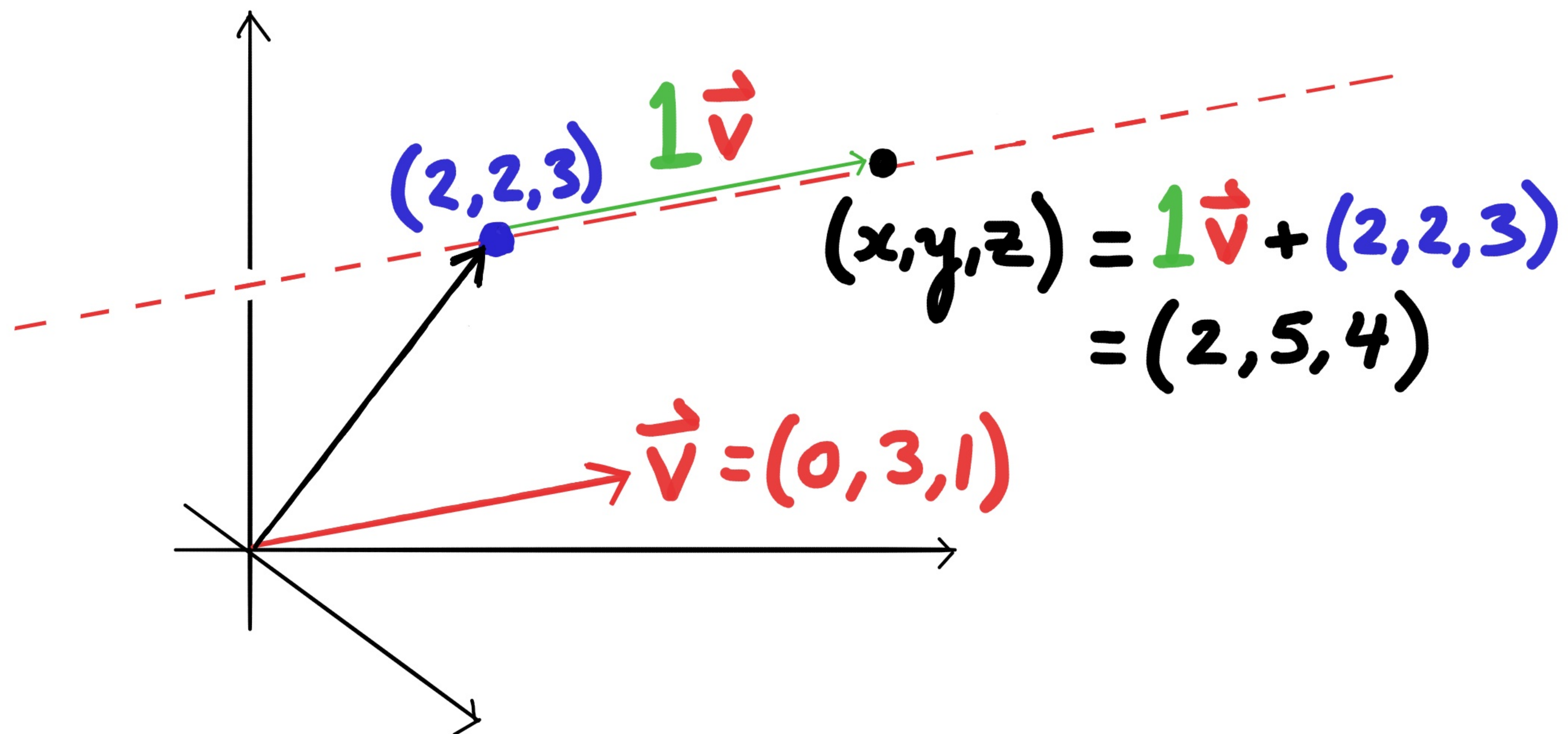


Points (x, y, z)
satisfying
 $(x, y, z) = t\vec{v} + \vec{p}$
for some $t \in \mathbb{R}$

Example:

The line of points (x, y, z) satisfying $(x, y, z) = t\vec{v} + p$ for some $t \in \mathbb{R}$.

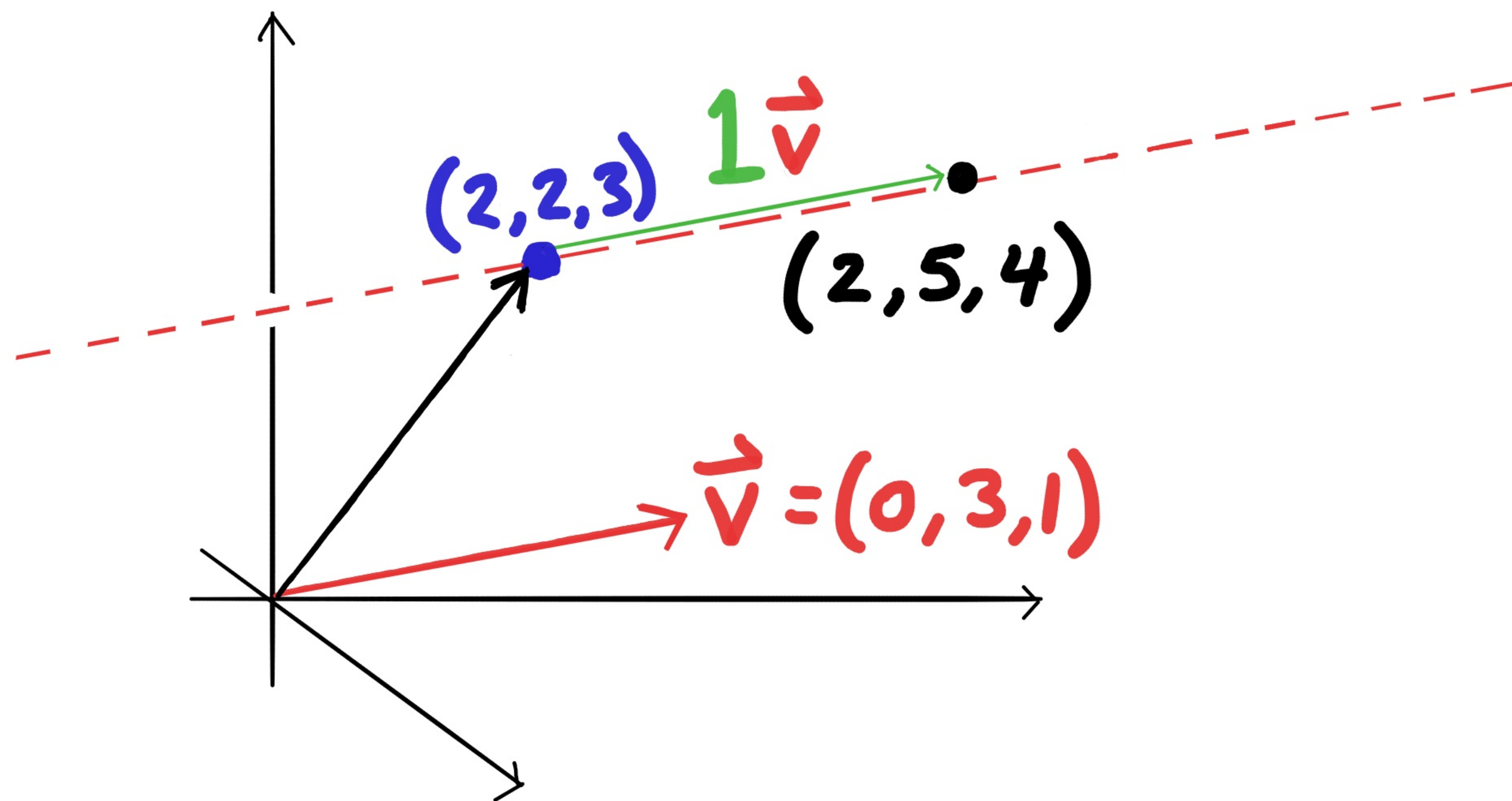
$$t=1$$



Example:

$t=1$

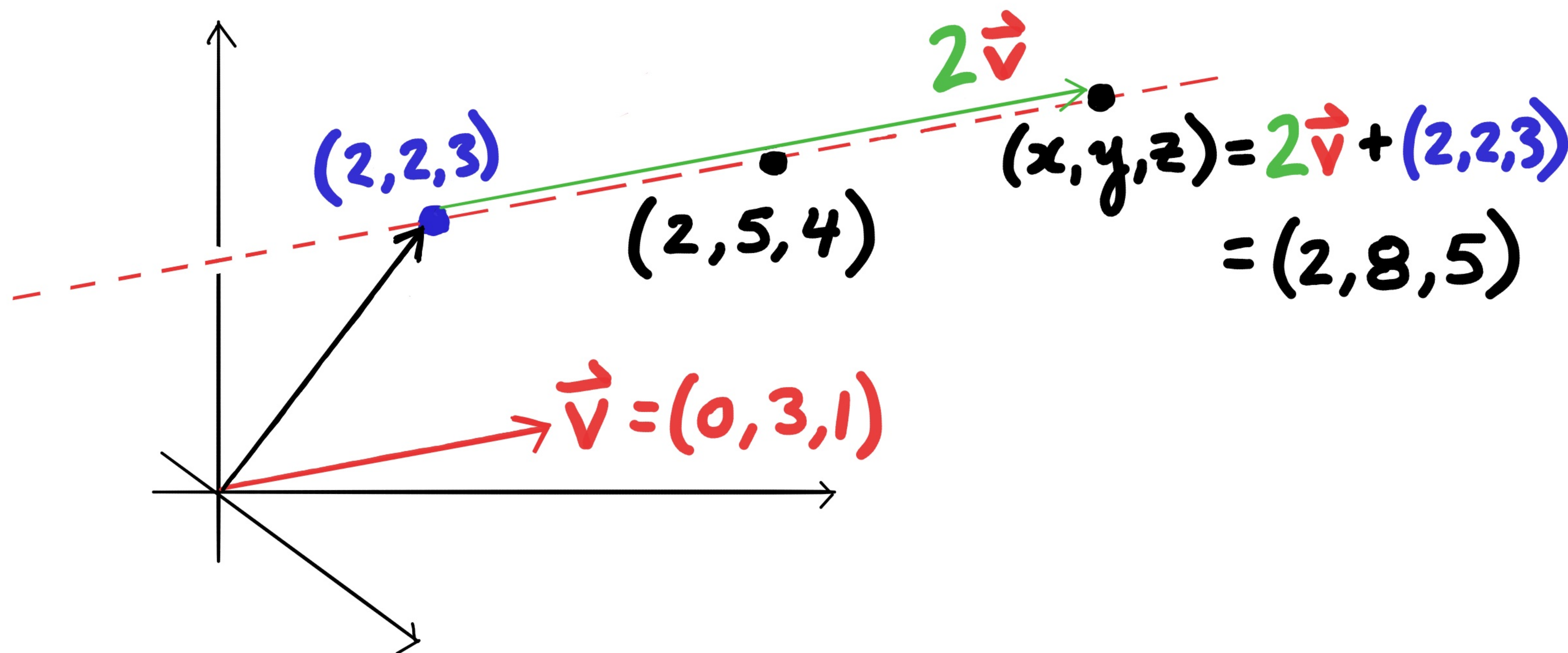
The line of points (x, y, z)
satisfying $(x, y, z) = t\vec{v} + p$
for some $t \in \mathbb{R}$.



Example:

The line of points (x, y, z) satisfying $(x, y, z) = t\vec{v} + p$ for some $t \in \mathbb{R}$.

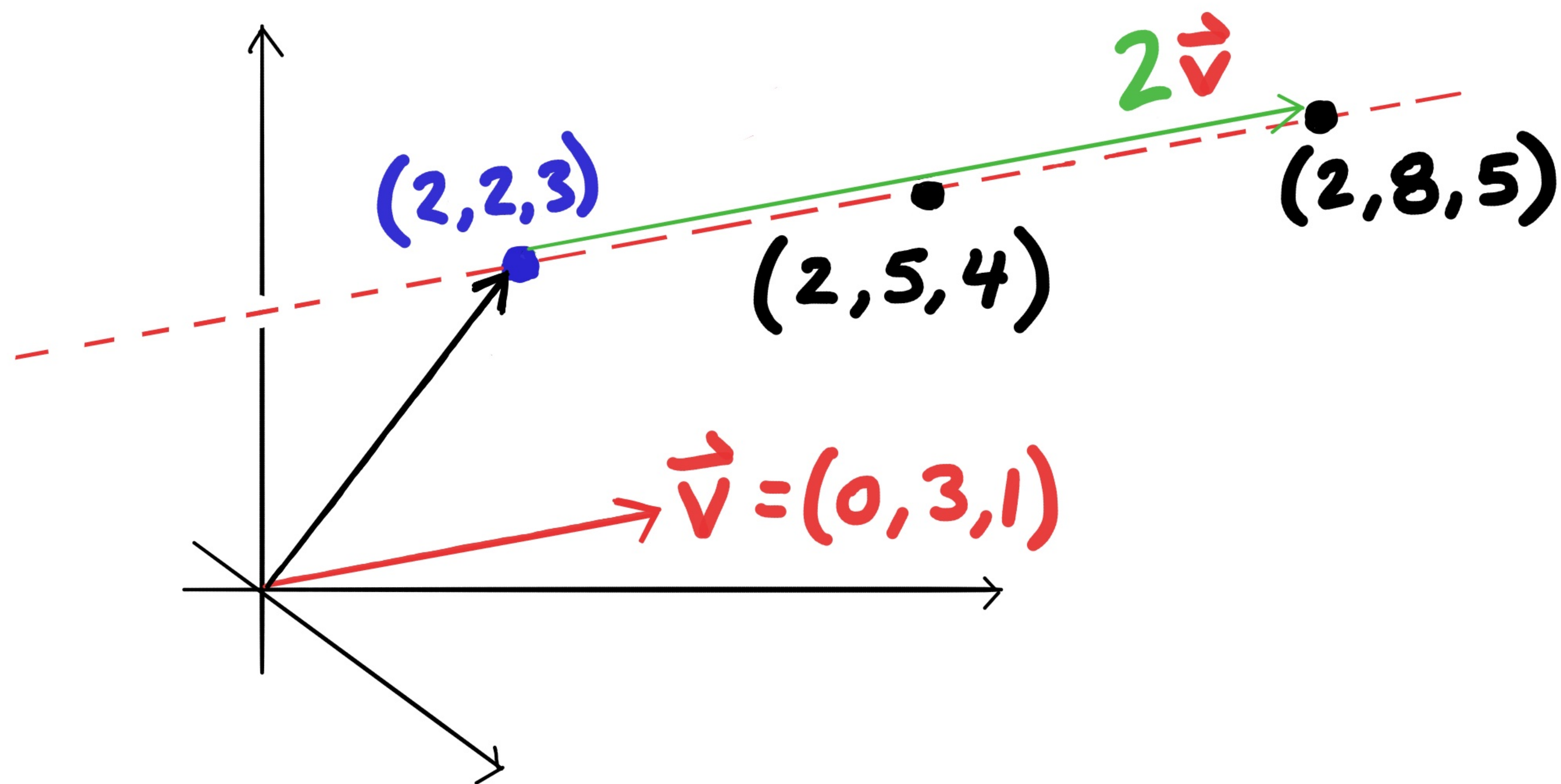
$$t = 2$$



Example:

The line of points (x, y, z)
satisfying $(x, y, z) = t\vec{v} + p$
for some $t \in \mathbb{R}$.

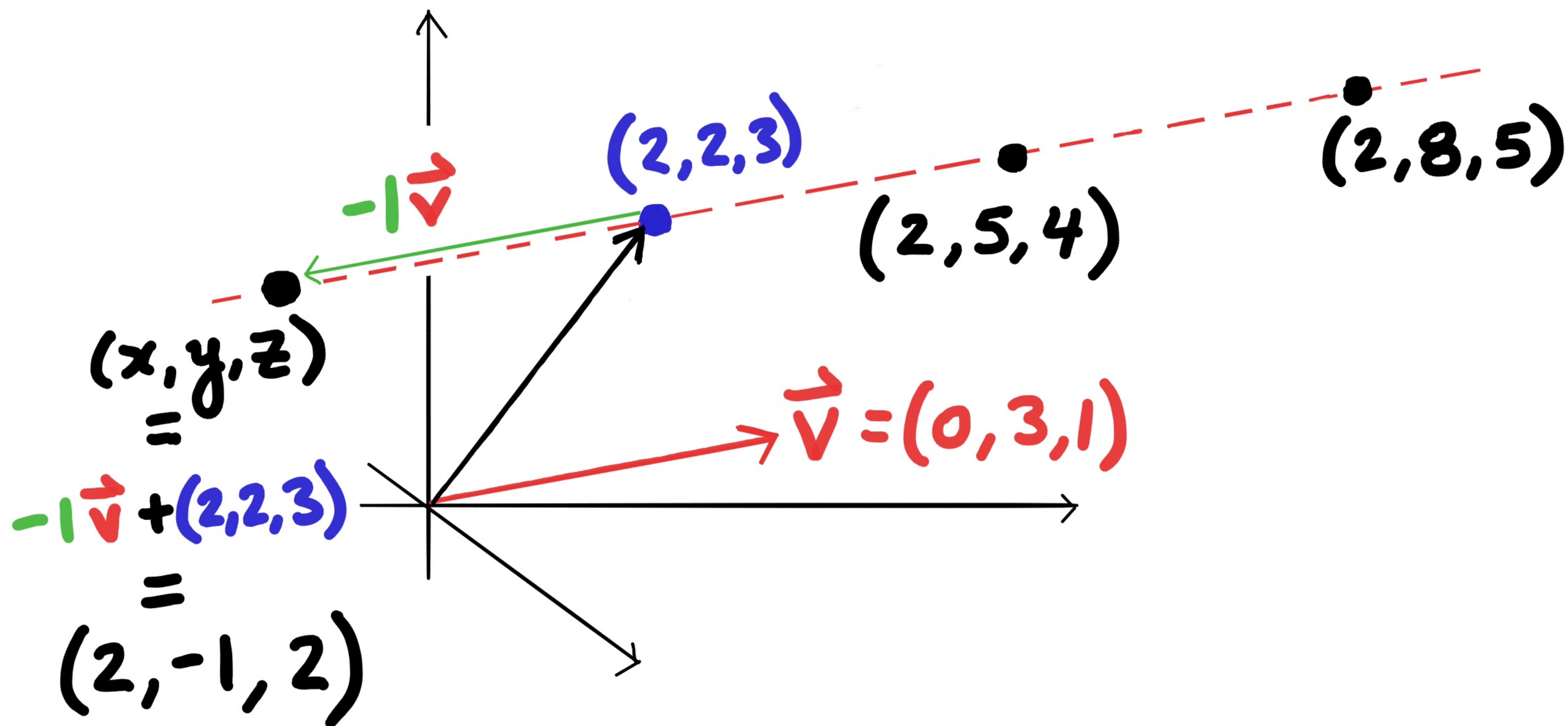
$$t=2$$



Example:

$$t = -1$$

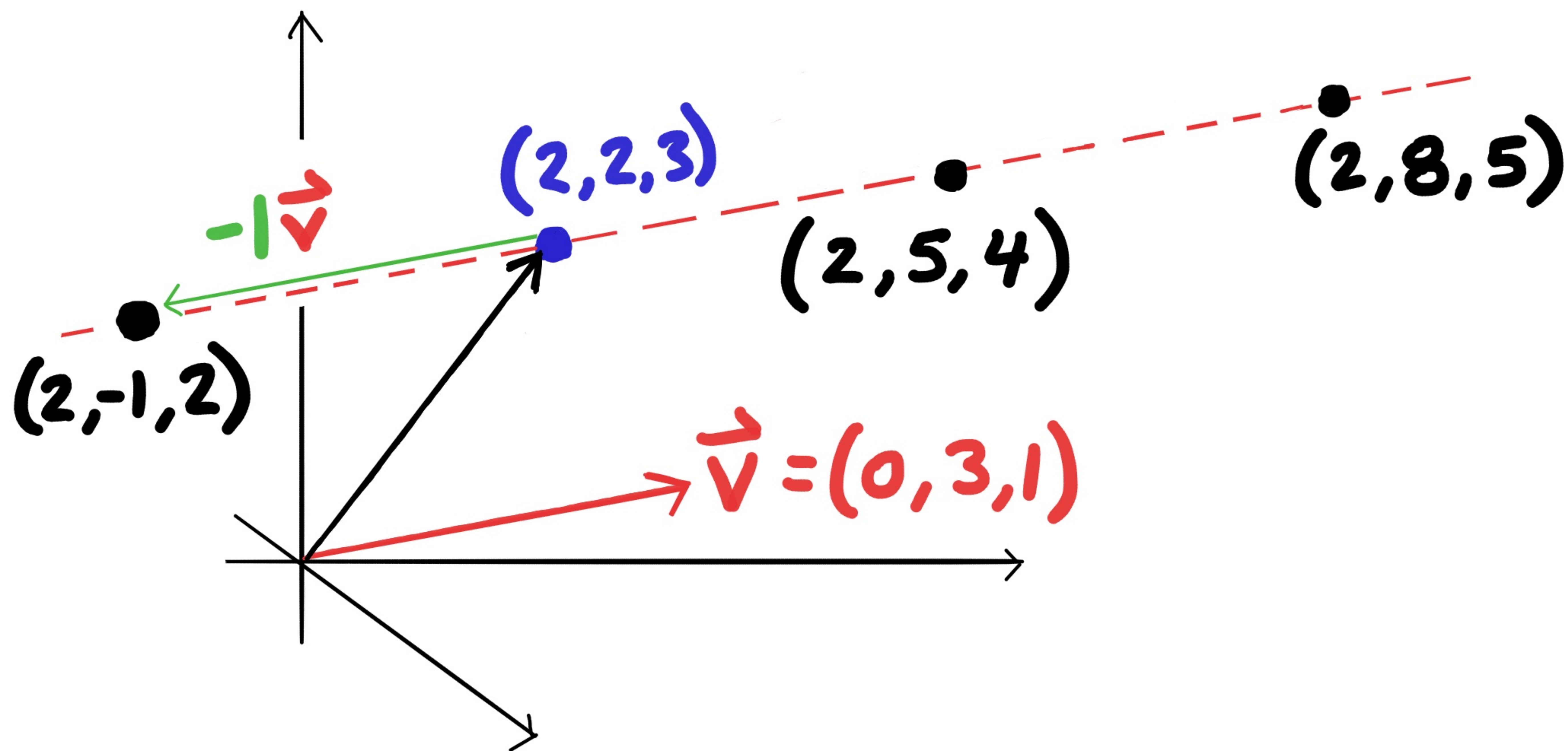
The line of points (x, y, z) satisfying $(x, y, z) = t\vec{v} + p$ for some $t \in \mathbb{R}$.



Example:

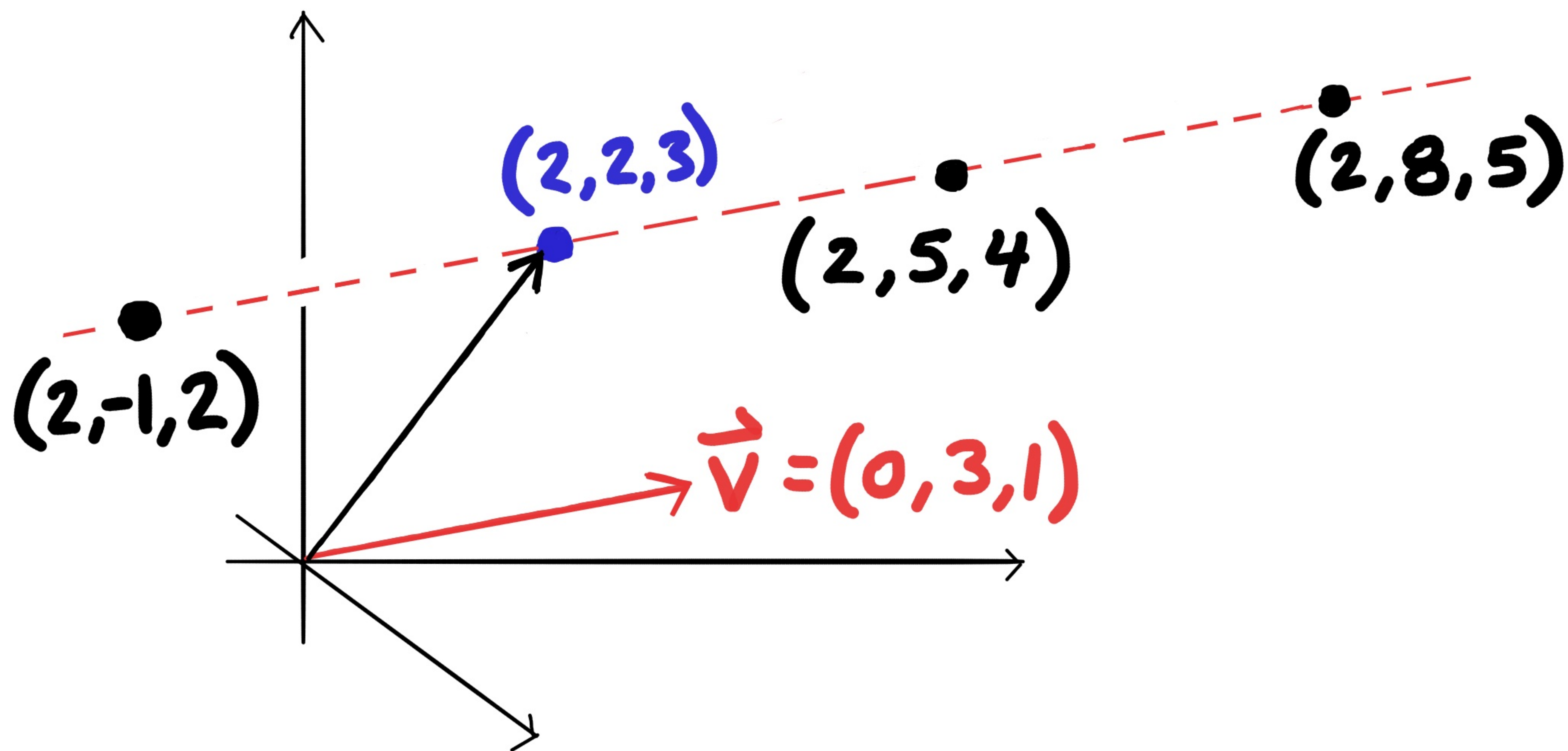
The line of points (x, y, z) satisfying $(x, y, z) = t\vec{v} + p$ for some $t \in \mathbb{R}$.

$$t = -1$$

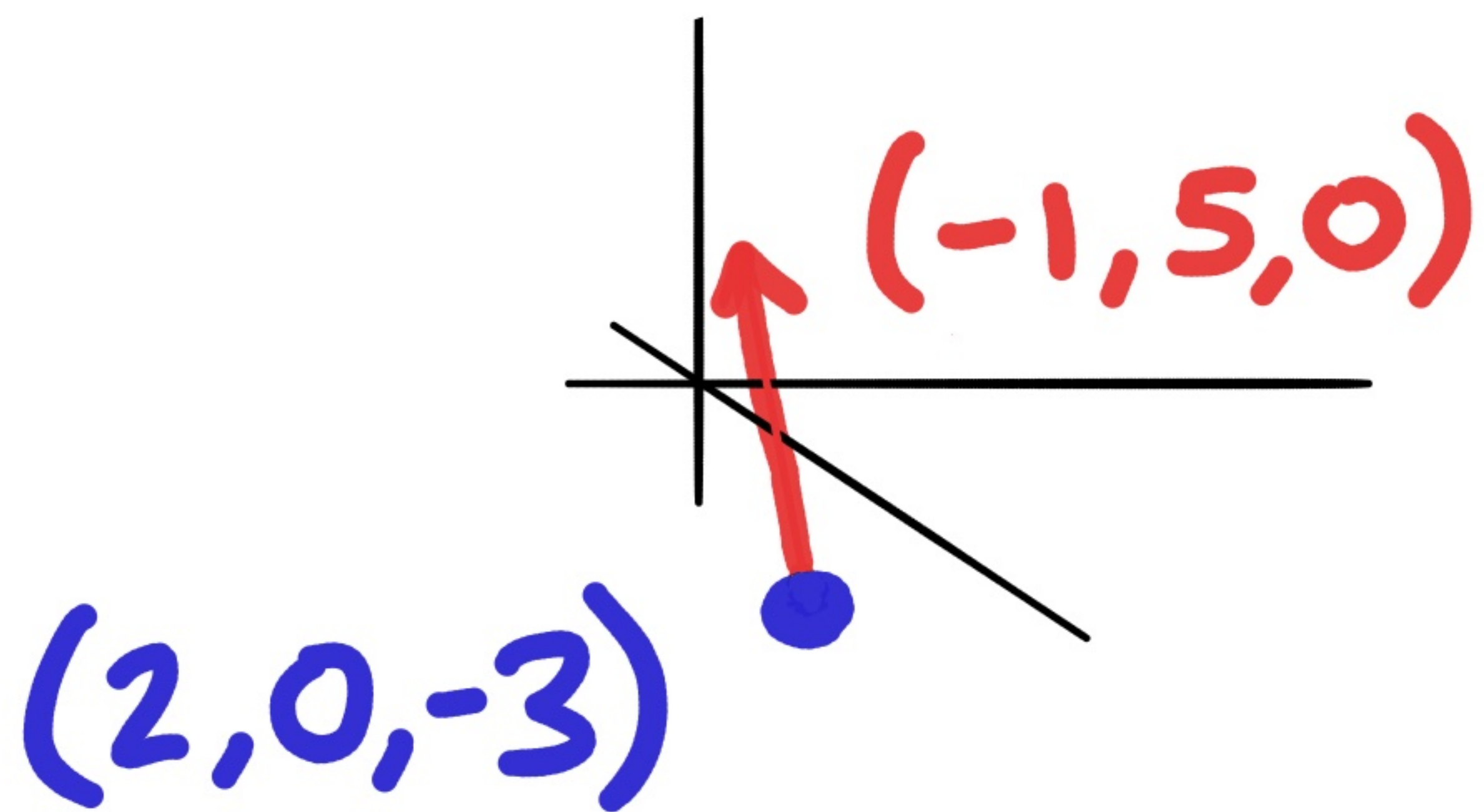


Example:

The line of points (x, y, z)
satisfying $(x, y, z) = t\vec{v} + p$
for some $t \in \mathbb{R}$.



Problem: Give the equation of the line containing $(2, 0, -3)$ and parallel to $(-1, 5, 0)$.

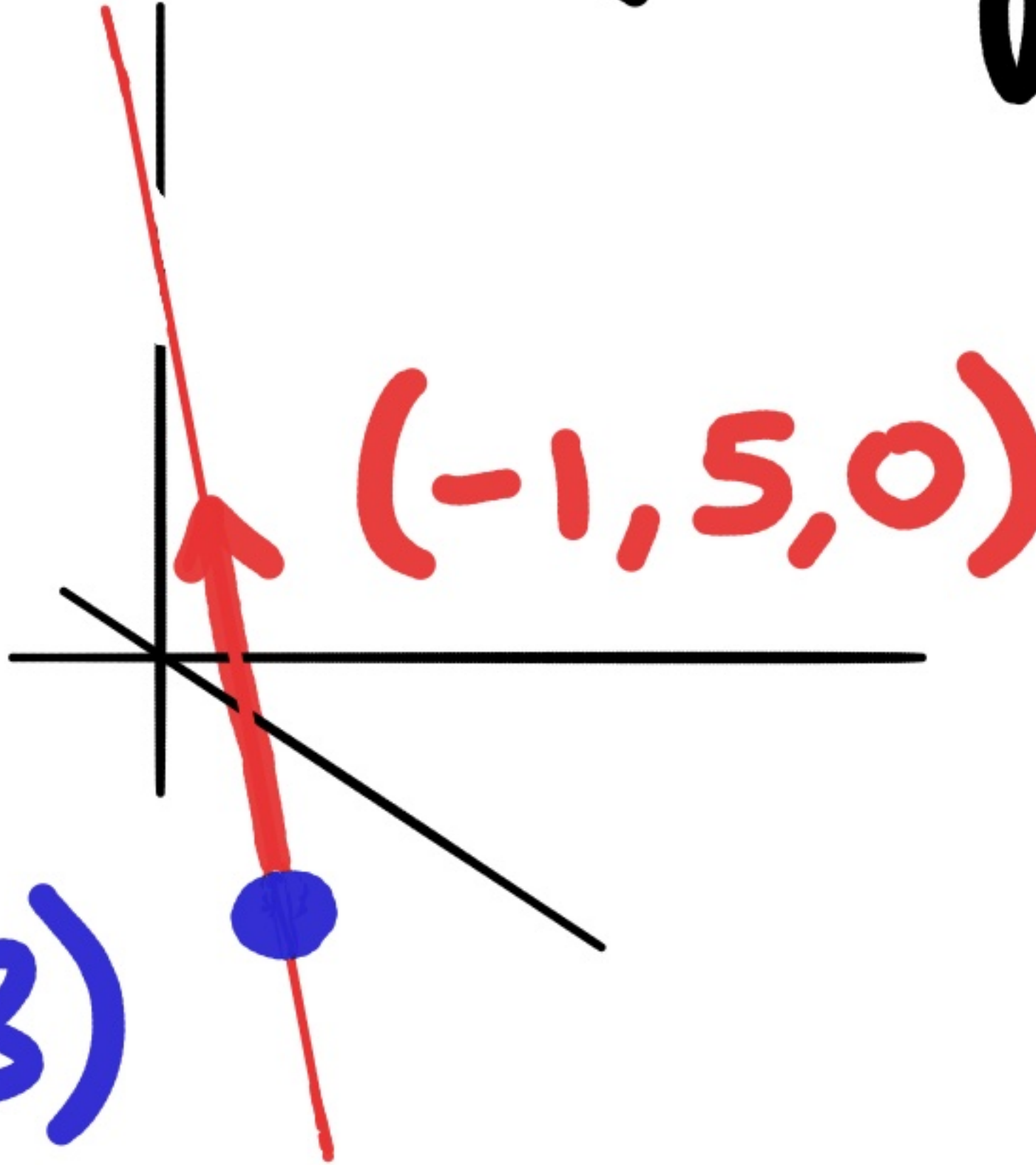


Problem: Give the equation of the line containing $(2, 0, -3)$ and parallel to $(-1, 5, 0)$.

Answer: $(x, y, z) = t(-1, 5, 0) + (2, 0, -3)$

$= (-t, 5t, 0) + (2, 0, -3)$

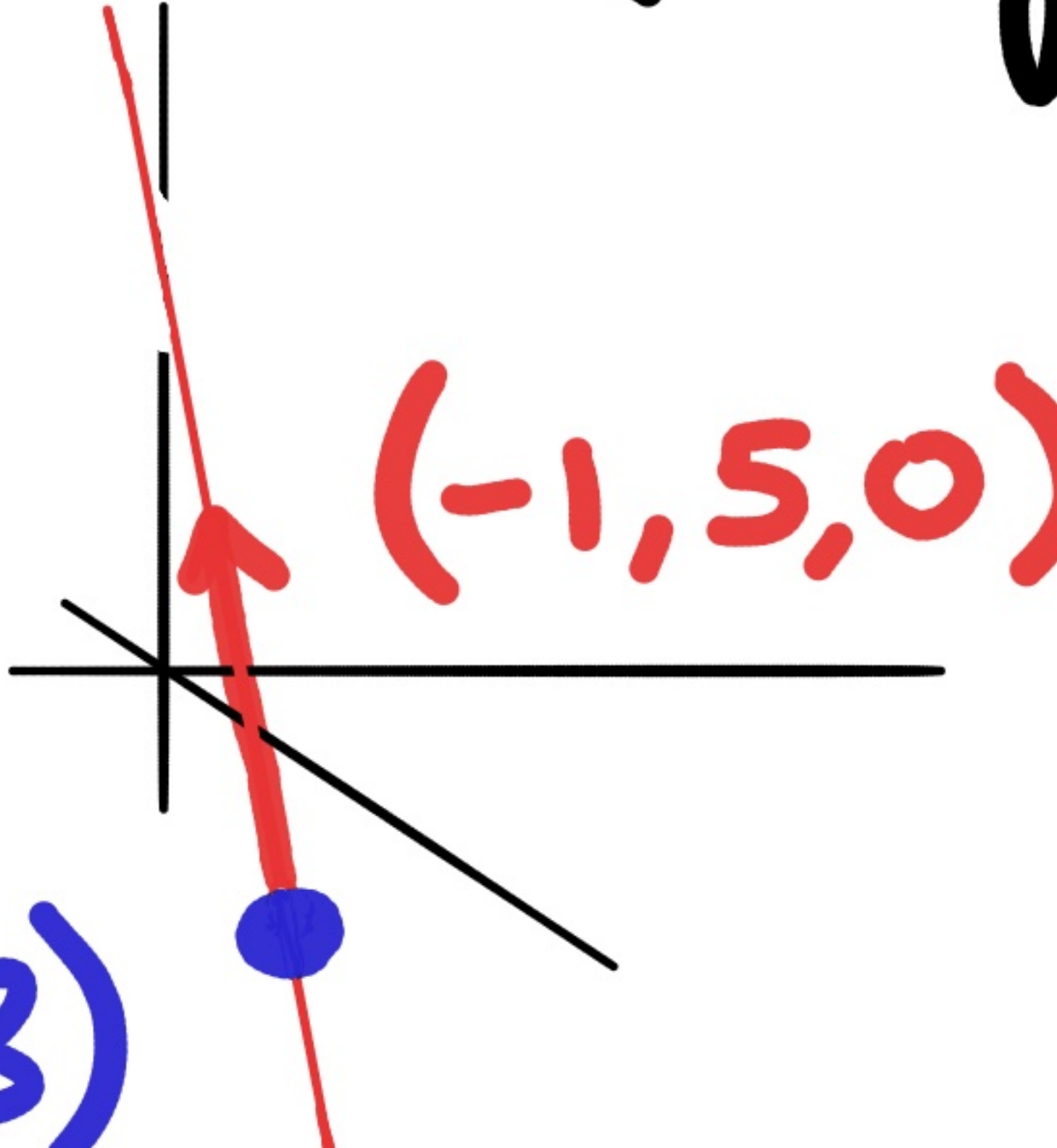
$= (-t + 2, 5t, -3)$



The diagram illustrates a 3D coordinate system with three axes. A red line passes through a blue point labeled $(2, 0, -3)$. A red arrow, representing a direction vector, starts at the origin and points towards the upper right, labeled $(-1, 5, 0)$. The line is parallel to this vector.

Problem: Give the equation of the line containing $(2, 0, -3)$ and parallel to $(-1, 5, 0)$.

Answer: $(x, y, z) = t(-1, 5, 0) + (2, 0, -3)$

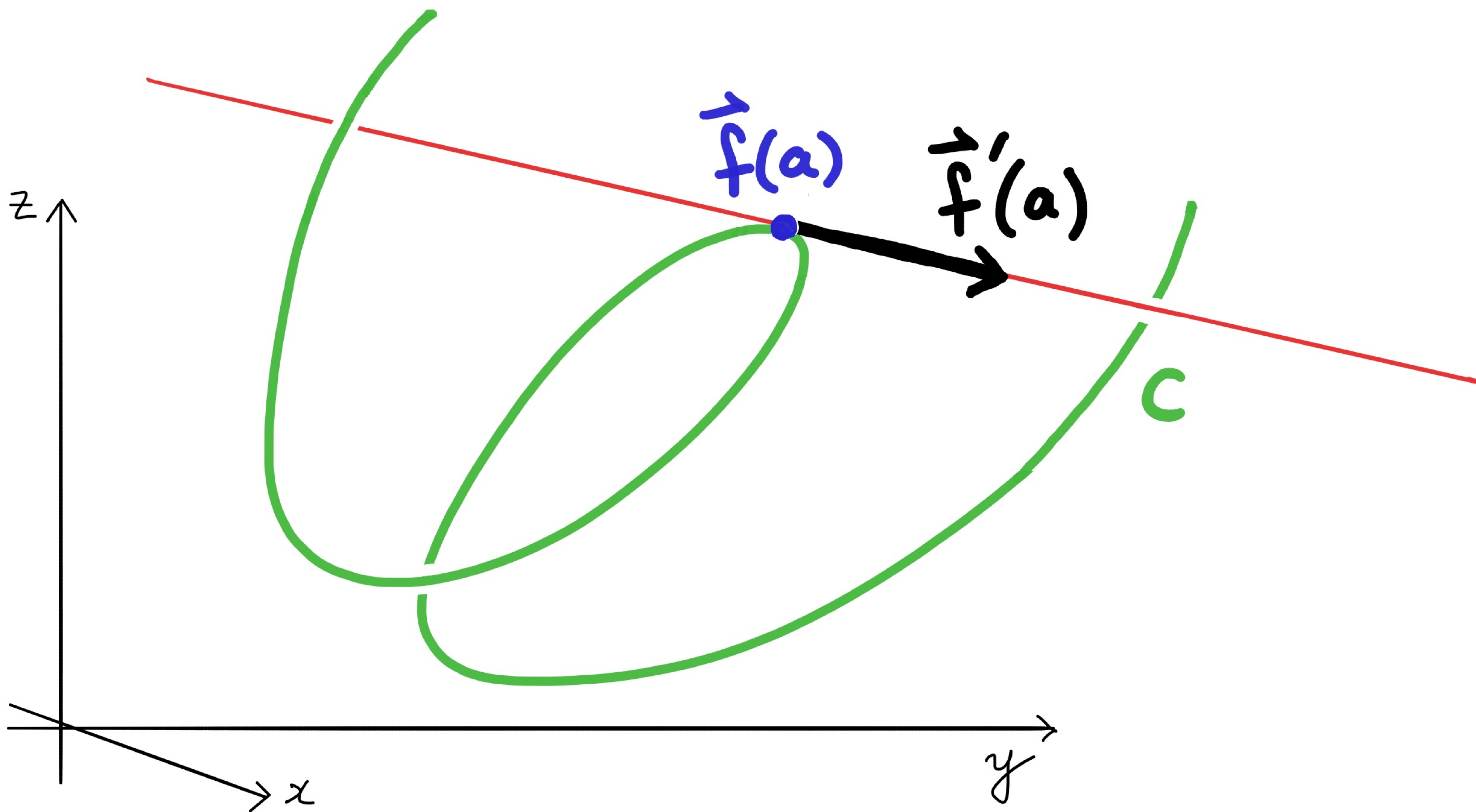


$= (-t, 5t, 0) + (2, 0, -3)$

$= (-t + 2, 5t, -3)$

Alternatively: $x = -t + 2$, $y = 5t$, $z = -3$

Equation of tangent line } $(x, y, z) = t\vec{f}'(a) + \vec{f}(a)$



Problem: What's the equation of the tangent line of the helix $\vec{h}(t) = \langle \cos t, \sin t, t \rangle$ at $\vec{h}(2\pi) = \langle 1, 0, 2\pi \rangle$?

Problem: What's the equation of the tangent line of the helix $\vec{h}(t) = \langle \cos t, \sin t, t \rangle$ at $\vec{h}(2\pi) = \langle 1, 0, 2\pi \rangle$?

Solution:

$$\vec{h}'(t) = \langle -\sin t, \cos t, 1 \rangle$$

$$\vec{h}'(2\pi) = \langle 0, 1, 1 \rangle$$

$$\begin{aligned} (x, y, z) &= t \langle 0, 1, 1 \rangle + \langle 1, 0, 2\pi \rangle \\ &= (1, t, t + 2\pi) \end{aligned}$$

