Twenty-seven

Ⅰ Cartesian coordinates for $\mathbb{R}^n$

Ⅱ Vector fields

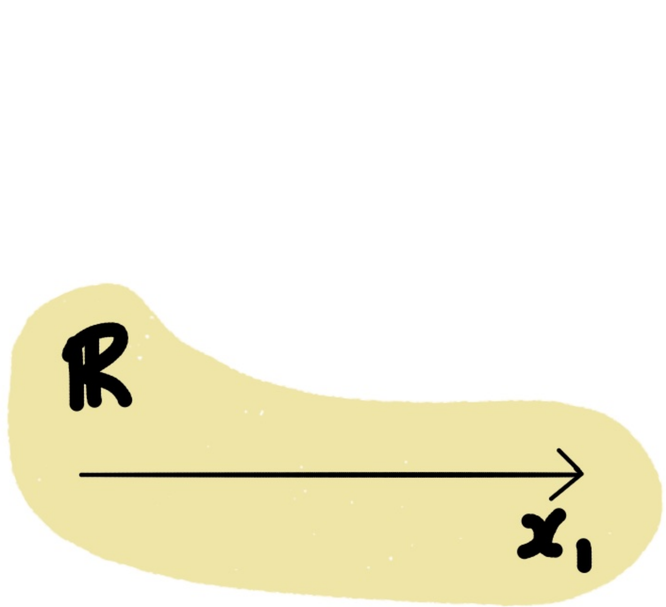
Ⅲ Conservative vector fields

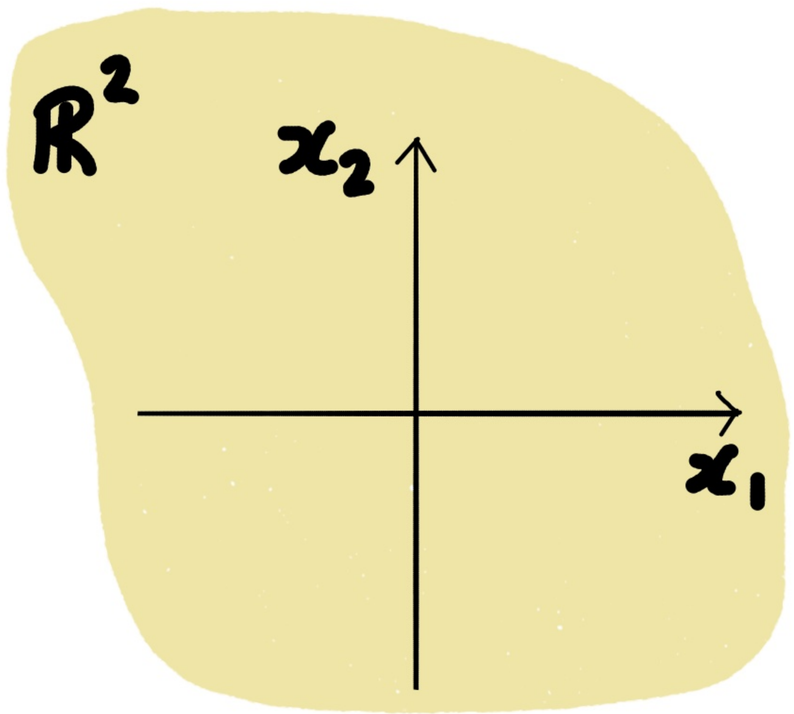
Ⅳ Test for being conservative

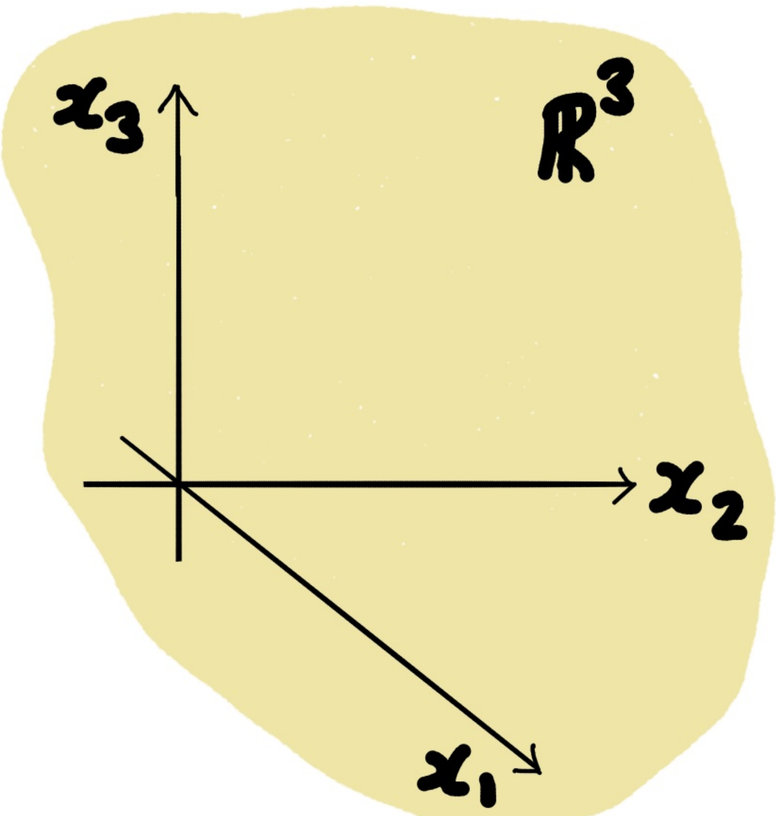
Ⅴ Divergence

① Cartesian coordinates for $\mathbb{R}^n$

Cartesian coordinates for $\mathbb{R}^n$ are $x_1, x_2, x_3, \ldots, x_n$.







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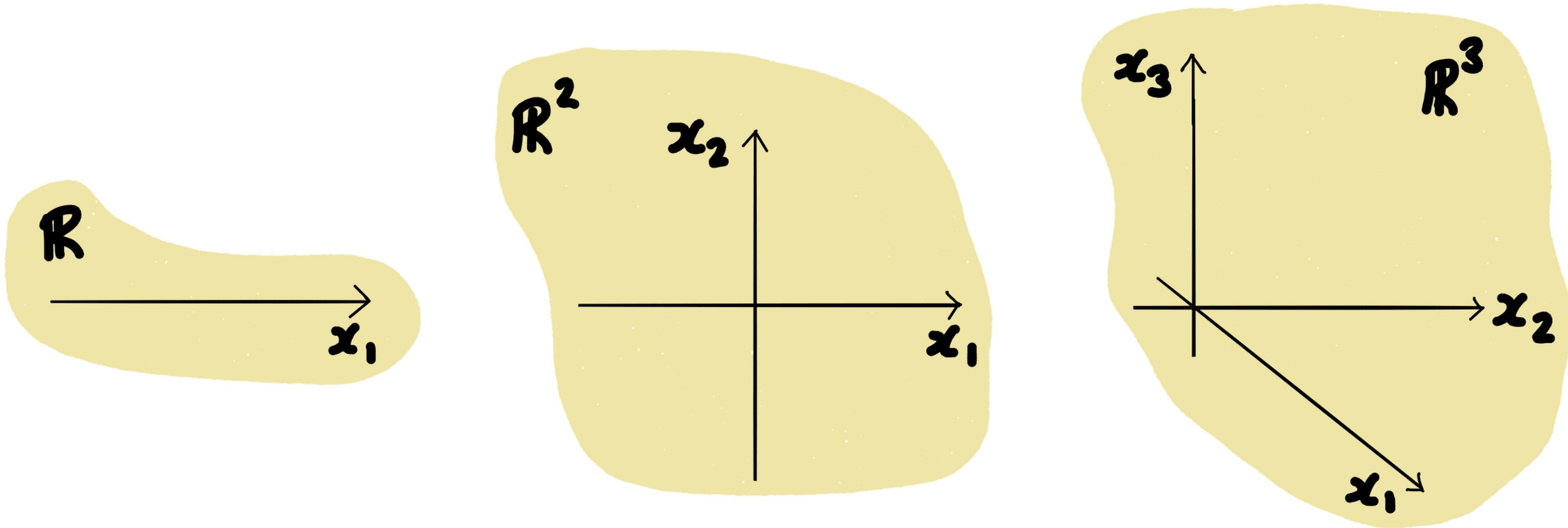

$h: \mathbb{R} \to \mathbb{R}$, $h(x_1) = 2x_1 + 3$

$f: \mathbb{R}^2 \to \mathbb{R}$, $f(x_1, x_2) = 3x_1^2 - x_1 x_2$

$g: \mathbb{R}^3 \to \mathbb{R}$, $g(x_1, x_2, x_3) = x_1 + x_2 + x_3$

Example:

$$f(x_1, x_2, x_3) = 2x_1x_2 - x_3$$

Find

$$\frac{\partial f}{\partial x_1}$$

$$\frac{\partial f}{\partial x_2}$$

$$\frac{\partial f}{\partial x_3}$$

Example:

$$f(x_1, x_2, x_3) = 2x_1x_2 - x_3$$

Find

$$\frac{\partial f}{\partial x_1} = 2x_2$$

$$\frac{\partial f}{\partial x_2} = 2x_1$$

$$\frac{\partial f}{\partial x_3} = -1$$

Example:

$$f(x,y,z) = xy^2 + xz + 4$$

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$$\frac{\partial f}{\partial x_2}$$

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Find

$$\frac{\partial f}{\partial x_2} \rightsquigarrow \frac{\partial f}{\partial y} = 2xy$$

Ⅱ Vector fields

A **vector field** is a function

$$\vec{F} : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

Examples:

① $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2, \quad \vec{F}(x,y) = (-y, x)$

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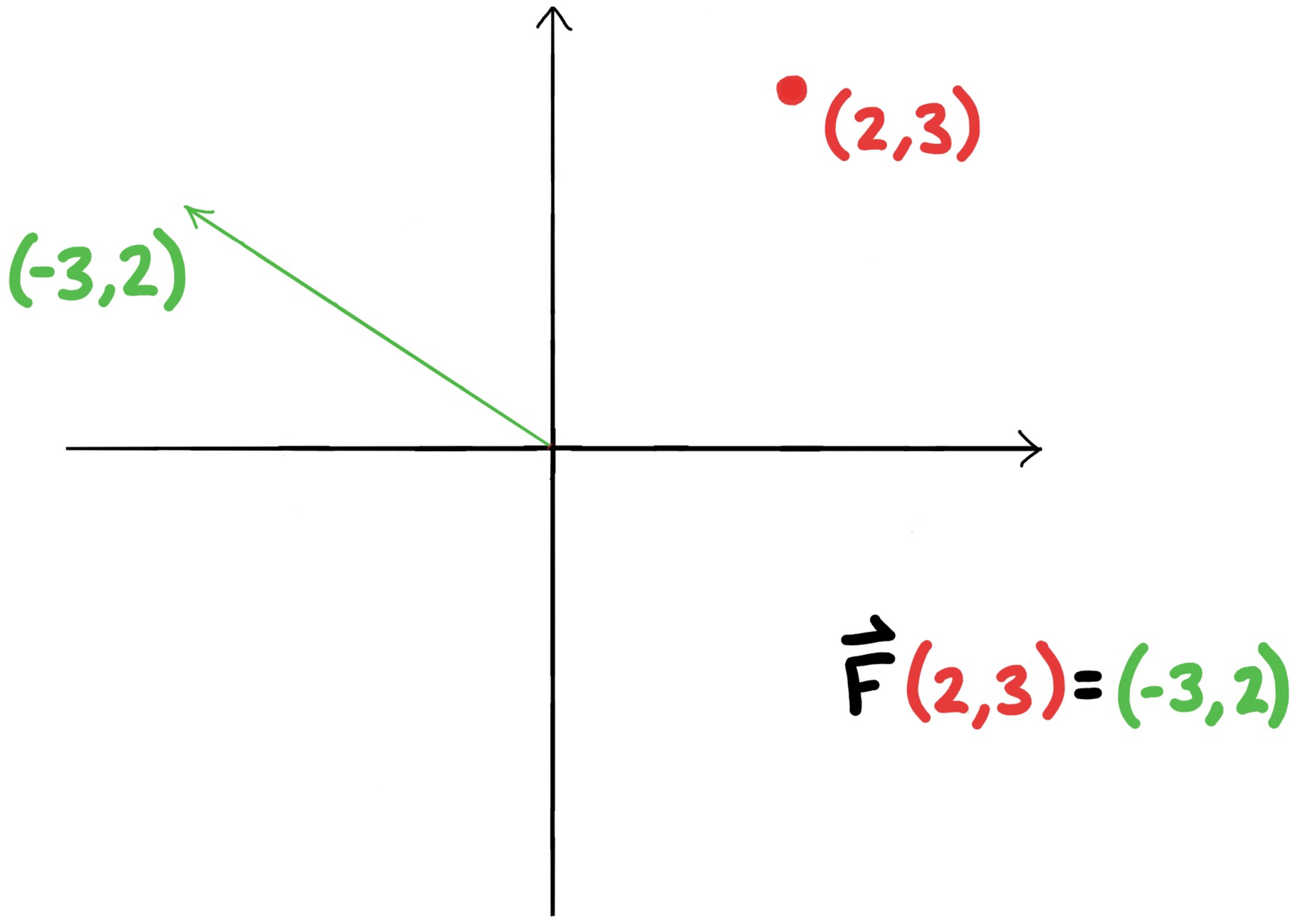
① $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$, $\vec{F}(x,y) = (-y, x)$, $\vec{F}(2,3) = (-3,2)$

② $\vec{F} : \mathbb{R}^2 \to \mathbb{R}^2$, $\vec{F}(x,y) = (2xy, x^2)$, $\vec{F}(-1,4) = (-8,1)$

③ $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$, $\vec{F}(x,y,z) = (z, 2yz, x+y^2)$,
$\vec{F}(1,0,2) = (2,0,1)$

④ $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$, $\vec{F}(x,y,z) = (x, z, yz)$,
$\vec{F}(0,1,-1) = (0,-1,-1)$

$$\vec{F}(x,y) = (-y,x)$$



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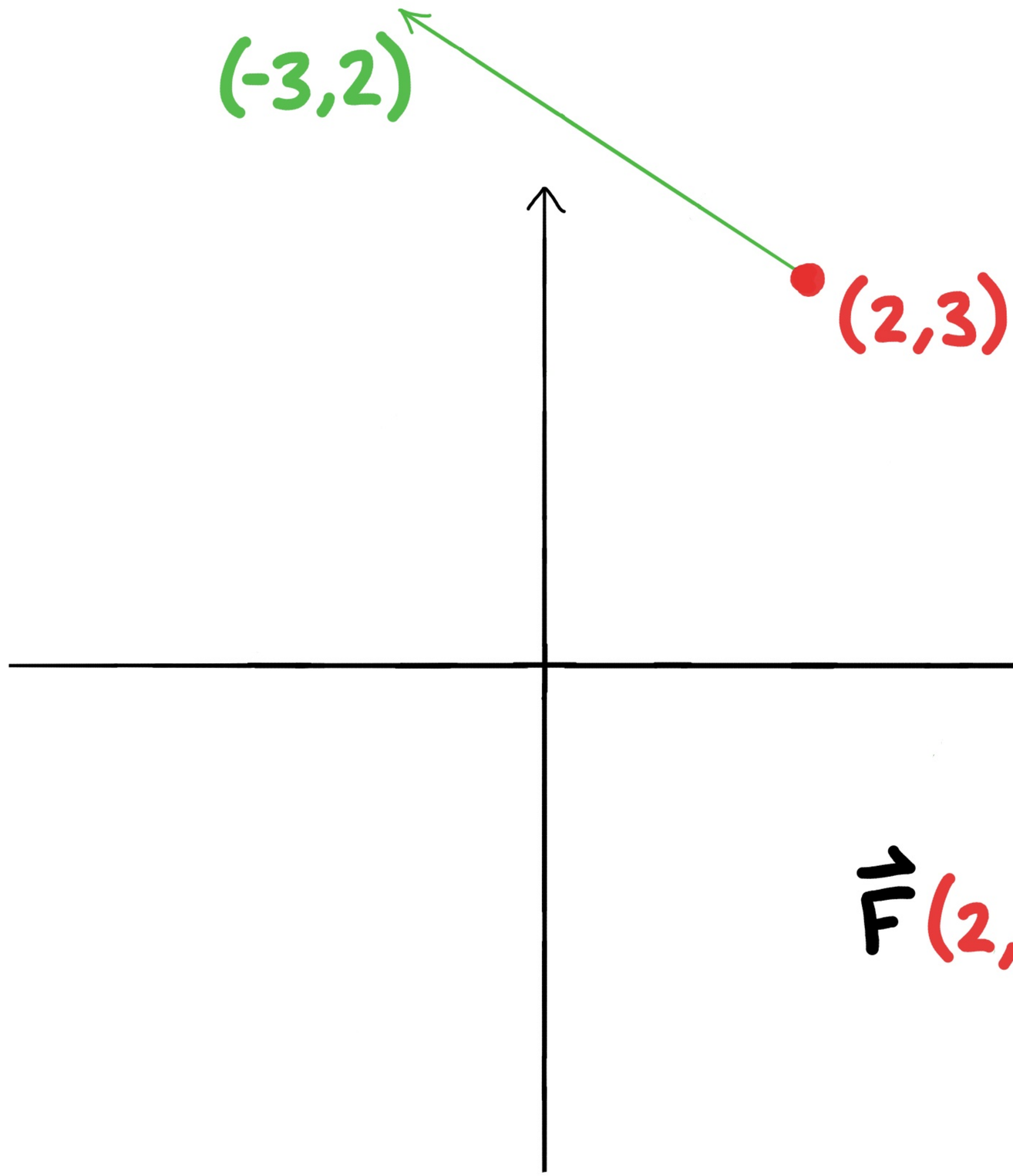

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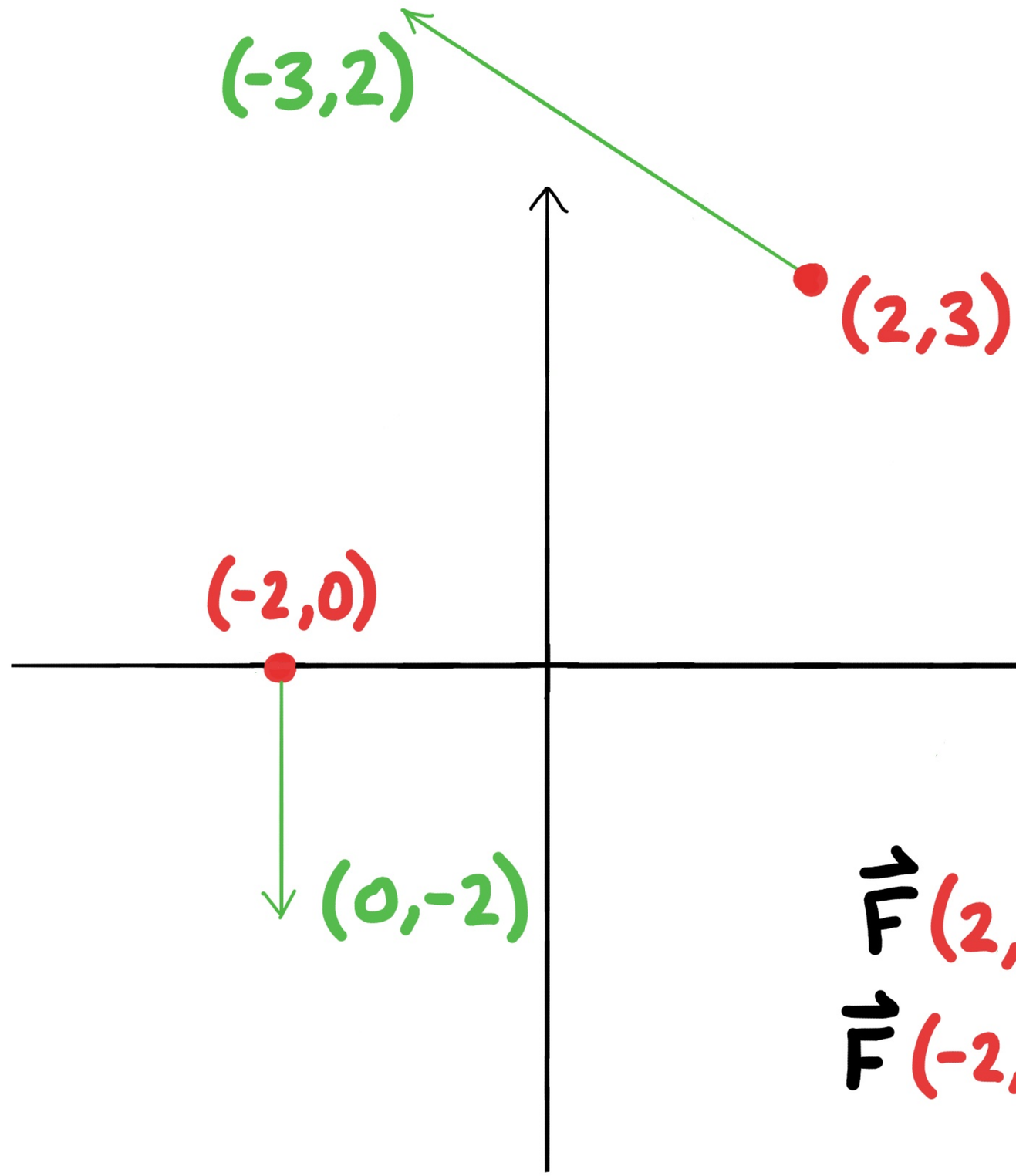

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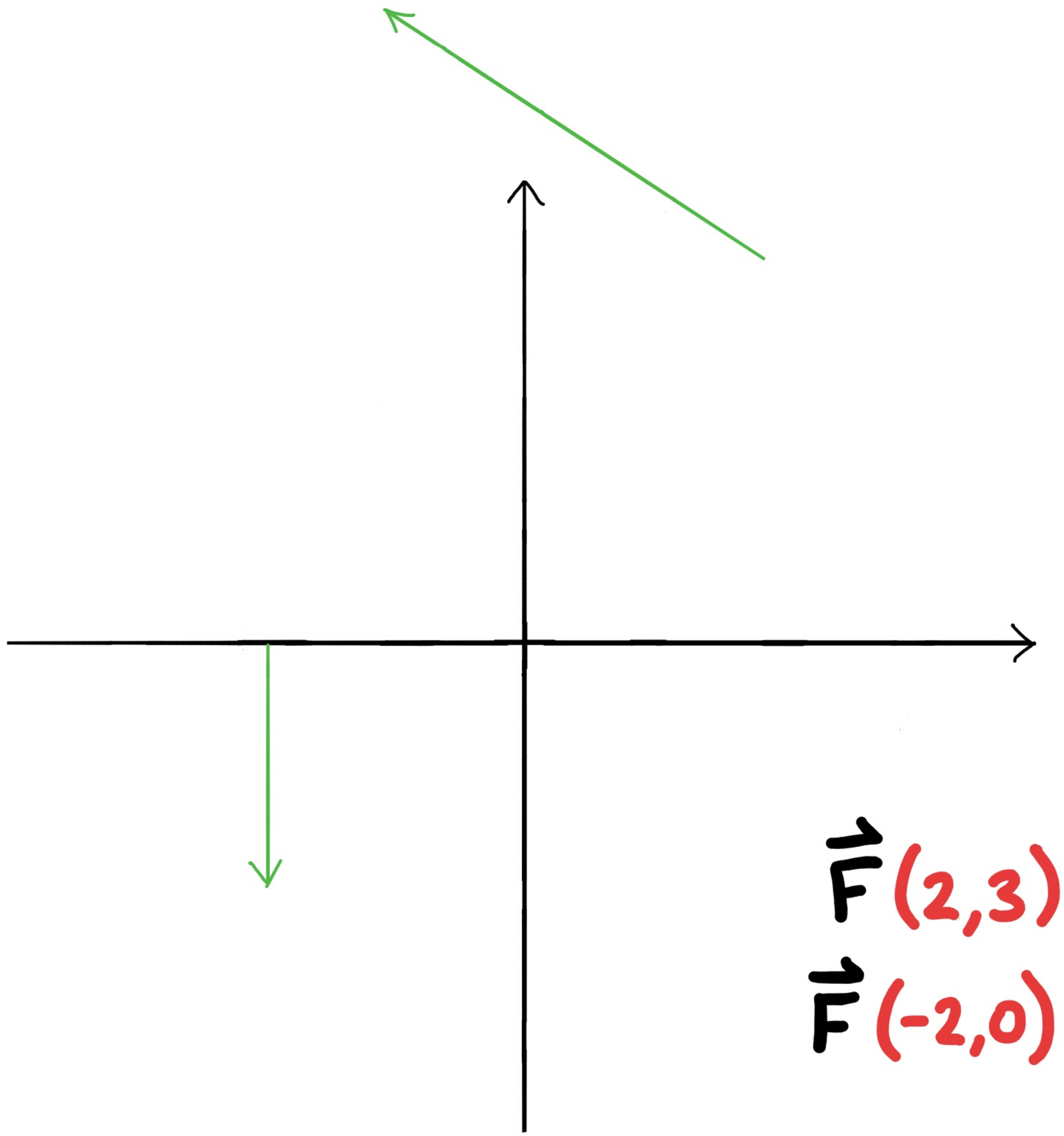

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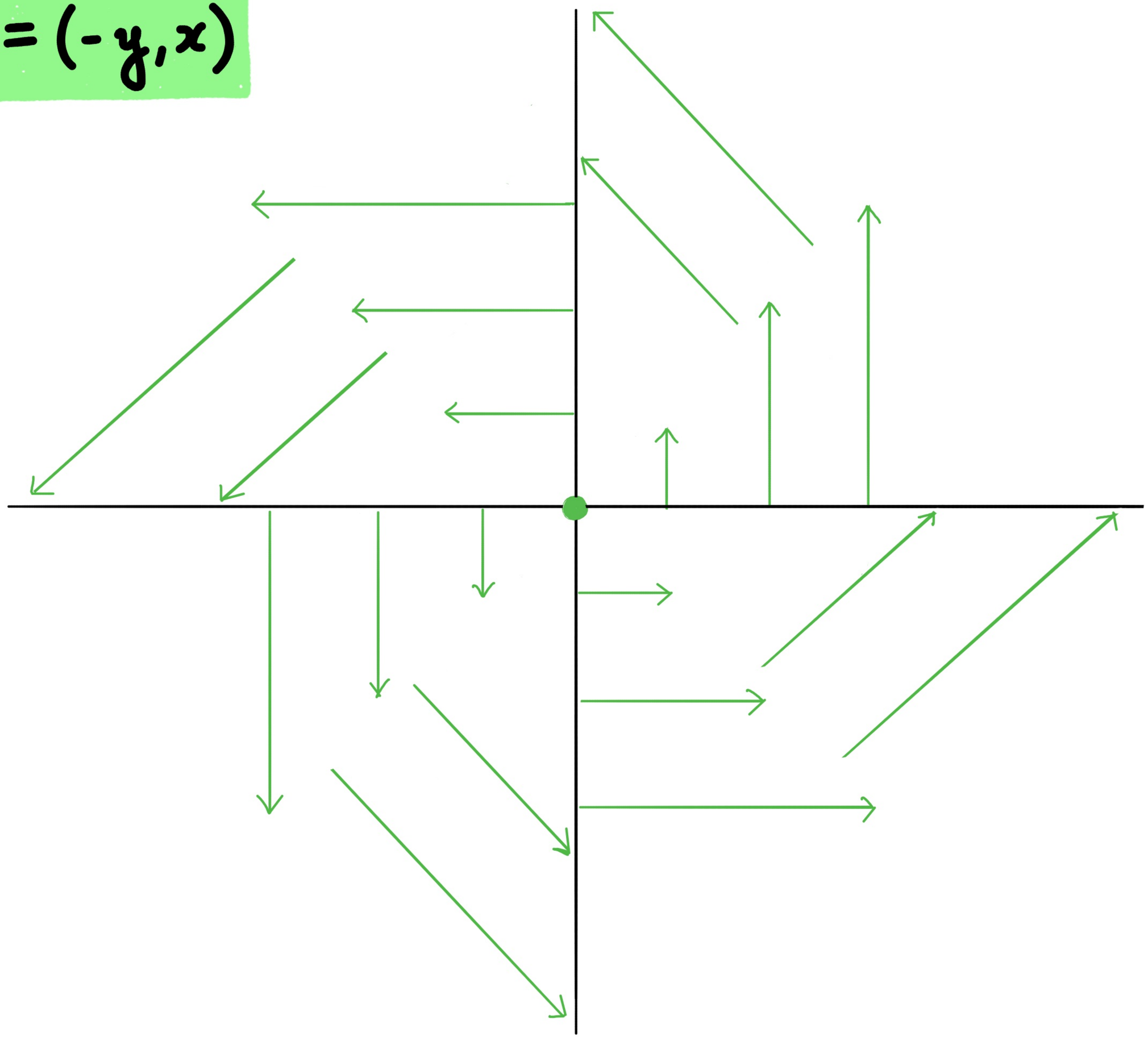
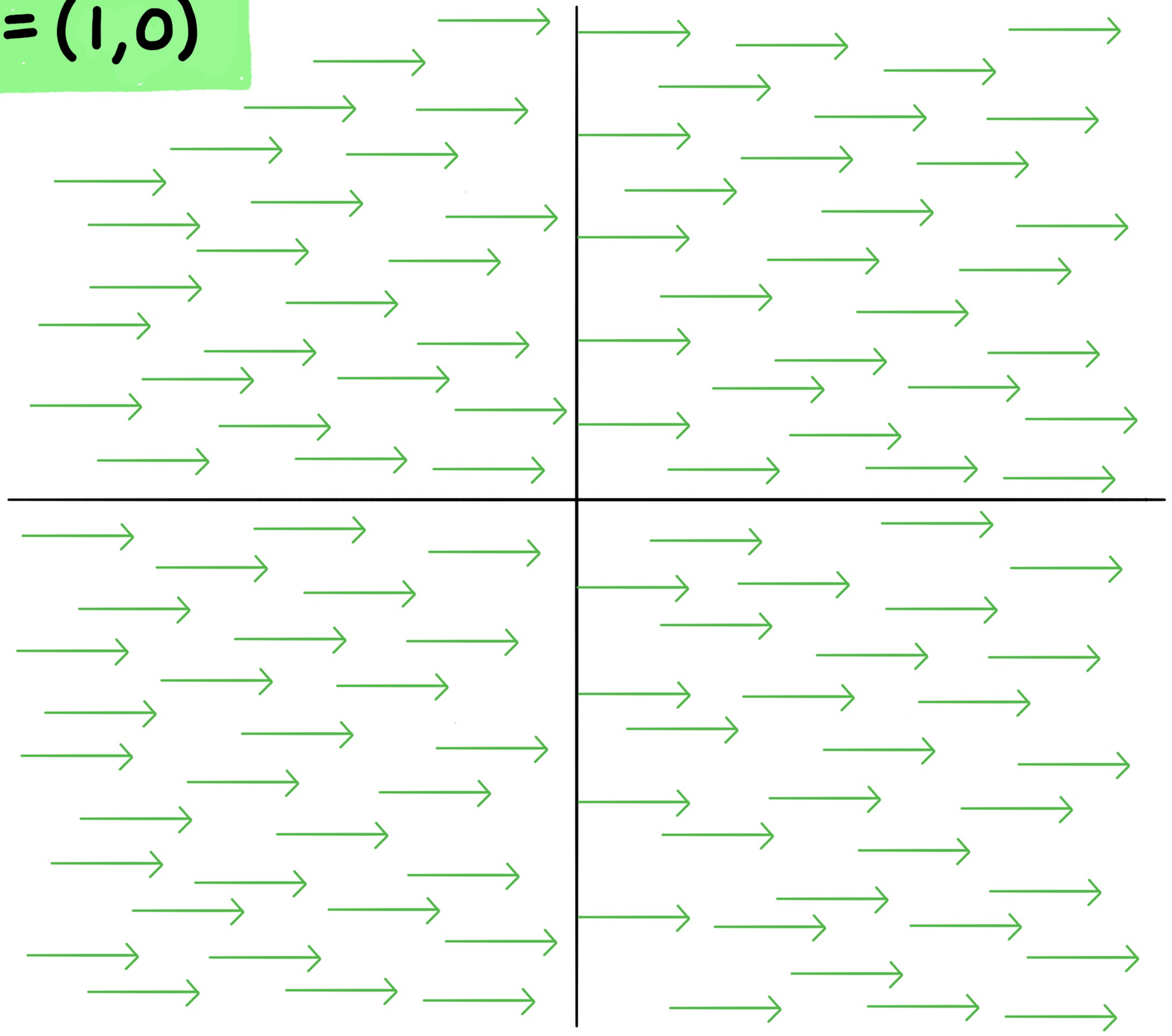
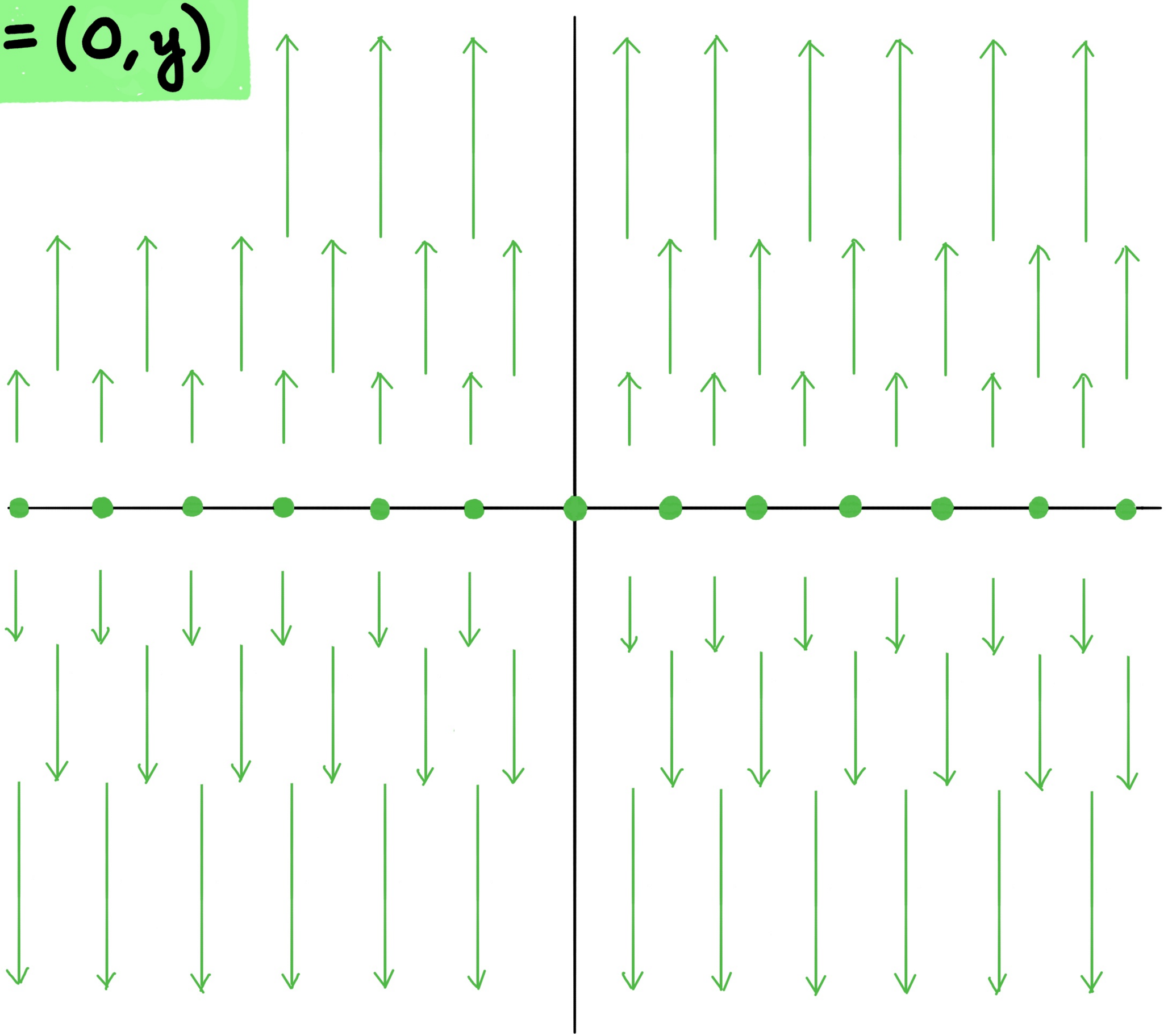


$$\vec{F}(x,y) = (1,0)$$



$$\vec{F}(x,y) = (0,y)$$



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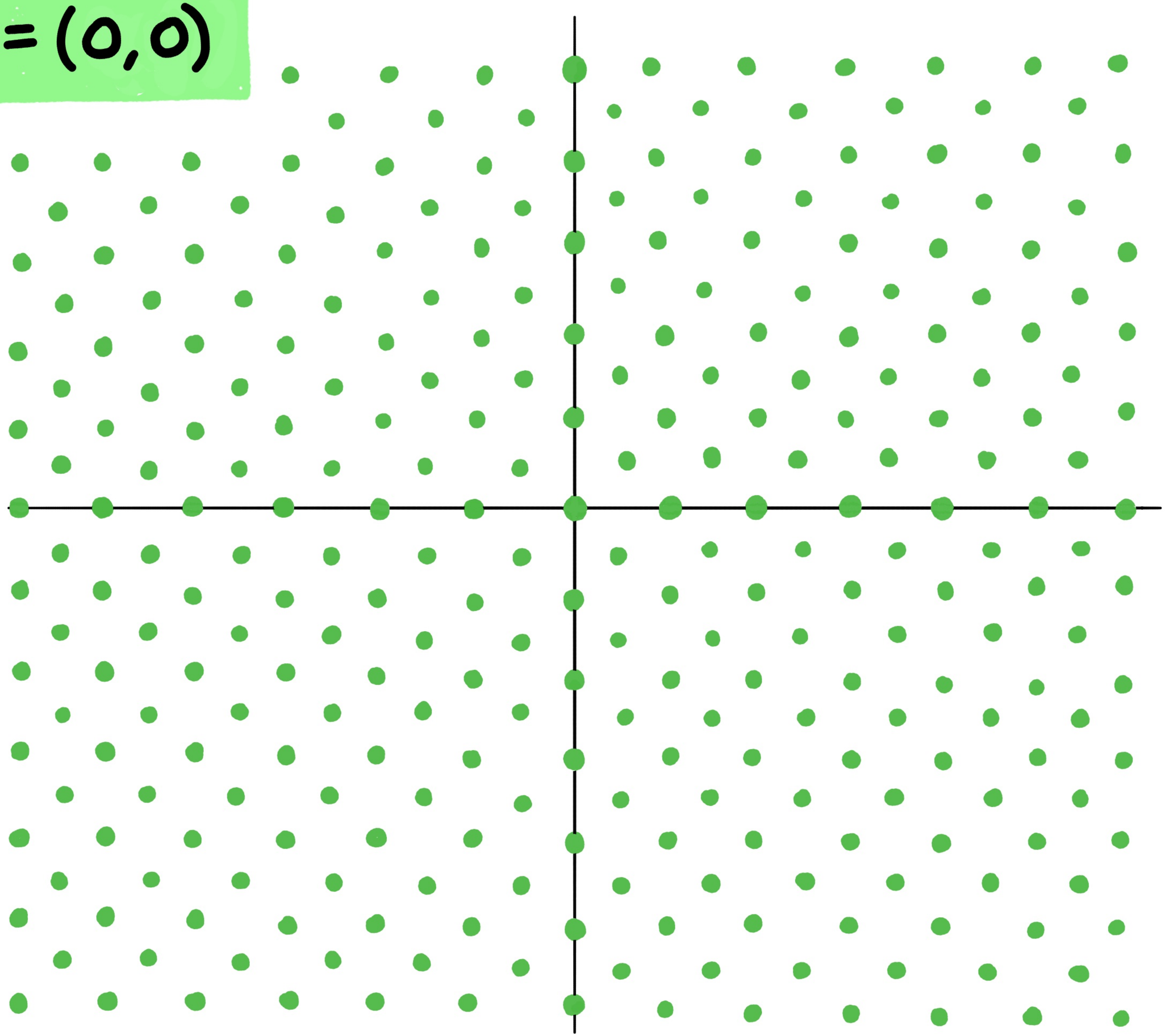
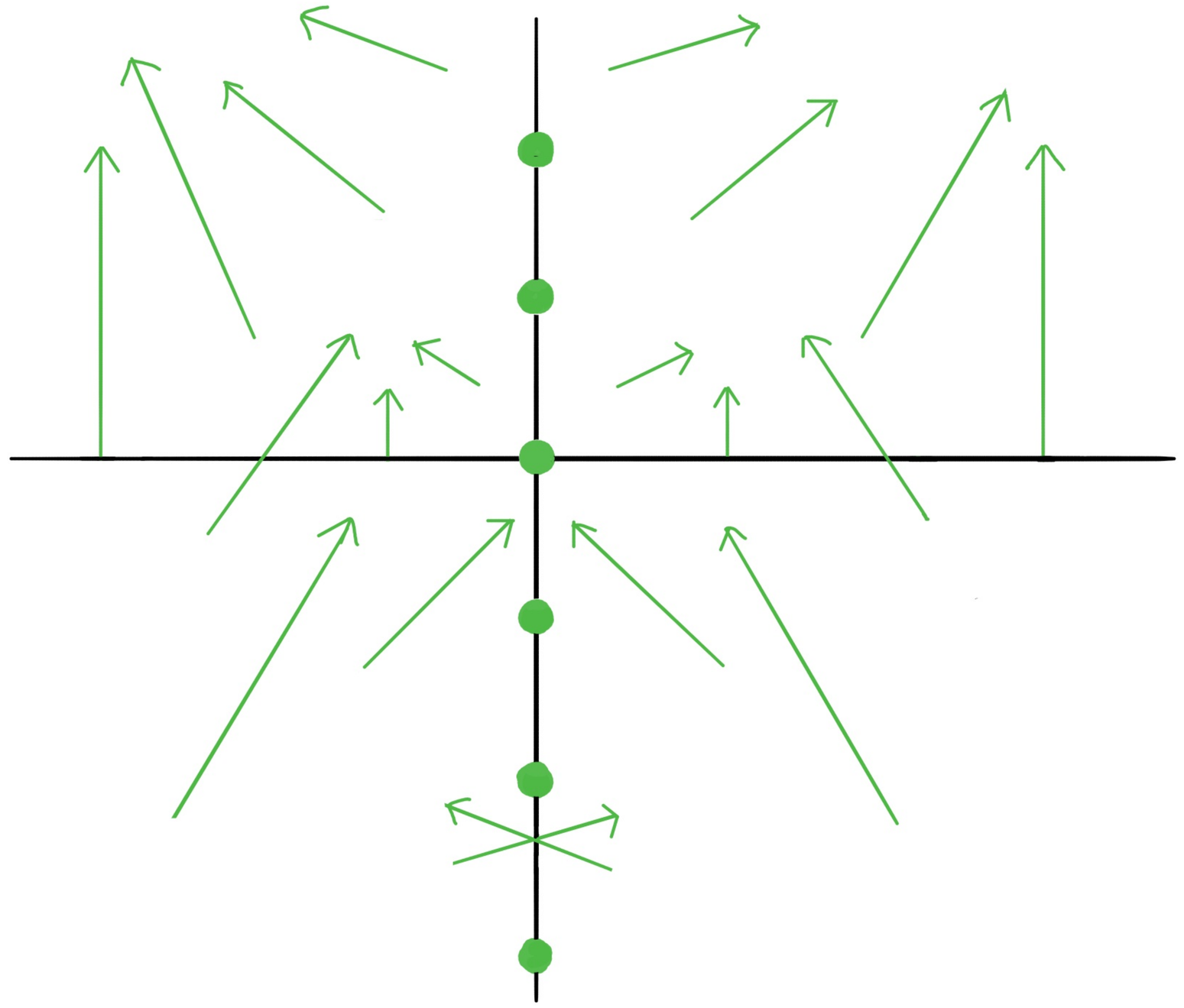


$$\vec{F}(x,y) = (2xy, x^2)$$



For $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$ we'll write

$$\vec{F}(x_1,\dots,x_n) = \left(F_1(x_1,\dots,x_n), F_2(x_1,\dots,x_n), \dots, F_n(x_1,\dots,x_n)\right)$$

where the $F_i: \mathbb{R}^n \to \mathbb{R}$ are coordinate functions.

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Example:

$$\vec{F}(x,y,z) = (z, 2yz, x+y^2)$$

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For short, $\vec{F} = (F_1, F_2, \dots, F_n)$.

Example:

$$\vec{F}(x,y) = (2xy, x^2)$$

$F_1(x,y) = 2xy$

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Ⅲ Conservative vector fields

Gradients are vector fields

Recall, if $f: \mathbb{R}^n \to \mathbb{R}$, then

$$\nabla f : \mathbb{R}^n \to \mathbb{R}^n, \quad \nabla f(x_1, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \right)$$

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Examples:

① $f: \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = 2x^2 + y$

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (4x, 1)$$

② $g: \mathbb{R}^3 \to \mathbb{R}$, $g(x,y,z) = xyz$

$$\nabla g(x,y,z) = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z} \right) = (yz, xz, xy)$$

Conservative vector fields

$\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$ is **conservative**

if $\vec{F} = \nabla f$ for some $f: \mathbb{R}^n \to \mathbb{R}$.

If so, f is the **potential function**.

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Examples:

① $\vec{F}(x,y) = (4x, 1)$ is conservative with potential $f(x,y) = 2x^2 + y$, since $\nabla f = \vec{F}$.

② $\vec{F}(x,y,z) = (yz, xz, xy)$ is conservative with potential $g(x,y,z) = xyz$, since $\nabla g = \vec{F}$.

Ⅳ Test for being conservative

Suppose $\vec{F} = \nabla f$

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F_1 F_2 F_n

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$$\frac{\partial}{\partial x_j} F_i =$$

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Summary:

If a vector field $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$ is the gradient of a function, then

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i} \qquad 1 \le i, j \le n$$

Central Theorem

A vector field $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$ is the gradient of a function exactly when

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}$$

for all i and j.

Example:

$$\vec{F}: \mathbb{R}^3 \to \mathbb{R}^3, \quad \vec{F}(x,y,z) = (z, 2yz, x+y^2)$$

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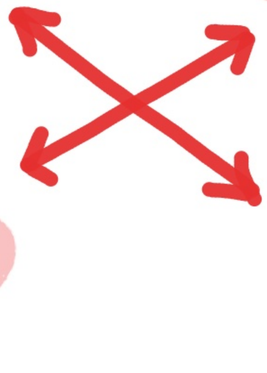

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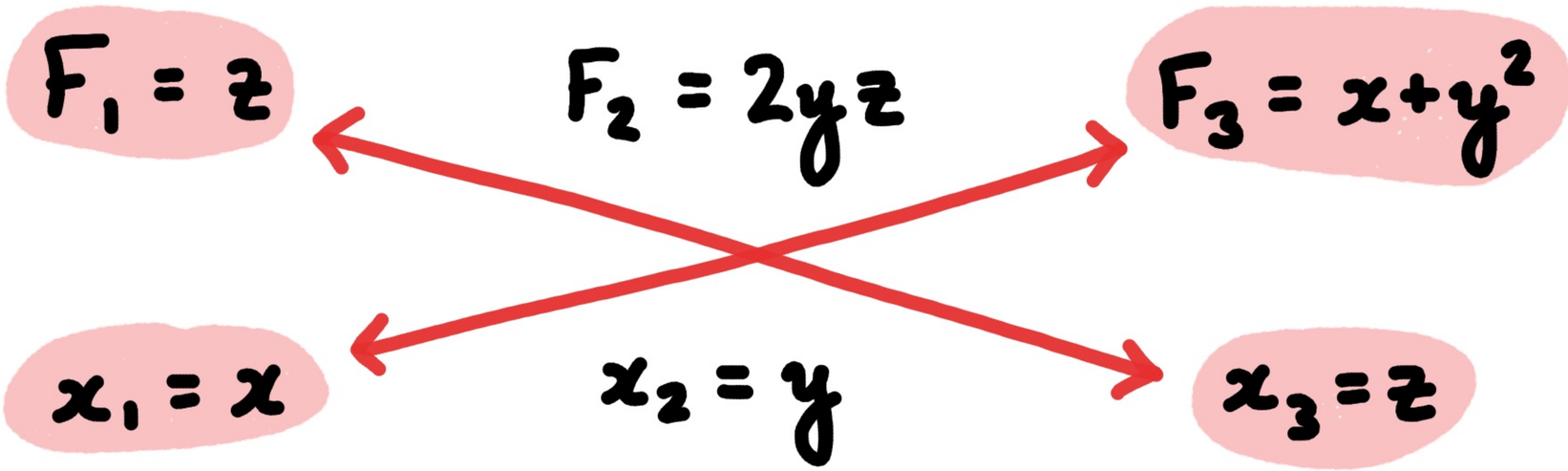

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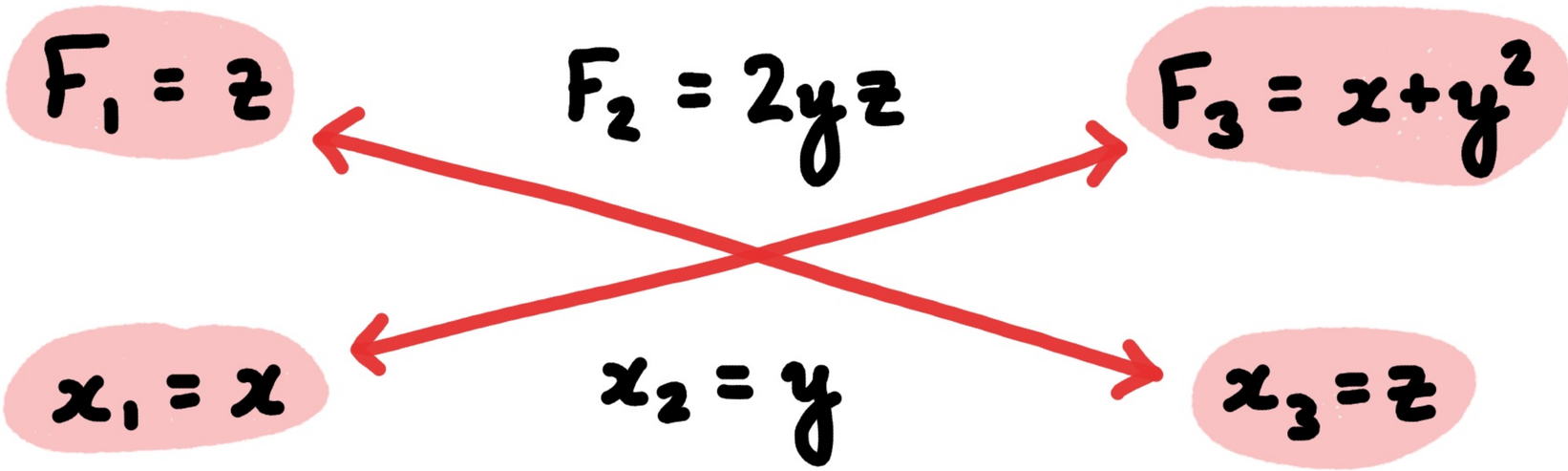

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for $1 \le i \ne j \le 3$.

so $\vec{F}$ is conservative.

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coordinate functions:	x	z	yz
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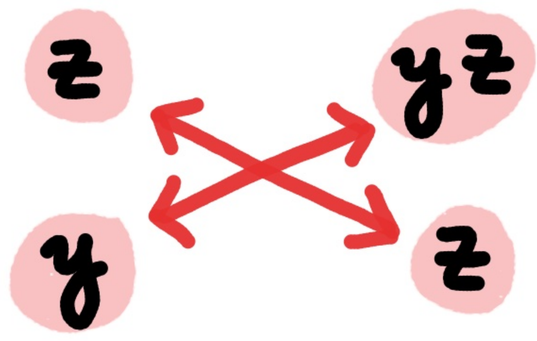
coordinate functions: x

variables: x



$\vec{F}$ is <u>not</u> conservative since

$$\frac{\partial z}{\partial z} = 1 \neq z = \frac{\partial yz}{\partial y}$$

Example:

$$\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2, \quad \vec{F}(x,y) = (-y, x)$$

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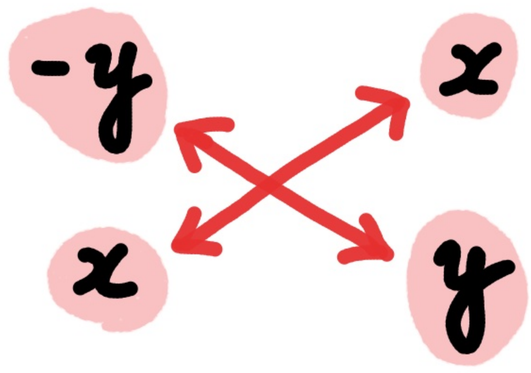

$\vec{F}$ is <u>not</u> conservative since

$$\frac{\partial(-y)}{\partial y} = -1 \neq 1 = \frac{\partial x}{\partial x}$$

Example:

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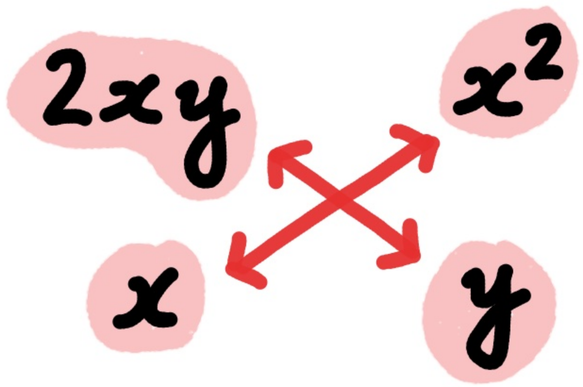

$\vec{F}$ is conservative since

$$\frac{\partial(2xy)}{\partial y} = 2x = \frac{\partial x^2}{\partial x}$$

Ⓥ Divergence

Divergence of a vector field

Let $\vec{F}: \mathbb{R}^n \to \mathbb{R}^n$, $\vec{F} = (F_1, \ldots, F_n)$.

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