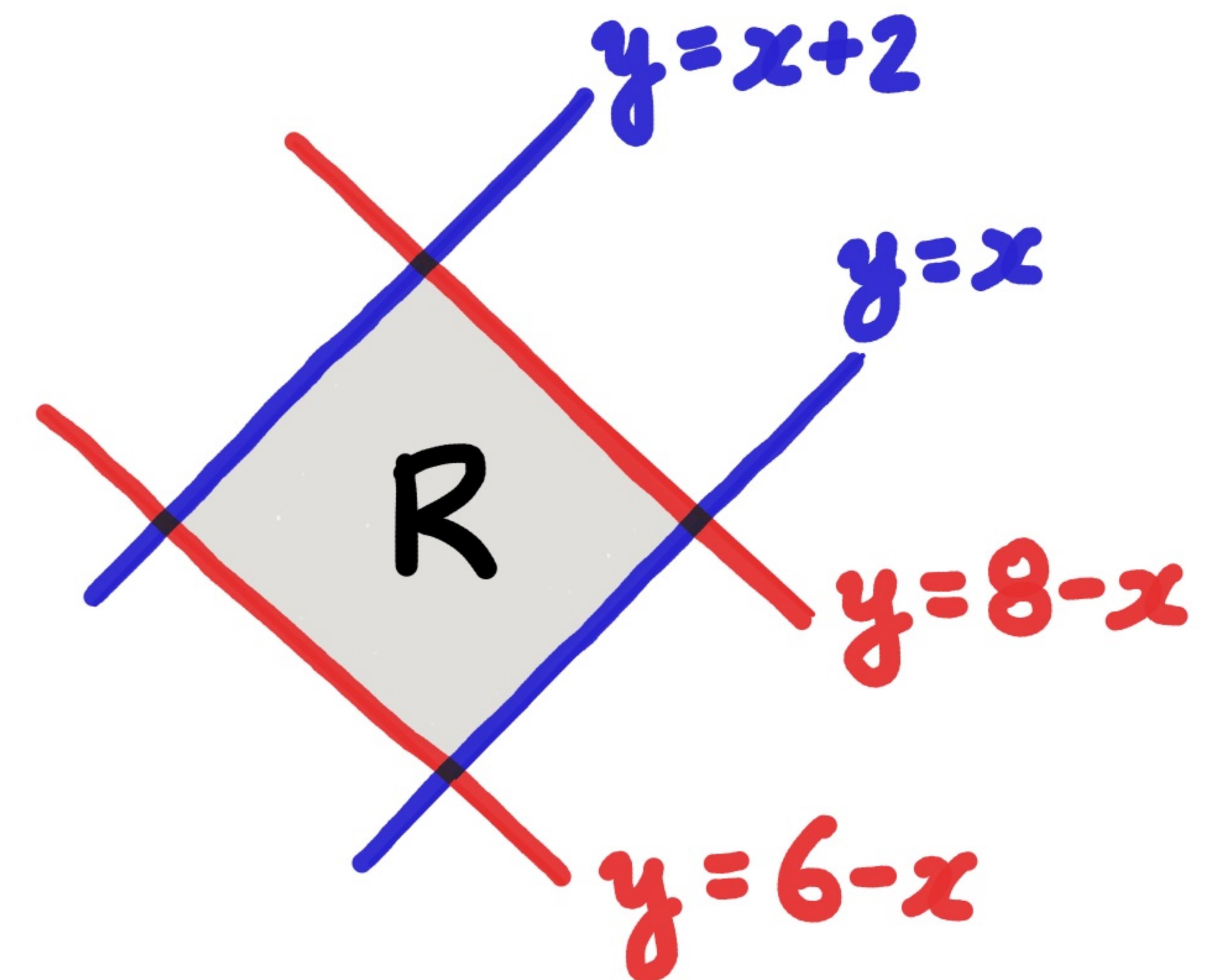


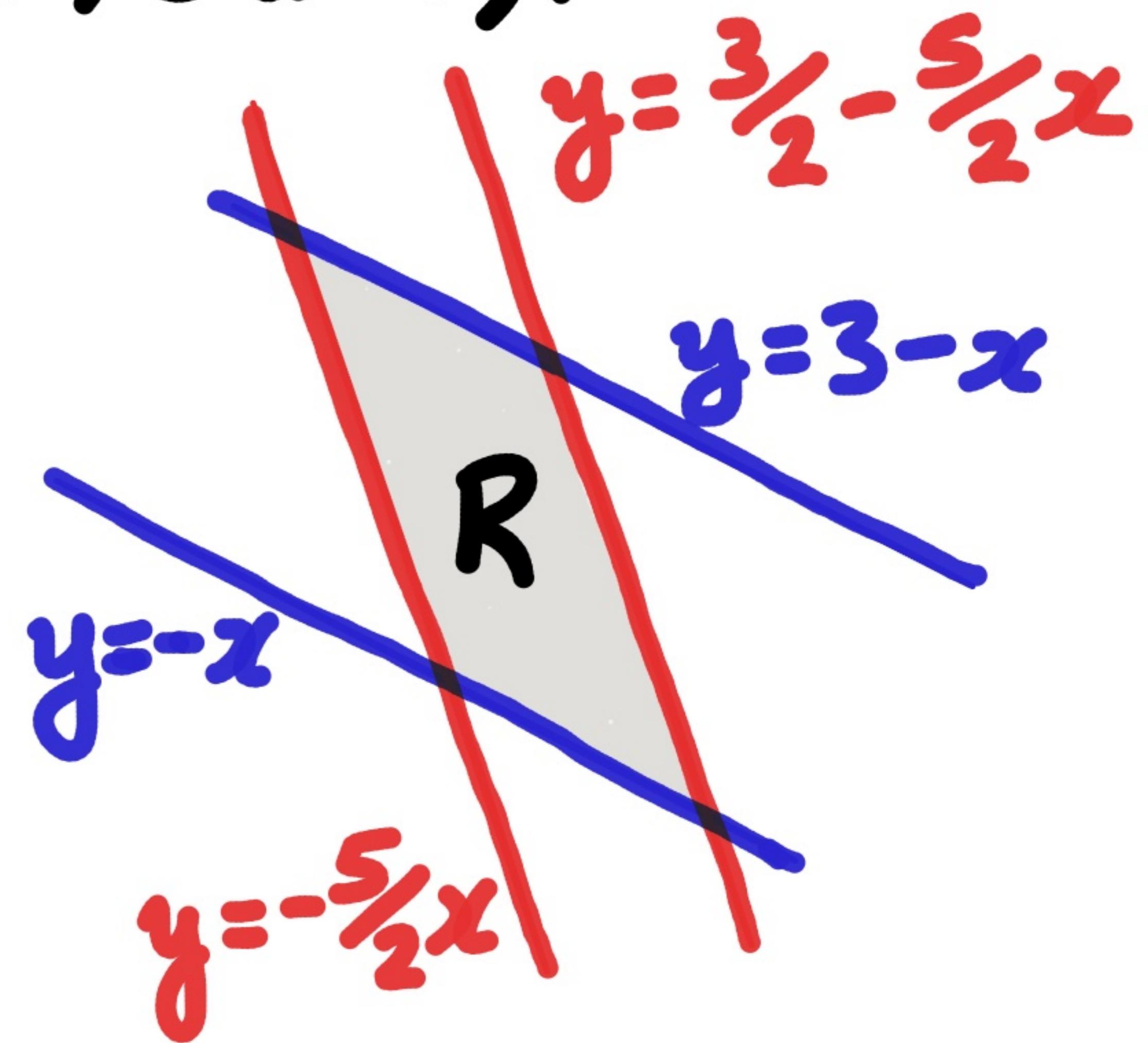
Twenty-six

Examples of Change of variables

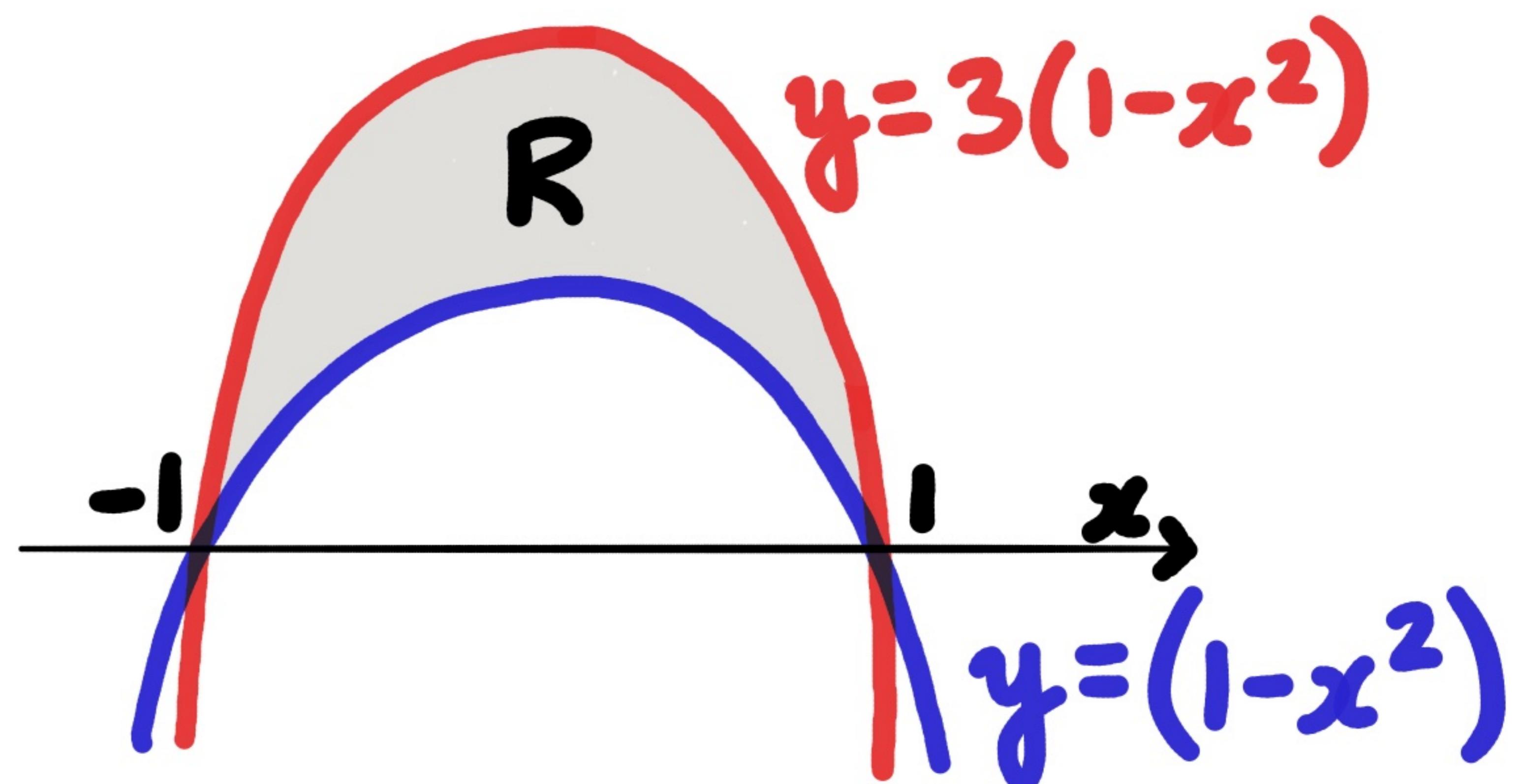
① Find $\iint_R \frac{x-y}{x+y} dx dy$ using change of variables with $G(u,v) = (u-v, u+v)$.



② Find $\iint_R \sin(x+y) dx dy$ using change of variables with $G(u,v) = (v-2u, 5u-v)$.

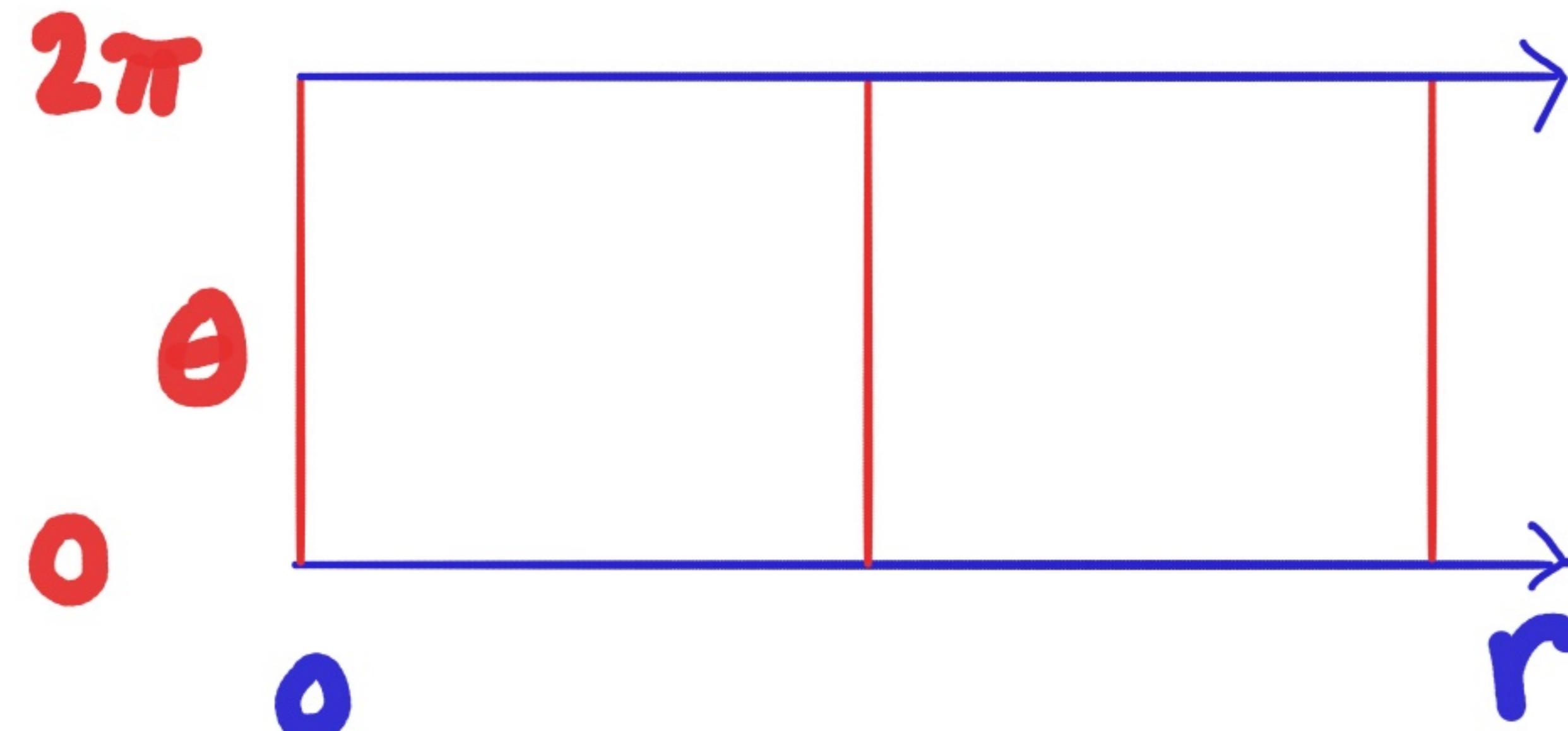


③ Find $\iint_R \frac{x^2}{y} dx dy$ using change of variables with $G(u,v) = (v, u(1-v^2))$.

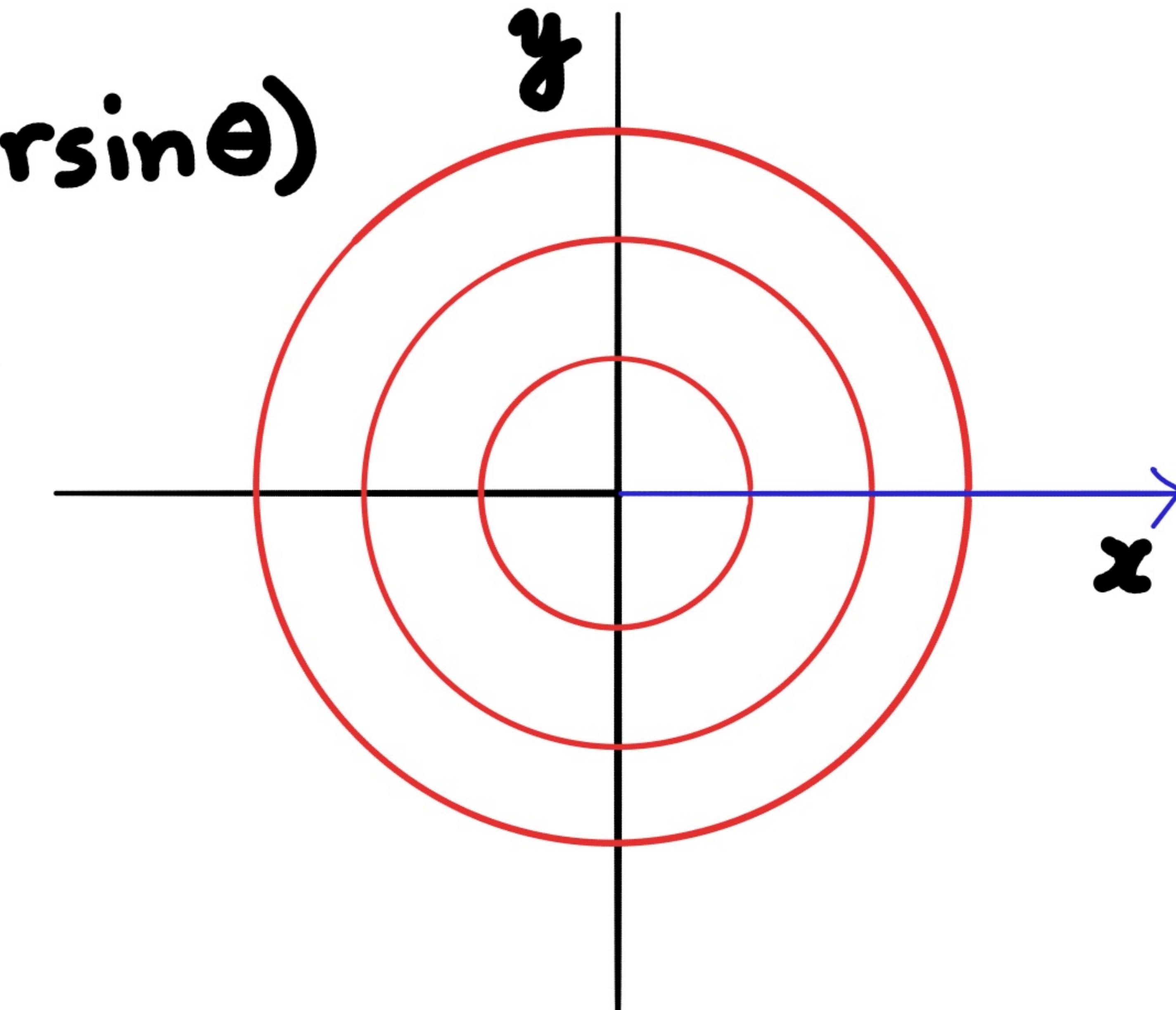


④ Polar COV

$$G(r, \theta) = (r \cos \theta, r \sin \theta)$$



$x = r \cos \theta$
 $y = r \sin \theta$



$$D_{(r, \theta)} G = \begin{pmatrix} \frac{\partial r \cos \theta}{\partial r} & \frac{\partial r \cos \theta}{\partial \theta} \\ \frac{\partial r \sin \theta}{\partial r} & \frac{\partial r \sin \theta}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{pmatrix}$$

$$|\det D_{(r, \theta)} G| = |r \cos^2 \theta + r \sin^2 \theta| = |r| = r$$

⑤ Cylindrical COV $G(r, \theta, z) = (\underbrace{r \cos \theta}_x, \underbrace{r \sin \theta}_y, \underbrace{z}_z)$

$$D_{(r, \theta, z)} G = \begin{pmatrix} \frac{\partial r \cos \theta}{\partial r} & \frac{\partial r \cos \theta}{\partial \theta} & \frac{\partial r \cos \theta}{\partial z} \\ \frac{\partial r \sin \theta}{\partial r} & \frac{\partial r \sin \theta}{\partial \theta} & \frac{\partial r \sin \theta}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{pmatrix}$$

$$= \begin{pmatrix} \cos \theta & -r \sin \theta & 0 \\ \sin \theta & r \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus,

$$|\det D_{(r, \theta, z)} G| = r$$

⑥ Spherical COV

$$G(\rho, \theta, \phi) = (\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho \cos\phi)$$

The equation is annotated with red curly braces under each term to indicate they correspond to the x, y, and z axes respectively.

$$|\det D_{(\rho, \theta, \phi)} G| = \rho^2 \sin\phi$$

7

$$\int_0^1 \sin(3x) dx = \int_0^3 \sin(u) \frac{1}{3} du$$

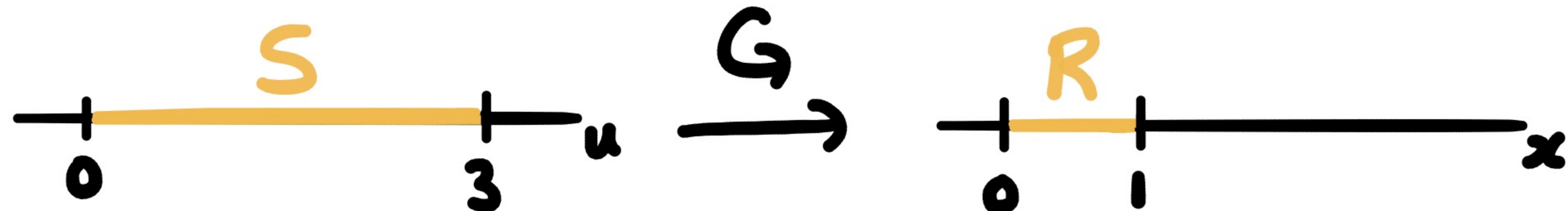
$$u = 3x$$

7

$$\int_0^1 \sin(3x) dx = \int_0^3 \sin(u) \frac{1}{3} du$$

$$u = 3x$$

$$G(u) = \underbrace{\frac{1}{3}u}_x$$



$$D_u G = G'(u) = \frac{1}{3}, \quad \text{so} \quad |D_u G| = \frac{1}{3}$$

7

$$\int_0^1 \sin(3x) dx = \int_0^3 \sin(u) \frac{1}{3} du$$

$3x \mapsto 3(\frac{1}{3}u)$

$$G(u) = \underbrace{\frac{1}{3}u}_x$$



$$D_u G = G'(u) = \frac{1}{3}, \quad \text{so} \quad |D_u G| = \frac{1}{3}$$