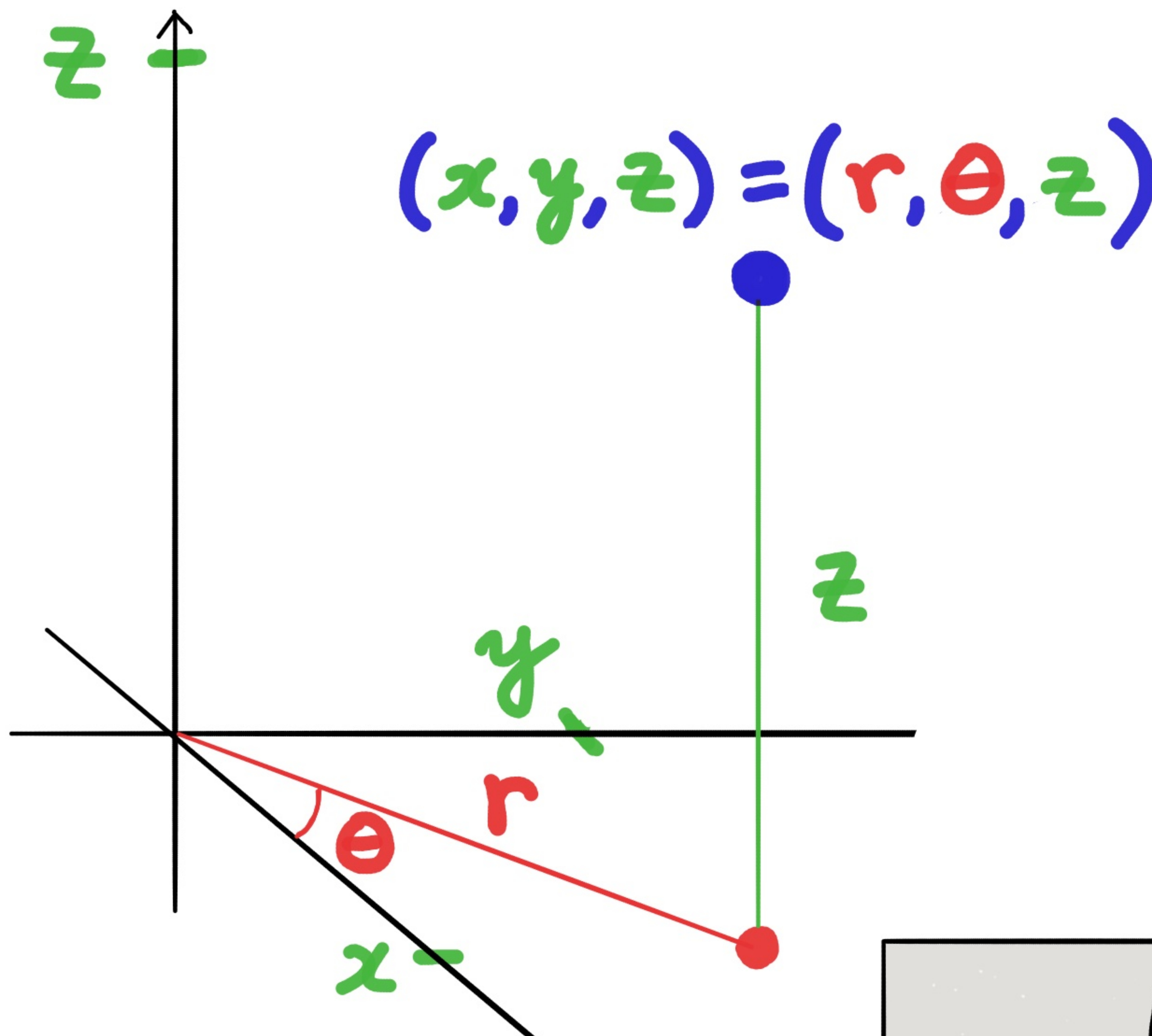
Twenty-four

I. Cylindrical and spherical coordinates for $\mathbb{R}^3$

II. Triple integrals

I Cylindrical and spherical coordinates for $\mathbb{R}^3$

Cylindrical coordinates



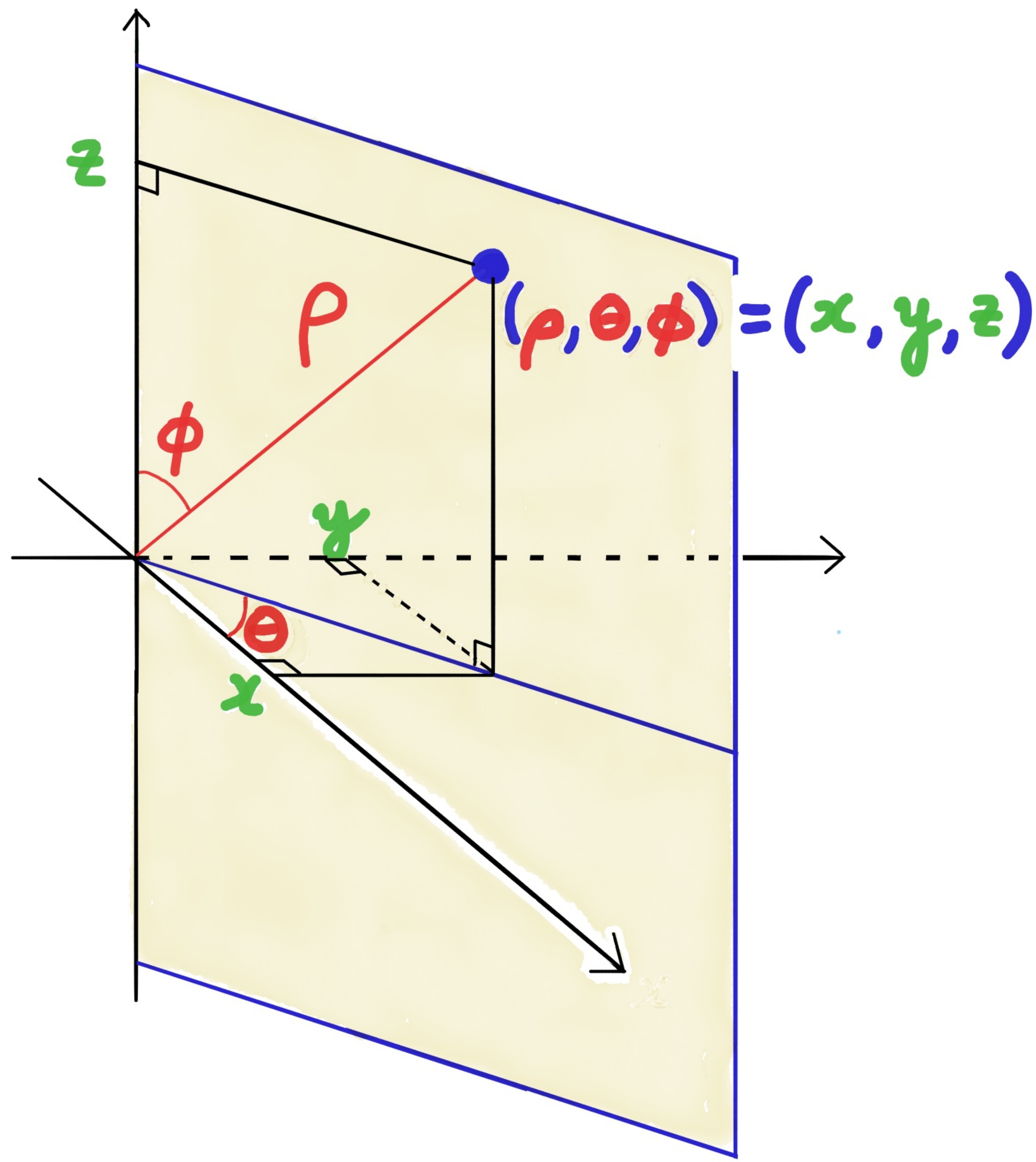
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

$r = \sqrt{x^2 + y^2}$
$0 \le r$
$0 \le \theta \le 2\pi$

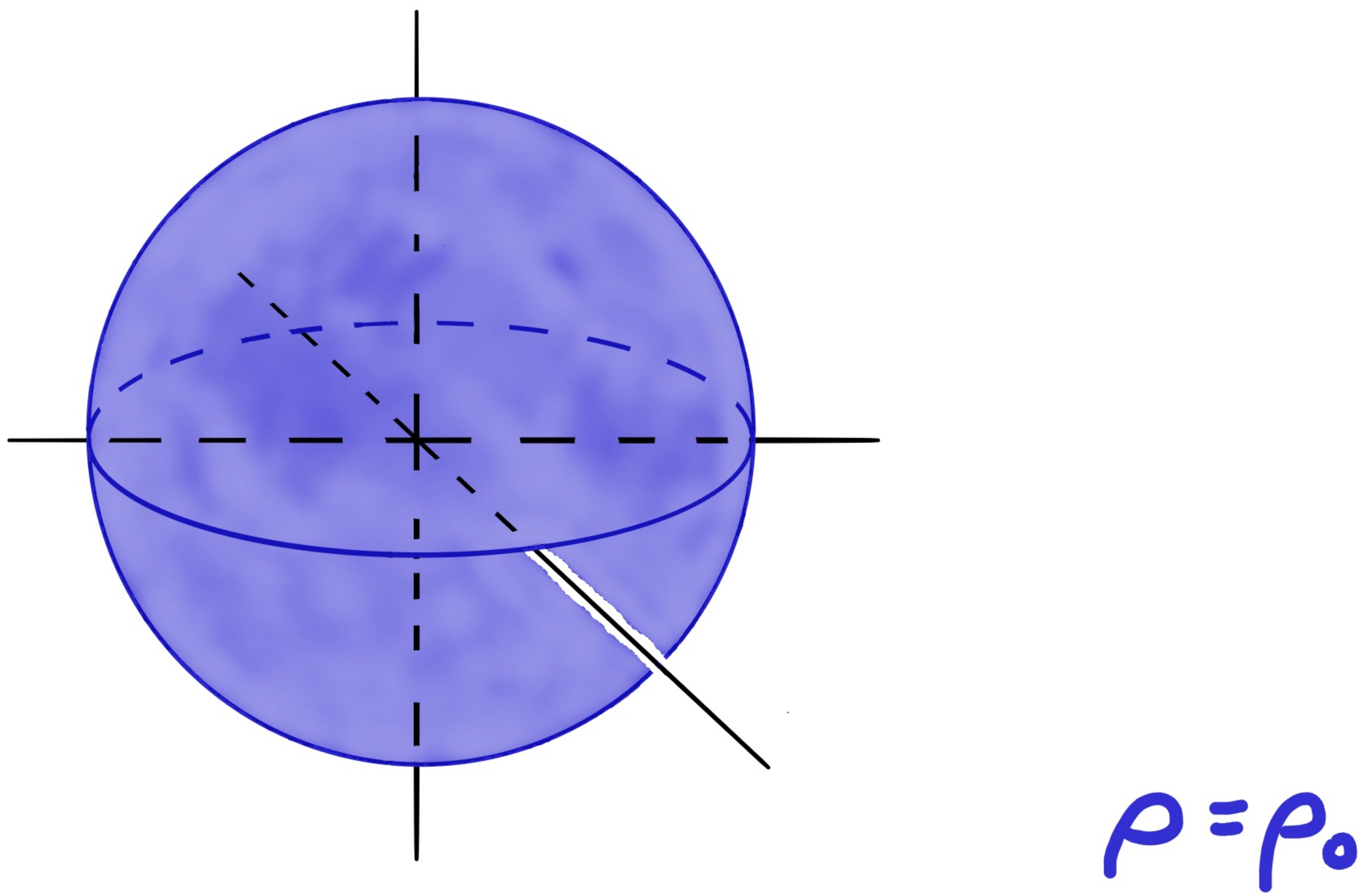
Spherical coordinates

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

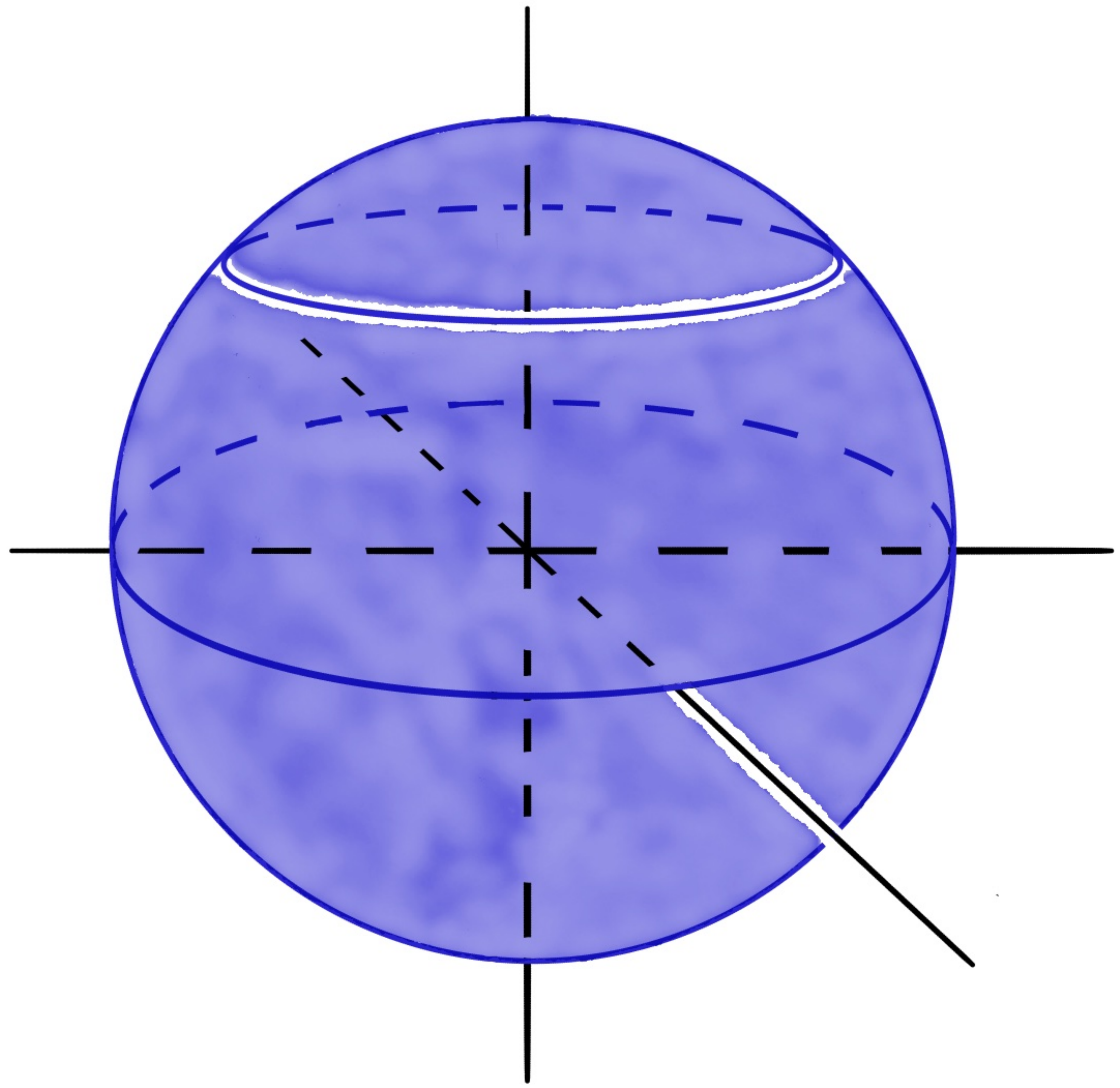
$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$0 \leq \rho$$
$$0 \leq \theta \leq 2\pi$$
$$0 \leq \phi \leq \pi$$



Spherical: $\rho = \rho_0$, $\phi = \phi_0$
is circle of radius $\rho_0 \sin\phi_0$



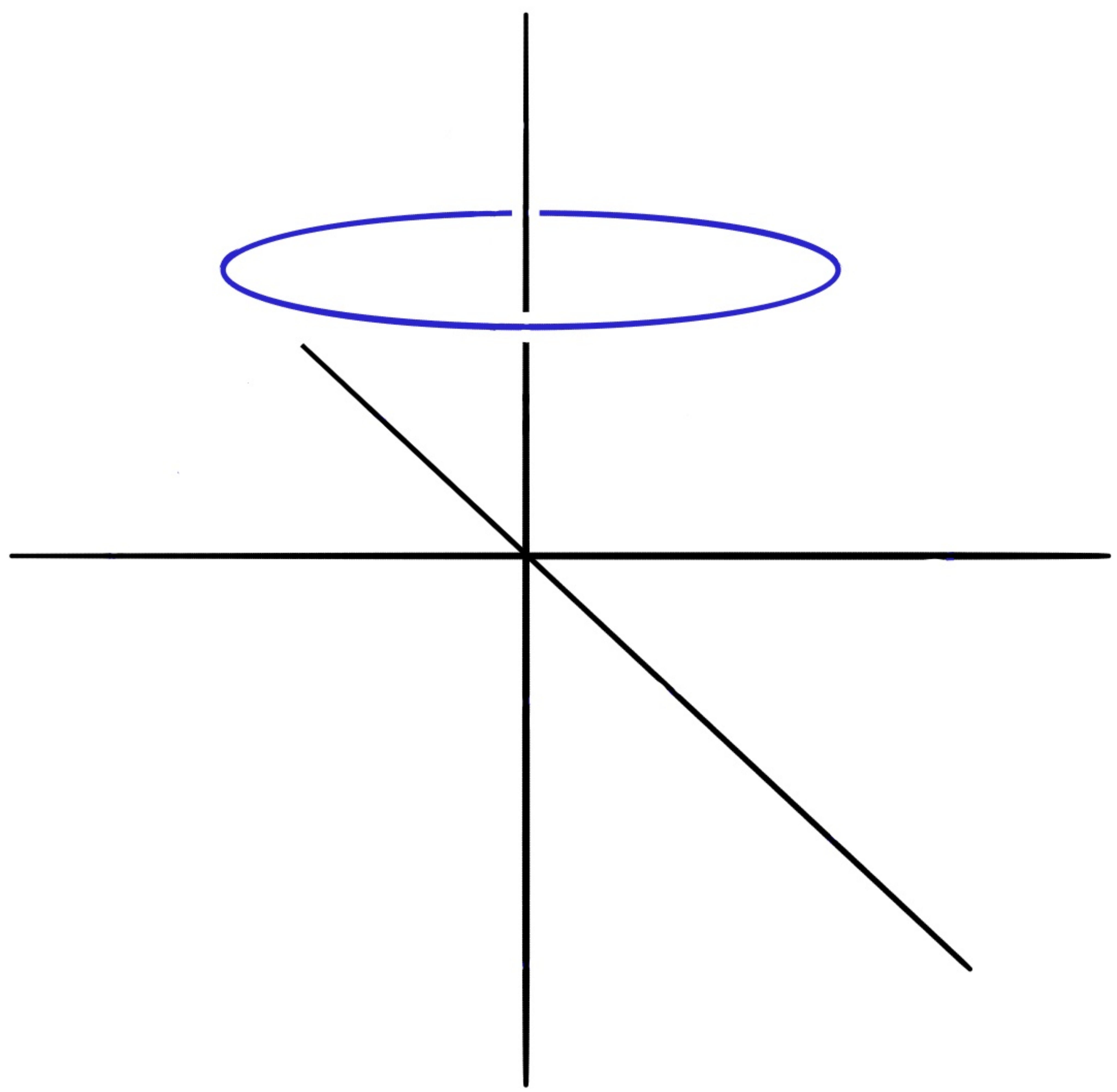
Spherical: $\rho=\rho_0,\ \phi=\phi_0$ is circle of radius $\rho_0 \sin\phi_0$



$\rho=\rho_0$

$\phi=\phi_0$

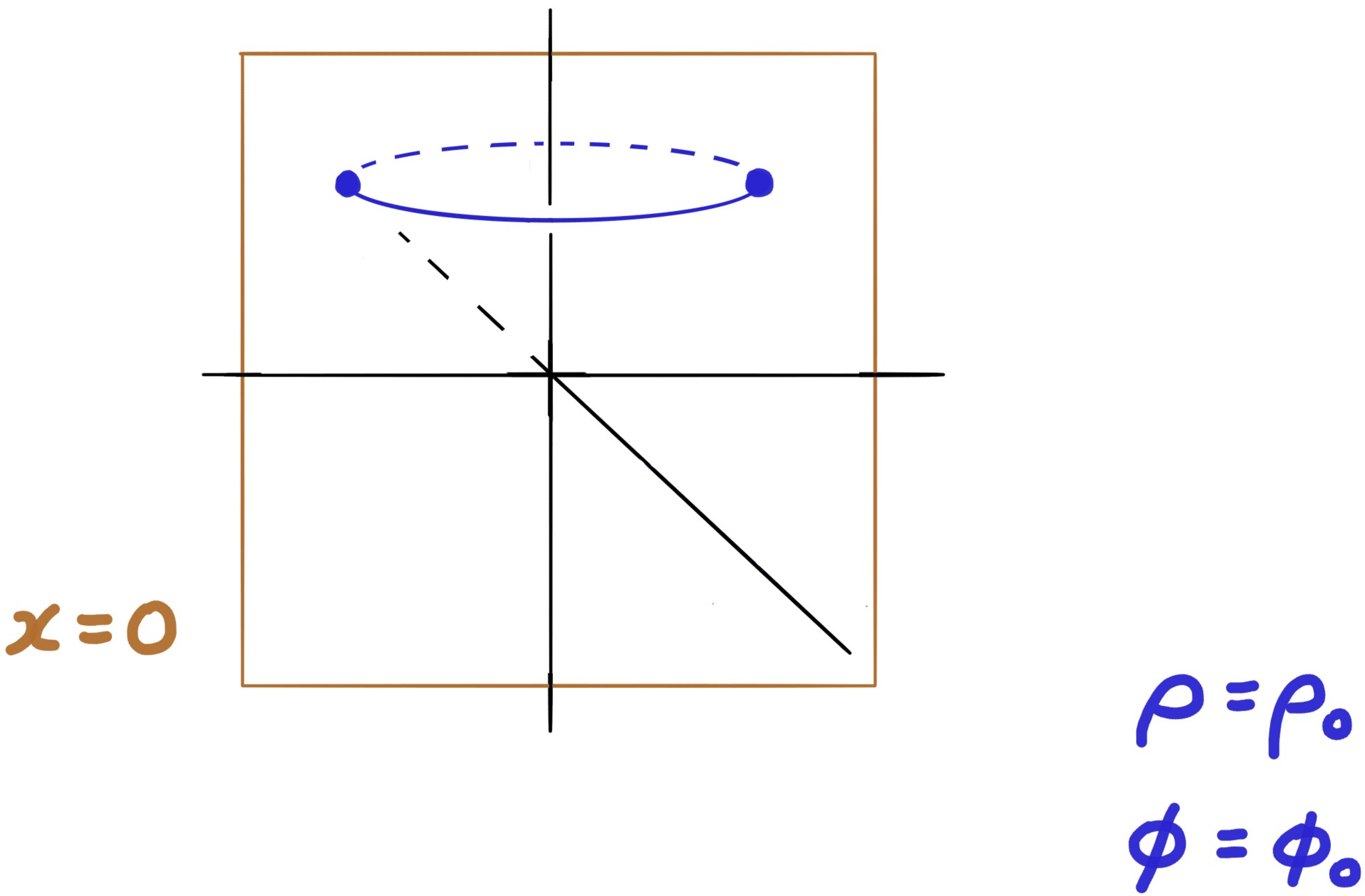
Spherical: $\rho = \rho_0$, $\phi = \phi_0$ is circle of radius $\rho_0 \sin\phi_0$



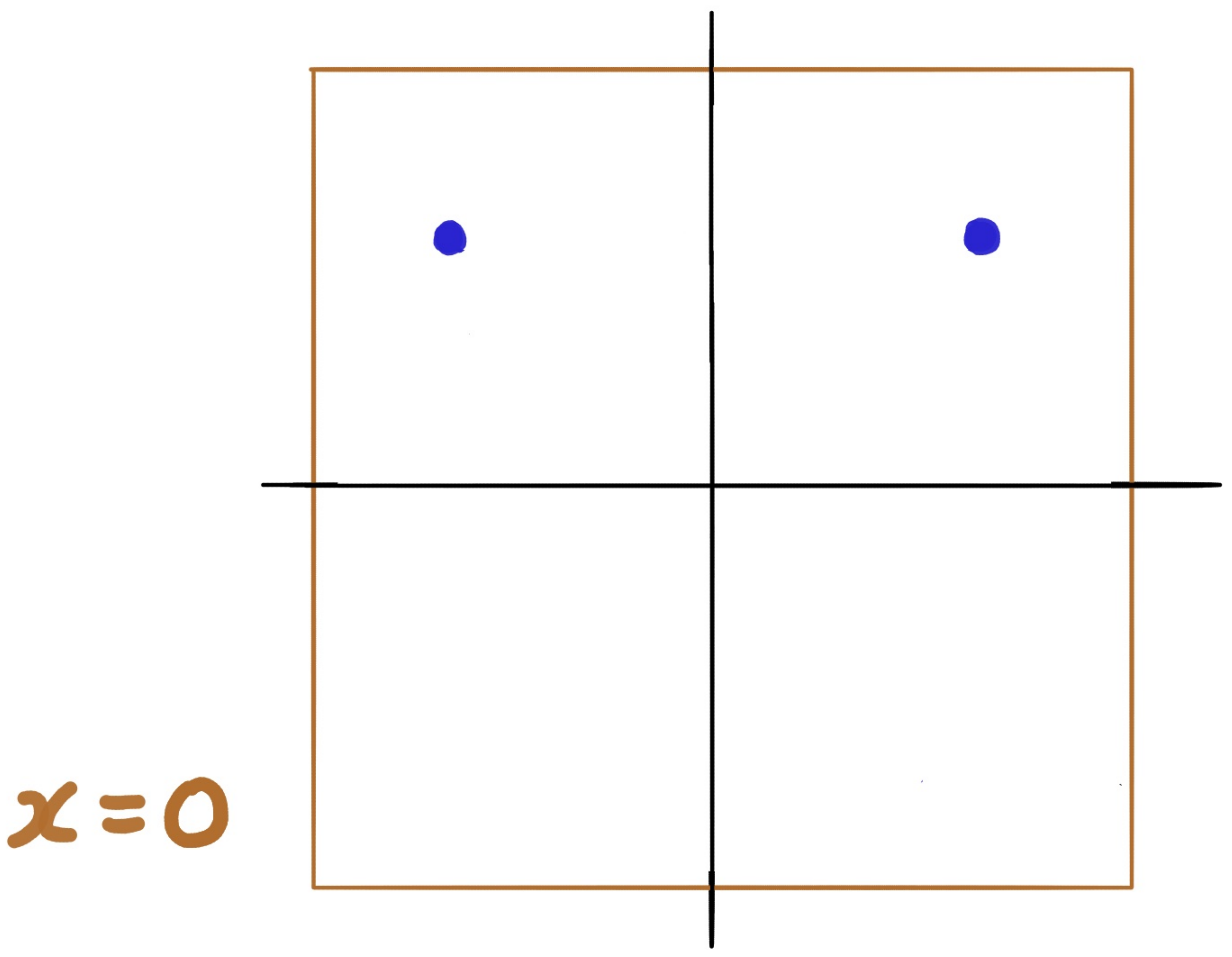
$\rho = \rho_0$

$\phi = \phi_0$

Spherical: $\rho = \rho_0,\ \phi = \phi_0$
is circle of radius $\rho_0 \sin\phi_0$



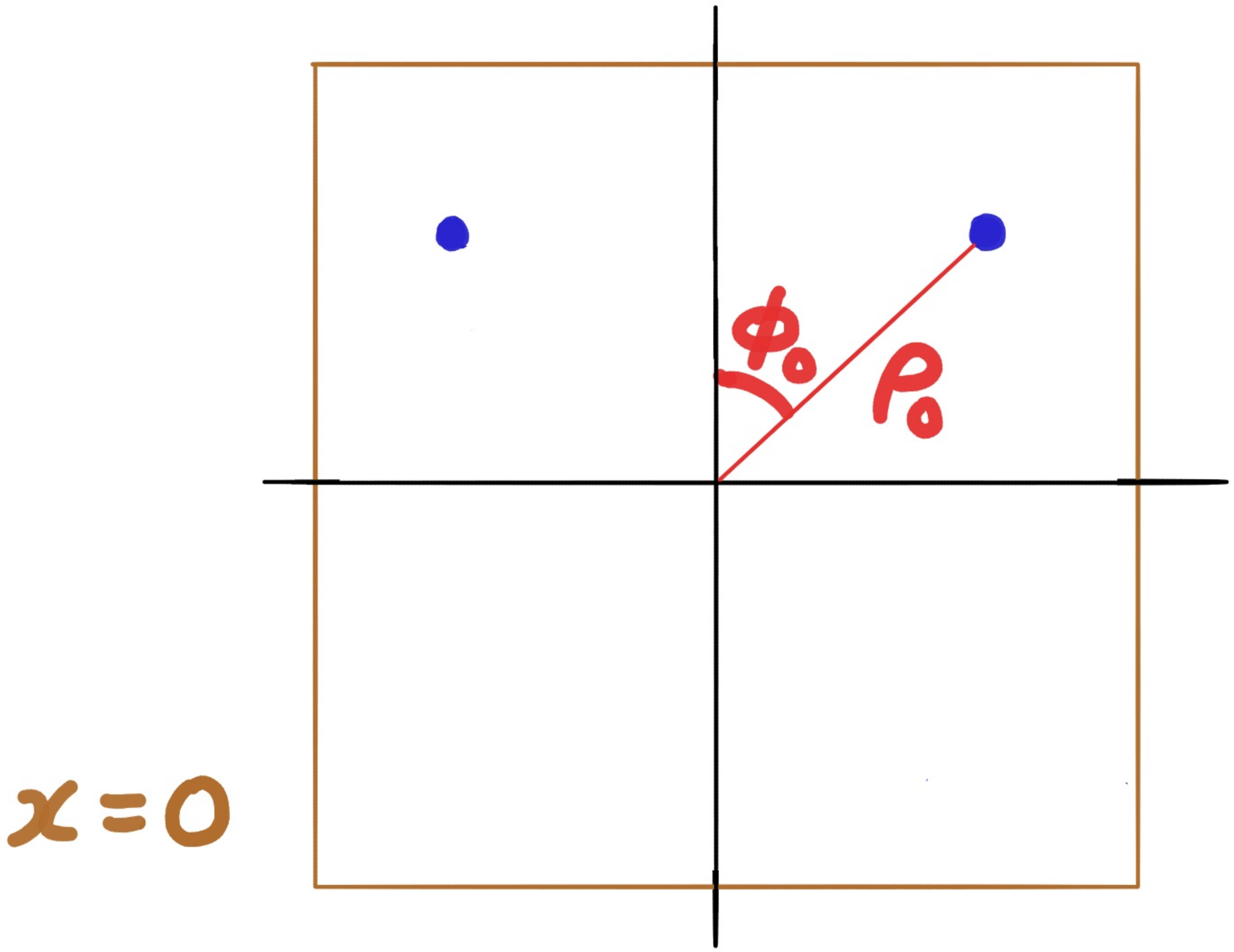
Spherical: $\rho = \rho_0$, $\phi = \phi_0$ is circle of radius $\rho_0 \sin\phi_0$



$\rho = \rho_0$

$\phi = \phi_0$

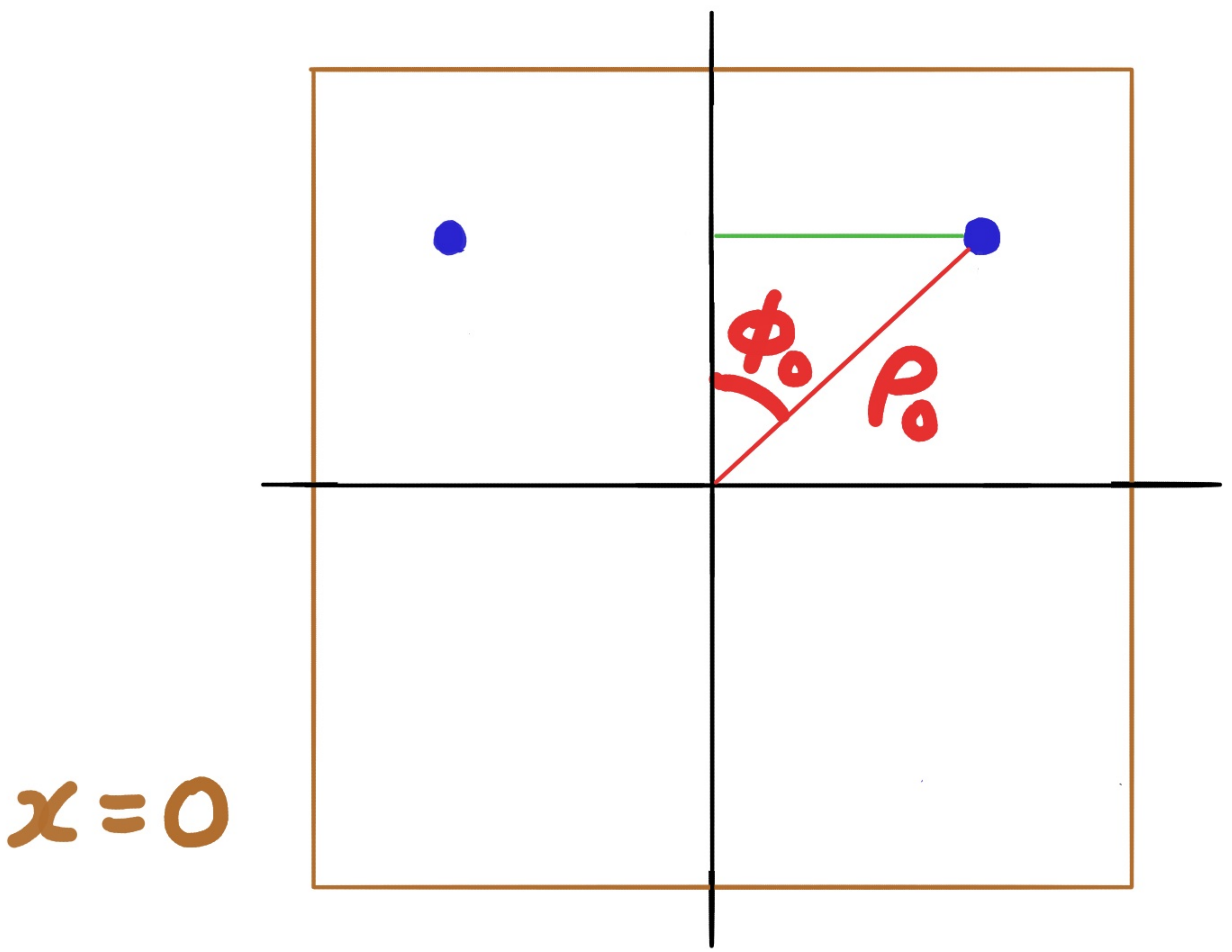
Spherical: $\rho = \rho_0$, $\phi = \phi_0$ is circle of radius $\rho_0 \sin\phi_0$



$\rho = \rho_0$

$\phi = \phi_0$

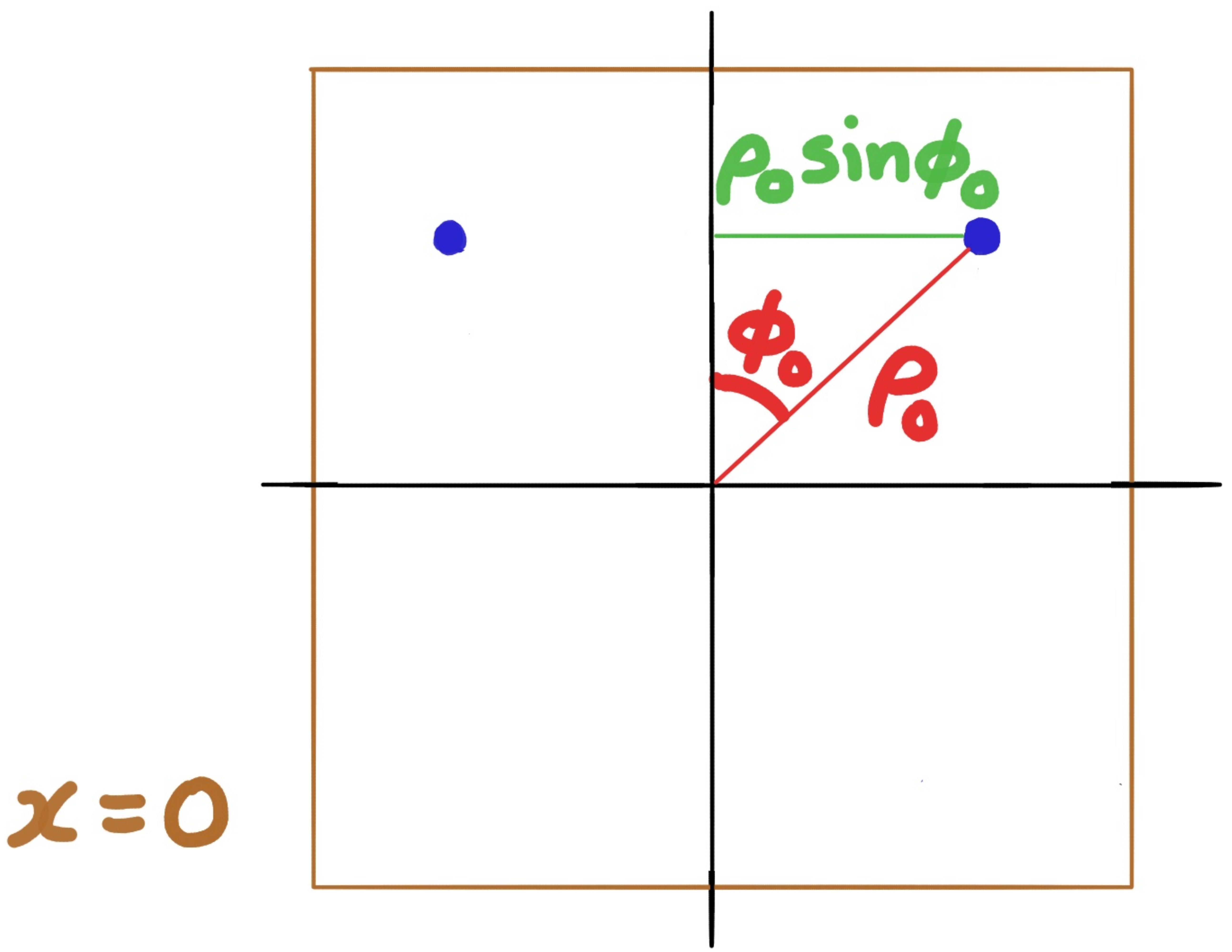
Spherical: $\rho = \rho_0$, $\phi = \phi_0$ is circle of radius $\rho_0 \sin\phi_0$



$\rho = \rho_0$

$\phi = \phi_0$

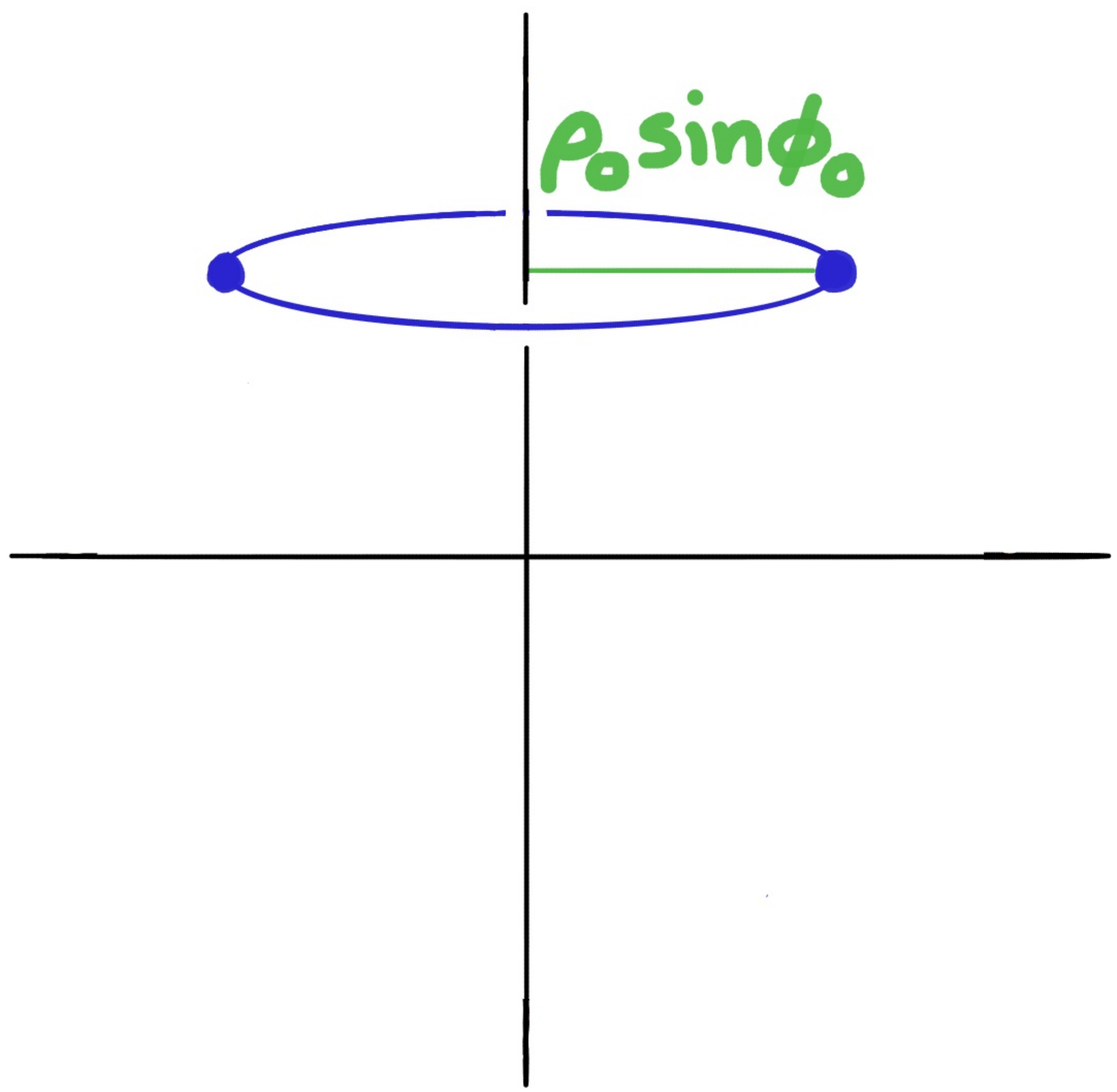
Spherical: $\rho=\rho_0$, $\phi=\phi_0$ is circle of radius $\rho_0\sin\phi_0$



$\rho=\rho_0$

$\phi=\phi_0$

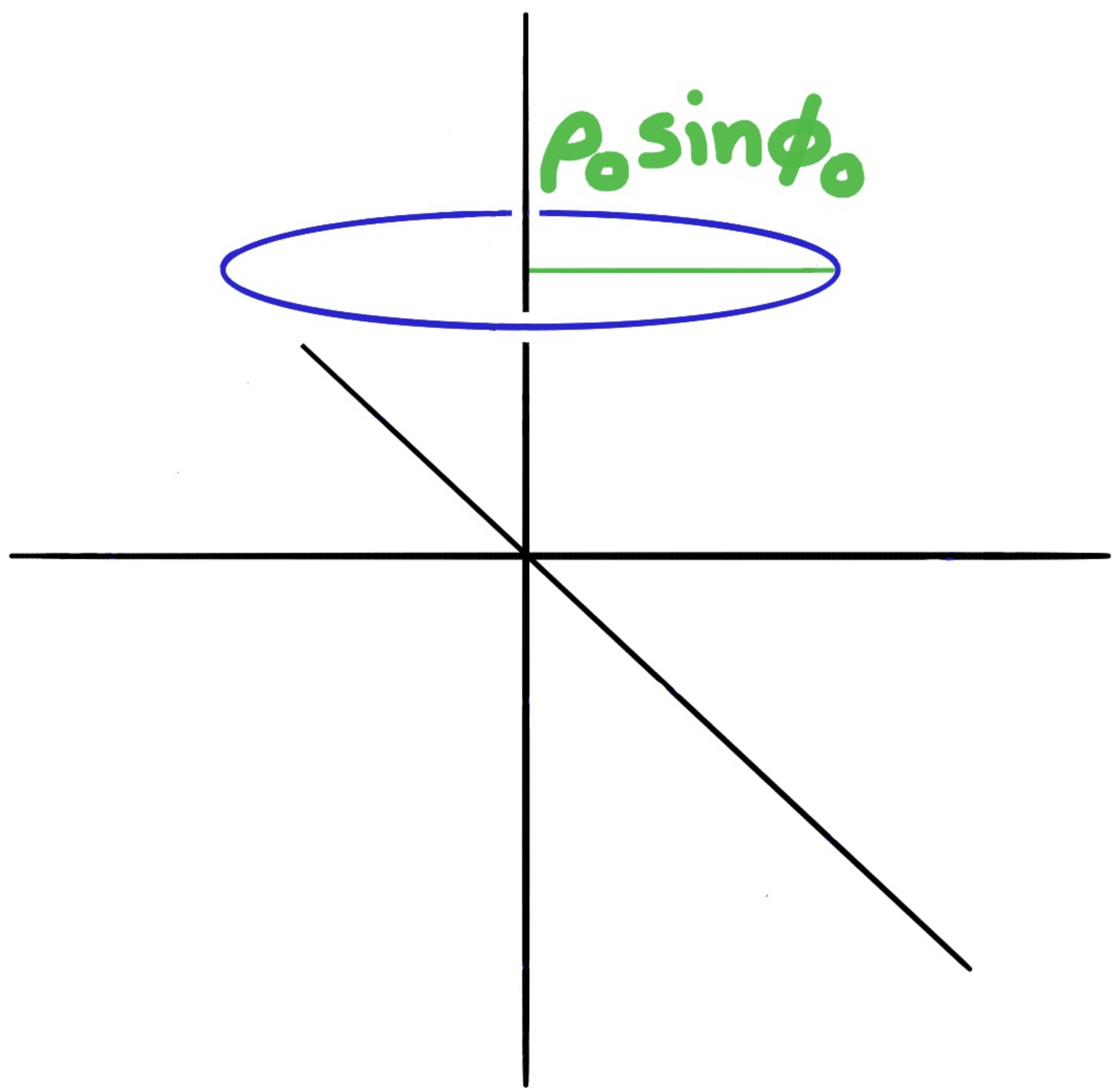
Spherical: $\rho = \rho_0$, $\phi = \phi_0$ is circle of radius $\rho_0 \sin\phi_0$



$\rho = \rho_0$

$\phi = \phi_0$

Spherical: $\rho=\rho_0,\ \phi=\phi_0$ is circle of radius $\rho_0\sin\phi_0$



$\rho=\rho_0$

$\phi=\phi_0$

II Triple integrals

$$\iiint_R f(x,y,z)\,dV$$

measures a 4-dimensional volume

$$\iiint_R f(x,y,z)\,dV$$

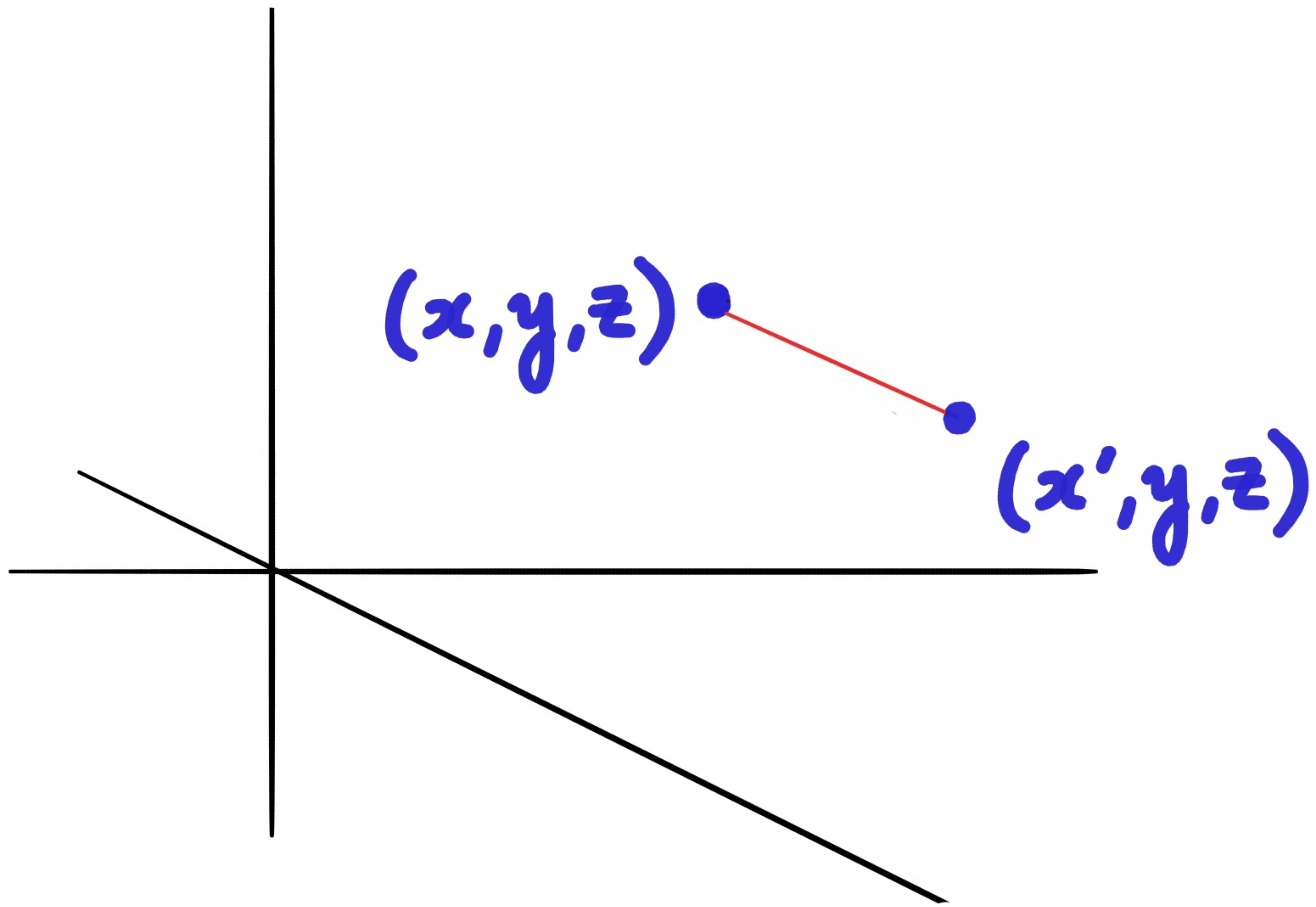
region in $\mathbb{R}^3$

height

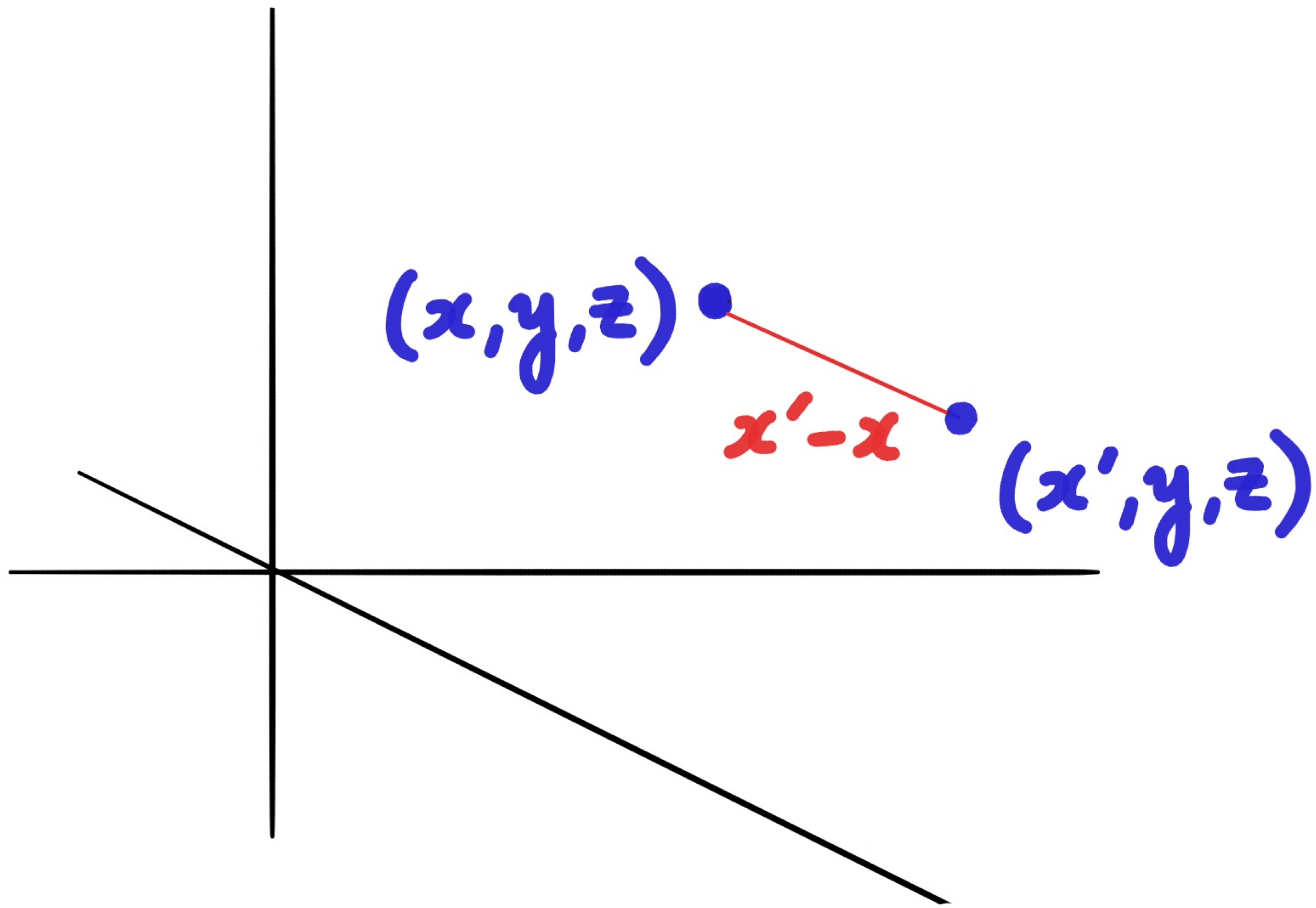
way to measure and multiply widths of 3 coordinates

measures a 4-dimensional volume

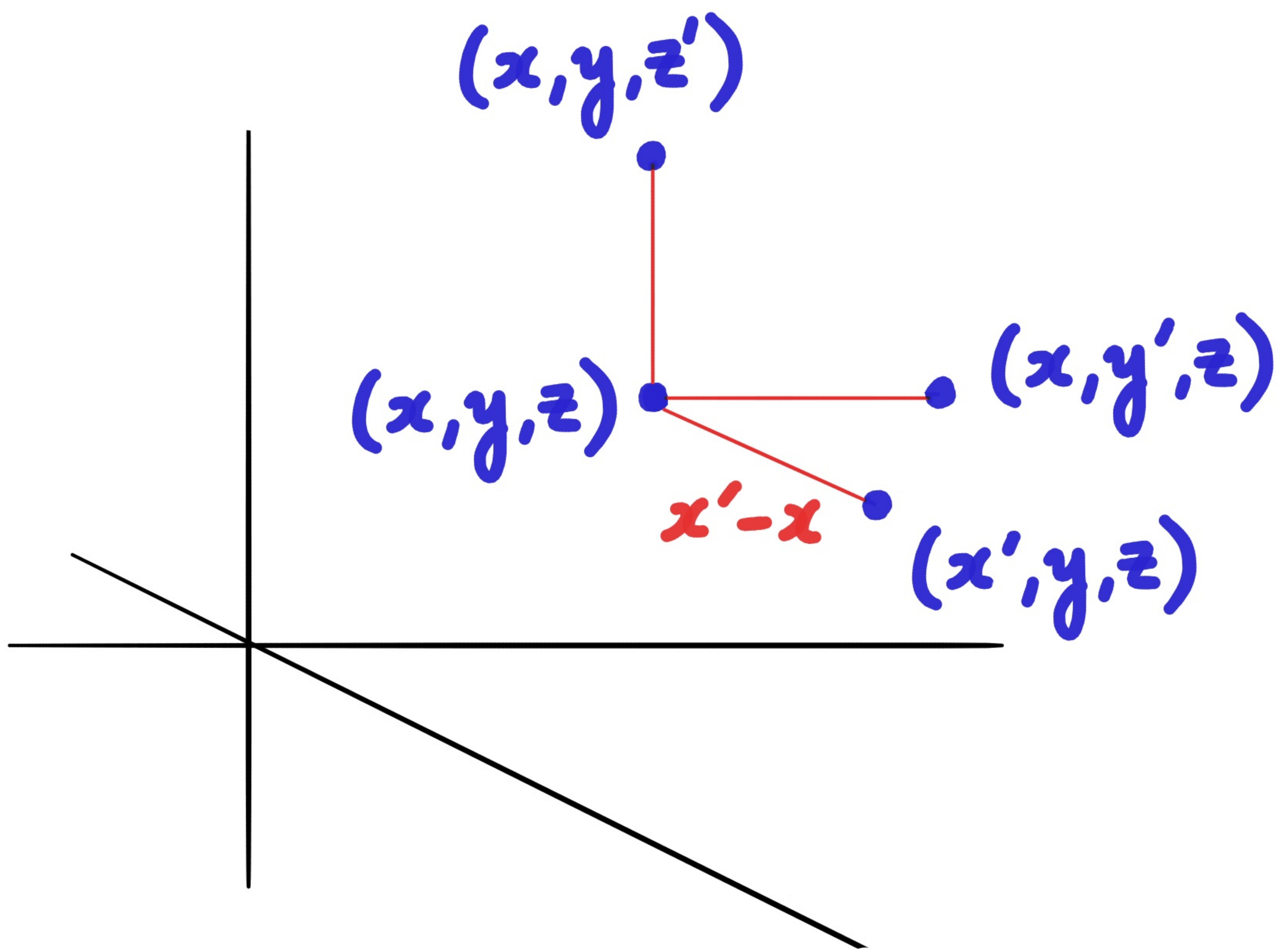
Widths and volumes in $\mathbb{R}^3$: Euclidean



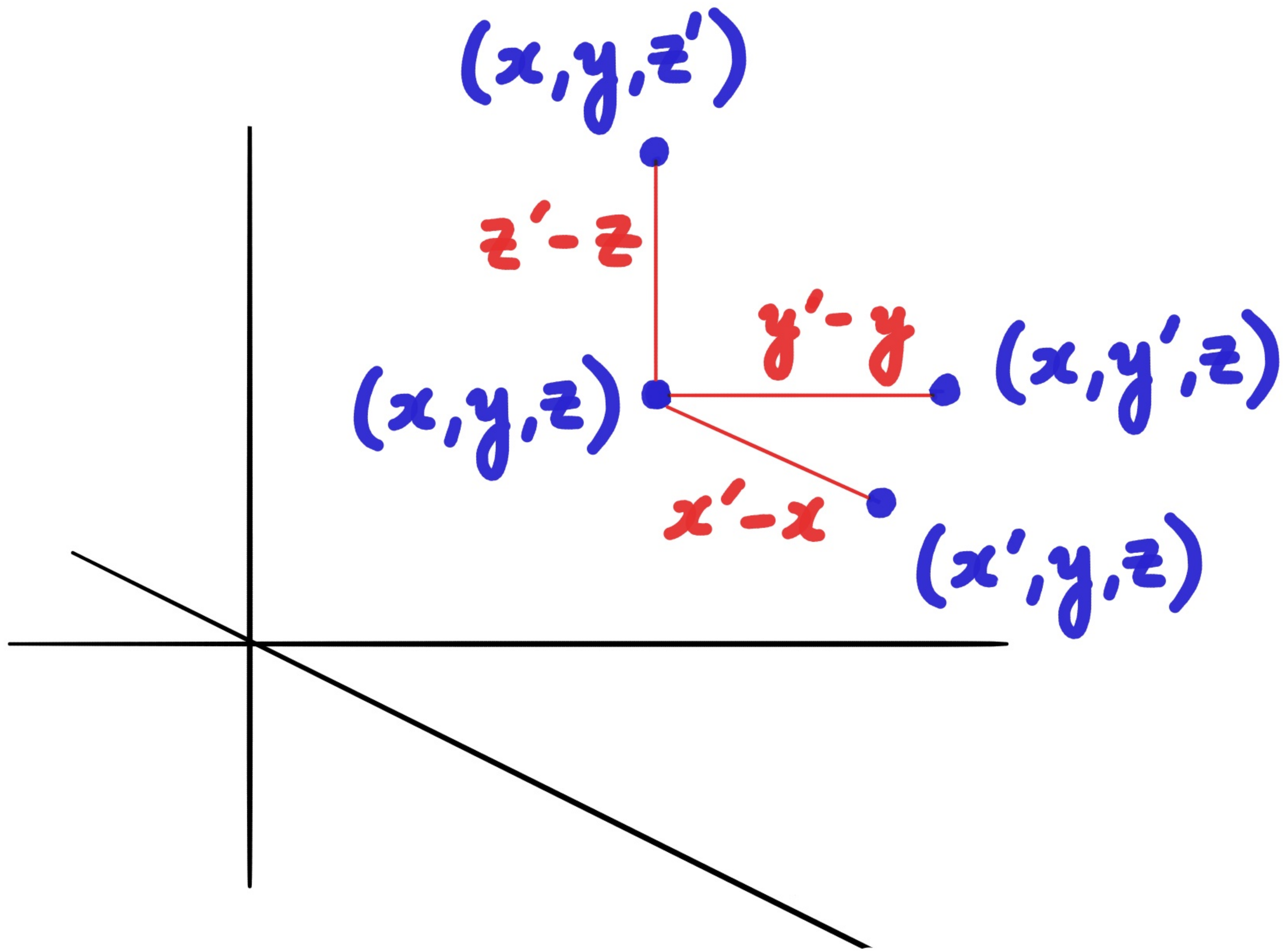
Widths and volumes in $\mathbb{R}^3$: Euclidean



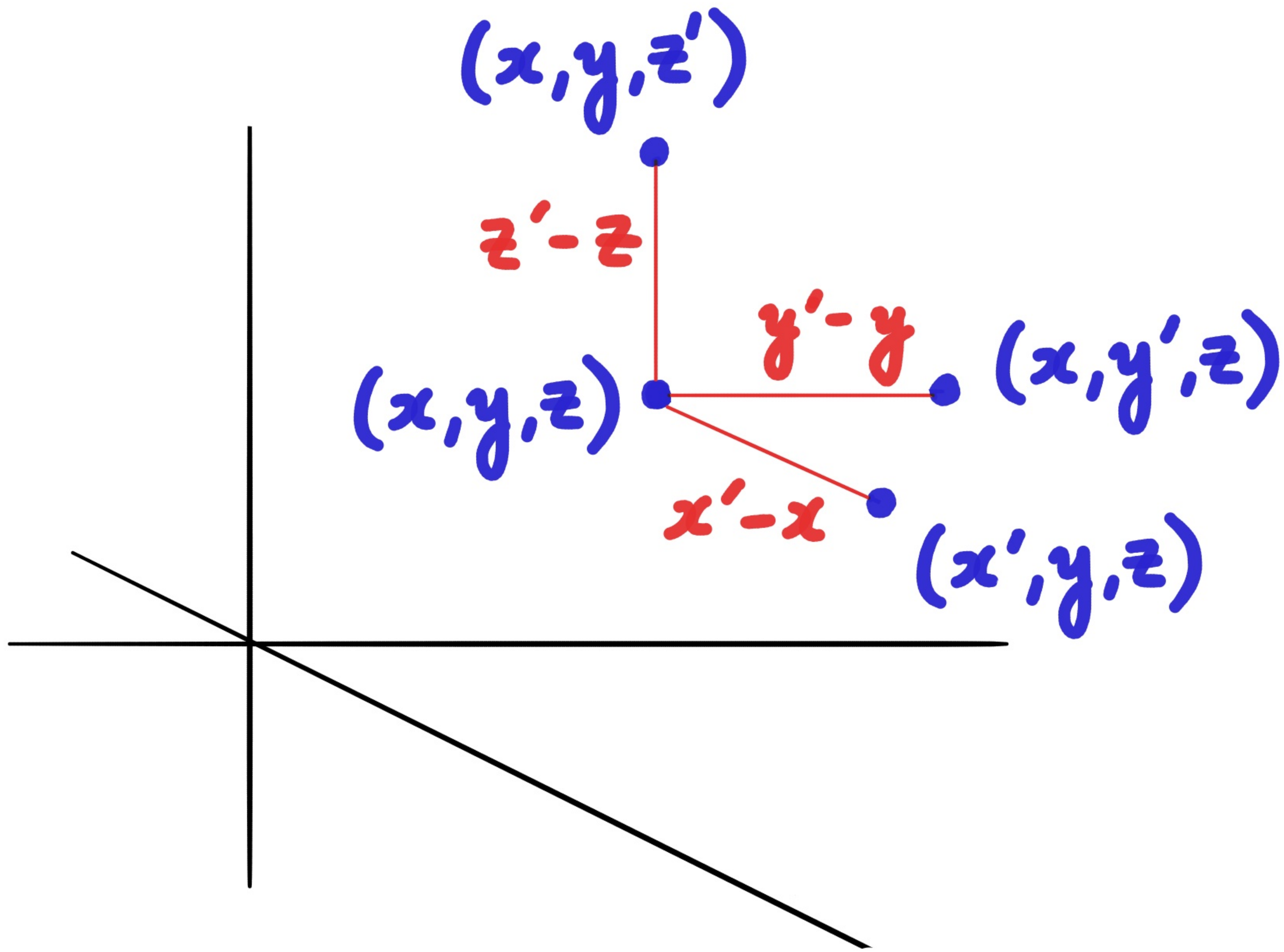
Widths and volumes in $\mathbb{R}^3$: Euclidean



Widths and volumes in $\mathbb{R}^3$: Euclidean



Widths and volumes in $\mathbb{R}^3$: Euclidean



$$\boxed{dV = dx\,dy\,dz}$$

Euclidean Coordinates

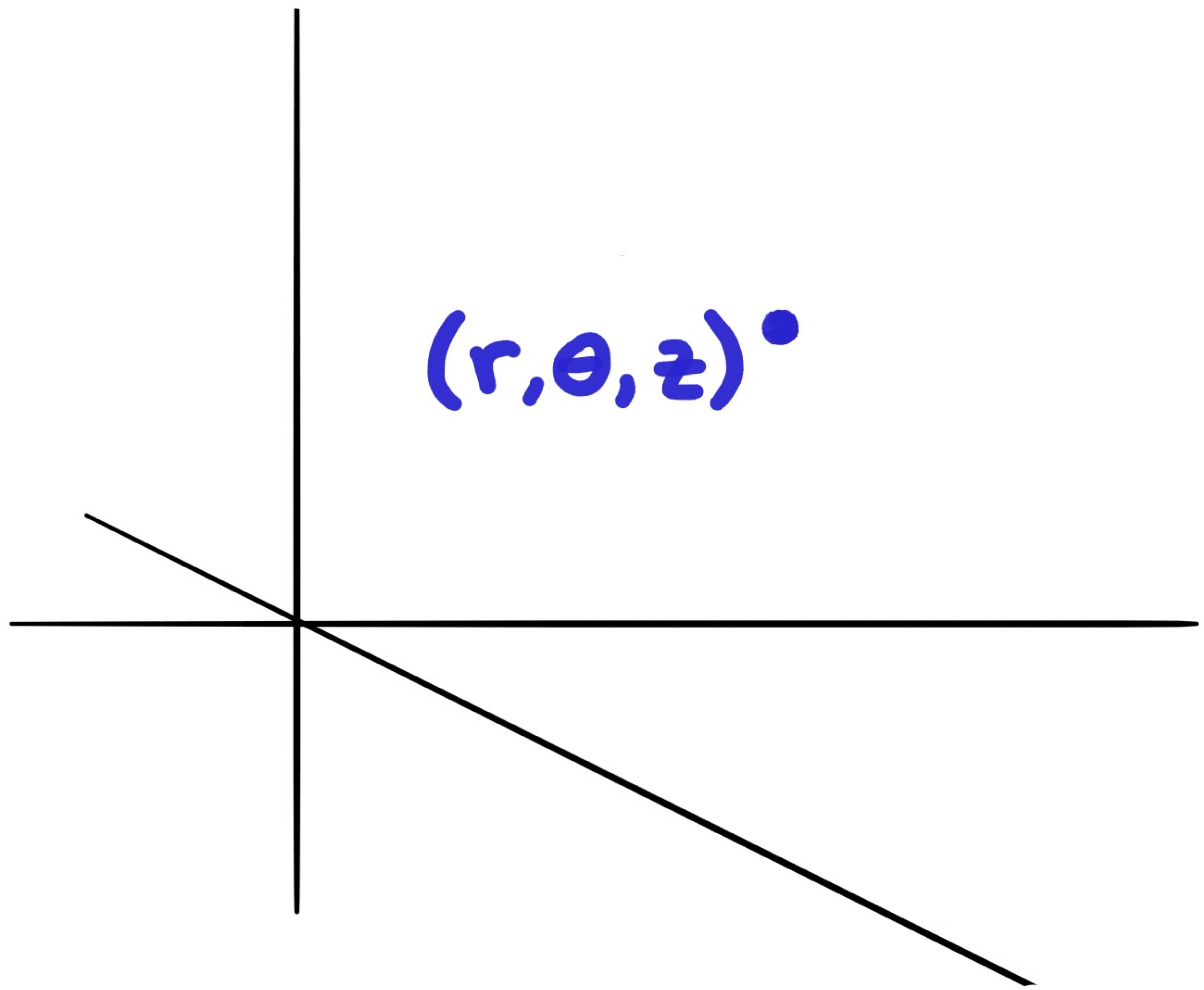
$$\iiint_R f(x,y,z)\,dV = \iiint_R f(x,y,z)\,dx\,dy\,dz$$

Euclidean Coordinates

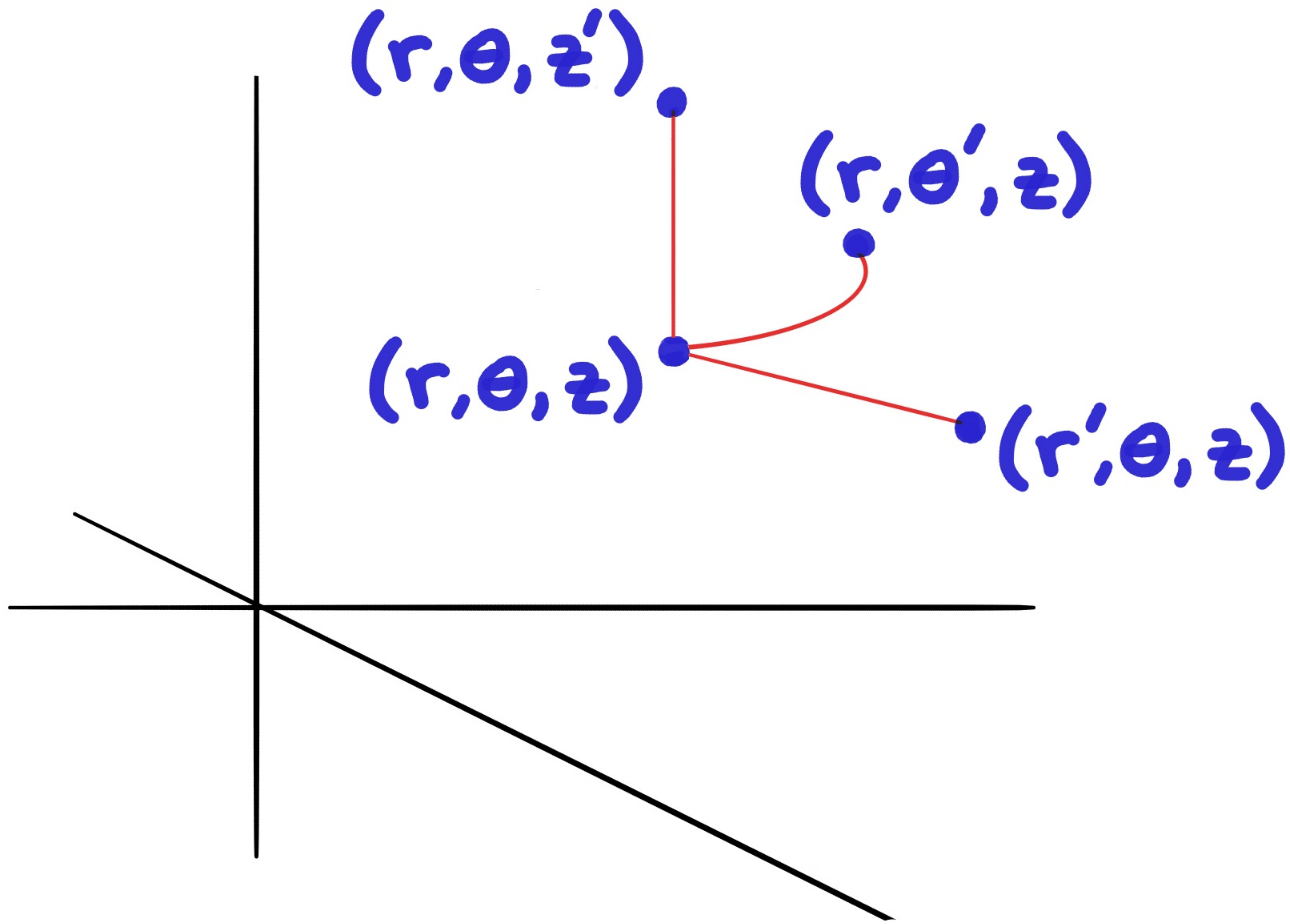
$$\iiint\limits_R f(x,y,z)\,dV = \iiint\limits_R f(x,y,z)\,dx\,dy\,dz$$

$$dV \longmapsto dx\,dy\,dz$$

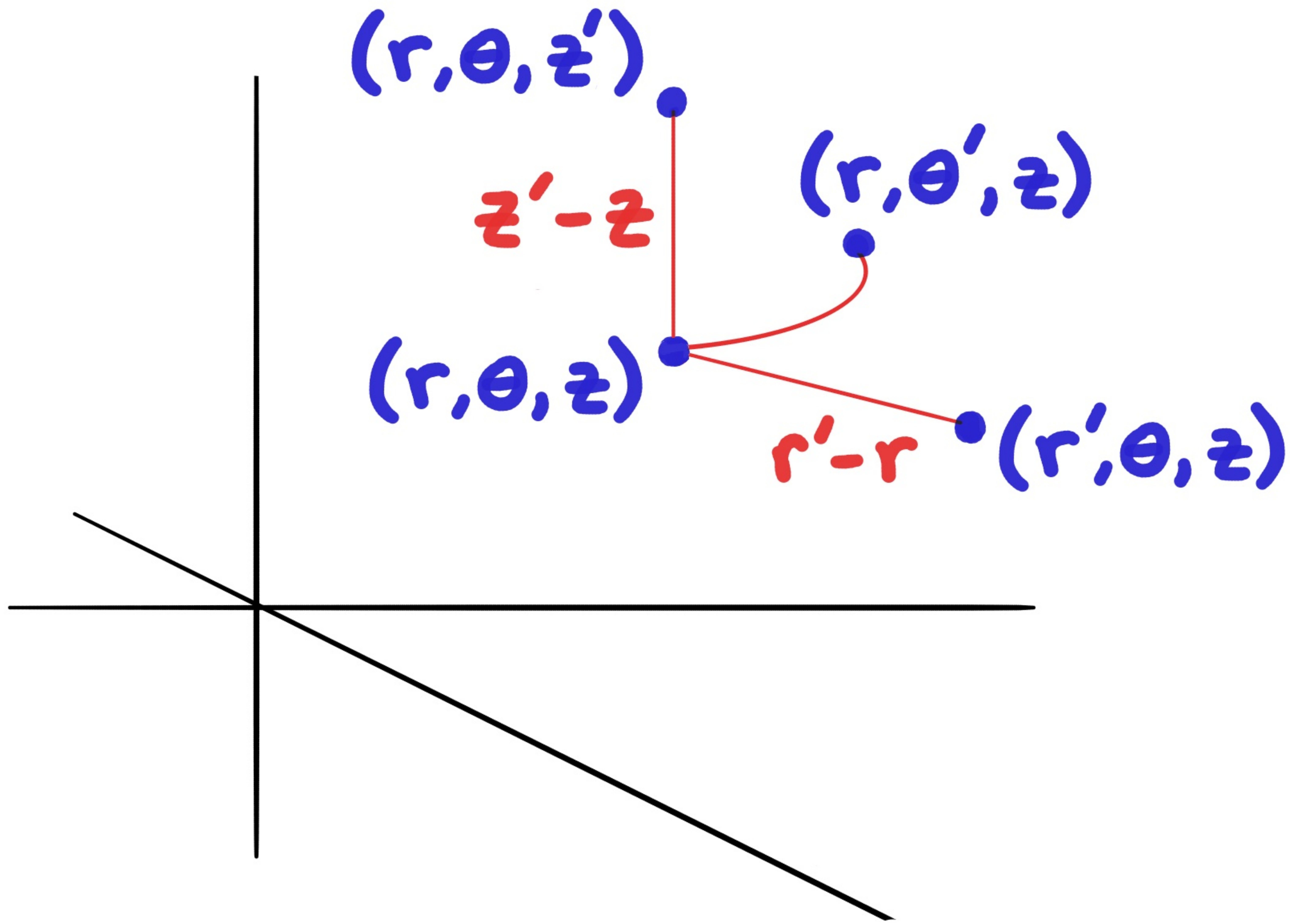
Widths and volumes in $\mathbb{R}^3$: Cylindrical



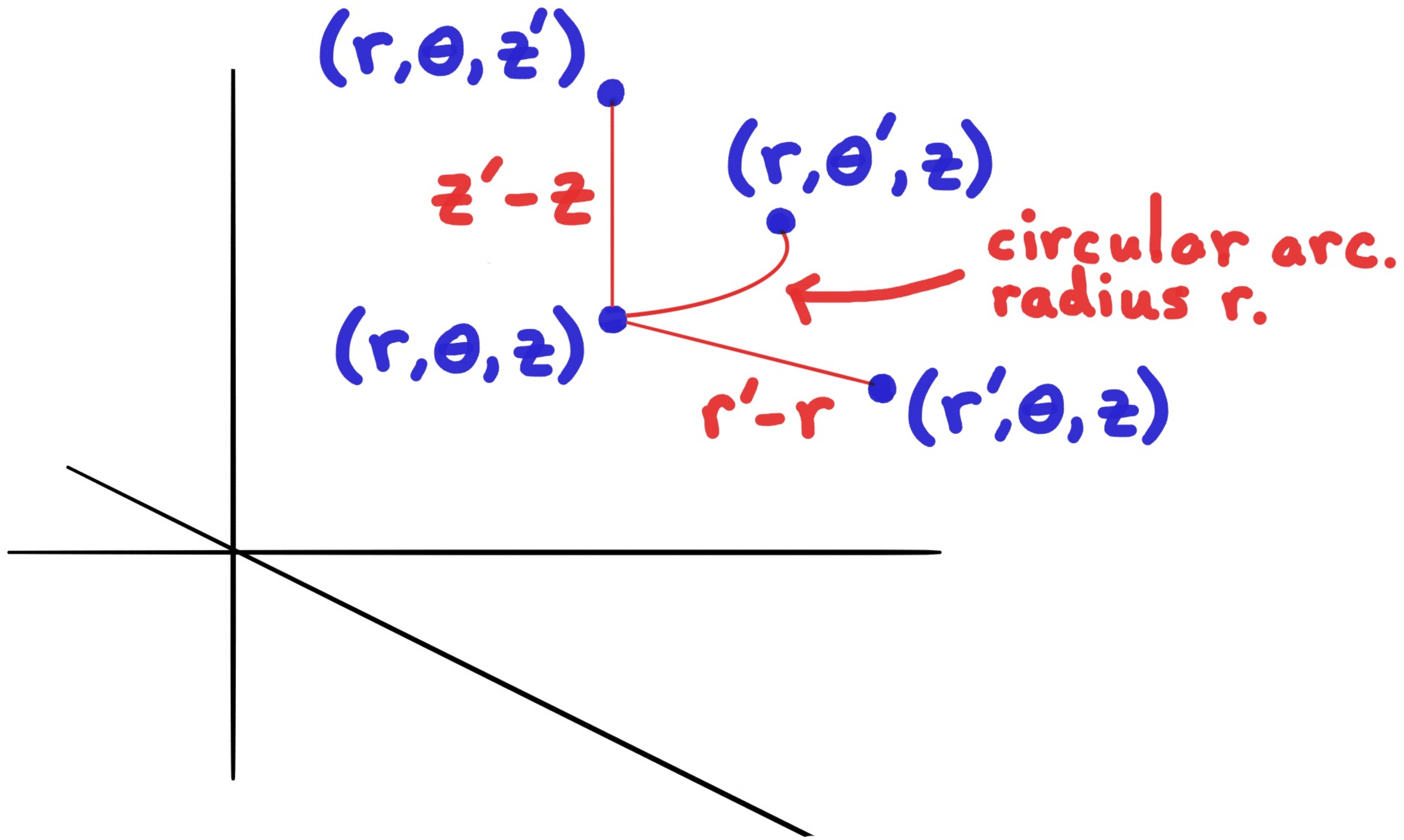
Widths and volumes in $\mathbb{R}^3$: Cylindrical



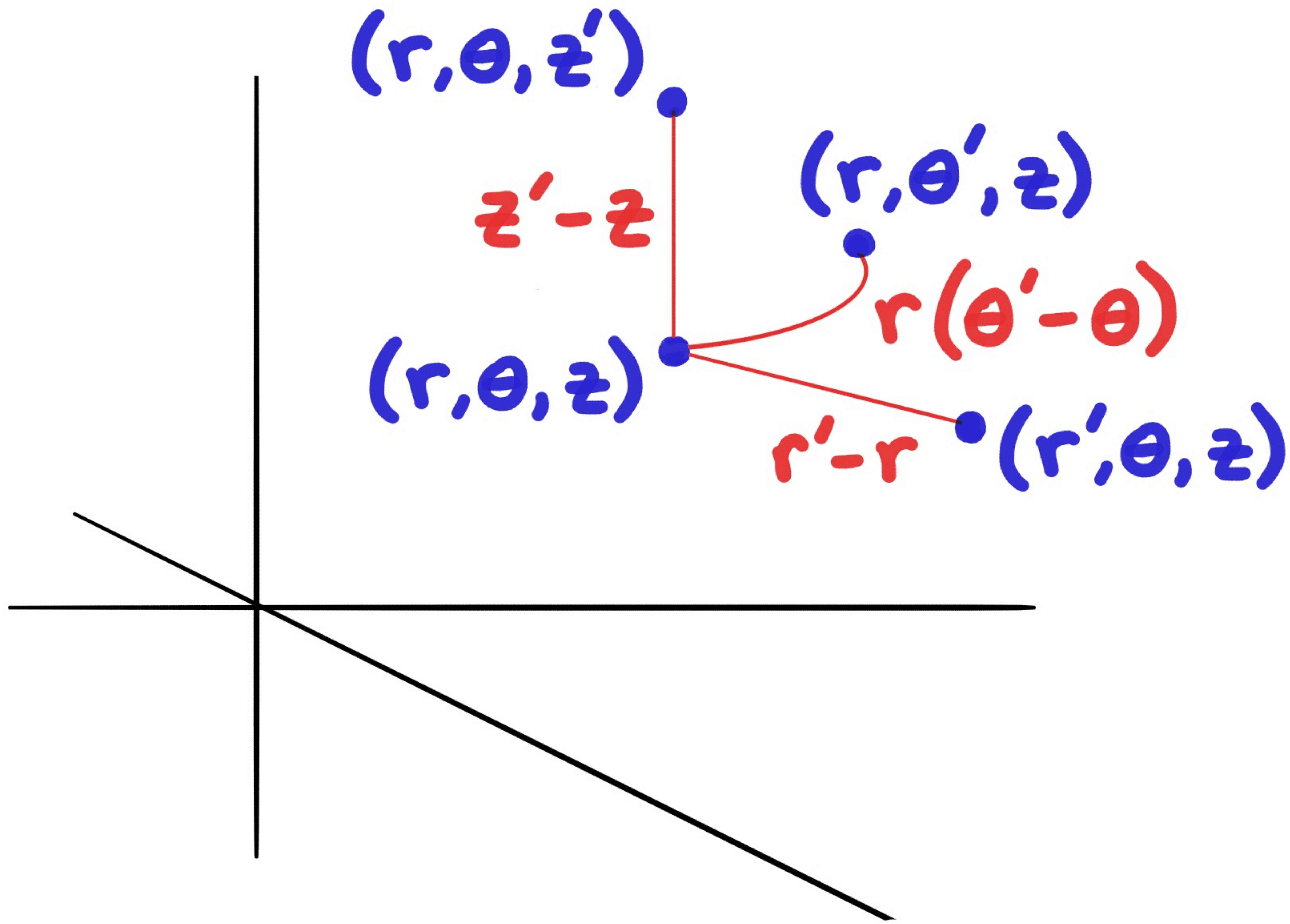
Widths and volumes in $\mathbb{R}^3$: Cylindrical



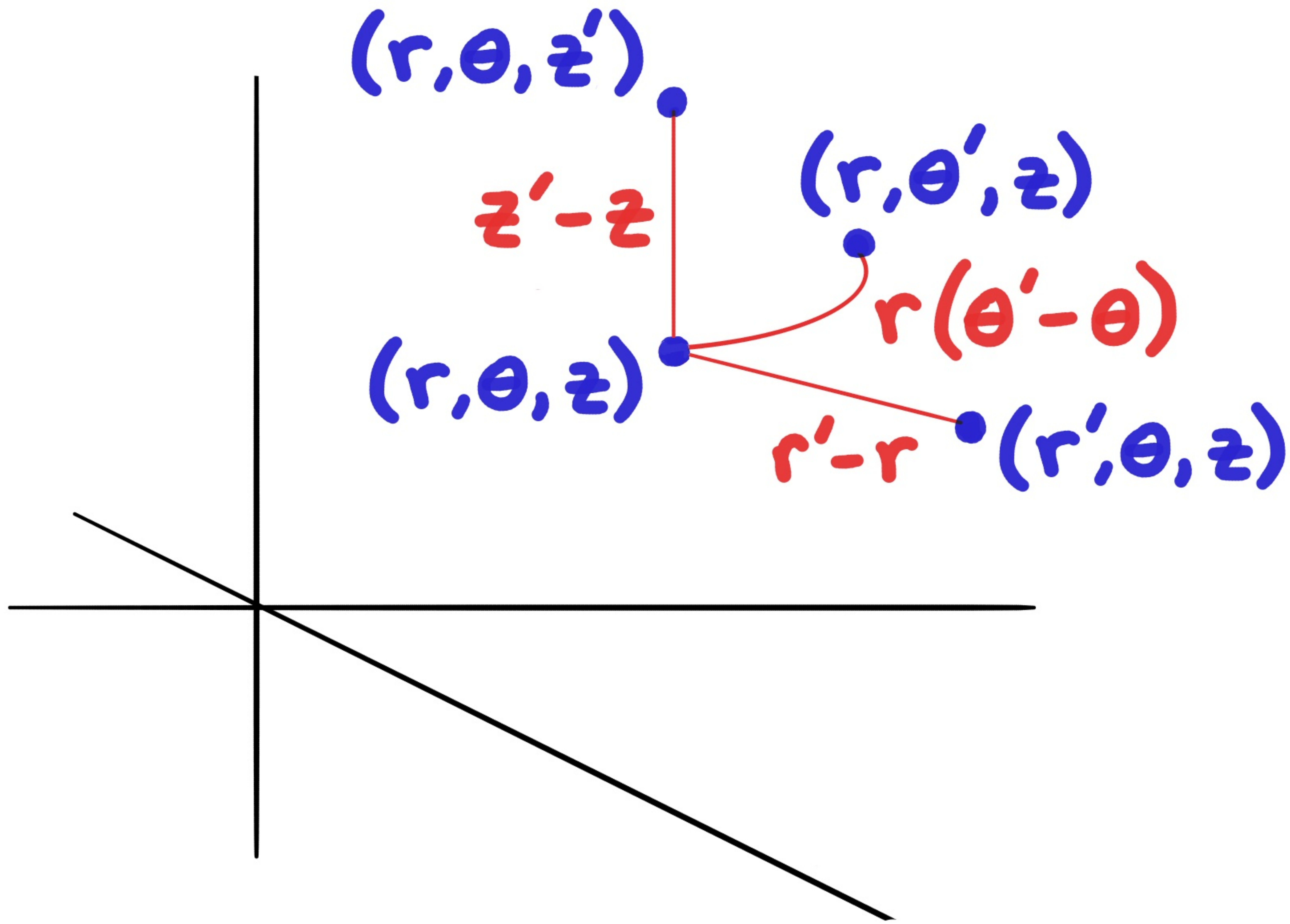
Widths and volumes in $\mathbb{R}^3$: Cylindrical



Widths and volumes in $\mathbb{R}^3$: Cylindrical



Widths and volumes in $\mathbb{R}^3$: Cylindrical



$$\boxed{dV = r\,dr\,d\theta\,dz}$$

Cylindrical Coordinates

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$$

Cylindrical Coordinates

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(r\cos\theta, r\sin\theta, z)\, r\, dr\, d\theta\, dz$$

$$dV \mapsto dr\,d\theta\,dz$$

$$x \mapsto r\cos\theta$$

$$y \mapsto r\sin\theta$$

$$z \mapsto z$$

Cylindrical Coordinates

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(r\cos\theta, r\sin\theta, z)\,r\,dr\,d\theta\,dz$$

$$dV \mapsto dr\,d\theta\,dz$$

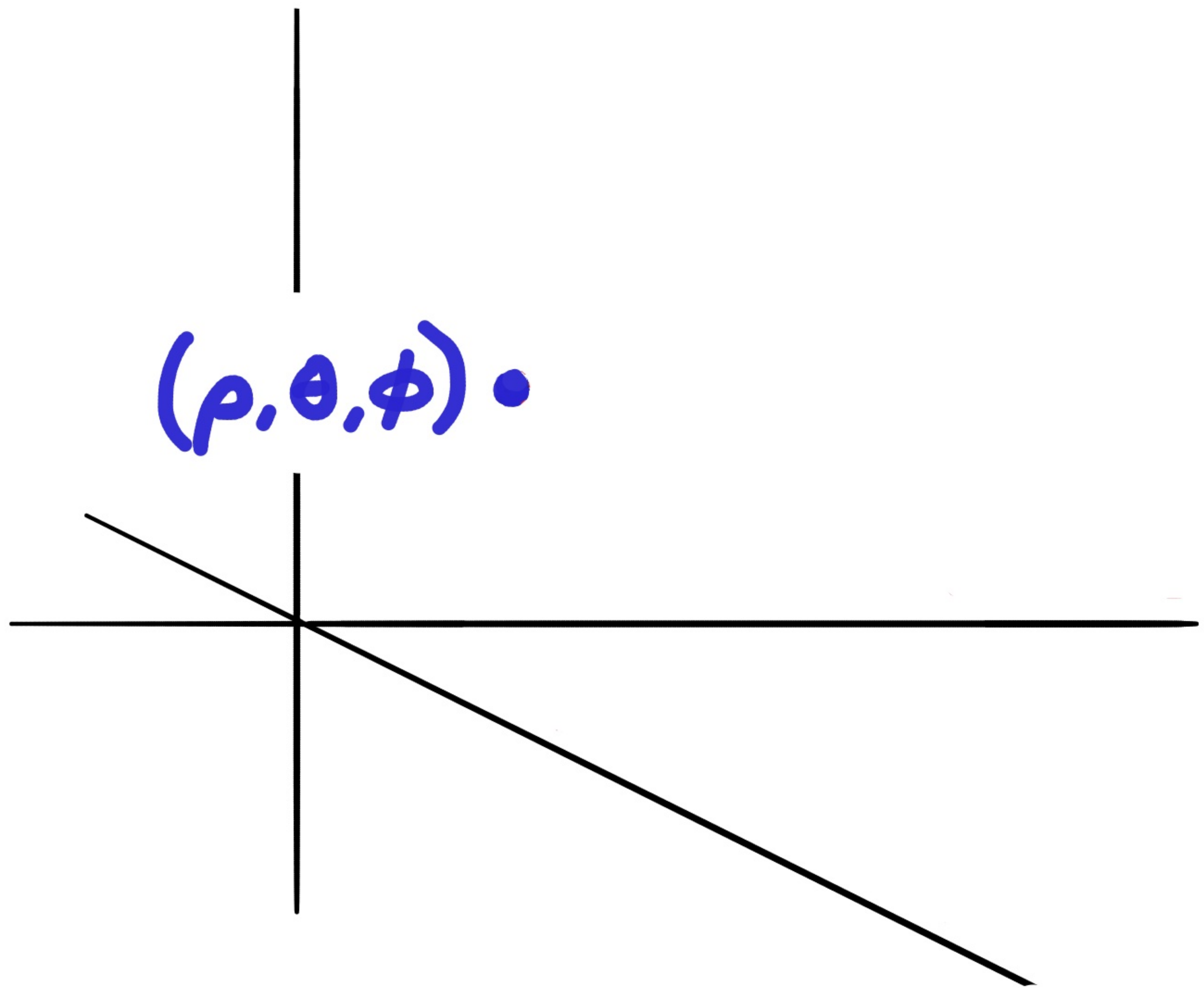
$$x \mapsto r\cos\theta$$

$$y \mapsto r\sin\theta$$

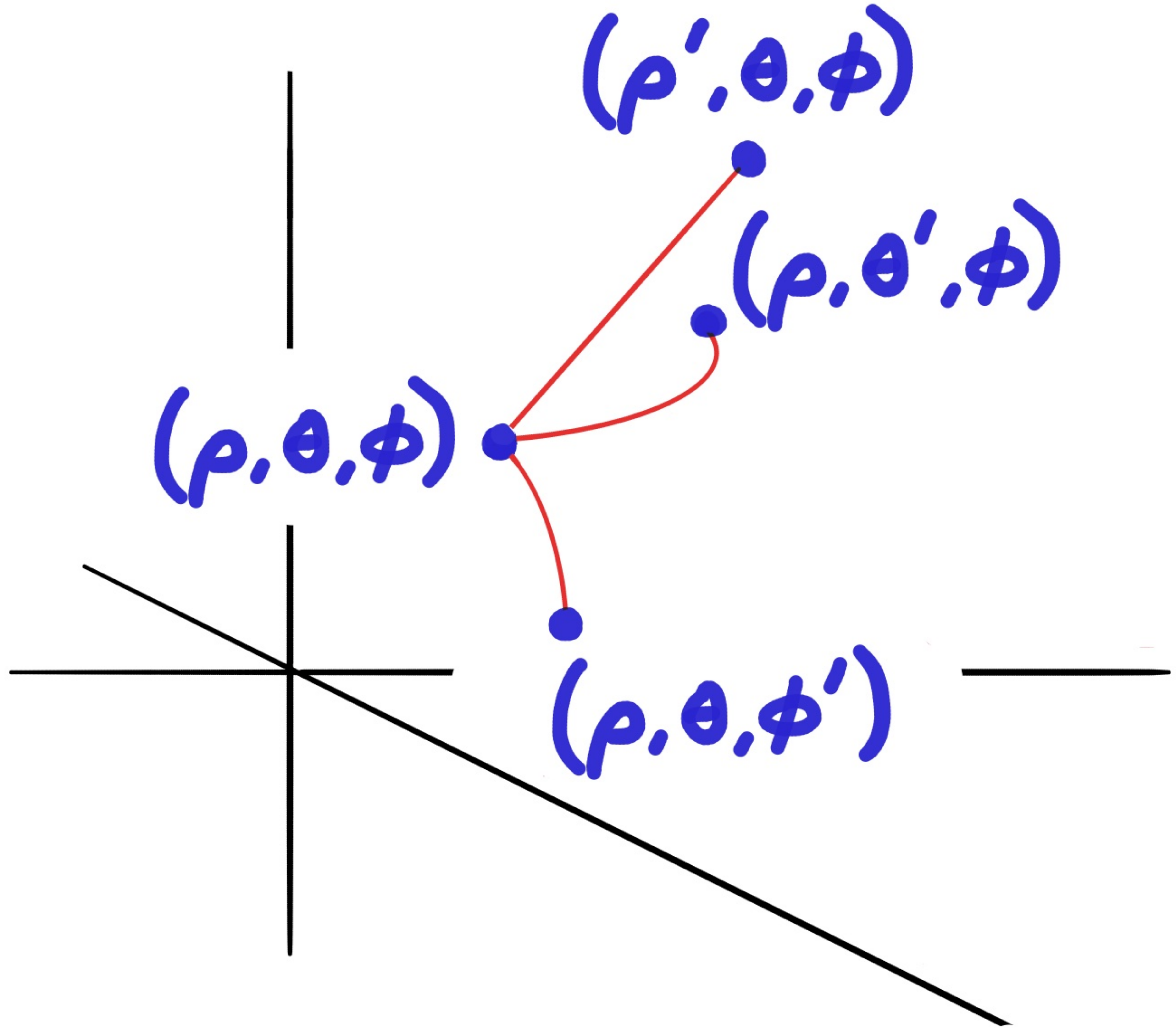
$$z \mapsto z$$

Throw in r.

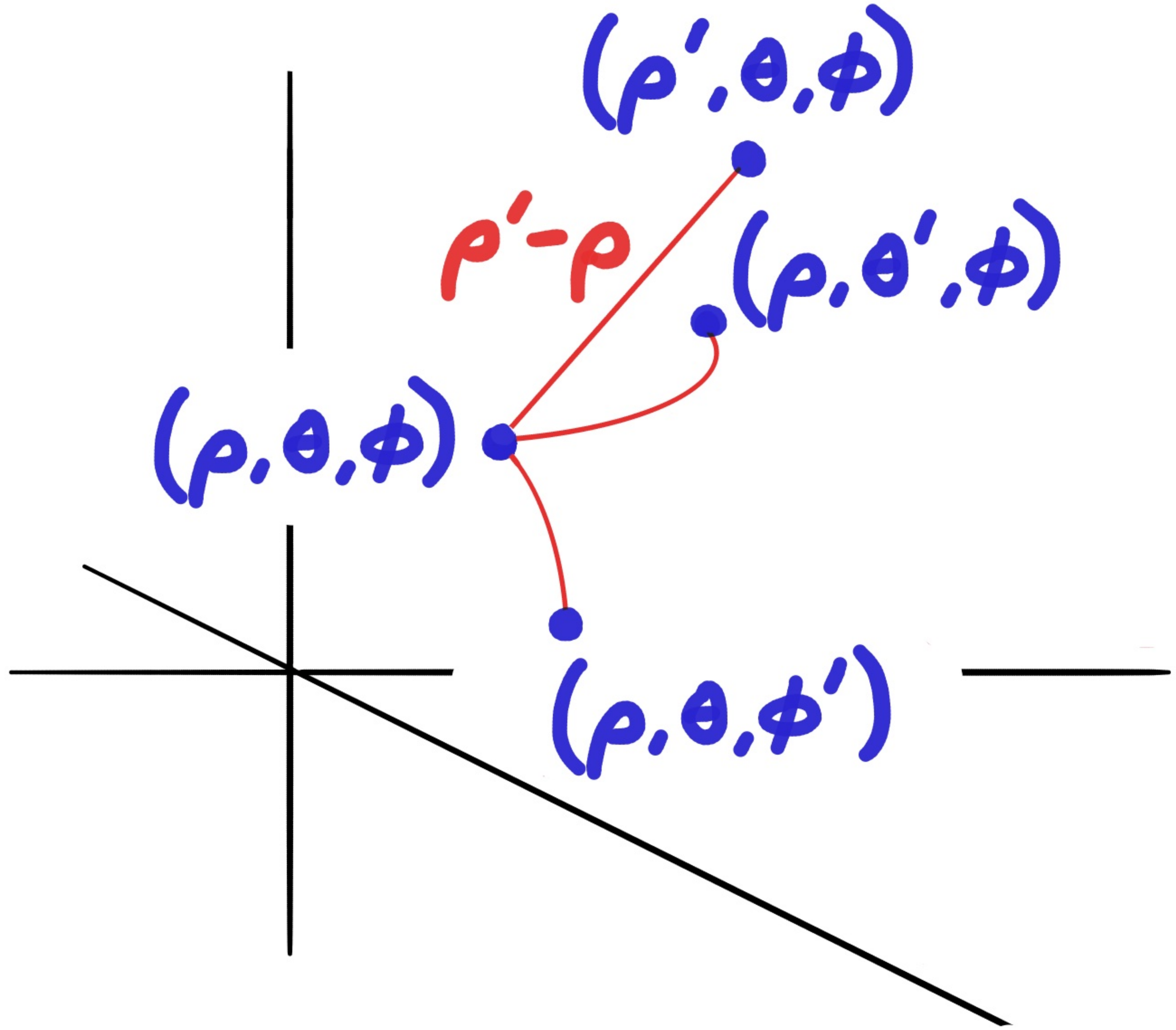
Widths and volumes in $\mathbb{R}^3$: Spherical



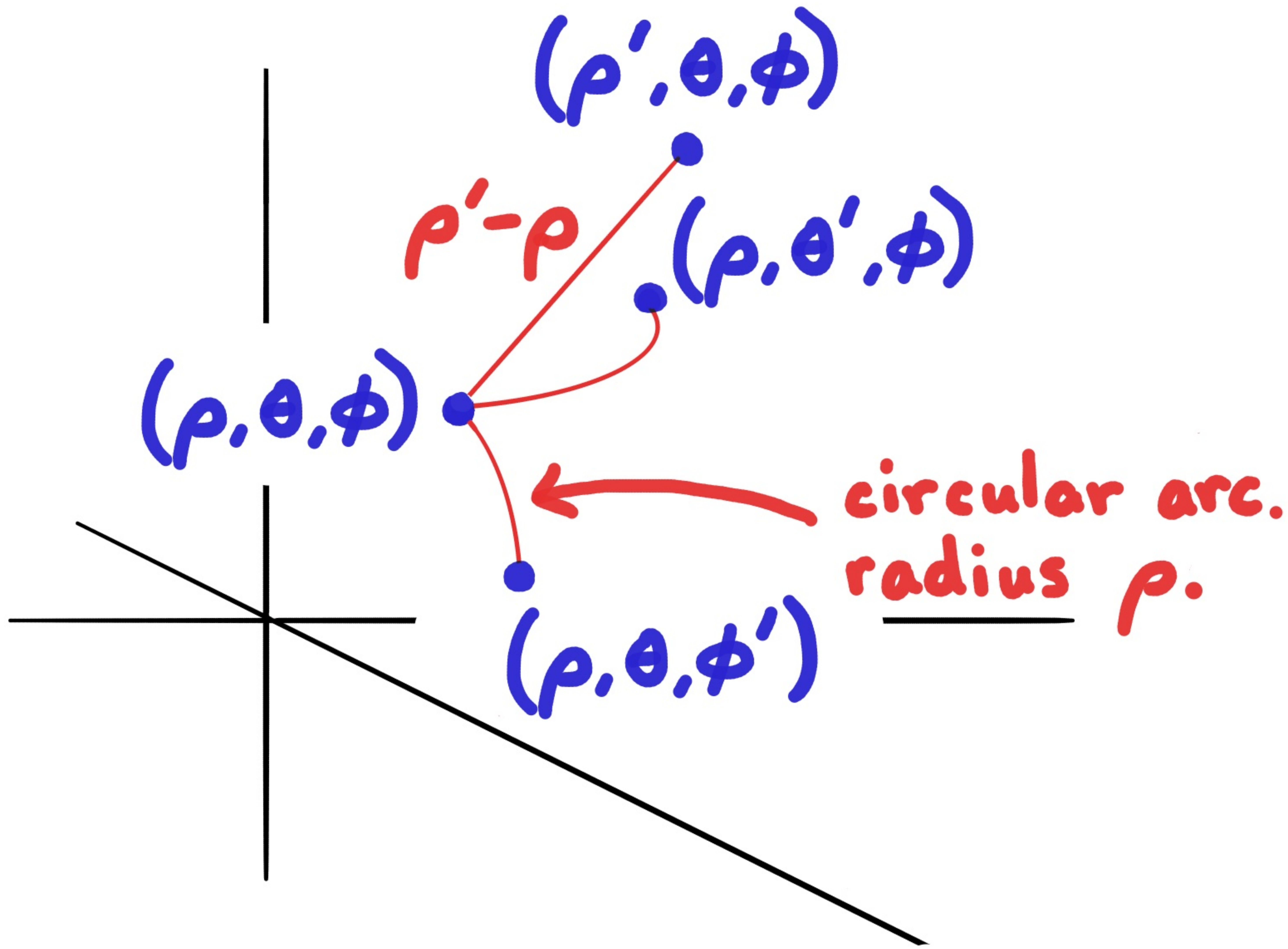
Widths and volumes in $\mathbb{R}^3$: Spherical



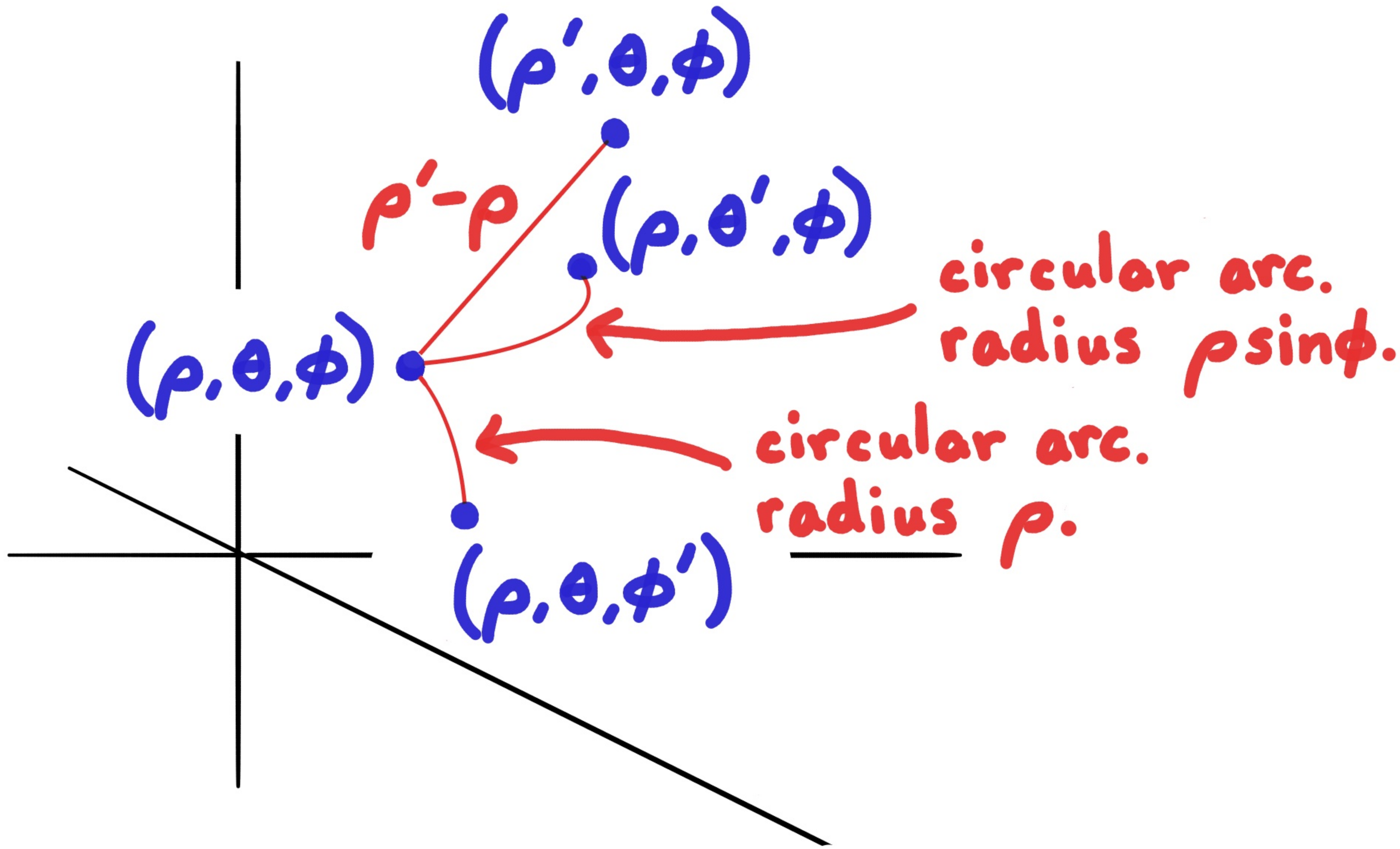
Widths and volumes in $\mathbb{R}^3$: Spherical



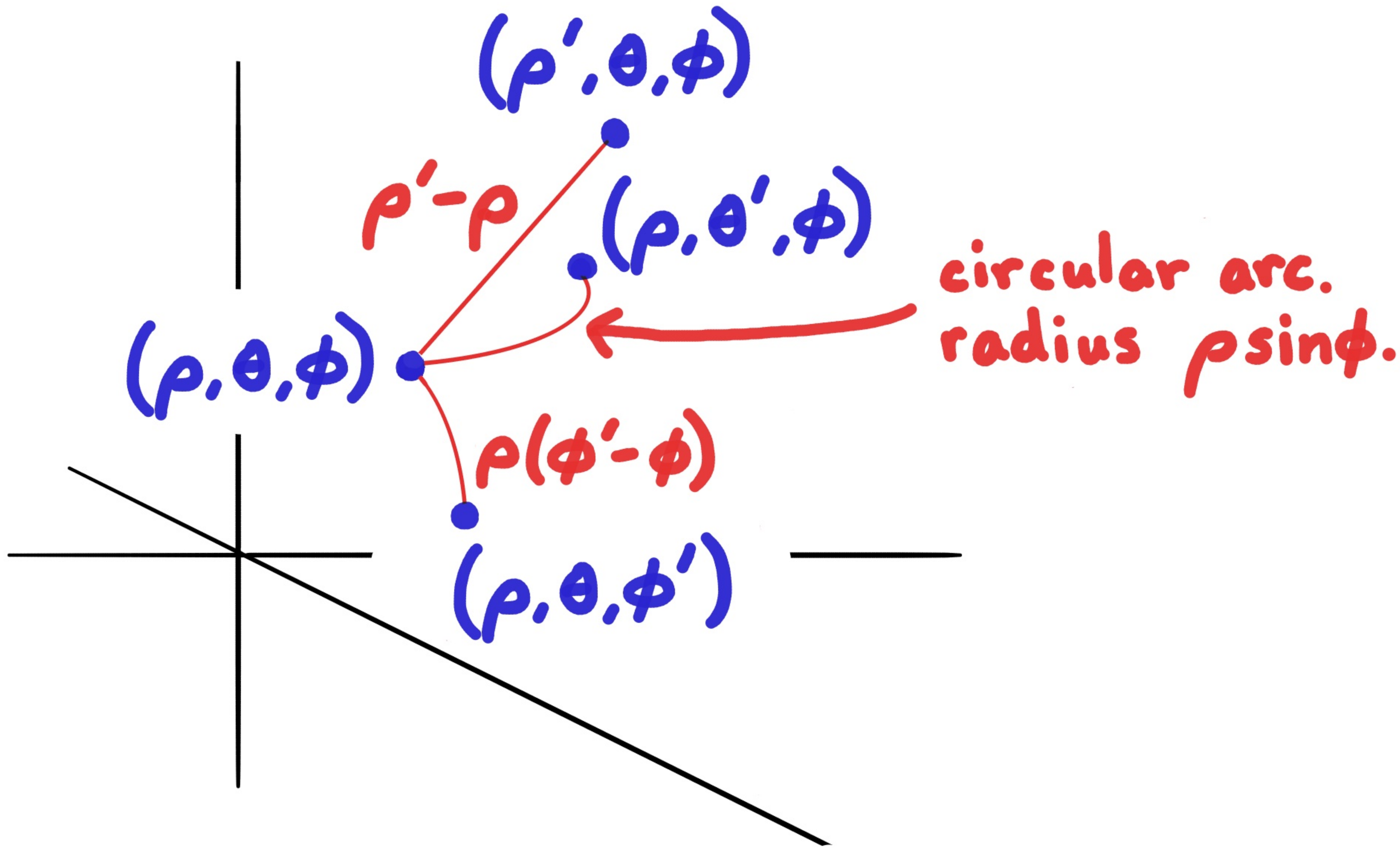
Widths and volumes in $\mathbb{R}^3$: Spherical



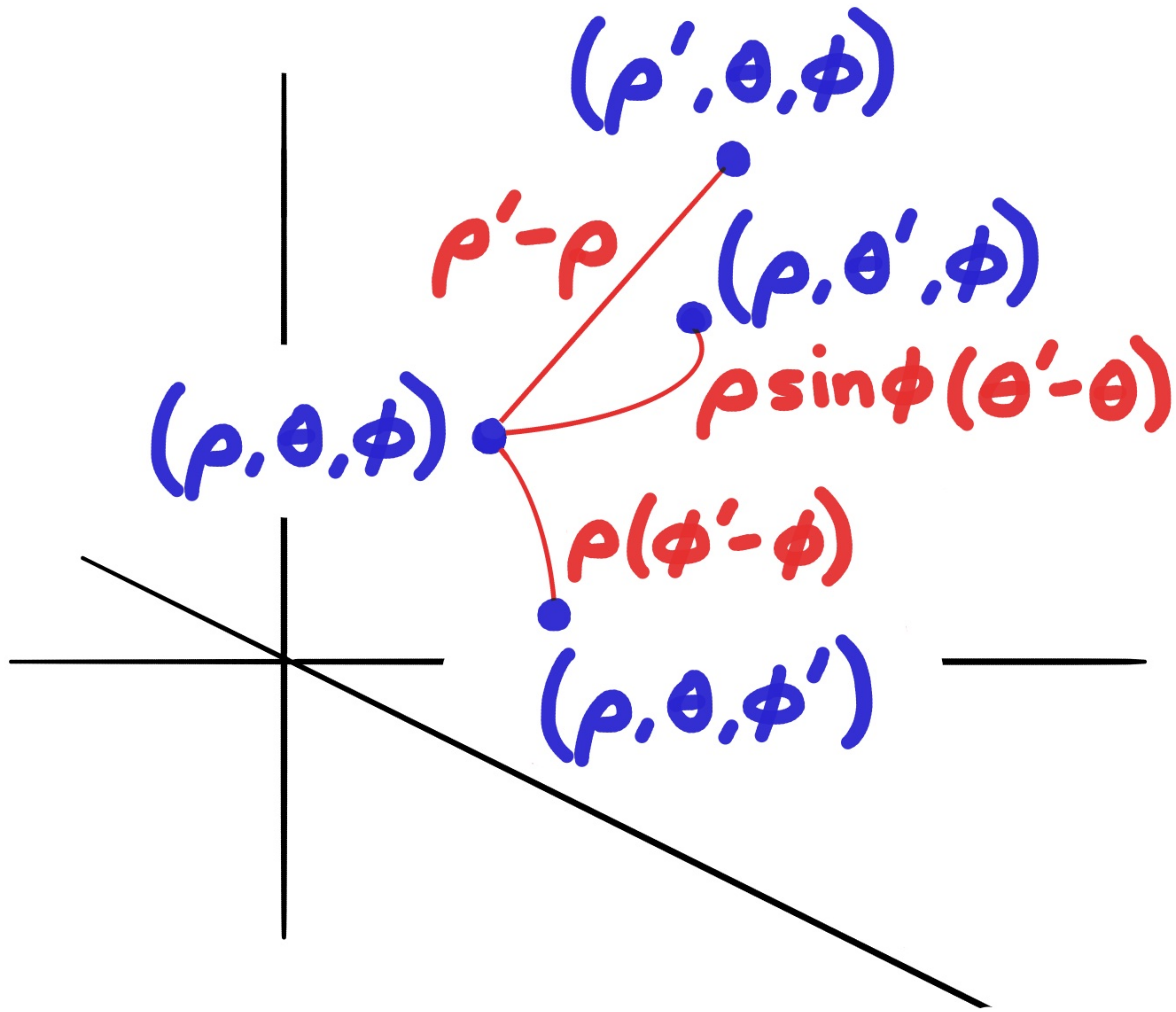
Widths and volumes in $\mathbb{R}^3$: Spherical



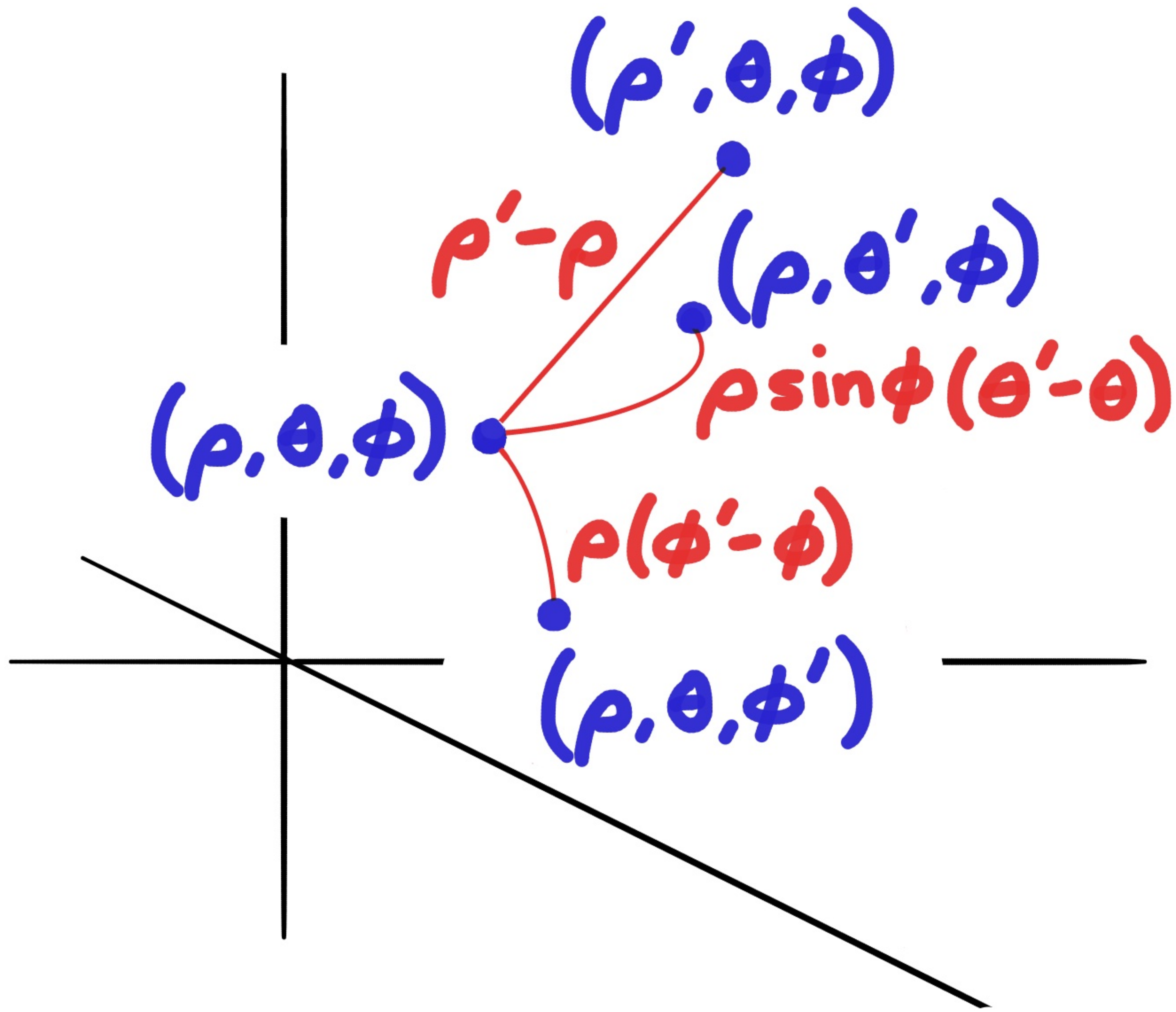
Widths and volumes in $\mathbb{R}^3$: Spherical



Widths and volumes in $\mathbb{R}^3$: Spherical



Widths and volumes in $\mathbb{R}^3$: Spherical



$$\boxed{dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi}$$

Spherical Coordinates

$$\iiint_R f(x,y,z)\,dV$$

$$= \iiint_R f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi\; d\rho\, d\theta\, d\phi$$

Spherical Coordinates

$$\iiint_R f(x,y,z)\,dV$$

$$= \iiint_R f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi\; d\rho\, d\theta\, d\phi$$

$$dV \longmapsto d\rho\, d\theta\, d\phi$$

$$x \longmapsto \rho\sin\phi\cos\theta$$

$$y \longmapsto \rho\sin\phi\sin\theta$$

$$z \longmapsto \rho\cos\phi$$

Spherical Coordinates

$$\iiint_R f(x,y,z)\,dV$$

$$= \iiint_R f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\,\rho^2\sin\phi \; d\rho\, d\theta\, d\phi$$

$dV \longmapsto d\rho\, d\theta\, d\phi$

$x \longmapsto \rho\sin\phi\cos\theta$

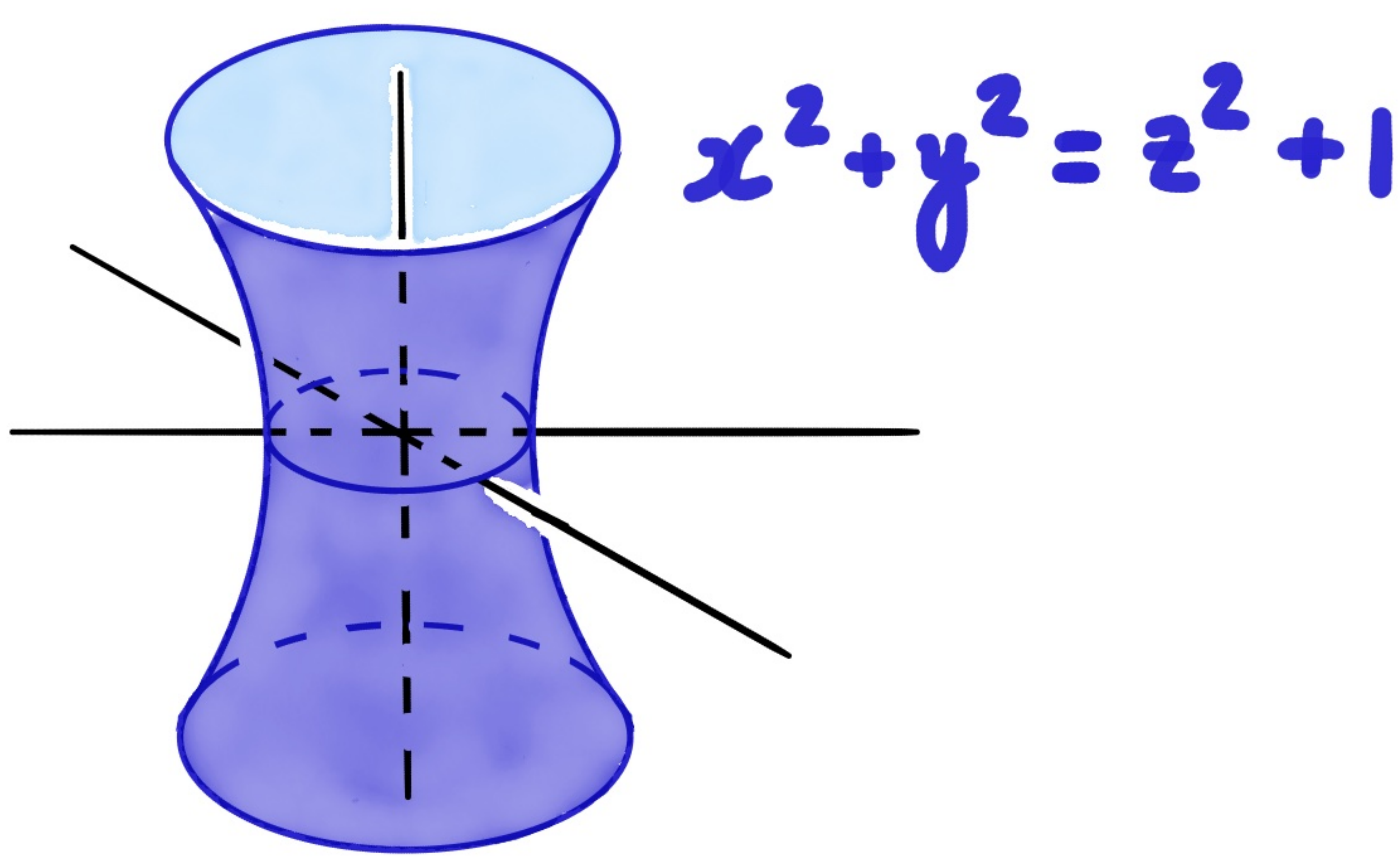
$y \longmapsto \rho\sin\phi\sin\theta$

$z \longmapsto \rho\cos\phi$

Throw in $\rho^2\sin\phi$.

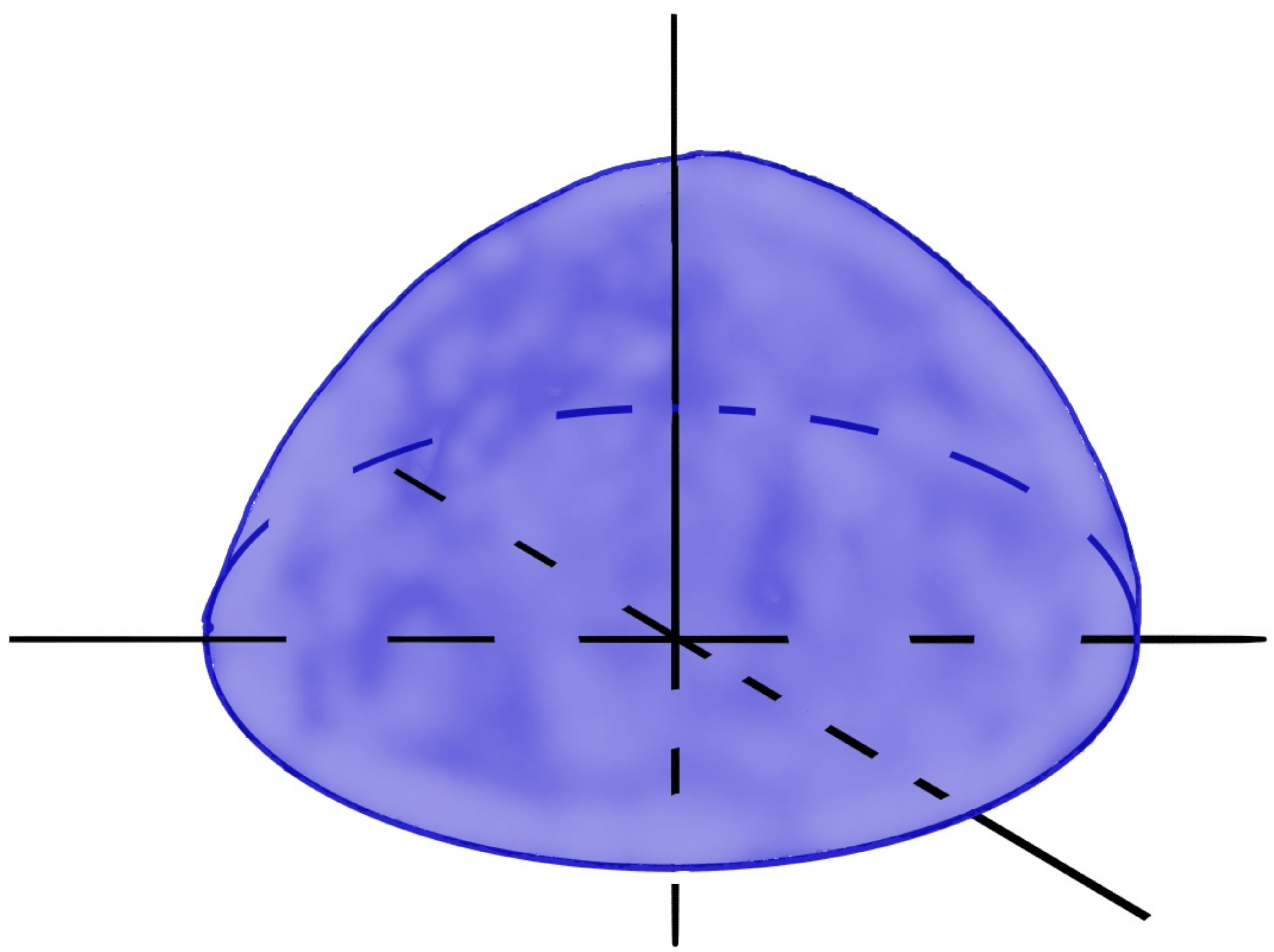
Examples:

① Find $\iiint_R y\,dV$ where R is the region above the xy-plane, with $z \le 4$, and bounded by the surface $x^2+y^2=z^2+1$.

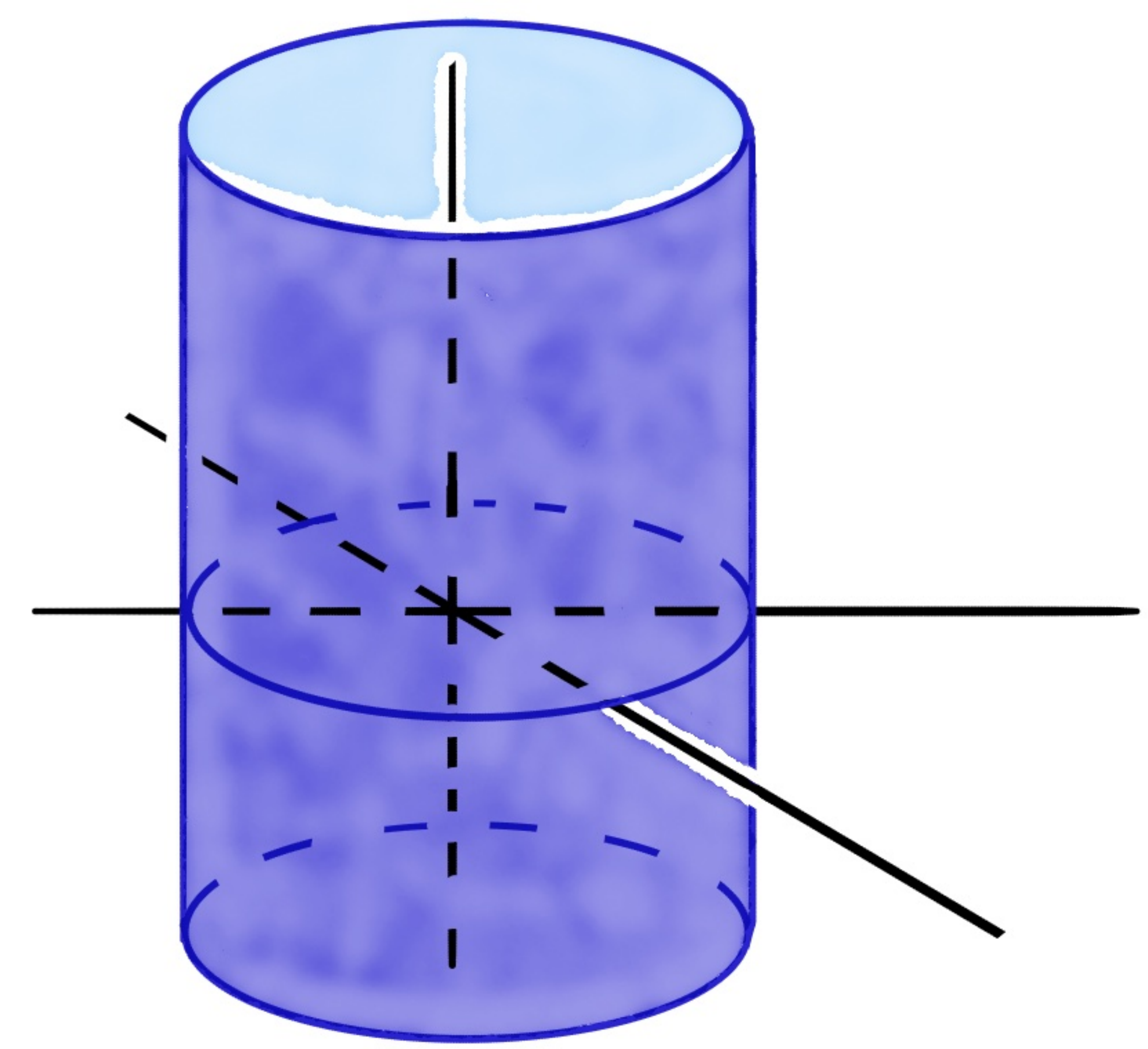


2 Find the volume of the region in $\mathbb{R}^3$ enclosed by a sphere of radius a.

3 Find the volume of the region in $\mathbb{R}^3$ above the xy-plane, below the surface $z = 9 - x^2 - y^2$, and contained in the cylinder $x^2 + y^2 = 4$.



$z = 9 - x^2 - y^2$



$x^2 + y^2 = 4$