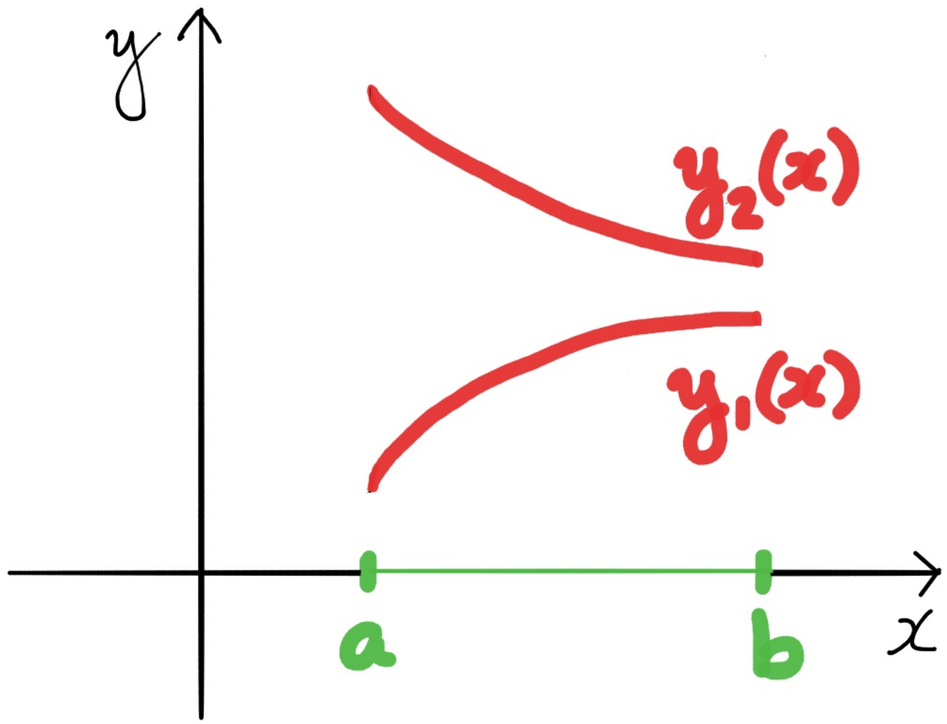
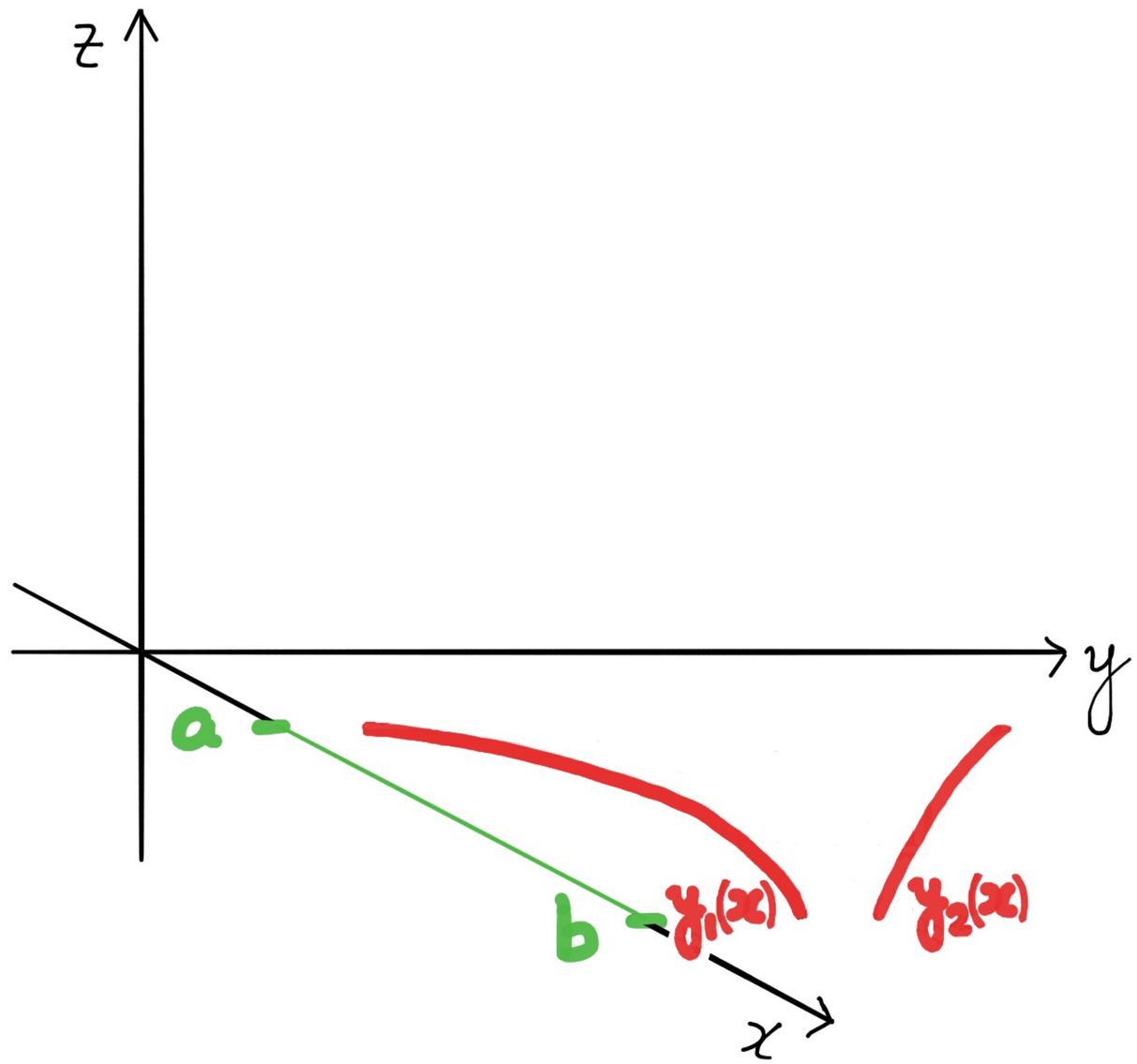
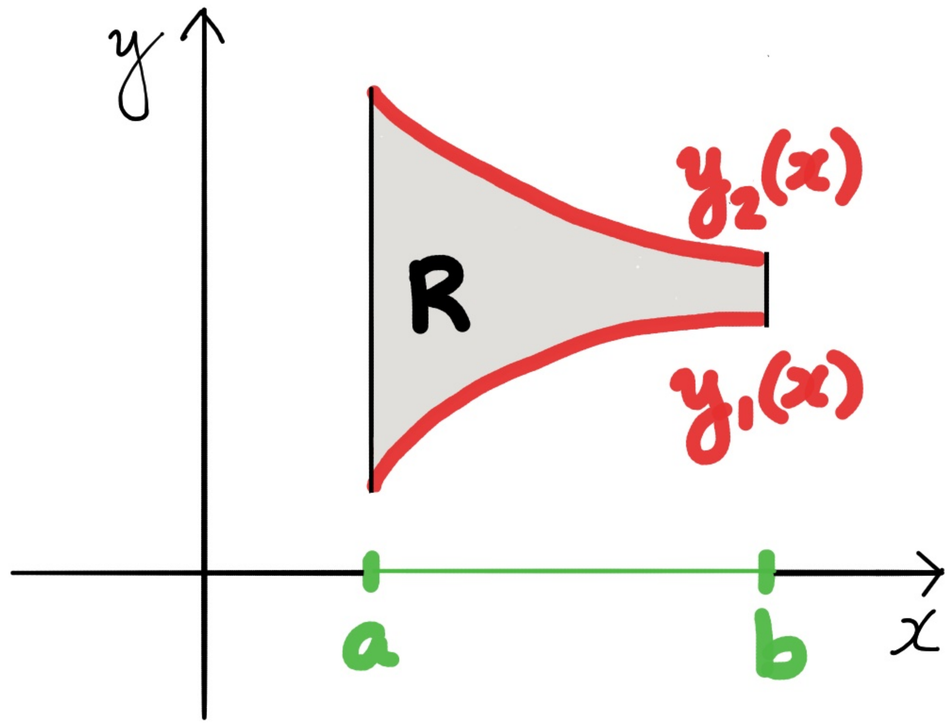
Twenty

Double integrals over nonrectangular regions

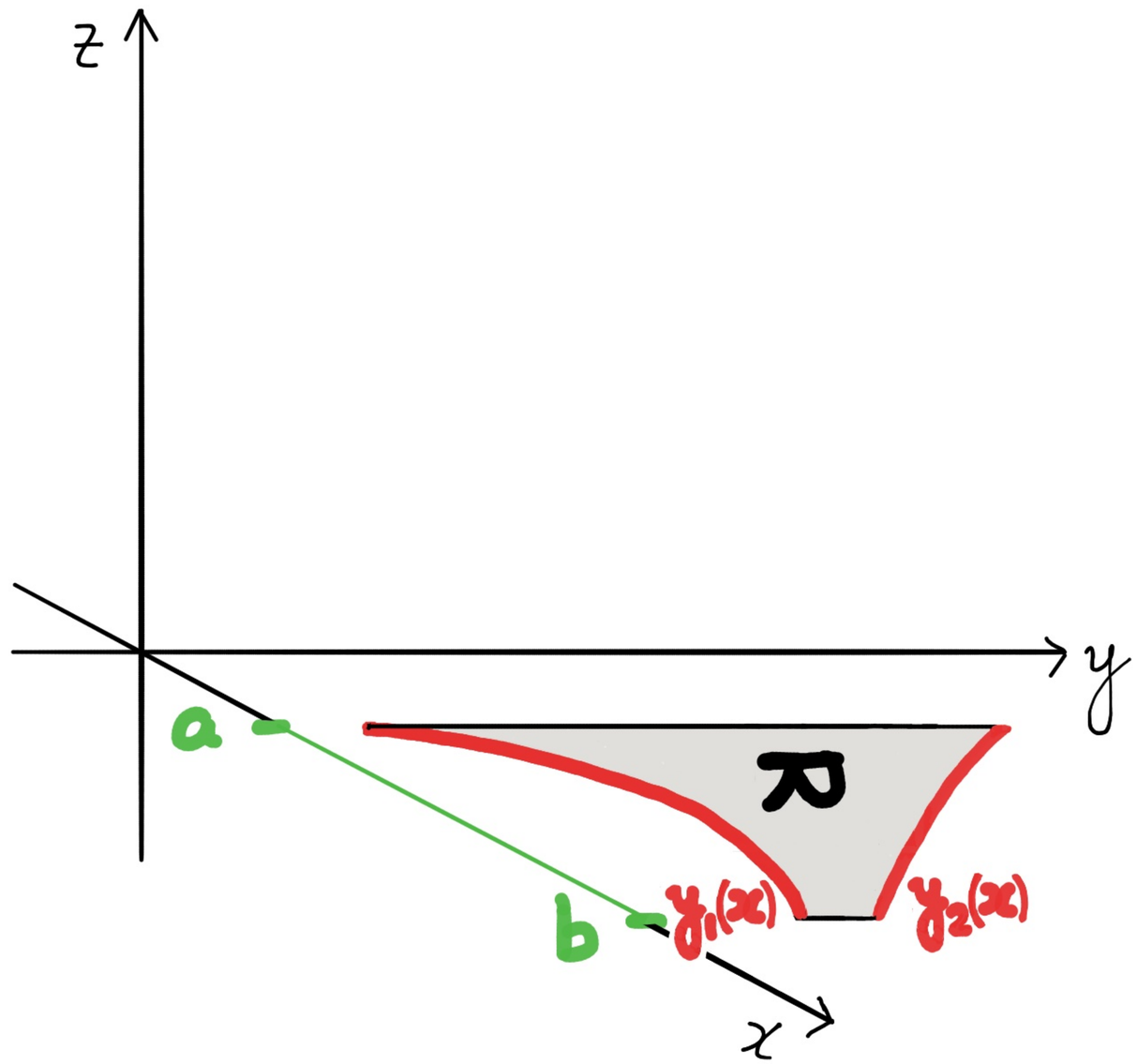

y
$y_2(x)$
$y_1(x)$
a
b
x

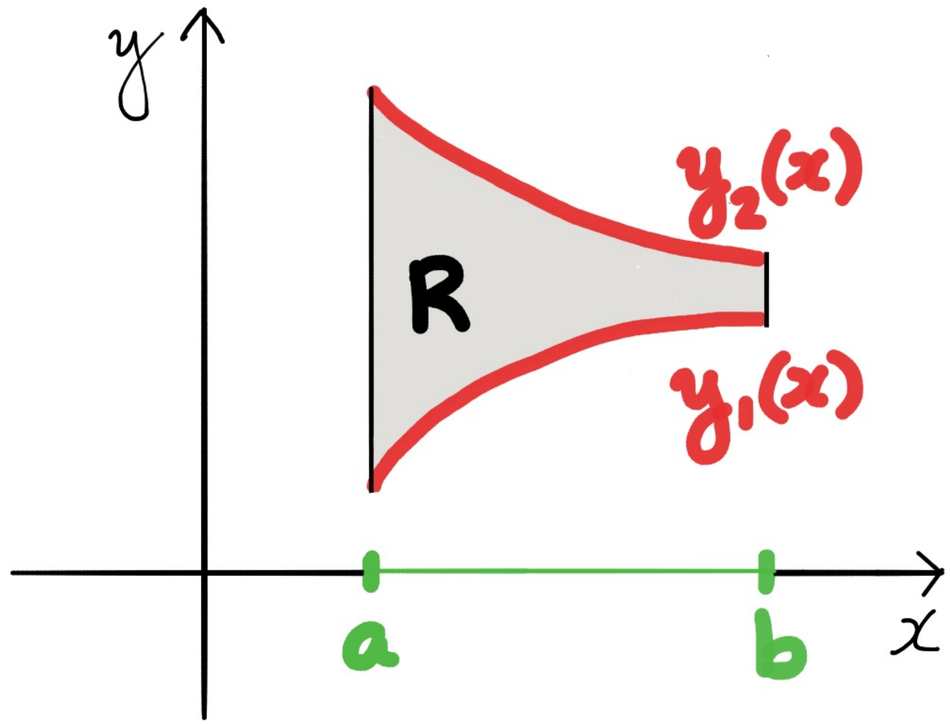

z
y
a
b
$y_1(x)$
$y_2(x)$
x



$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

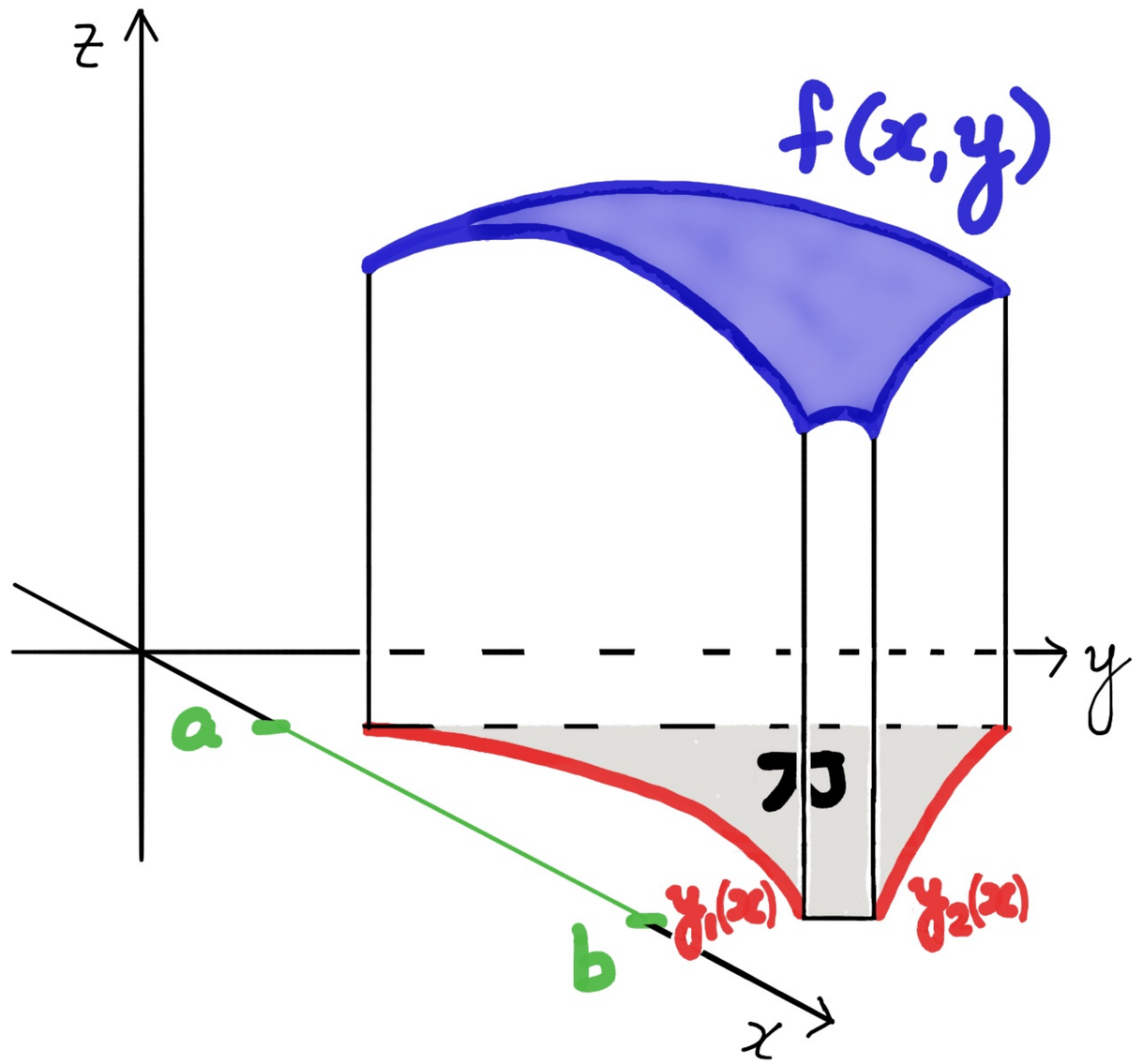
"y-simple"



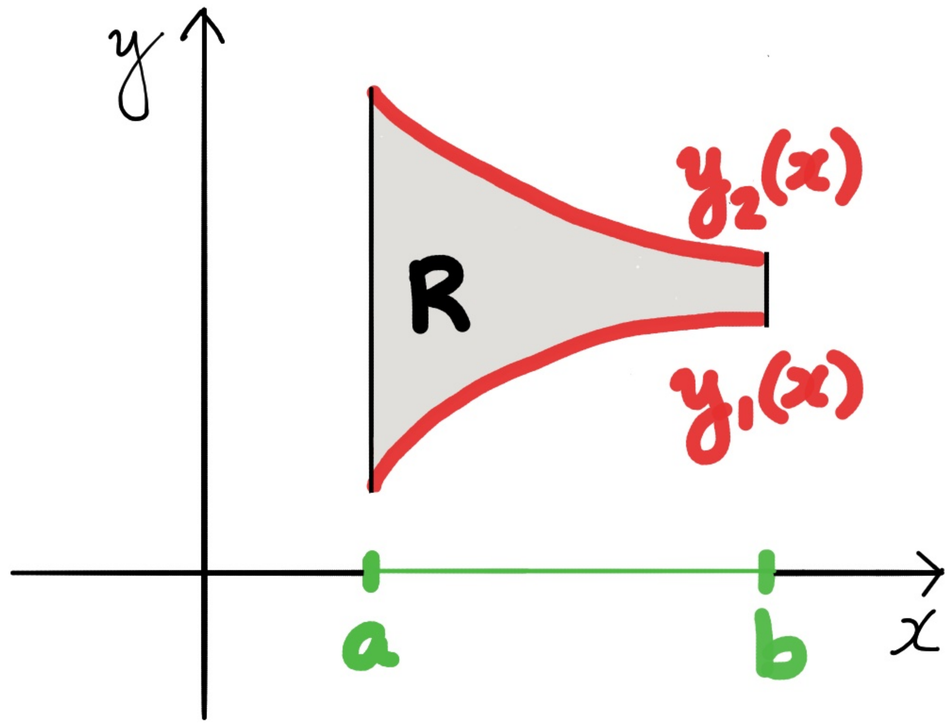


$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"

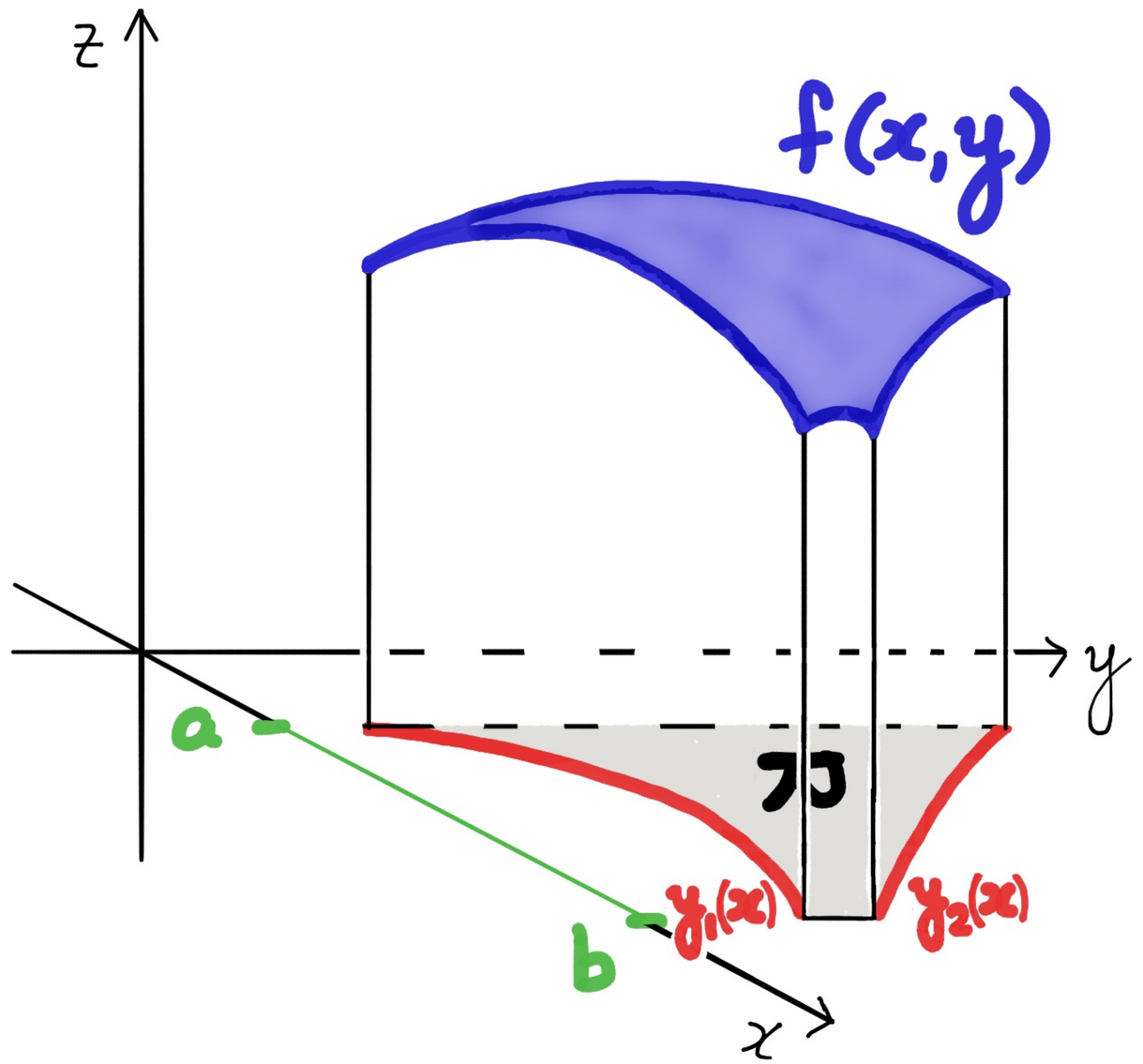


$$\iint_R f(x,y)\,dA$$

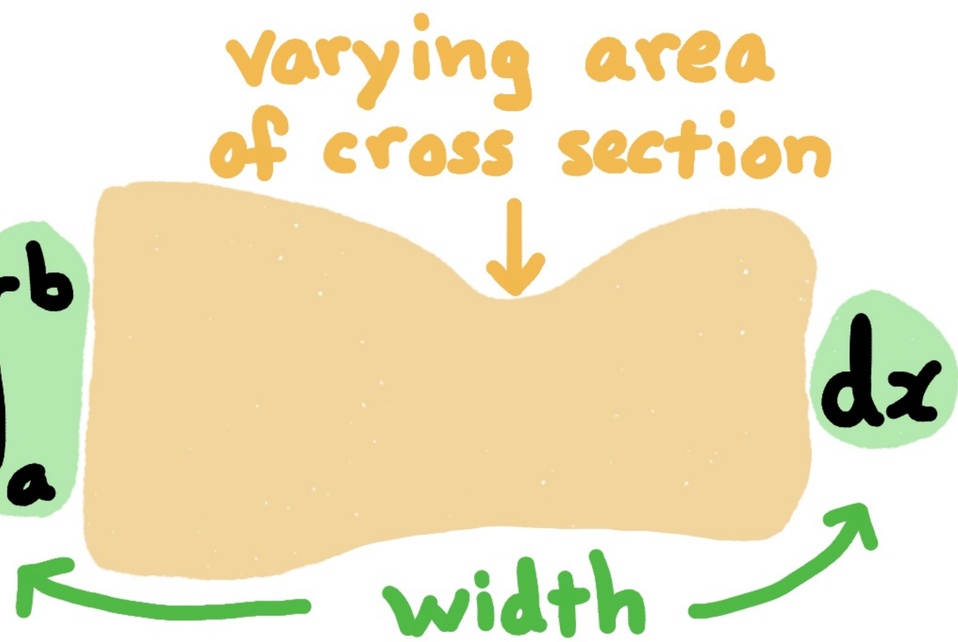


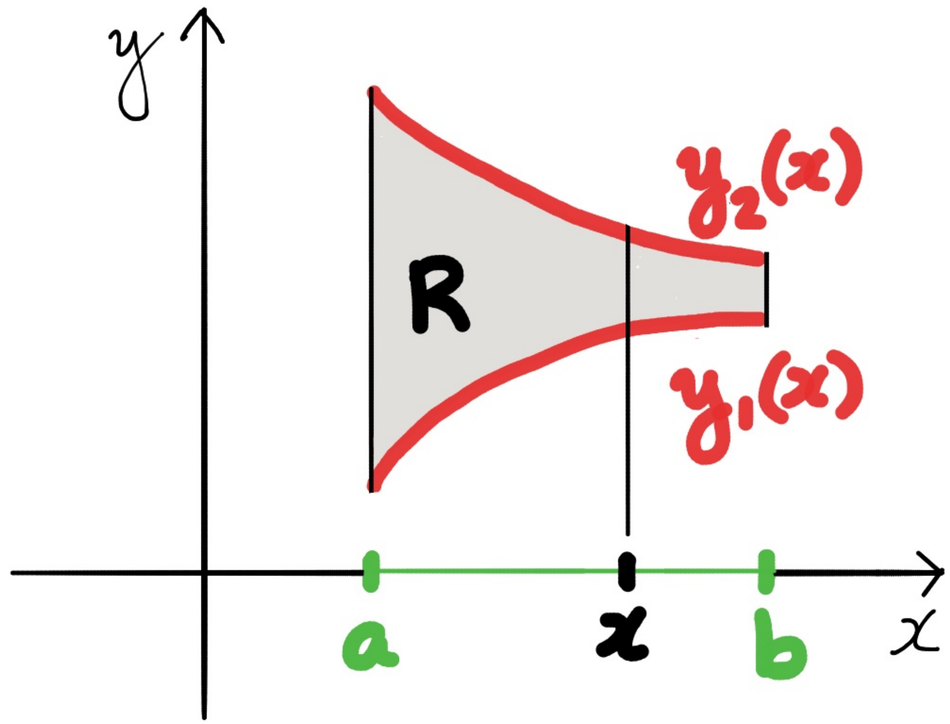
$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"



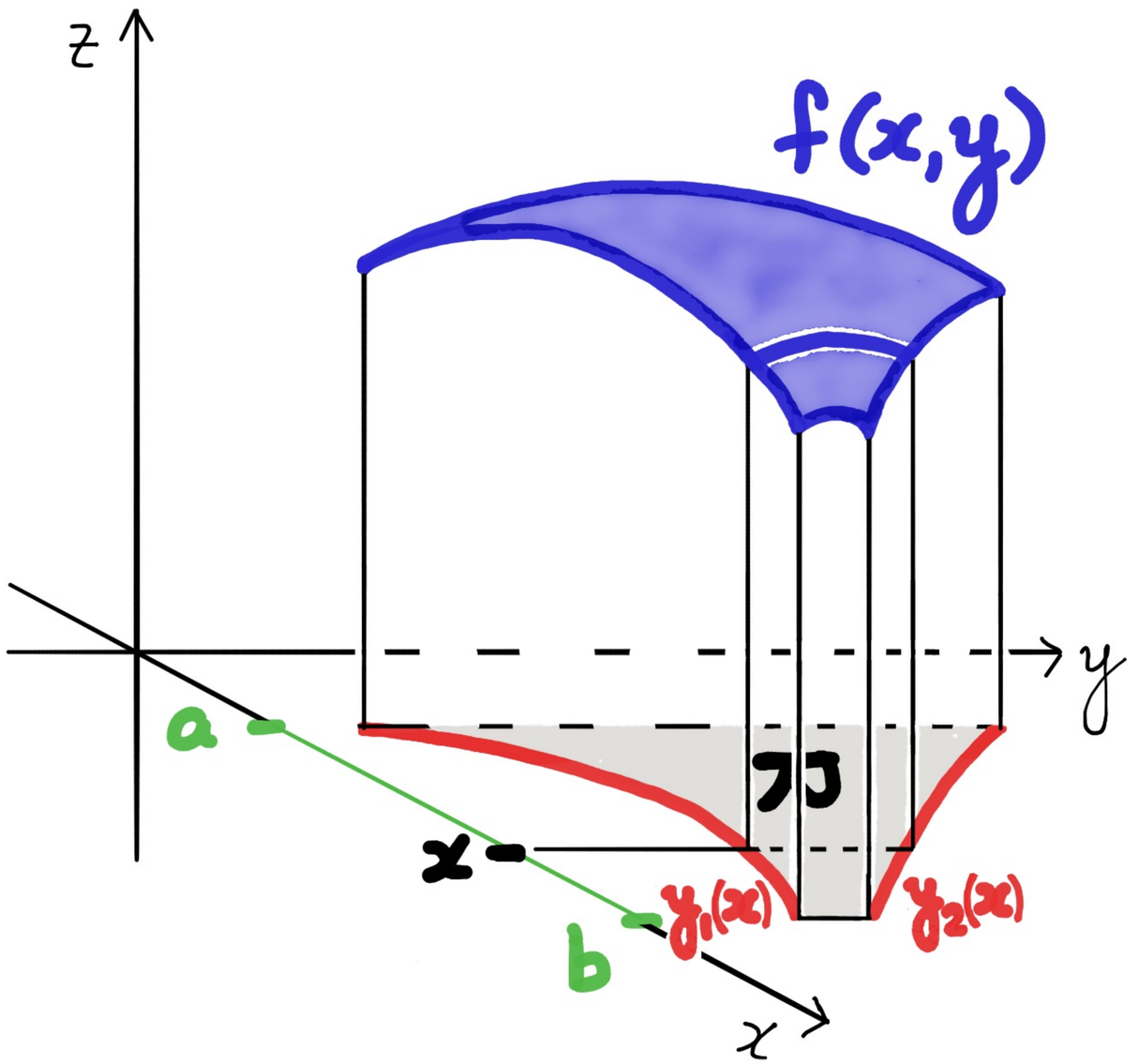
$$\iint_R f(x,y)\,dA = \int_a^b$$



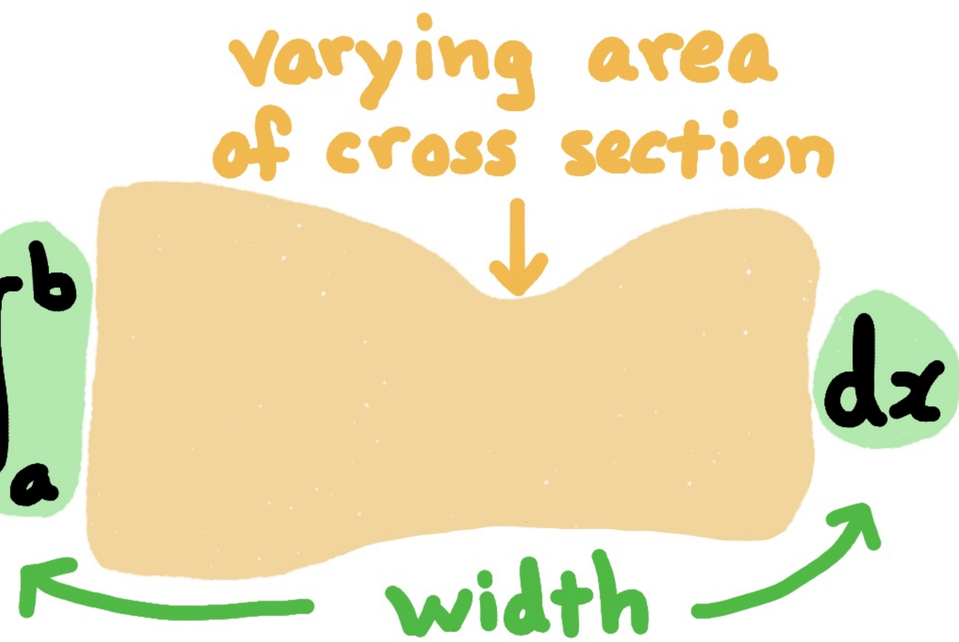


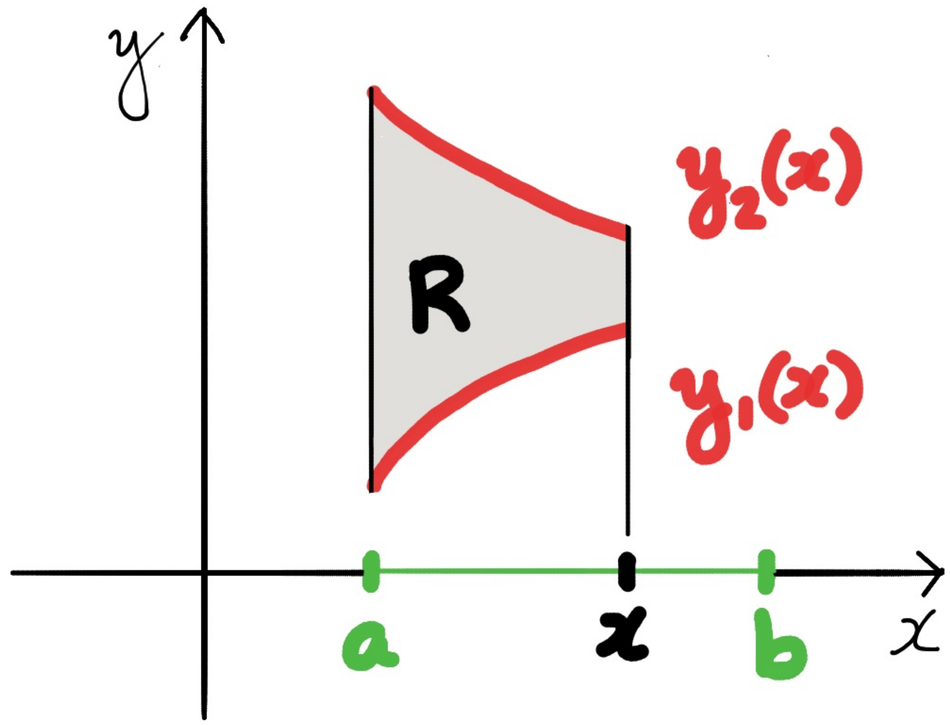
$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"



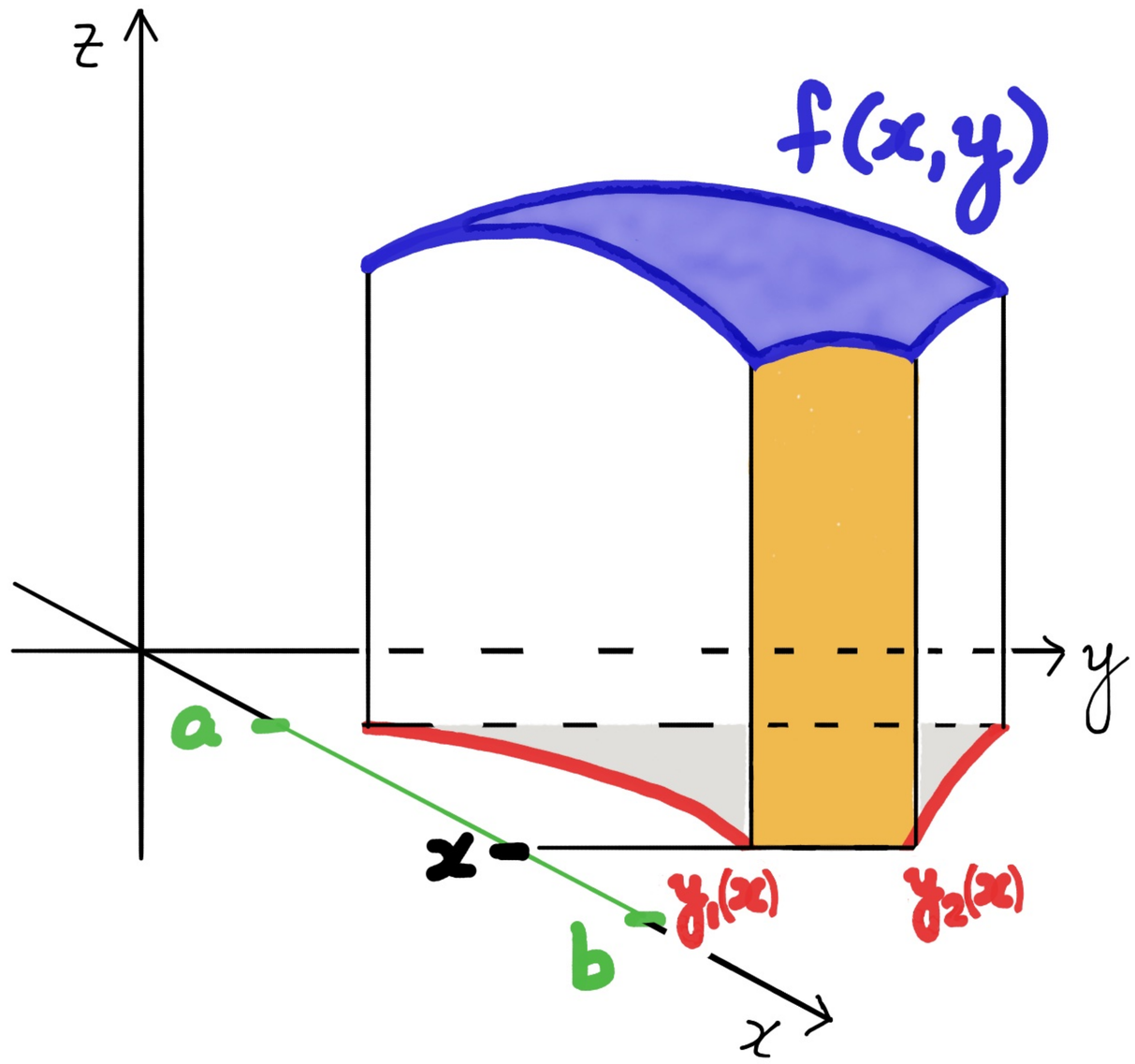
$$\iint_R f(x,y)\,dA = \int_a^b \text{(varying area of cross section)}\,dx$$



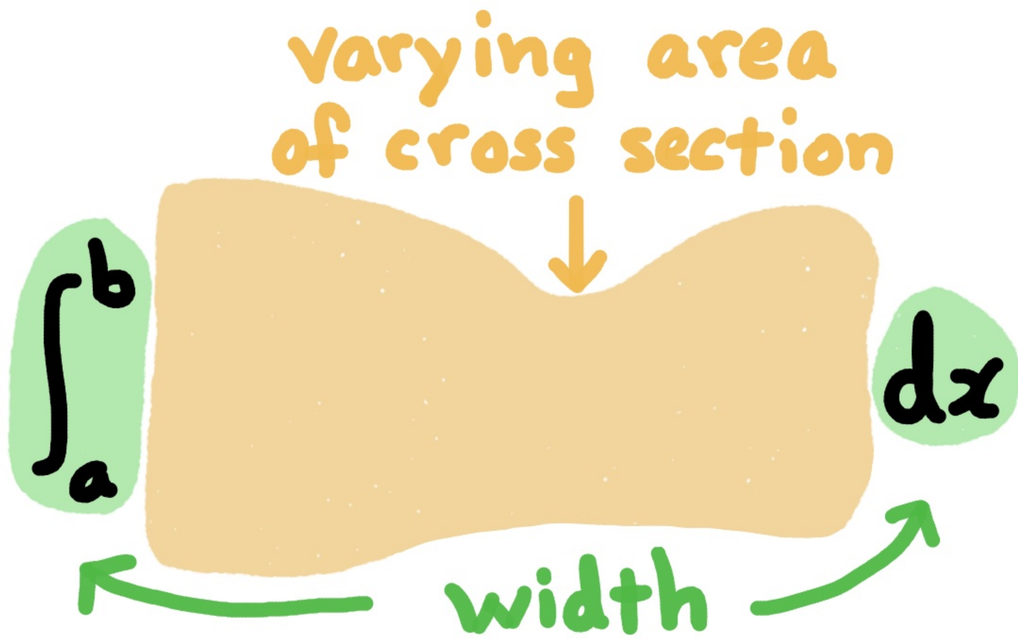


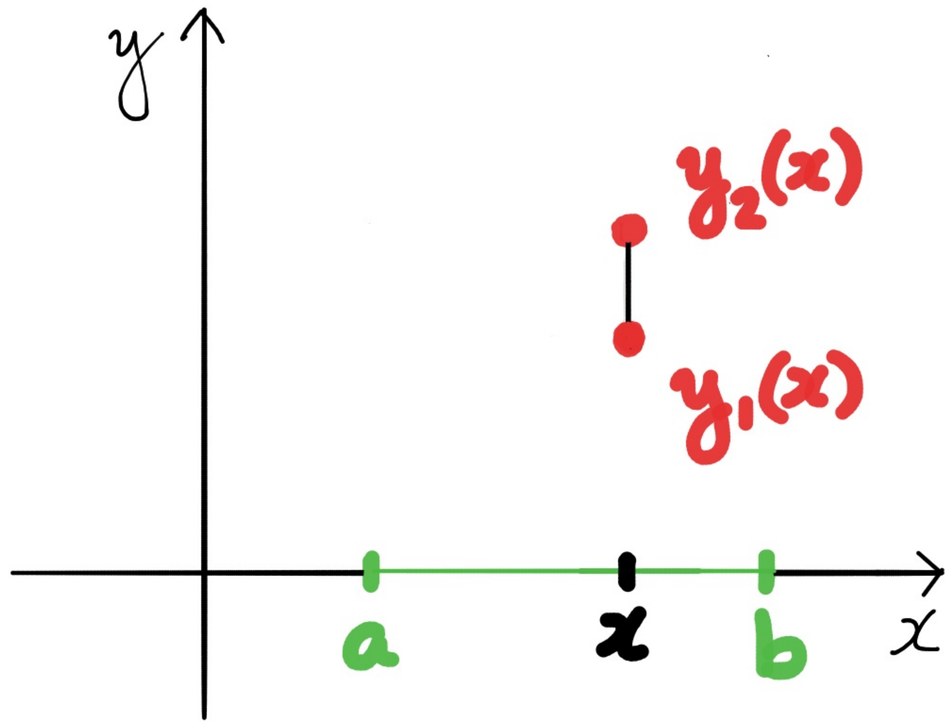
$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"



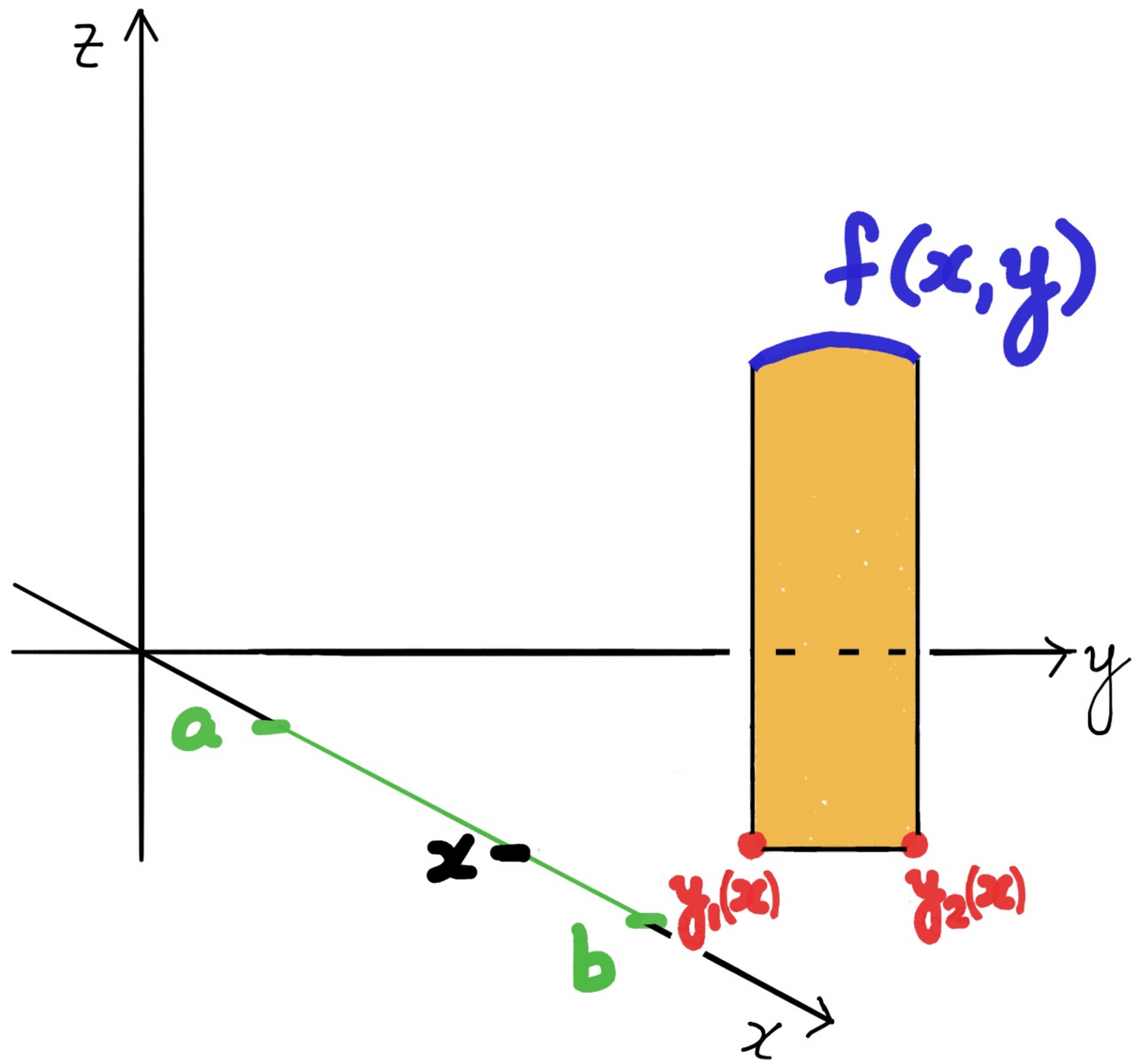
$$\iint_R f(x,y)\,dA = \int_a^b \text{(varying area of cross section)}\,dx$$



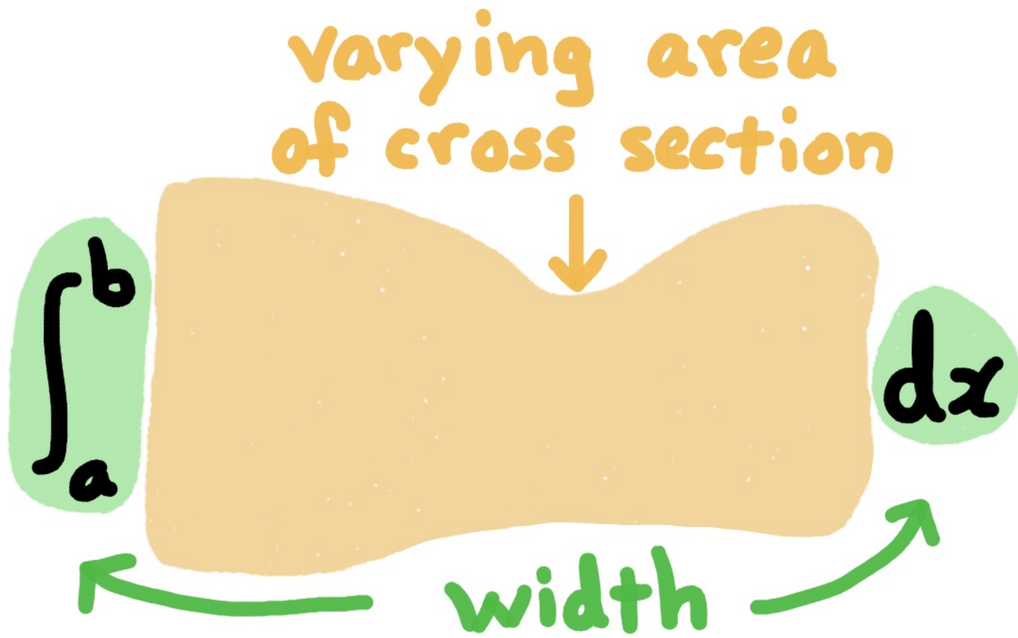


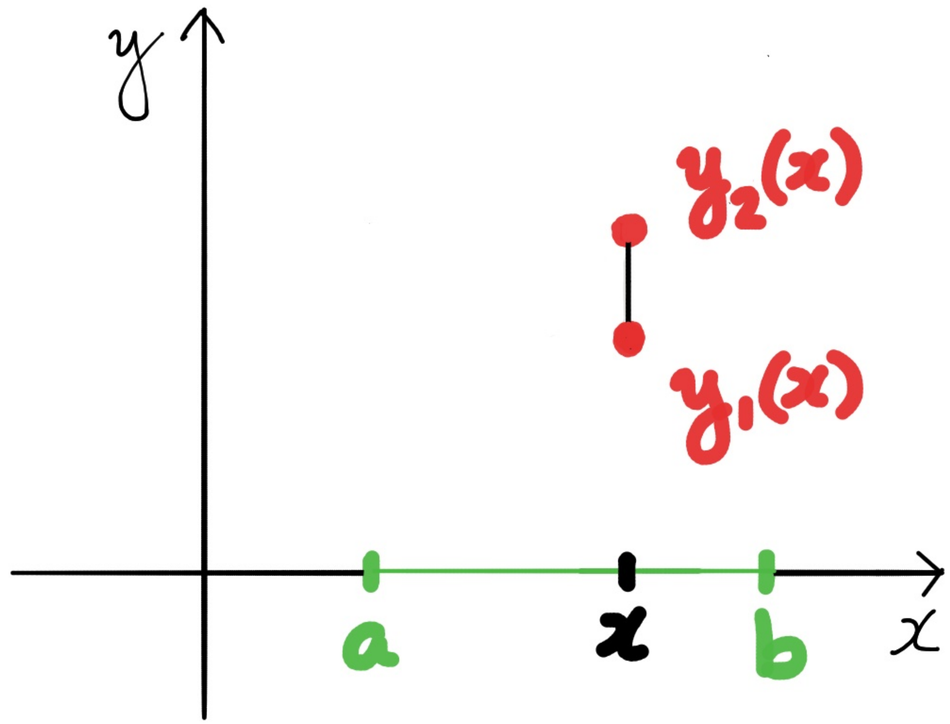
$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"



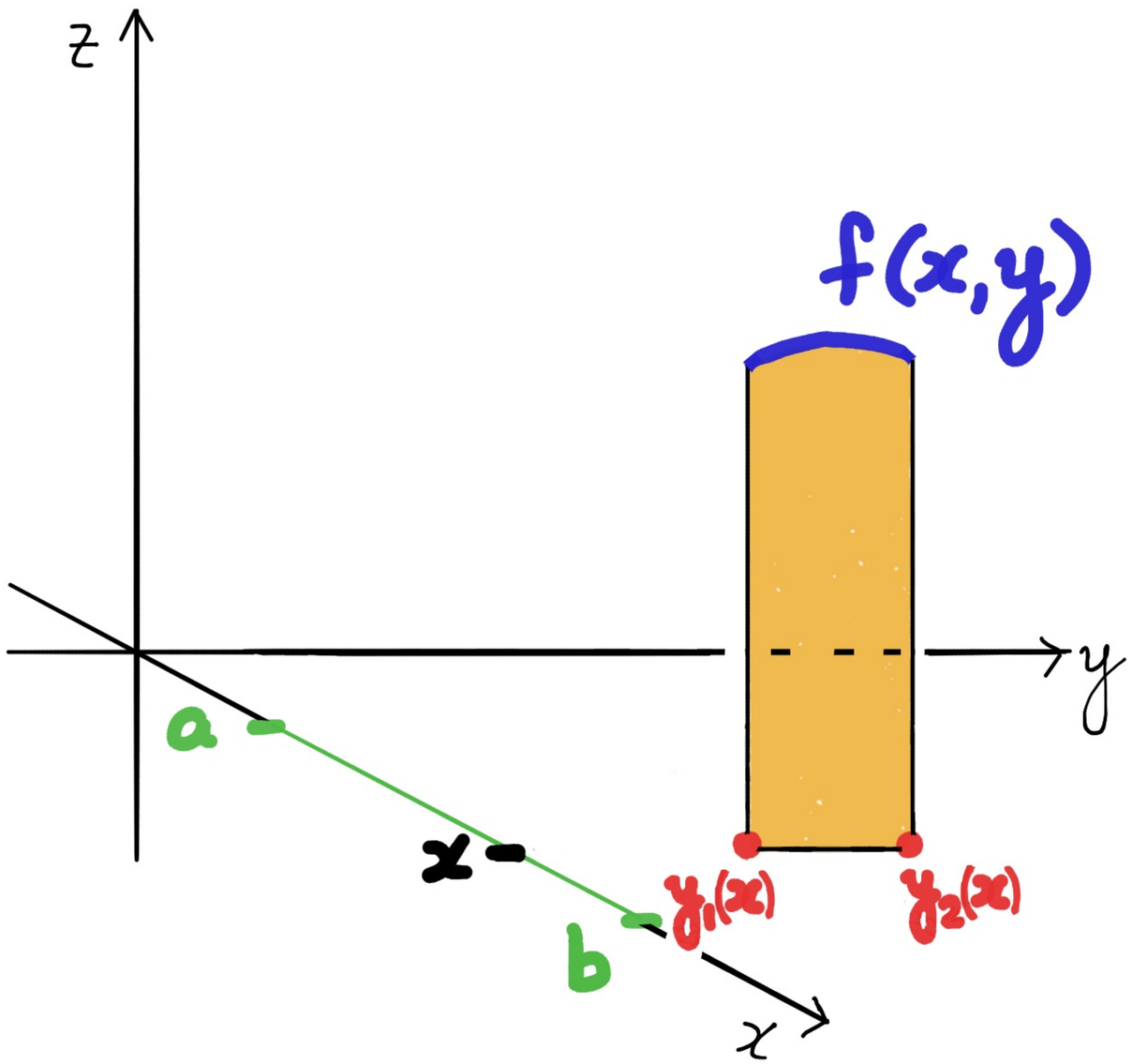
$$\iint_R f(x,y)\,dA = \int_a^b$$





$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

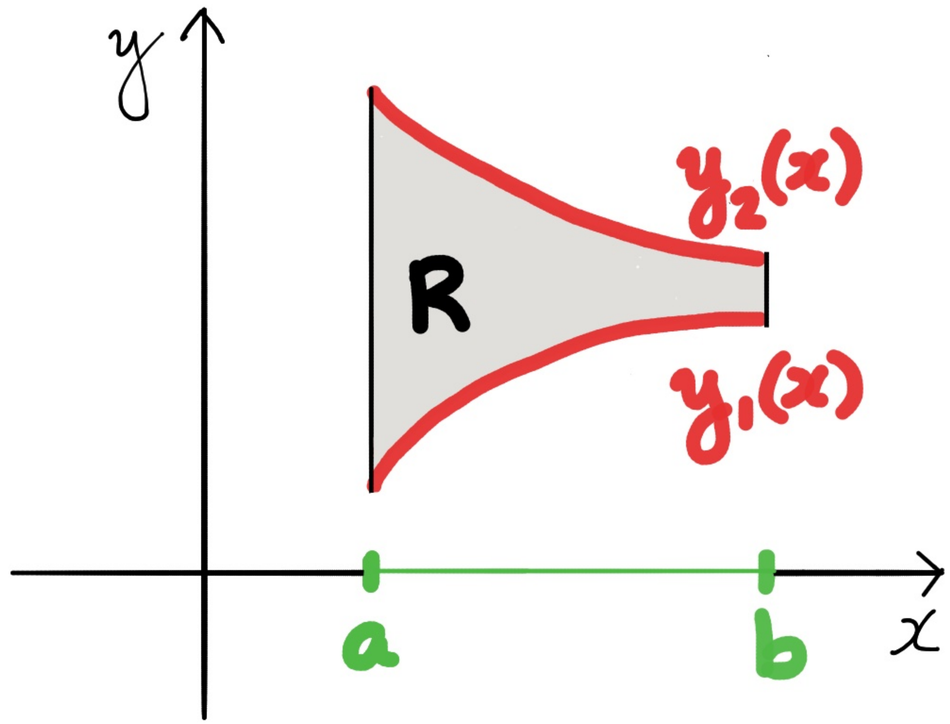
"y-simple"



$$\iint_R f(x,y)\,dA = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\,dx$$

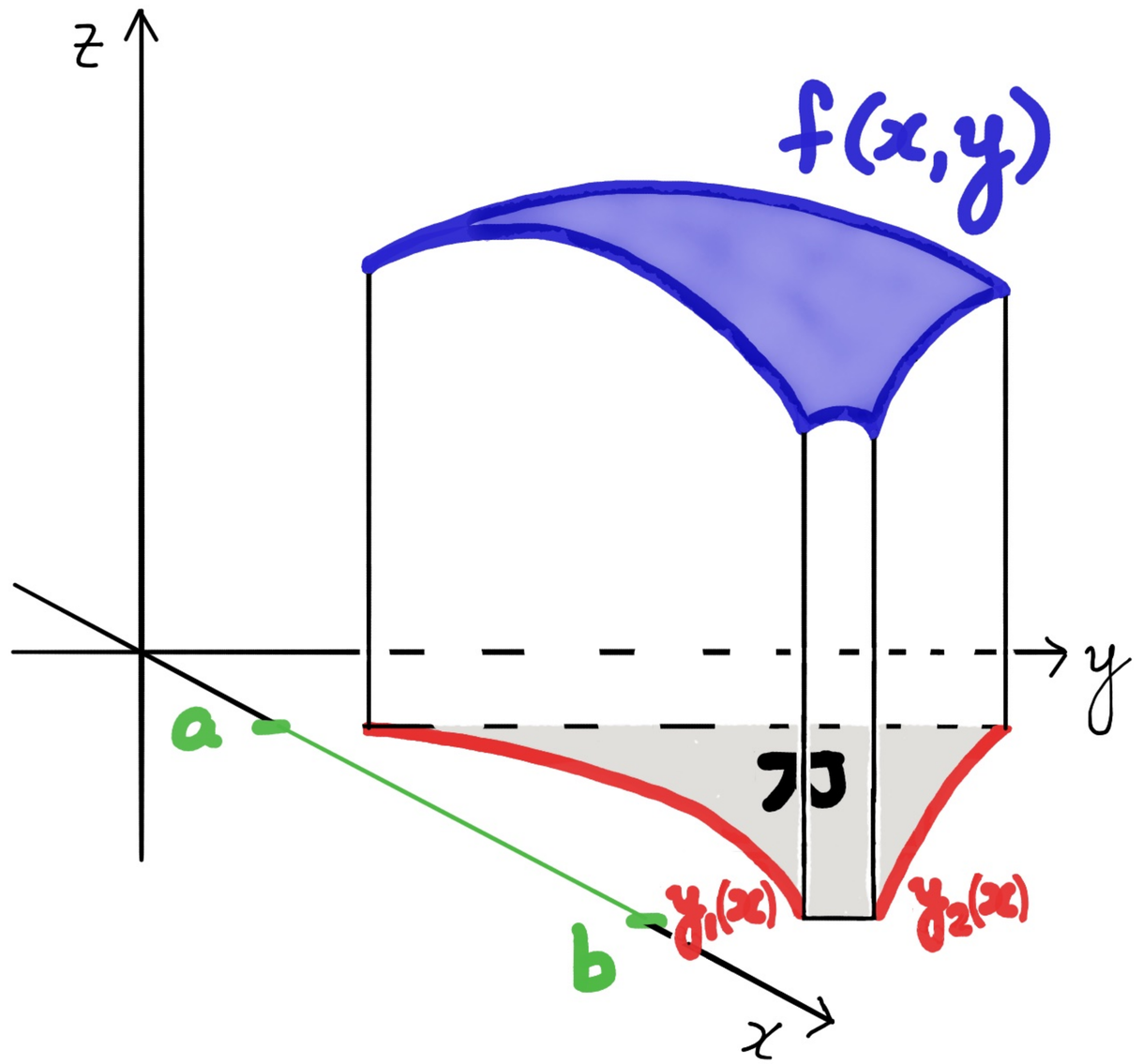
varying area of cross section

width



$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

"y-simple"



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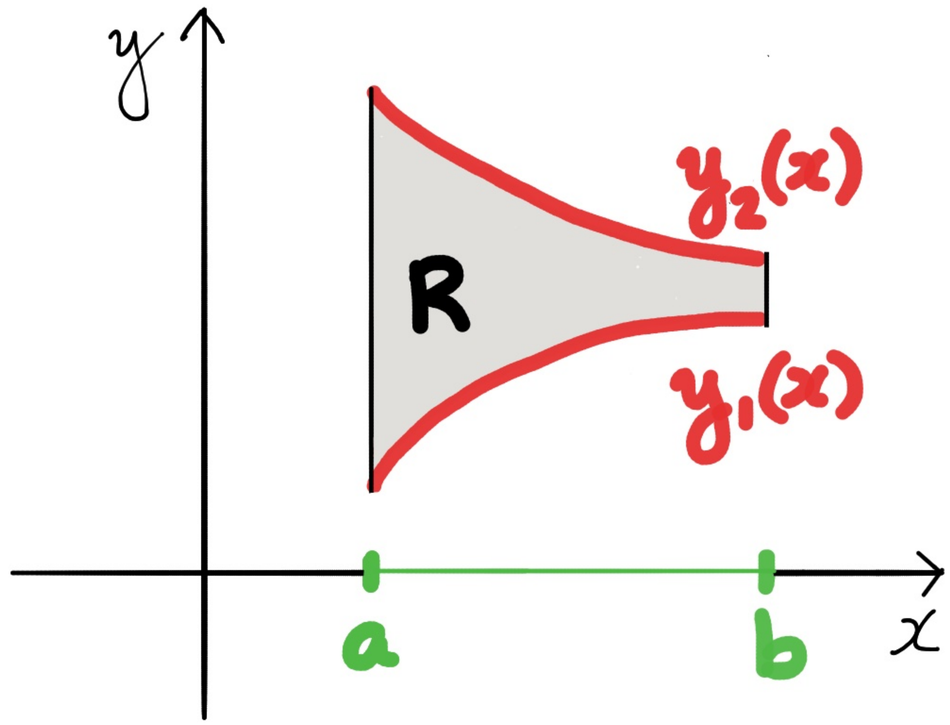
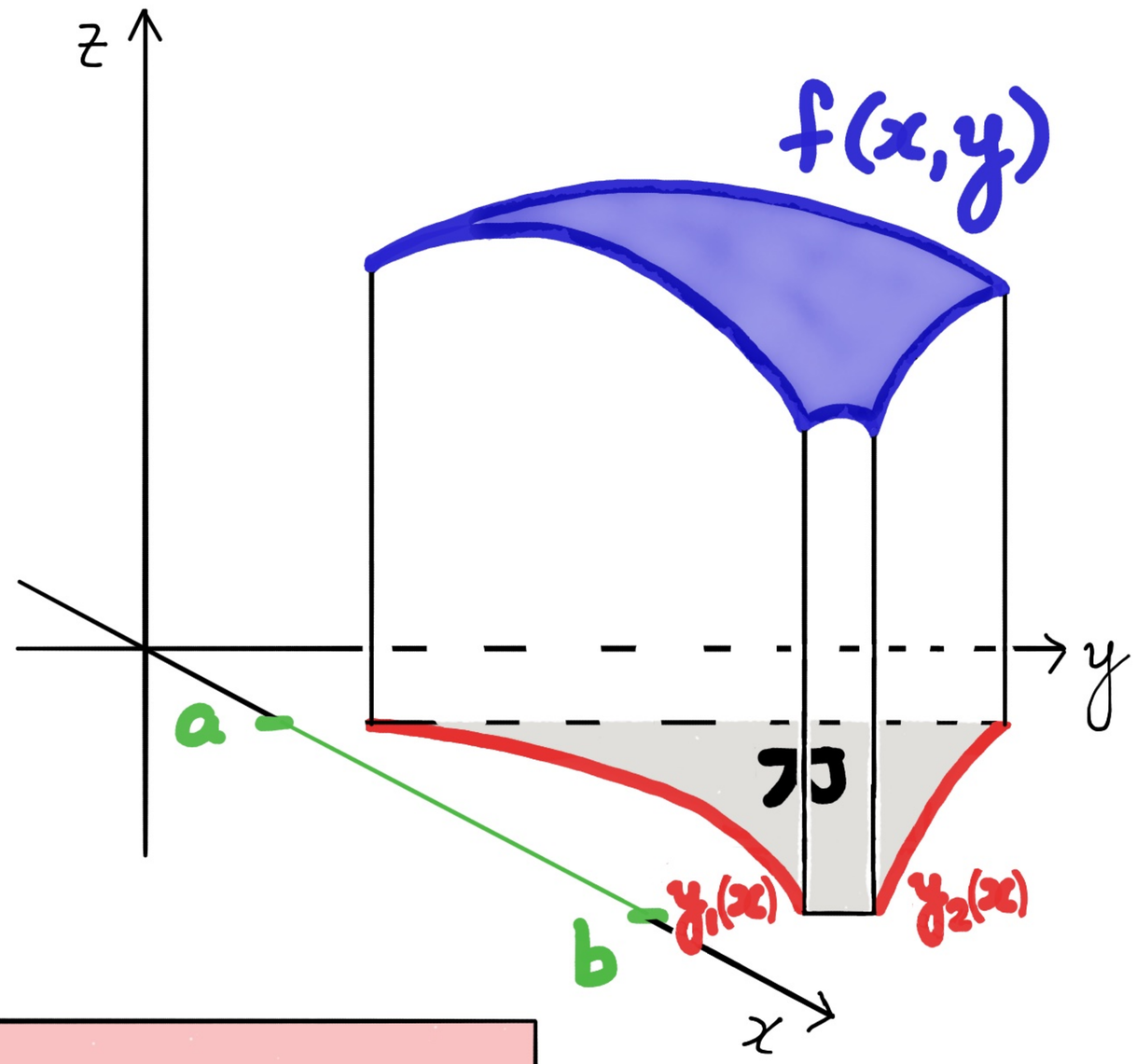
varying area of cross section

width



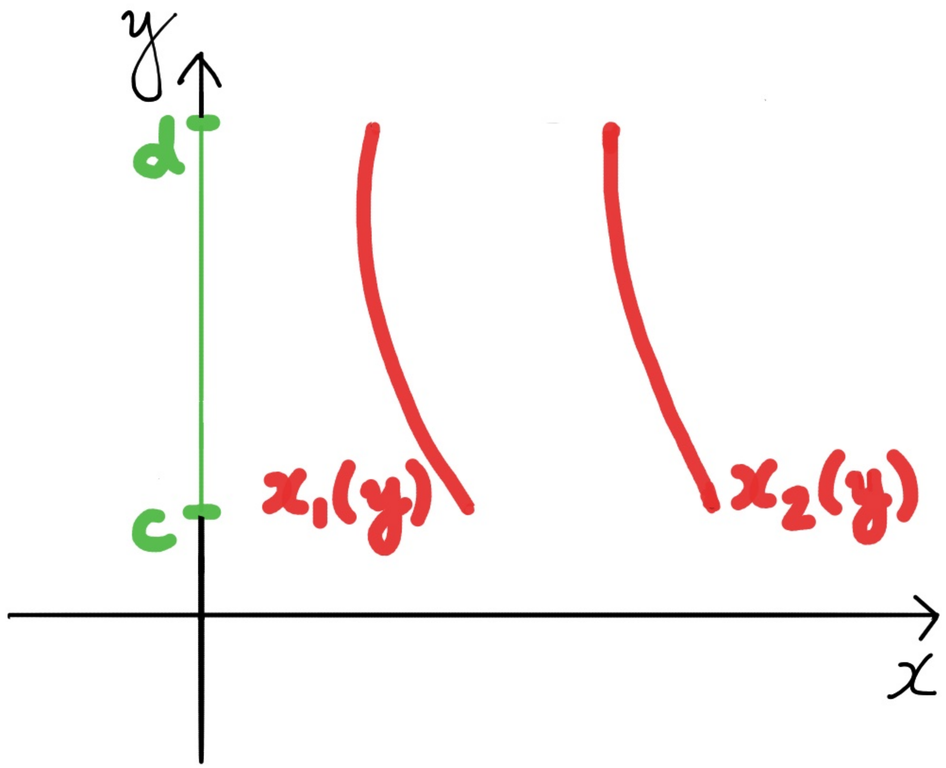


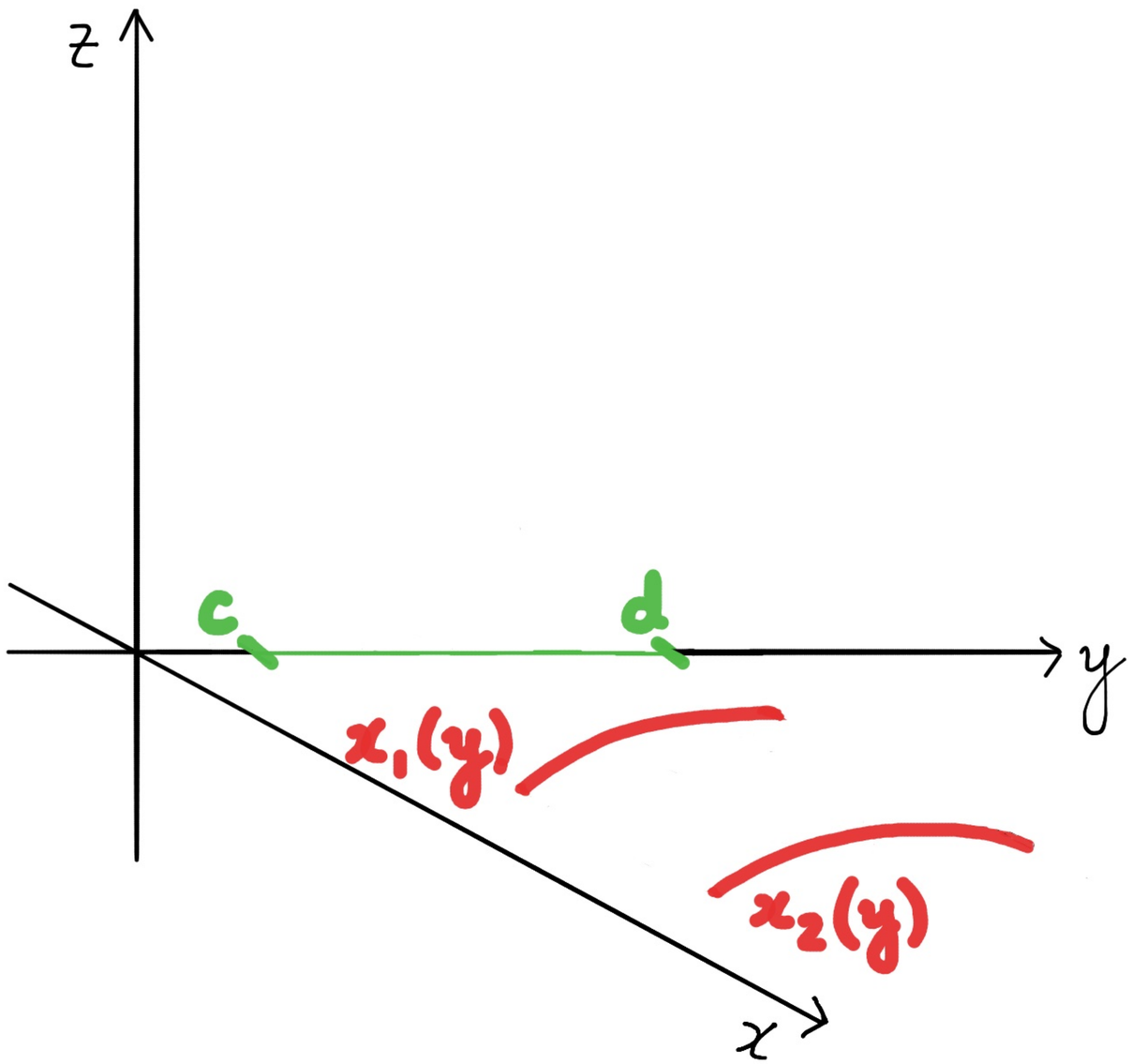
$$R=\{(x,y): a\le x\le b,\ y_1(x)\le y\le y_2(x)\}$$

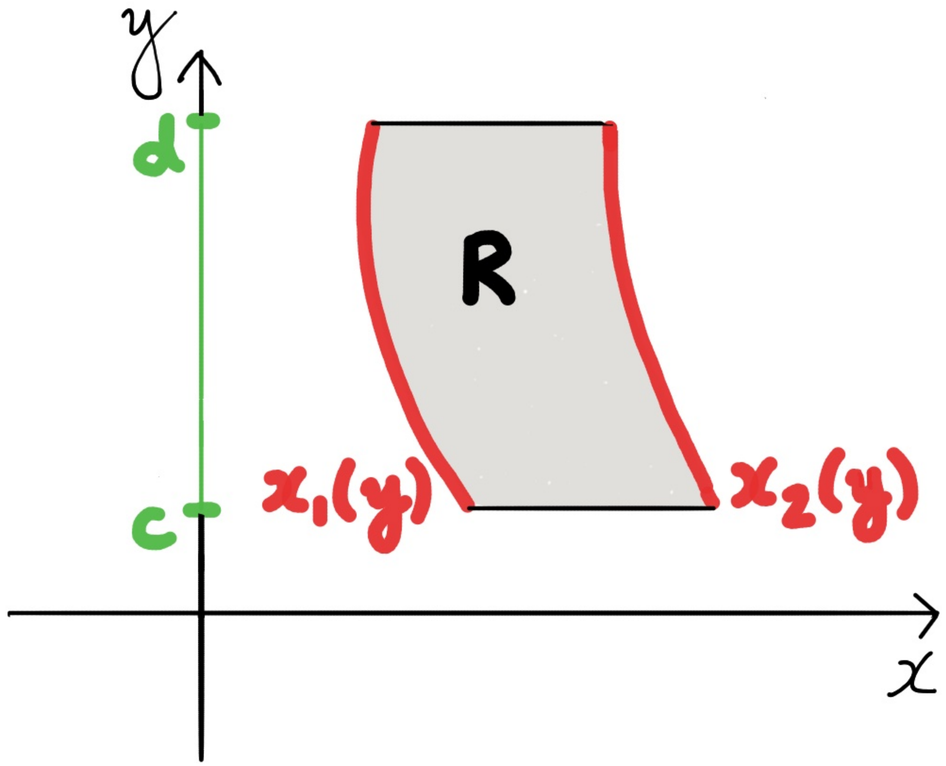
$$\iint_R f(x,y)\,dA = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\,dx$$

Example:

① $\int_0^1 \int_{-x^2}^{x^2} xy^2 \, dy \, dx$

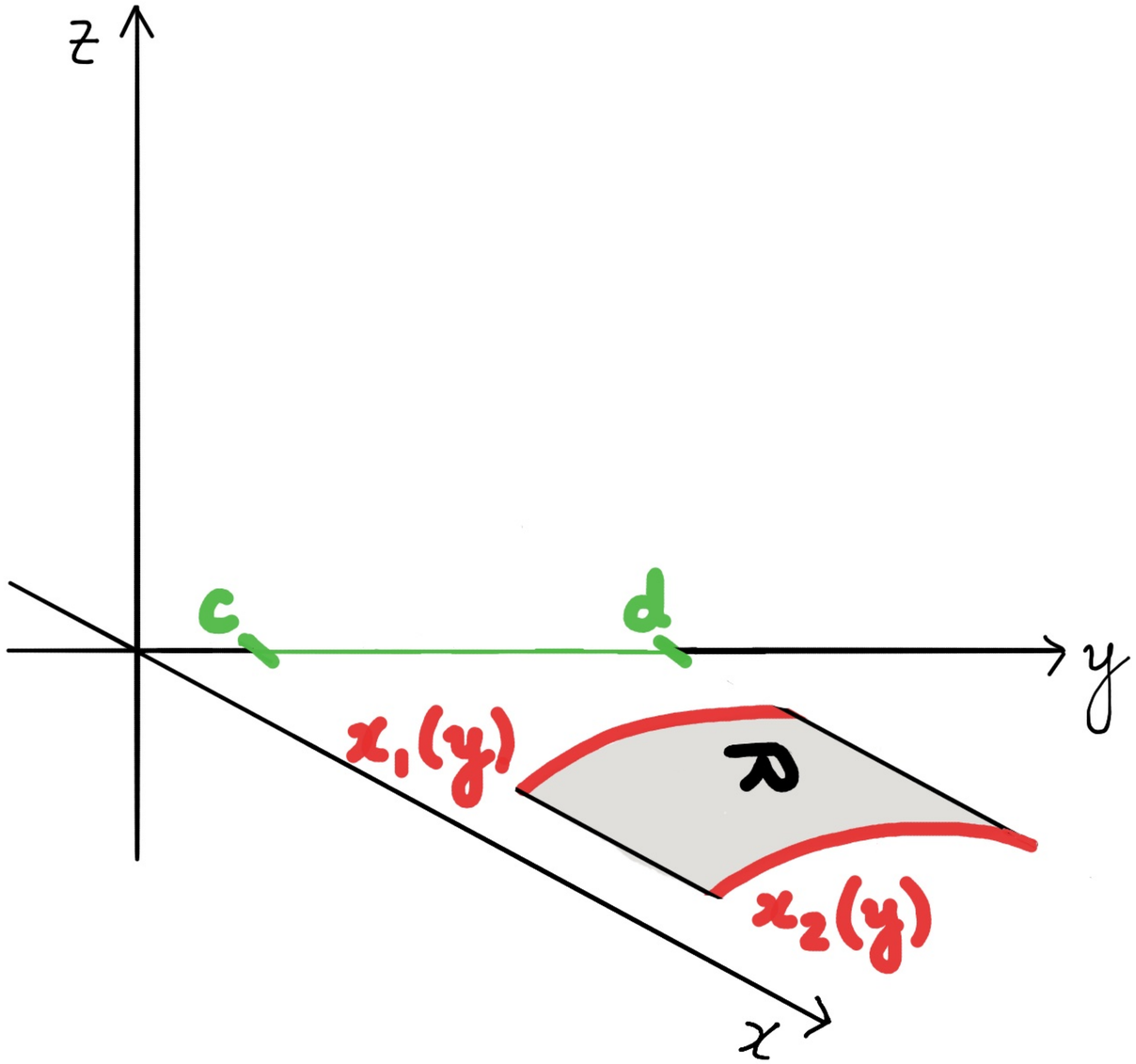

y
d
c
$x_1(y)$
$x_2(y)$
x

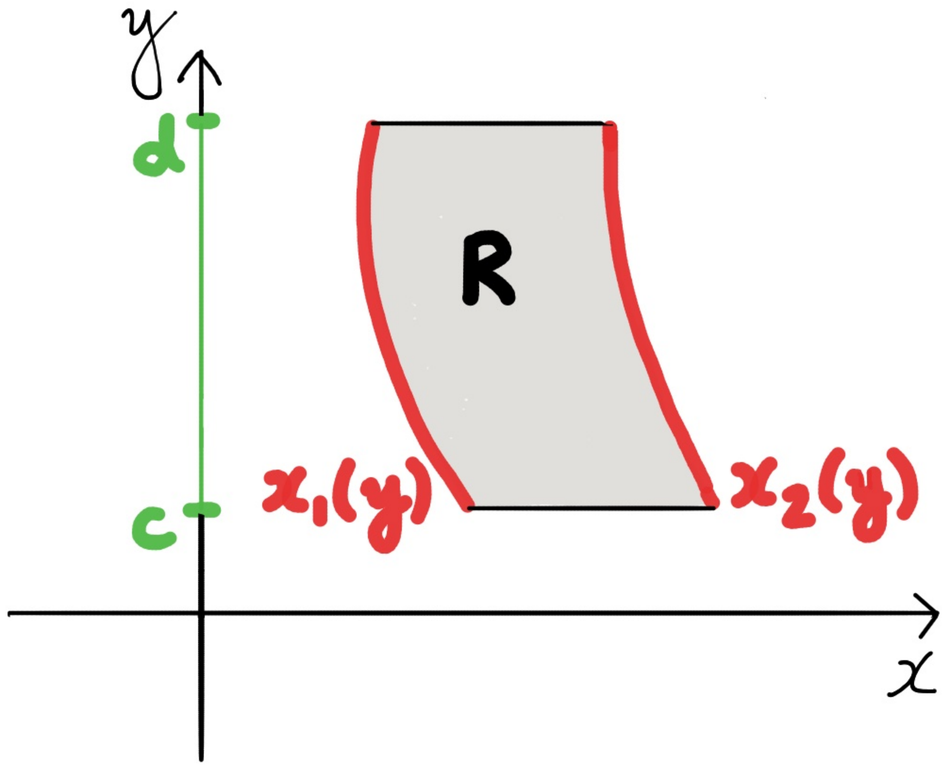

z
c
d
y
$x_1(y)$
$x_2(y)$
x



$$R=\{(x,y): c\le y\le d,\ x_1(y)\le x\le x_2(y)\}$$

"x-simple"





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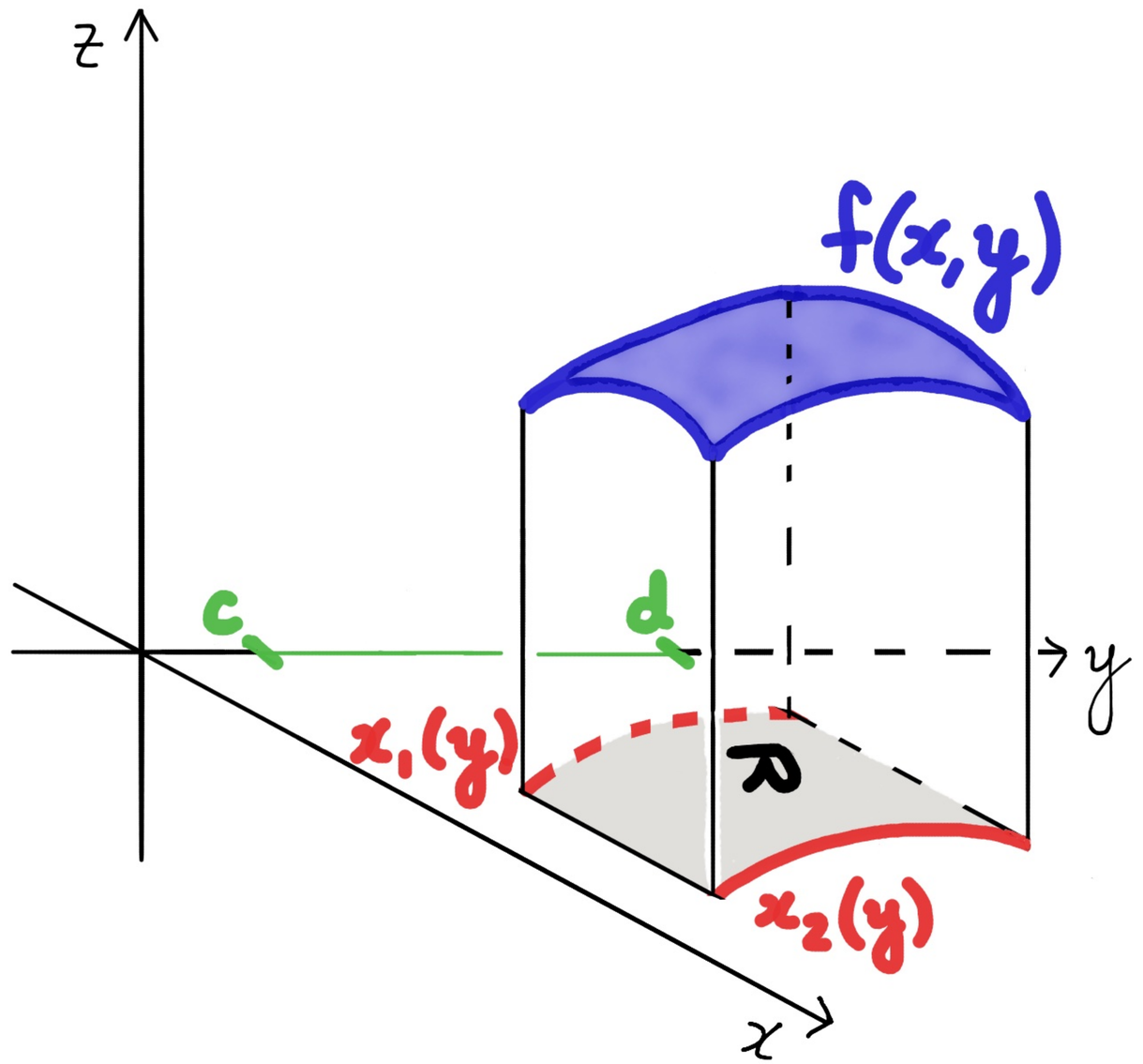
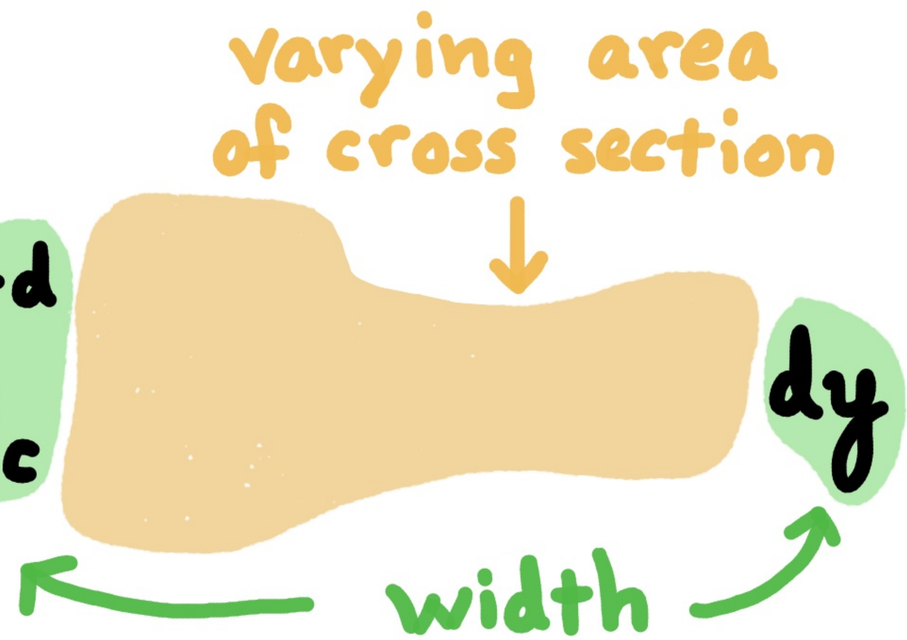
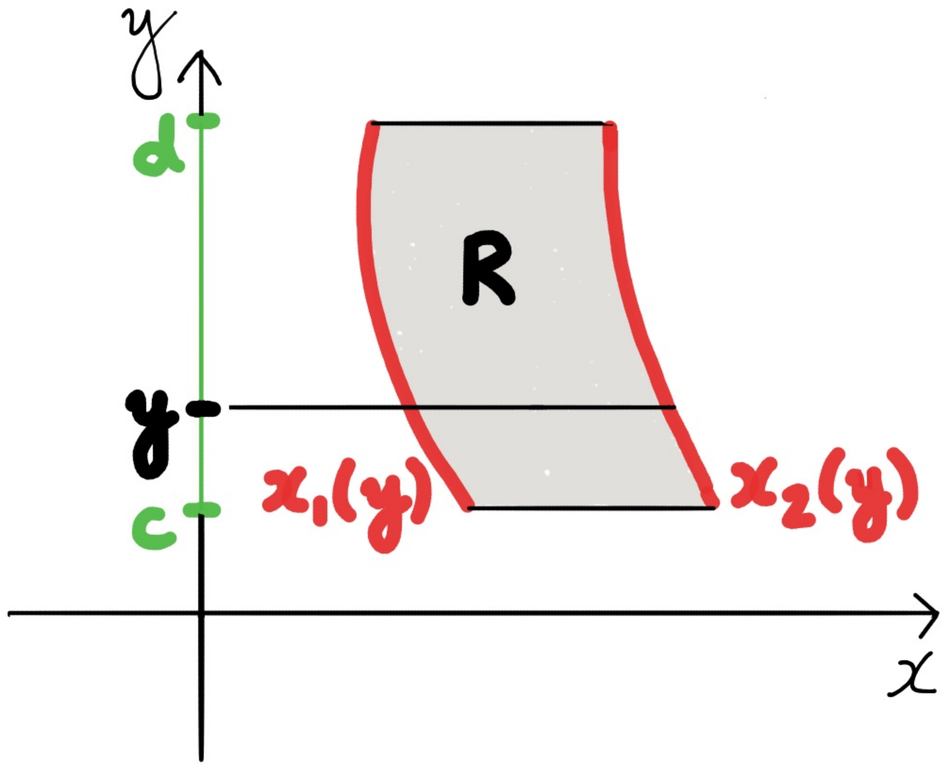


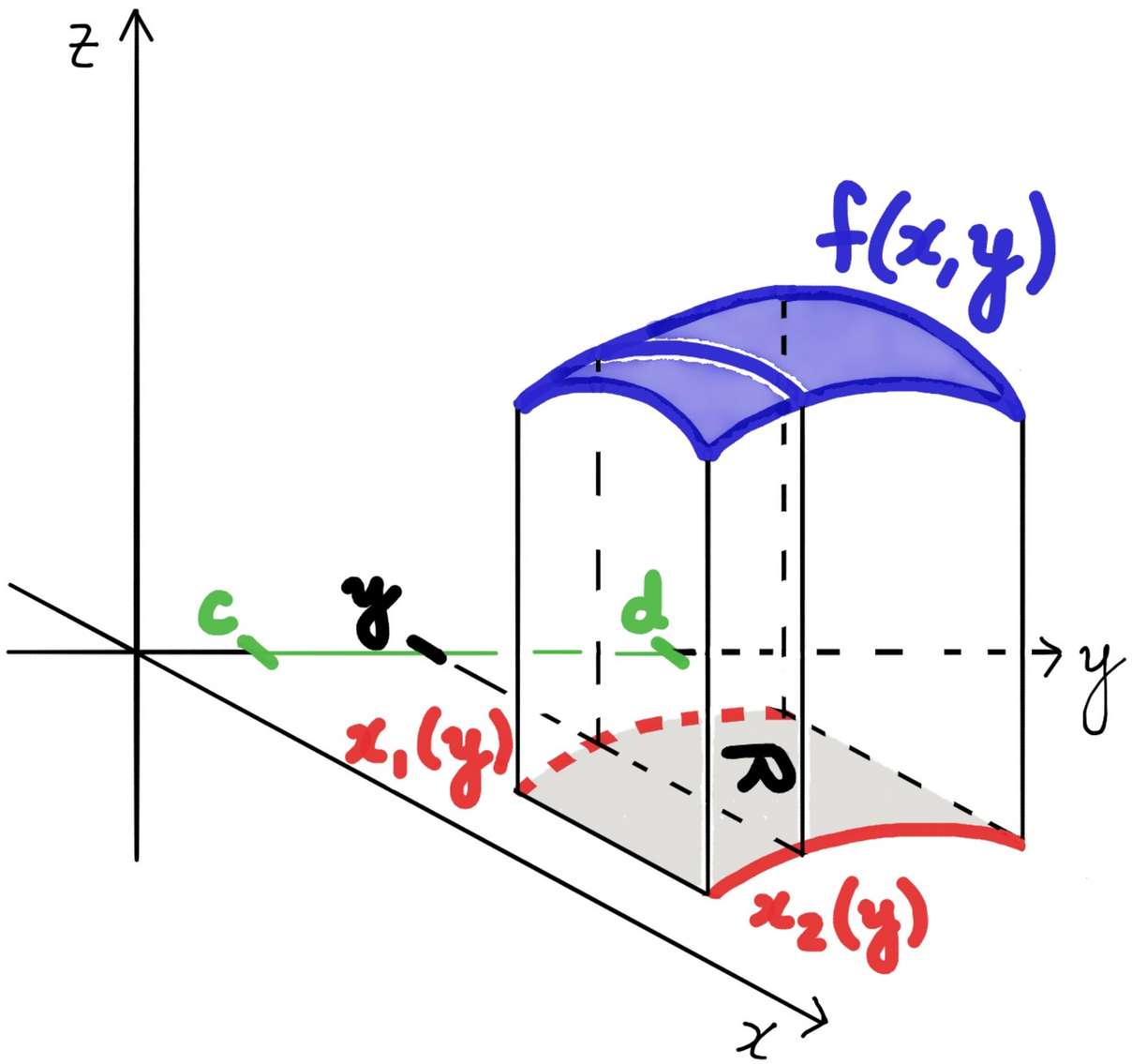
$$\iint_R f(x,y)\,dA = \int_c^d \text{(varying area of cross section)}\,dy$$



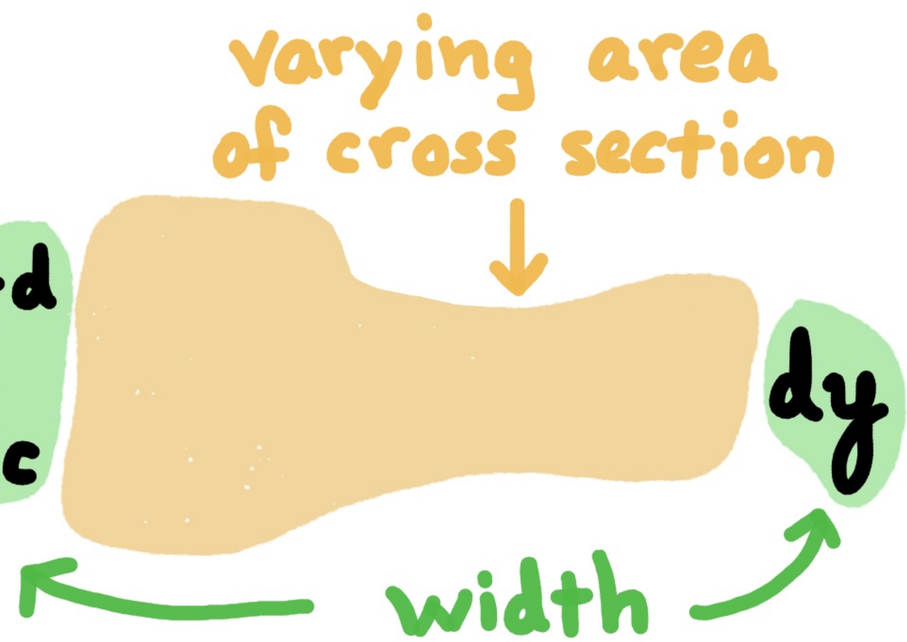


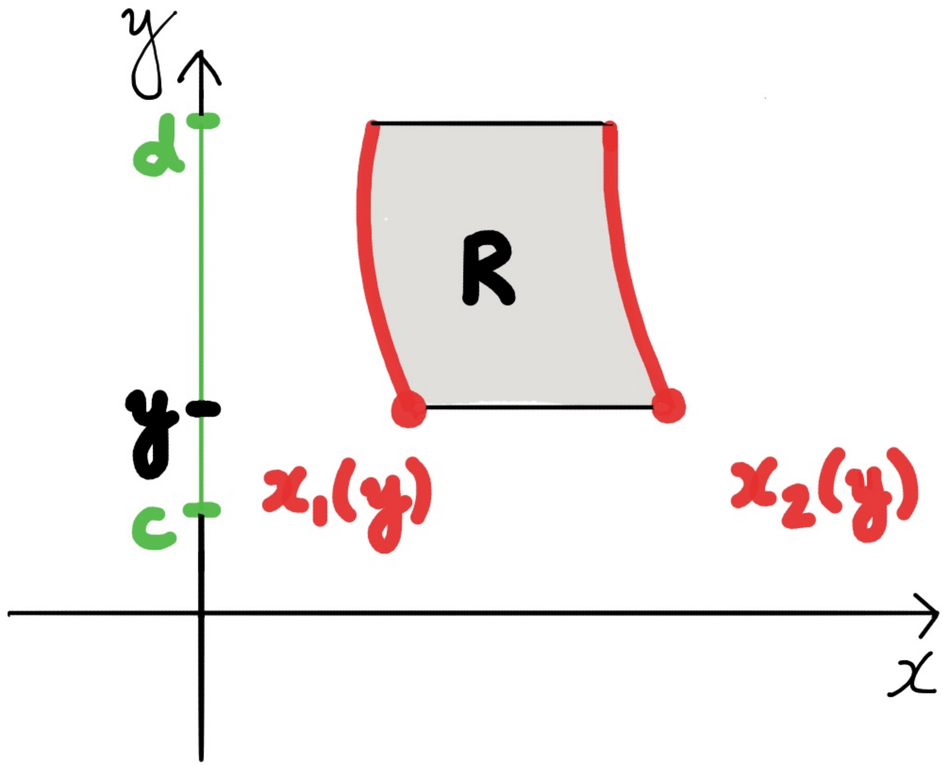
$$R=\{(x,y): c\le y\le d,\ x_1(y)\le x\le x_2(y)\}$$

"x-simple"



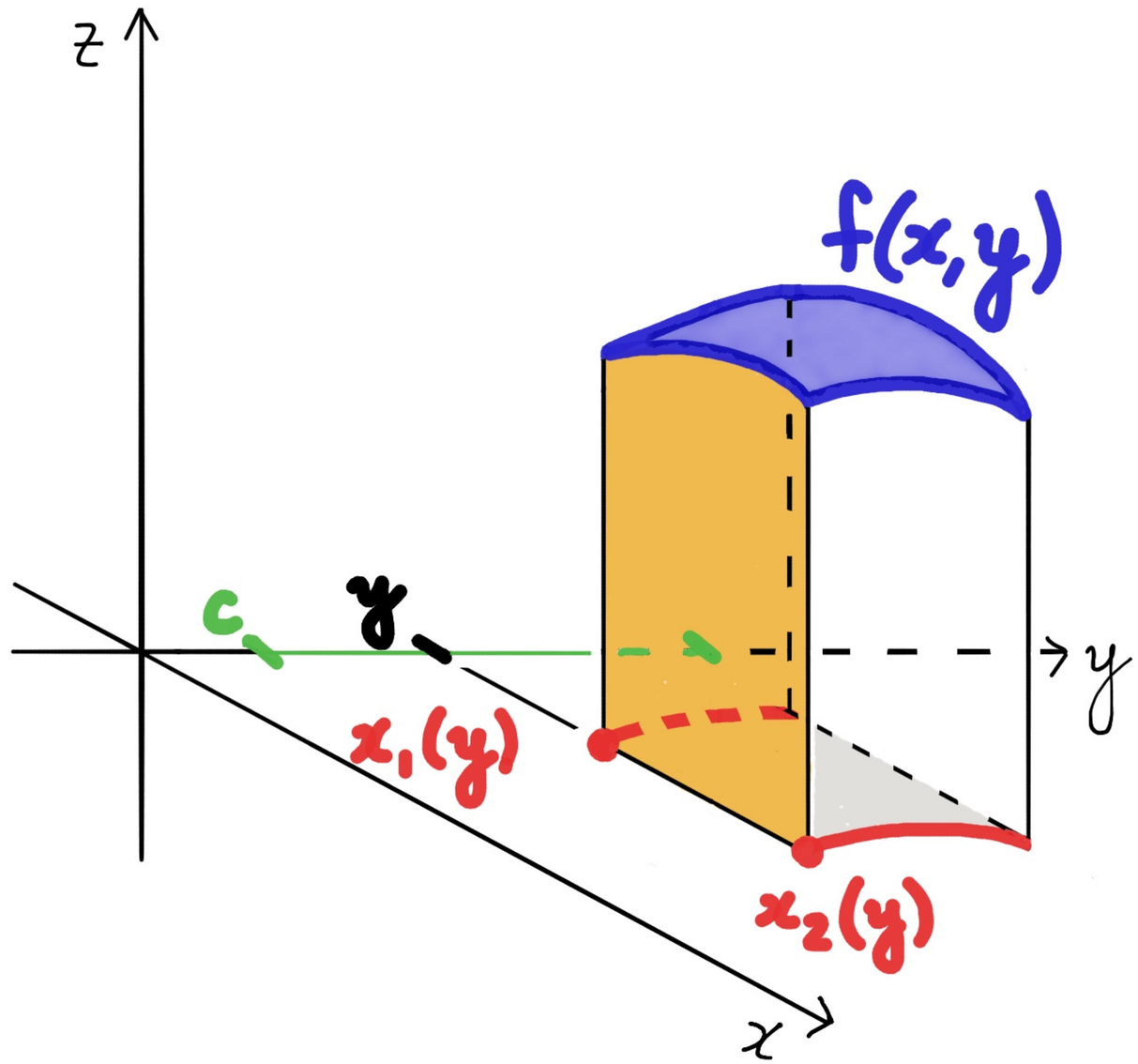
$$\iint_R f(x,y)\,dA = \int_c^d \text{(varying area of cross section)}\,dy$$



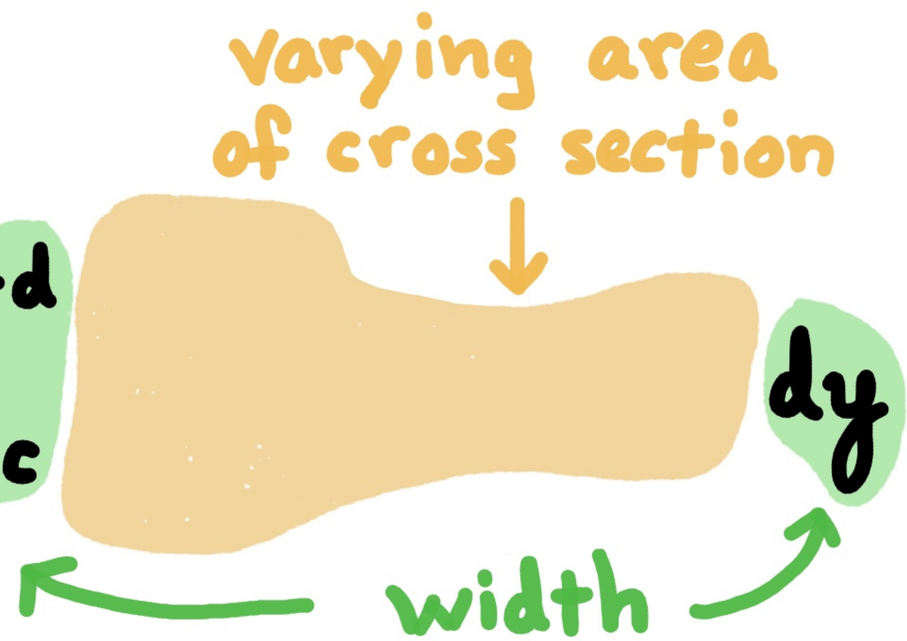


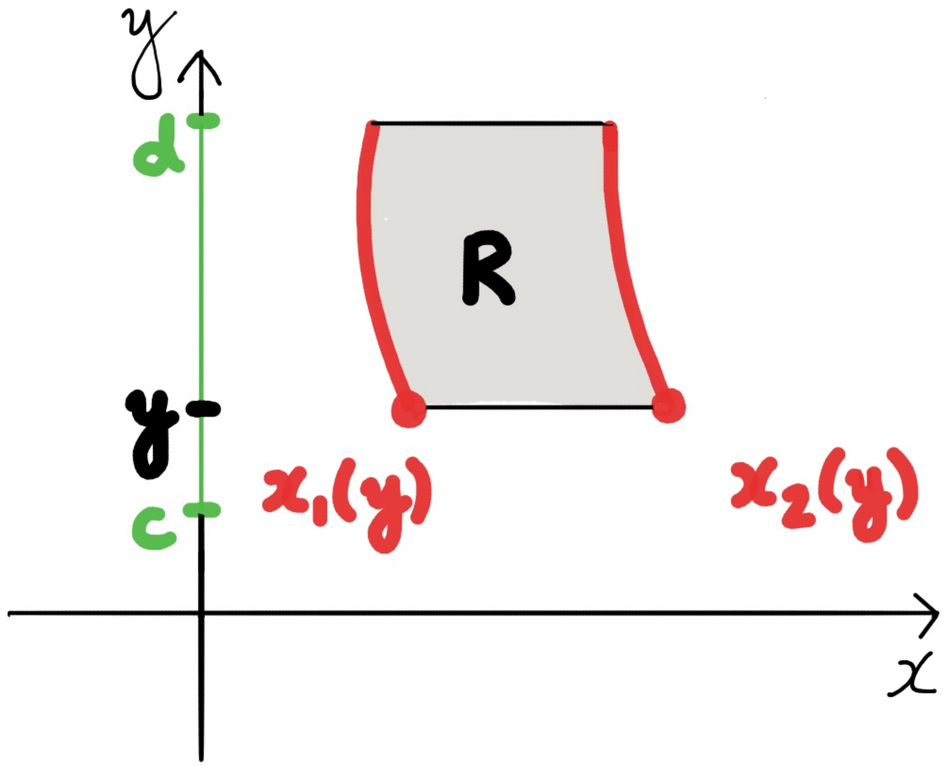
$$R=\{(x,y): c\le y\le d,\ x_1(y)\le x\le x_2(y)\}$$

"x-simple"



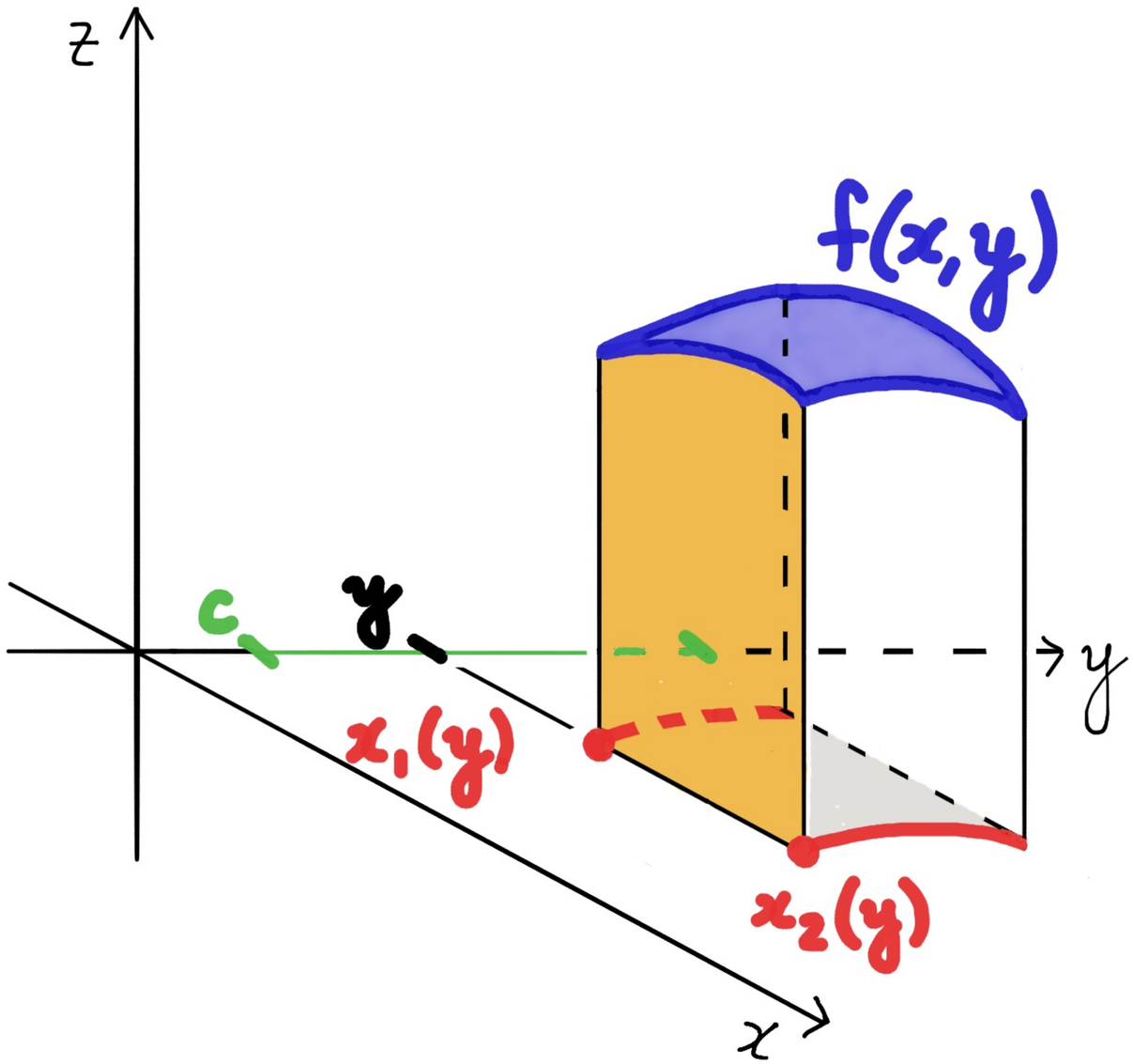
$$\iint_R f(x,y)\,dA = \int_c^d \text{(varying area of cross section)}\,dy$$





$$R=\{(x,y): c\le y\le d,\ x_1(y)\le x\le x_2(y)\}$$

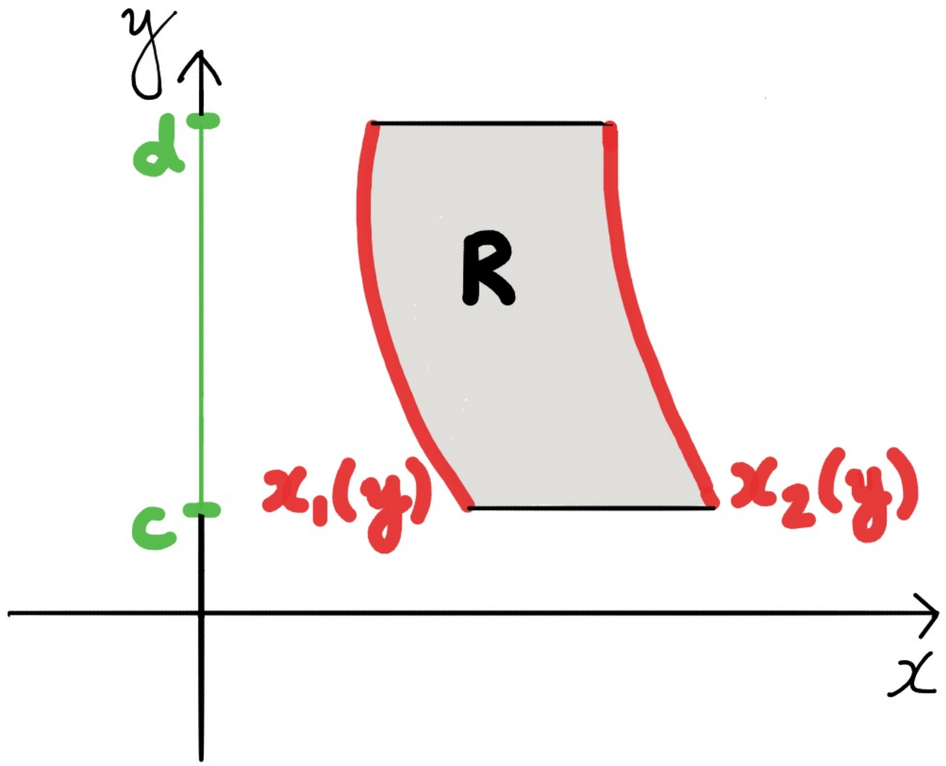
"x-simple"



$$\iint_R f(x,y)\,dA = \int_c^d \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx\,dy$$

varying area of cross section

width



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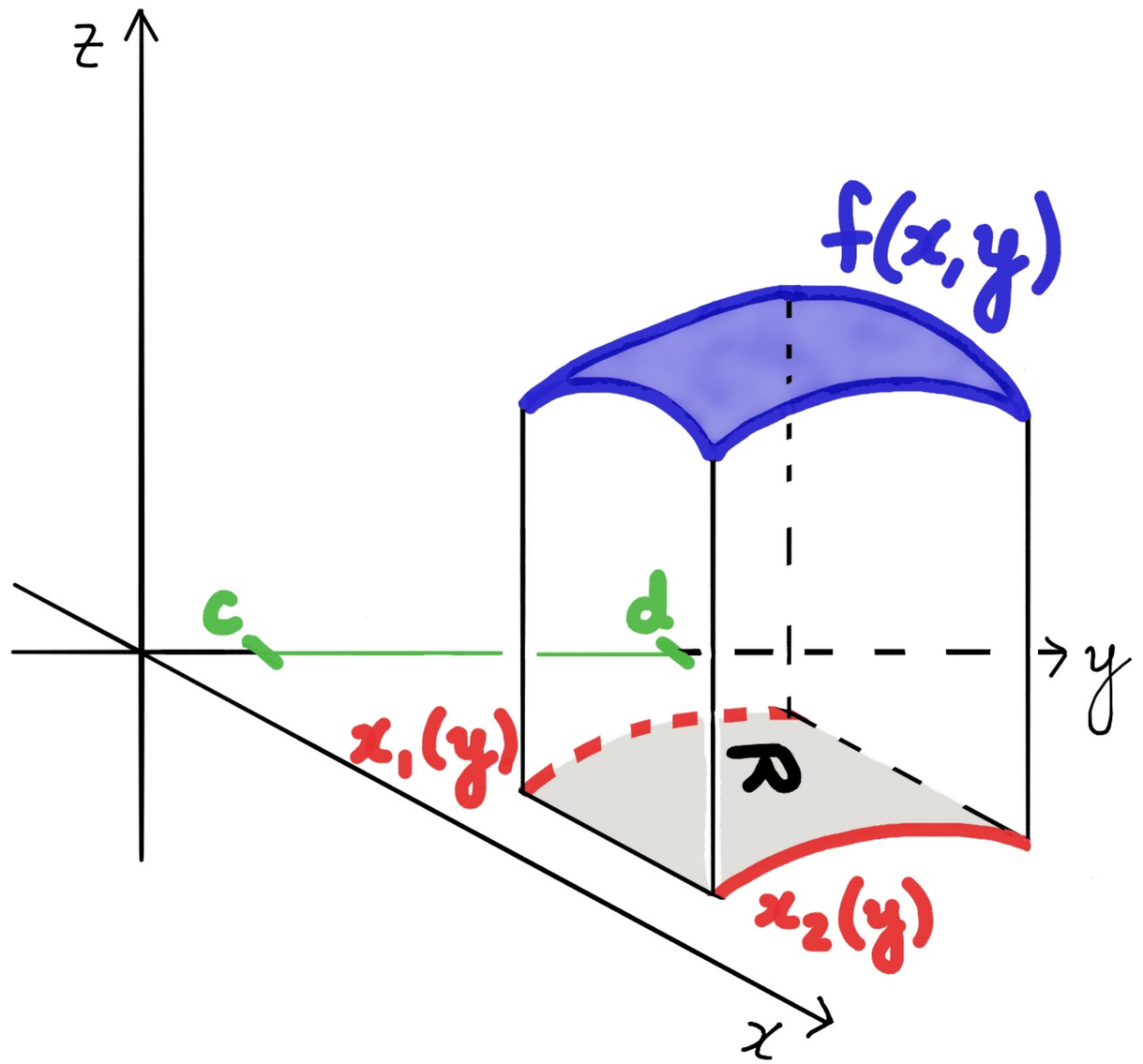

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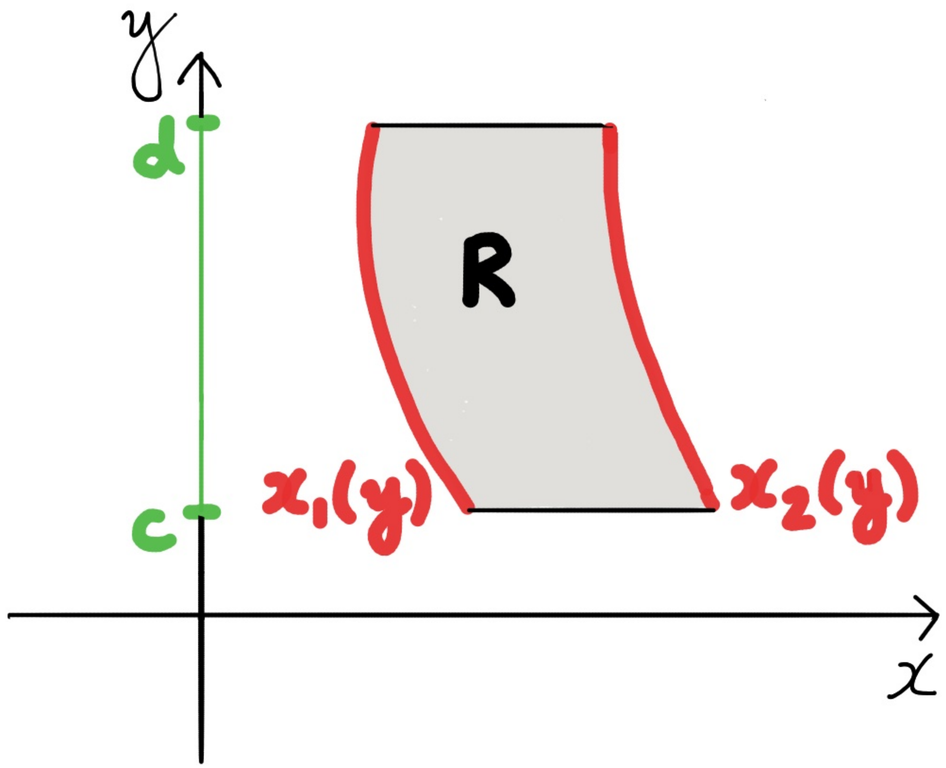
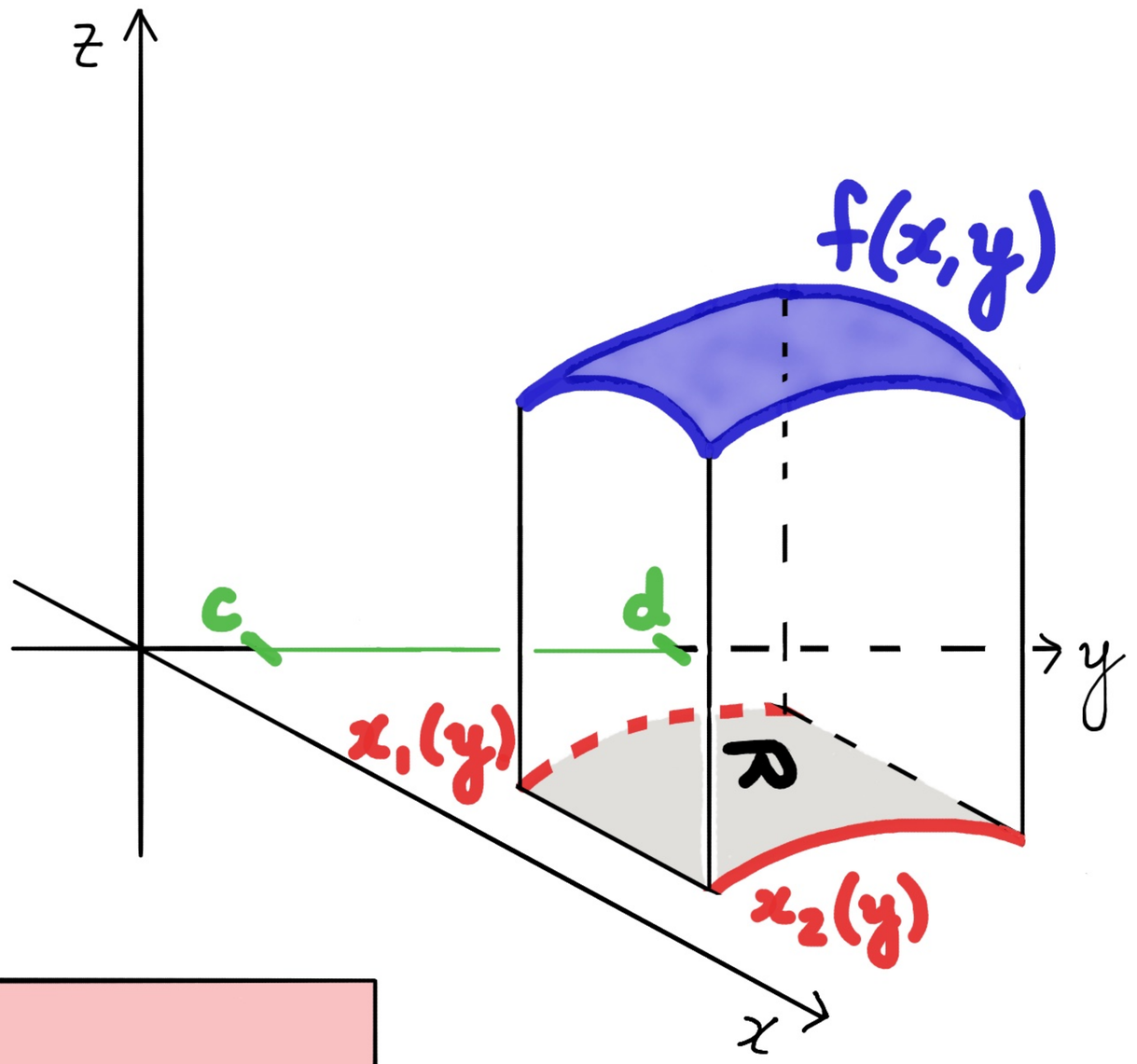
varying area of cross section

width





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$$\iint_R f(x,y)\,dA = \int_c^d \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx\,dy$$

Examples:

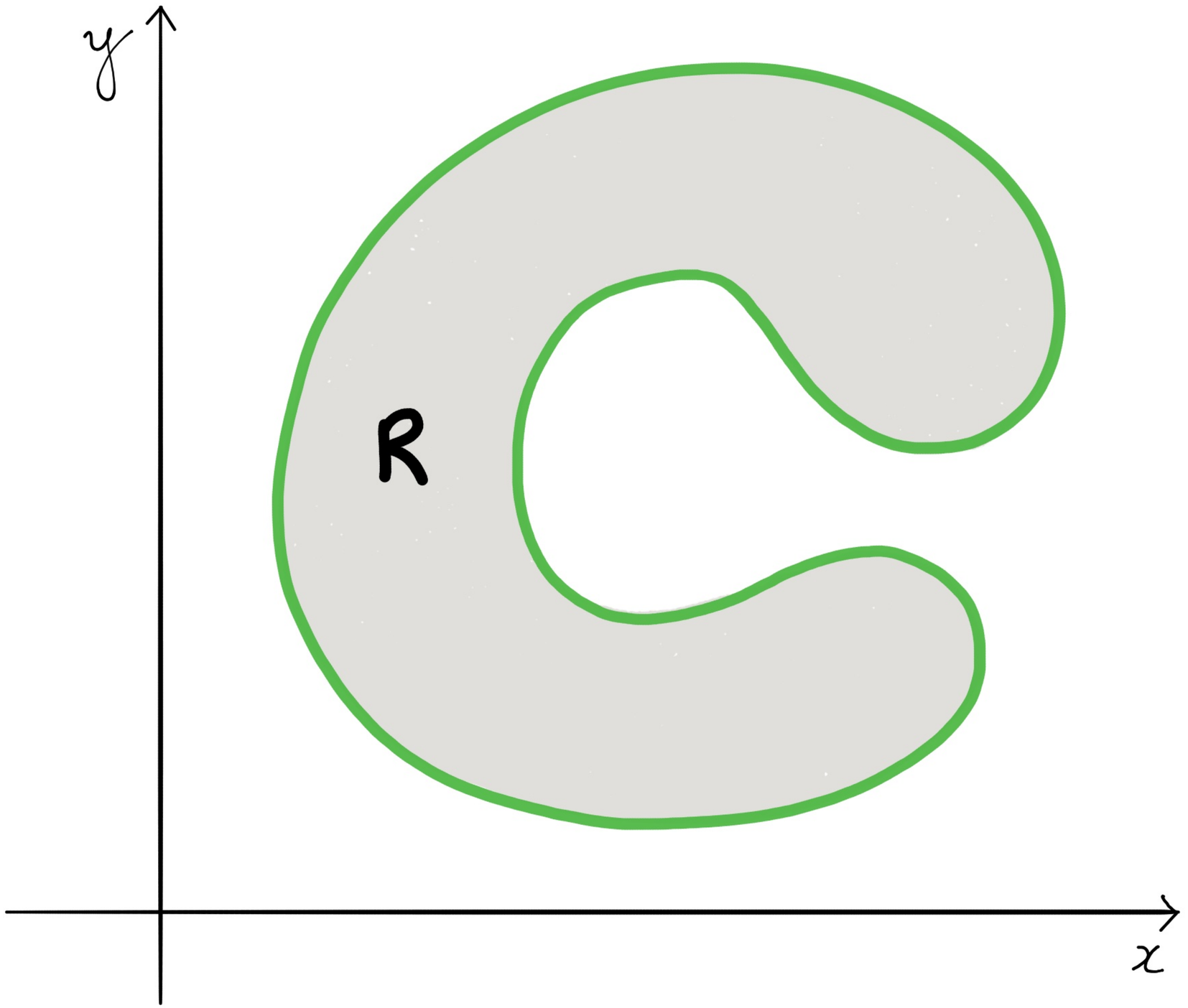
① $\int_0^1 \int_{-x}^{x^2} xy^2 \, dy \, dx$

② $\int_0^{\pi} \int_{\sin y}^{2\sin y} 1 \, dx \, dy$

③ $\iint_R x - 2y \, dA$ where R is the triangular region with vertices (0,0), (1,-1), and (1,3).

④ $\int_0^2 \int_{x/2}^1 e^{y^2} \, dy \, dx$

$$\iint_R f(x,y)\,dA$$



$$\iint_R f(x,y)\,dA$$