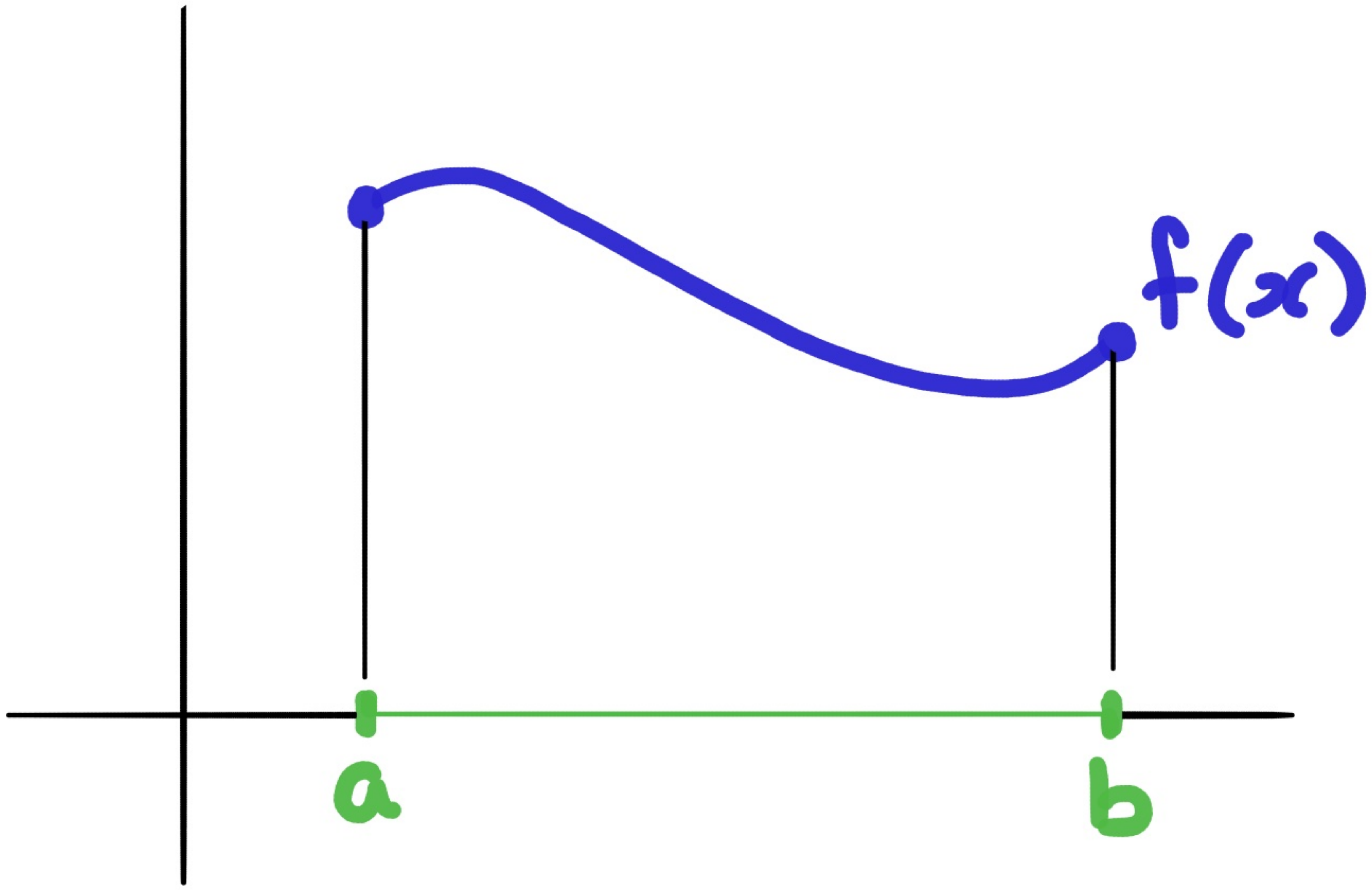
Nineteen

(I) Iterated integrals

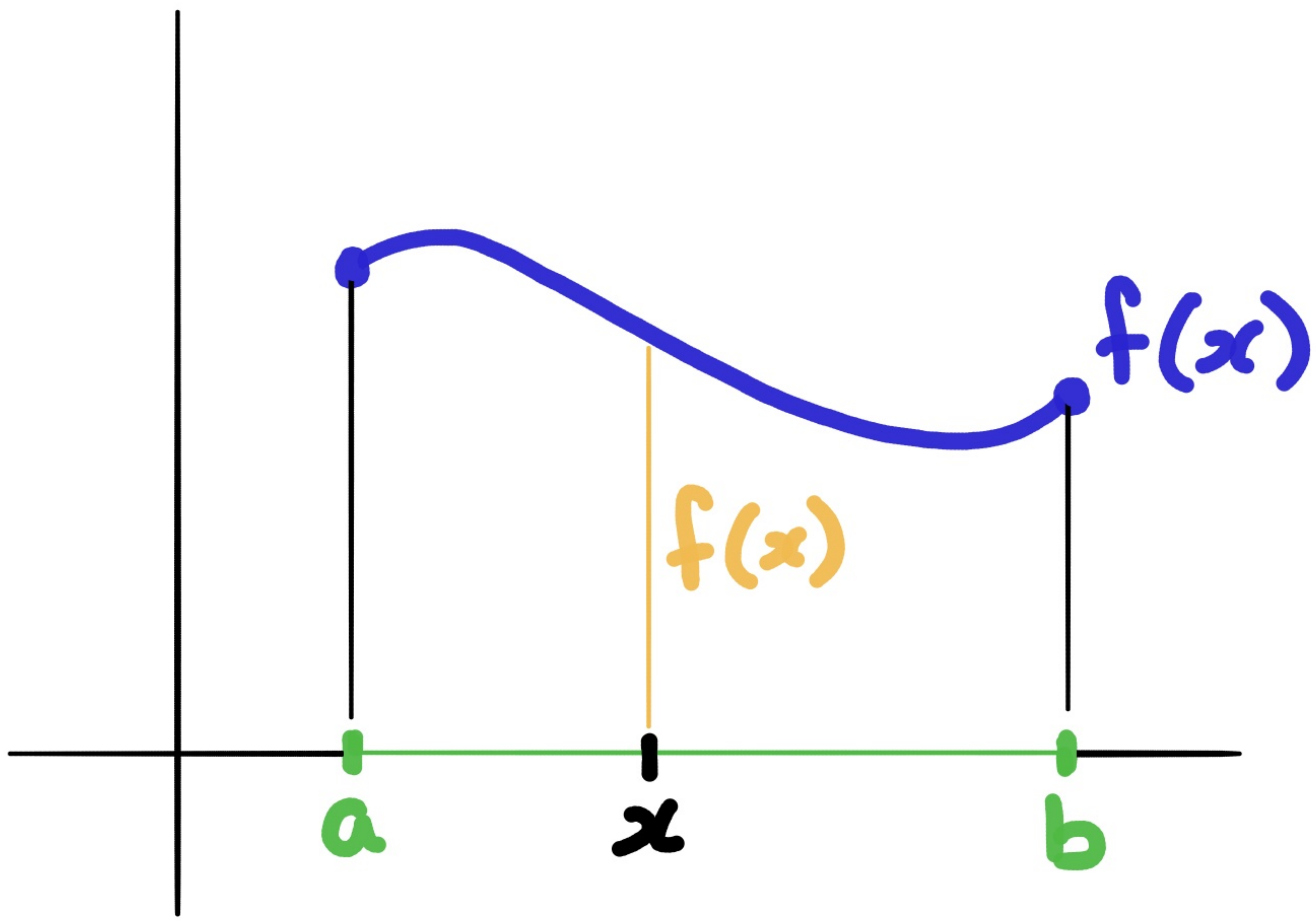
(II) Simple regions in polar coordinates

I Iterated integrals

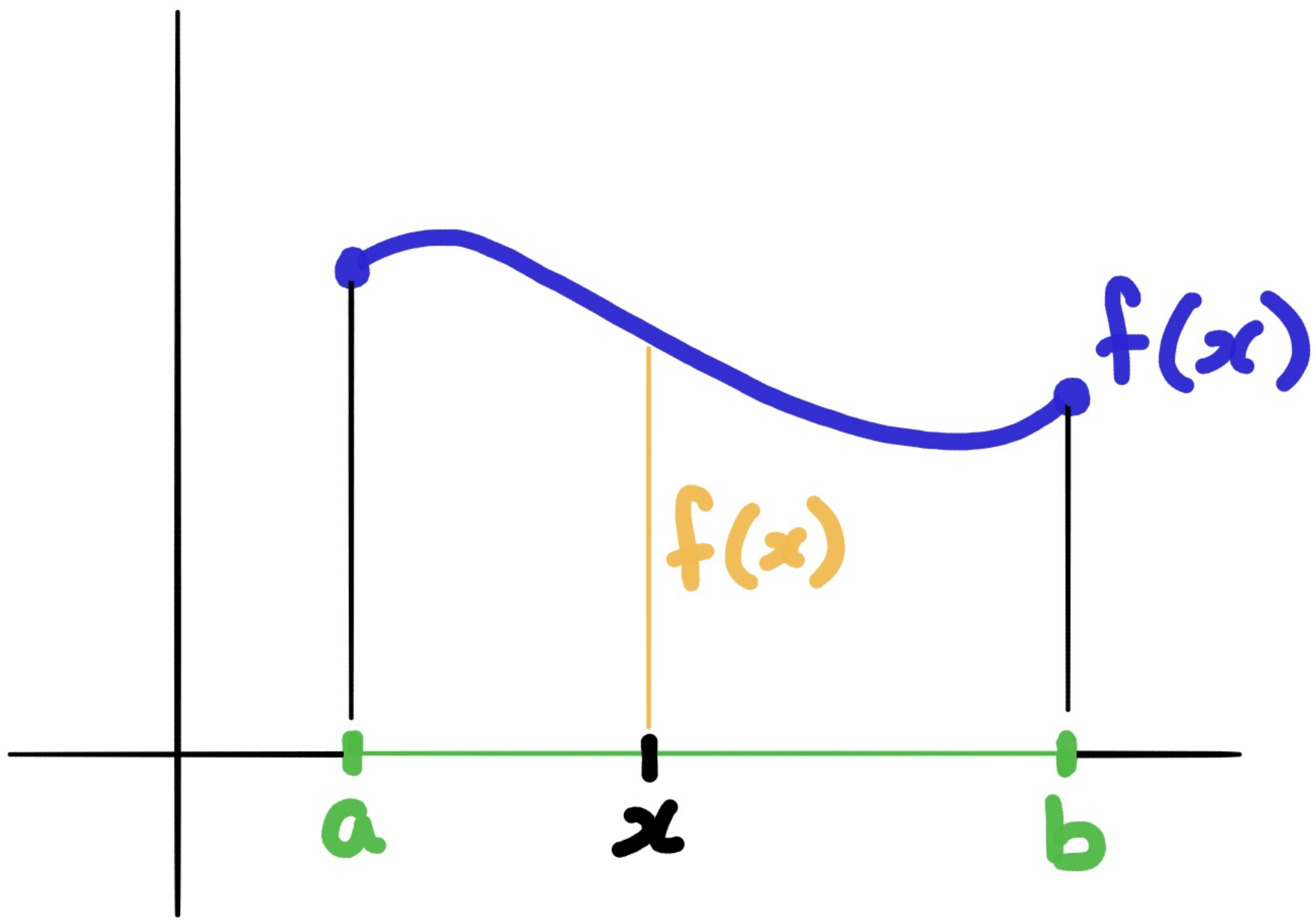
Area: width times height



$$\int_a^b f(x)\,dx$$



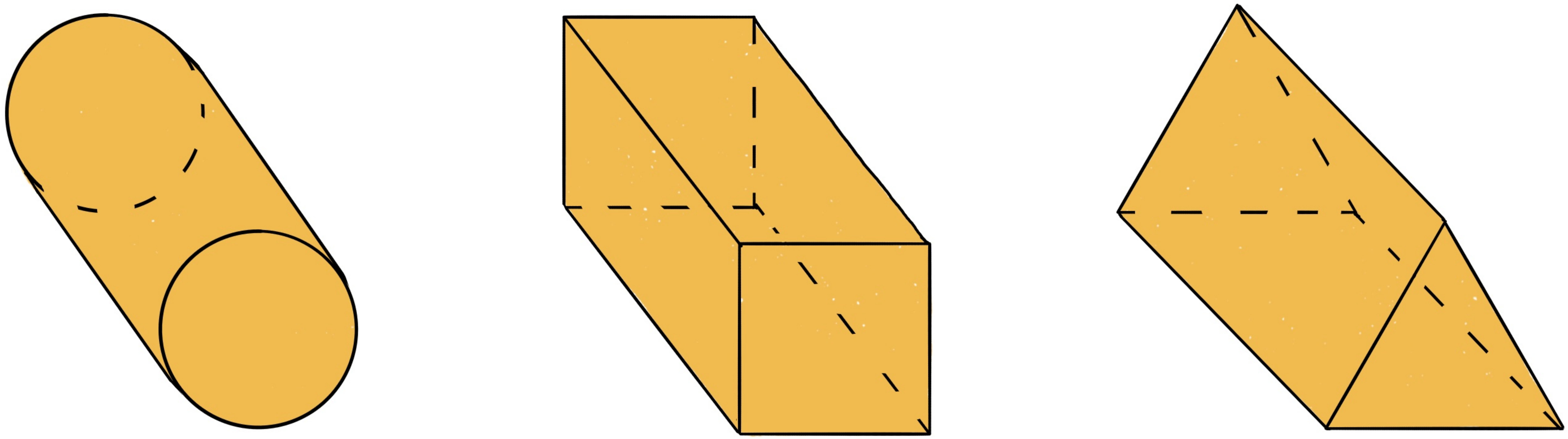
$$\int_a^b f(x)\,dx$$



varying height

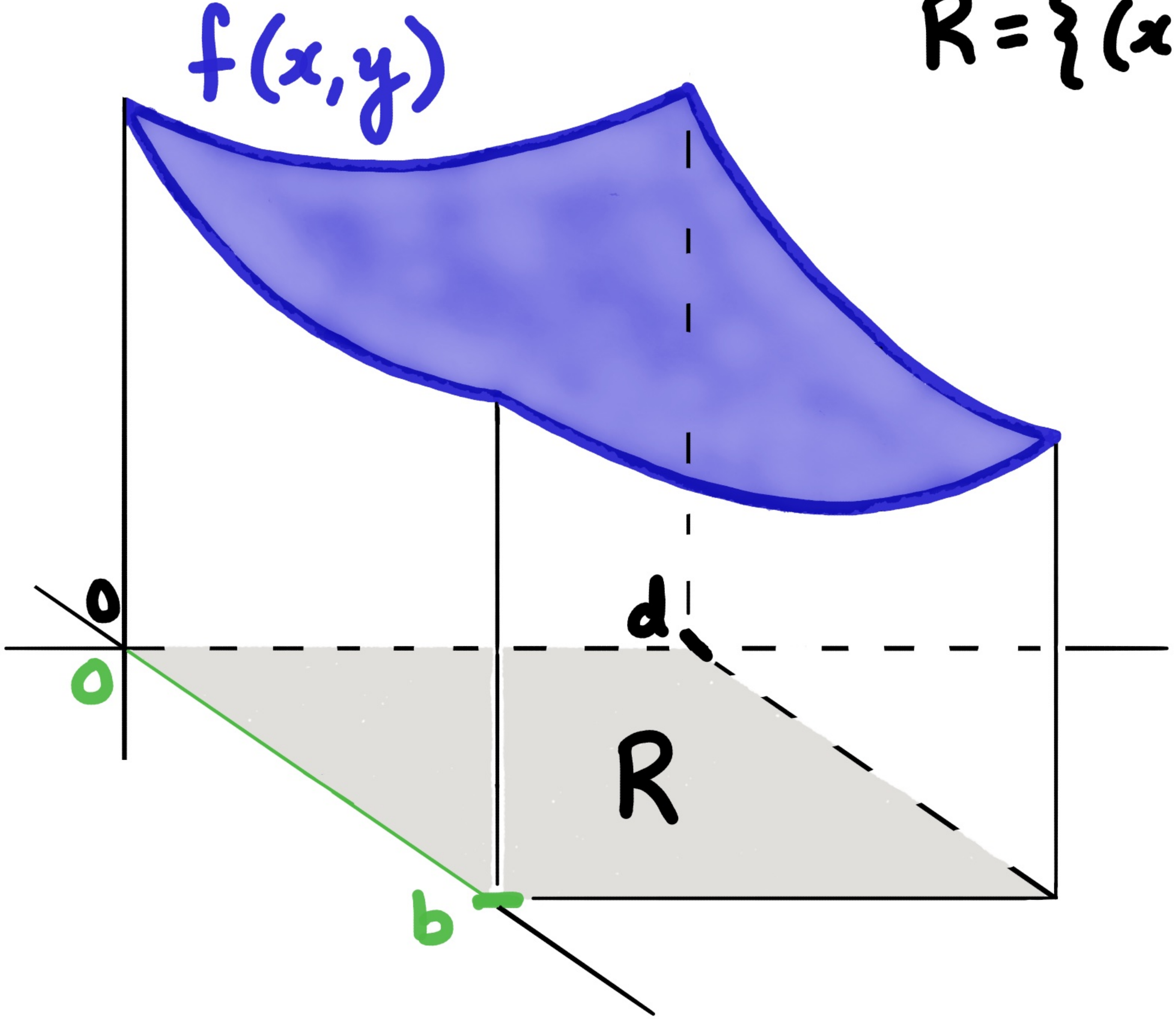
$$\int_a^b f(x)\,dx$$

width



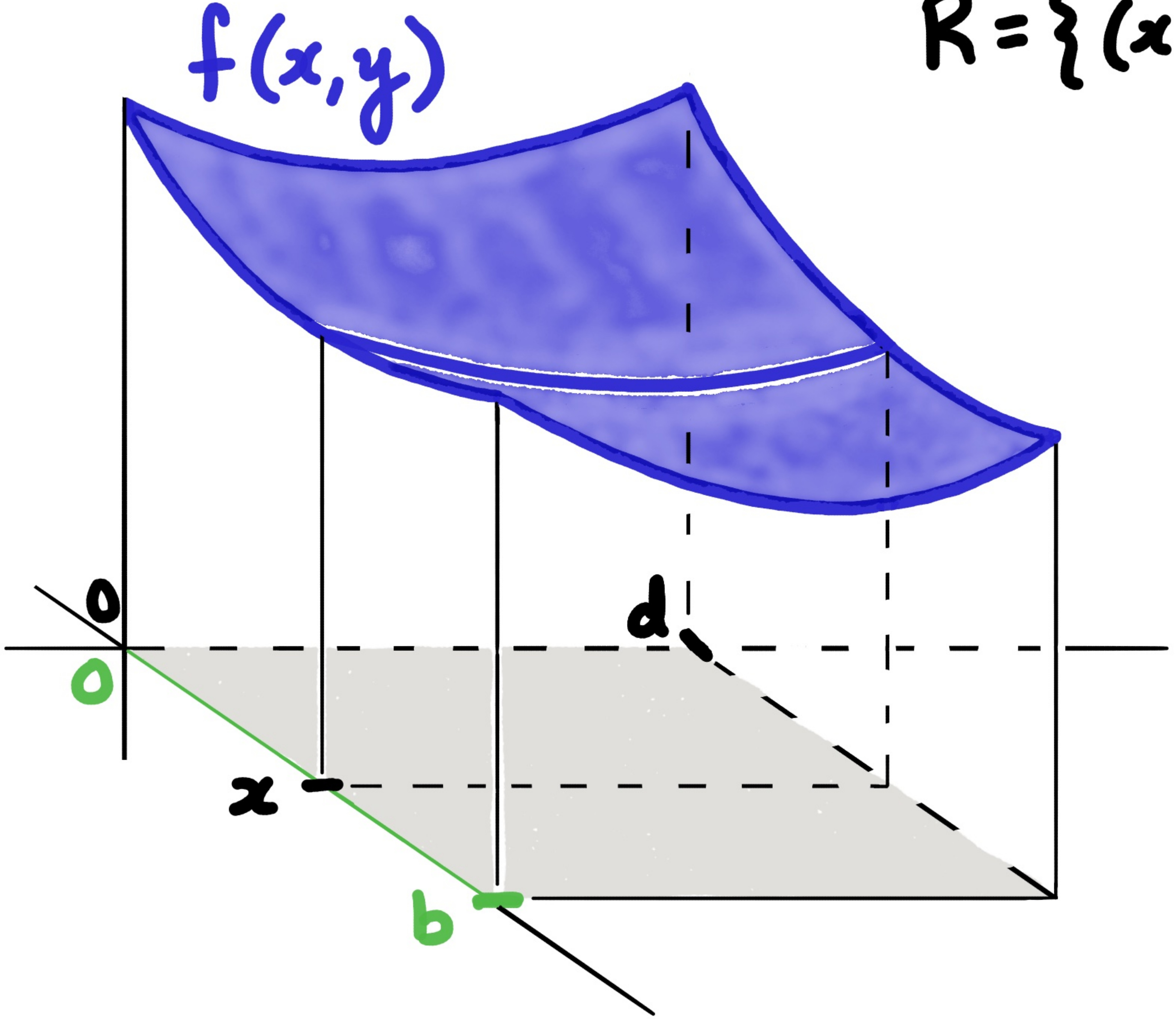
Volume: width times area of cross section

$$R = \{(x,y) : 0 \le x \le b, 0 \le y \le d\}$$



$$\iint_R f(x,y)\,dA$$

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$



$$\iint_R f(x,y)\, dA$$

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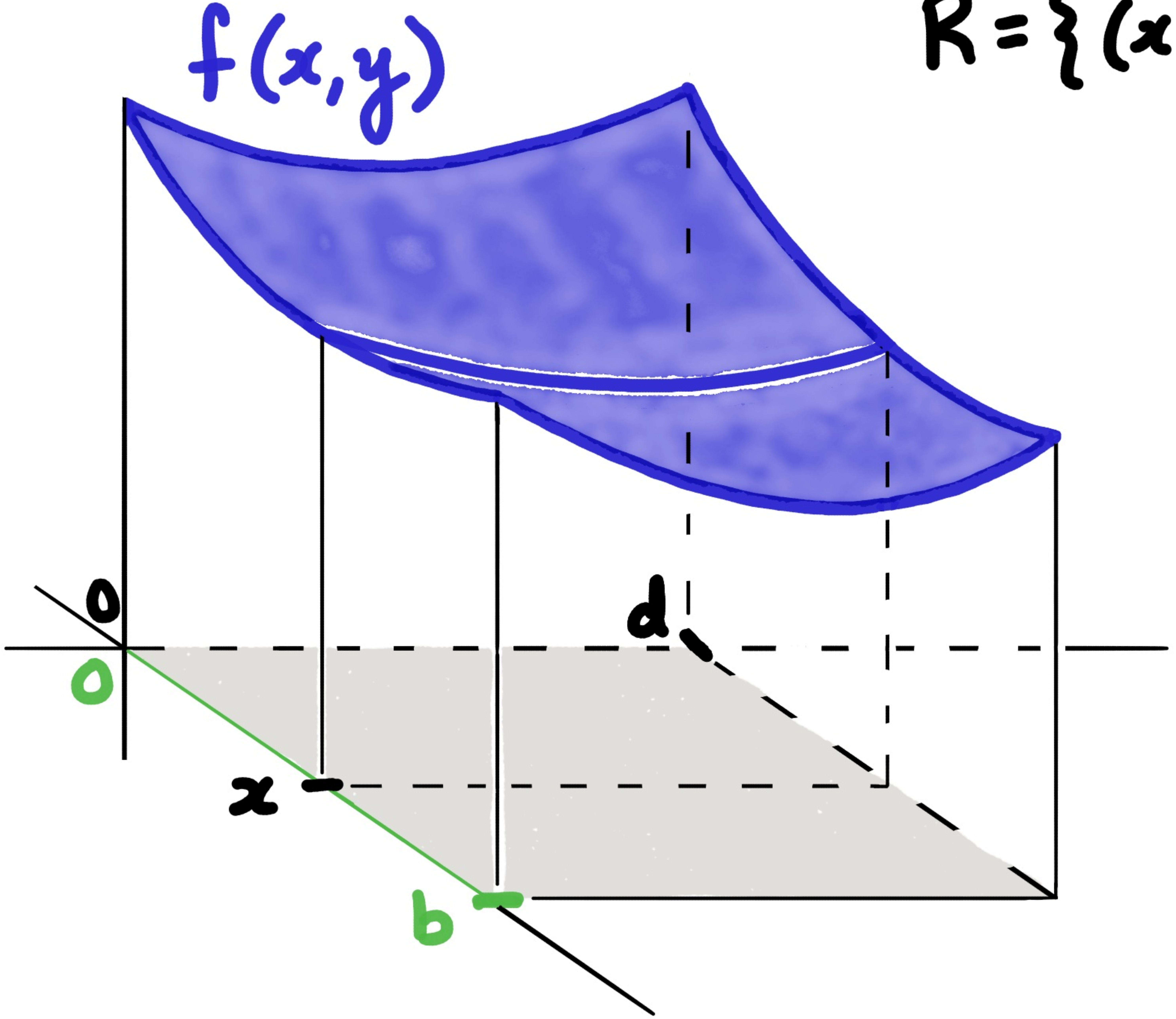
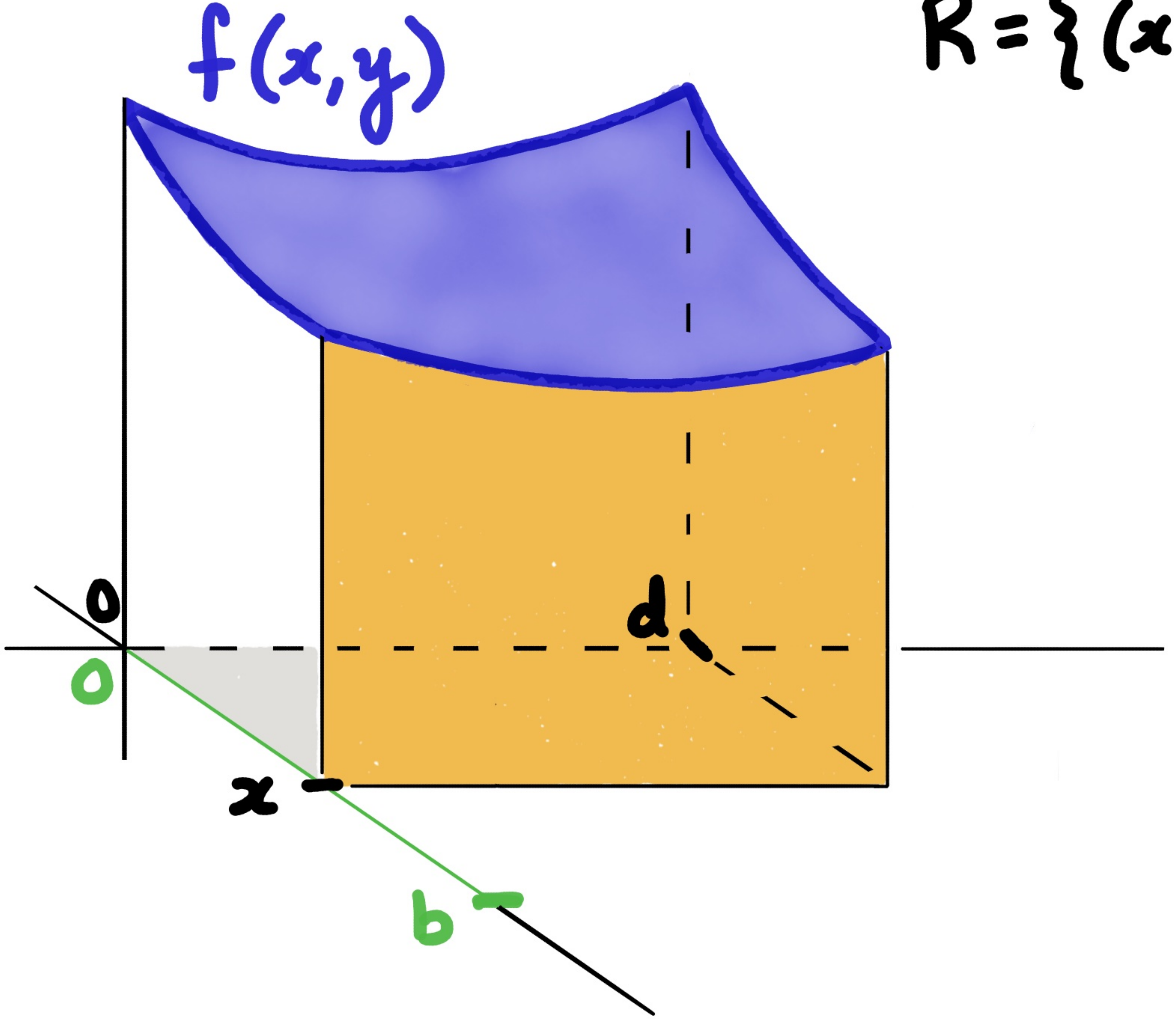


varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{}\,dx$$

width

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$

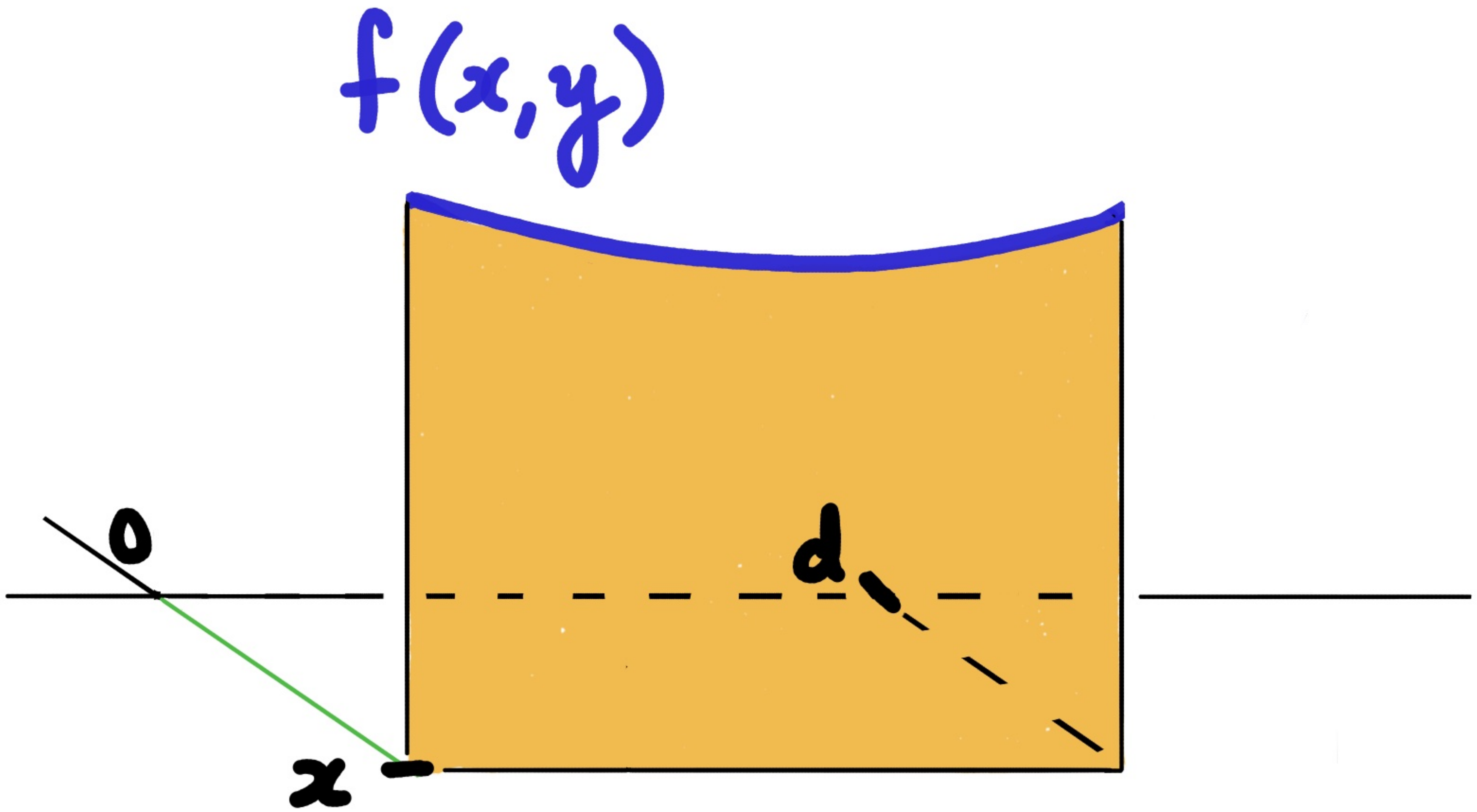


varying
area of
cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{}\,dx$$

width

$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$



varying area of cross section

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width

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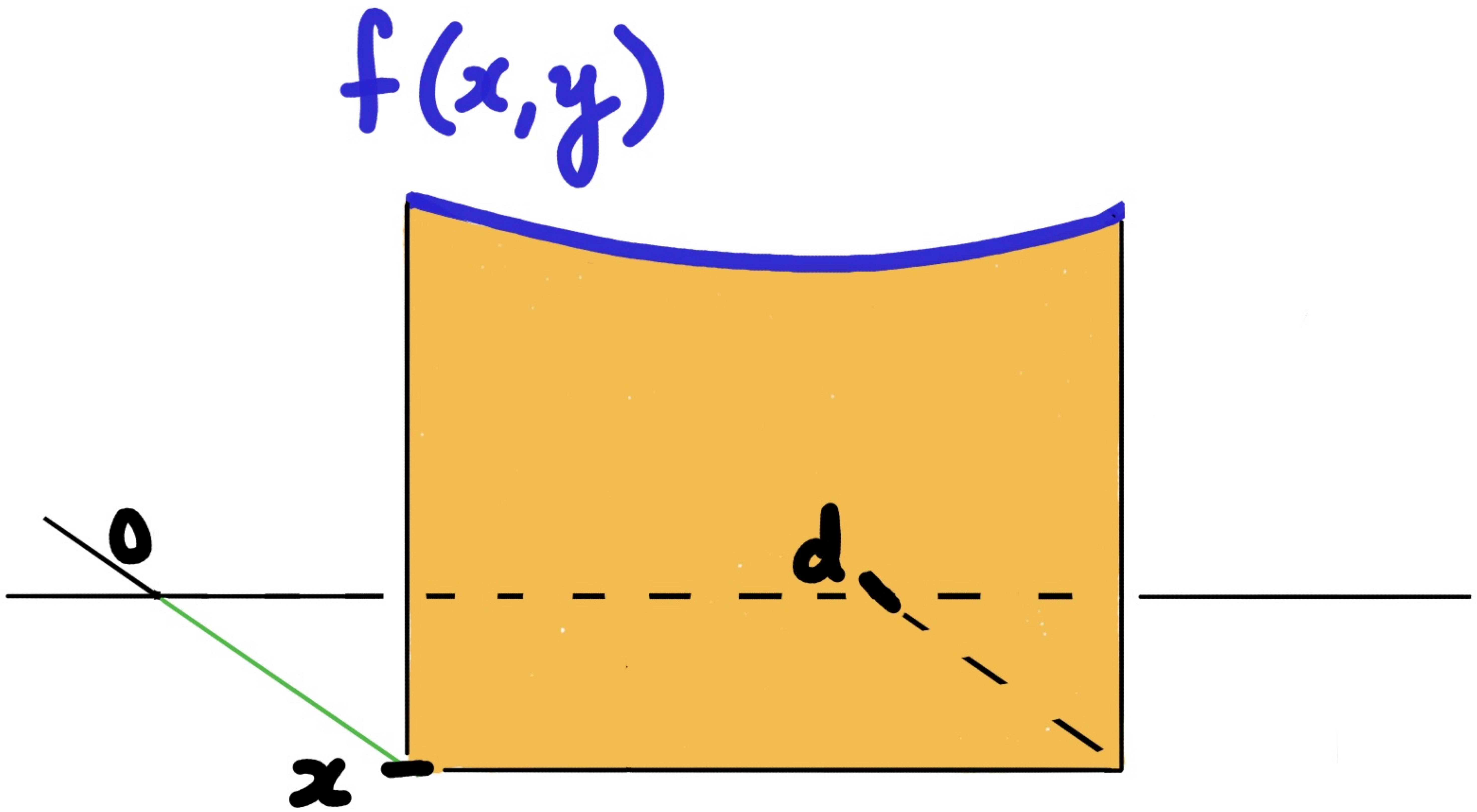


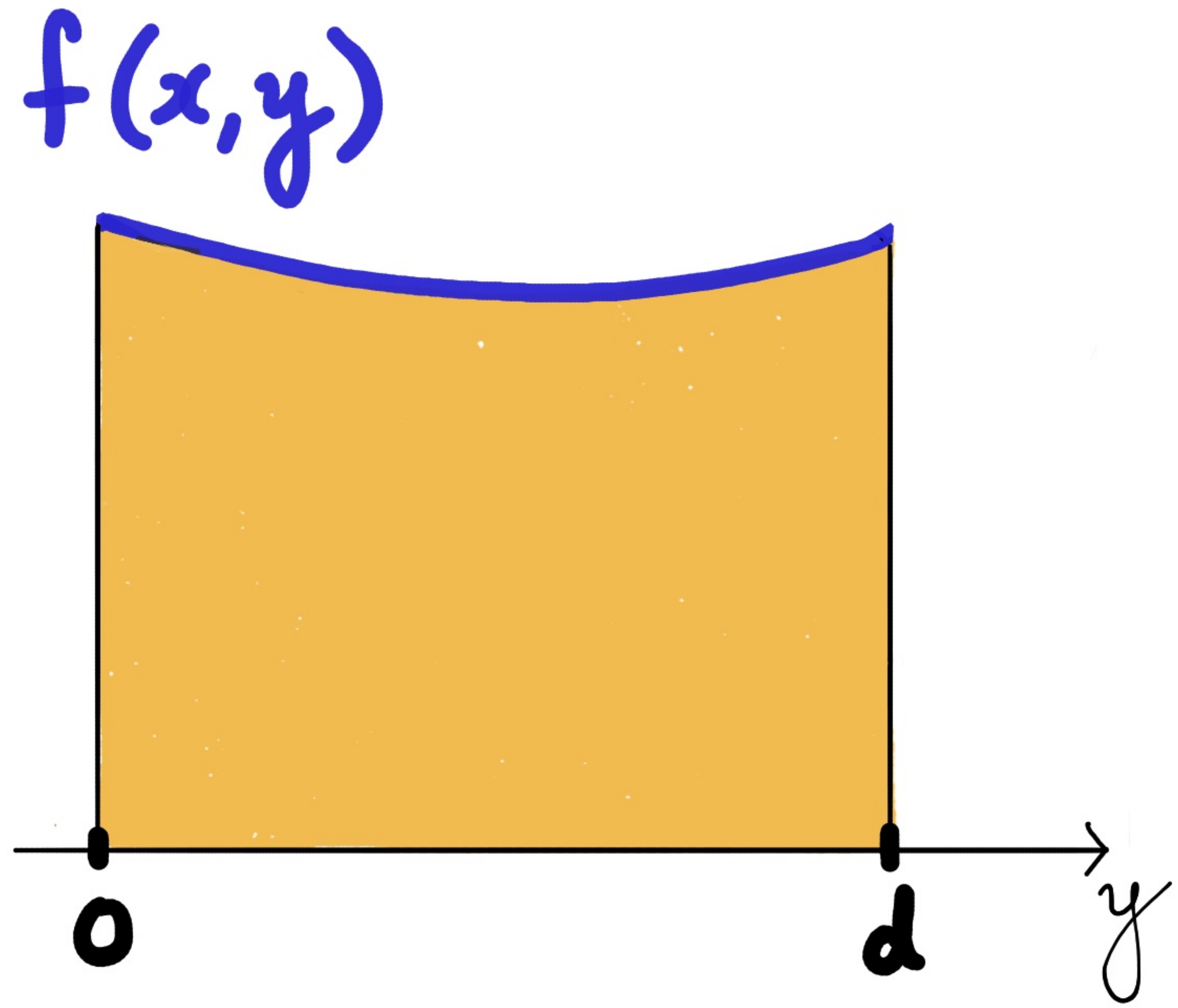
$x = \text{const.}$

varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{}\,dx$$

width

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$



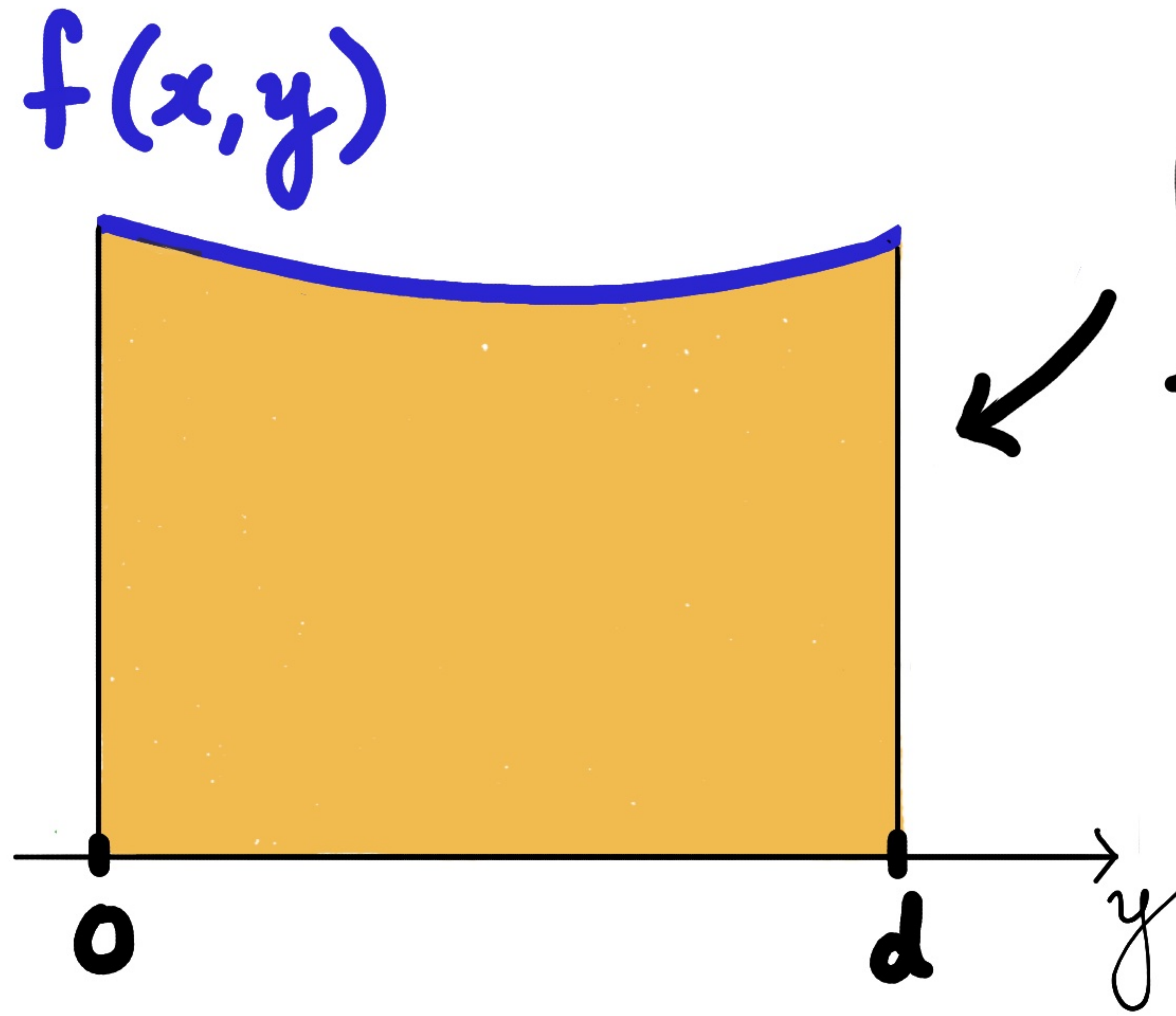
$x = \text{const.}$

varying
area of
cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{}\,dx$$

width

$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$



$$\int_0^d f(x,y)\,dy$$

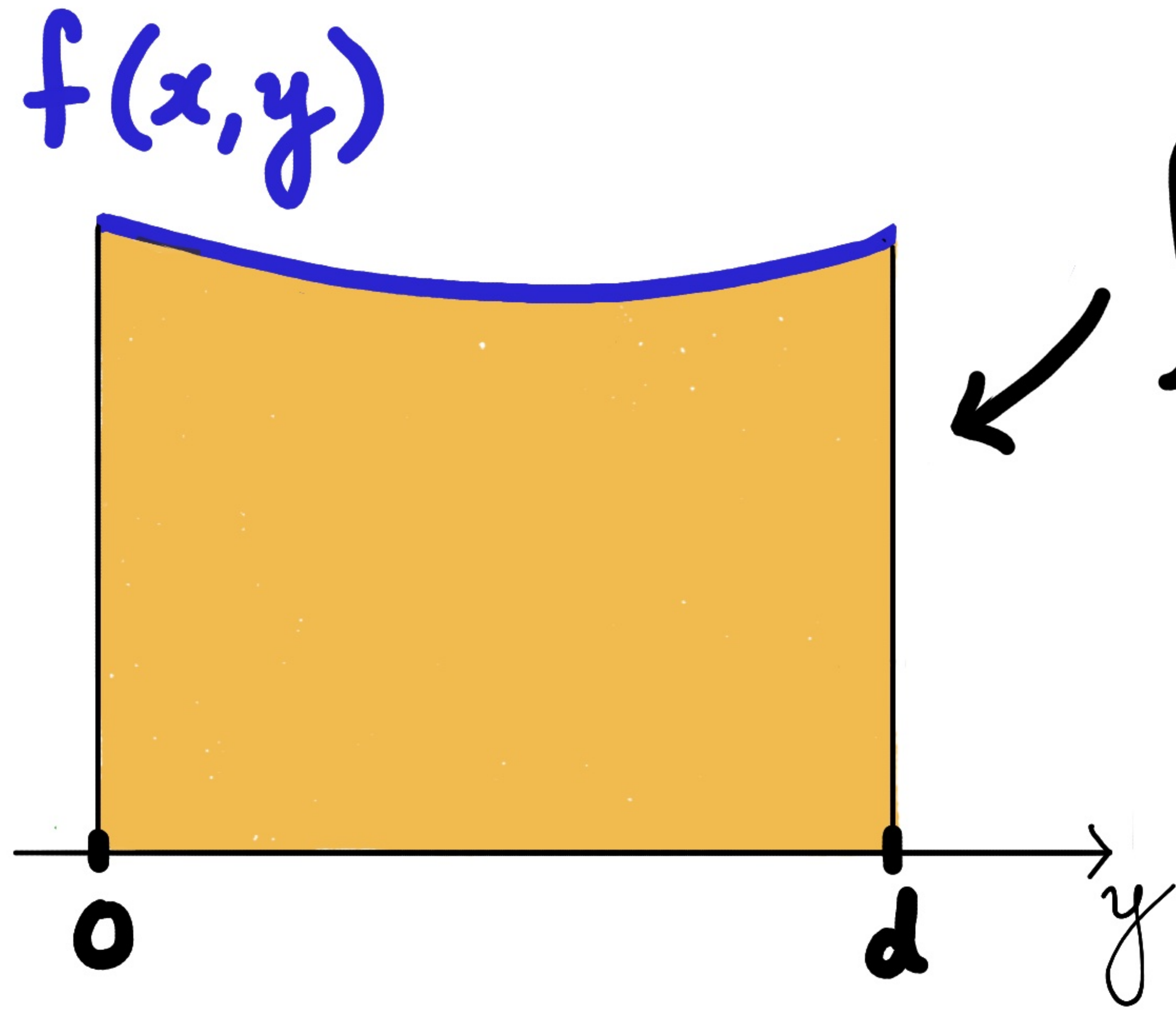
$x = \text{const.}$

varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{}\,dx$$

width

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$



$$\int_0^d f(x,y)\,dy$$

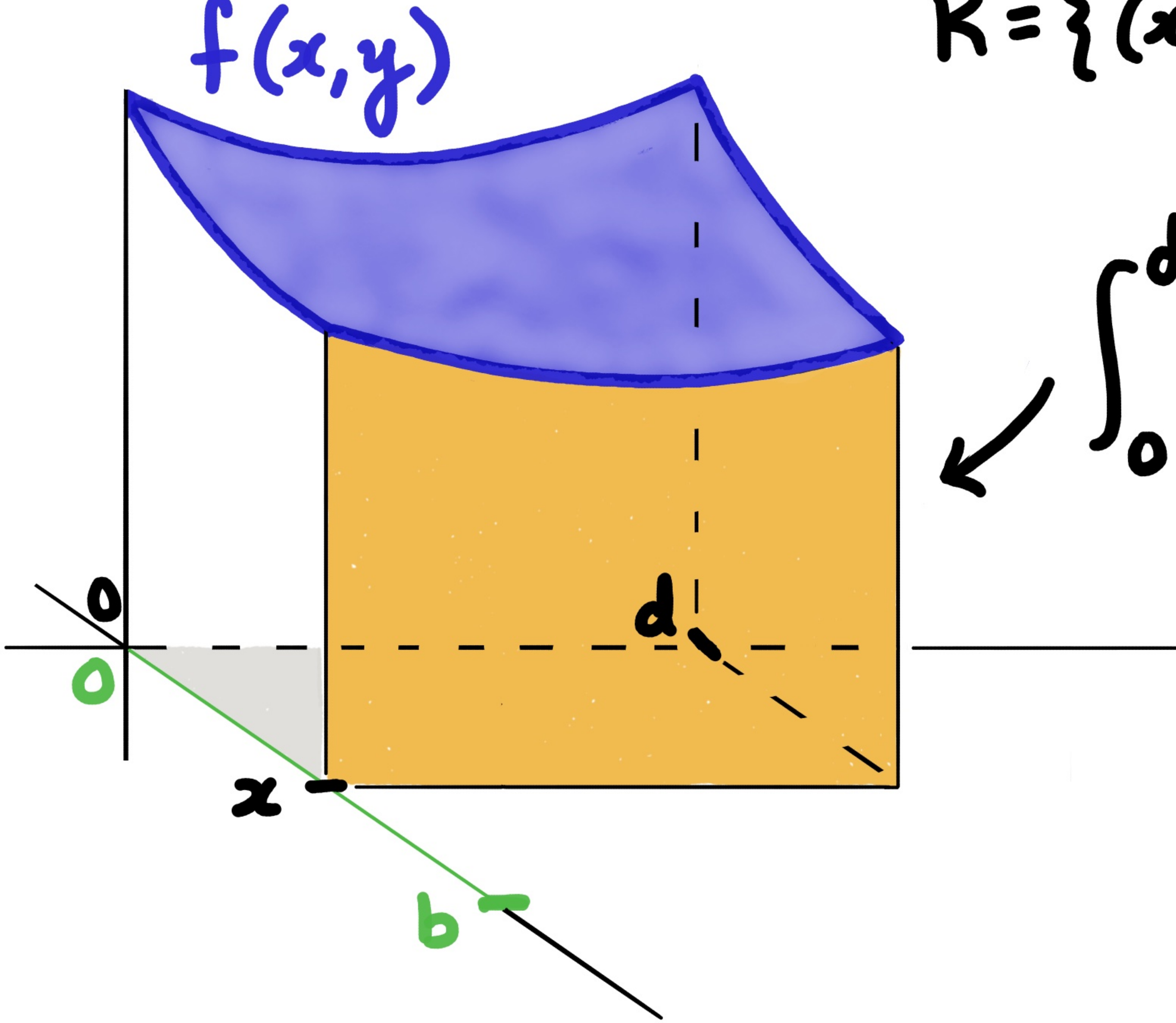
$x = \text{const.}$

varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \boxed{\int_0^d f(x,y)\,dy}\,dx$$

width

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$



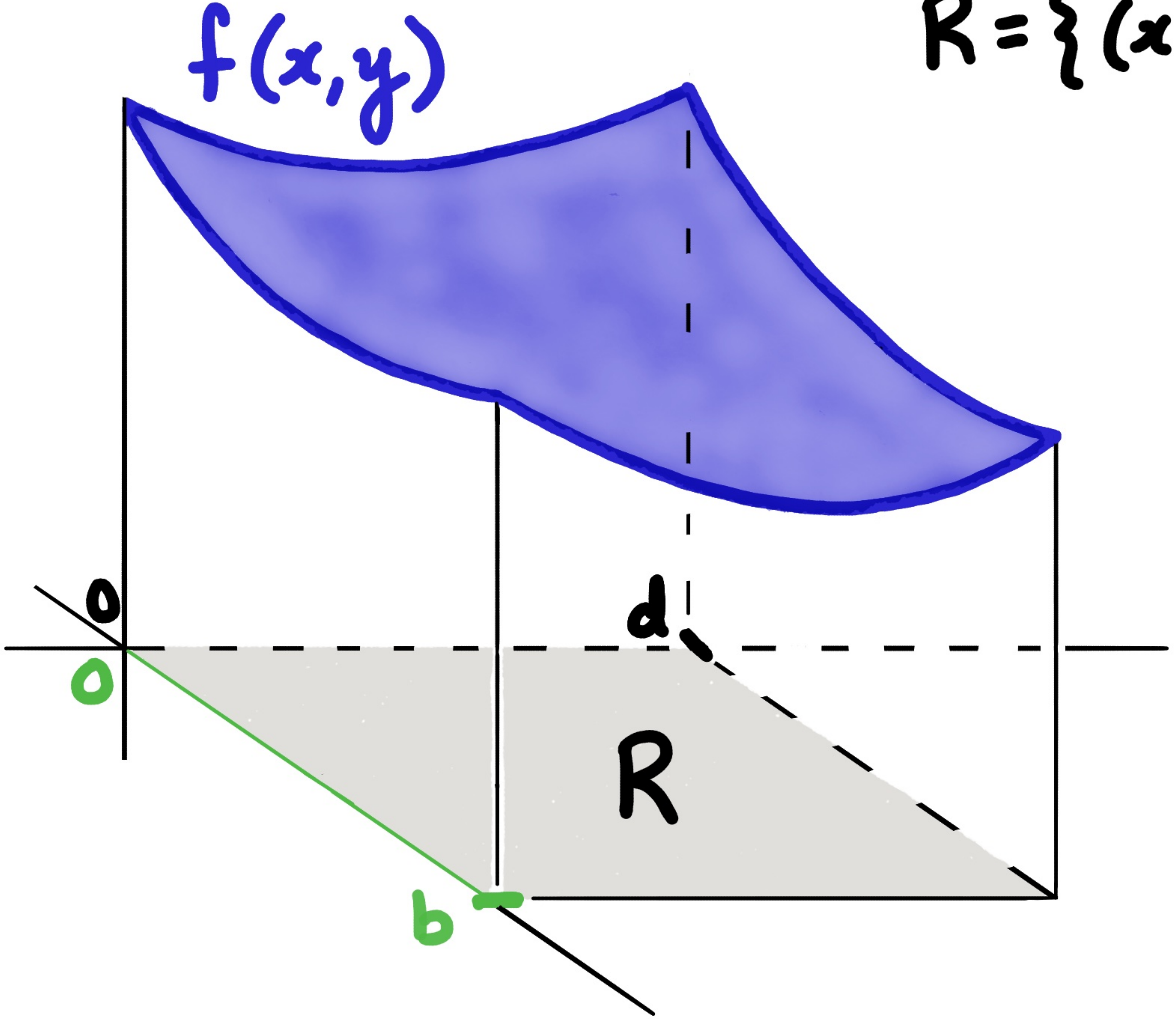
$$\int_0^d f(x,y)\,dy$$

varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \int_0^d f(x,y)\,dy\,dx$$

width

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$



varying area of cross section

$$\iint_R f(x,y)\,dA = \int_0^b \int_0^d f(x,y)\,dy\,dx$$

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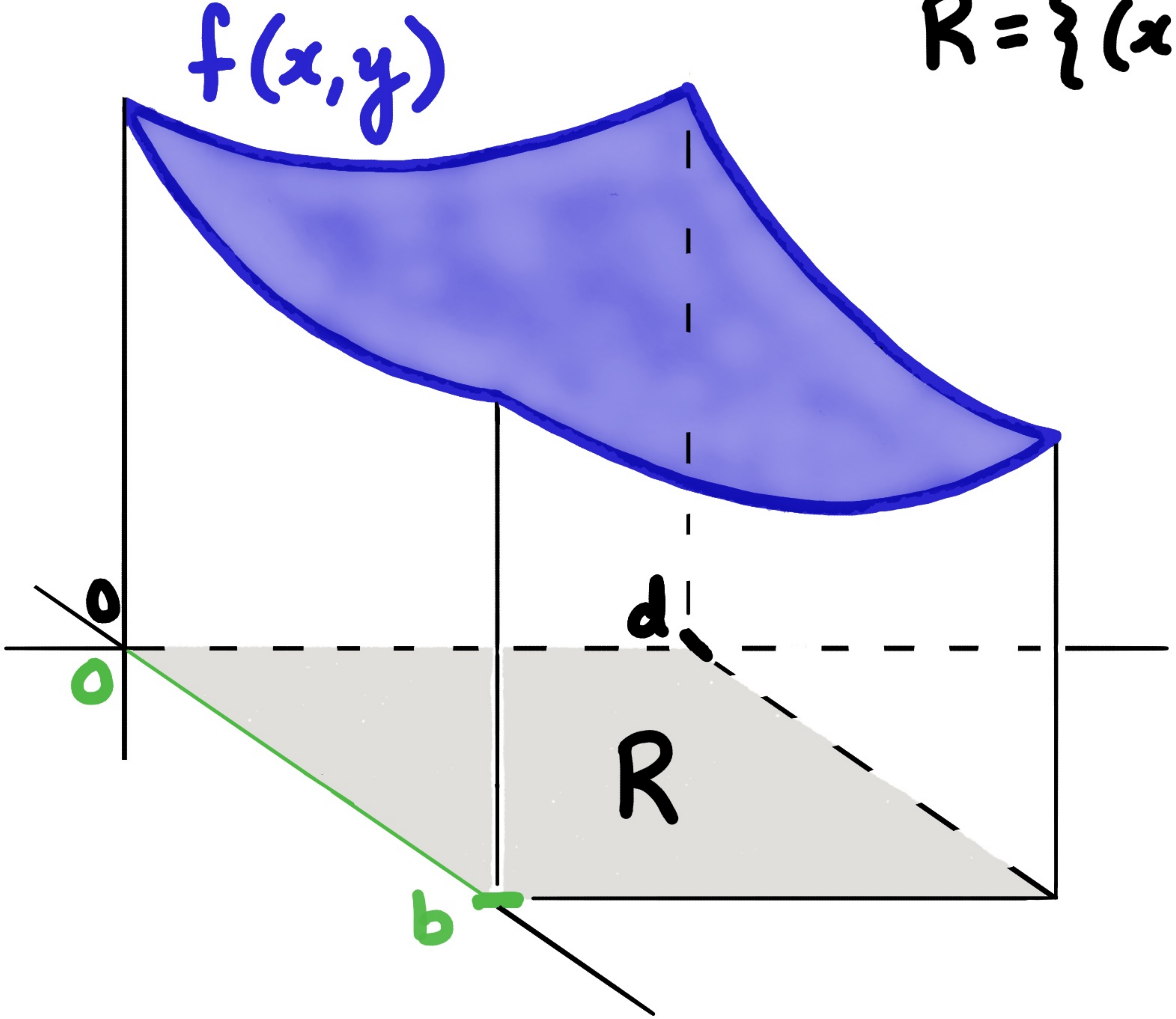
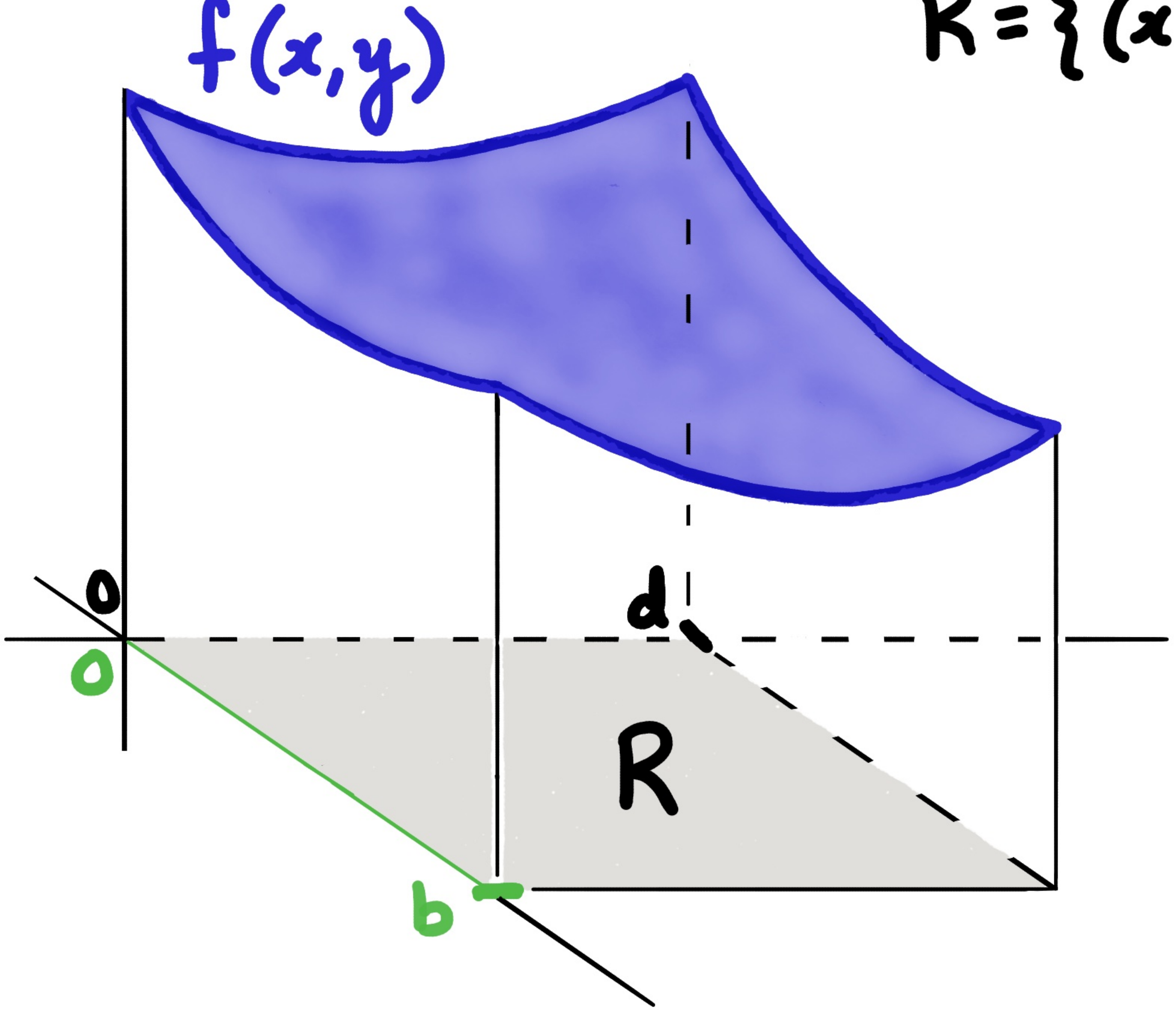


$$\iint_R f(x,y)\,dA = \int_0^b \int_0^d f(x,y)\,dy\,dx$$

$$R = \{(x,y) : 0 \le x \le b,\ 0 \le y \le d\}$$

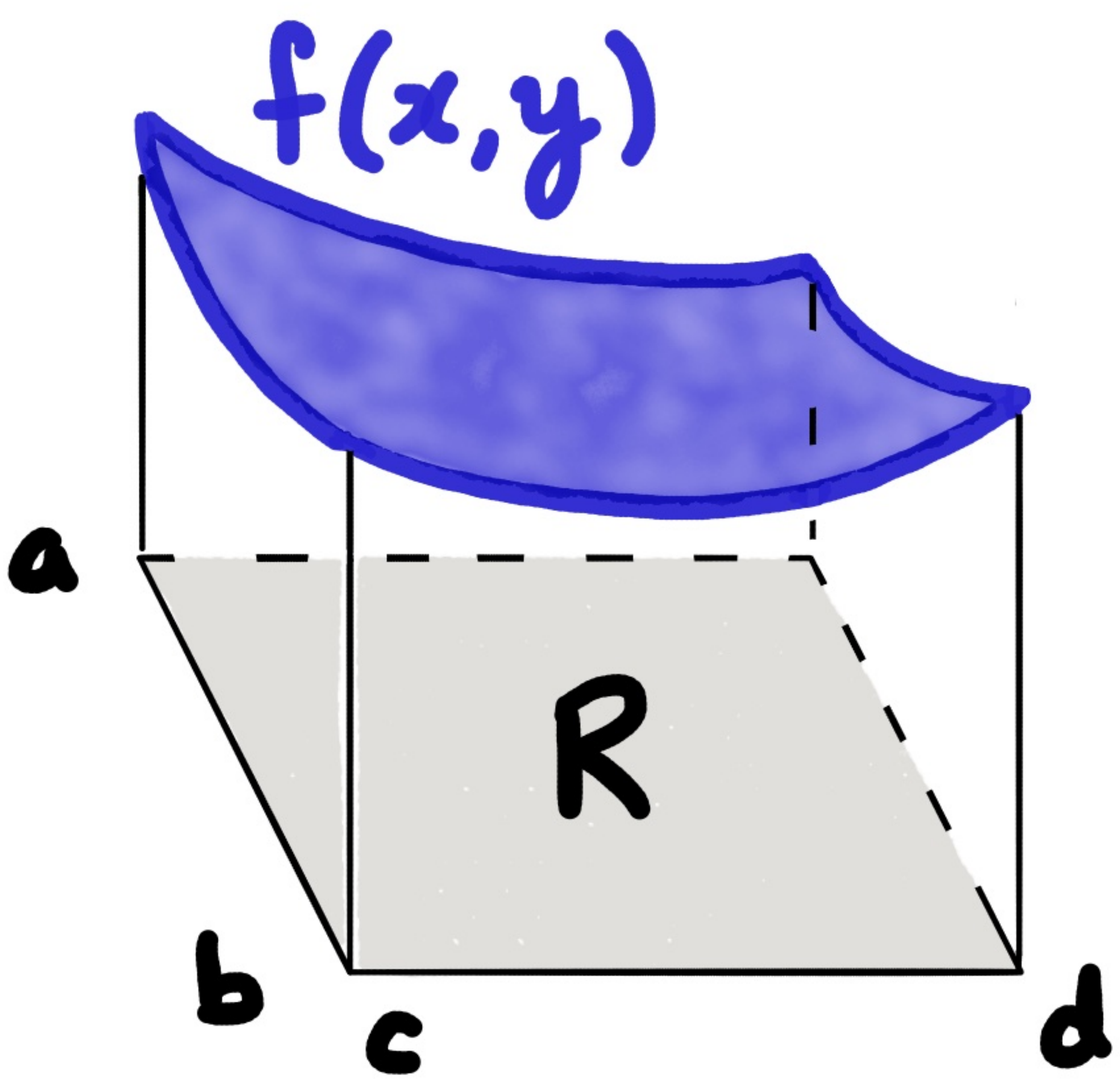


$$\iint_R f(x,y)\,dA = \int_0^b \int_0^d f(x,y)\,dy\,dx$$

$$dA \longmapsto dy\,dx$$

Iterated integrals

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

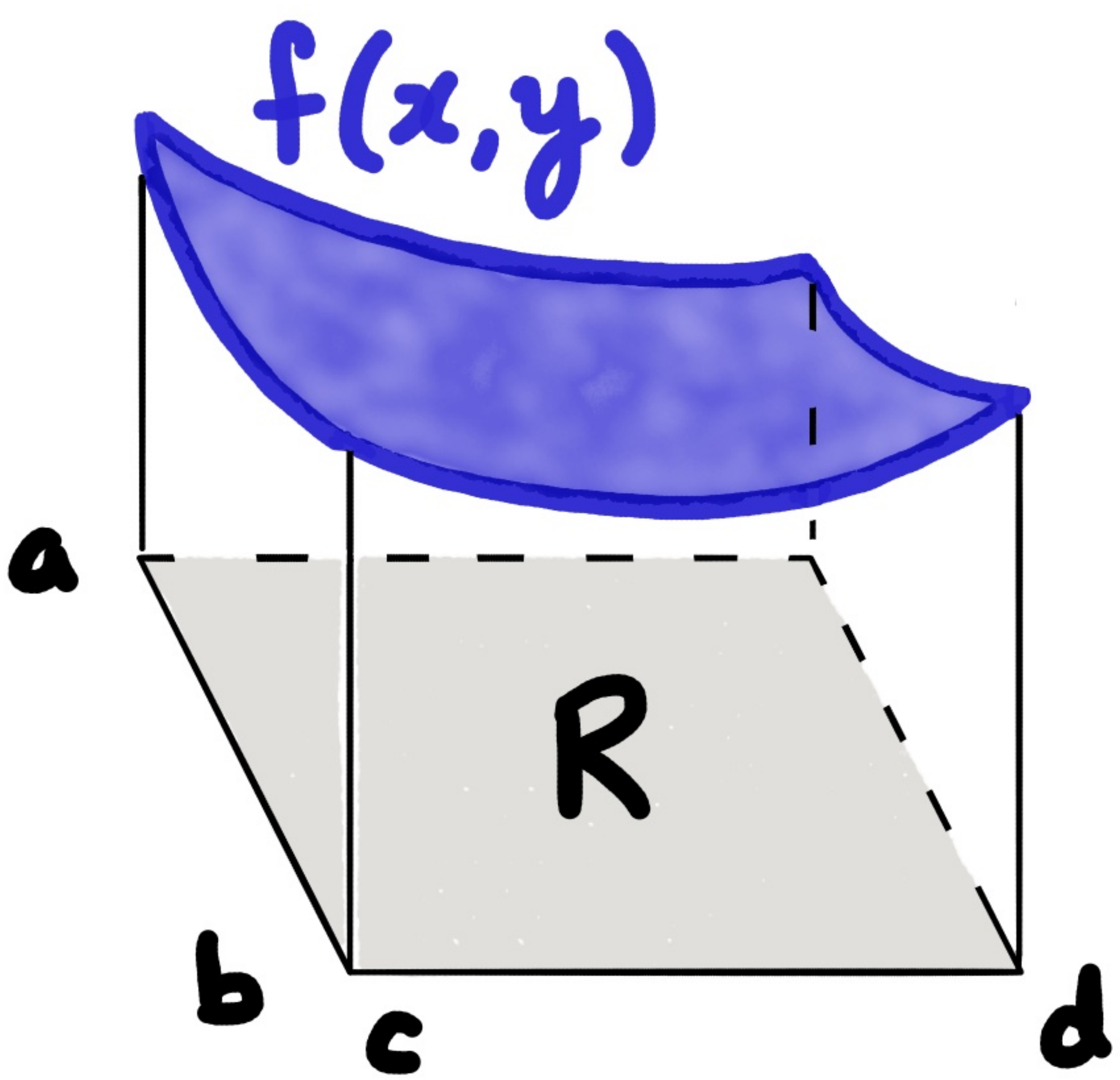


$R = \{(x,y) : a \le x \le b \;,\; c \le y \le d\}$

f is continuous

Iterated integrals

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$



$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$R = \{(x,y) : a \le x \le b \ , \ c \le y \le d\}$

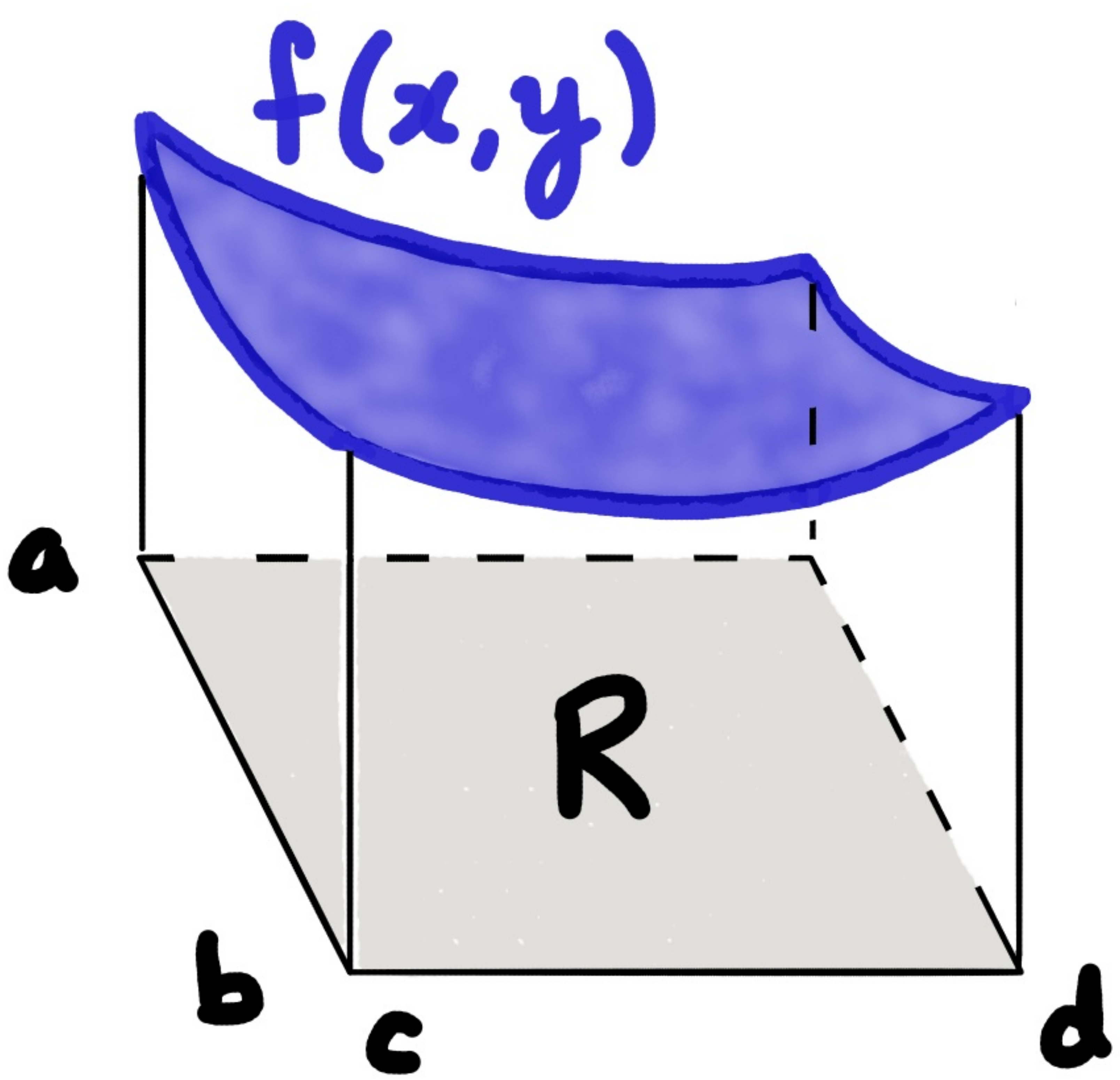
f is continuous

Iterated integrals

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

inner integral

$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$



$R = \{(x,y) : a \le x \le b\ ,\ c \le y \le d\}$

f is continuous

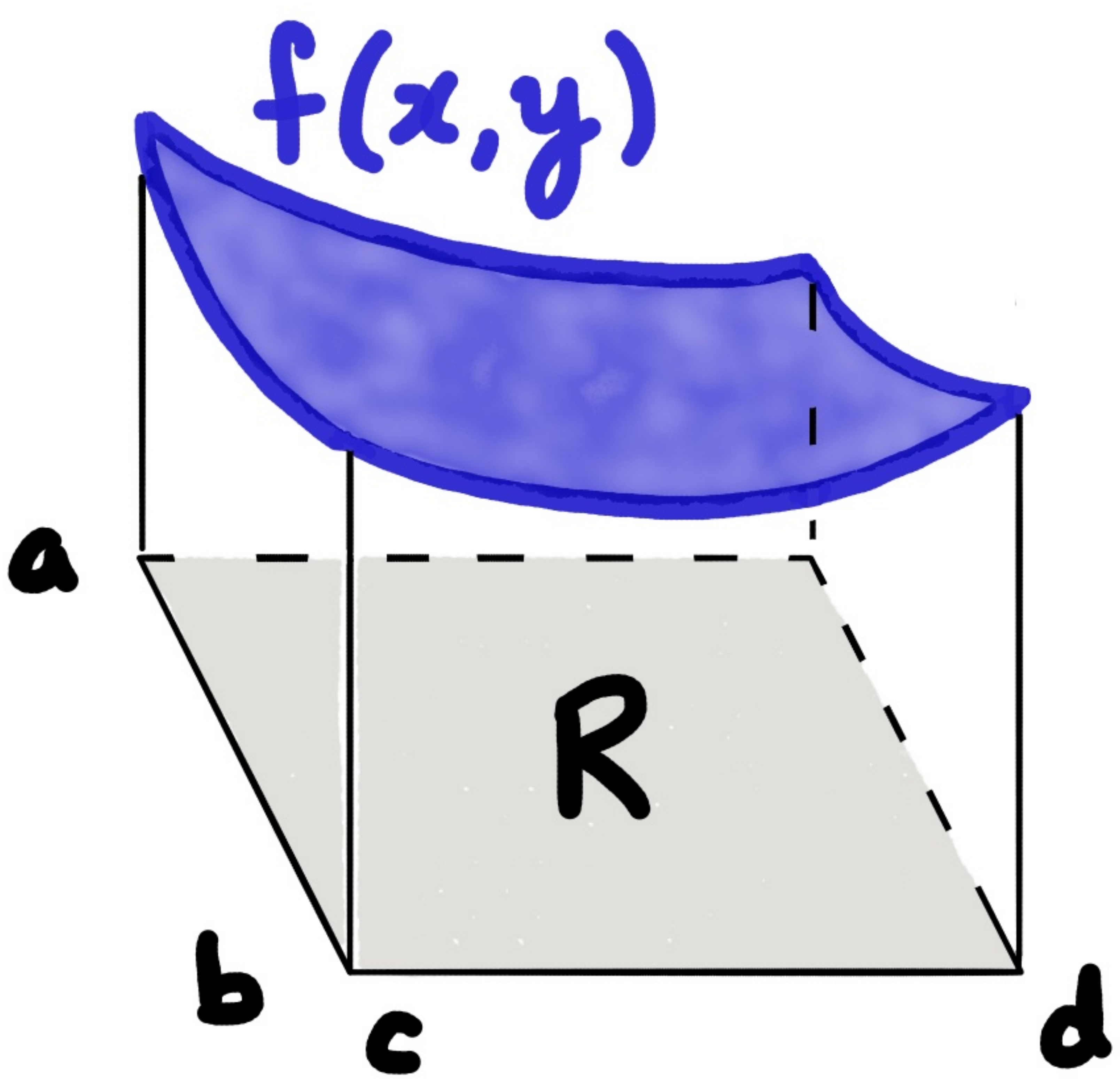
Iterated integrals

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

inner integral

$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$

outer integral



$R = \{(x,y) : a \le x \le b \ , \ c \le y \le d\}$

f is continuous

Examples:

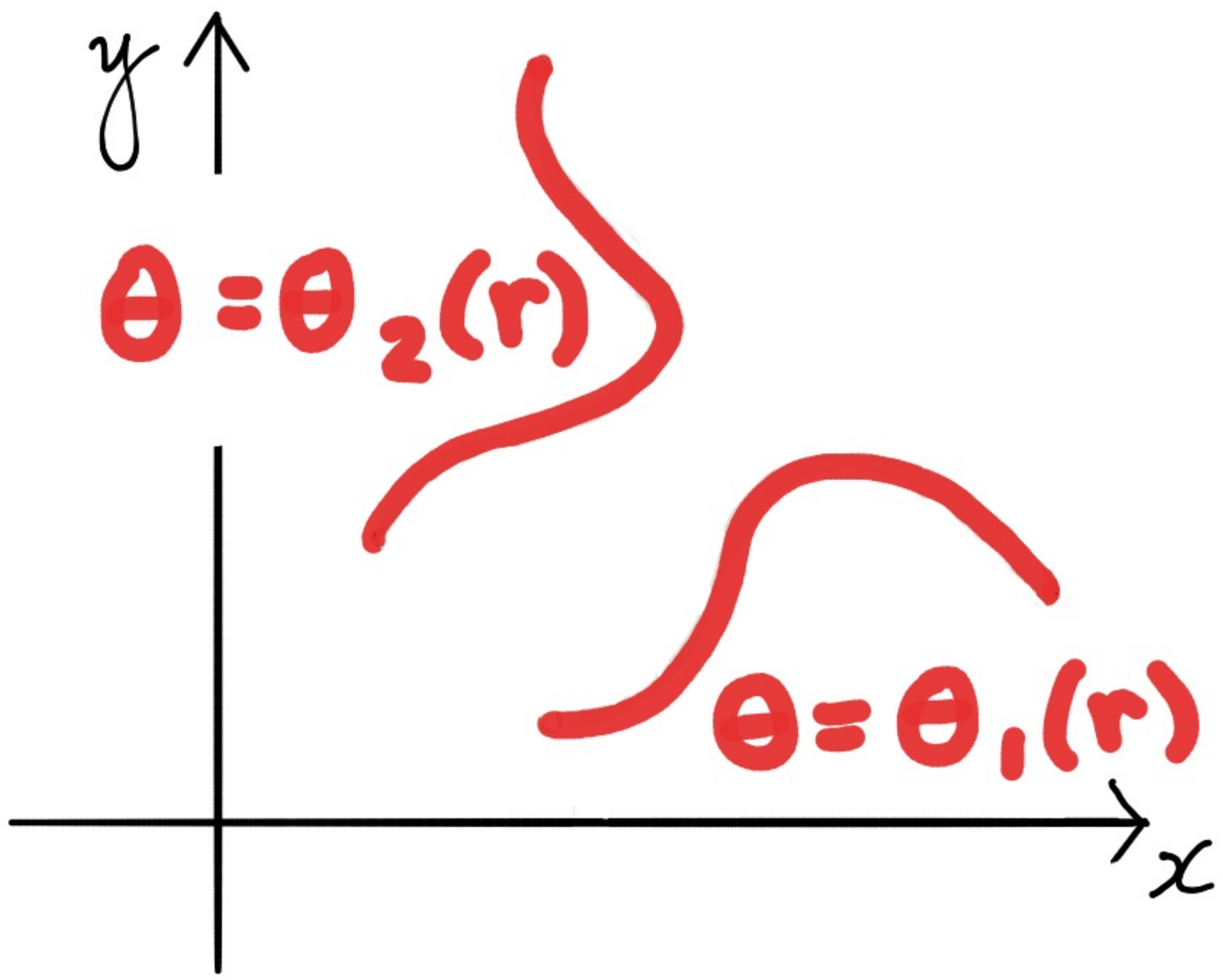
① $\int_0^2 \int_1^3 (x-y)\,dx\,dy$

② $\int_1^3 \int_0^2 (x-y)\,dy\,dx$

③ $\iint_R x e^{xy}\,dA$ where $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$

II Simple regions in polar coordinates

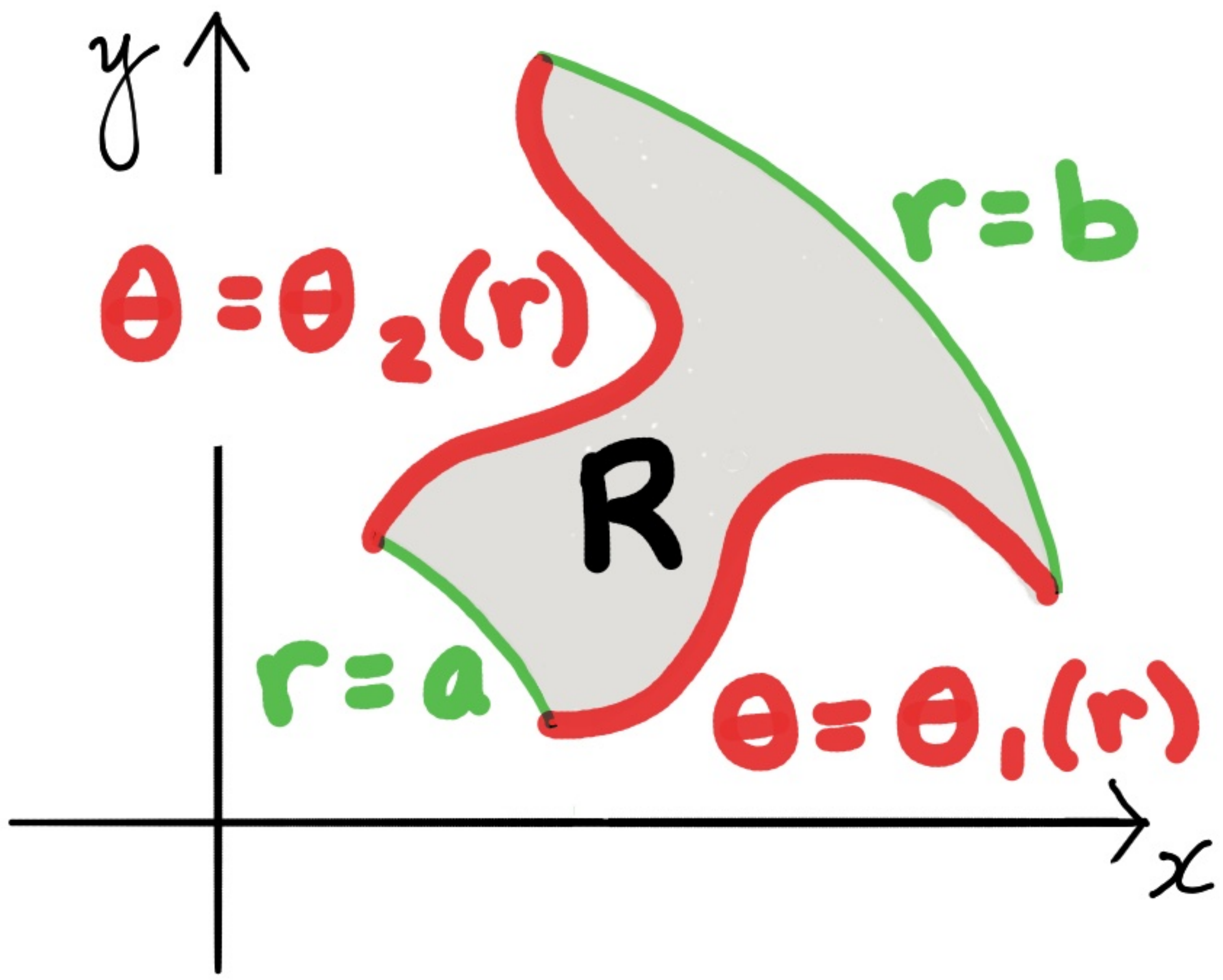
θ - simple regions



$$\theta_1 : \mathbb{R} \to \mathbb{R}$$

$$\theta_2 : \mathbb{R} \to \mathbb{R}$$

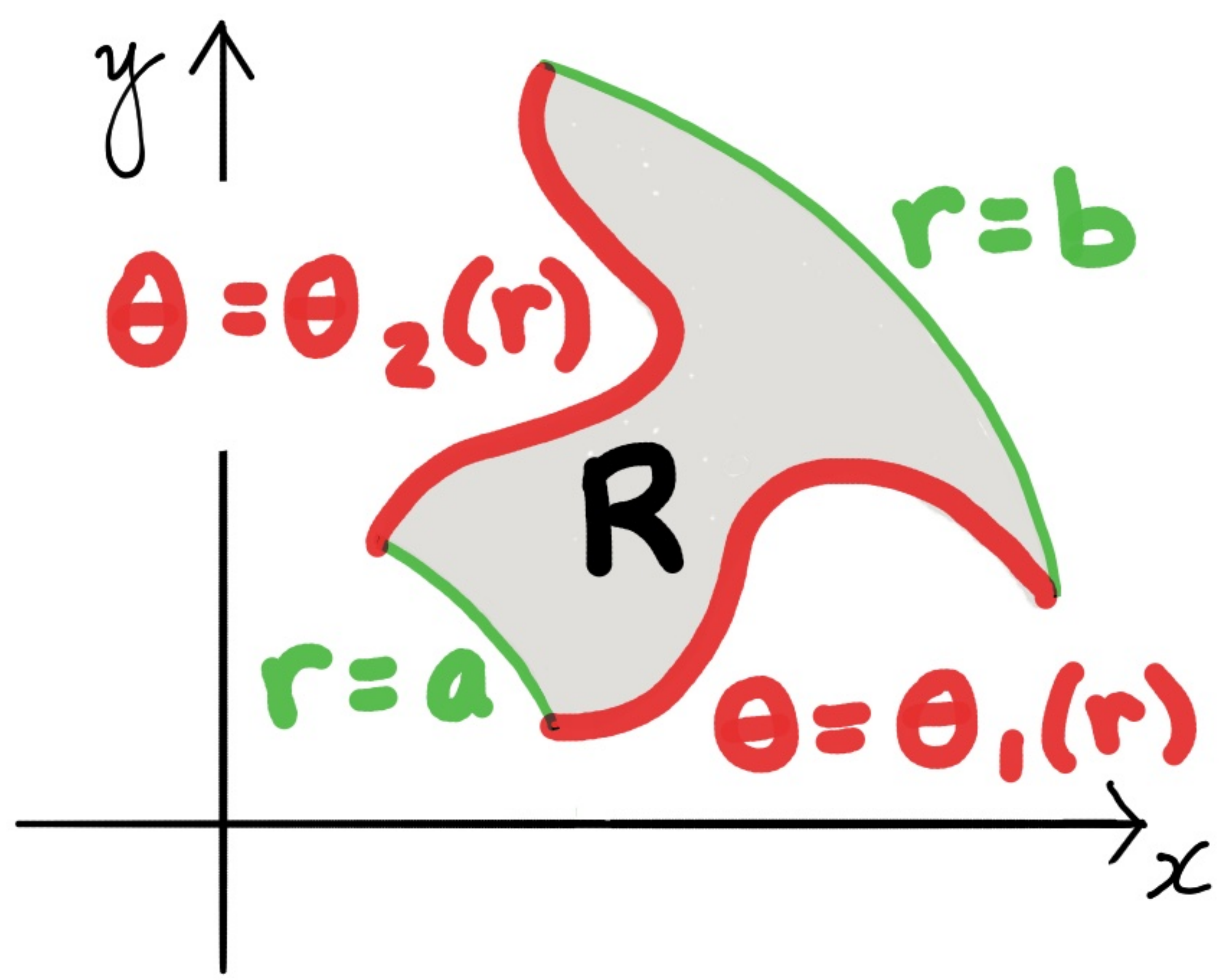
θ-simple regions



$\theta_1 : \mathbb{R} \to \mathbb{R}$

$\theta_2 : \mathbb{R} \to \mathbb{R}$

θ- simple regions



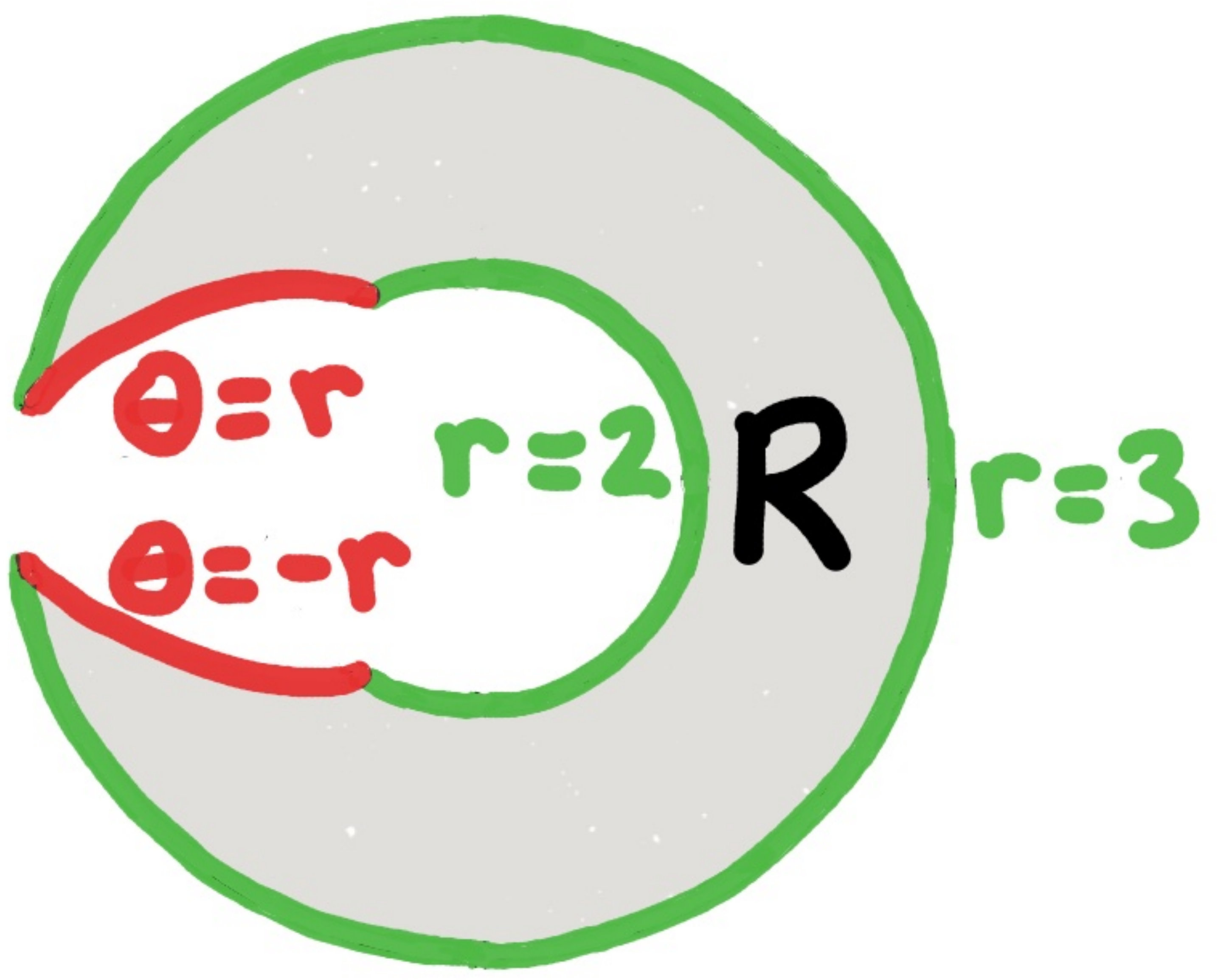
$\theta_1 : \mathbb{R} \to \mathbb{R}$

$\theta_2 : \mathbb{R} \to \mathbb{R}$

$$R = \{(r,\theta) : a \le r \le b,\ \theta_1(r) \le \theta \le \theta_2(r)\}$$

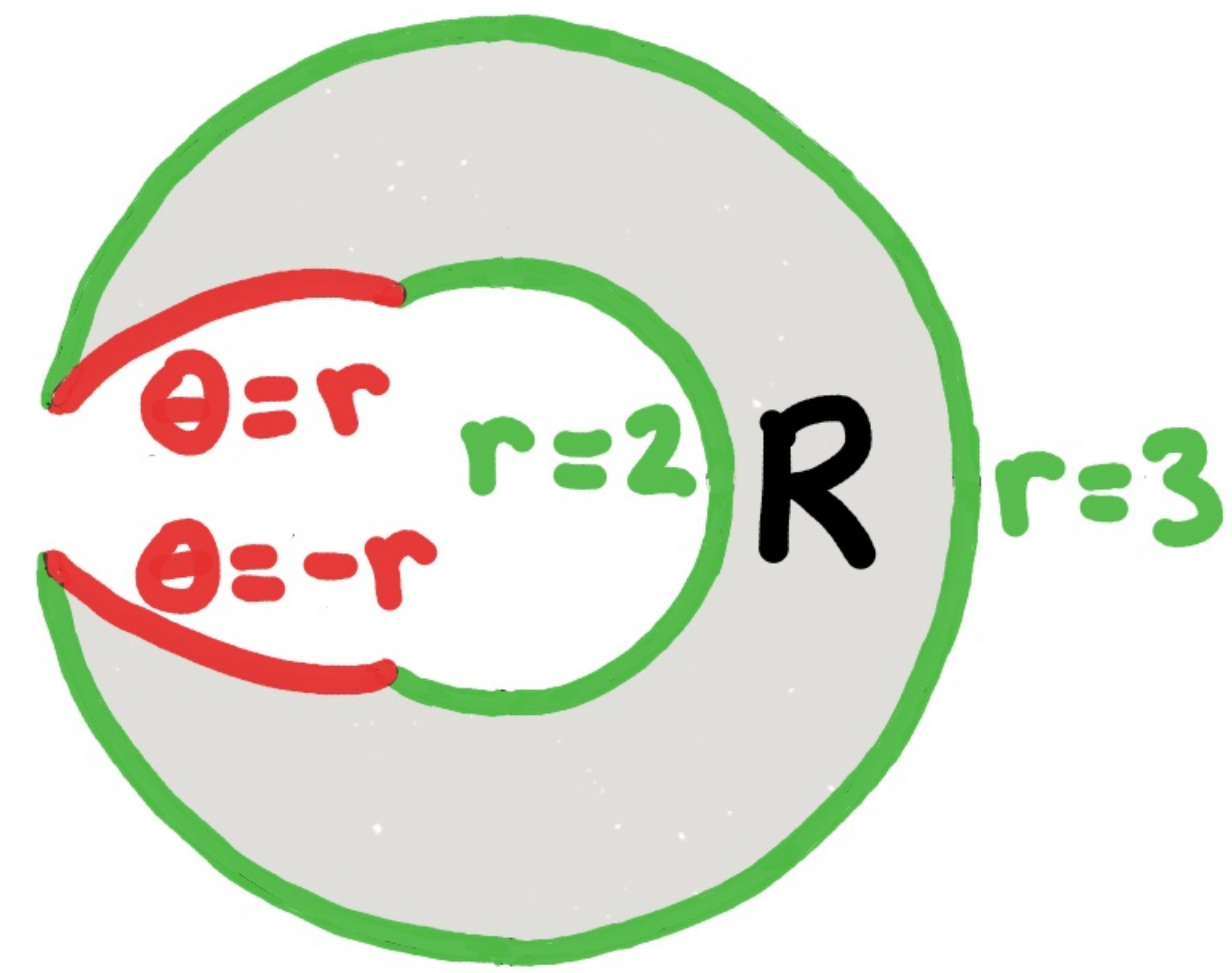
Problem:

Write the region R as a θ-simple set.



Problem:

Write the region R as a θ-simple set.

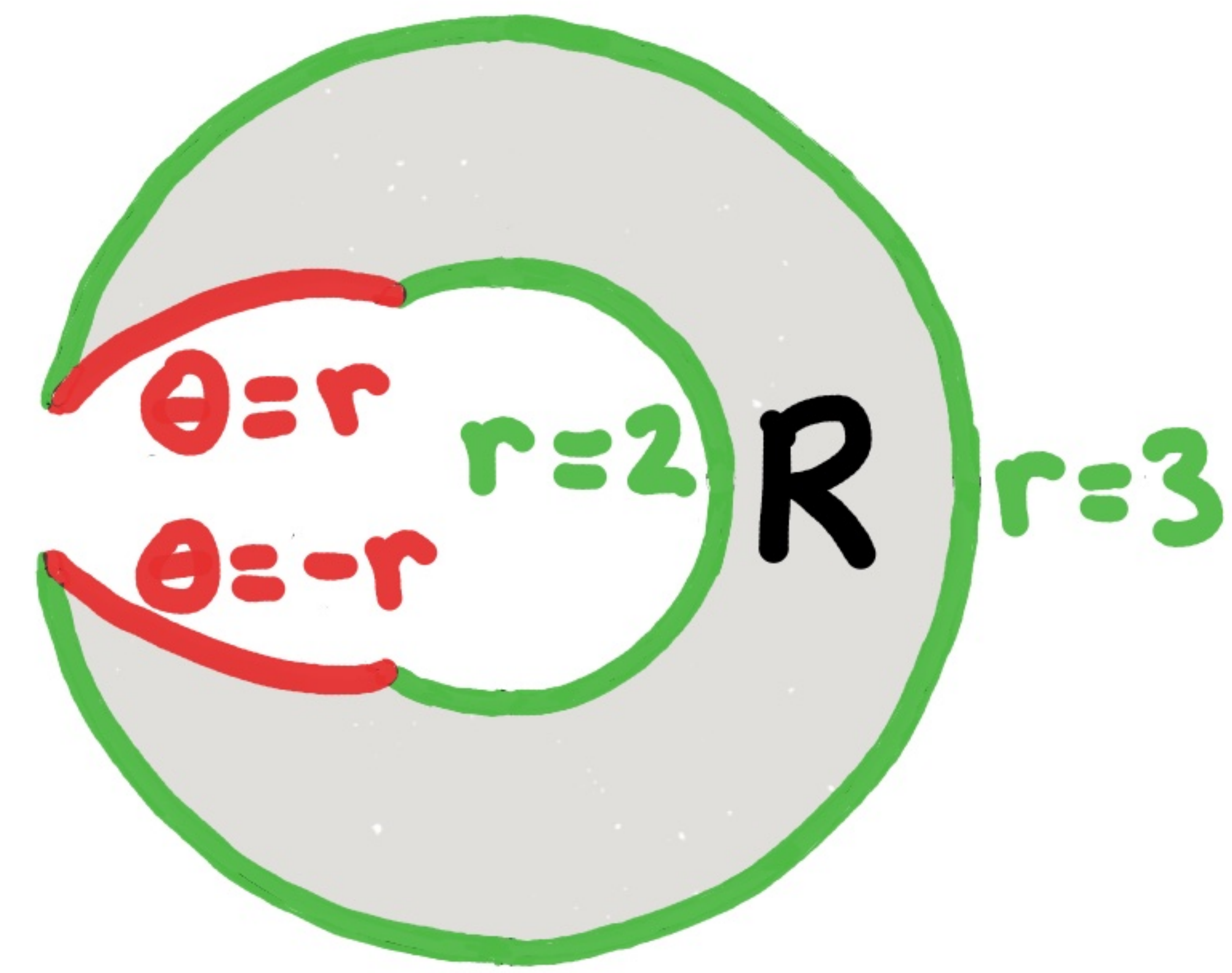


Answer:

$$R = \{(r,\theta) : 2 \le r \le 3,$$

Problem:

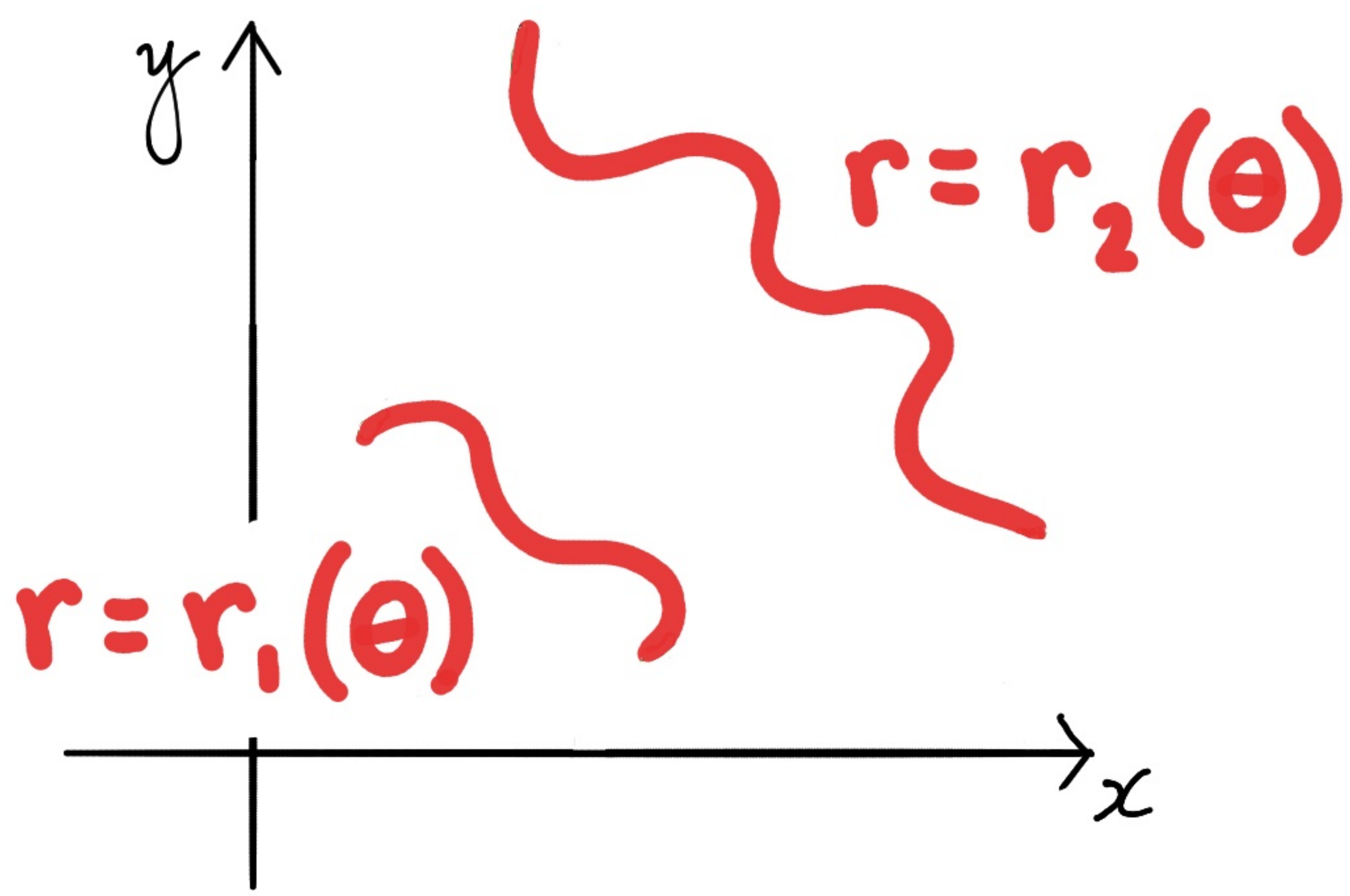
Write the region R as a θ-simple set.



Answer:

$$R = \{(r,\theta) : 2 \le r \le 3 \,,\; -r \le \theta \le r\}$$

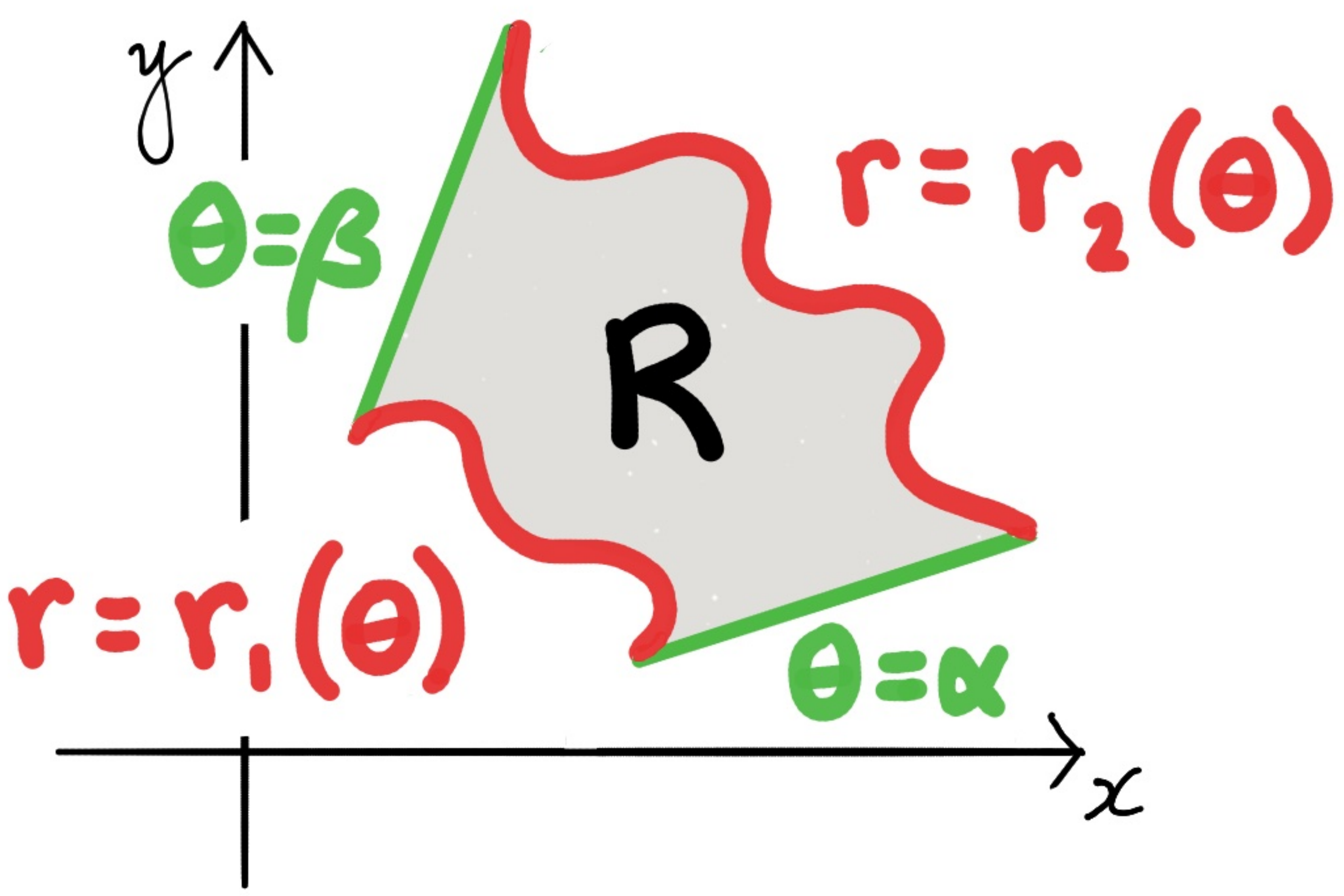
r-simple regions



$$r_1 : \mathbb{R} \to \mathbb{R}$$

$$r_2 : \mathbb{R} \to \mathbb{R}$$

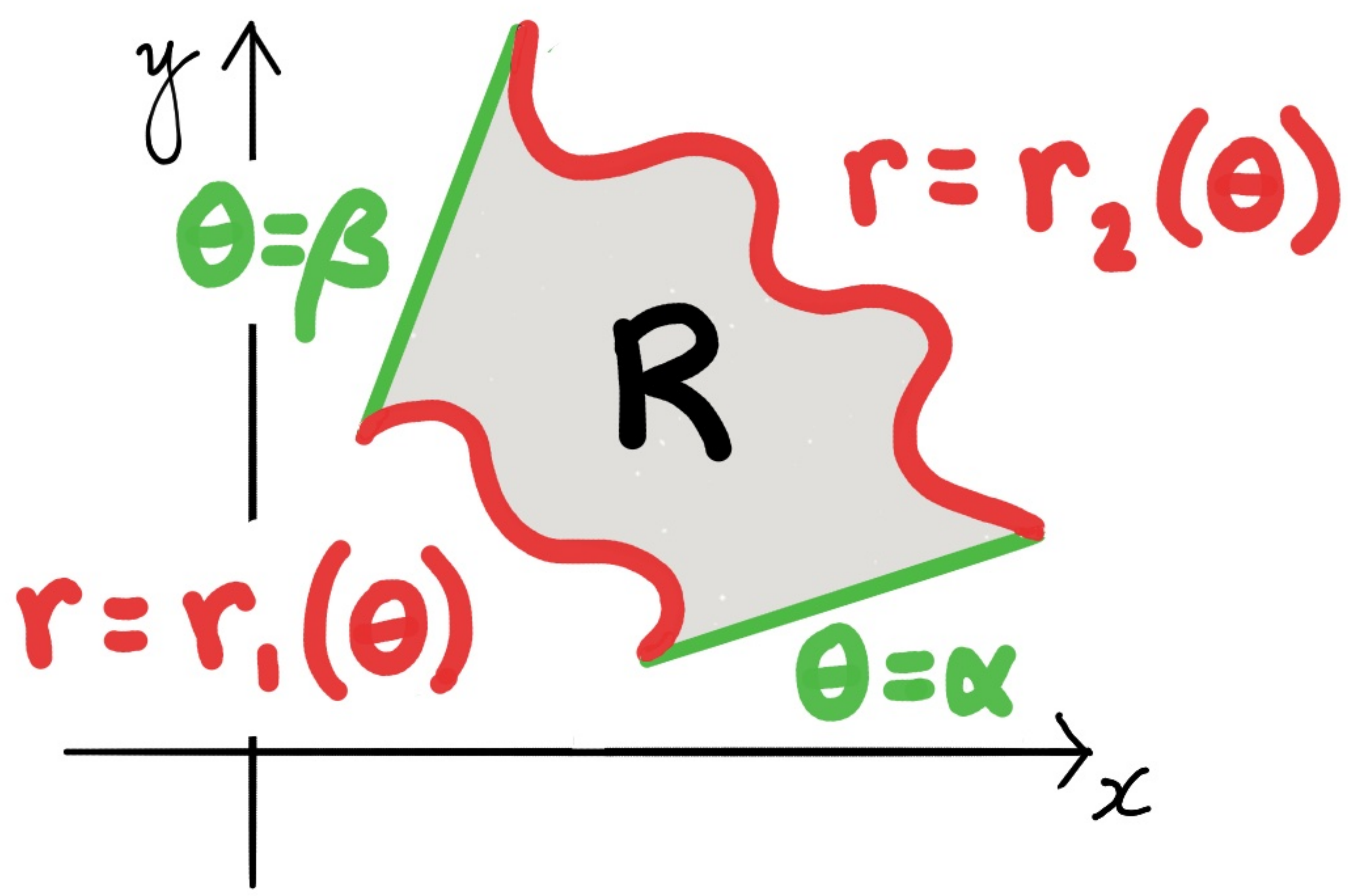
r-simple regions



$r_1 : \mathbb{R} \to \mathbb{R}$

$r_2 : \mathbb{R} \to \mathbb{R}$

r-simple regions



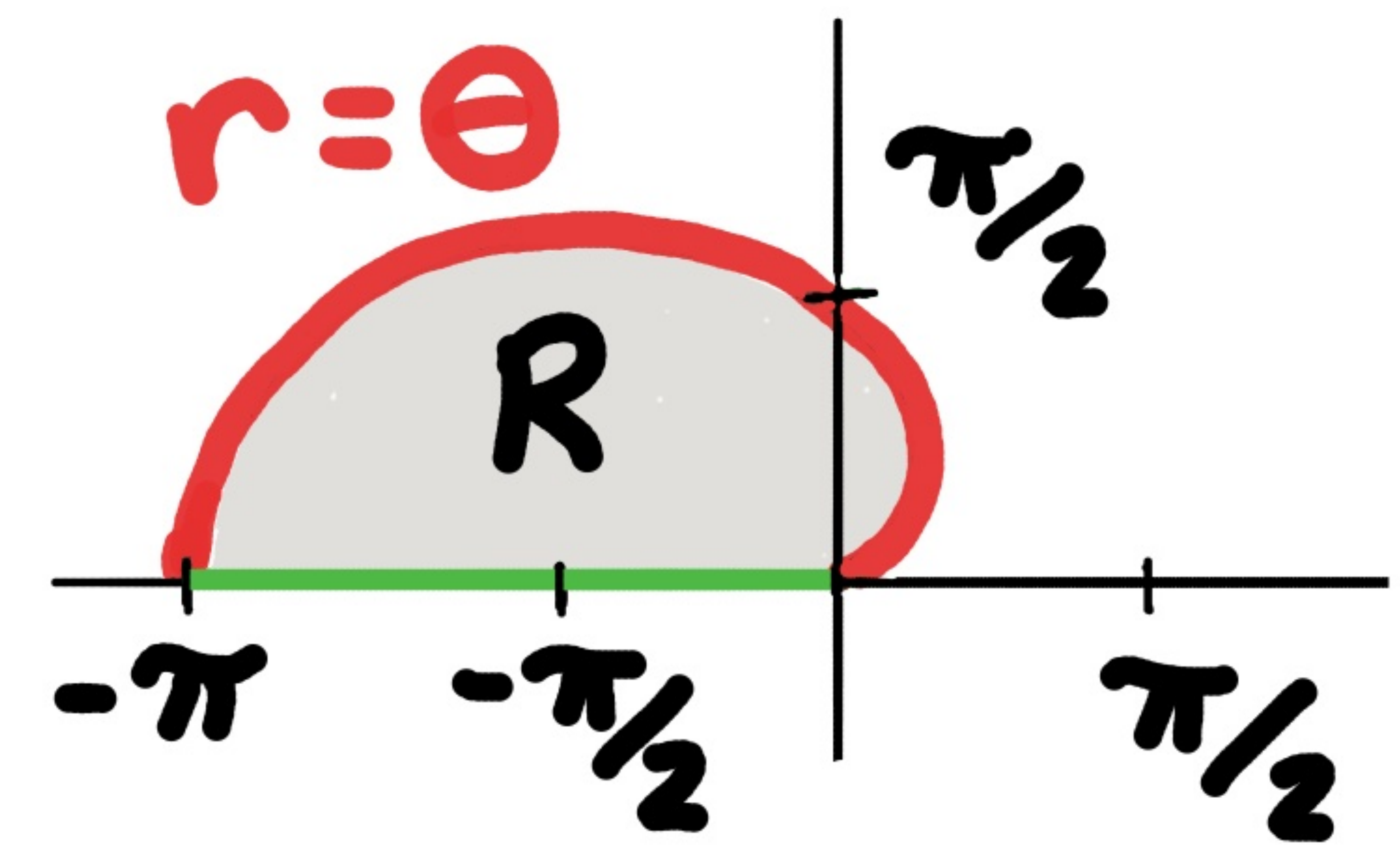
$r_1 : \mathbb{R} \to \mathbb{R}$

$r_2 : \mathbb{R} \to \mathbb{R}$

$$R = \{(r,\theta) : \alpha \le \theta \le \beta,\ r_1(\theta) \le r \le r_2(\theta)\}$$

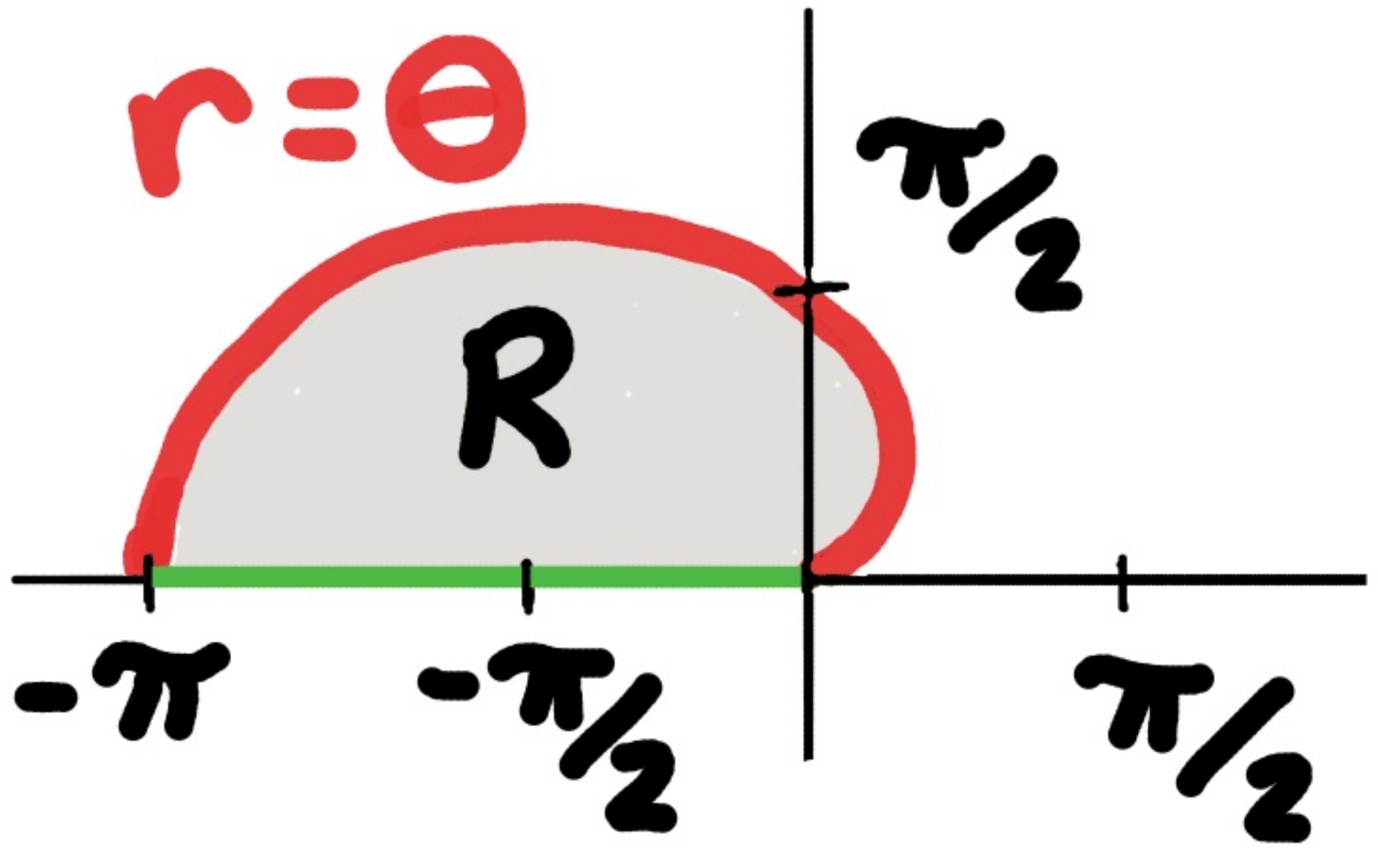
Problem:

Write the region R as an r-simple set.



Problem:

Write the region R as an r-simple set.

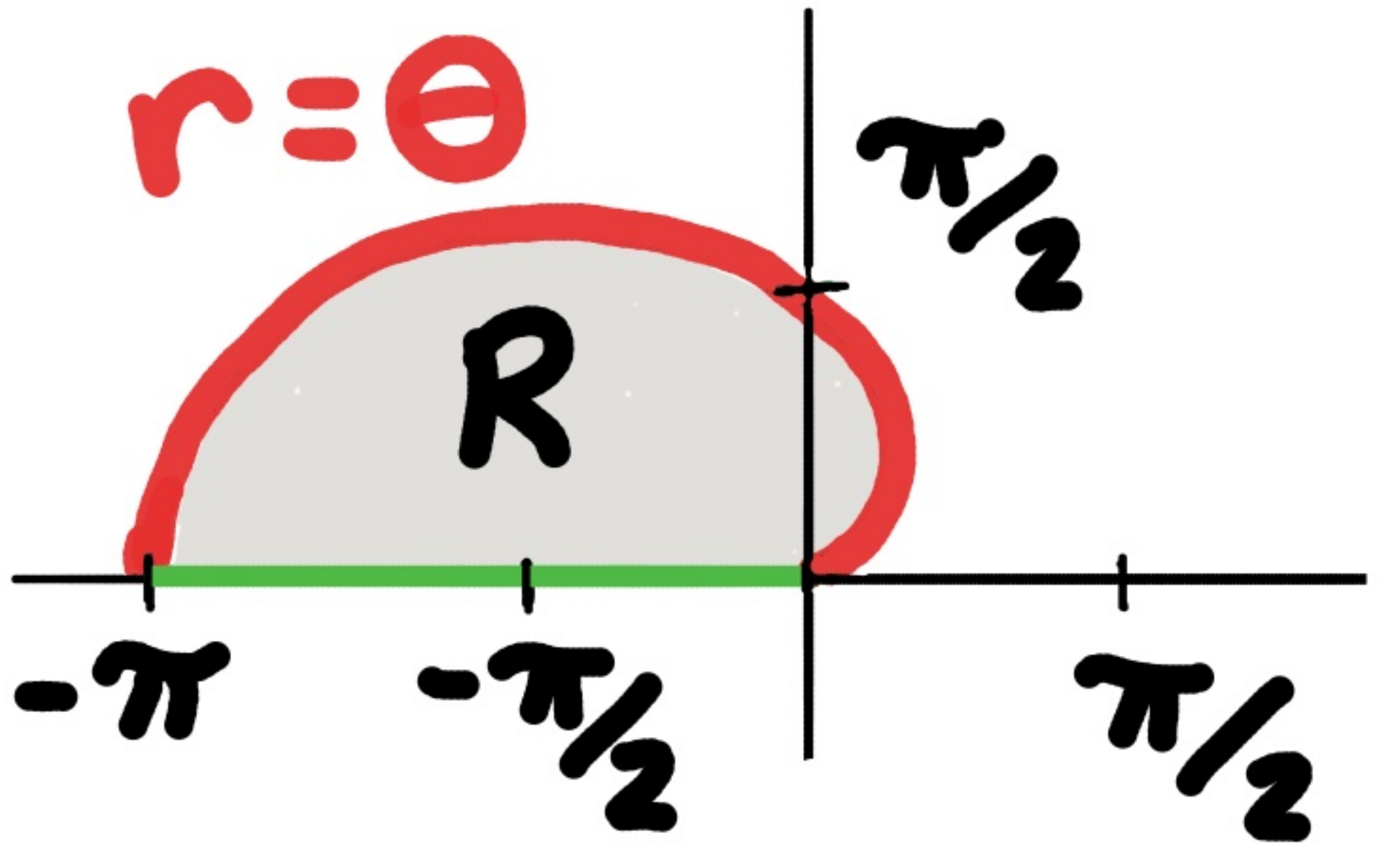


Answer:

$$R = \{(r,\theta) : 0 \leq \theta \leq \pi ,$$

Problem:

Write the region R as an r-simple set.



Answer:

$$R = \{(r,\theta) : 0 \le \theta \le \pi,\ 0 \le r \le \theta\}$$