

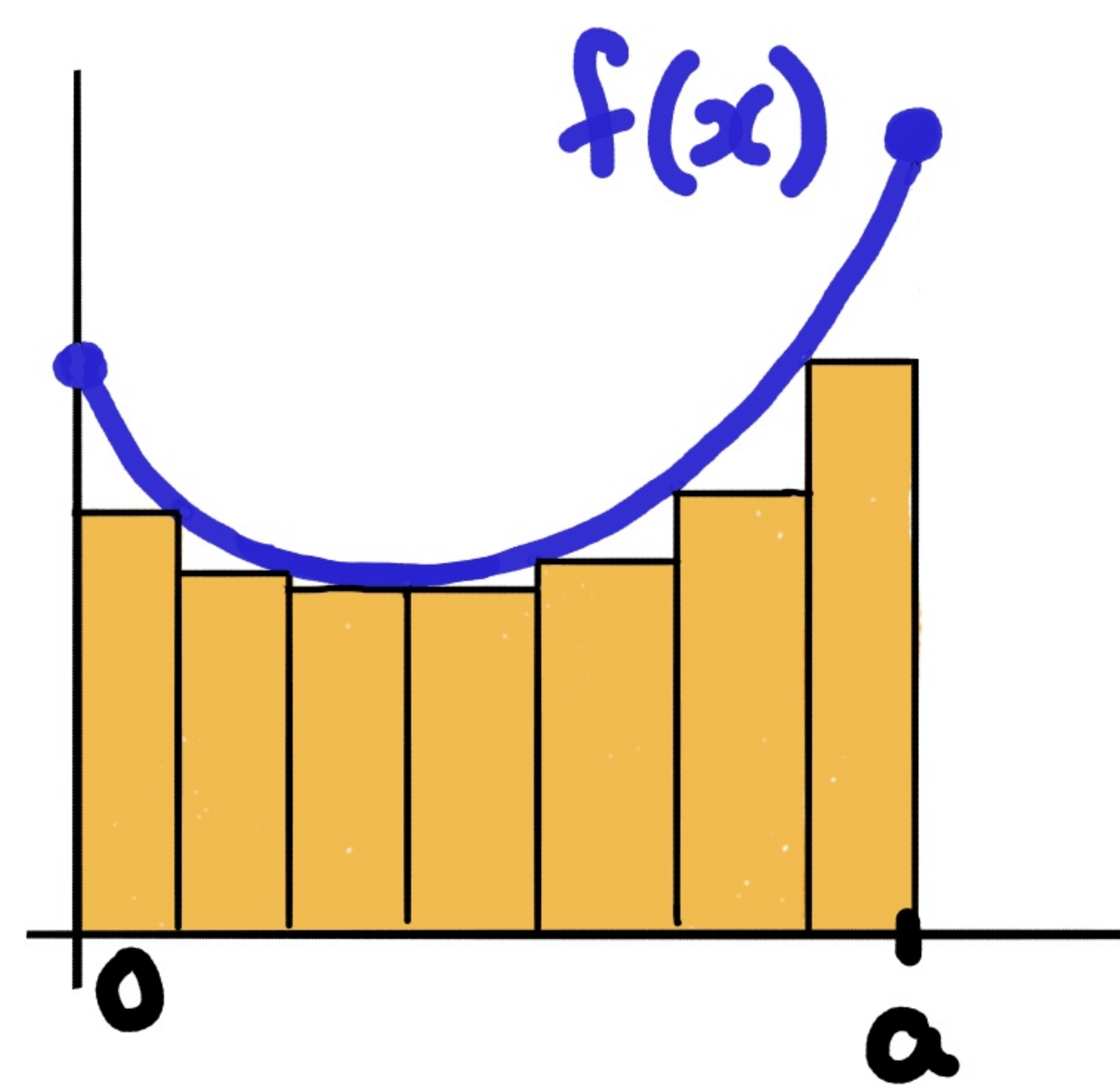
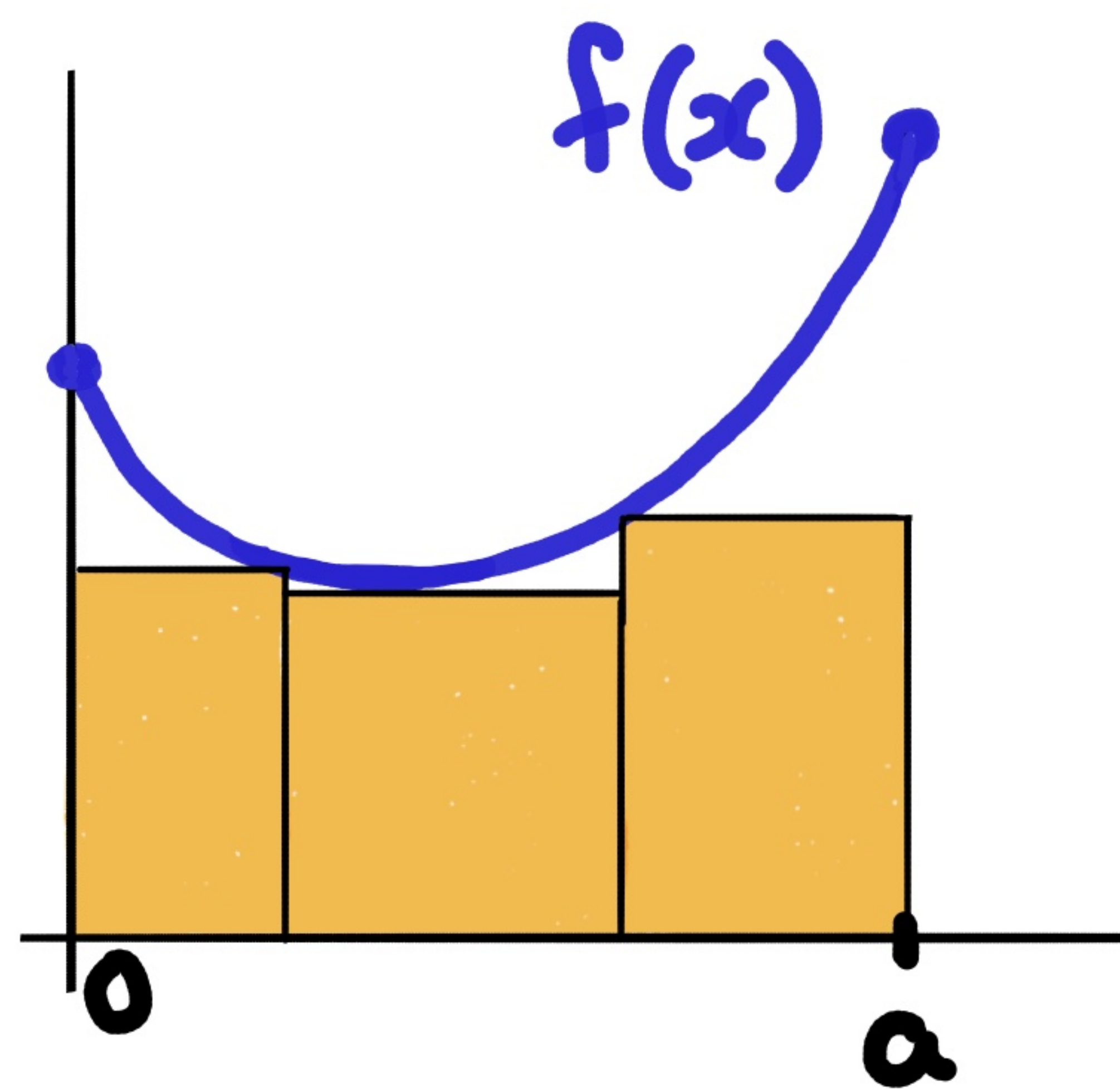
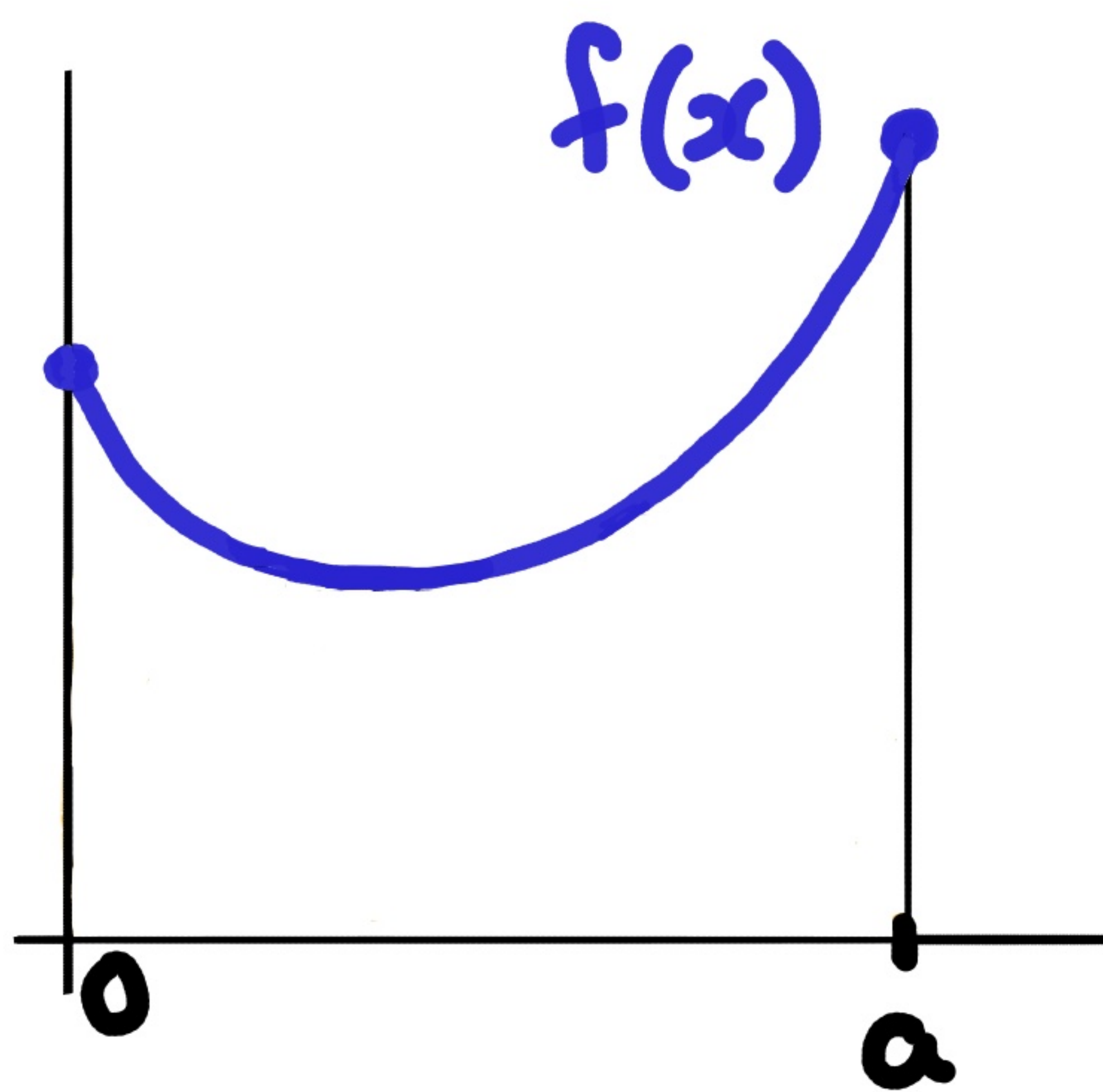
Eighteen

- ① One variable integrals
- ② Two variable integrals
- ③ Rules for integrals
- ④ Simple regions in the xy -plane

① One variable
integrals

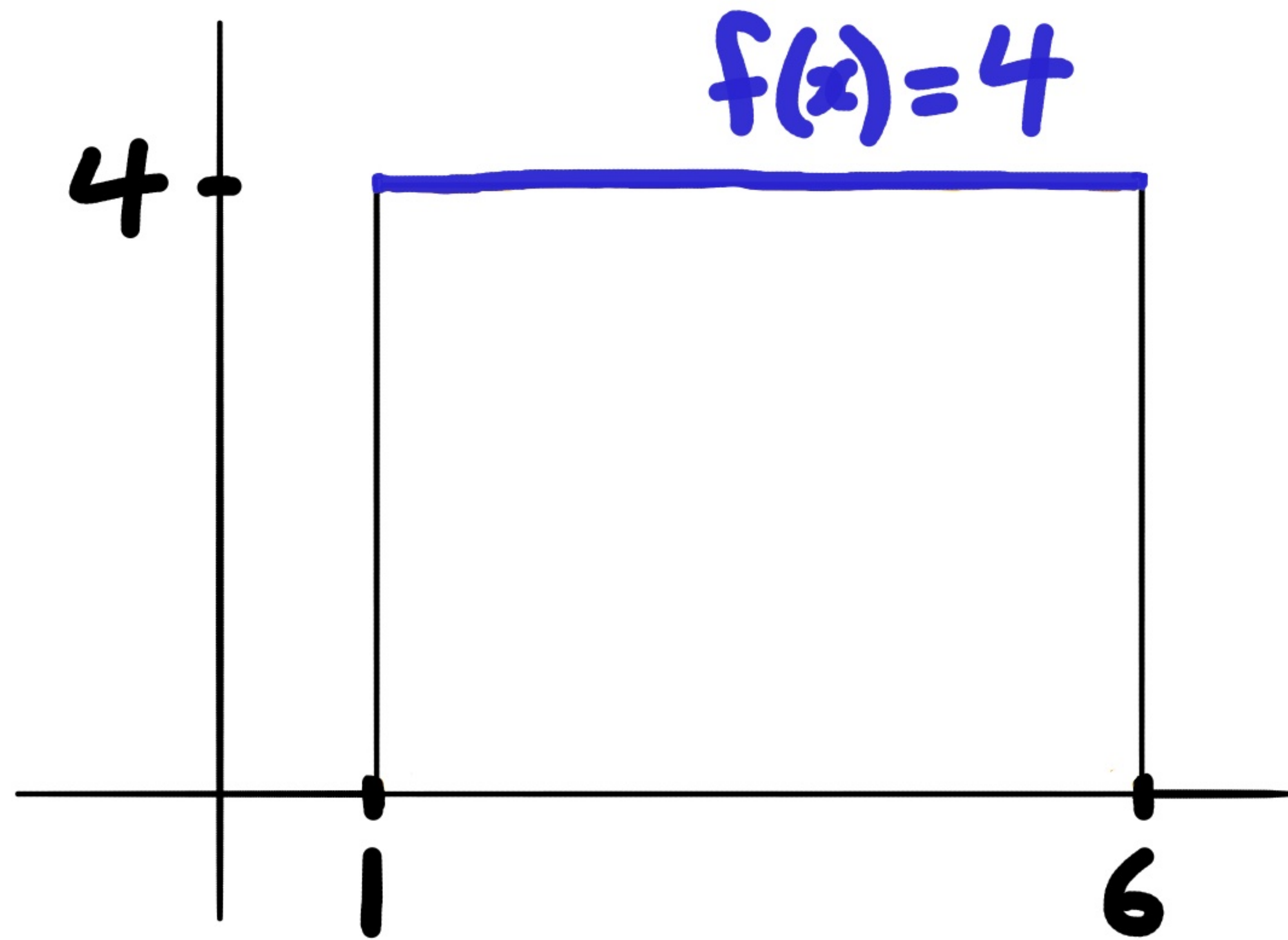


Area : width times height

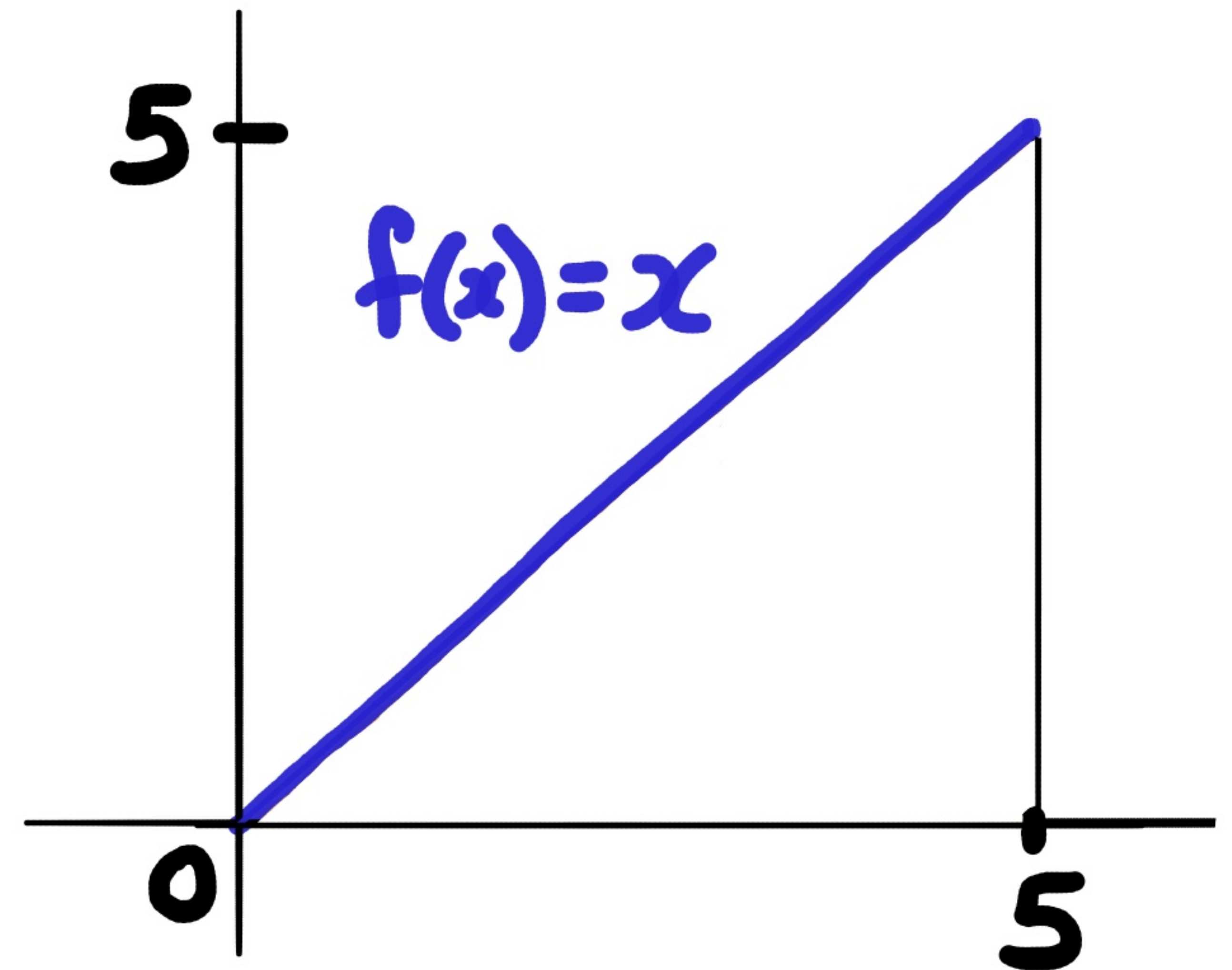


$$\int_0^a f(x) dx$$

Examples:



$$\int_1^6 4 dx = 5 \cdot 4 = 20$$

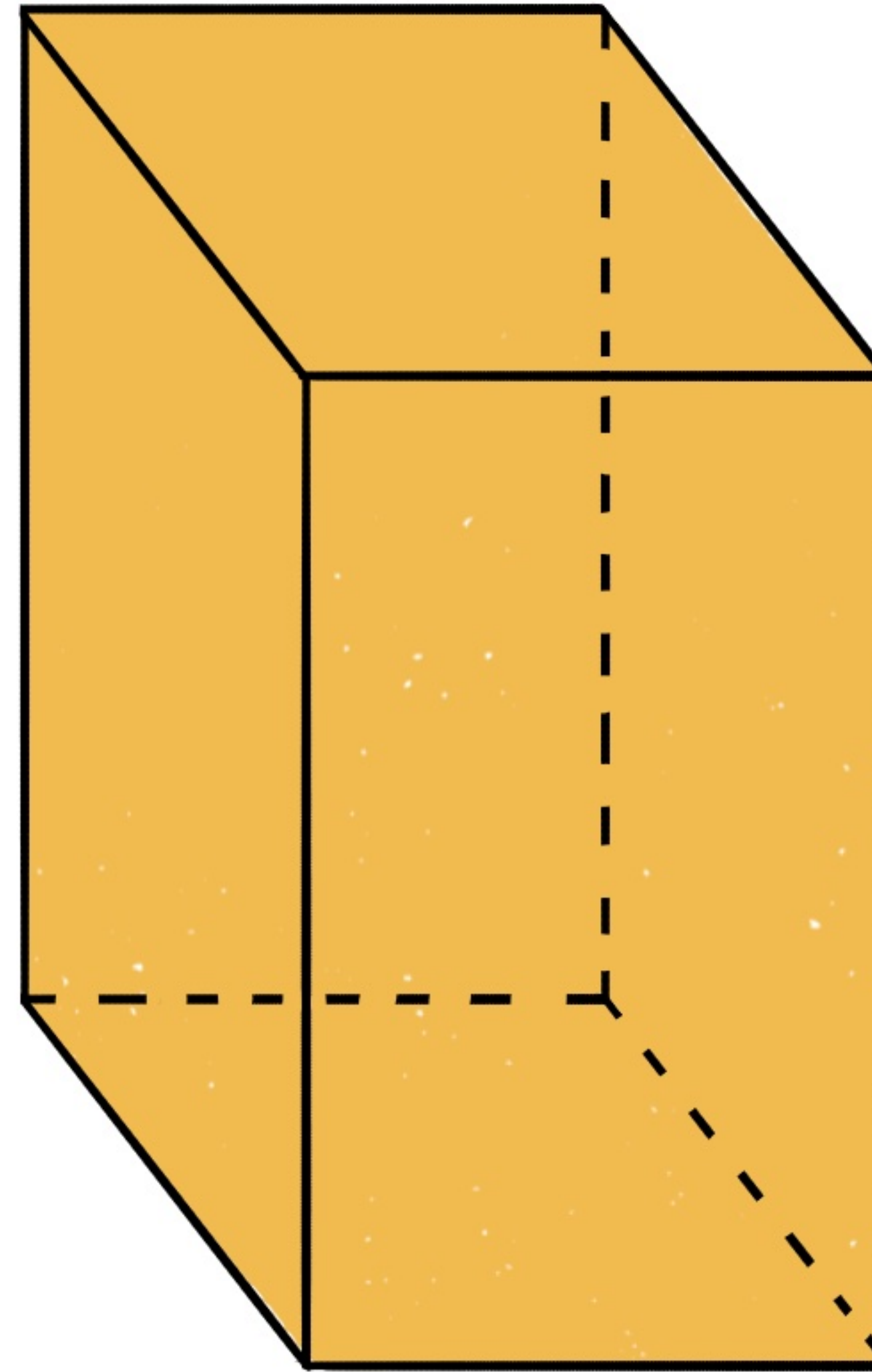


$$\int_0^5 x dx = \frac{1}{2} 5 \cdot 5 = \frac{25}{2}$$

Fundamental Theorem of Calculus

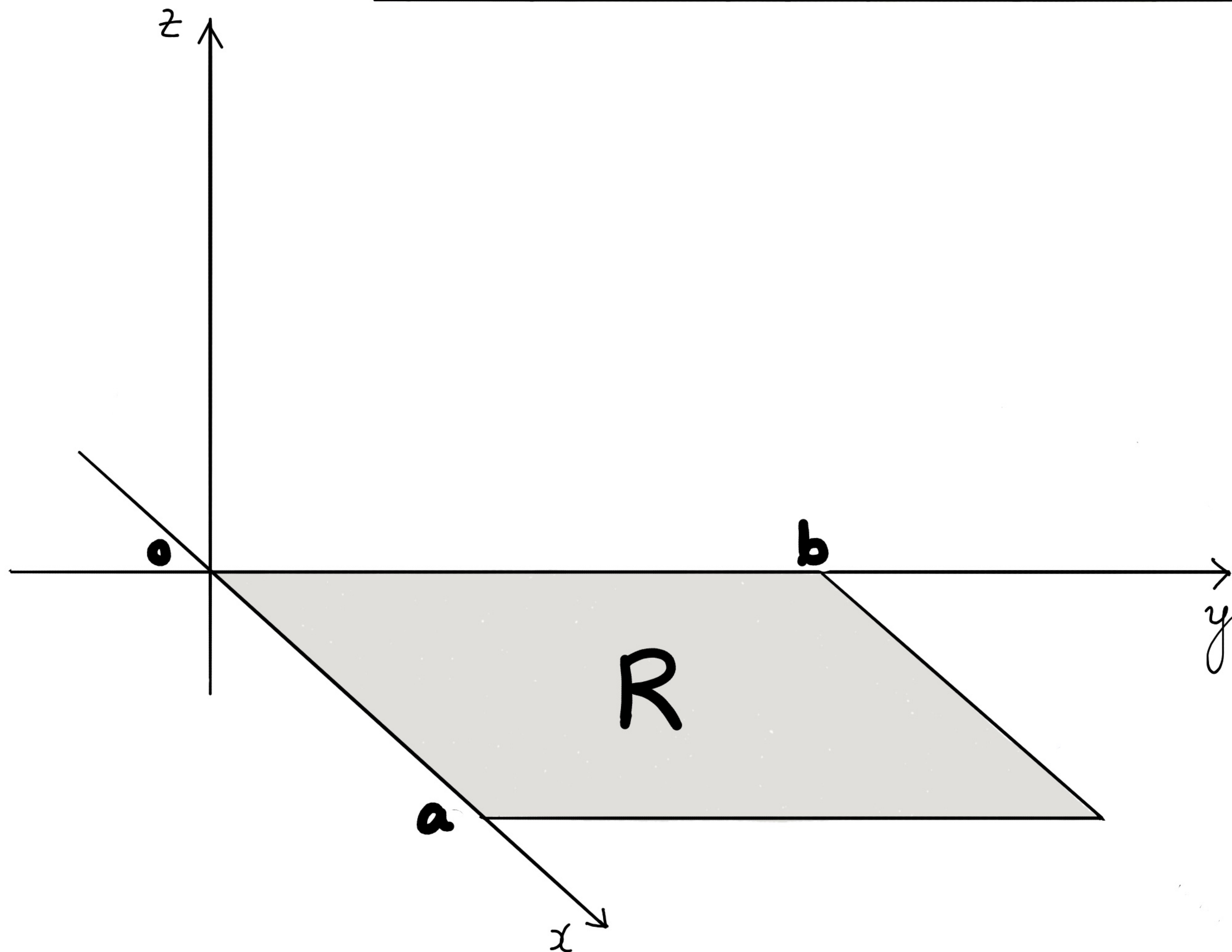
$$\int_a^b f'(x) dx = f(b) - f(a)$$

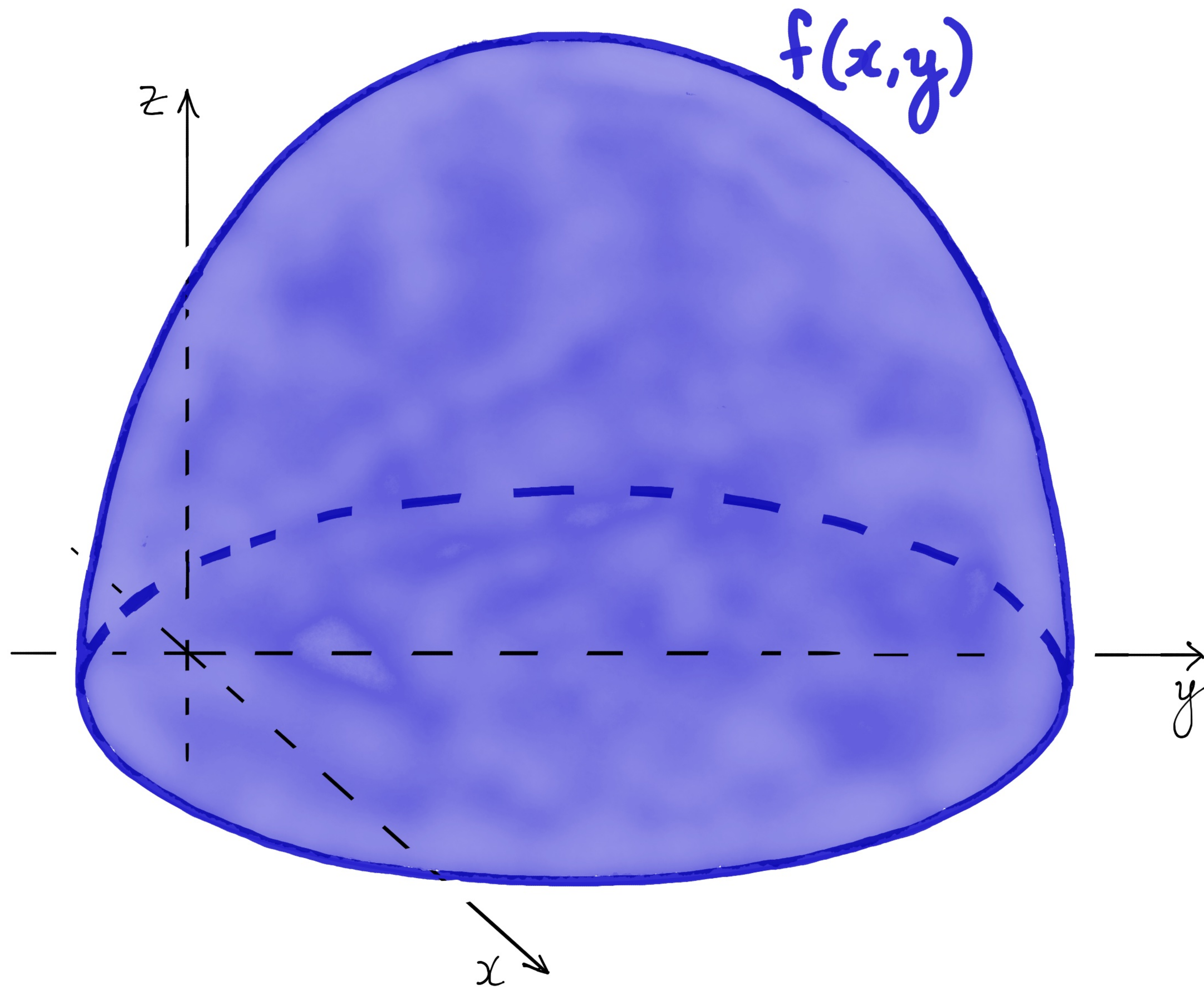
② Two variable integrals

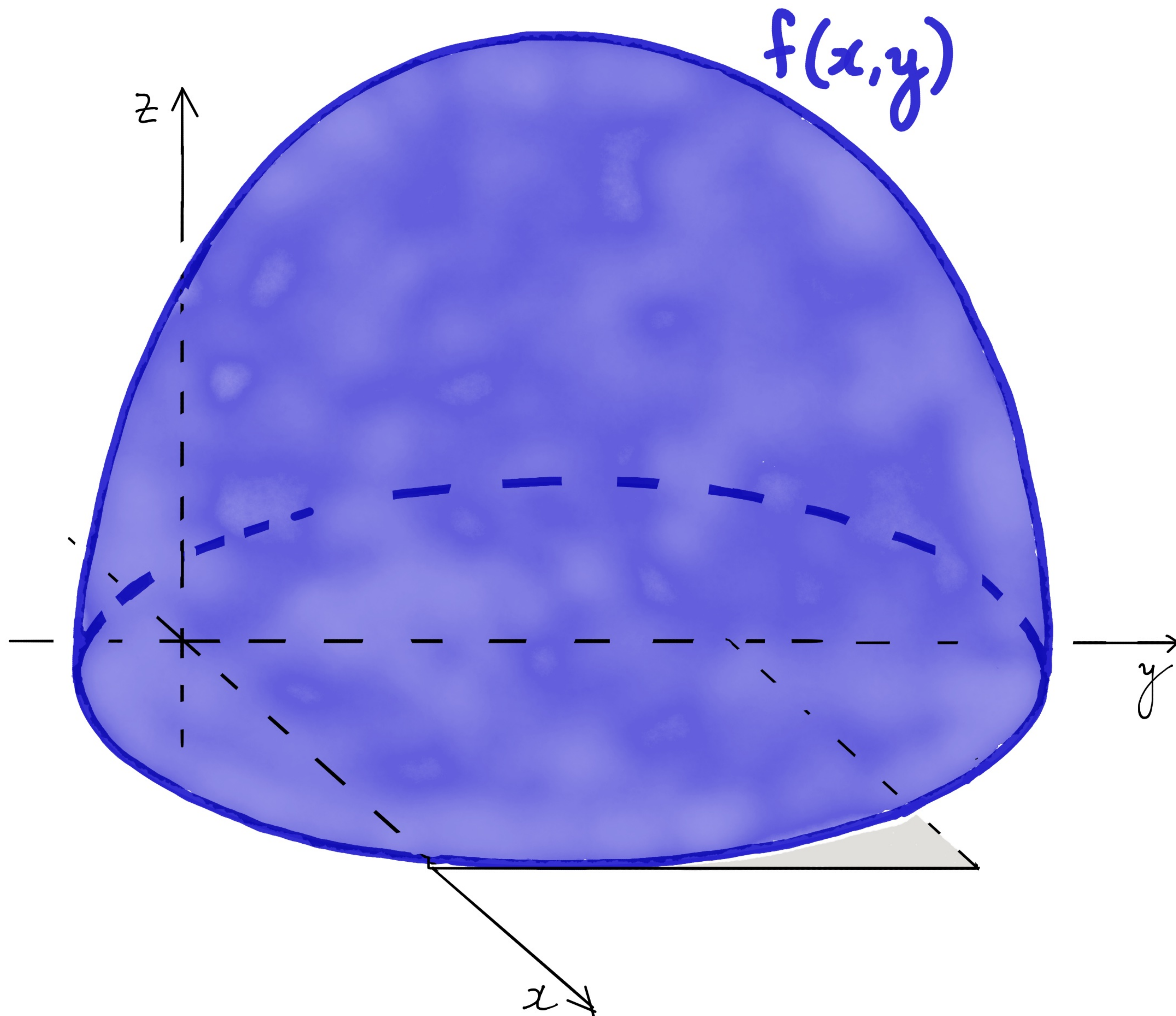


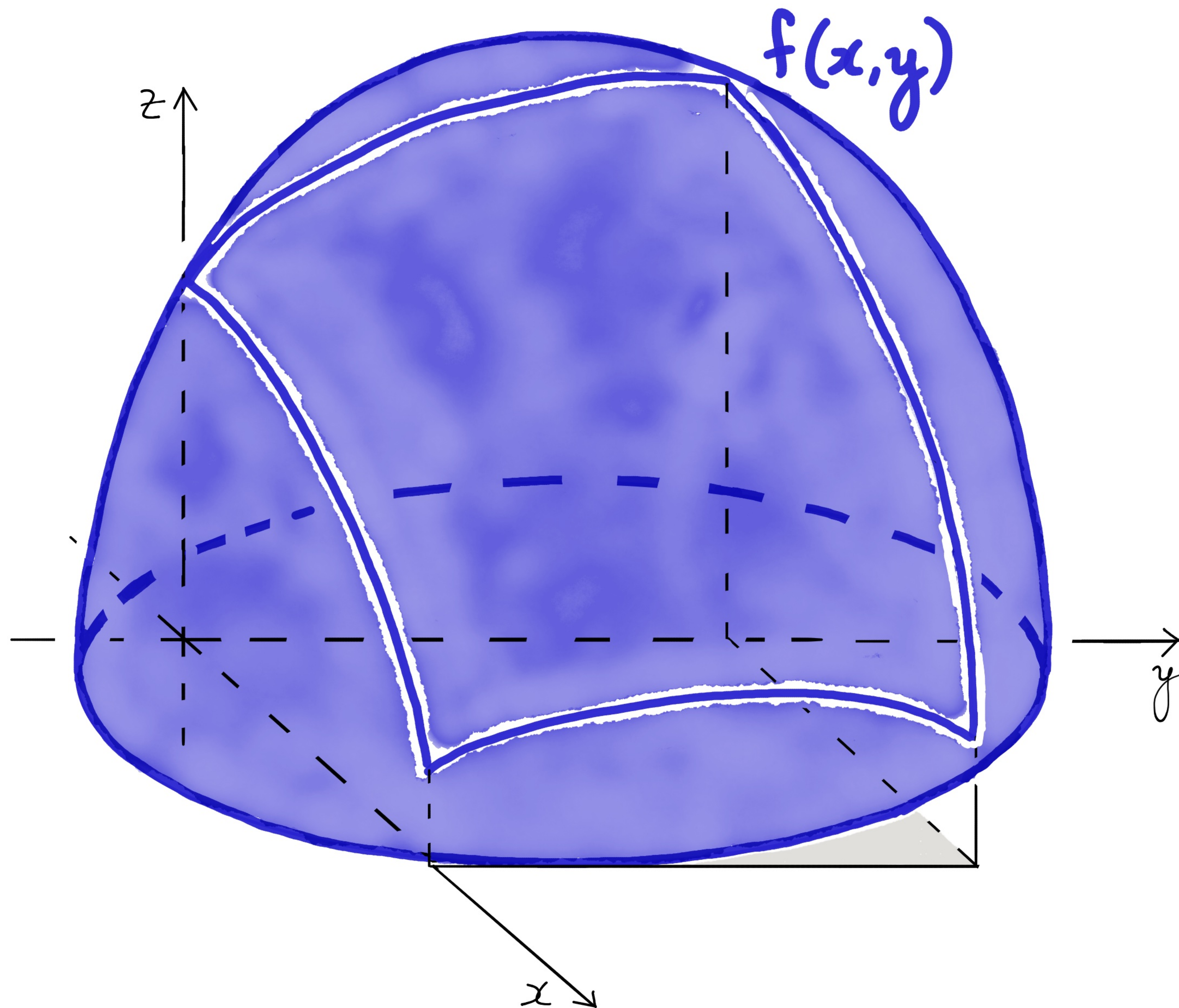
Volume: width times depth times height

$$R = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq b\}$$

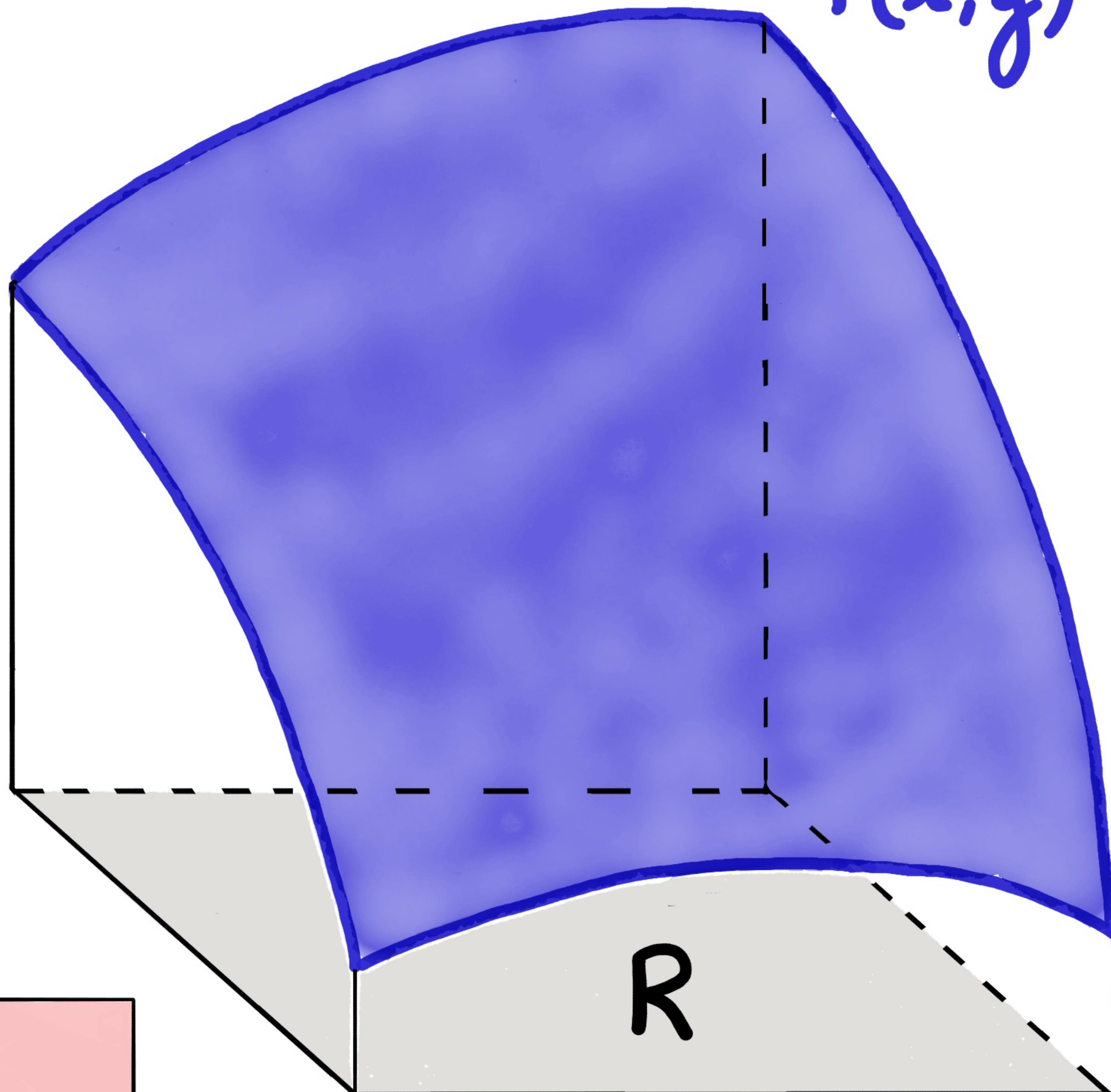




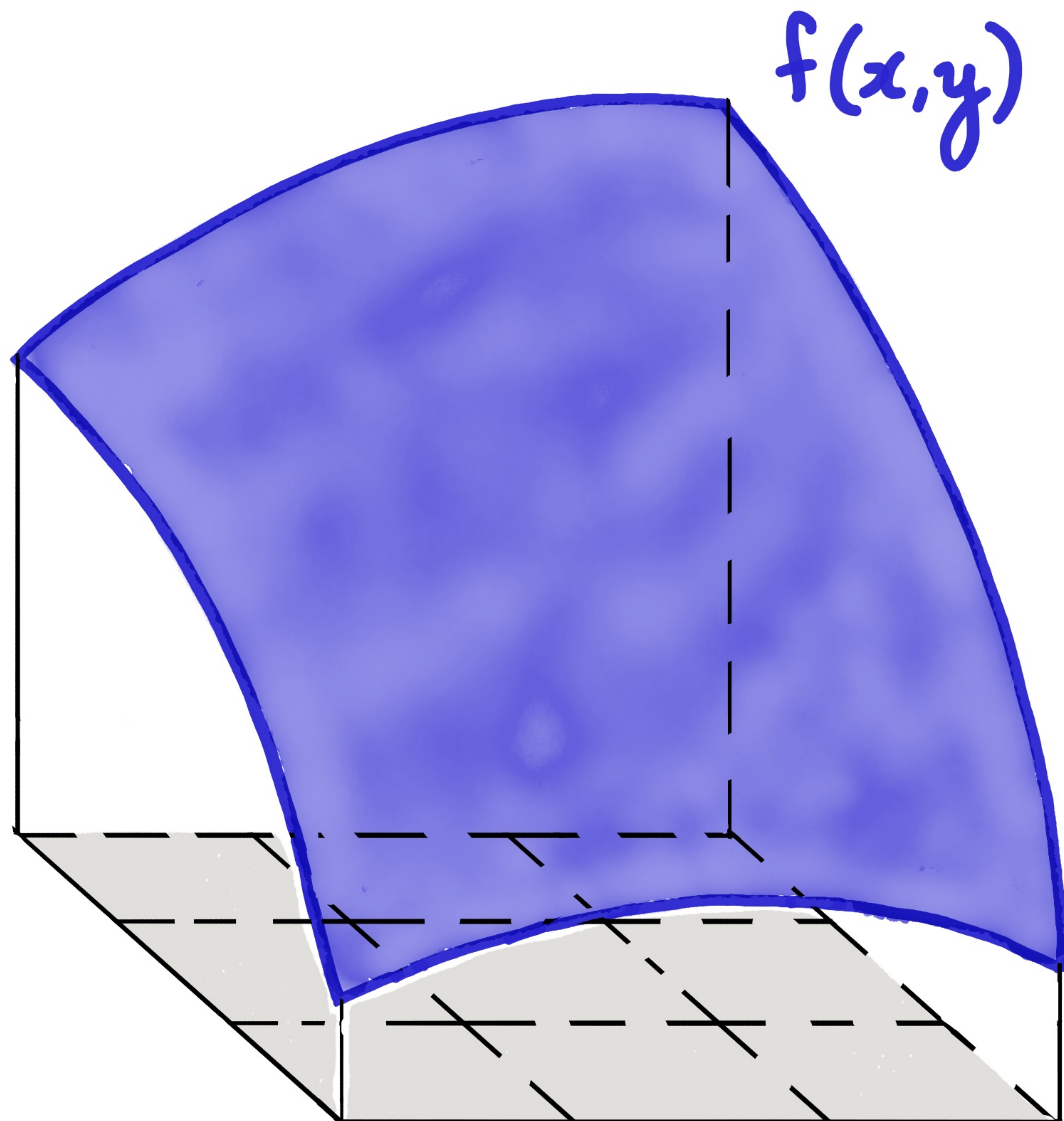


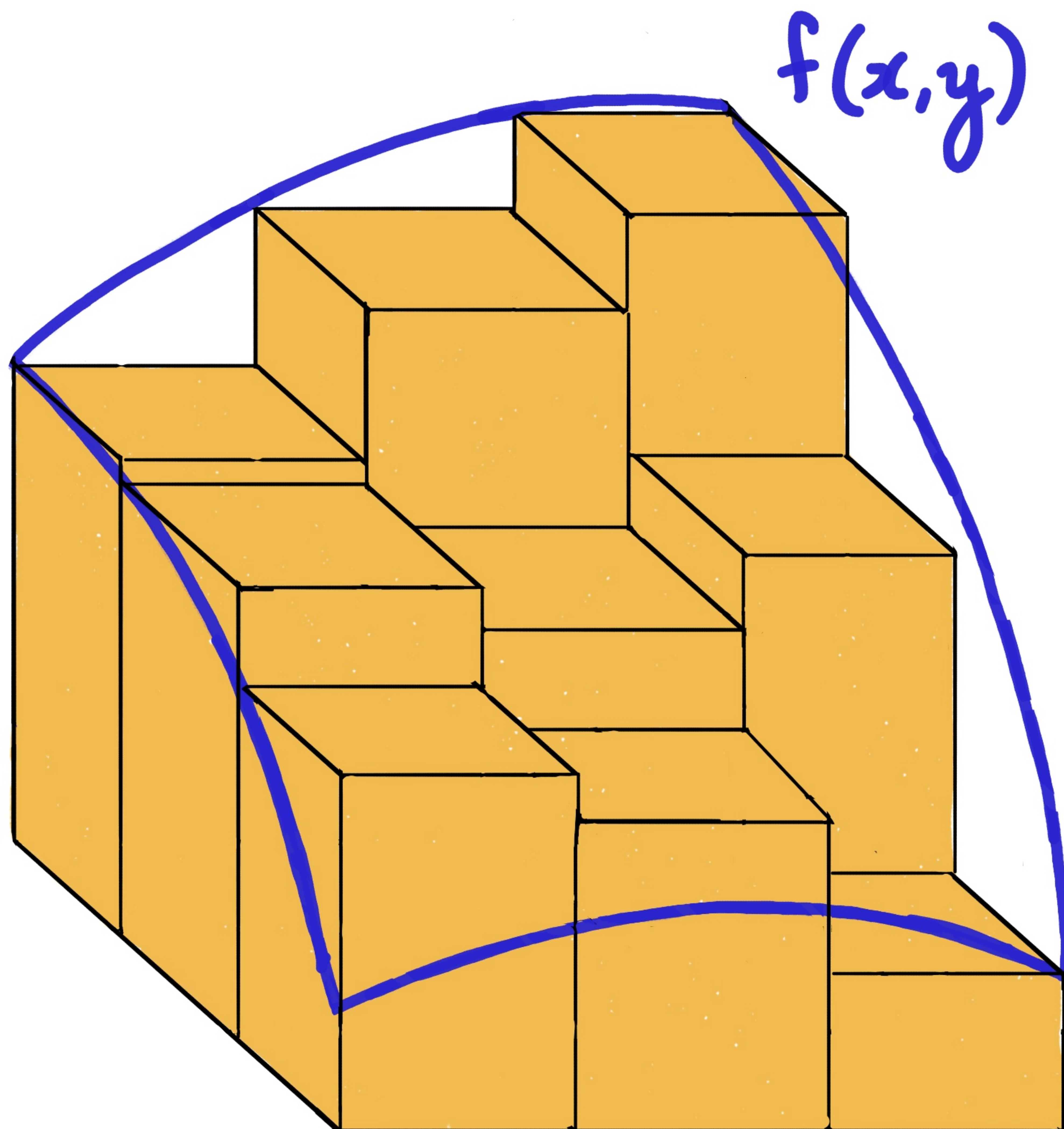


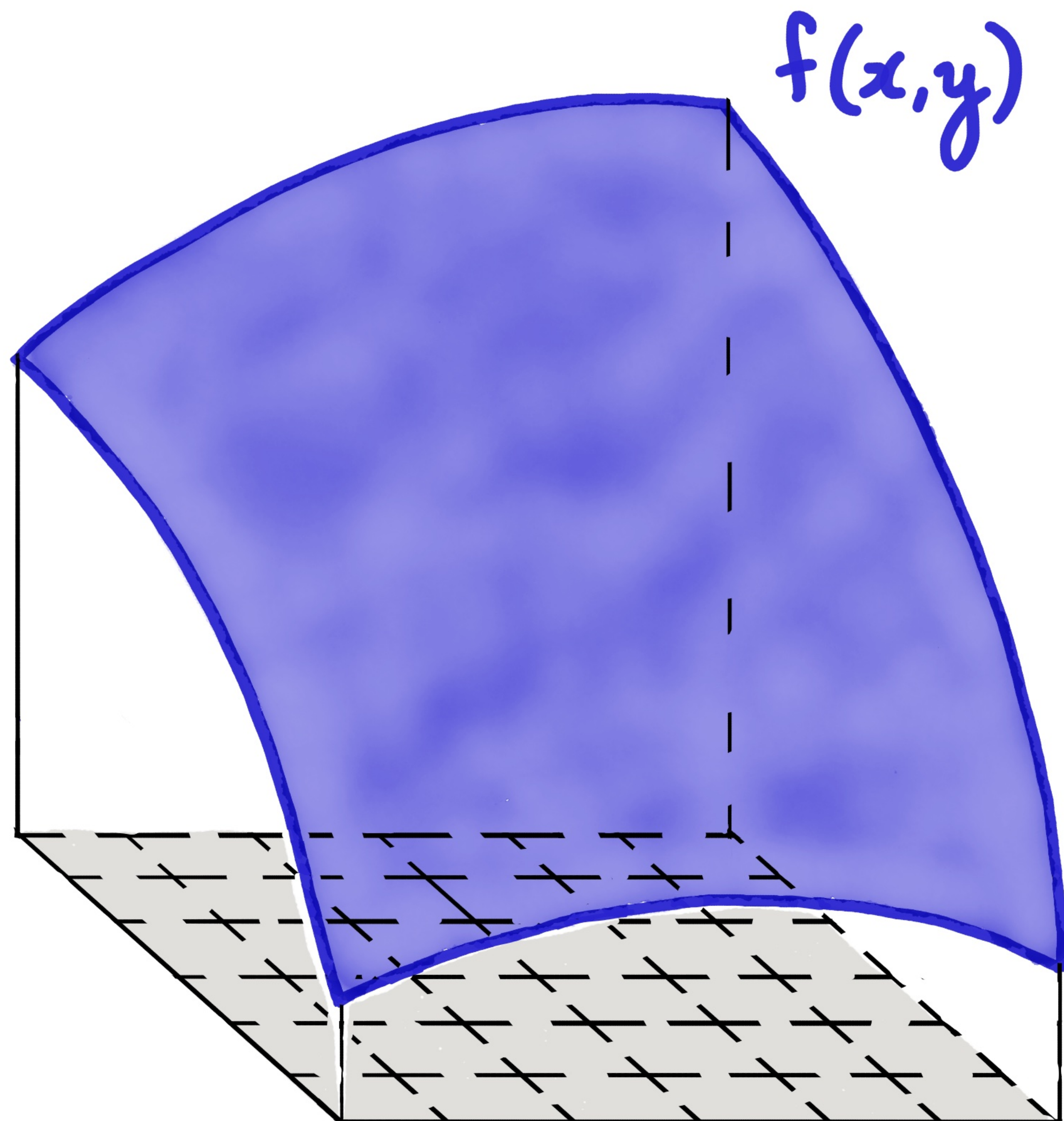
$f(x,y)$

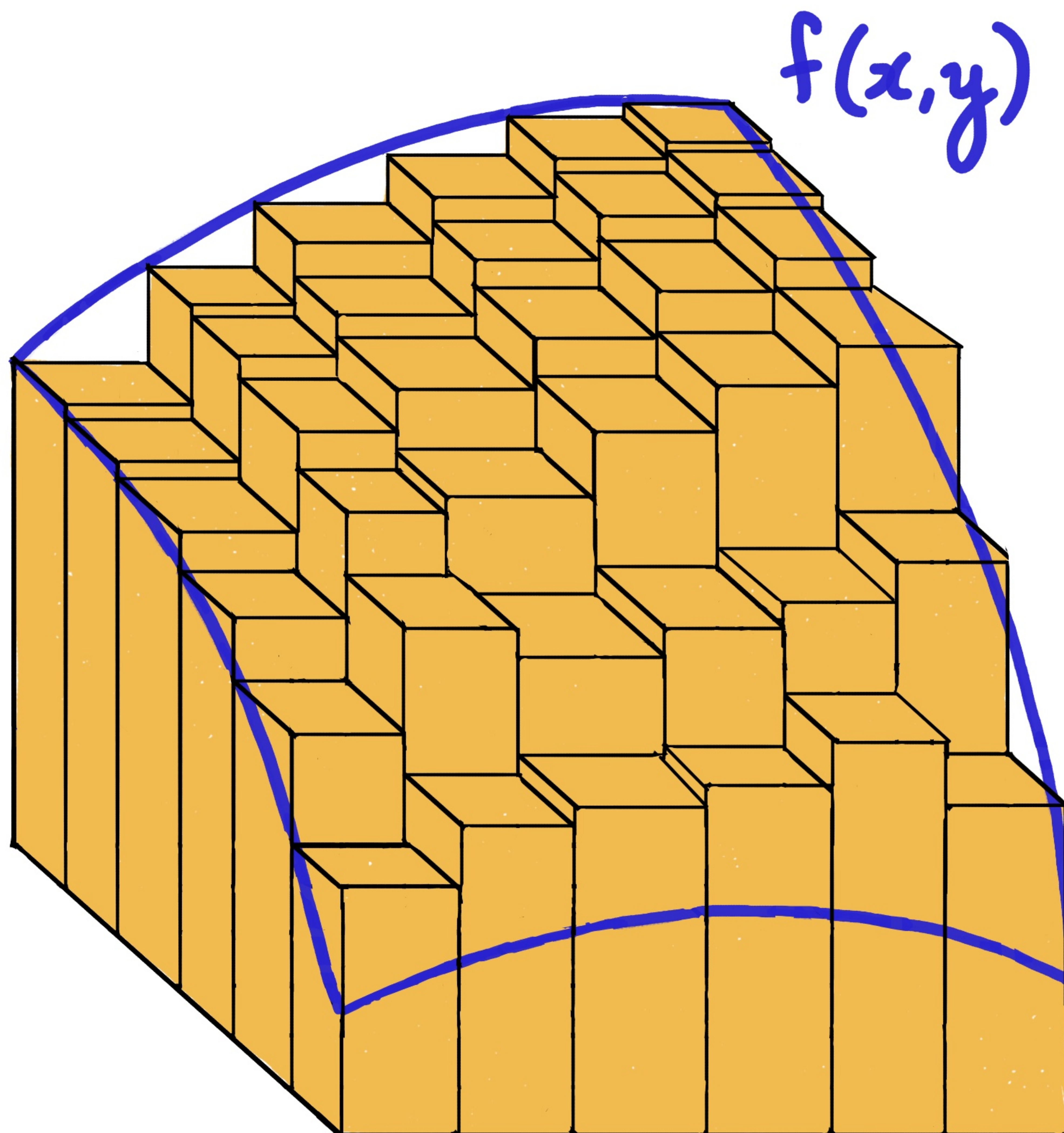


$$\iint_R f(x,y) dA$$



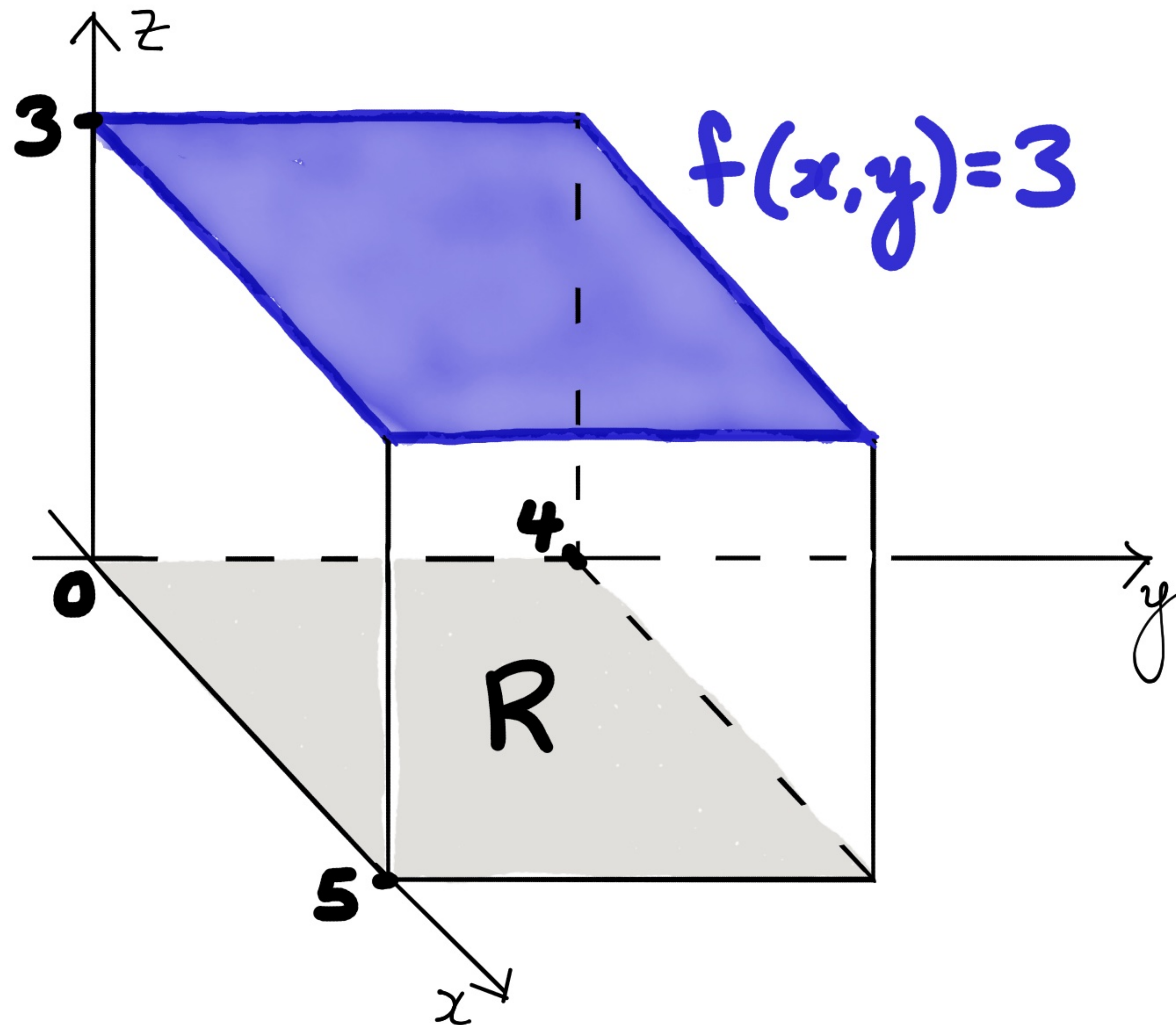






Examples:

①

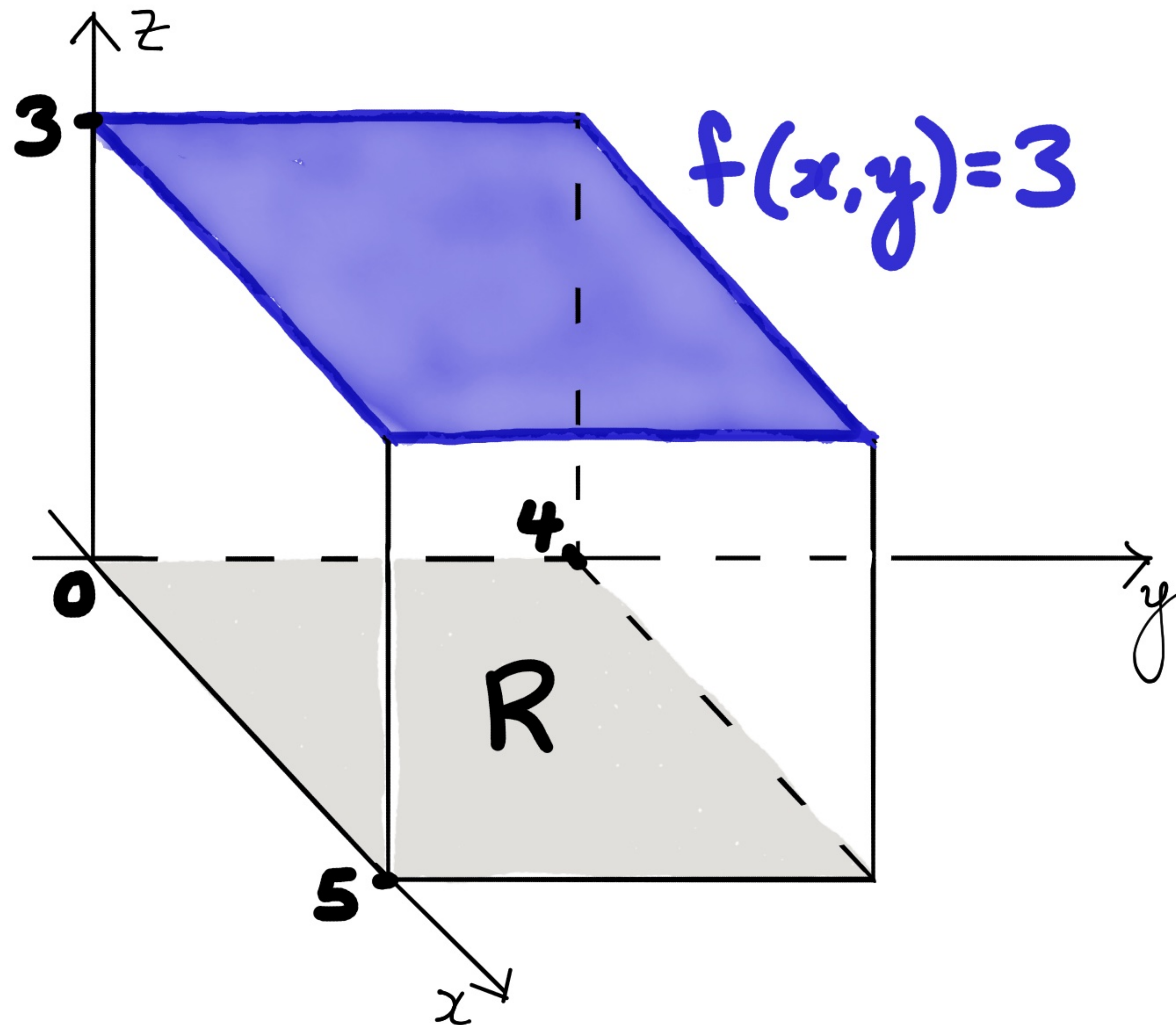


$$R = \{(x, y) : 0 \leq x \leq 5, 0 \leq y \leq 4\}$$

$$\iint_R 3 \, dA$$

Examples:

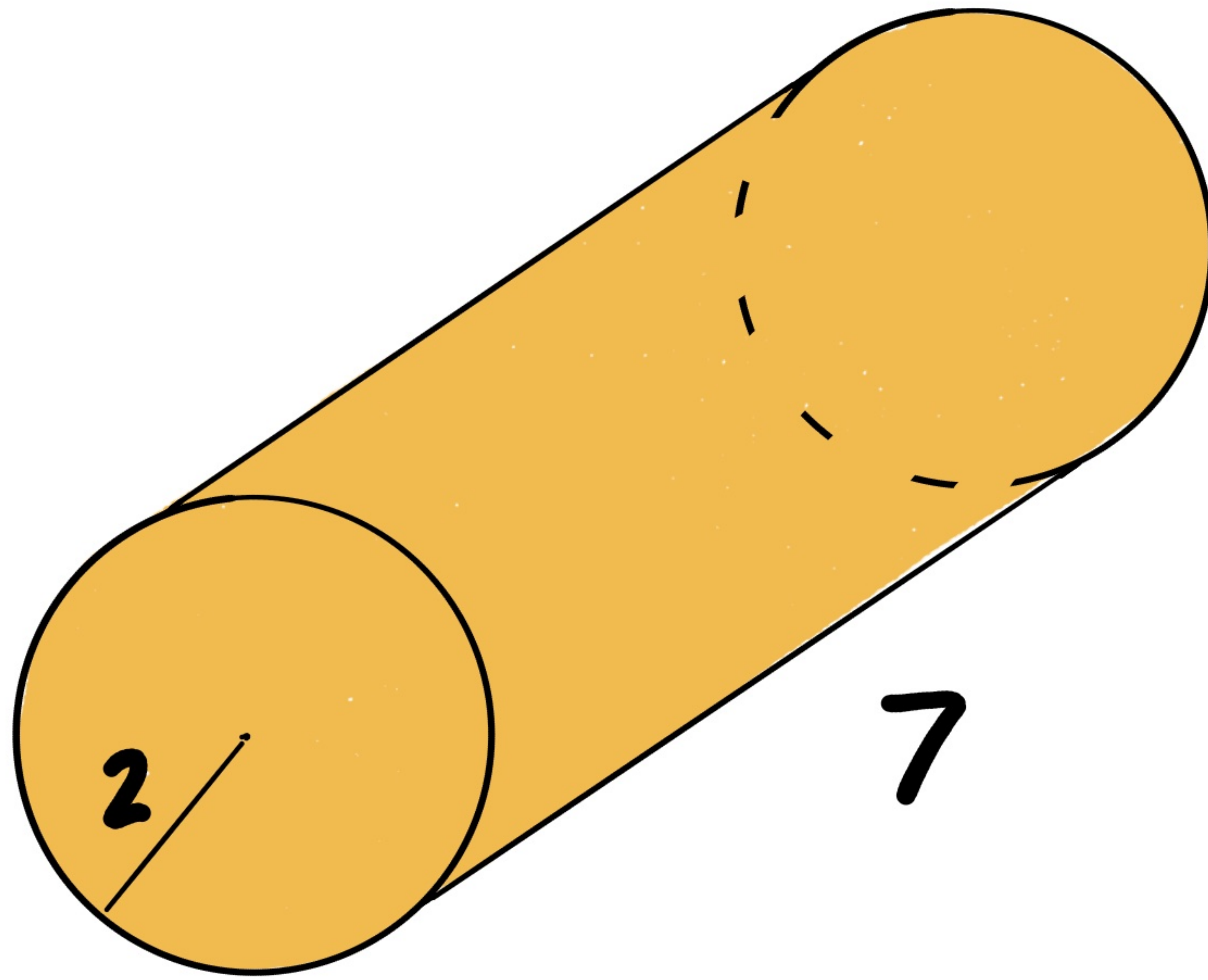
①



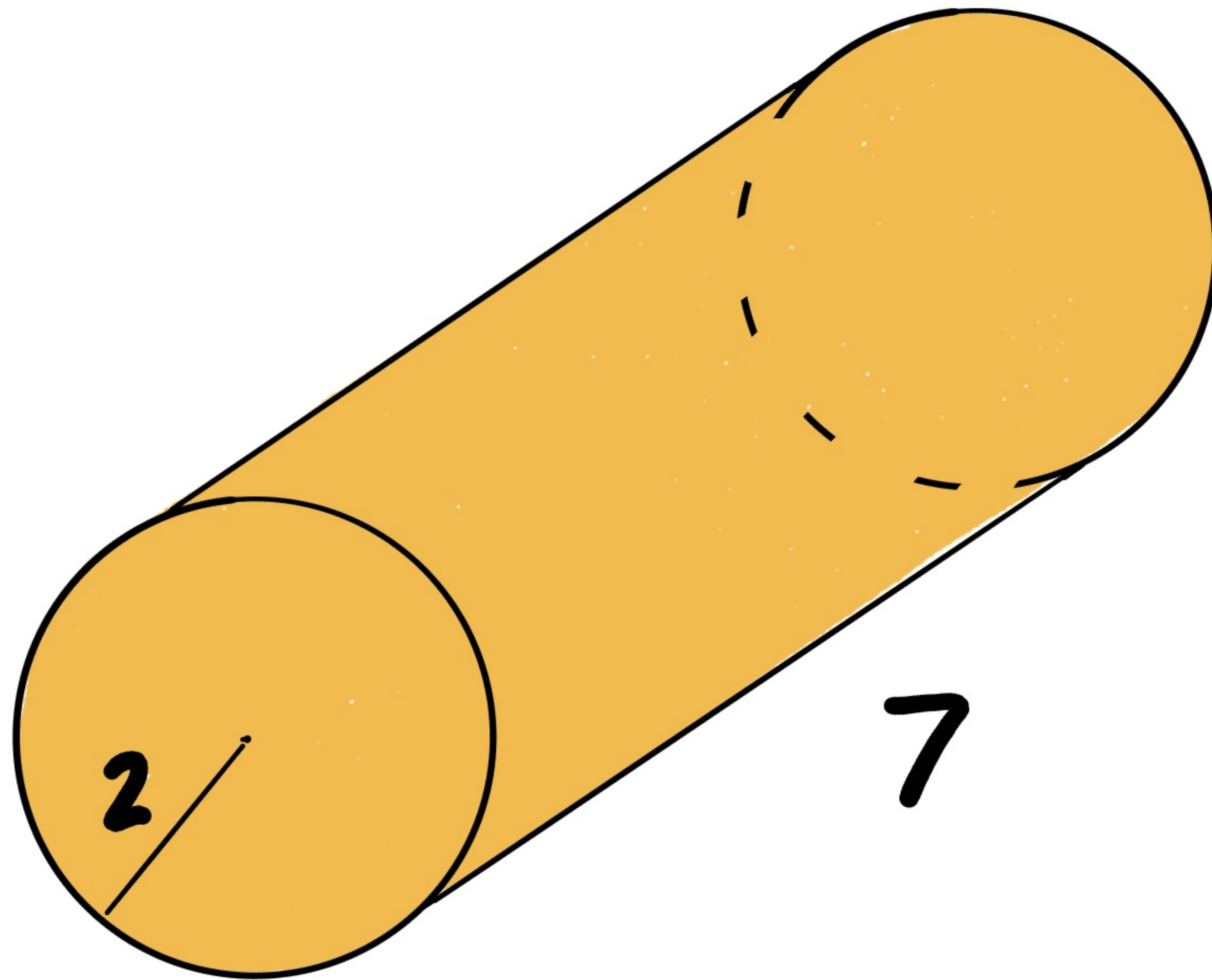
$$R = \{(x,y) : 0 \leq x \leq 5, 0 \leq y \leq 4\}$$

$$\iint_R 3 \, dA = 4 \cdot 5 \cdot 3 = 60$$

Volume of solids with constant area
cross sections: (Area of cross sections) $\times$ (length)

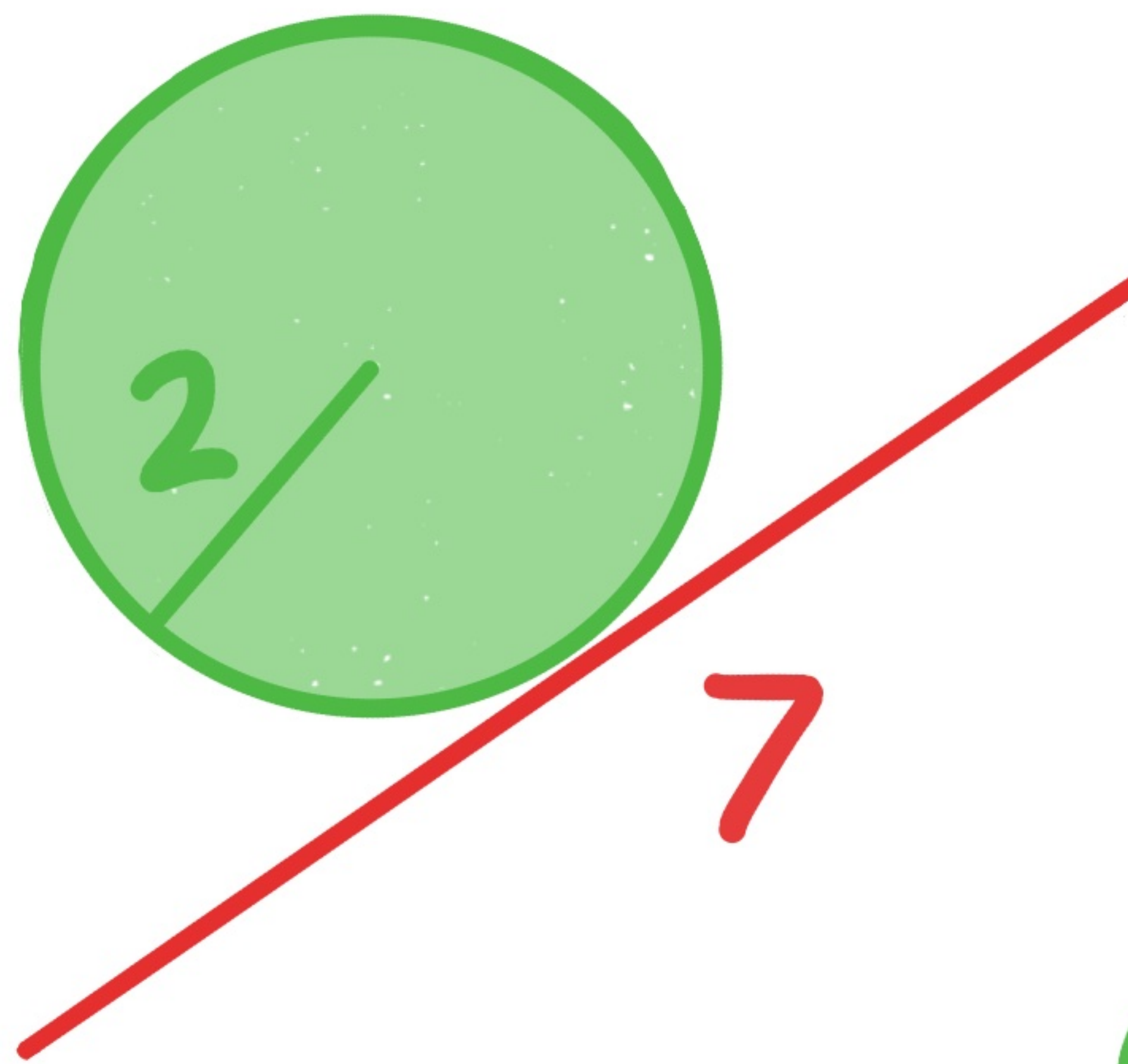


Volume of solids with constant area
cross sections: (Area of cross sections) $\times$ (length)



$$\left(\begin{array}{c} \text{Volume} \\ \text{of} \\ \text{cylinder} \end{array} \right) =$$

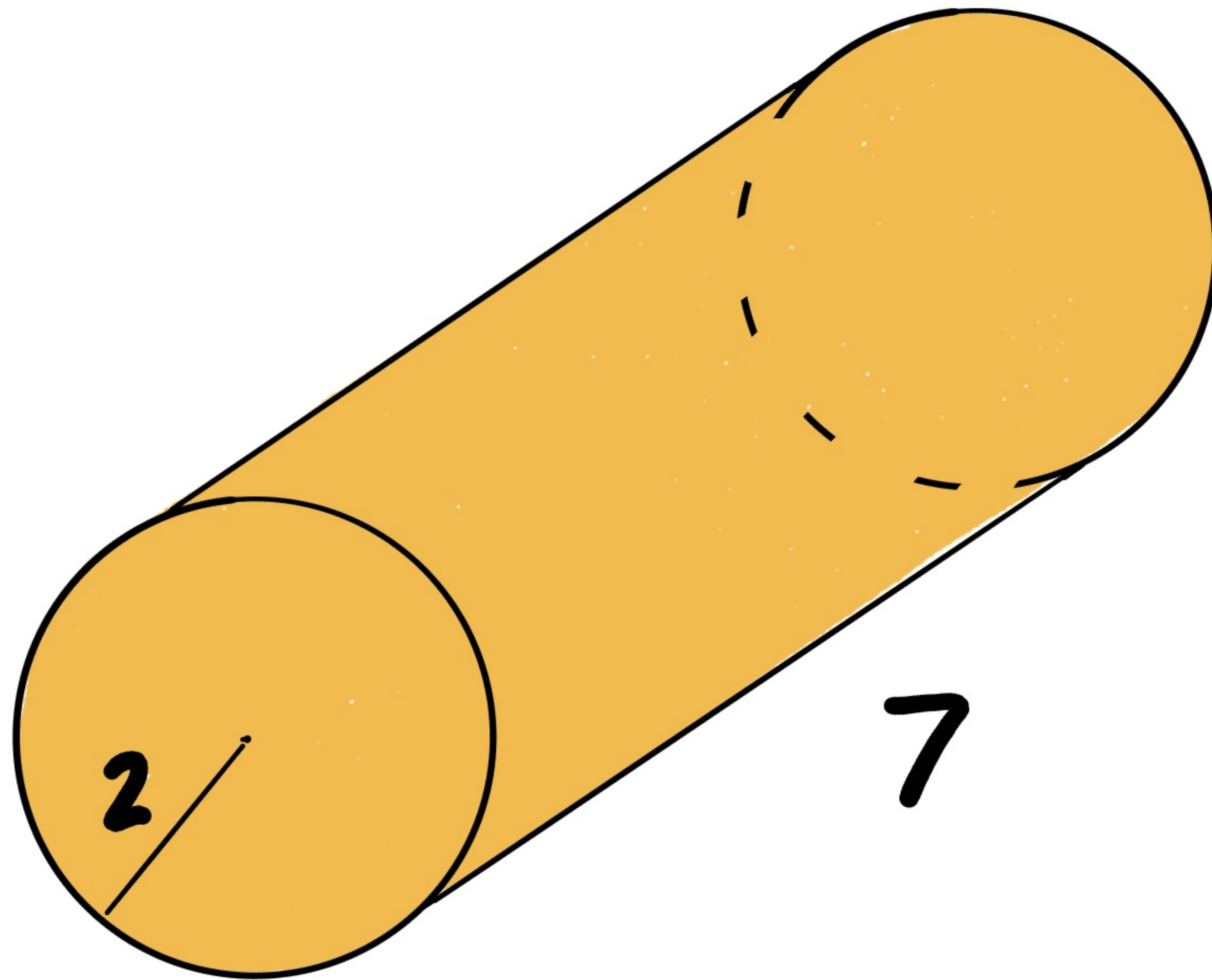
Volume of solids with constant area
cross sections: (Area of cross sections) $\times$ (length)



area of cross sections

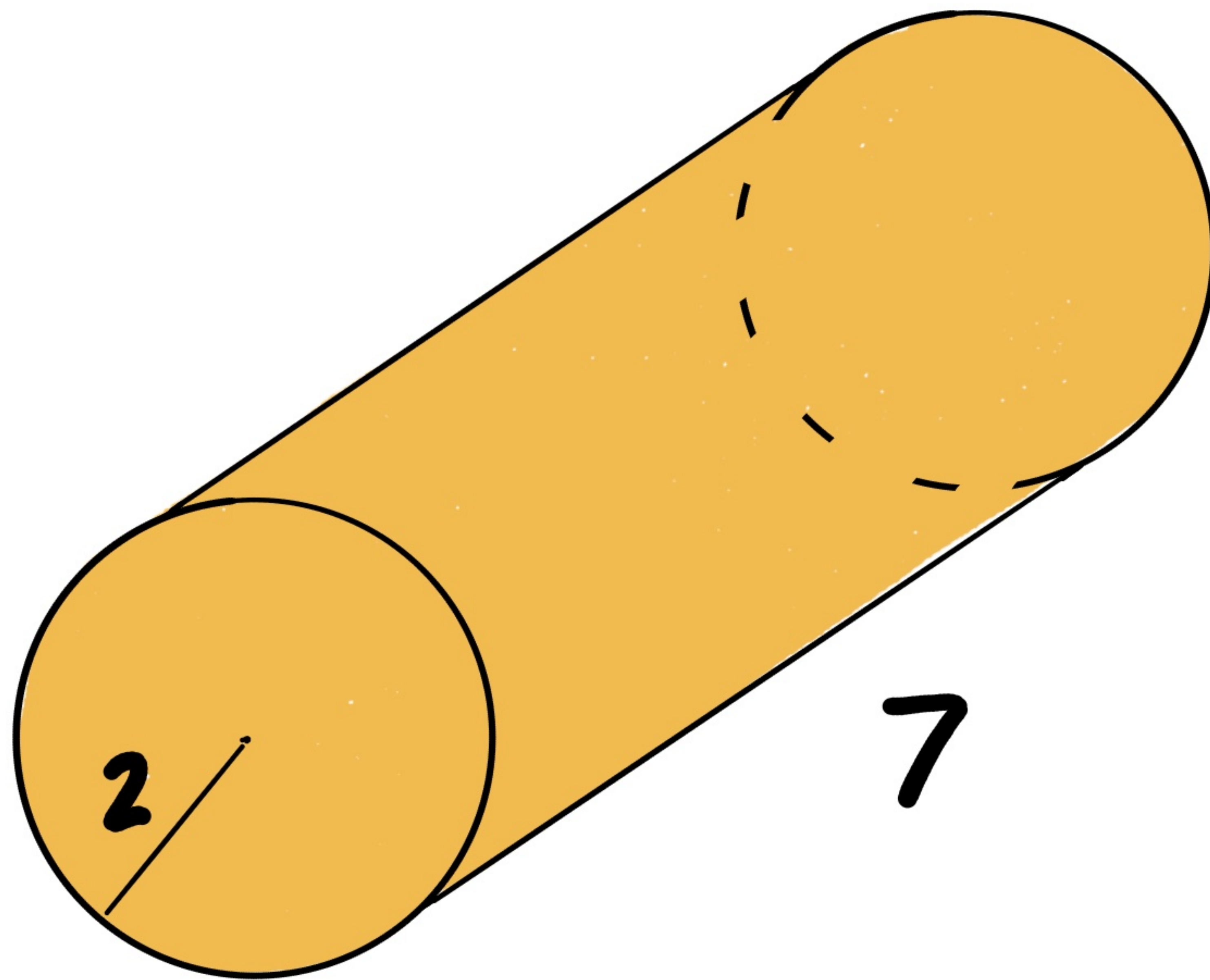
$\left(\begin{array}{c} \text{Volume} \\ \text{of} \\ \text{cylinder} \end{array} \right) = \pi \cdot 2^2 \cdot 7 \leftarrow \text{length}$

Volume of solids with constant area
cross sections: (Area of cross sections) $\times$ (length)



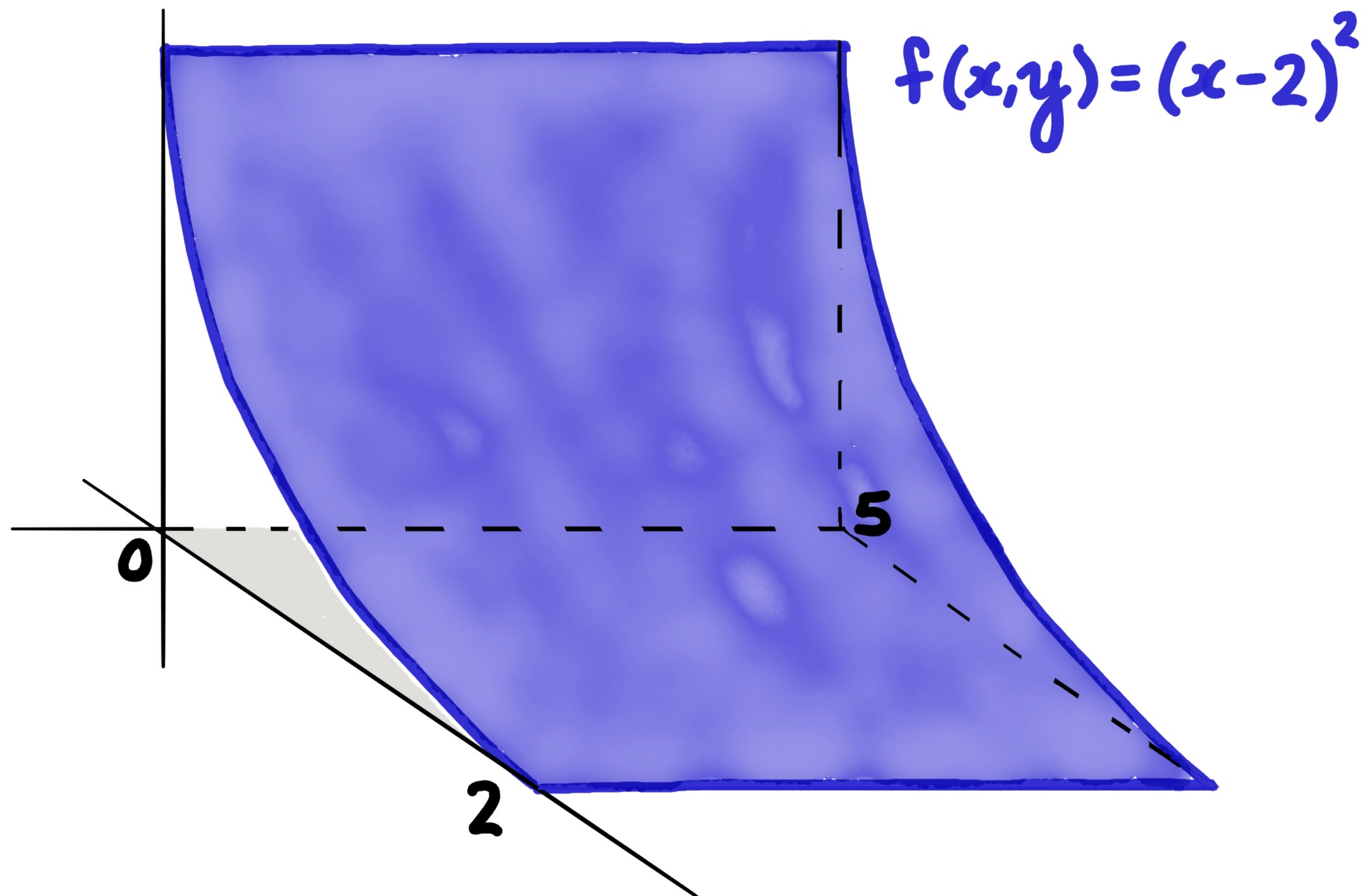
$$\left(\begin{array}{c} \text{Volume} \\ \text{of} \\ \text{cylinder} \end{array} \right) = \pi \cdot 2^2 \cdot 7$$

Volume of solids with constant area
cross sections: (Area of cross sections) $\times$ (length)



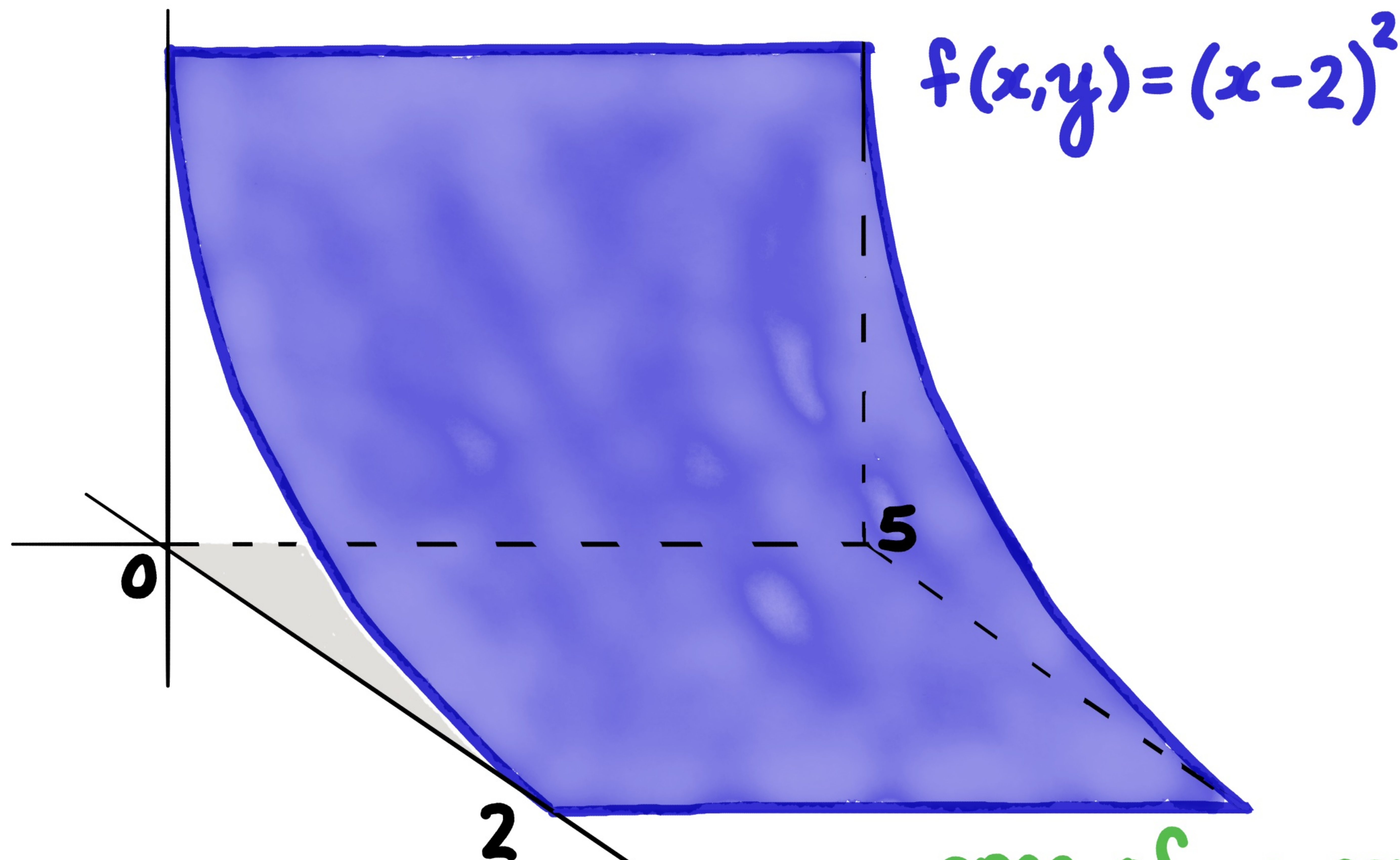
$$\left(\begin{array}{c} \text{Volume} \\ \text{of} \\ \text{cylinder} \end{array} \right) = \pi \cdot 2^2 \cdot 7 = 28\pi$$

②



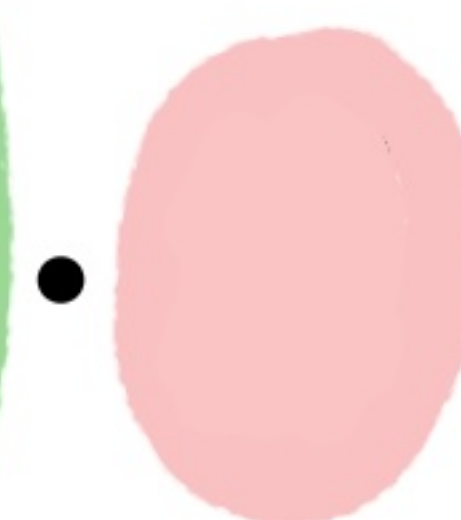
$$\iint_R (x-2)^2 dA =$$

②



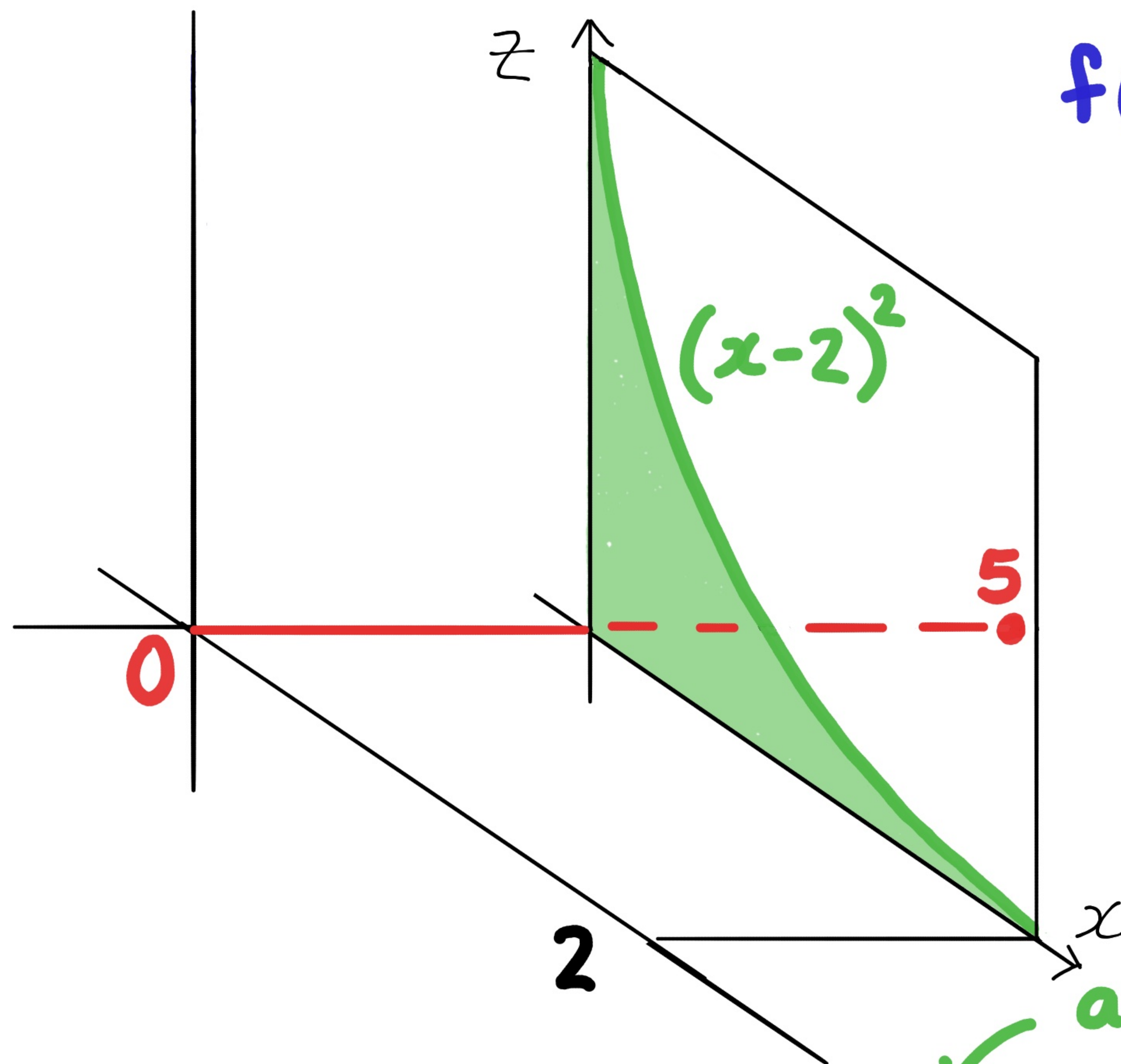
area of $y = \text{constant}$
cross sections

$$\iint_R (x-2)^2 dA =$$



length in
 y -direction

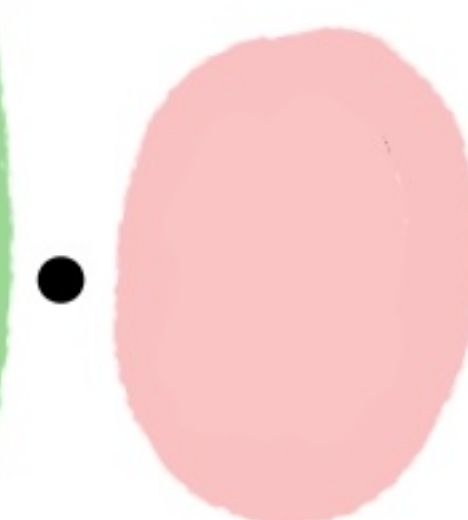
②



$$f(x,y) = (x-2)^2$$

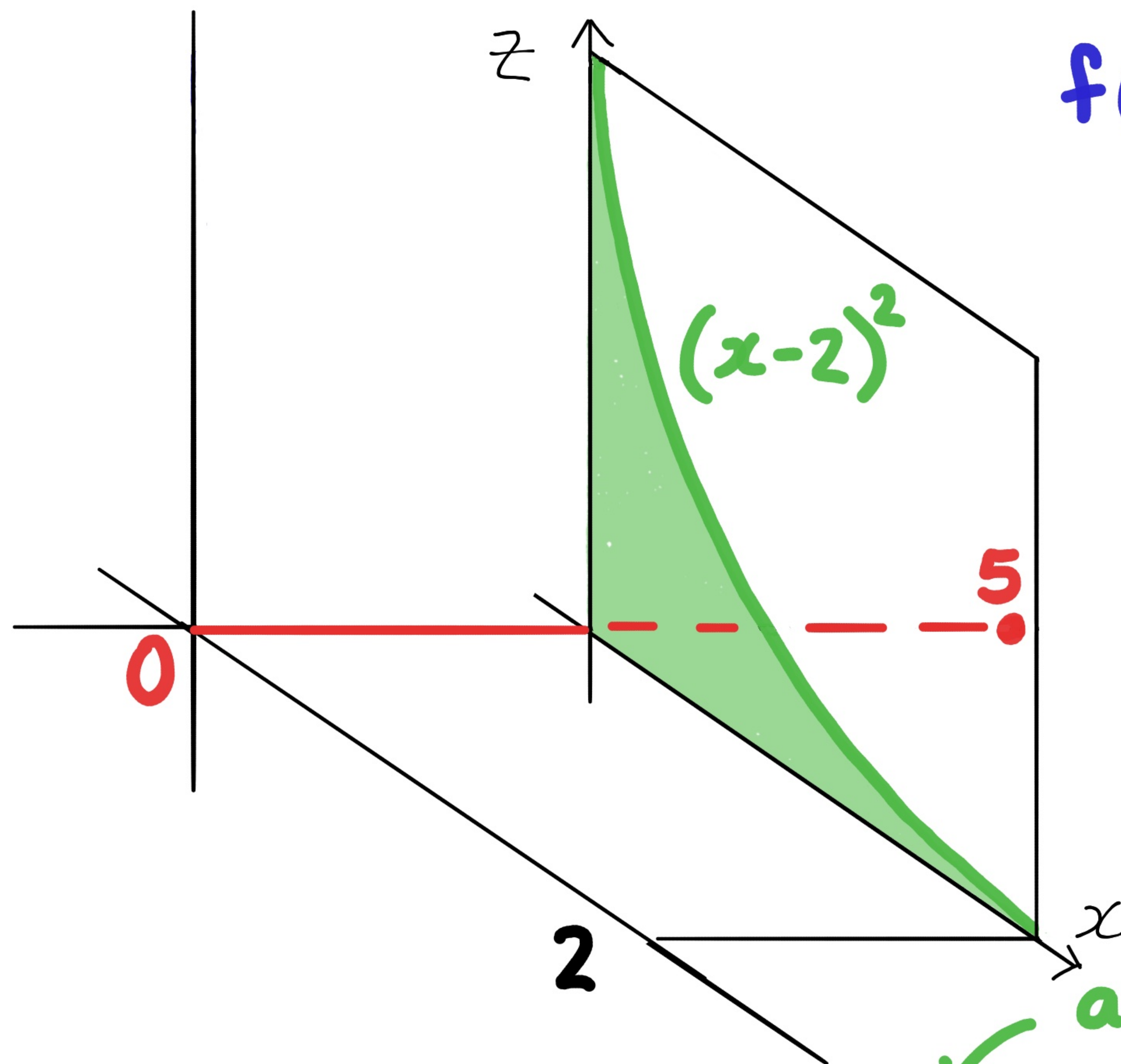
area of $y = \text{constant}$
cross sections

$$\iint_R (x-2)^2 dA =$$



length in
 y -direction

②



$$f(x,y) = (x-2)^2$$

$$(x-2)^2$$

5

2

area of $y=\text{constant}$
cross sections

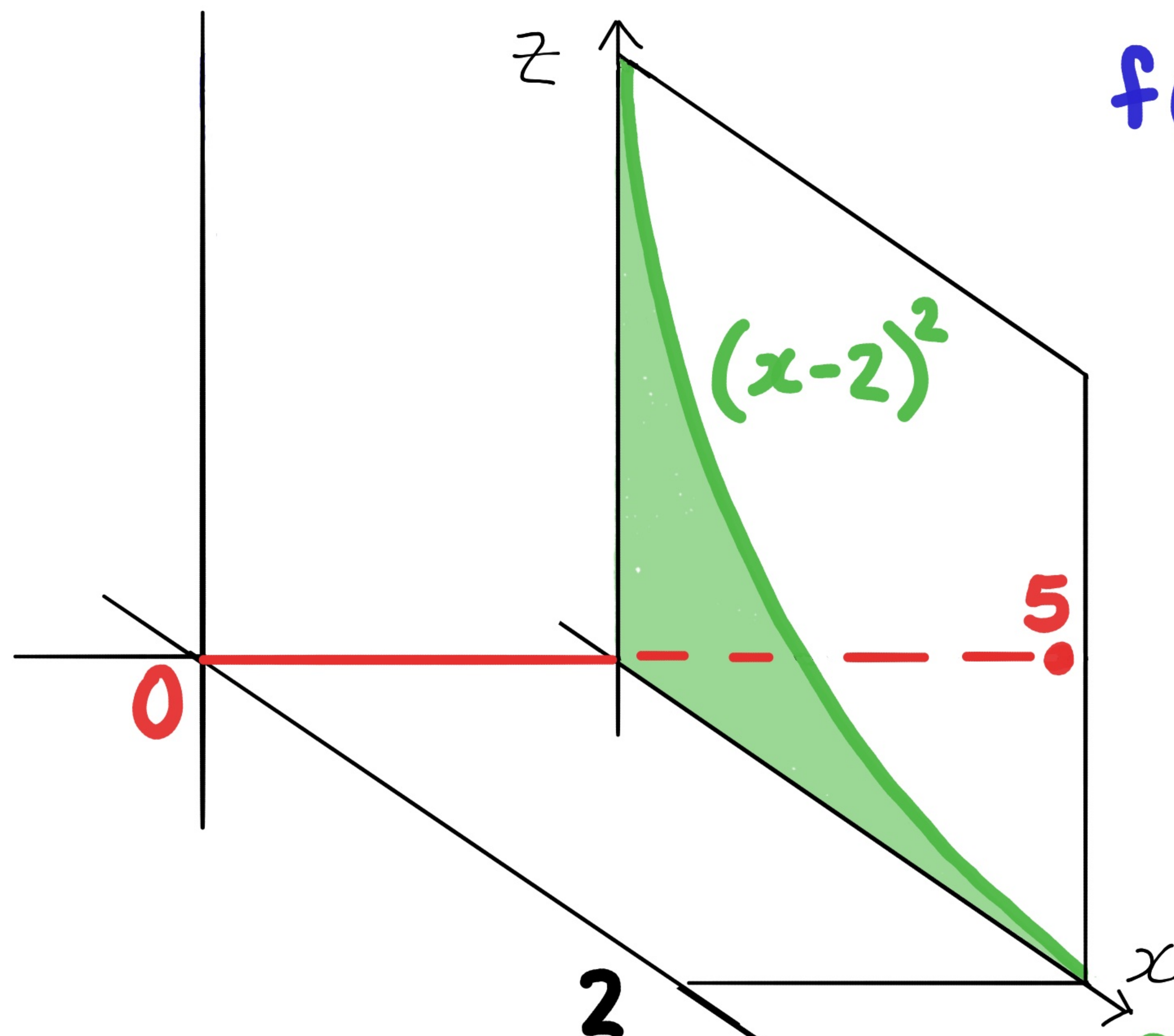
$$\iint_R (x-2)^2 dA =$$



5

length in
 y -direction

②



$$f(x,y) = (x-2)^2$$

$$(x-2)^2$$

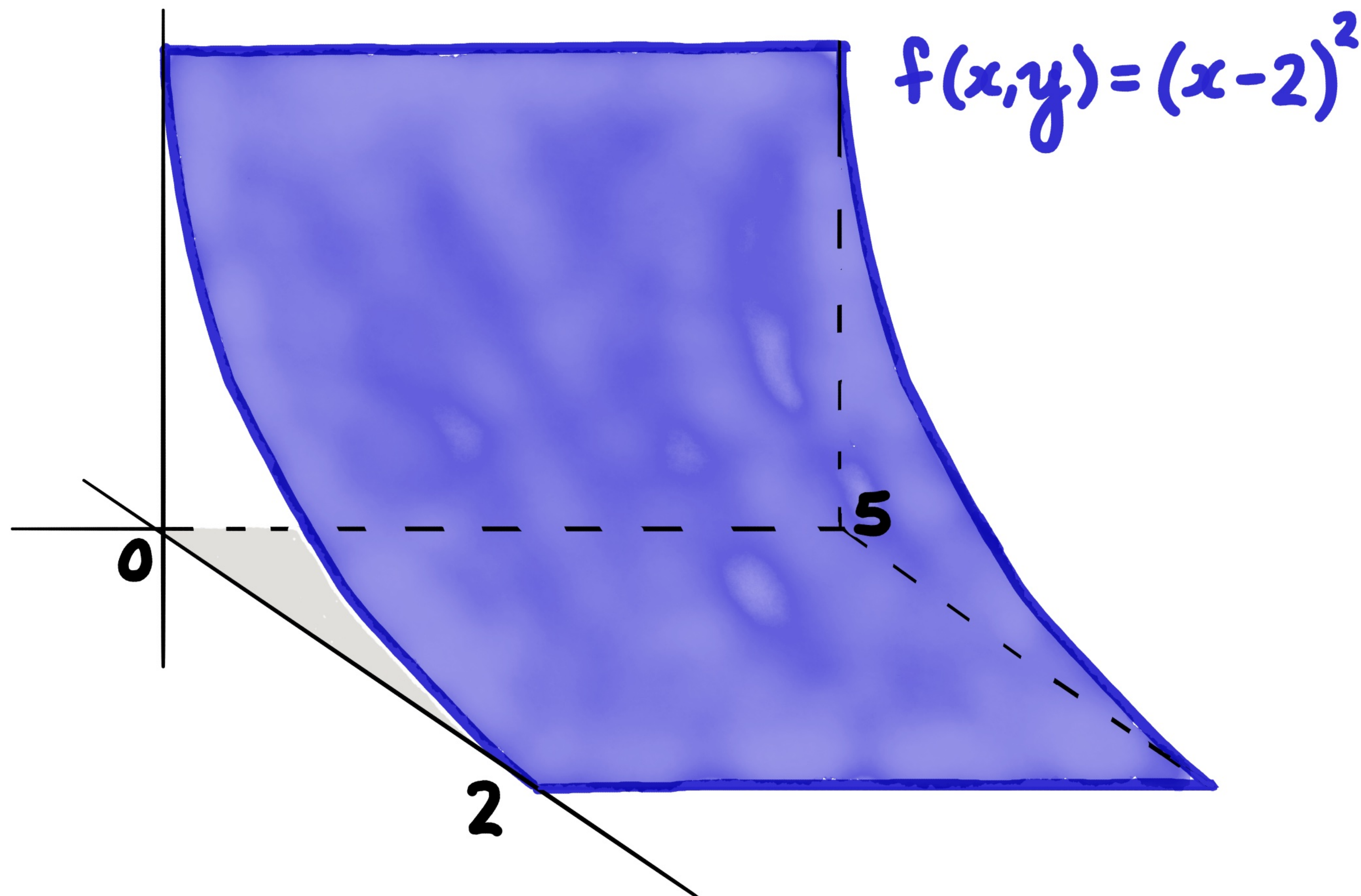
5

2

area of $y=\text{constant}$
cross sections

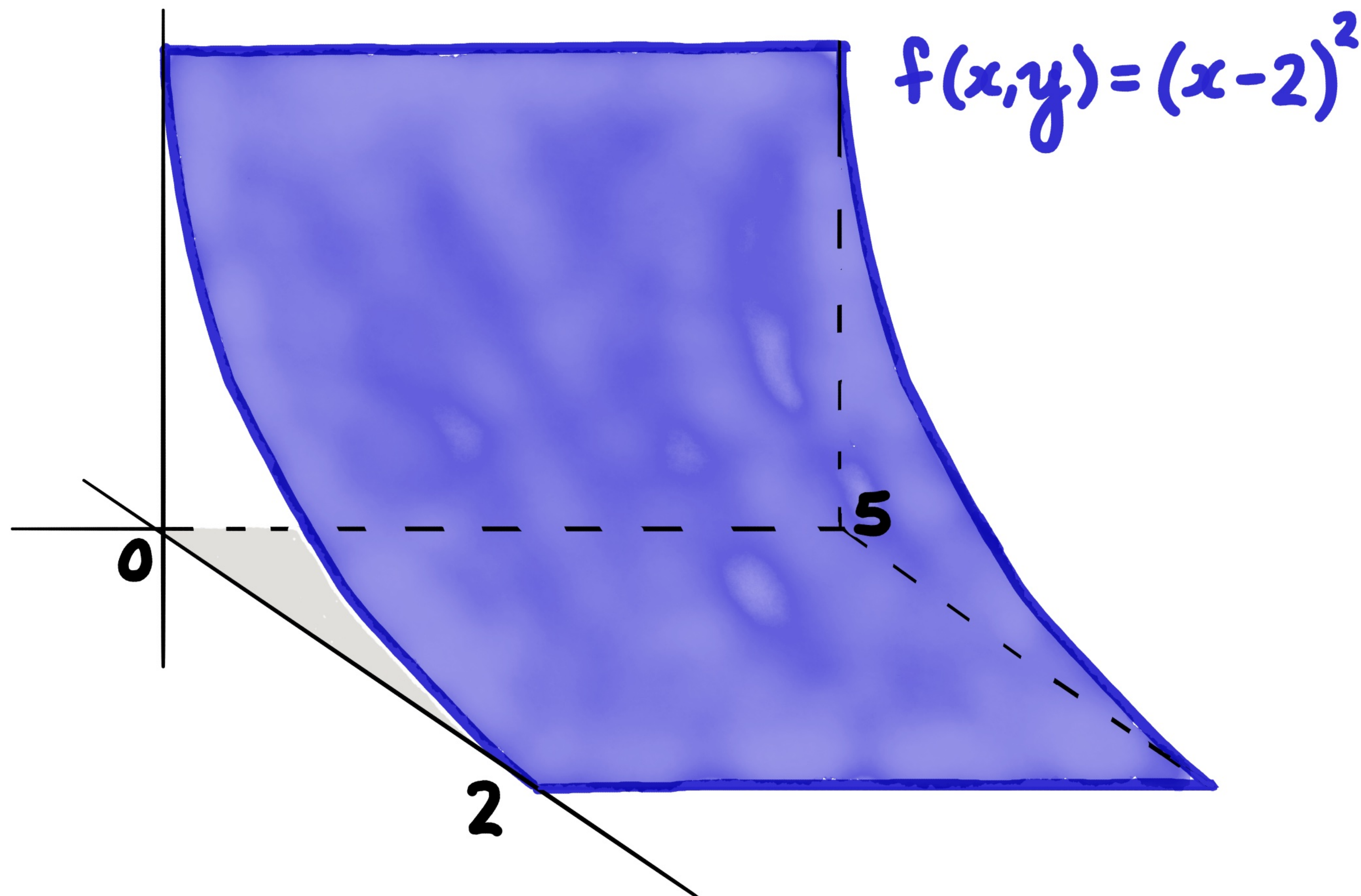
$$\iint_R (x-2)^2 dA = \left(\int_0^2 (x-2)^2 dx \right) \cdot 5 \leftarrow \text{length in } y\text{-direction}$$

②



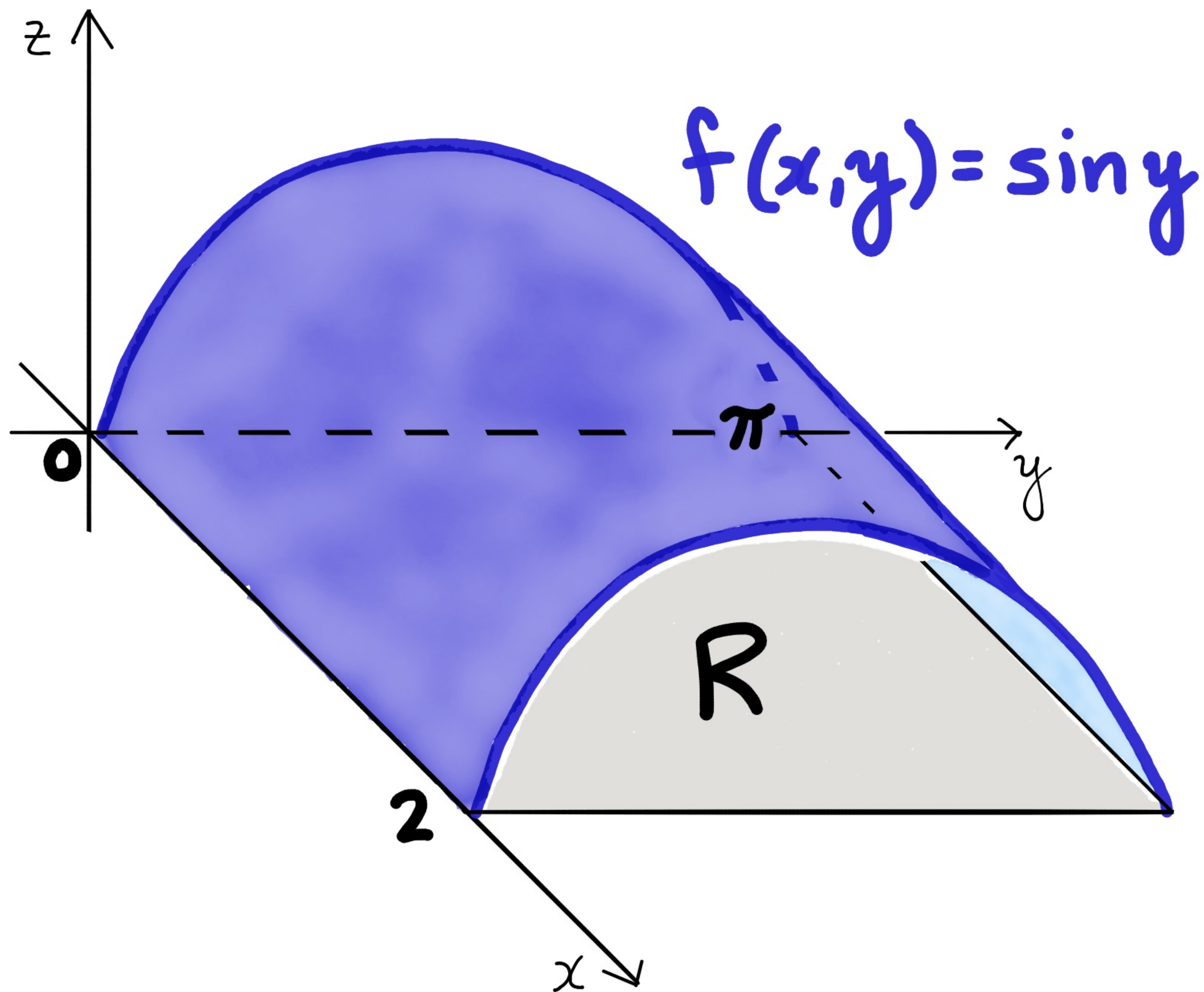
$$\iint_R (x-2)^2 dA = \left(\int_0^2 (x-2)^2 dx \right) \cdot 5$$

②



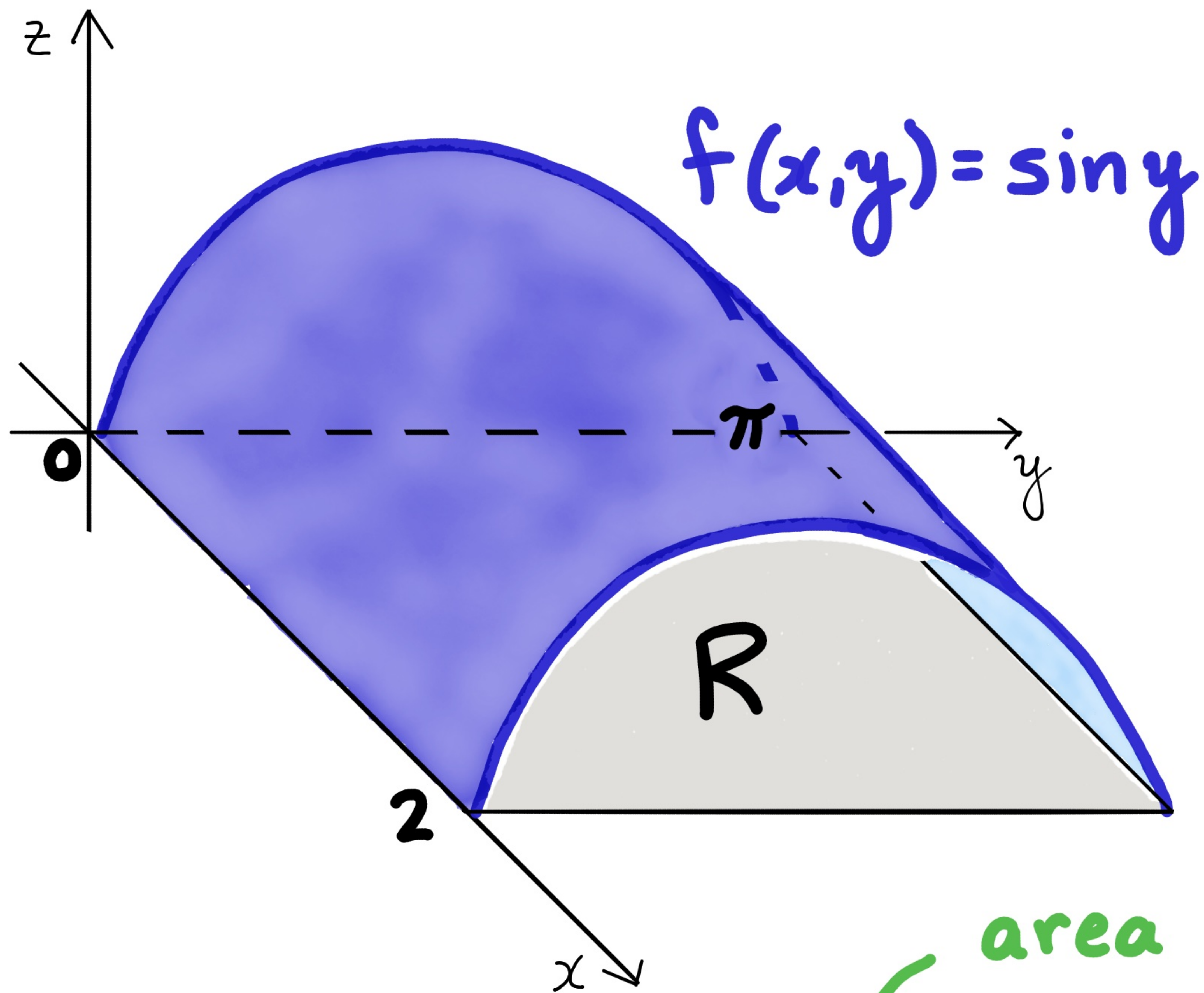
$$\iint_R (x-2)^2 dA = \left(\int_0^2 (x-2)^2 dx \right) \cdot 5 = \frac{8}{3} \cdot 5 = \frac{40}{3}$$

③



$$\iint_R \sin y \, dA =$$

③



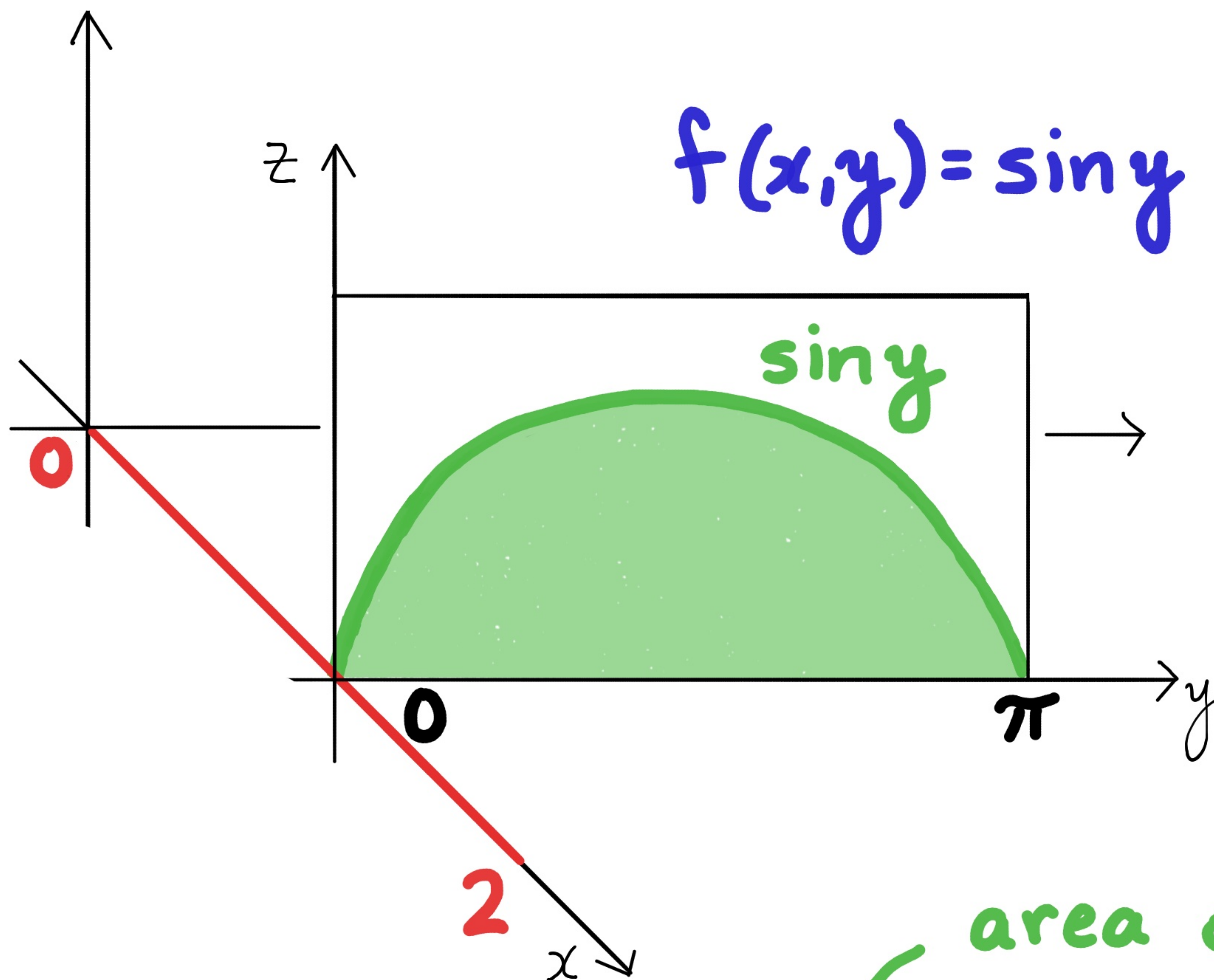
area of $x=\text{constant}$
cross sections

$$\iint_R \sin y \, dA =$$



length in
 x -direction

③



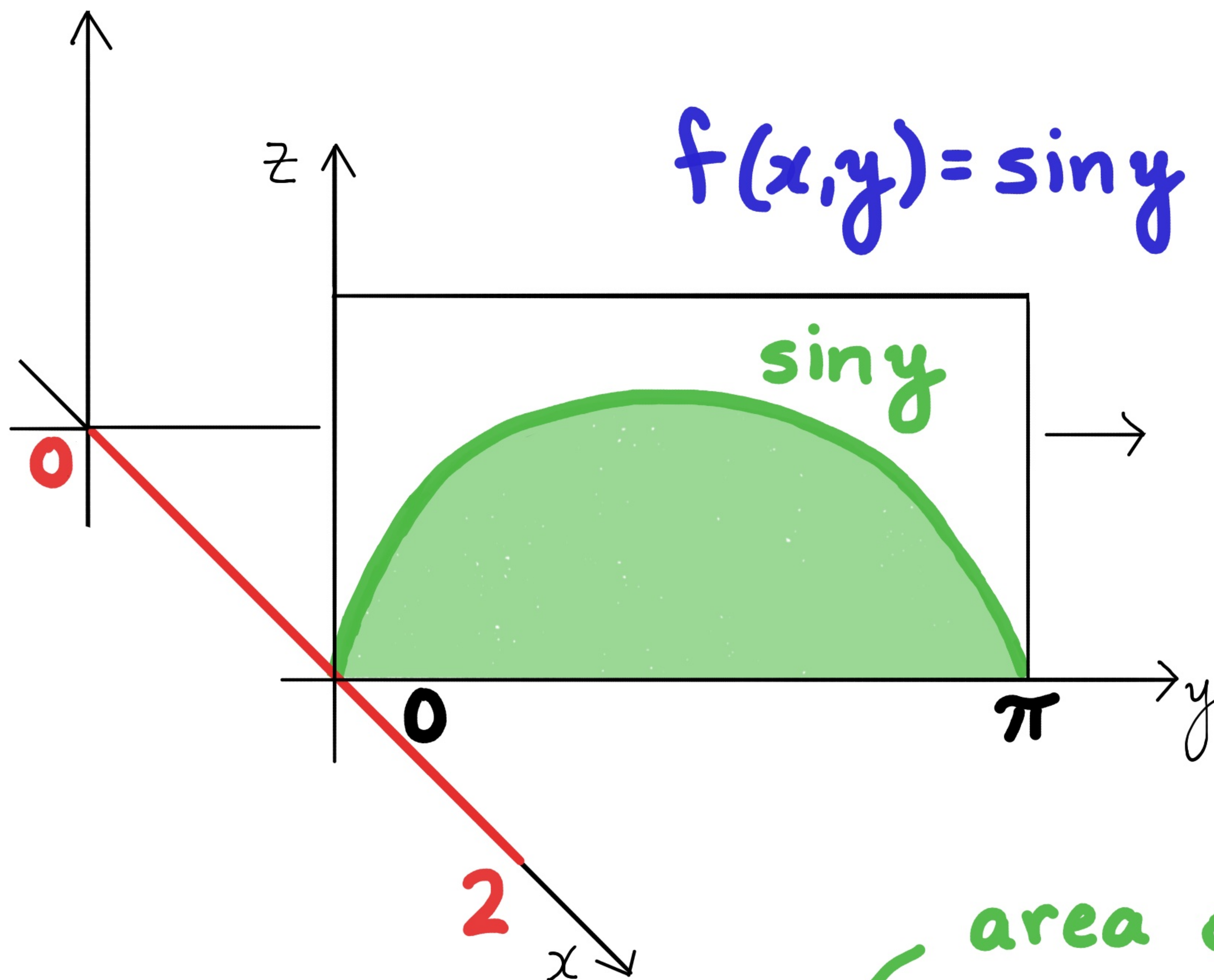
area of $x=\text{constant}$
cross sections

$$\iint_R \sin y \, dA =$$



length in
 x -direction

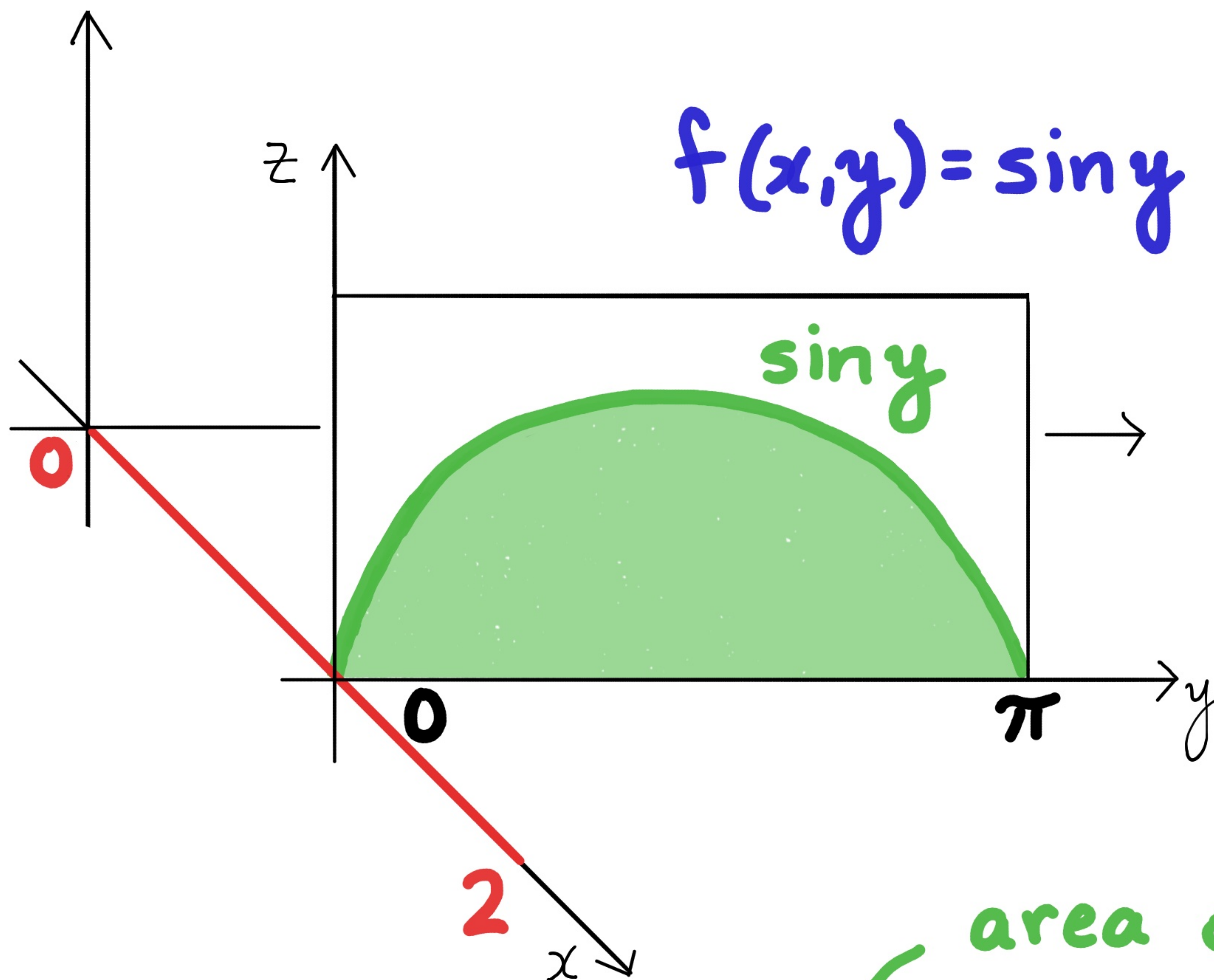
③



area of $x=\text{constant}$
cross sections

$$\iint_R \sin y \, dA = \text{[green blob]} \cdot 2 \leftarrow \text{length in } x\text{-direction}$$

③

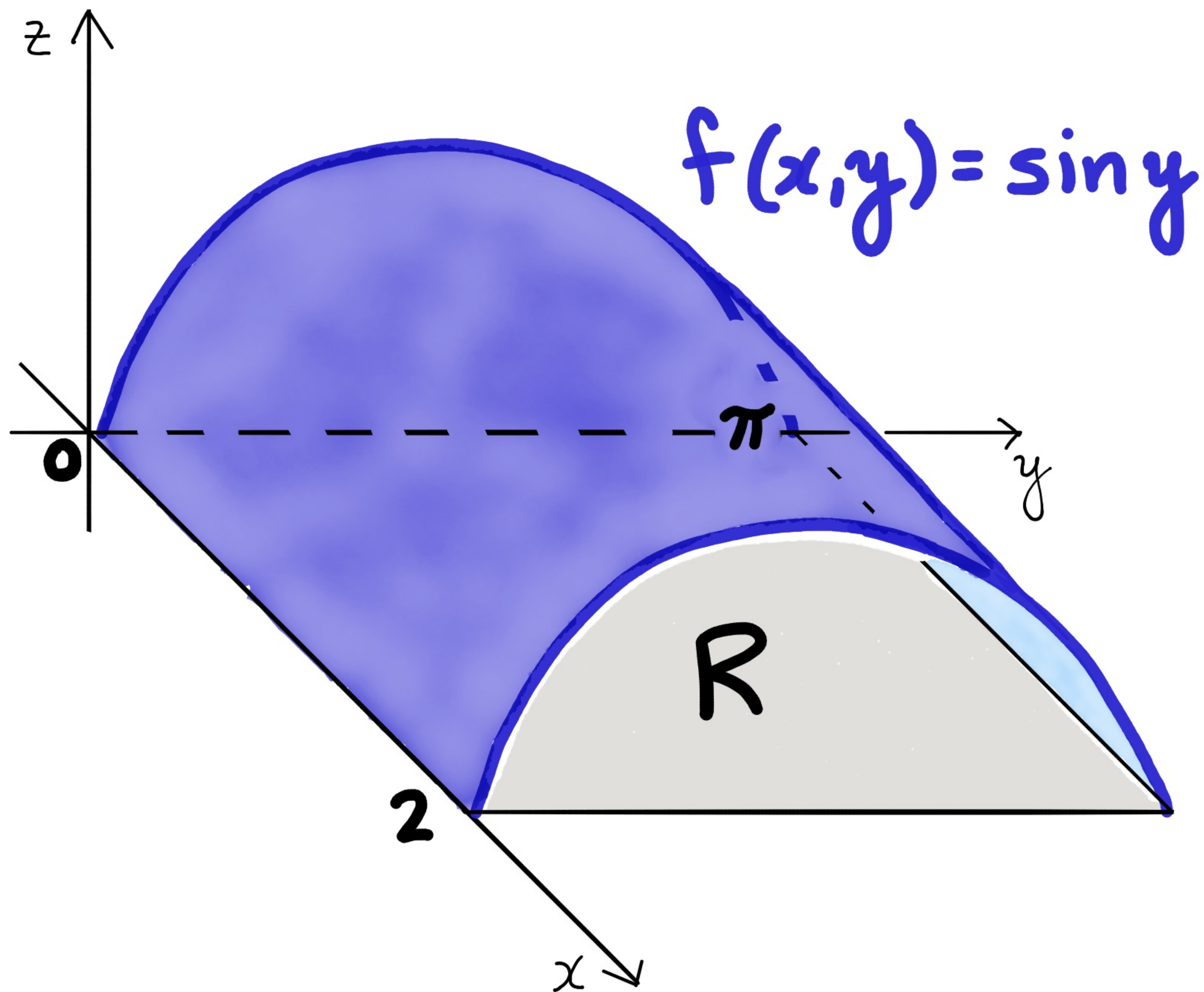


area of $x=\text{constant}$
cross sections

$$\iint_R \sin y \, dA = \left(\int_0^\pi \sin y \, dy \right) \cdot 2$$

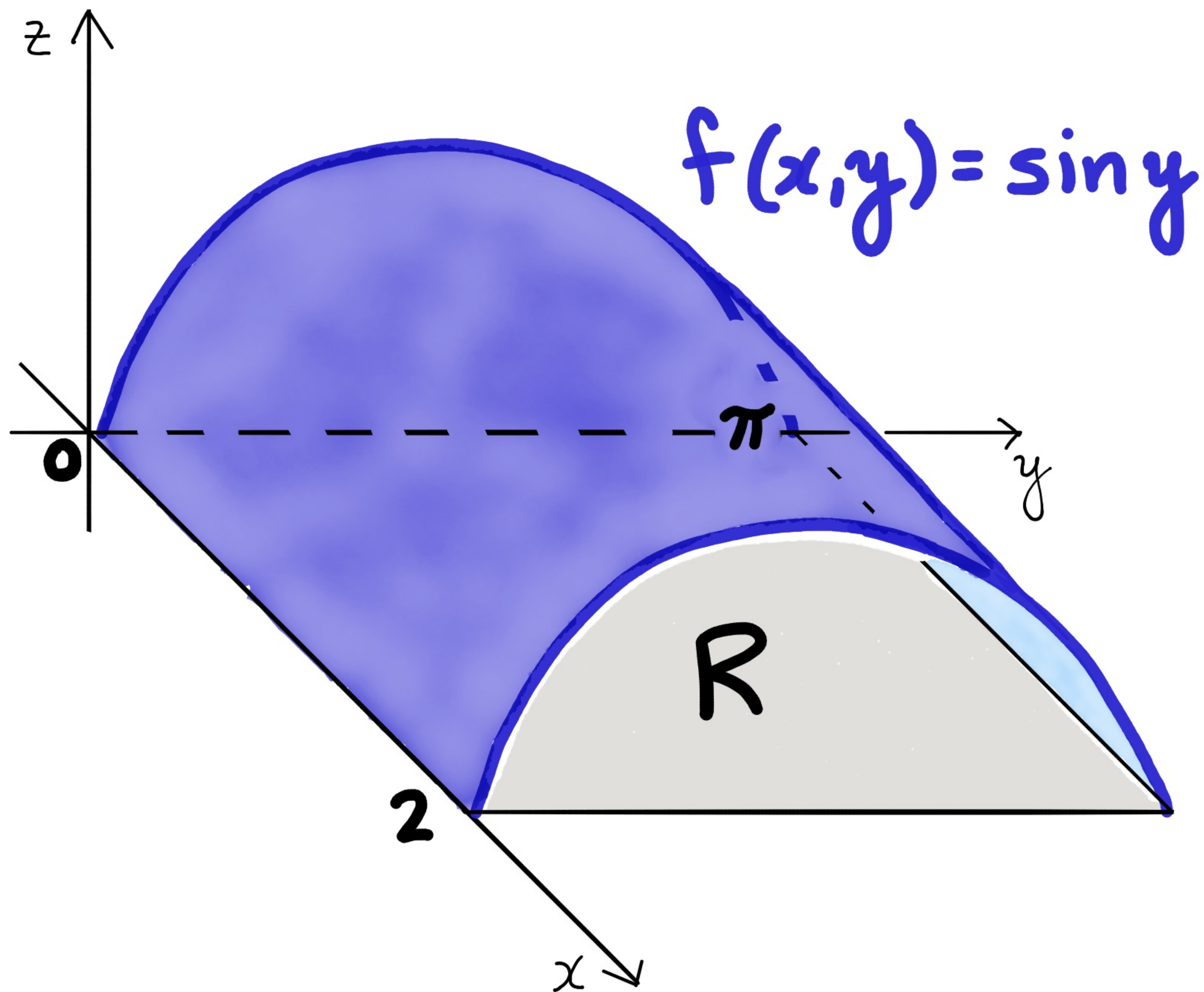
length in x -direction

③



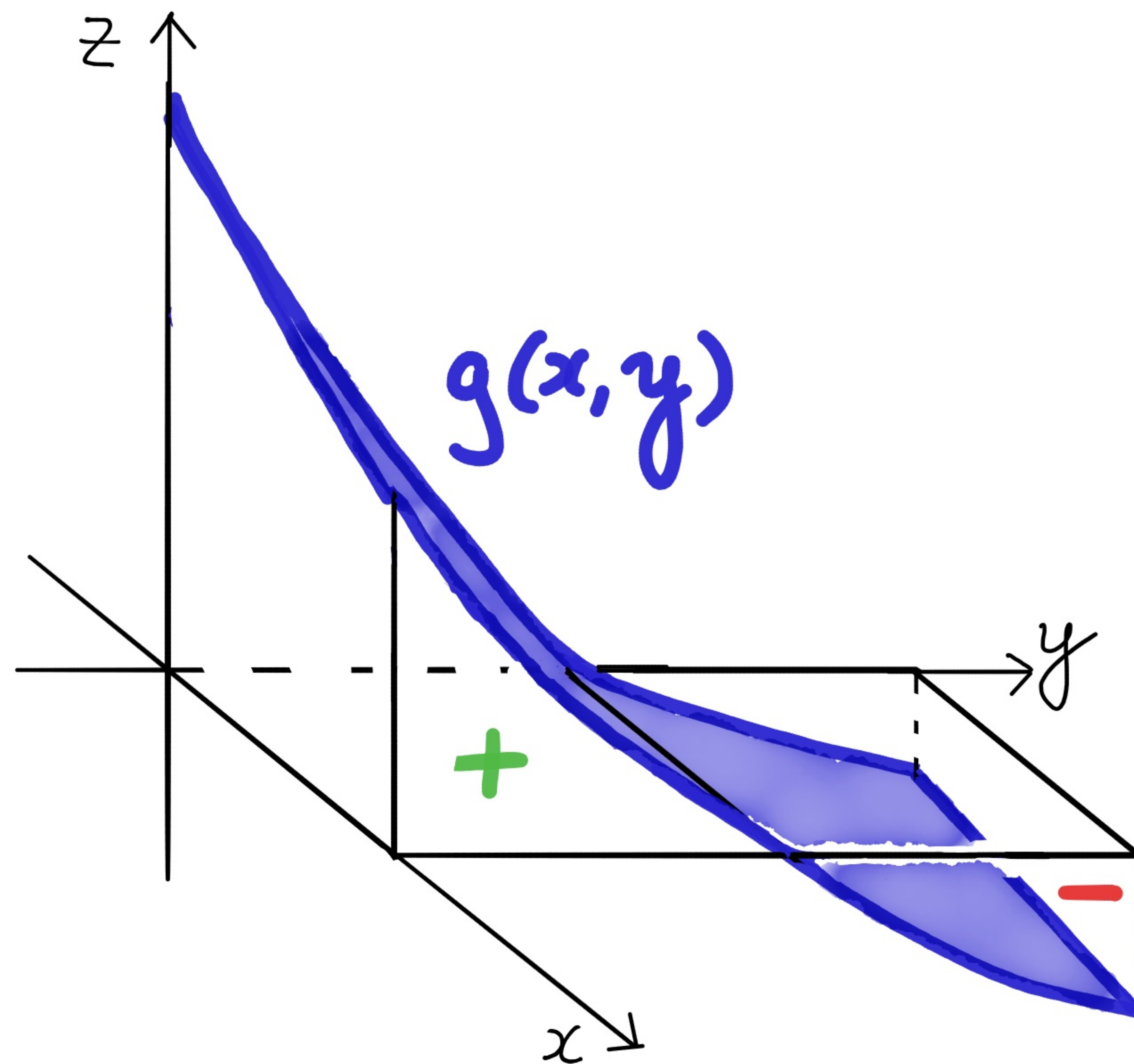
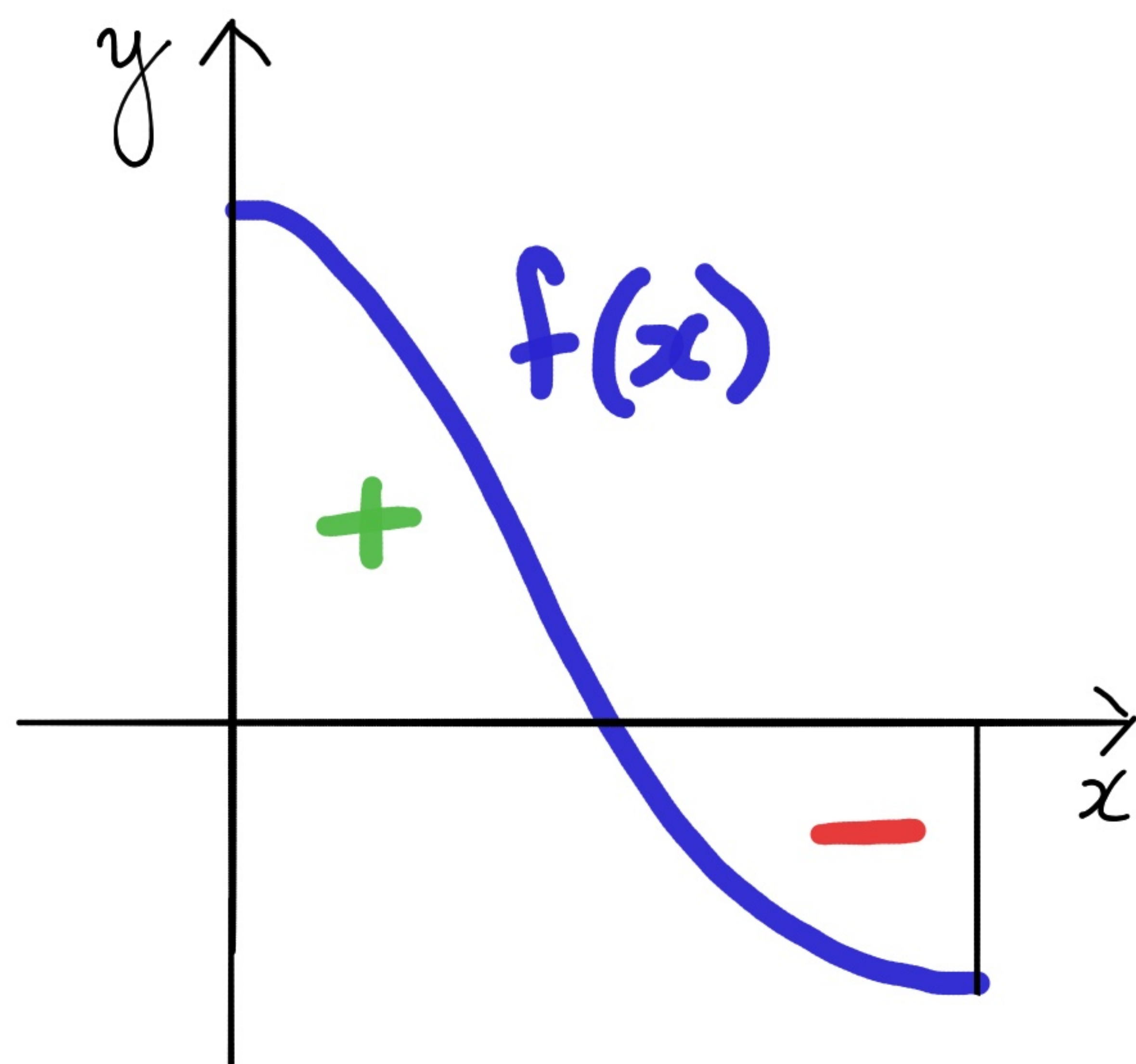
$$\iint_R \sin y \, dA = \left(\int_0^\pi \sin y \, dy \right) \cdot 2$$

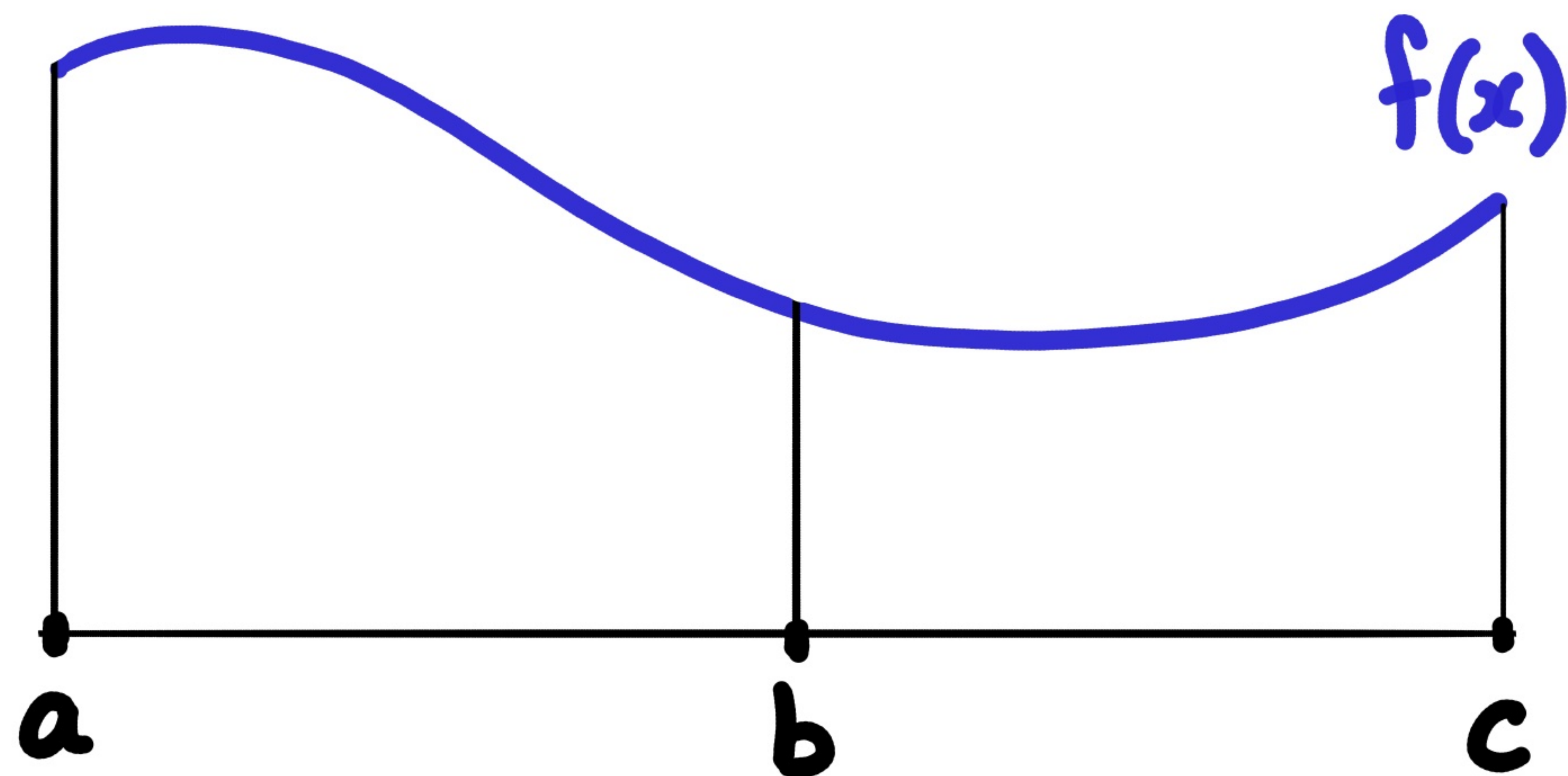
③



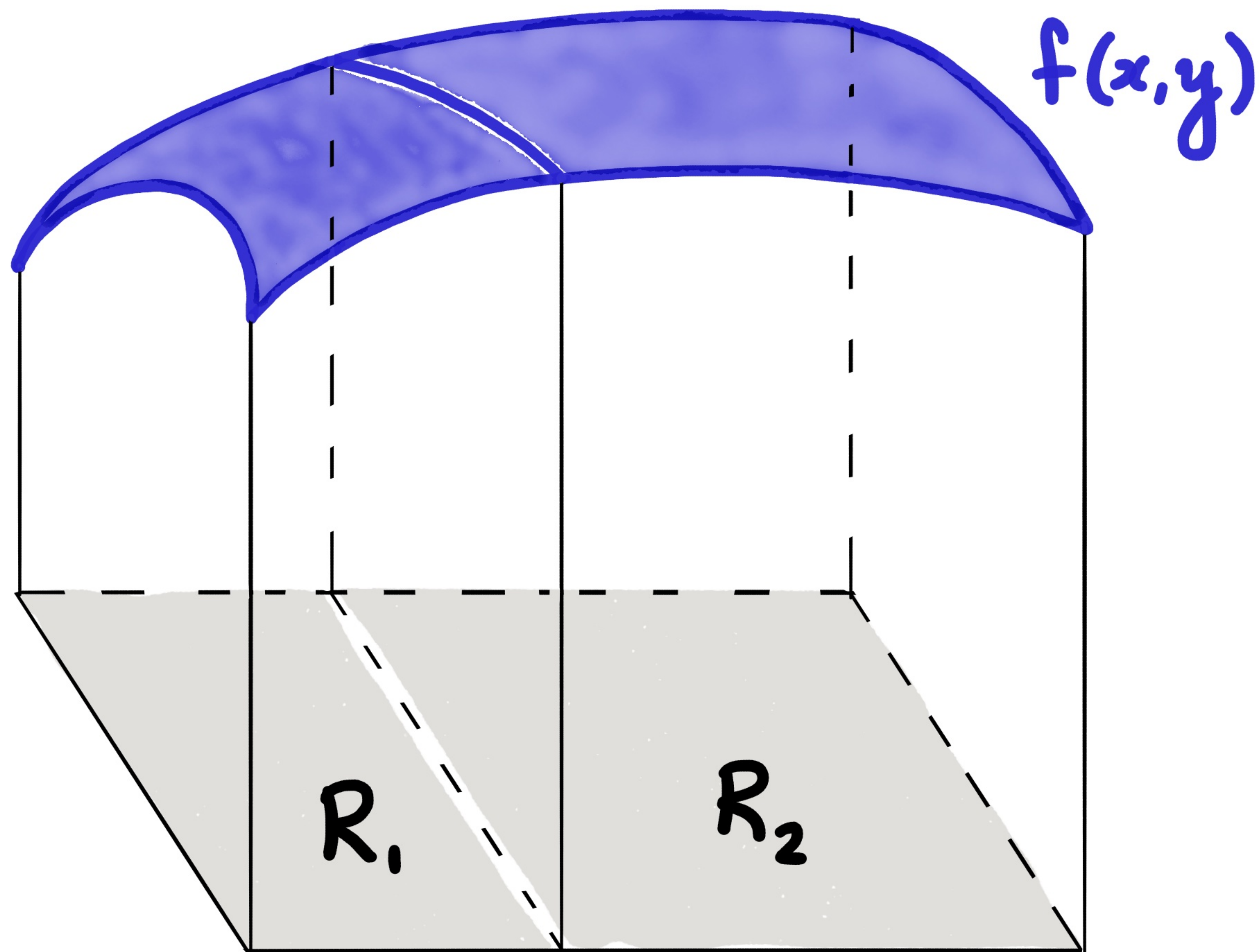
$$\iint_R \sin y \, dA = \left(\int_0^\pi \sin y \, dy \right) \cdot 2 = 2 \cdot 2 = 4$$

③ Rule for integrals

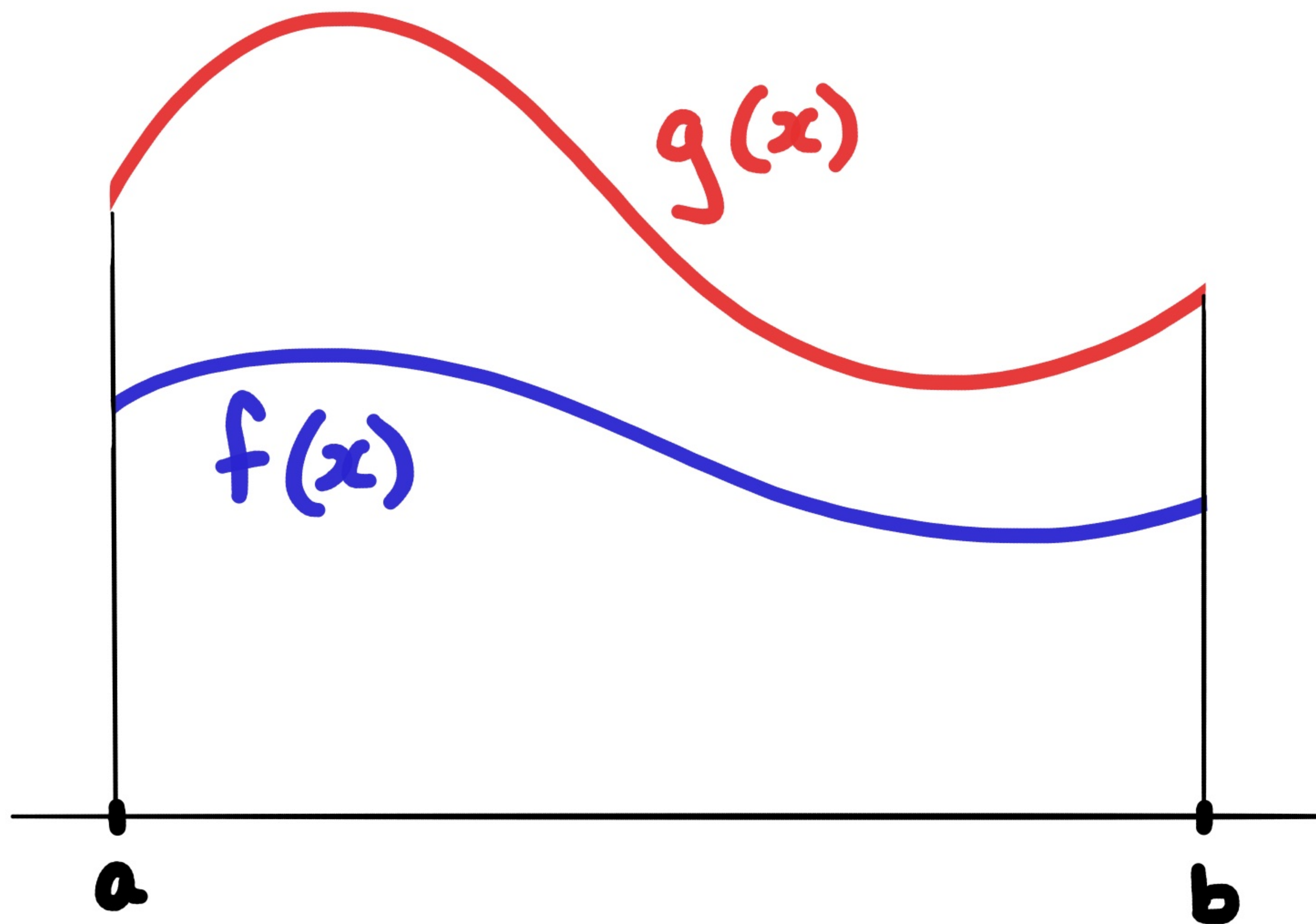




$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

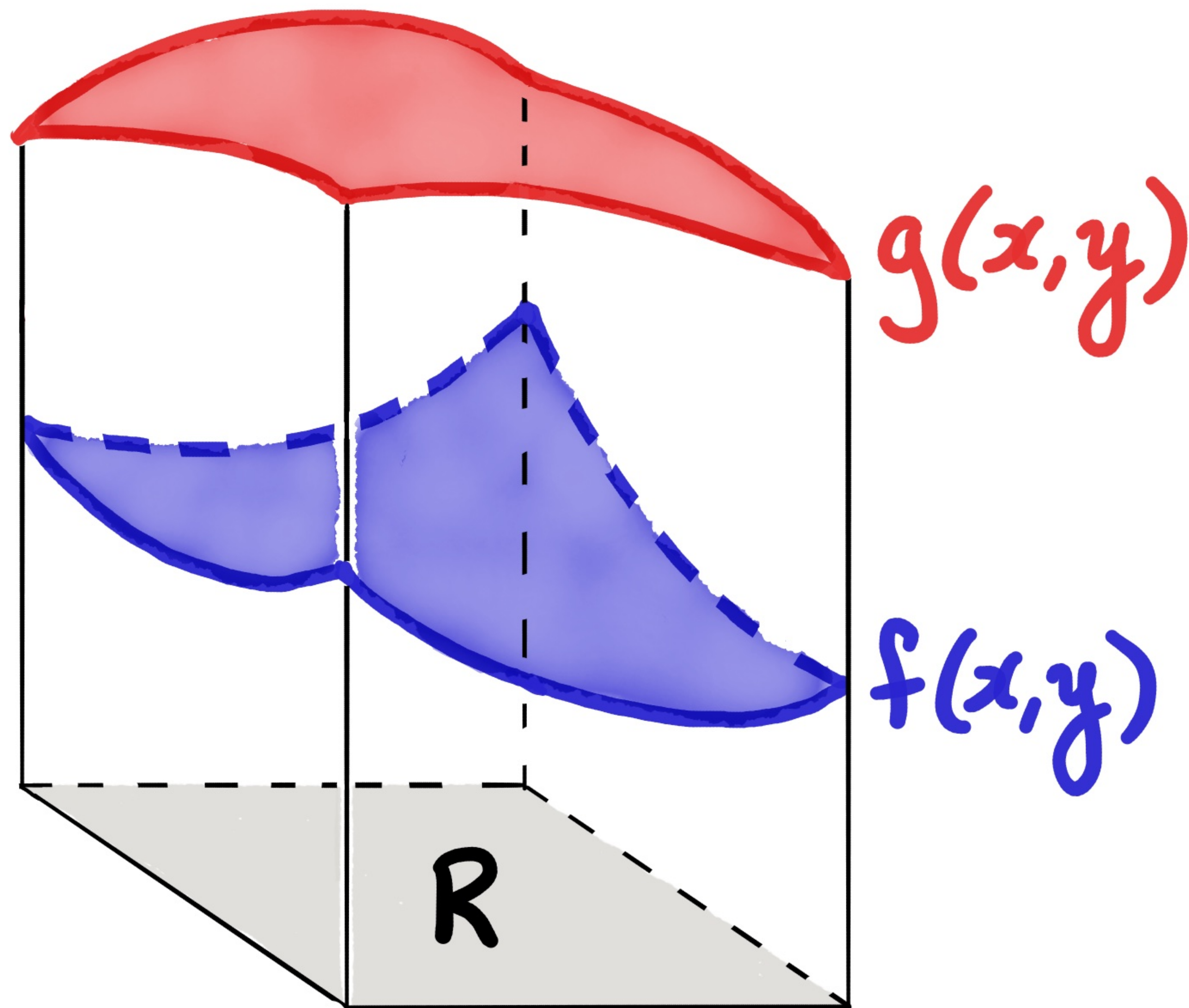


$$\iint_{R_1} f(x,y) dA + \iint_{R_2} f(x,y) dA = \iint_{R_1 \text{ and } R_2} f(x,y) dA$$



If $f(x) \leq g(x)$,

$$\text{then } \int_a^b f(x) dx \leq \int_a^b g(x) dx.$$



If $f(x, y) \leq g(x, y)$,

$$\text{then } \iint_R f(x, y) dA \leq \iint_R g(x, y) dA.$$

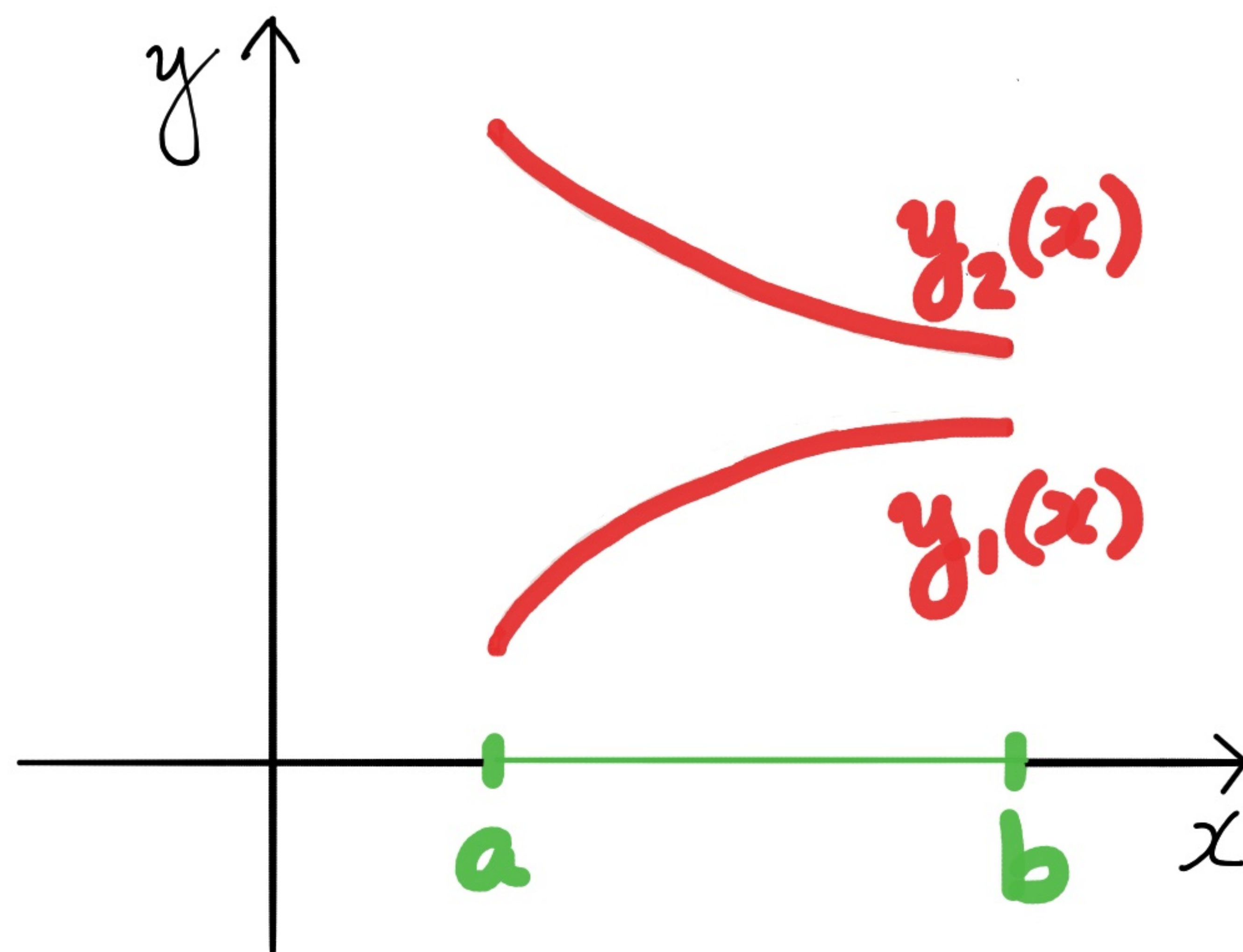
• If $c \in \mathbb{R}$,

then
$$\iint_R c f(x,y) dA = c \iint_R f(x,y) dA .$$

•
$$\iint_R (f(x,y) + g(x,y)) dA = \iint_R f(x,y) dA + \iint_R g(x,y) dA$$

④ Simple regions
in the xy -plane

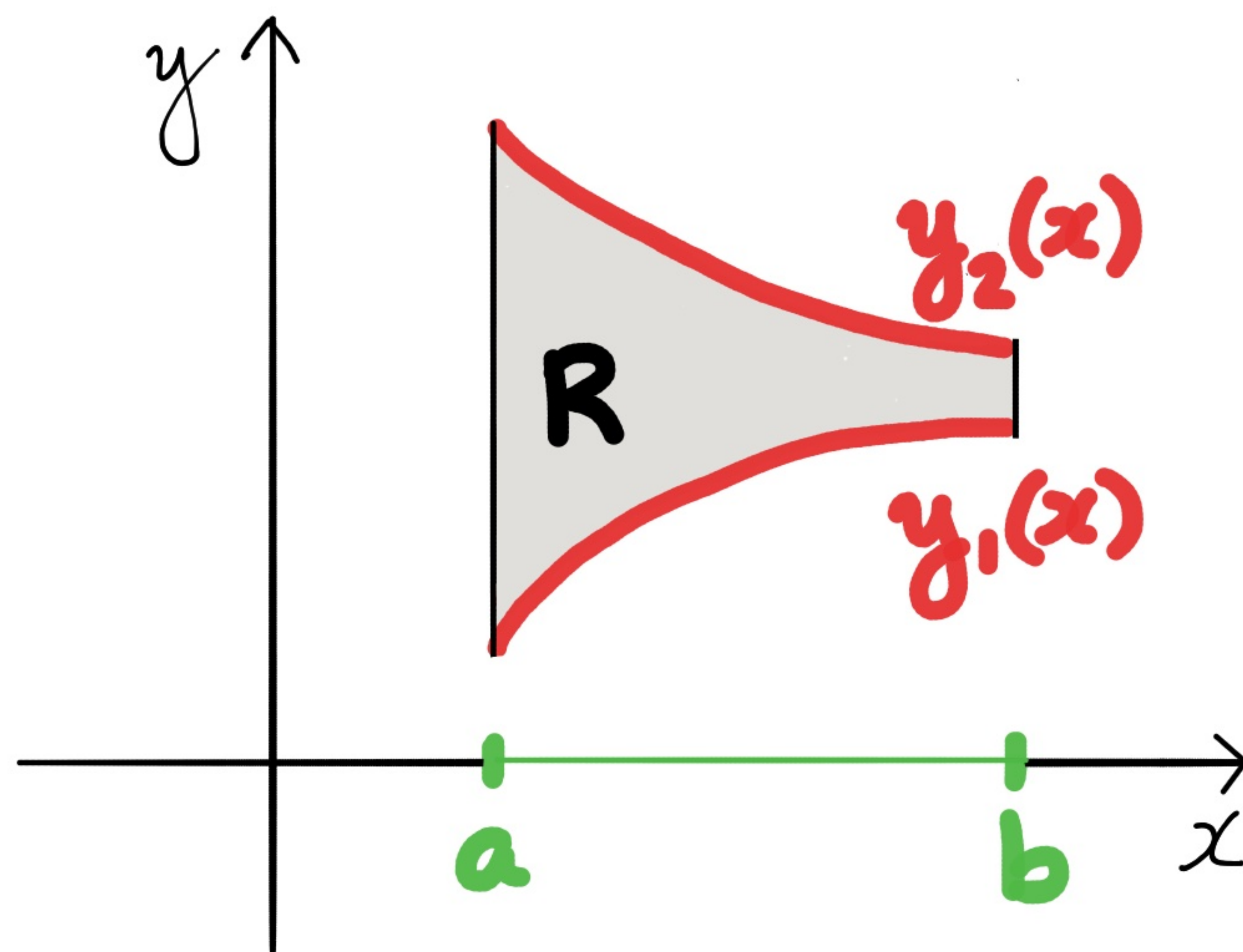
y-simple regions



$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

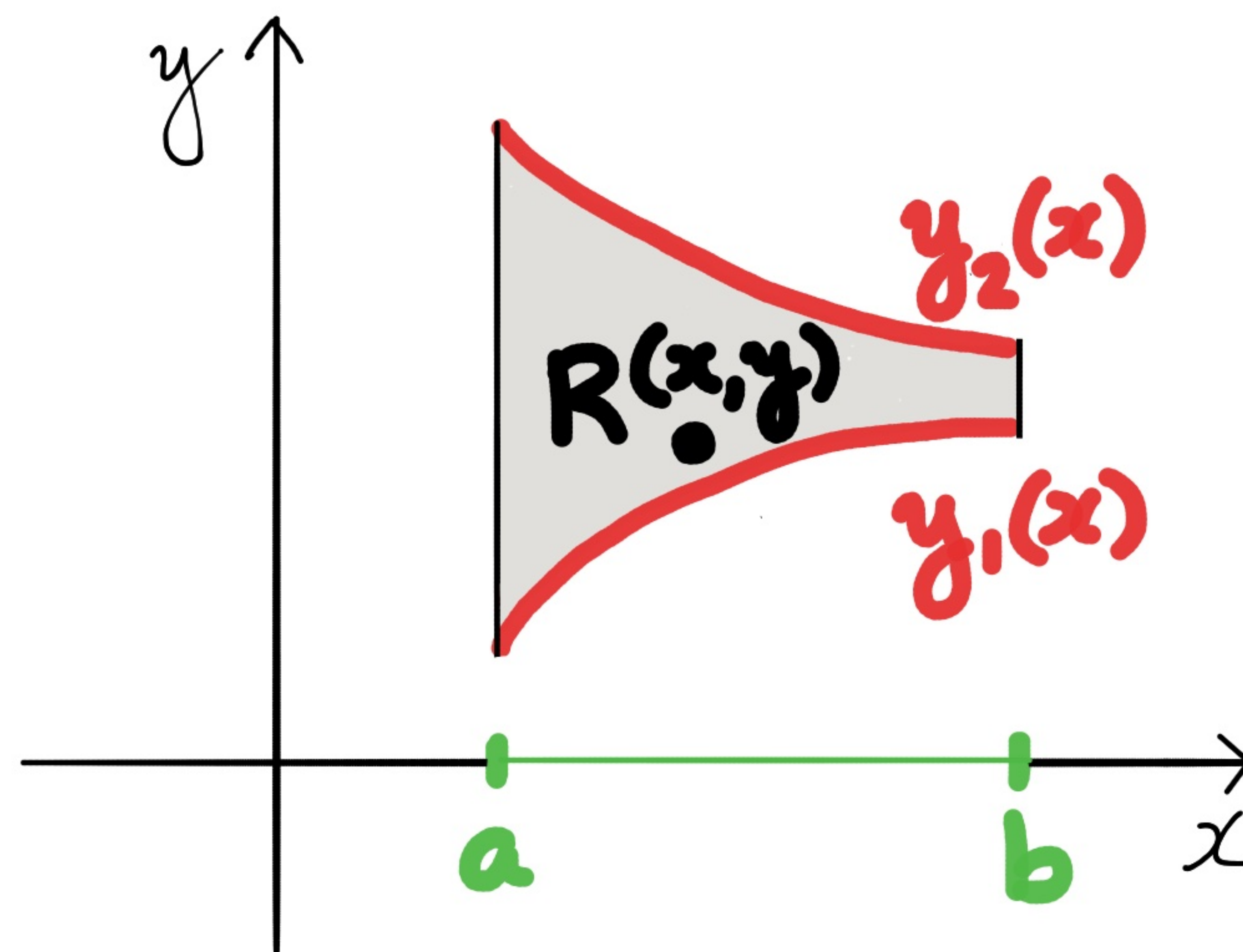
y-simple regions



$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

y-simple regions

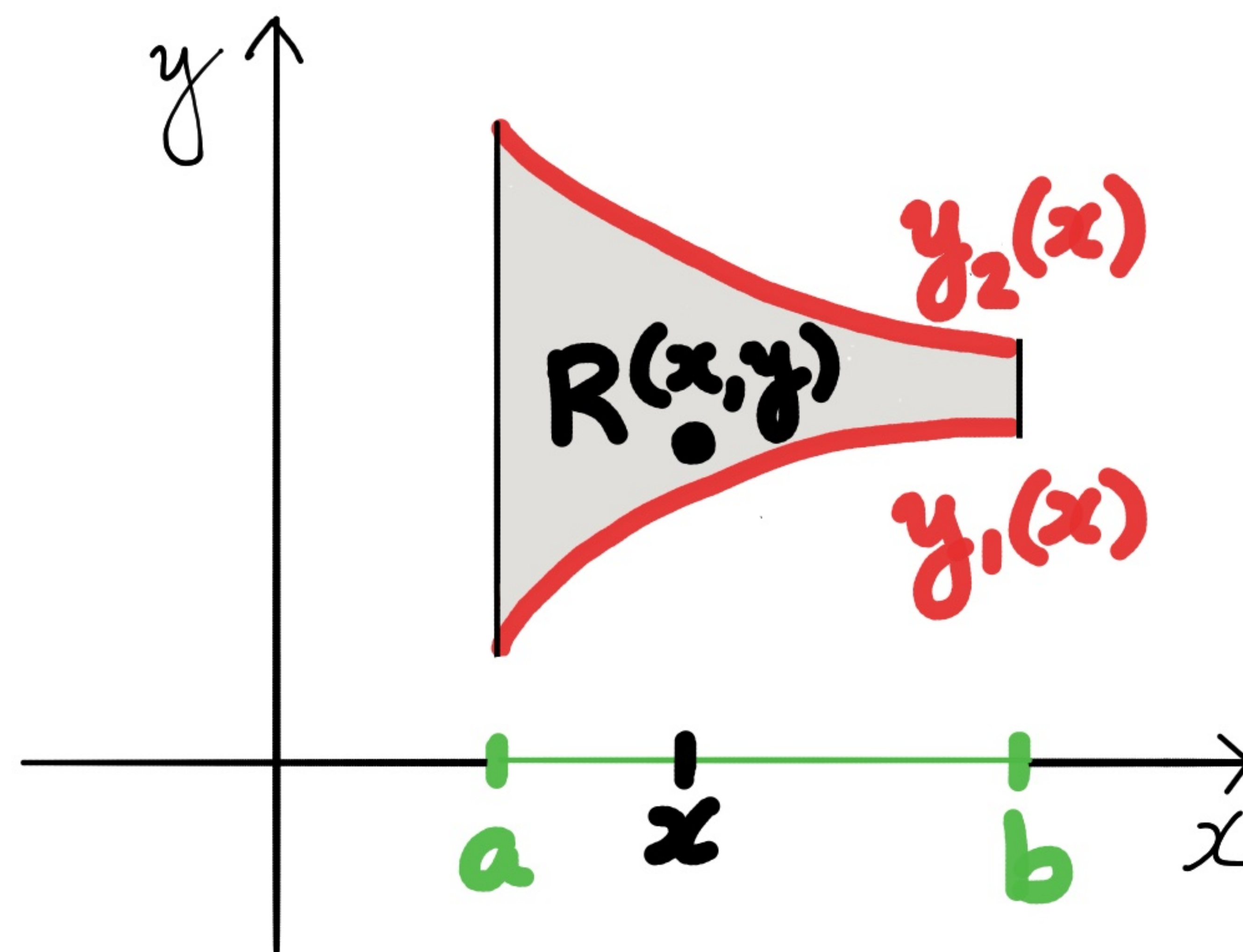


$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$R = \{(x,y):$$

y-simple regions

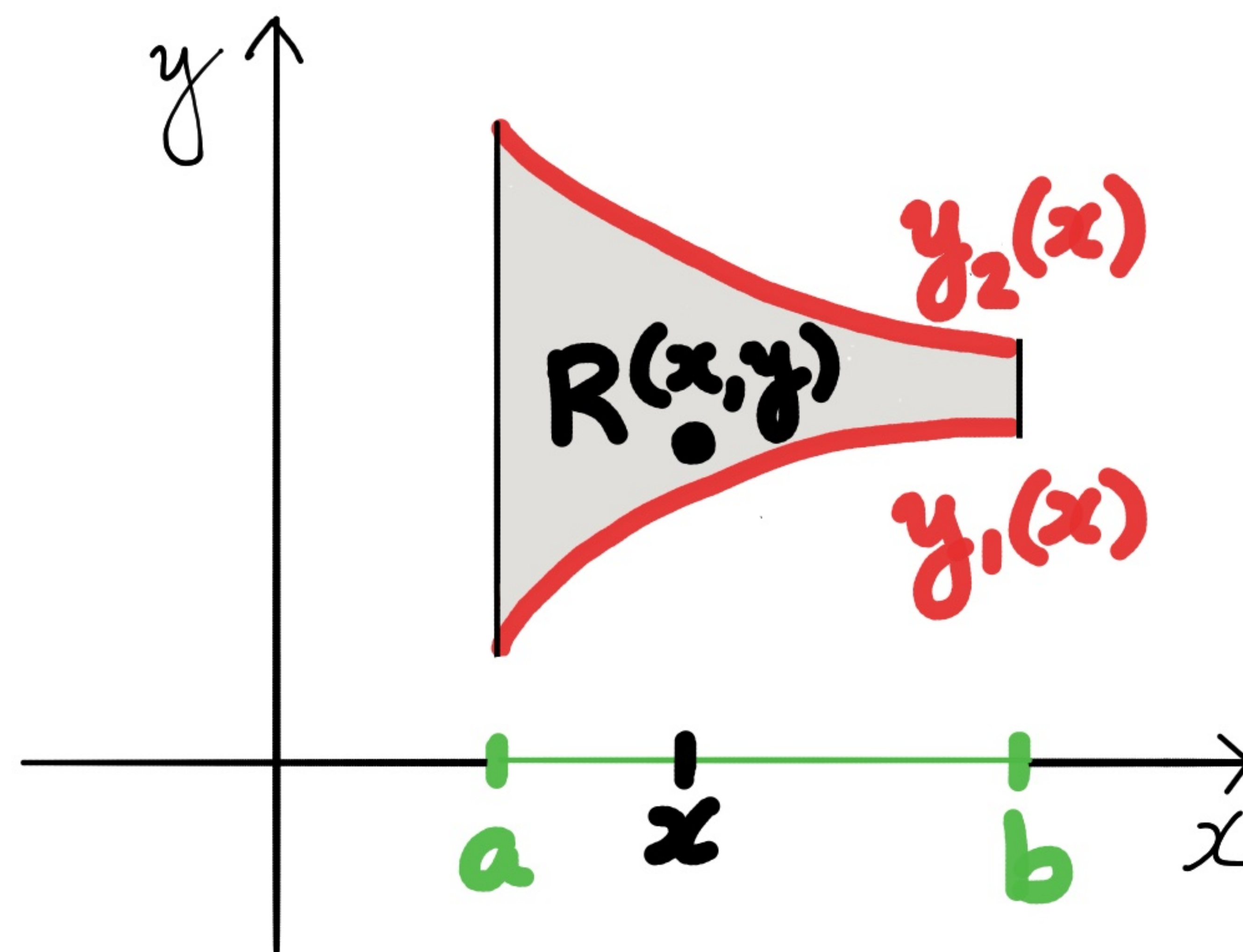


$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$R = \{(x, y) : a \leq x \leq b,$$

y-simple regions

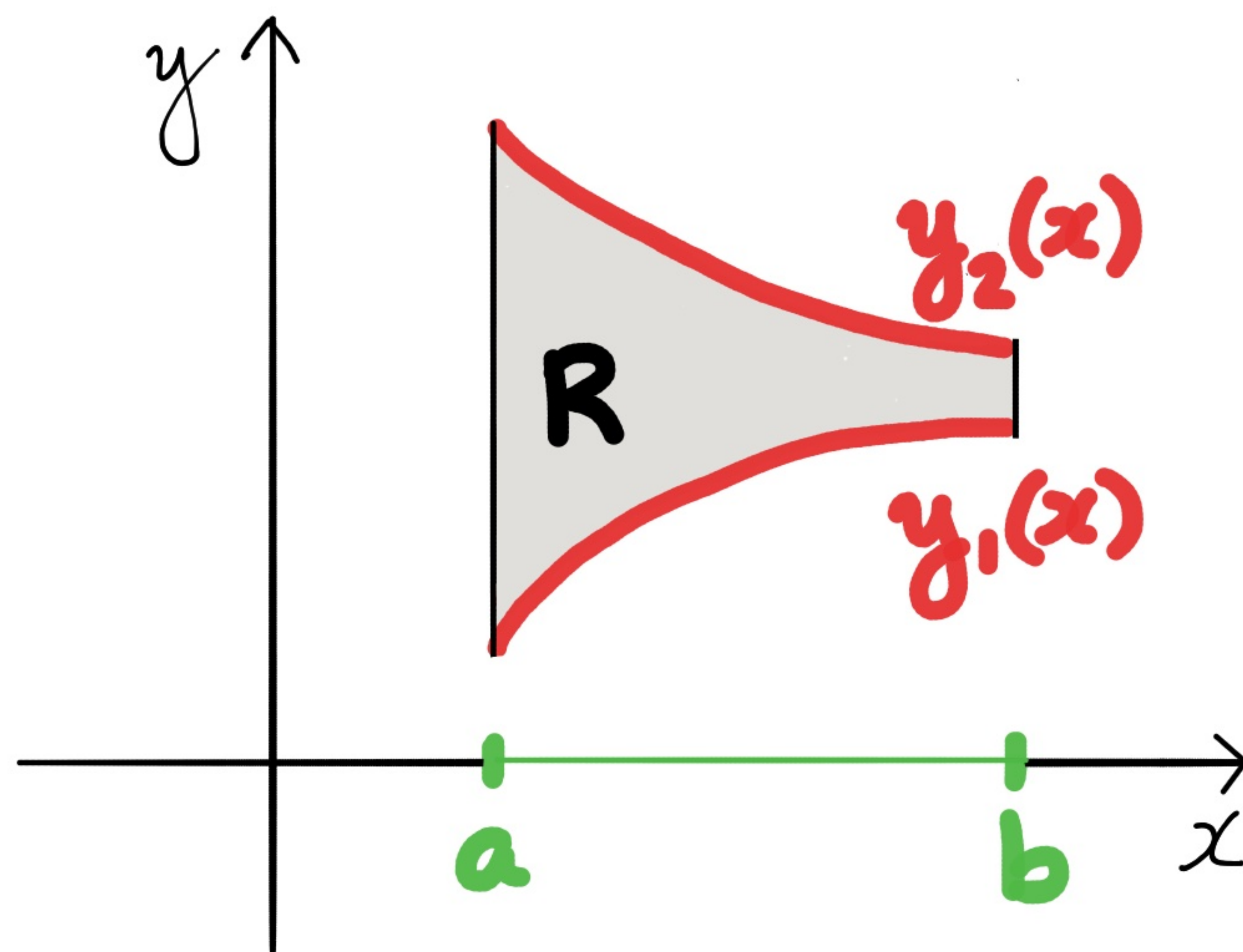


$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$R = \{(x, y) : a \leq x \leq b, y_1(x) \leq y \leq y_2(x)\}$$

y-simple regions

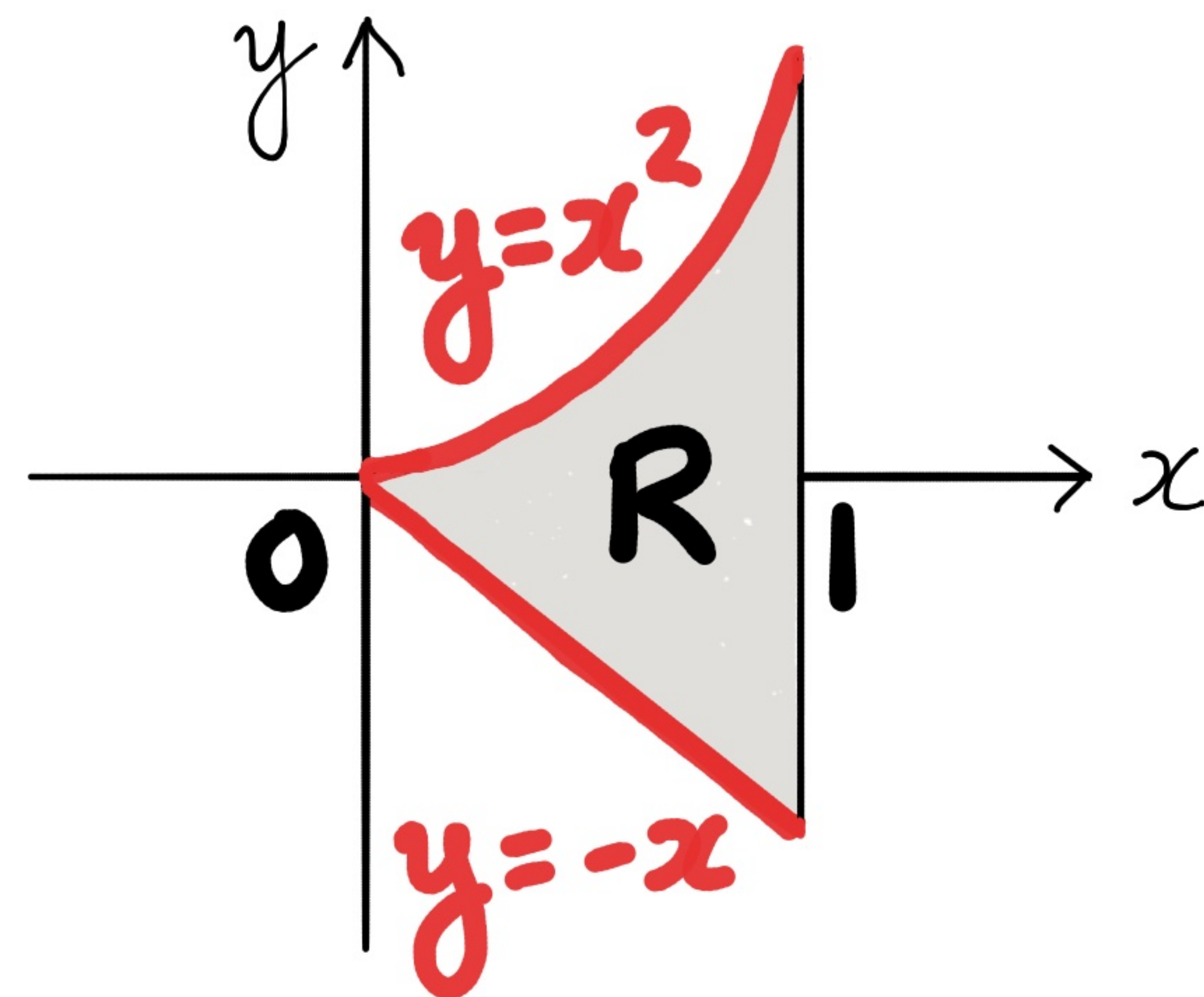


$$y_1: \mathbb{R} \rightarrow \mathbb{R}$$
$$y_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$R = \{(x, y) : a \leq x \leq b, y_1(x) \leq y \leq y_2(x)\}$$

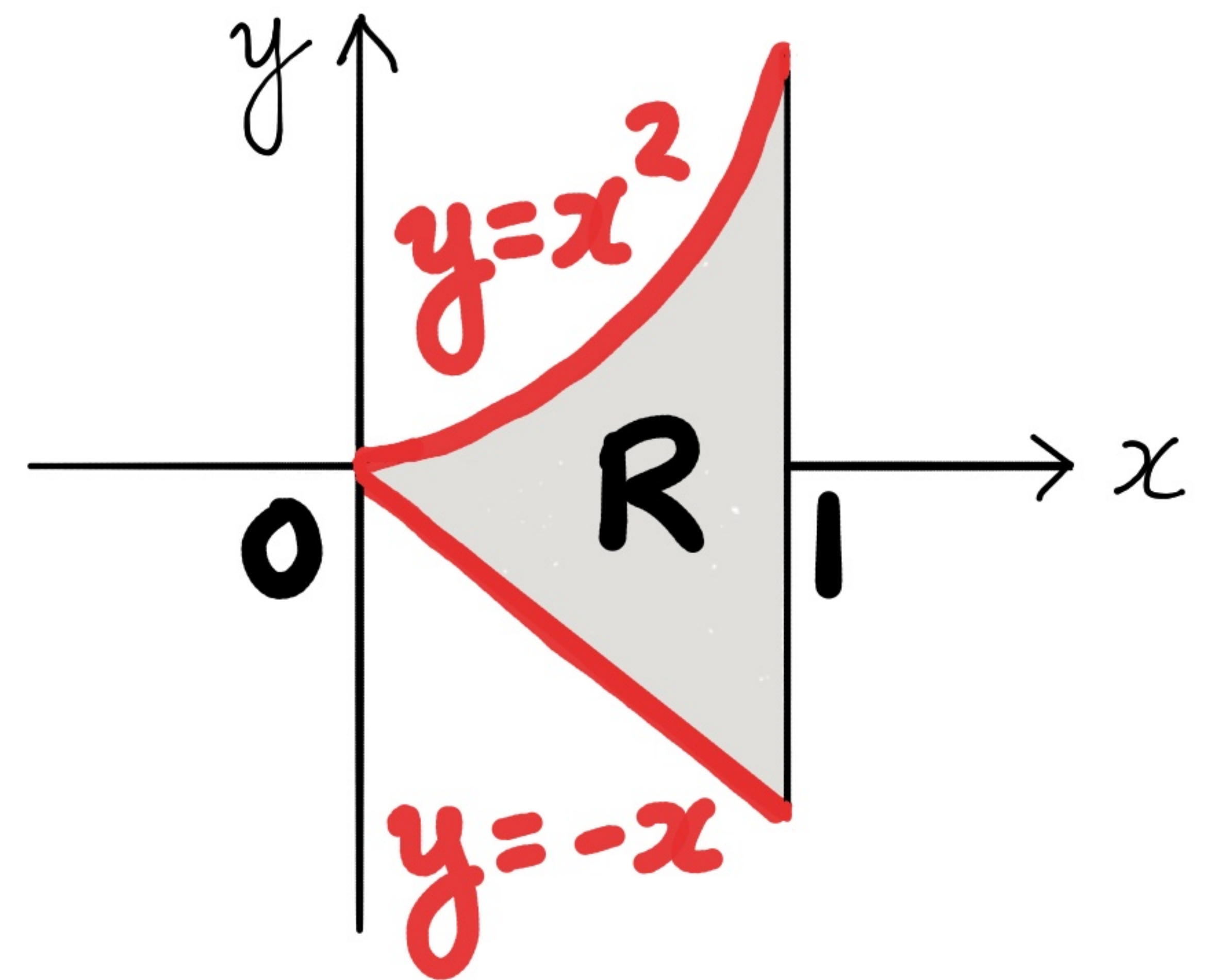
Problem:

Write the region R
as a y -simple set.



Problem:

Write the region R as a y -simple set.

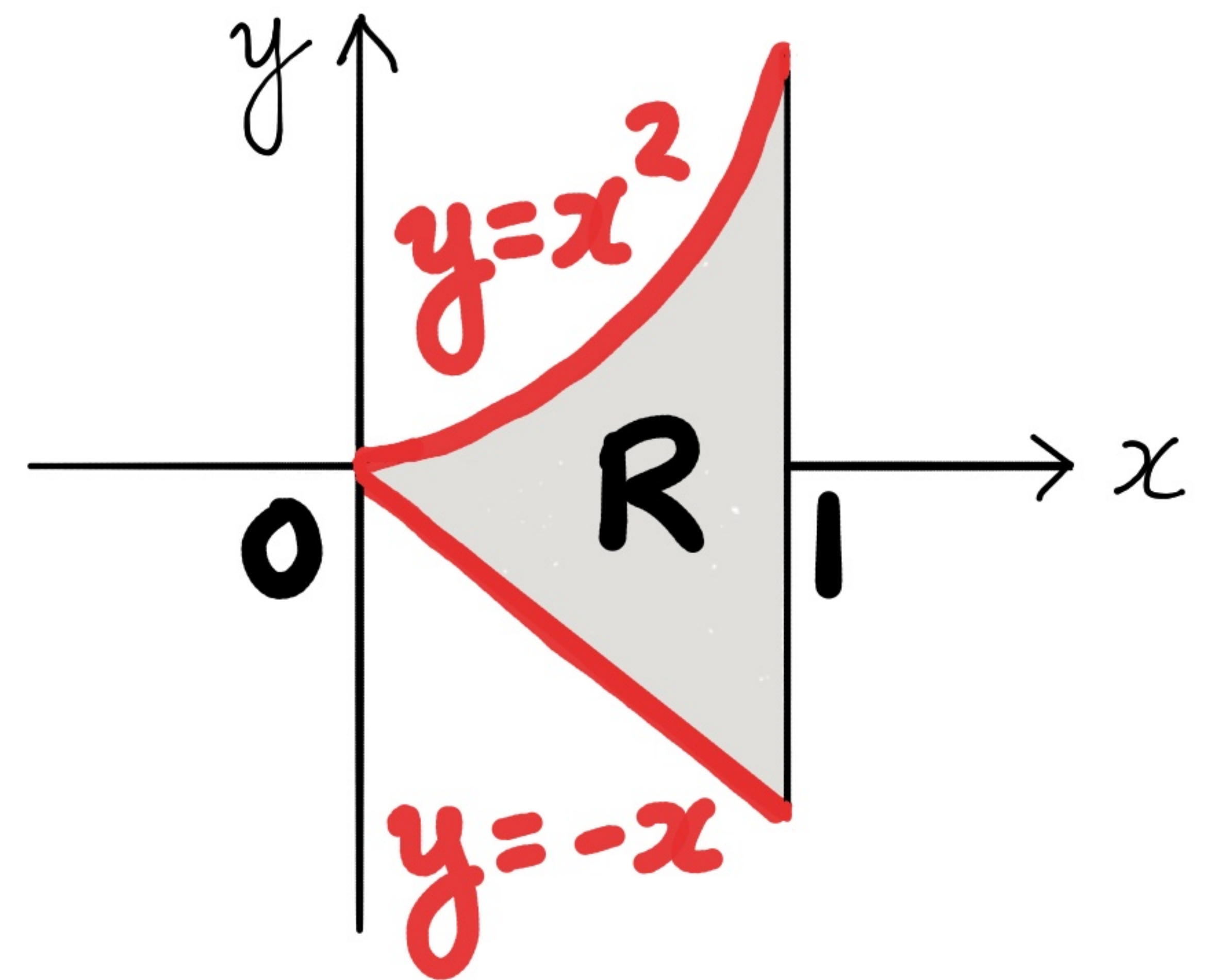


Answer:

$$R = \{(x, y) : \quad \leq x \leq \quad ,$$

Problem:

Write the region R as a y -simple set.

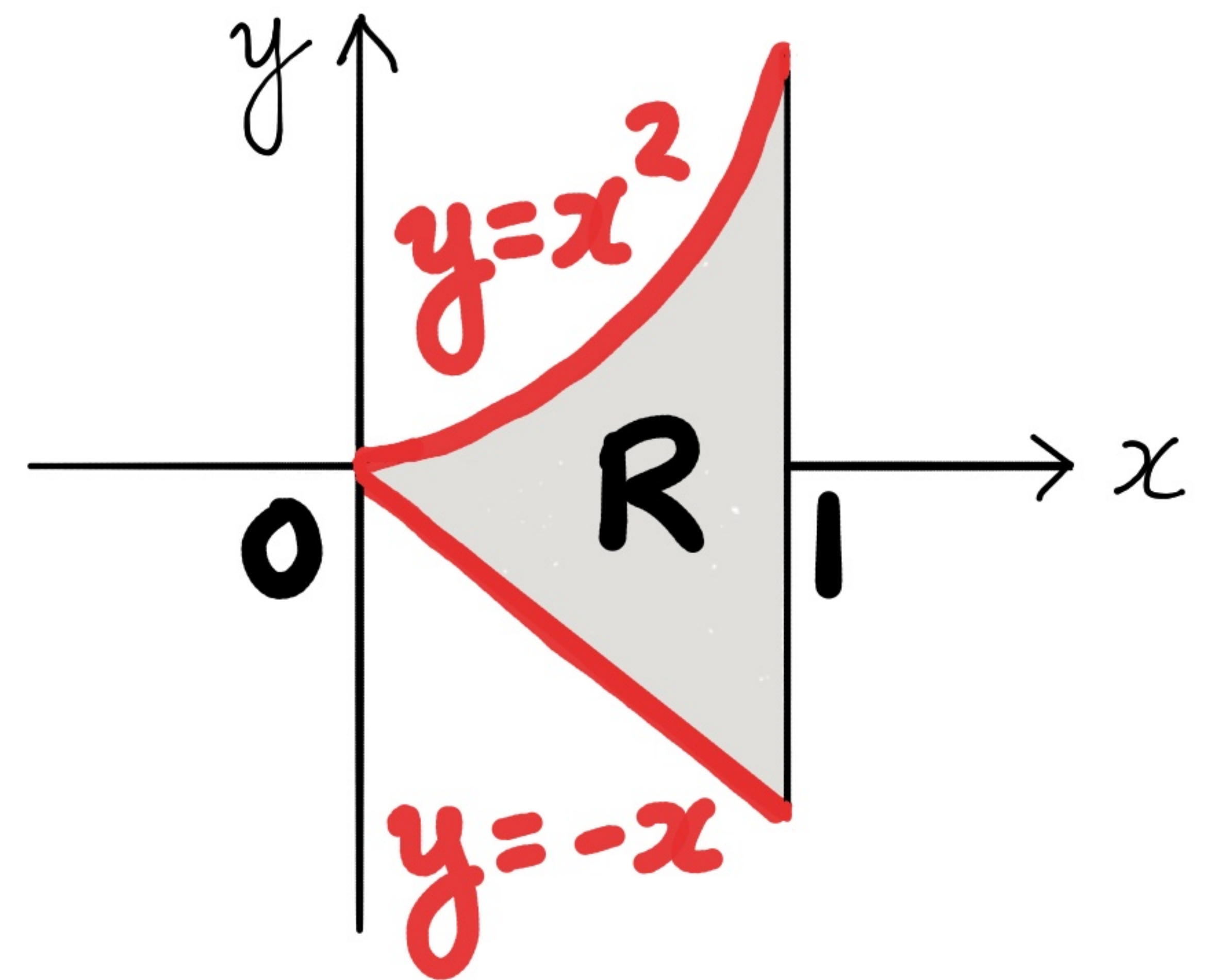


Answer:

$$R = \{(x, y) : 0 \leq x \leq 1, \quad -x \leq y \leq x^2\}$$

Problem:

Write the region R as a y -simple set.

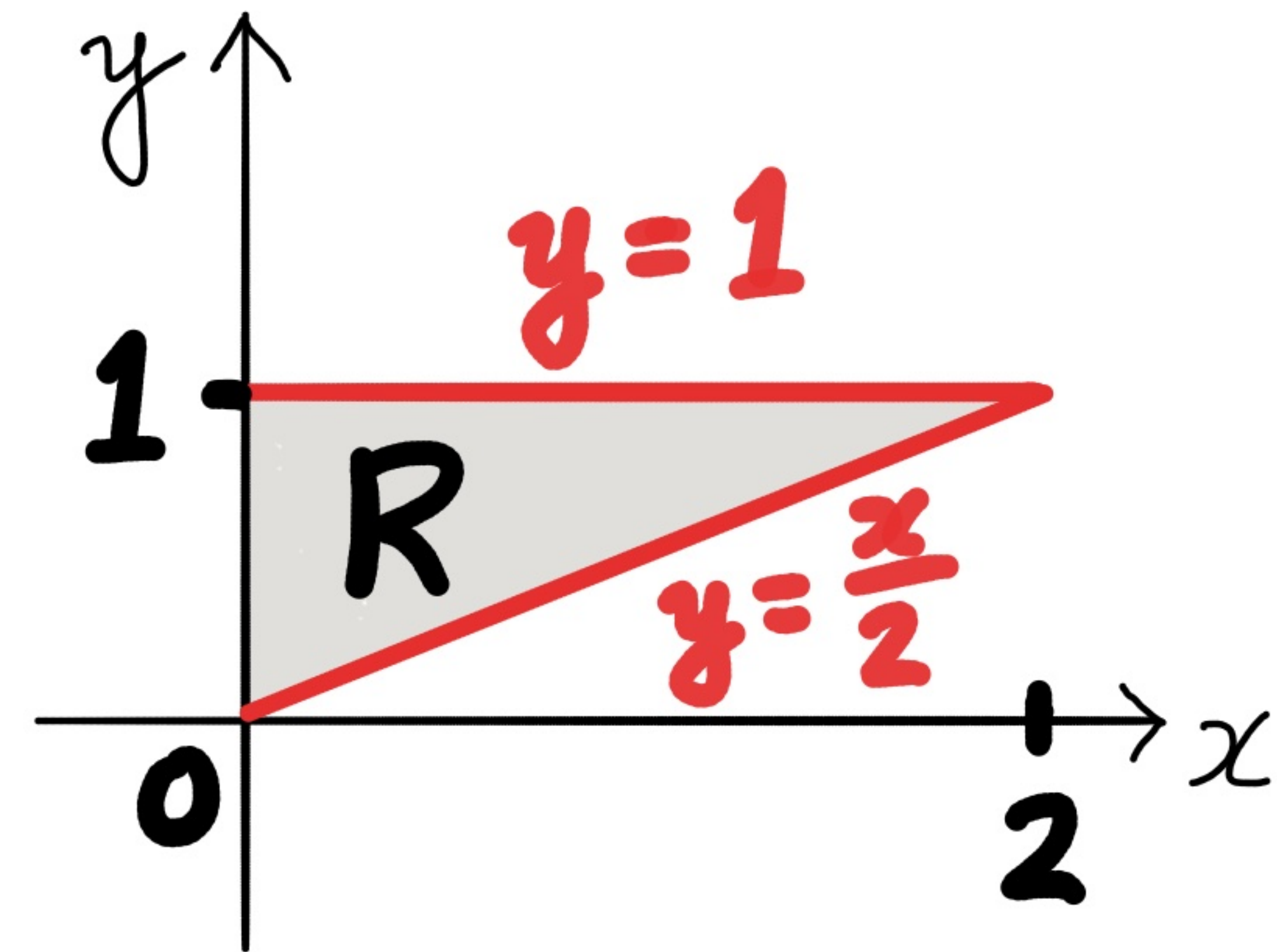


Answer:

$$R = \{(x, y) : 0 \leq x \leq 1, -x \leq y \leq x^2\}$$

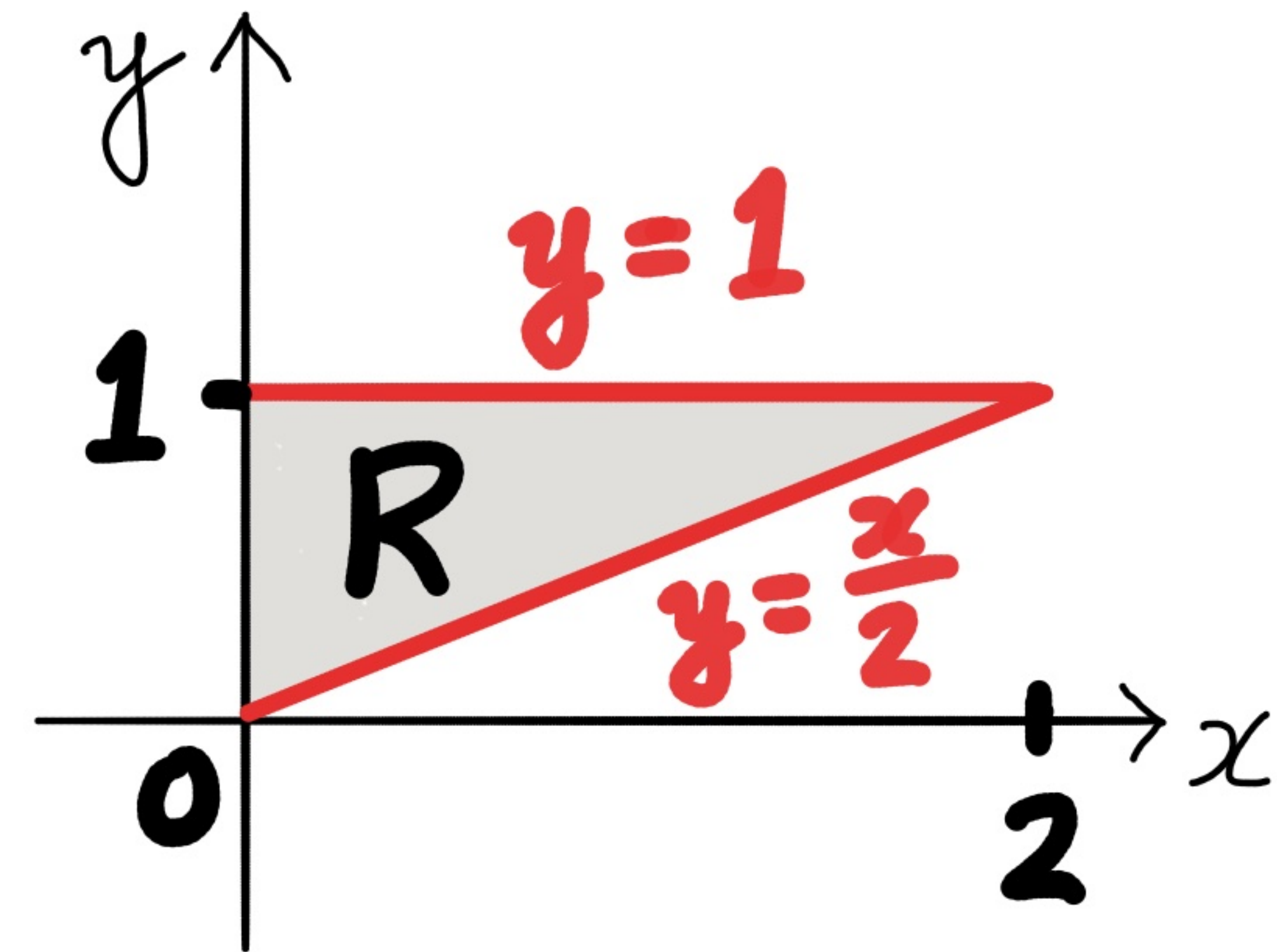
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Problem:

Write the region R as a y -simple set.

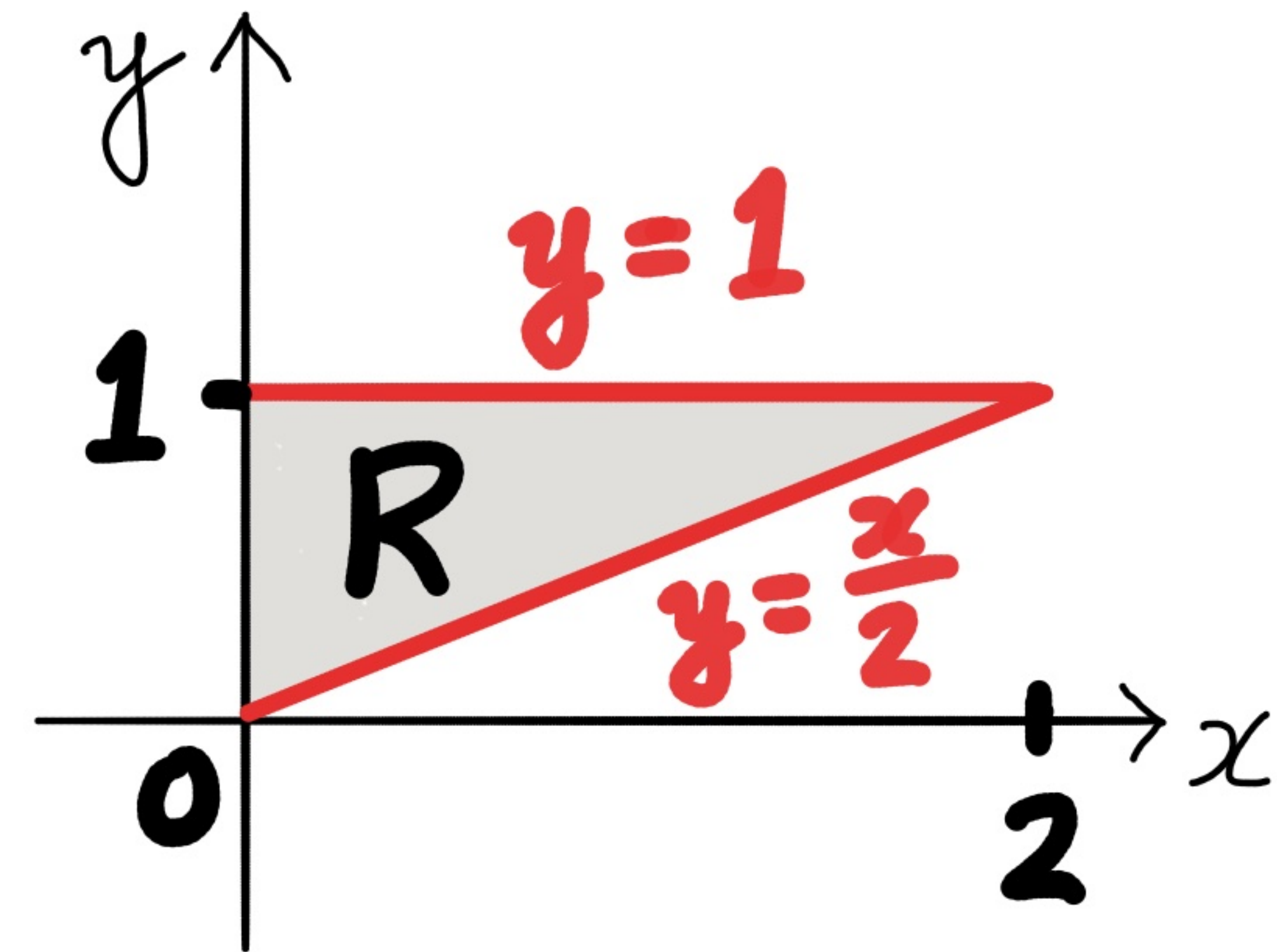


Answer:

$$R = \{(x, y) : 0 \leq x \leq 2, \quad y = \frac{x}{2} \leq y \leq 1\}$$

Problem:

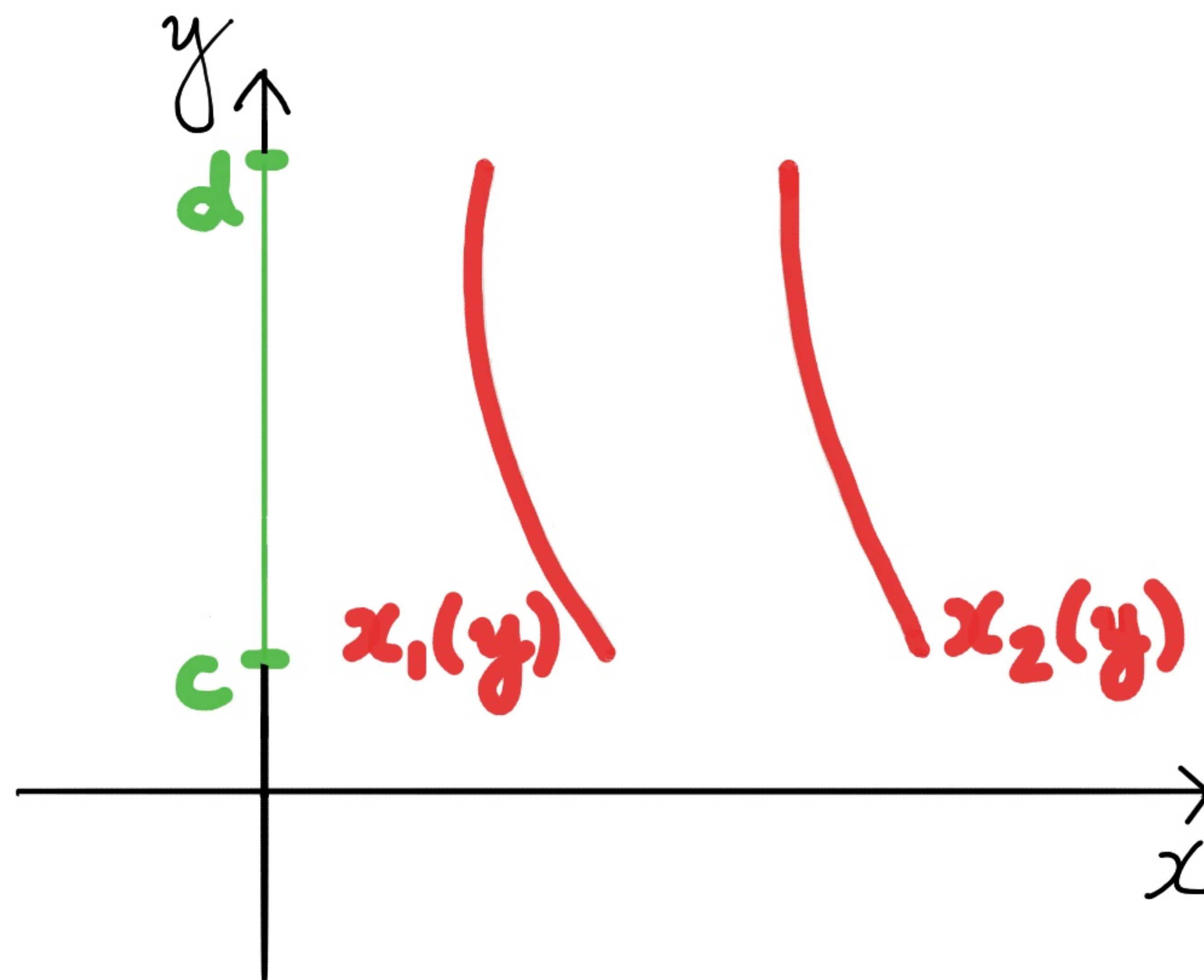
Write the region R as a y -simple set.



Answer:

$$R = \{(x, y) : 0 \leq x \leq 2, \frac{x}{2} \leq y \leq 1\}$$

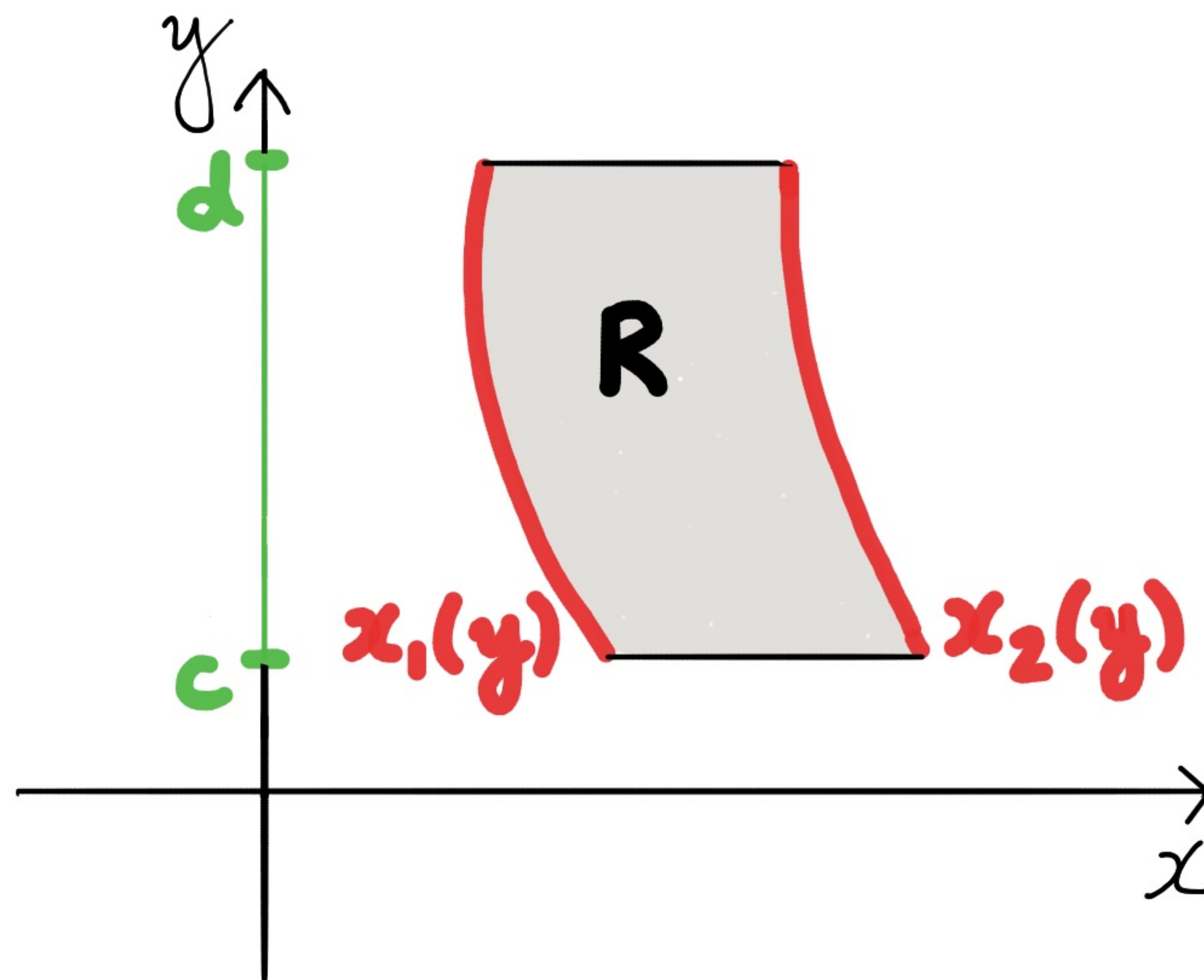
x -simple regions



$$x_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$x_2: \mathbb{R} \rightarrow \mathbb{R}$$

x -simple regions



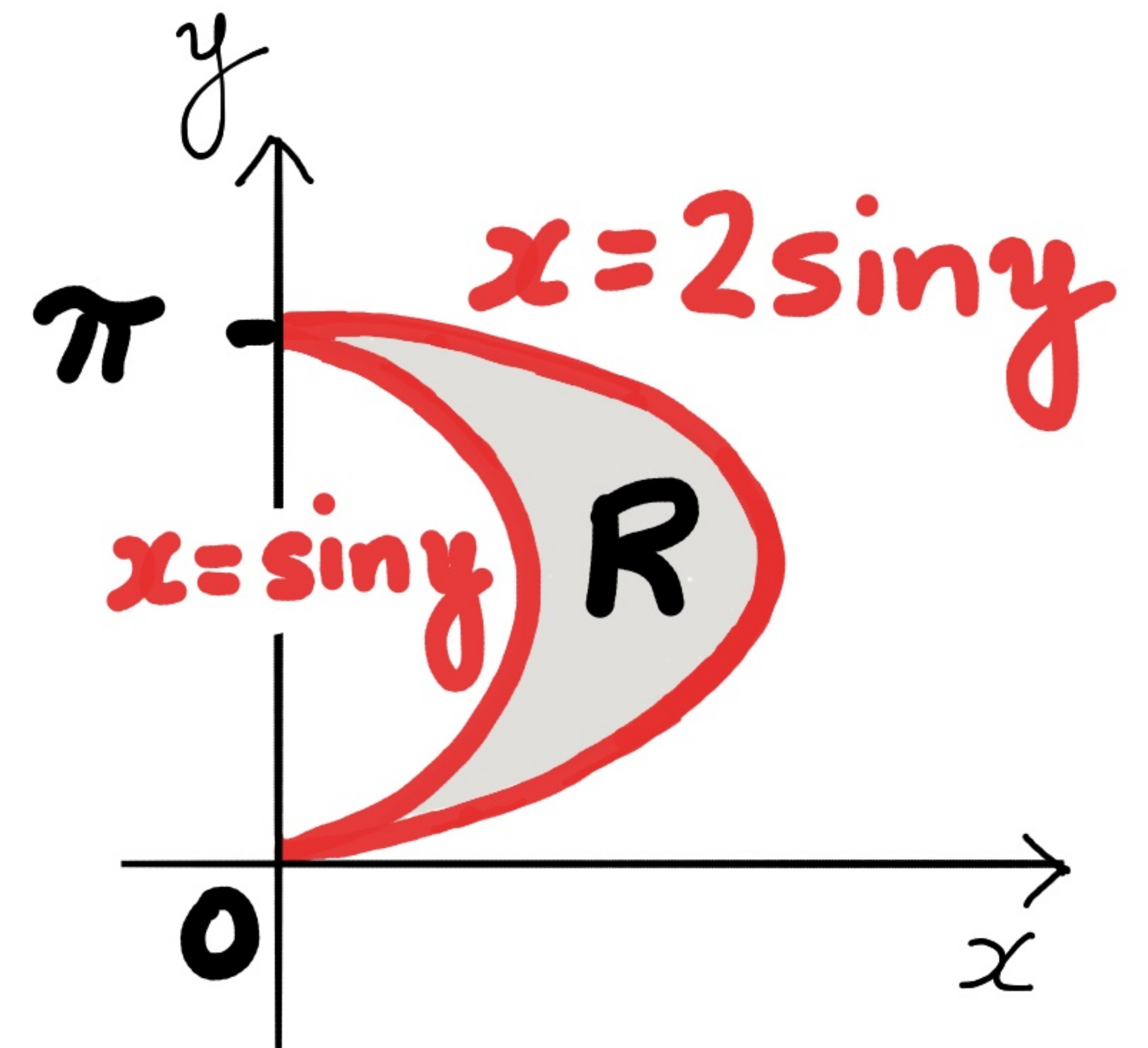
$$x_1: \mathbb{R} \rightarrow \mathbb{R}$$

$$x_2: \mathbb{R} \rightarrow \mathbb{R}$$

$$R = \{(x, y) : c \leq y \leq d, x_1(y) \leq x \leq x_2(y)\}$$

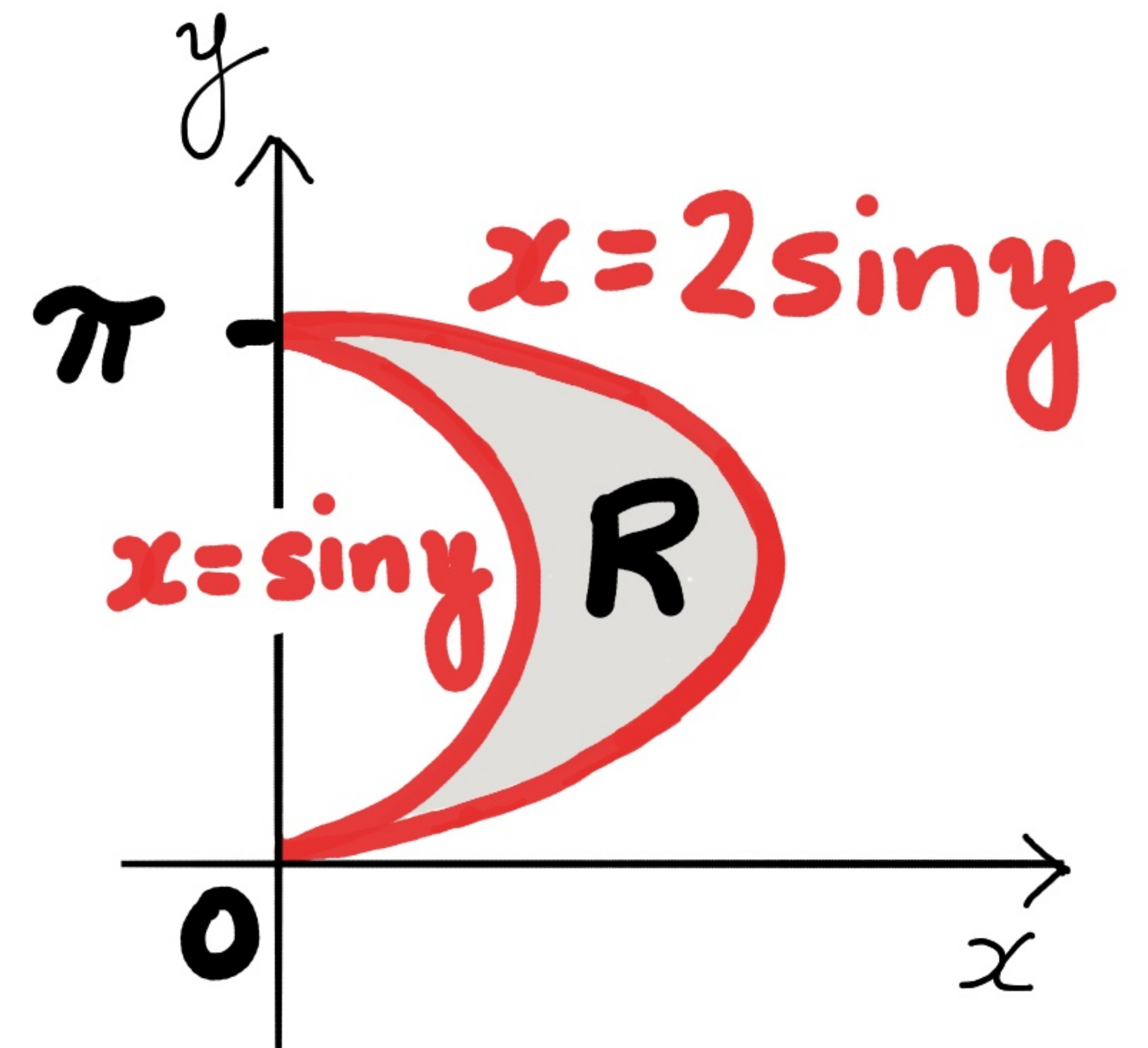
Problem:

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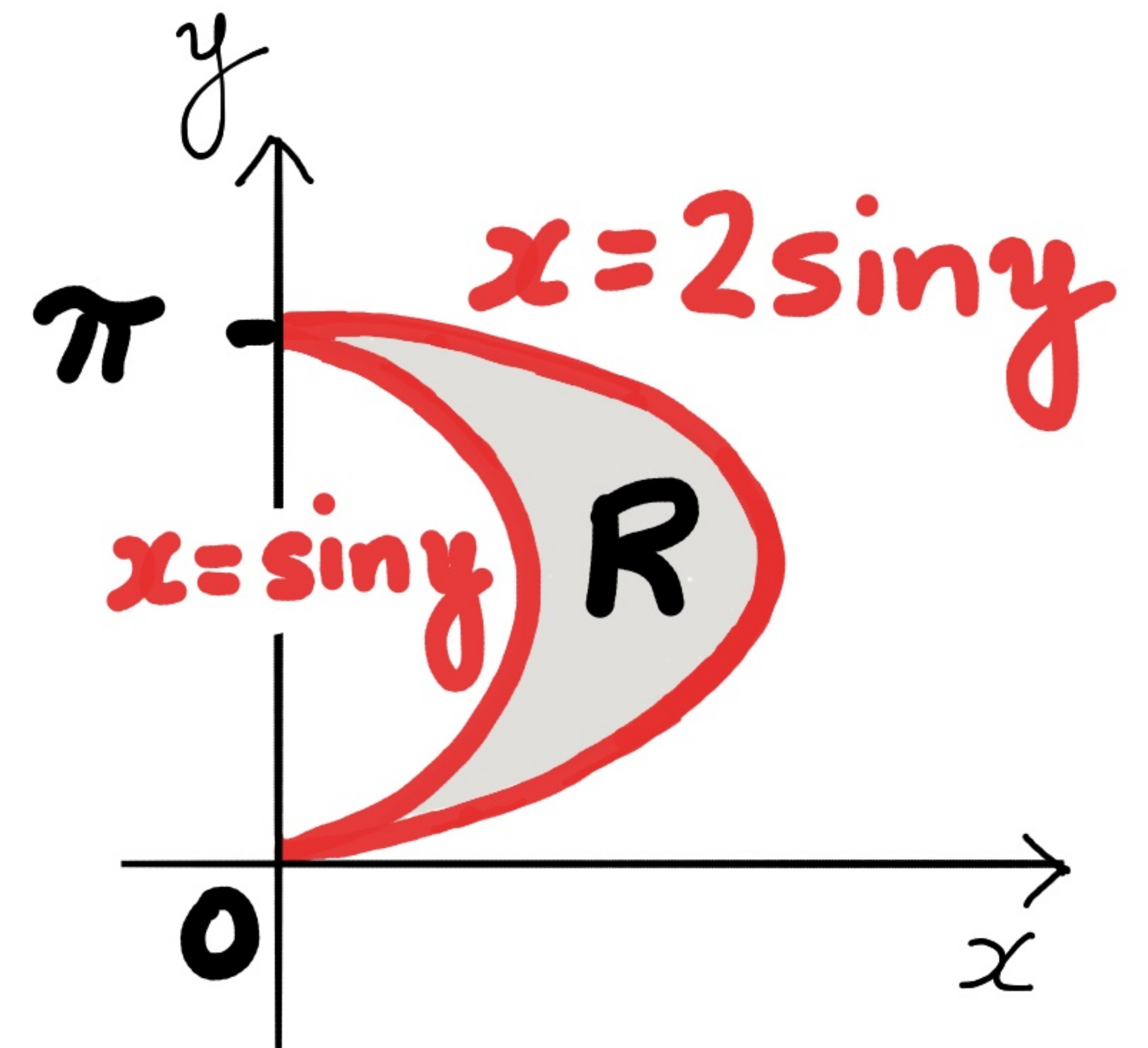


Answer:

$$R = \{(x, y) : \sin y \leq x \leq 2\sin y, 0 \leq y \leq \pi\}$$

Problem:

Write the region R
as an x -simple set.

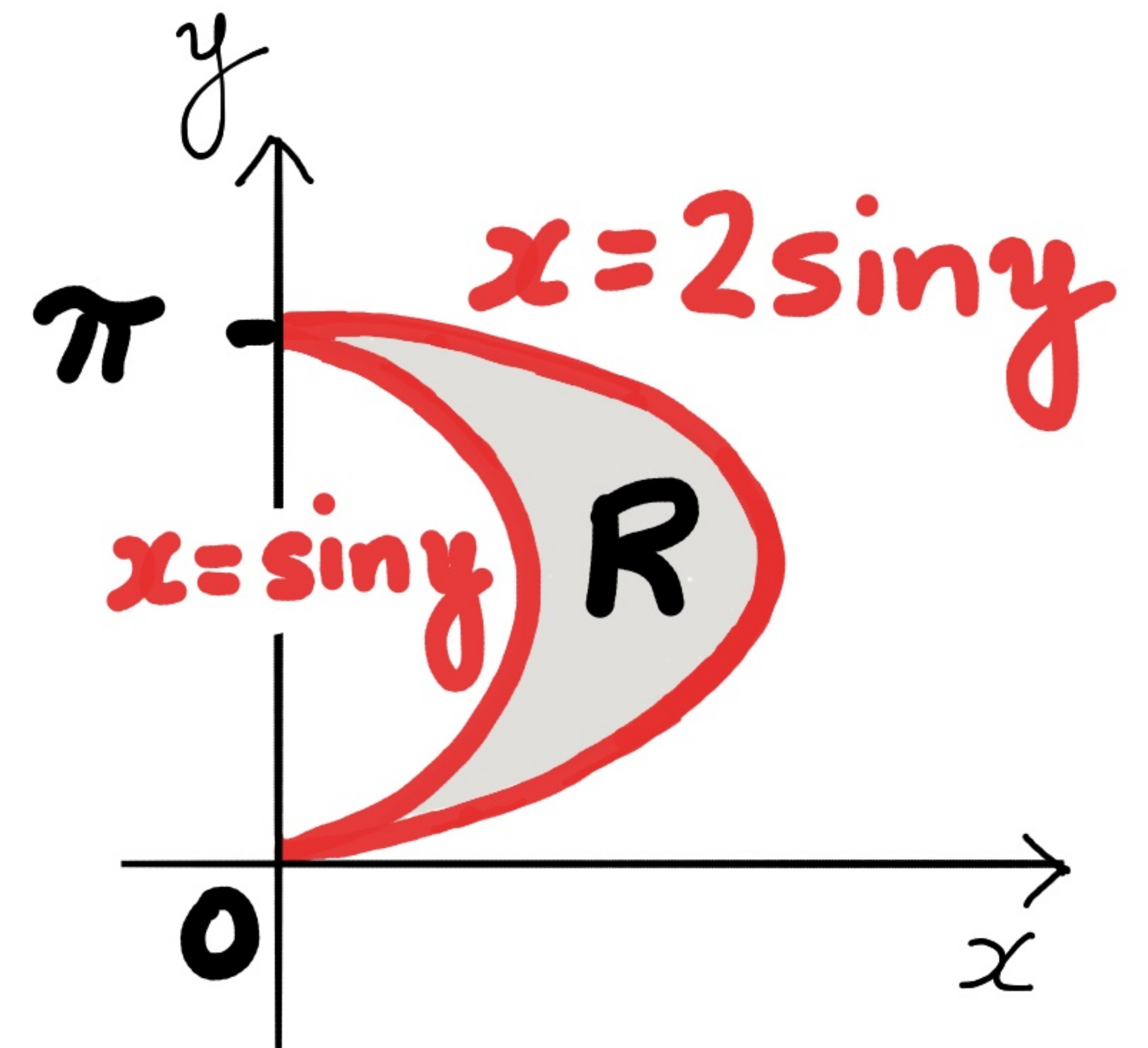


Answer:

$$R = \{(x, y) : 0 \leq y \leq \pi, \sin y \leq x \leq 2\sin y\}$$

Problem:

Write the region R as an x -simple set.

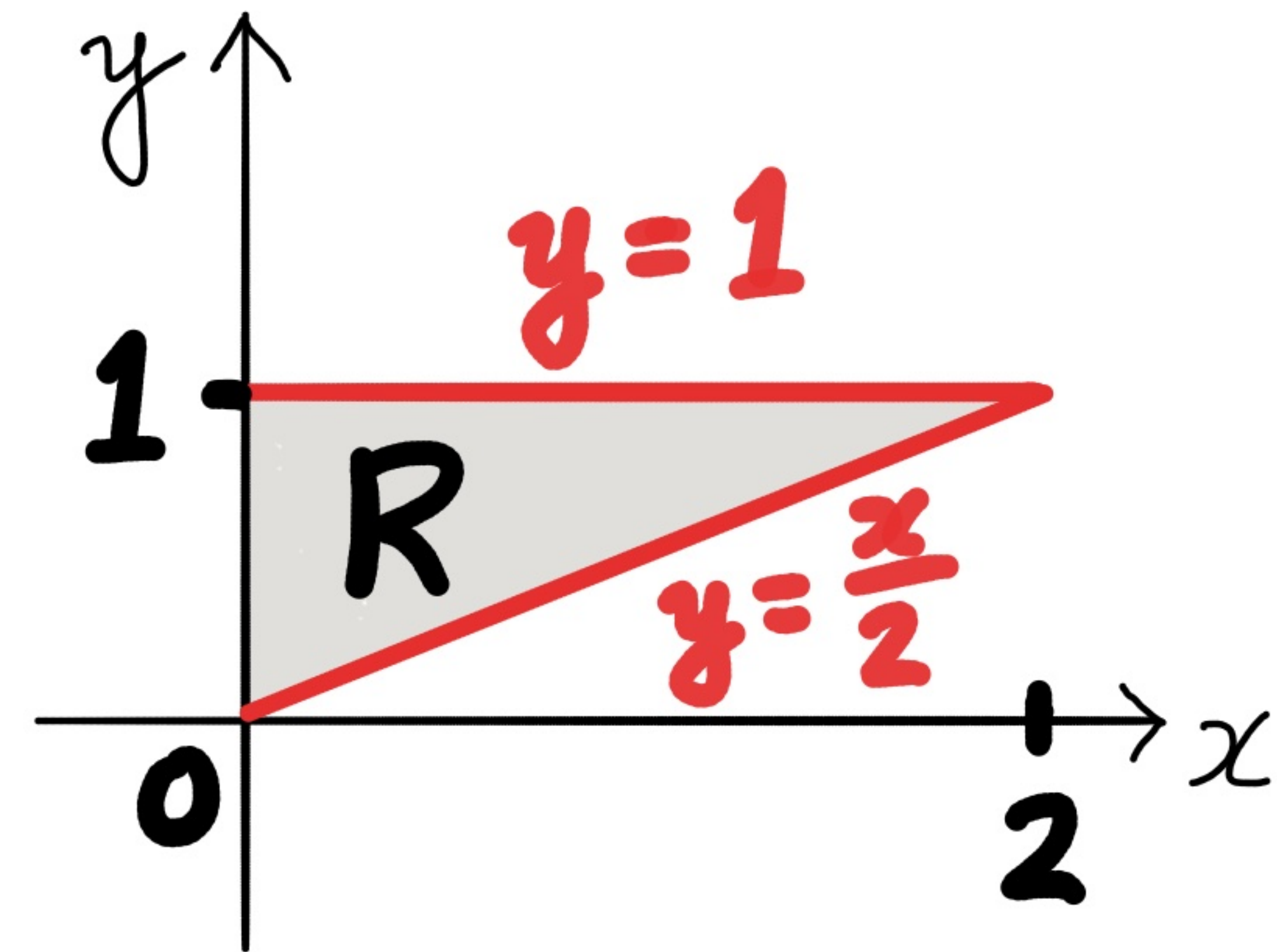


Answer:

$$R = \{(x, y) : 0 \leq y \leq \pi, \sin y \leq x \leq 2\sin y\}$$

Problem:

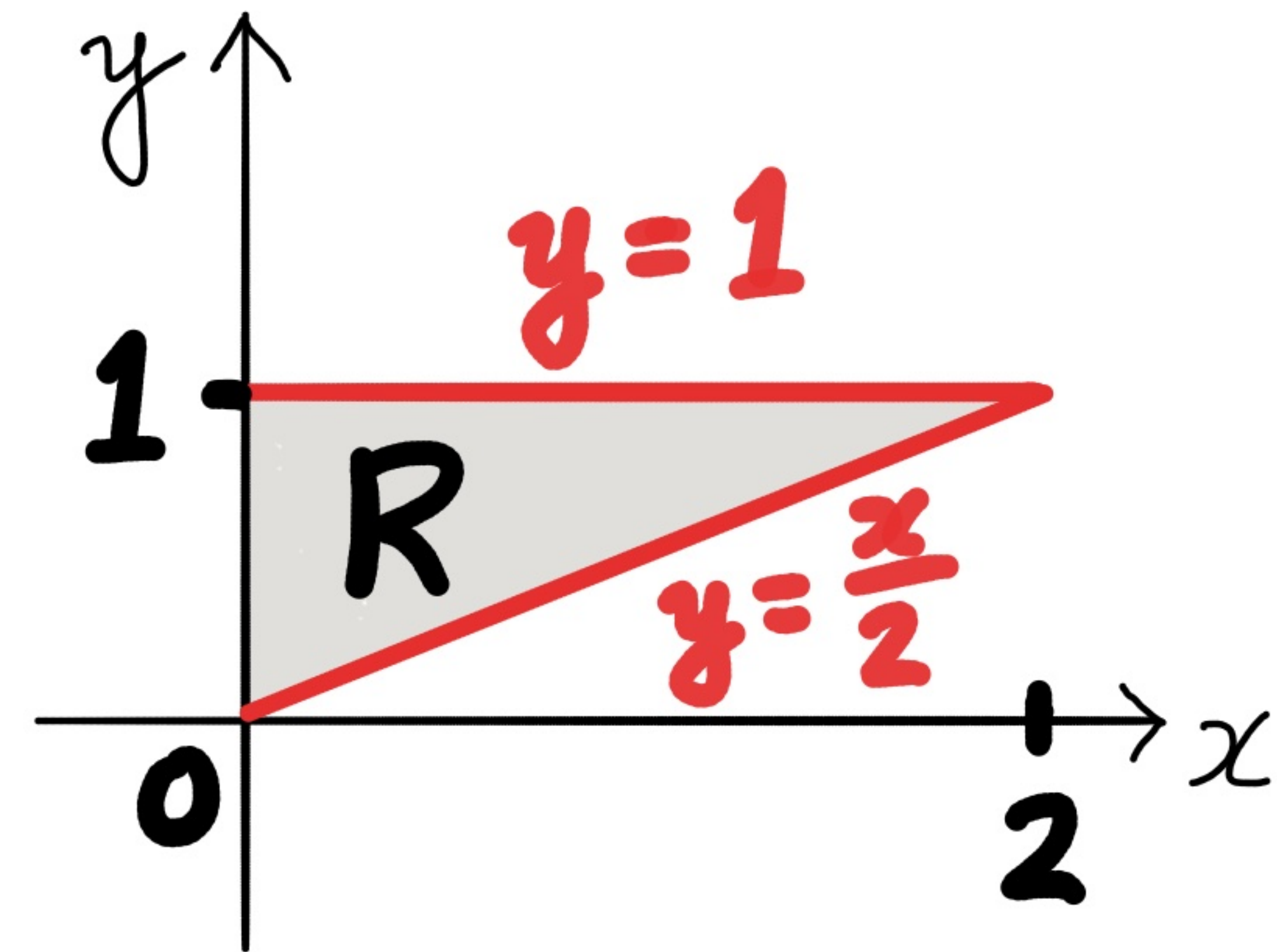
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Answer:

Problem:

Write the region R
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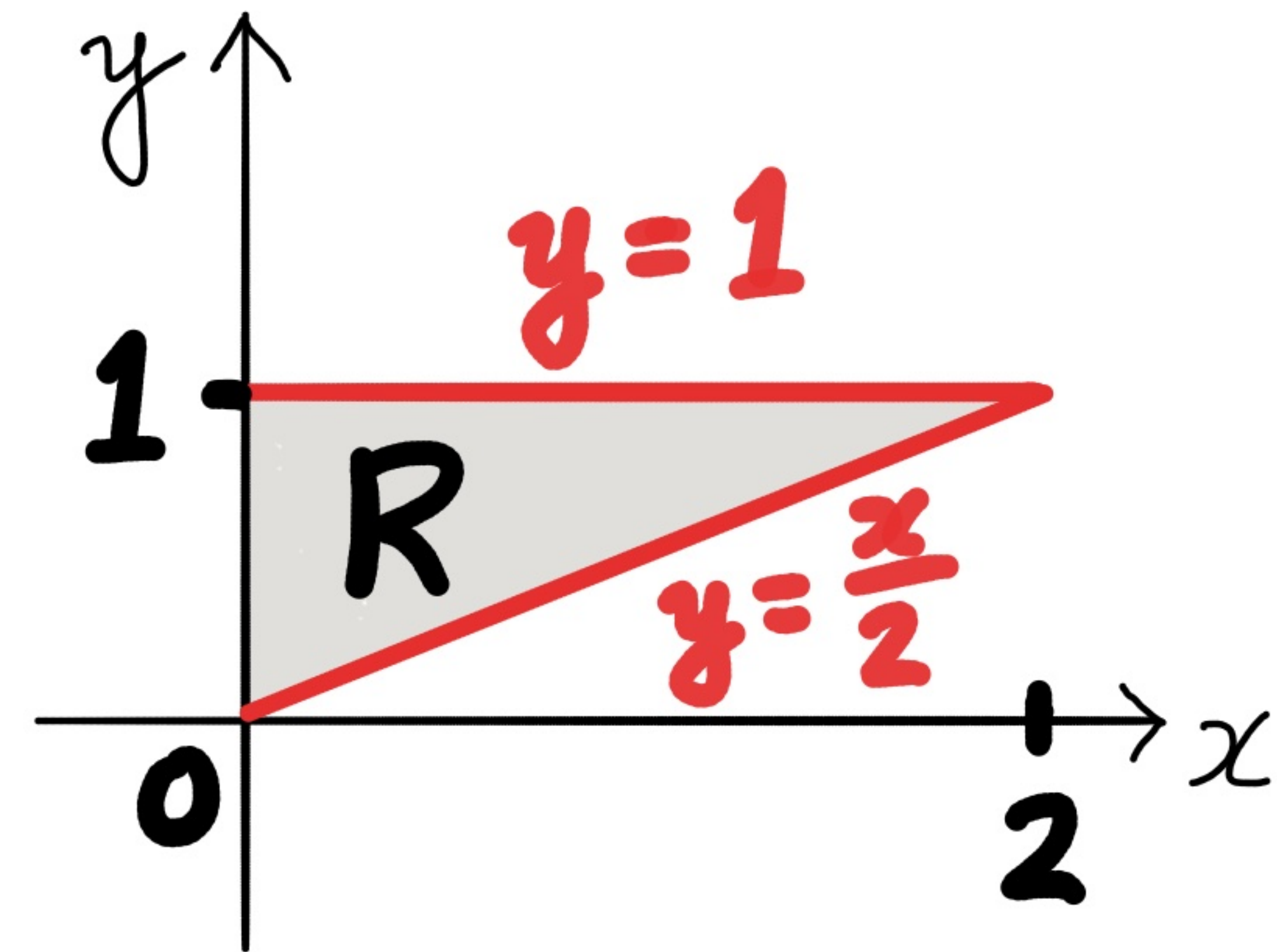


Answer:

$$R = \{(x, y) : \quad \leq y \leq \quad ,$$

Problem:

Write the region R as an x -simple set.

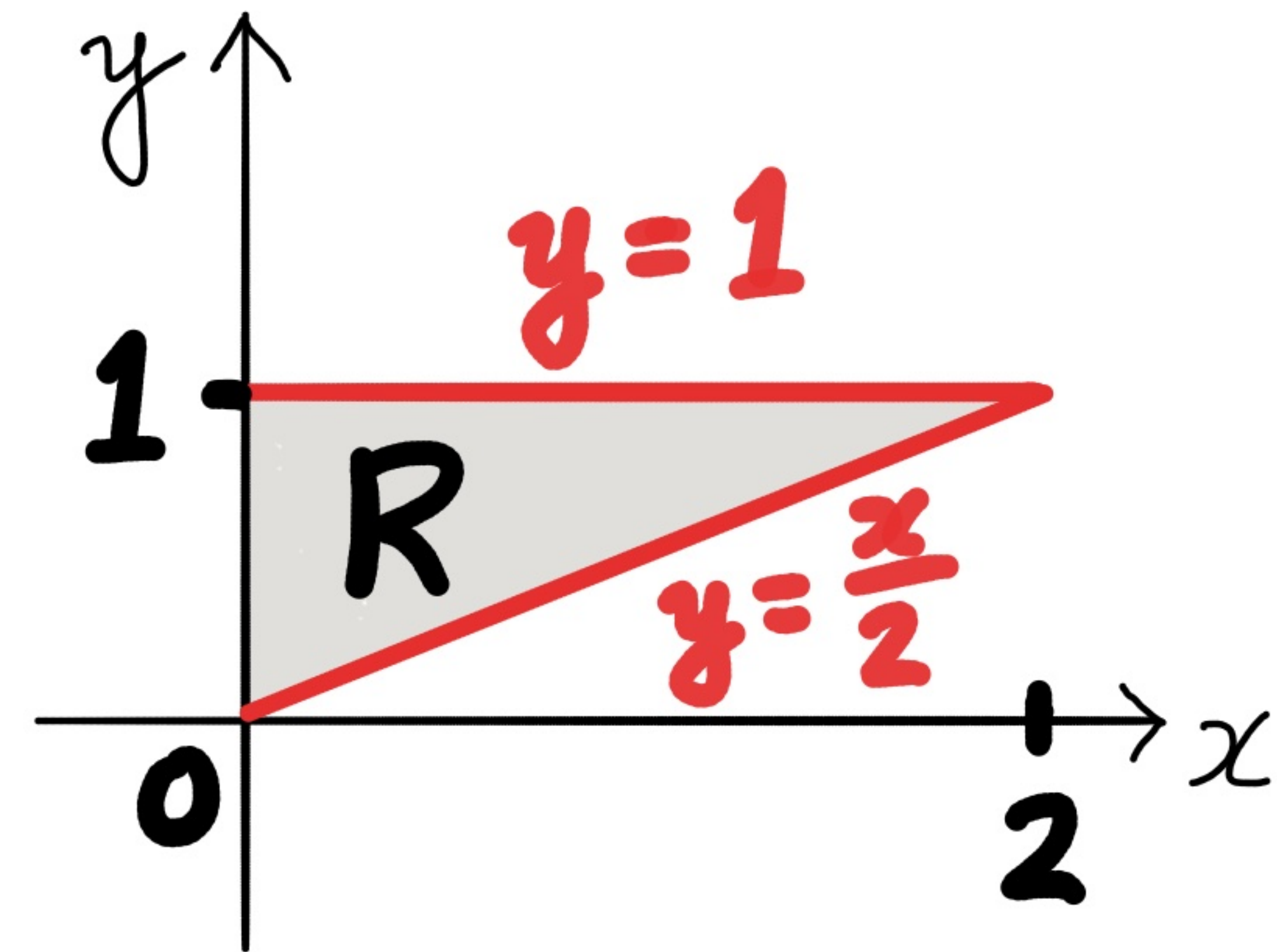


Answer:

$$R = \{(x,y) : 0 \leq y \leq 1,$$

Problem:

Write the region R as an x -simple set.

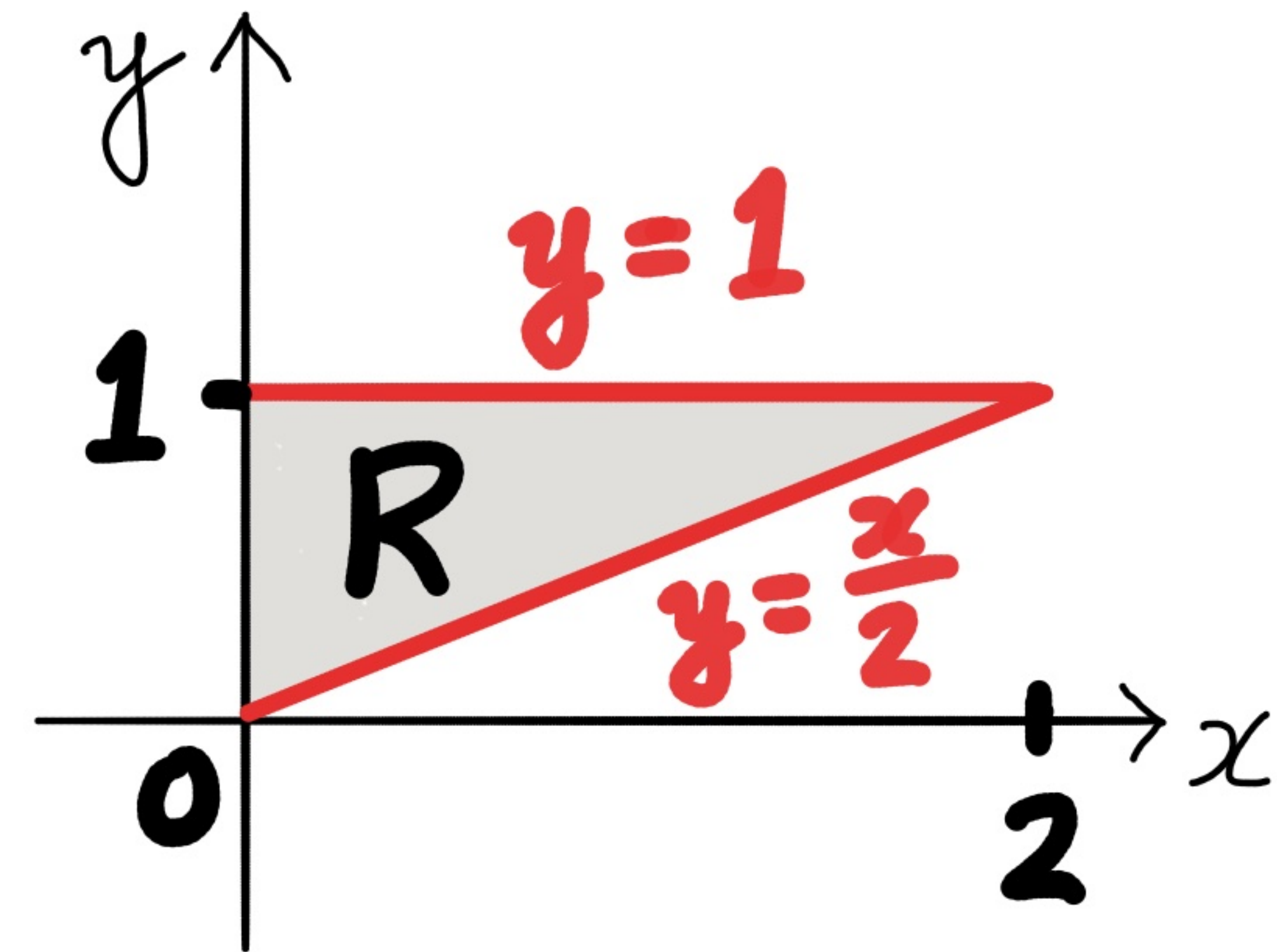


Answer:

$$R = \{(x, y) : 0 \leq y \leq 1, \quad 0 \leq x \leq 2\}$$

Problem:

Write the region R as an x -simple set.

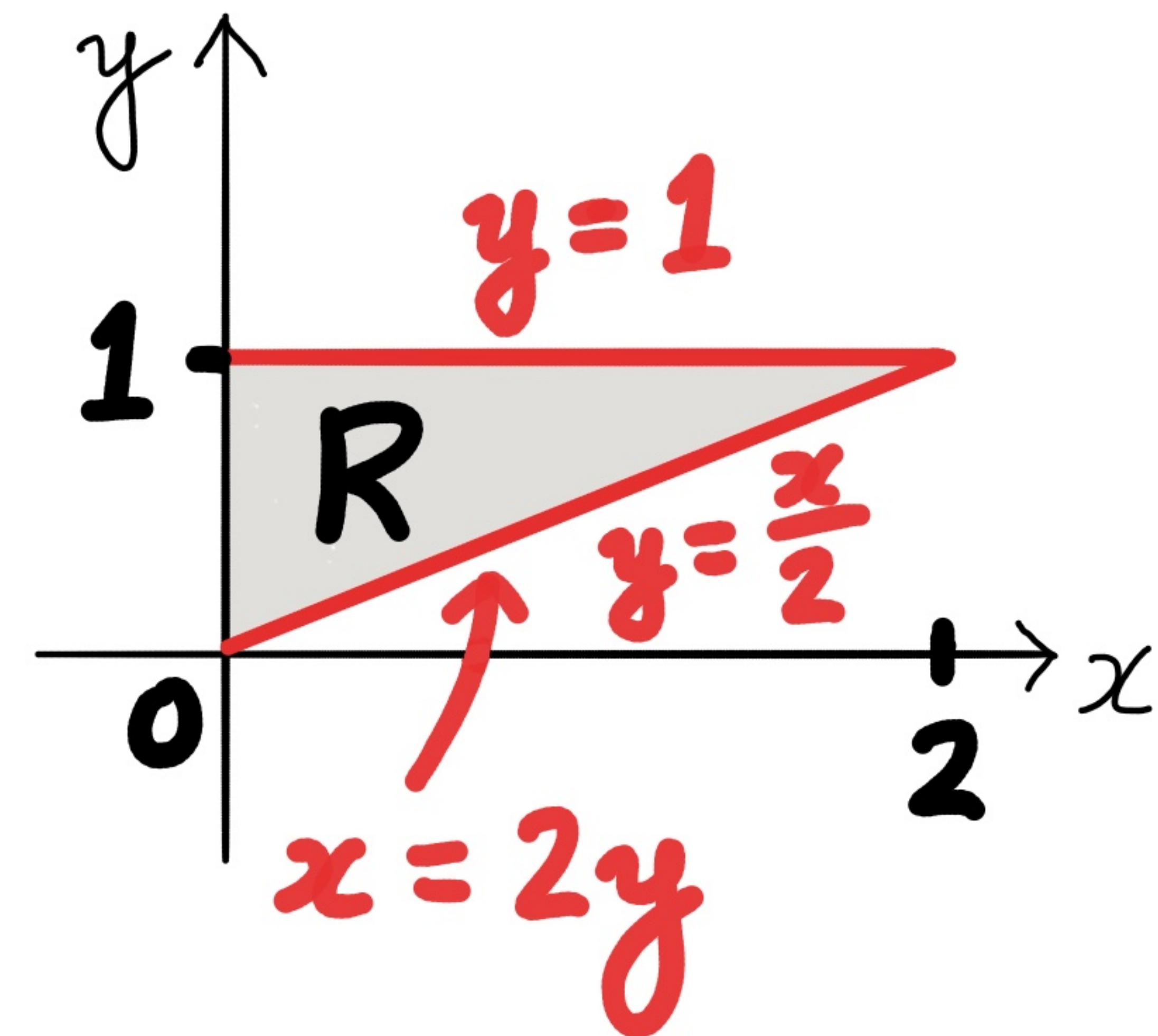


Answer:

$$R = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq$$

Problem:

Write the region R as an x -simple set.

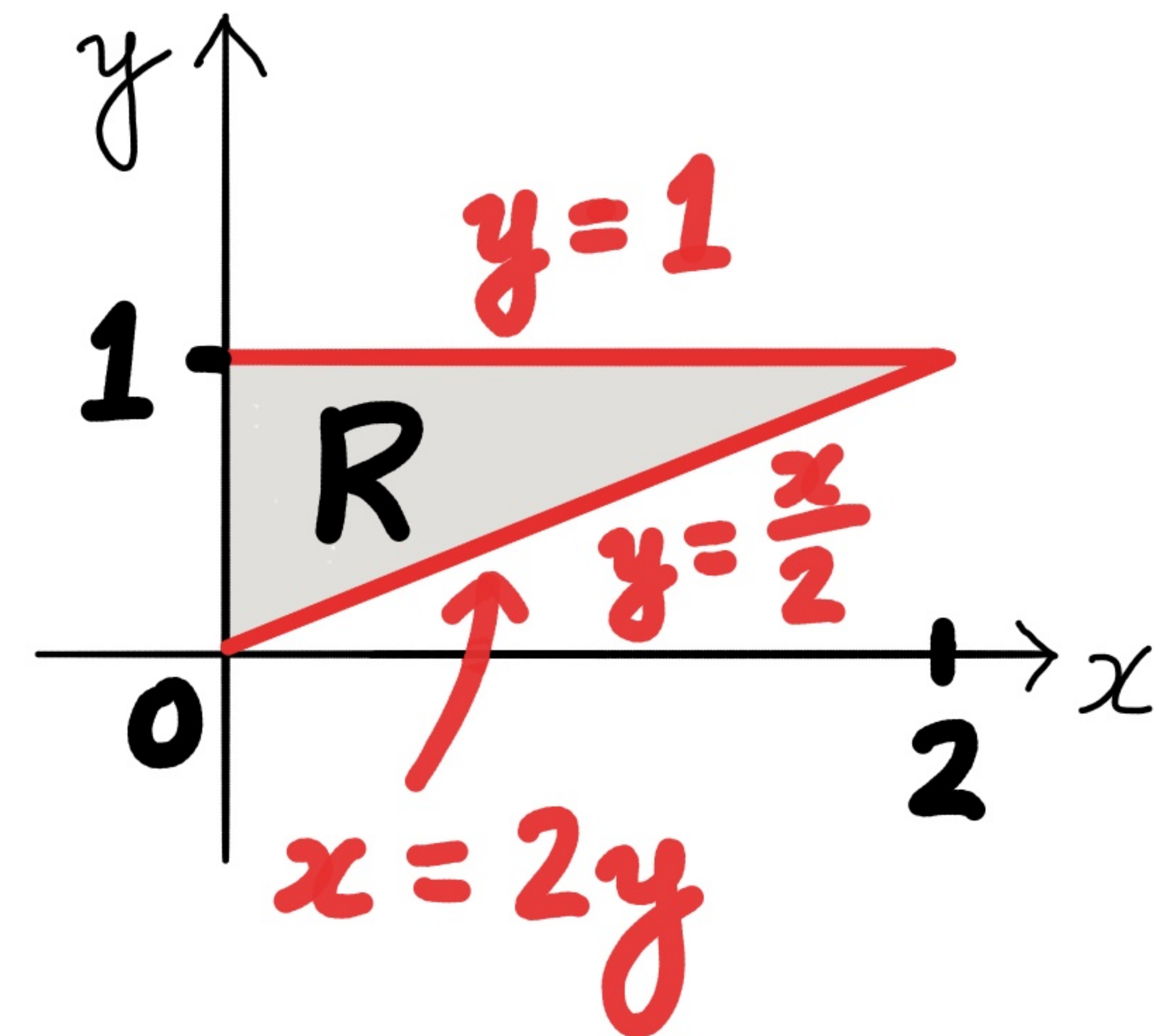


Answer:

$$R = \{(x, y) : 0 \leq y \leq 1, 0 \leq x \leq$$

Problem:

Write the region R as an x -simple set.



Answer:

$$R = \{(x,y) : 0 \leq y \leq 1, 0 \leq x \leq 2y\}$$