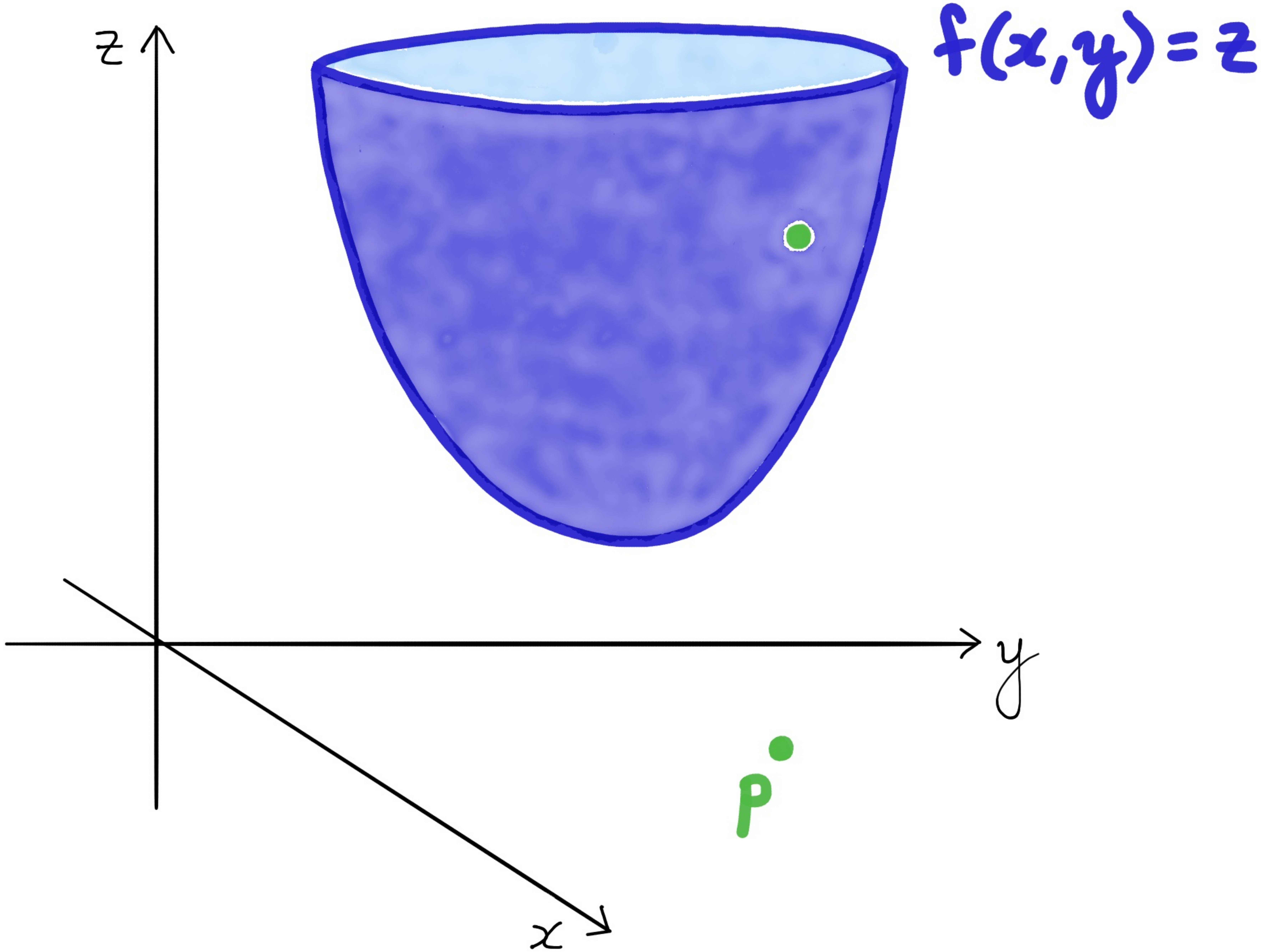
Sixteen

Max/Min

If $\nabla f(p) \neq \vec{0}$, then p is <u>not</u> a local max or min



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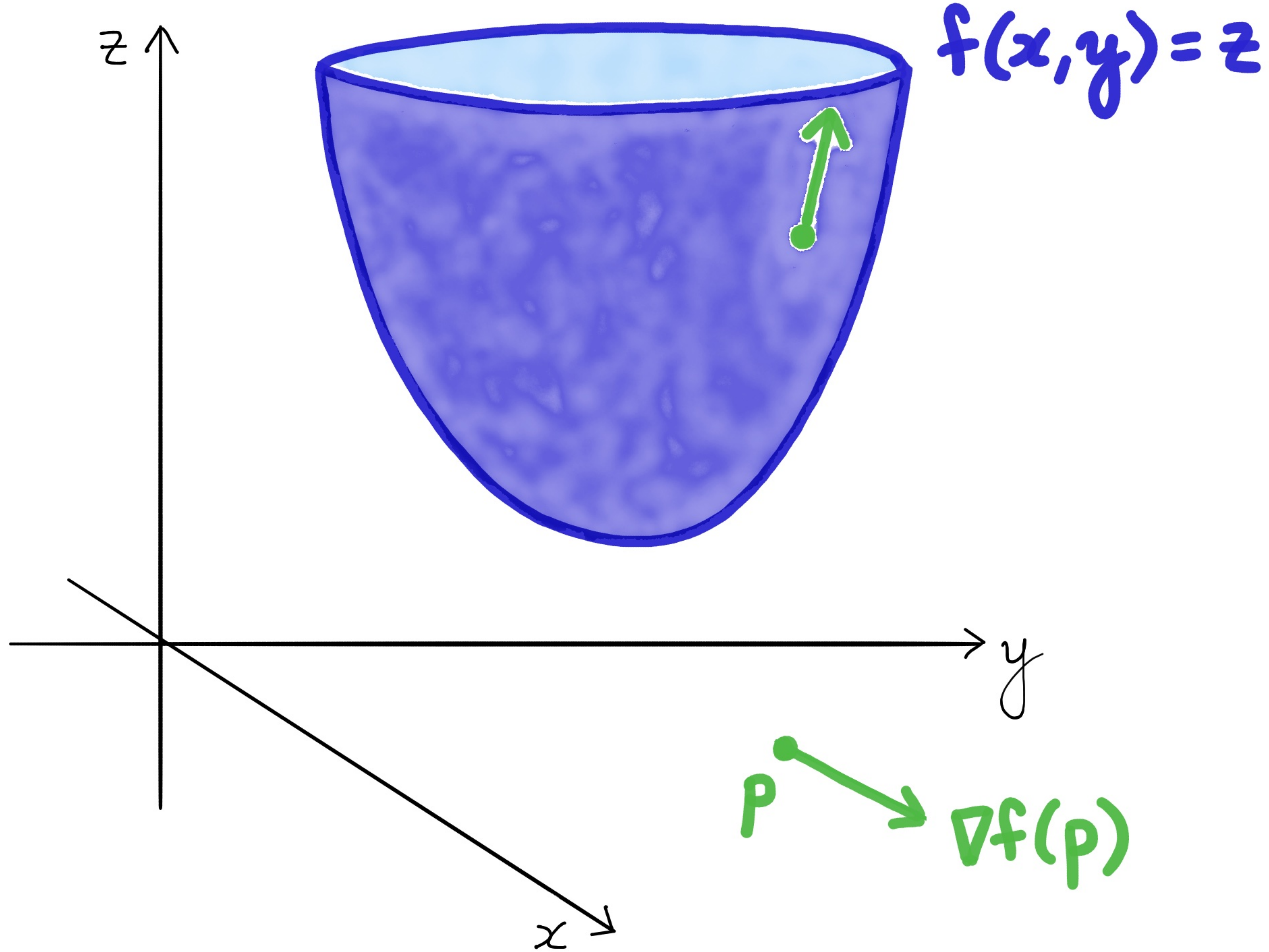

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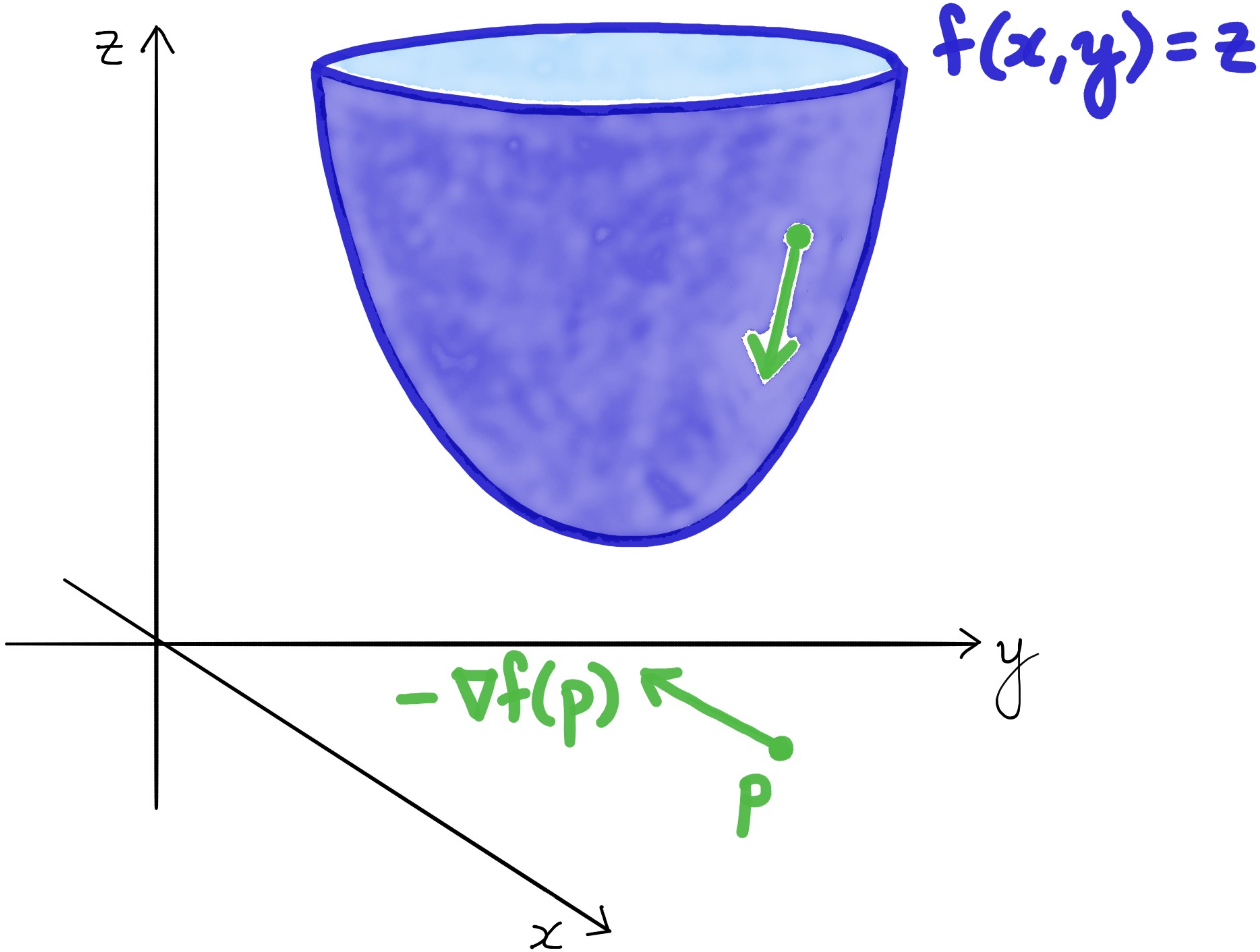
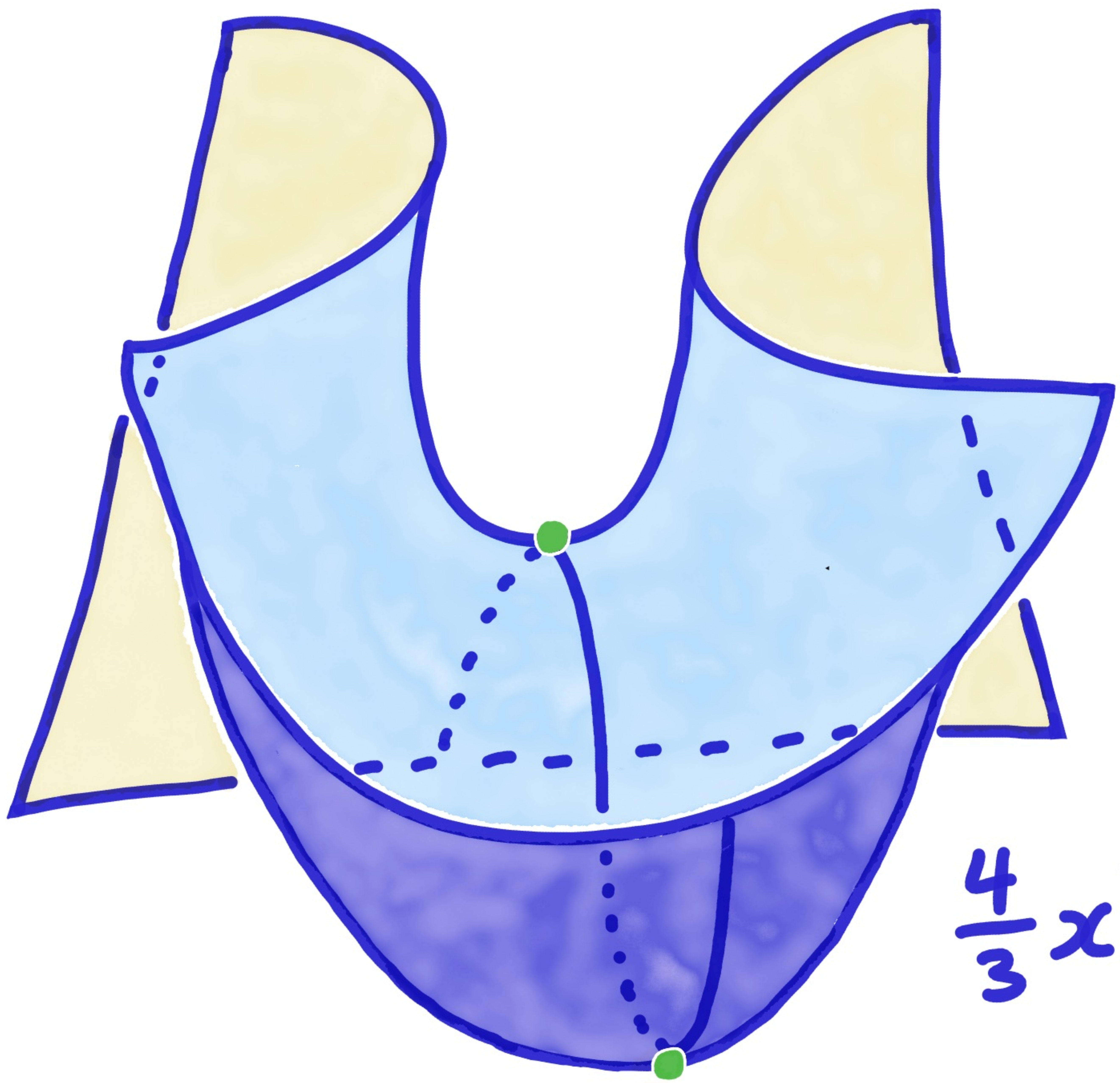


Problems:

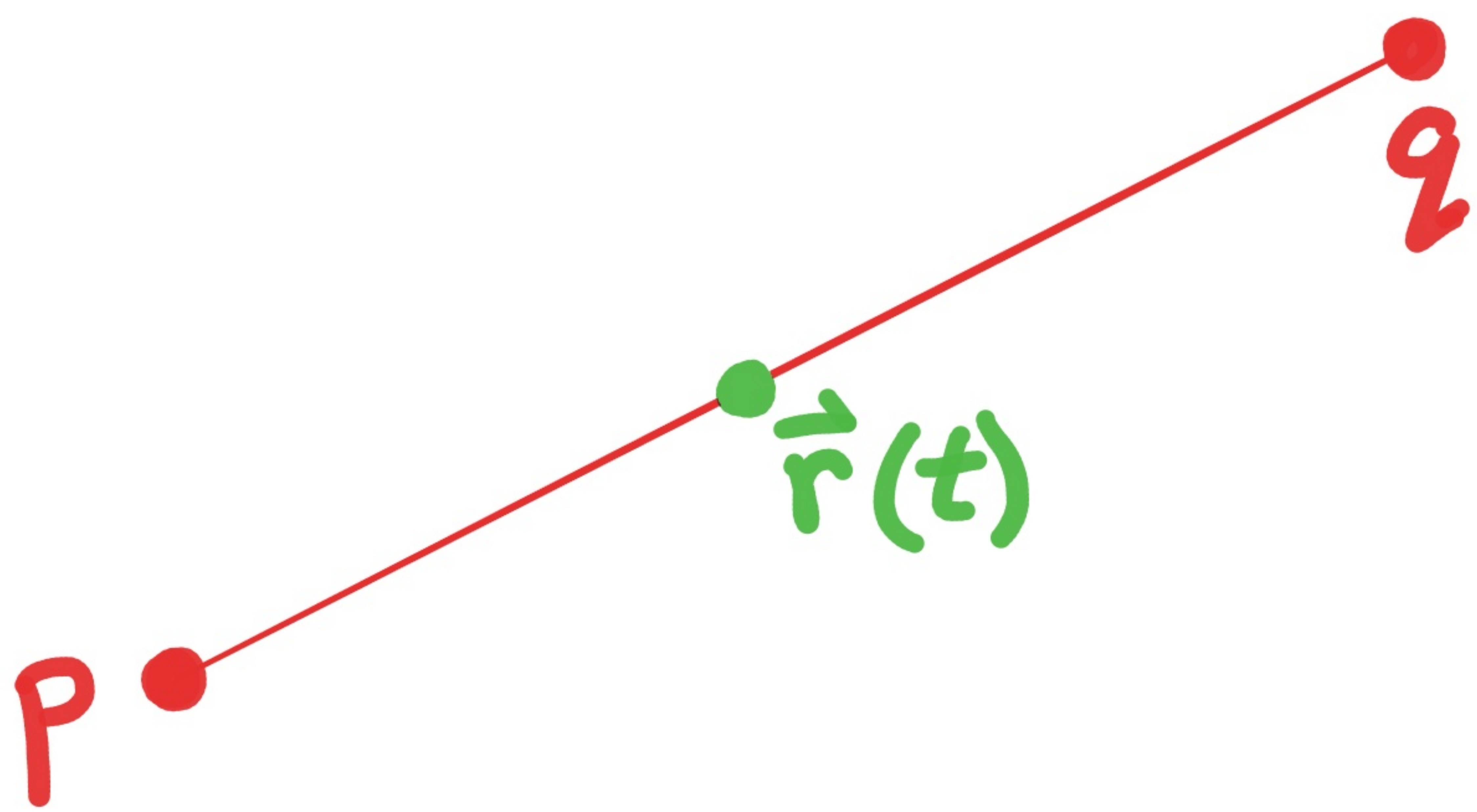
① $f(x,y) = \frac{4}{3}x^3 - x + y^2 + 3y$.

Find all critical points. Indicate whether each critical point gives a local max, local min, or a saddle point.



$$\frac{4}{3}x^3 - x + y^2 + 3y$$

To parametrize a straight line between $p, q \in \mathbb{R}^2$, use



$$\vec{r}(t) = t(q-p) + p$$

$$0 \le t \le 1$$

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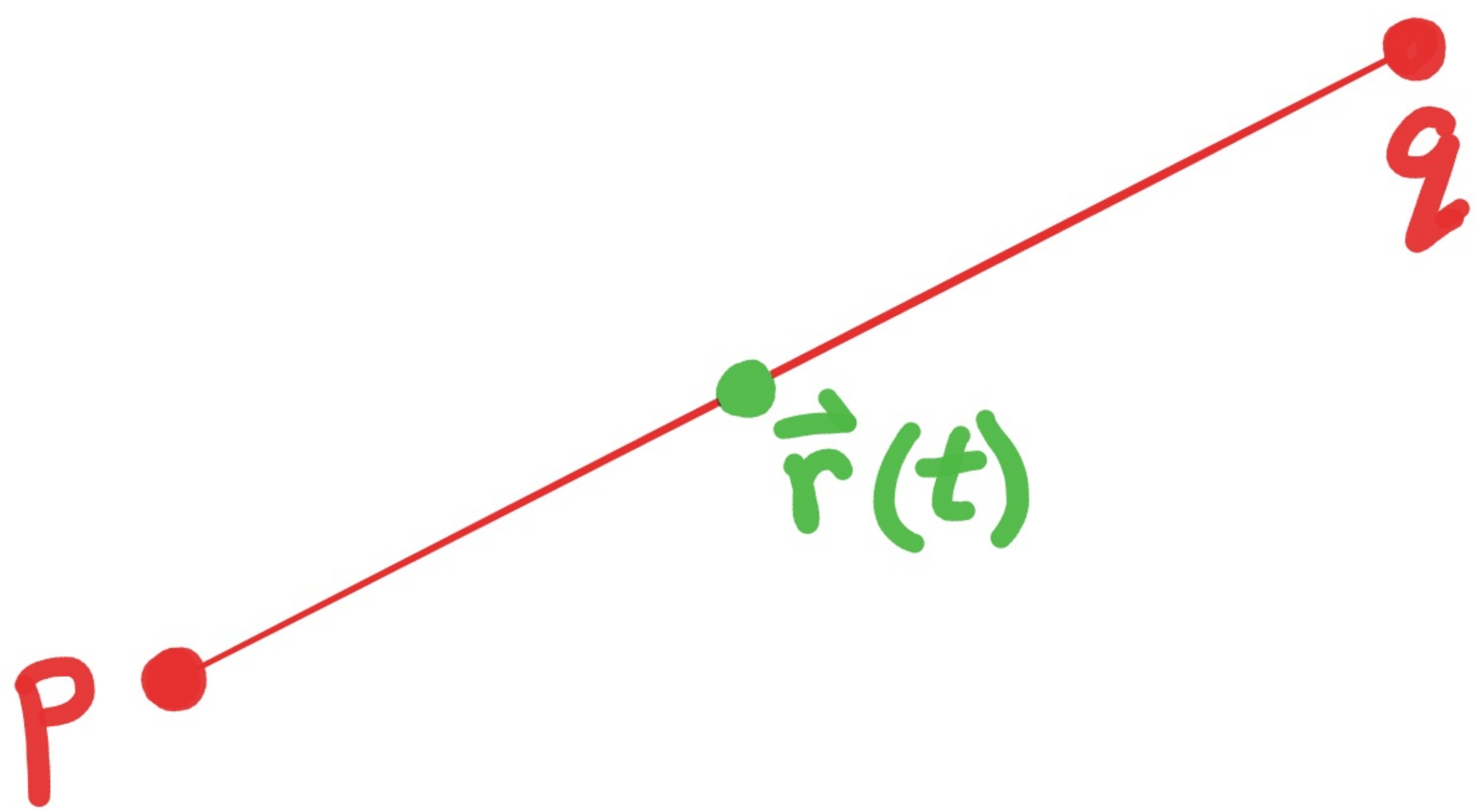

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<u>**Example**</u>: Parametrize the straight line between $(2,3)$ and $(5,7)$.

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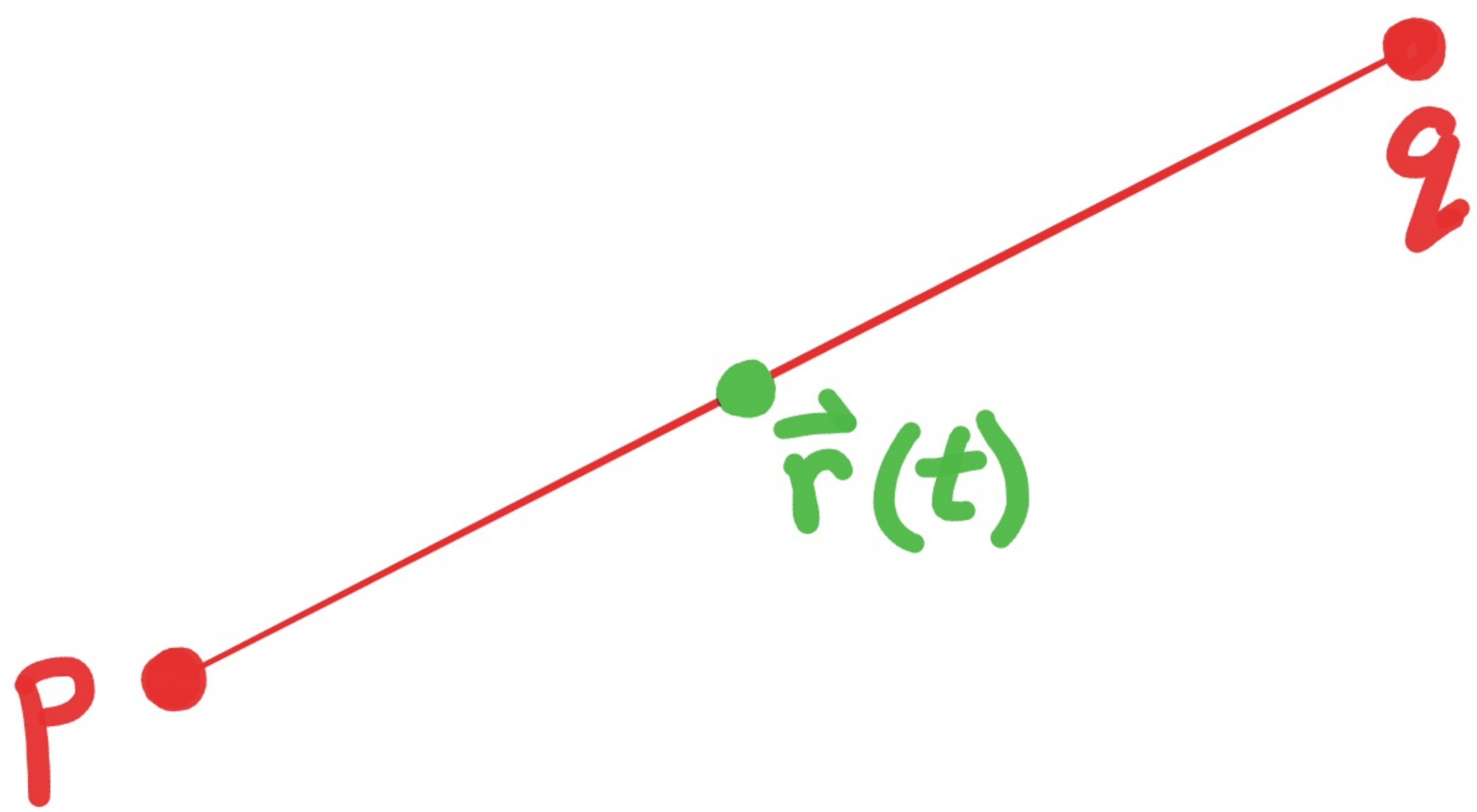
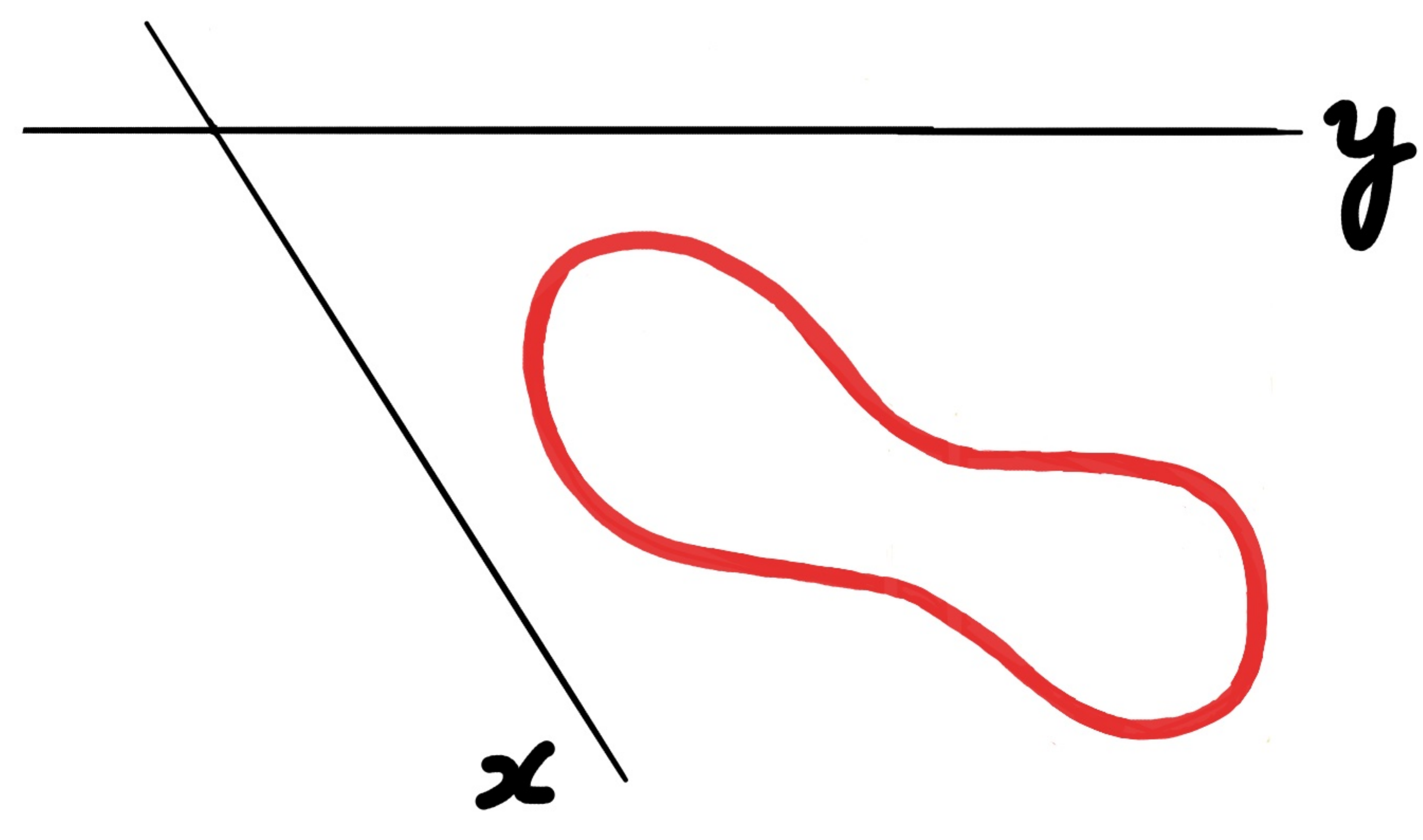


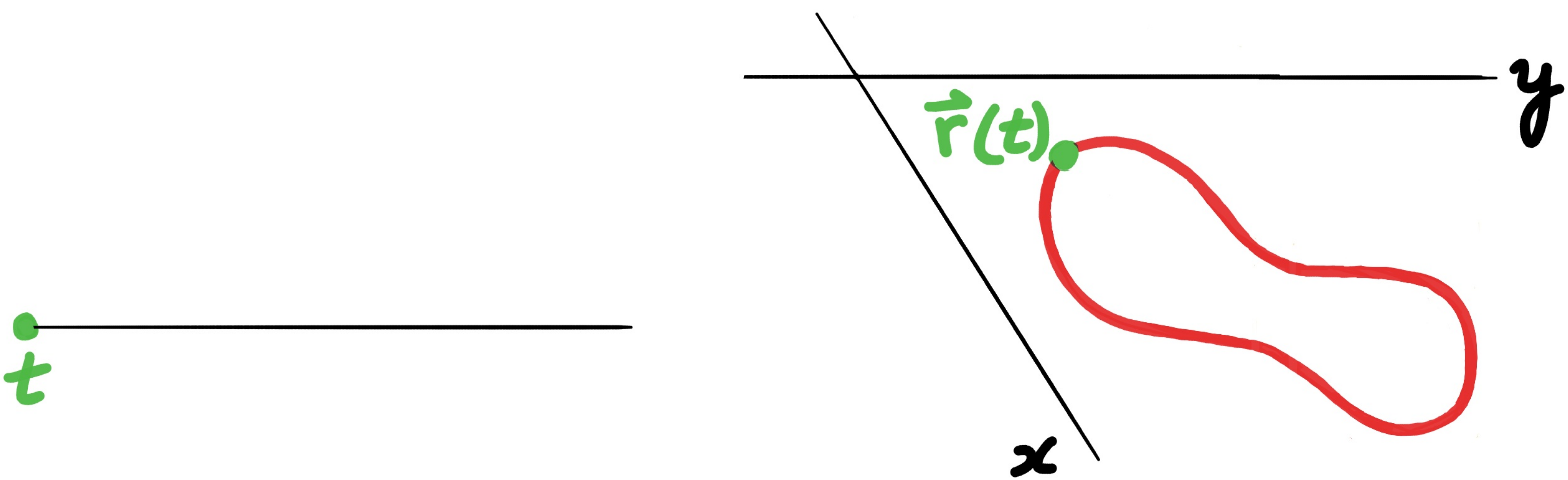
$$\vec{r}(t) = t(q-p) + p$$

$$0 \leq t \leq 1$$

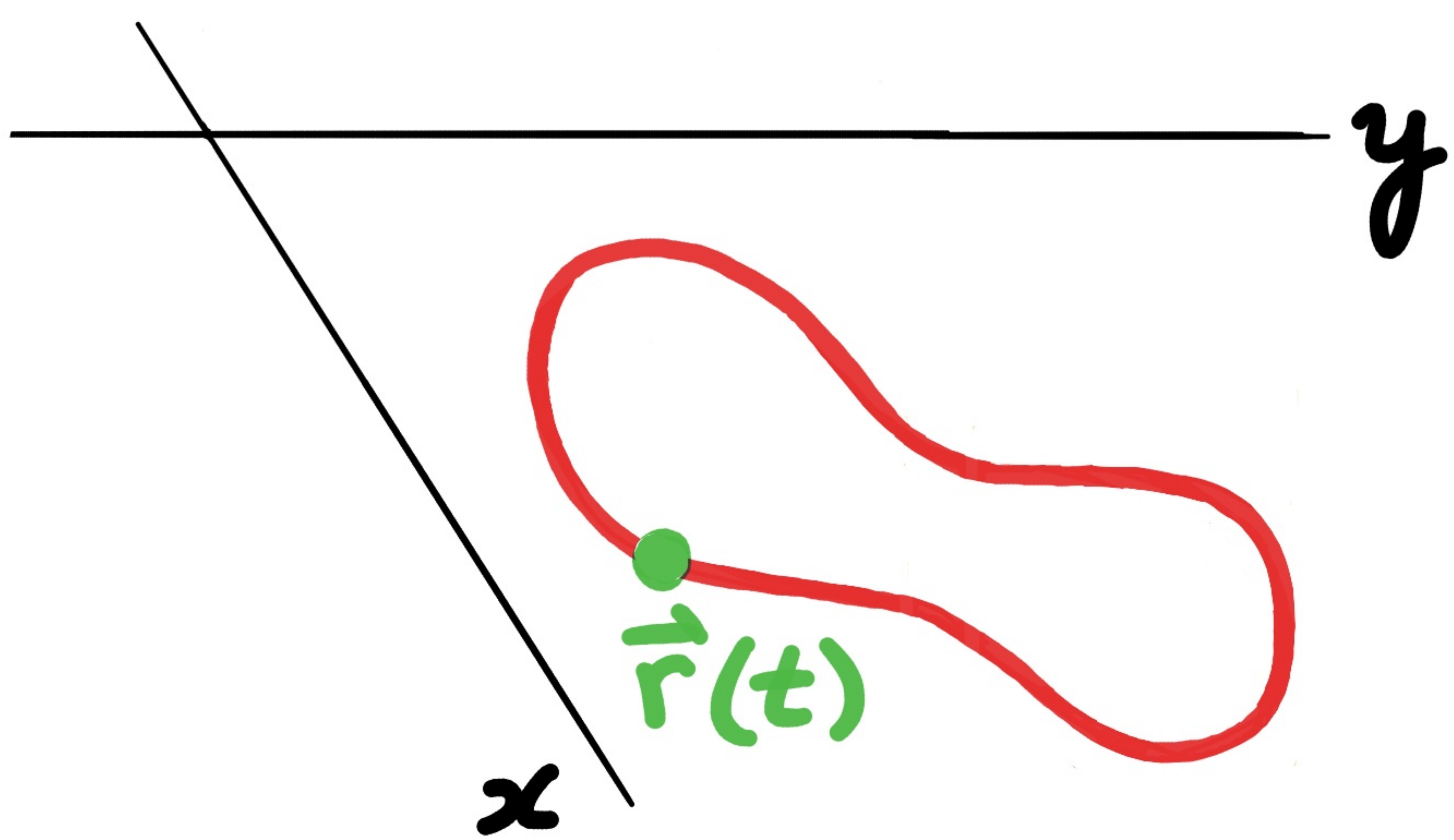
Example: Parametrize the straight line between $(2,3)$ and $(5,7)$.

$$0 \leq t \leq 1, \quad t\big((5,7)-(2,3)\big)+(2,3) = (3t+2, 4t+3)$$

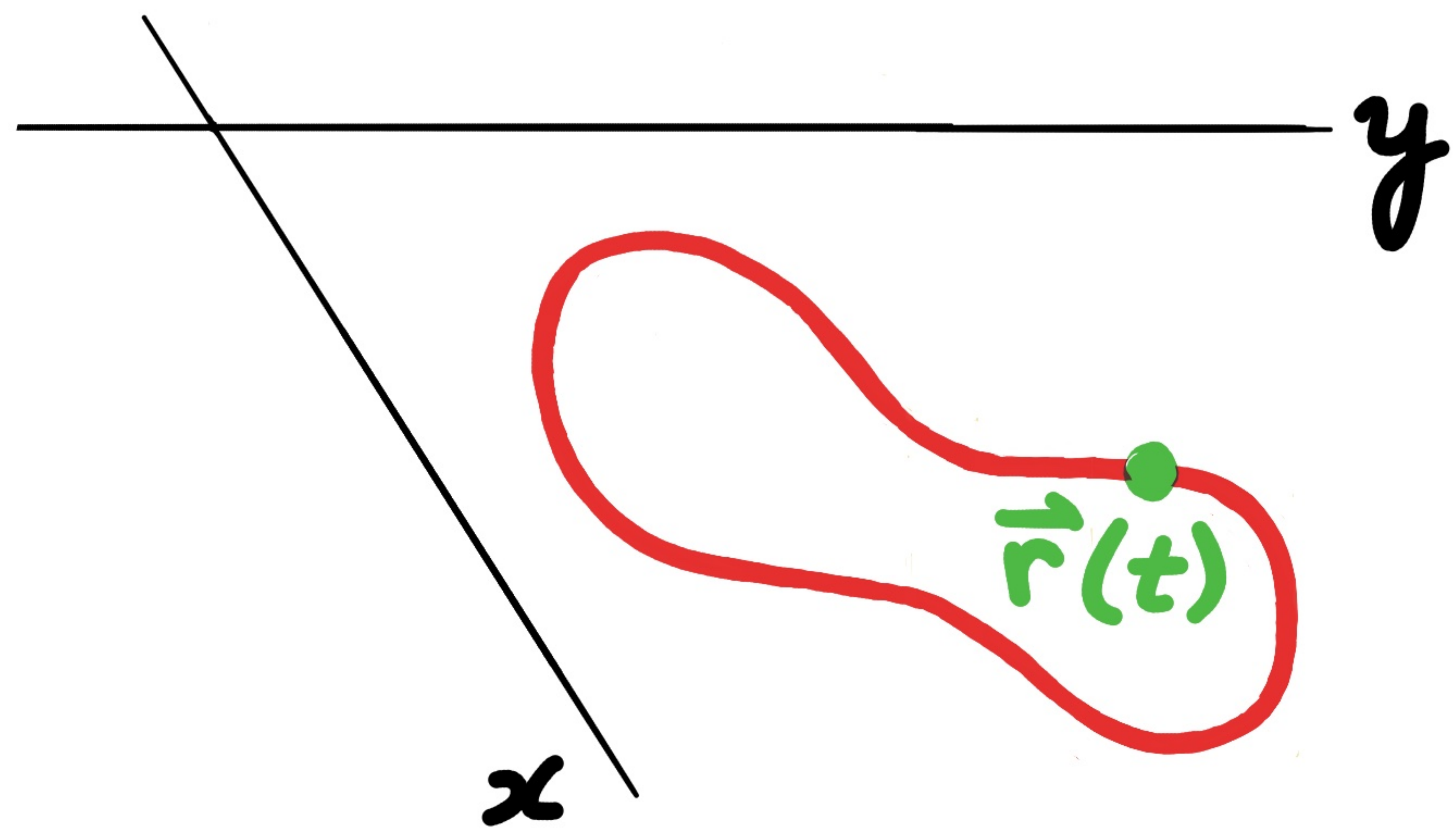

y
x

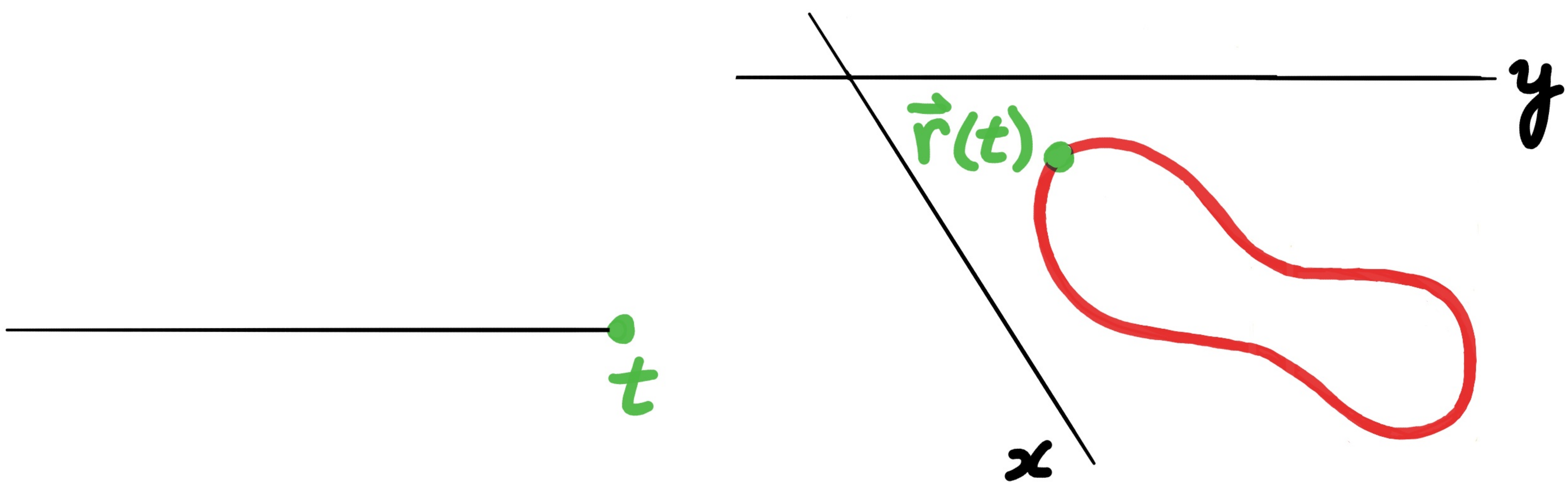

t
$\vec{r}(t)$
y
x

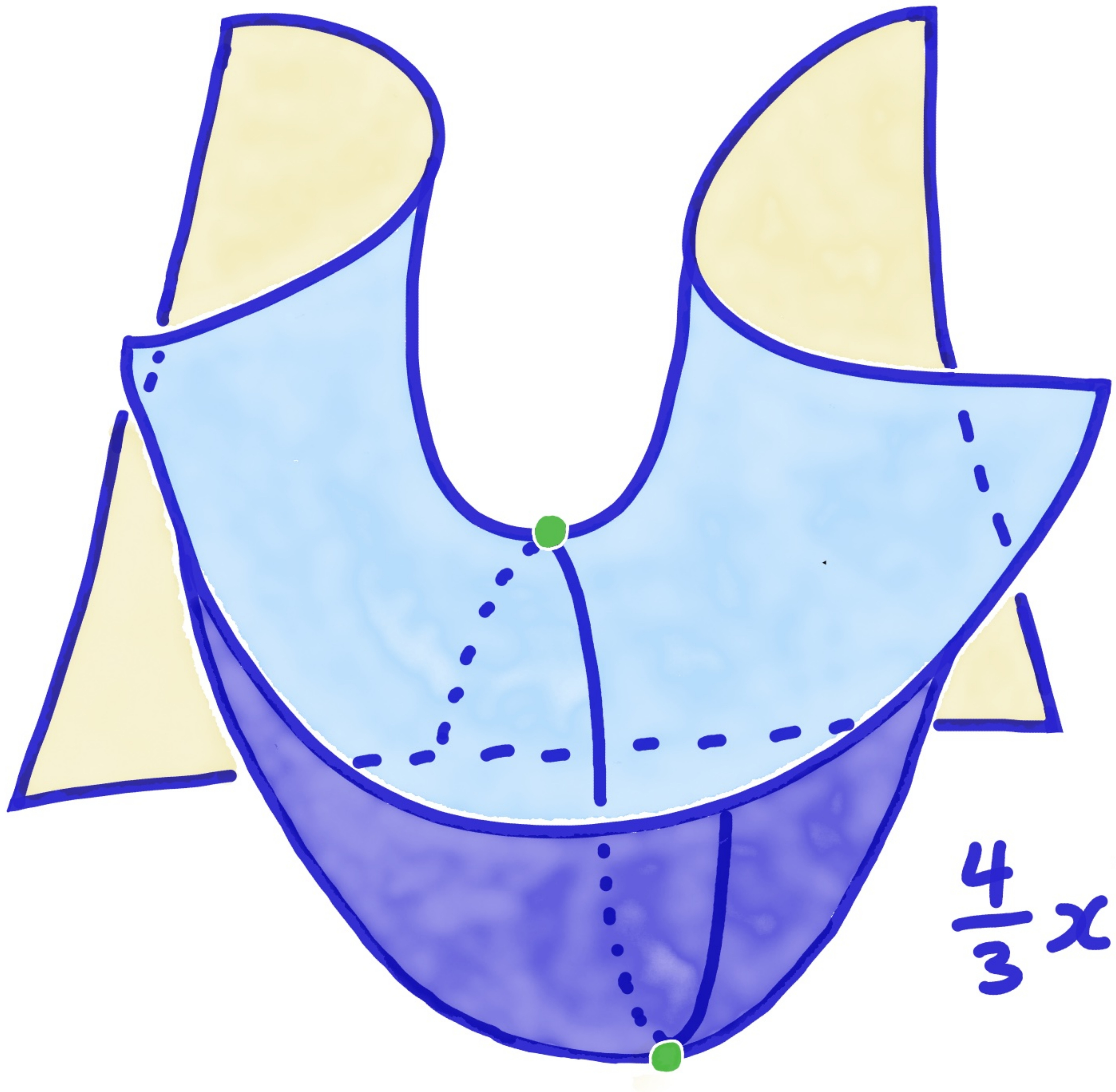

t


y
x
$\vec{r}(t)$

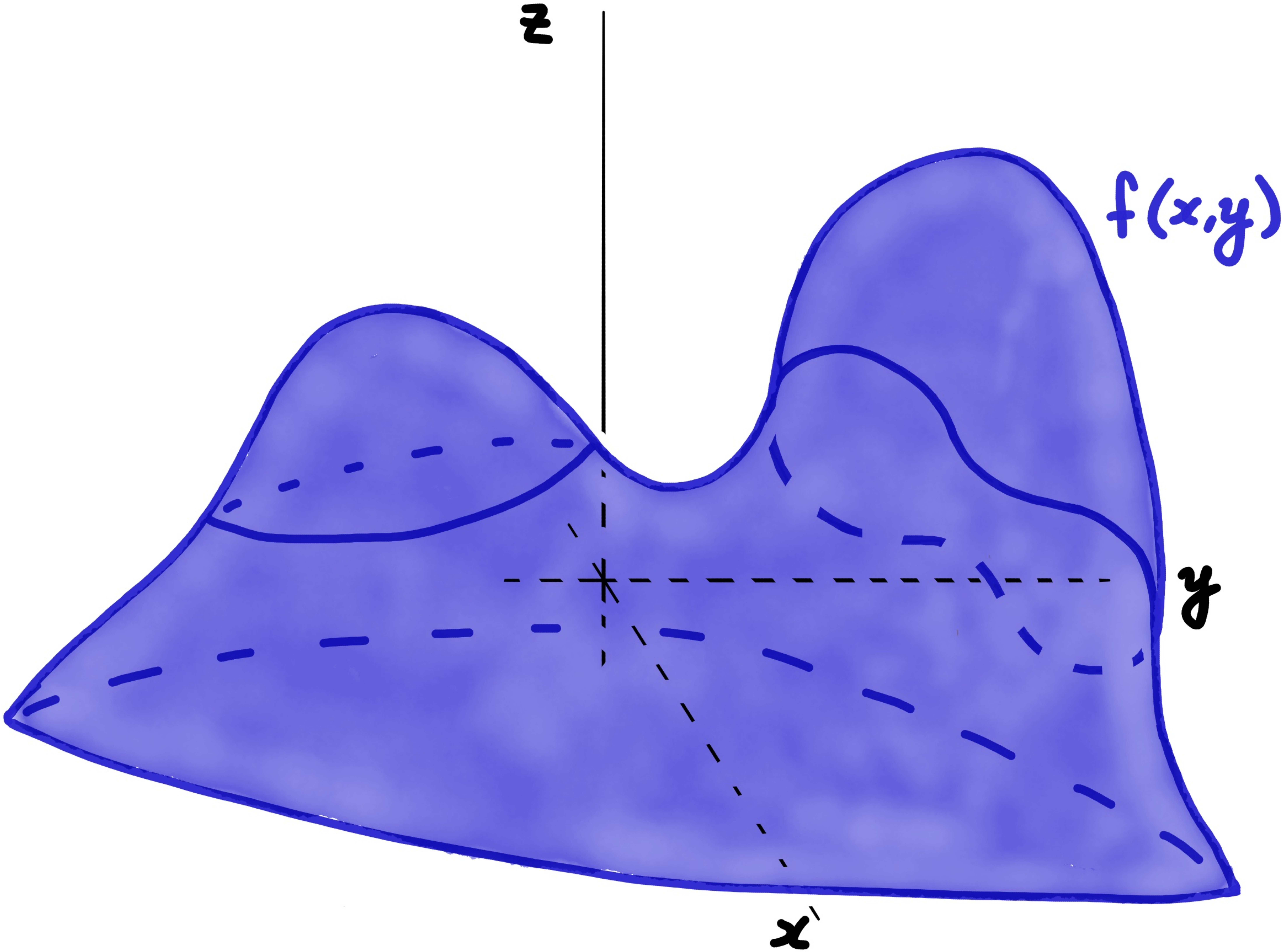

t

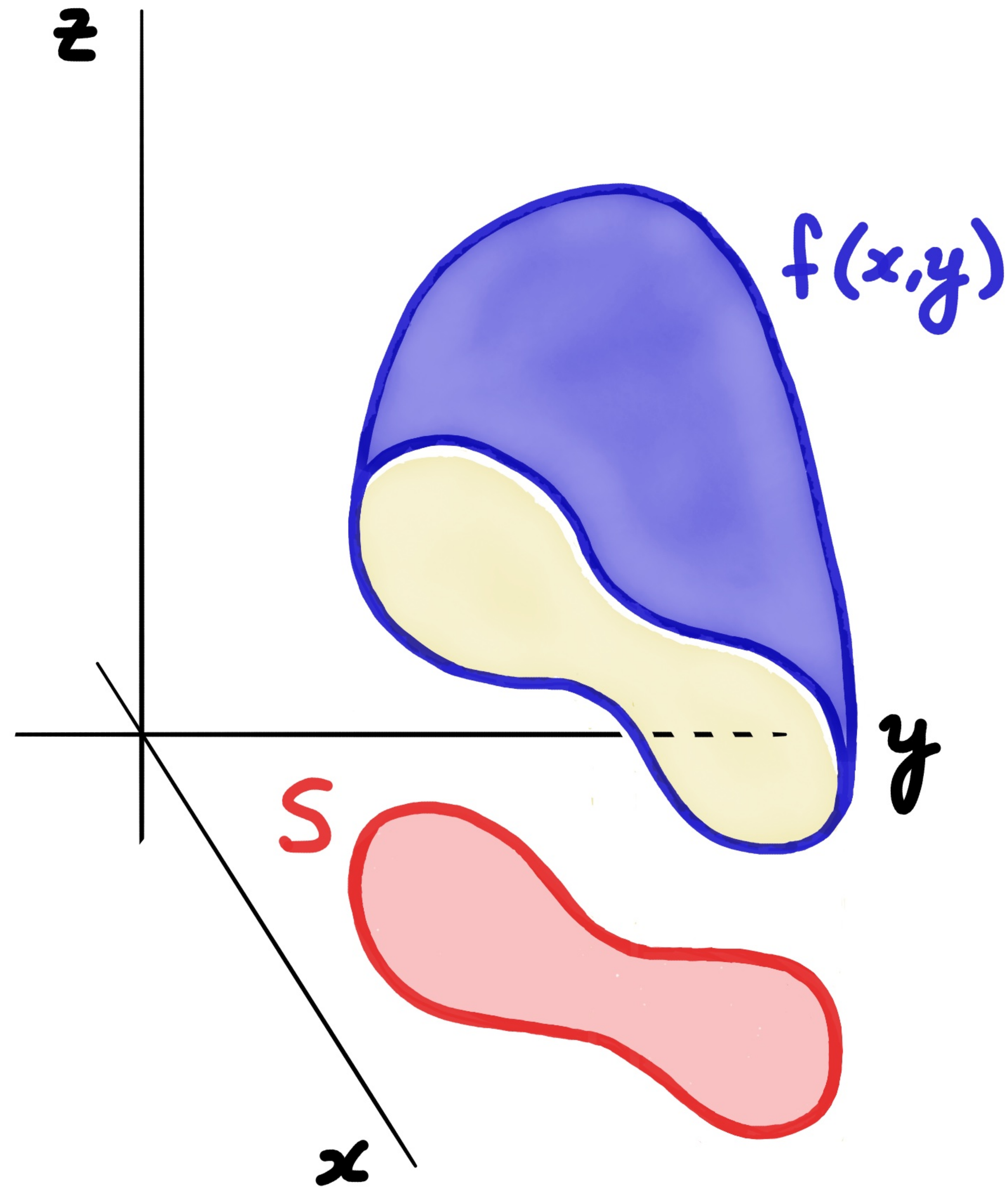

y
$\vec{r}(t)$
x

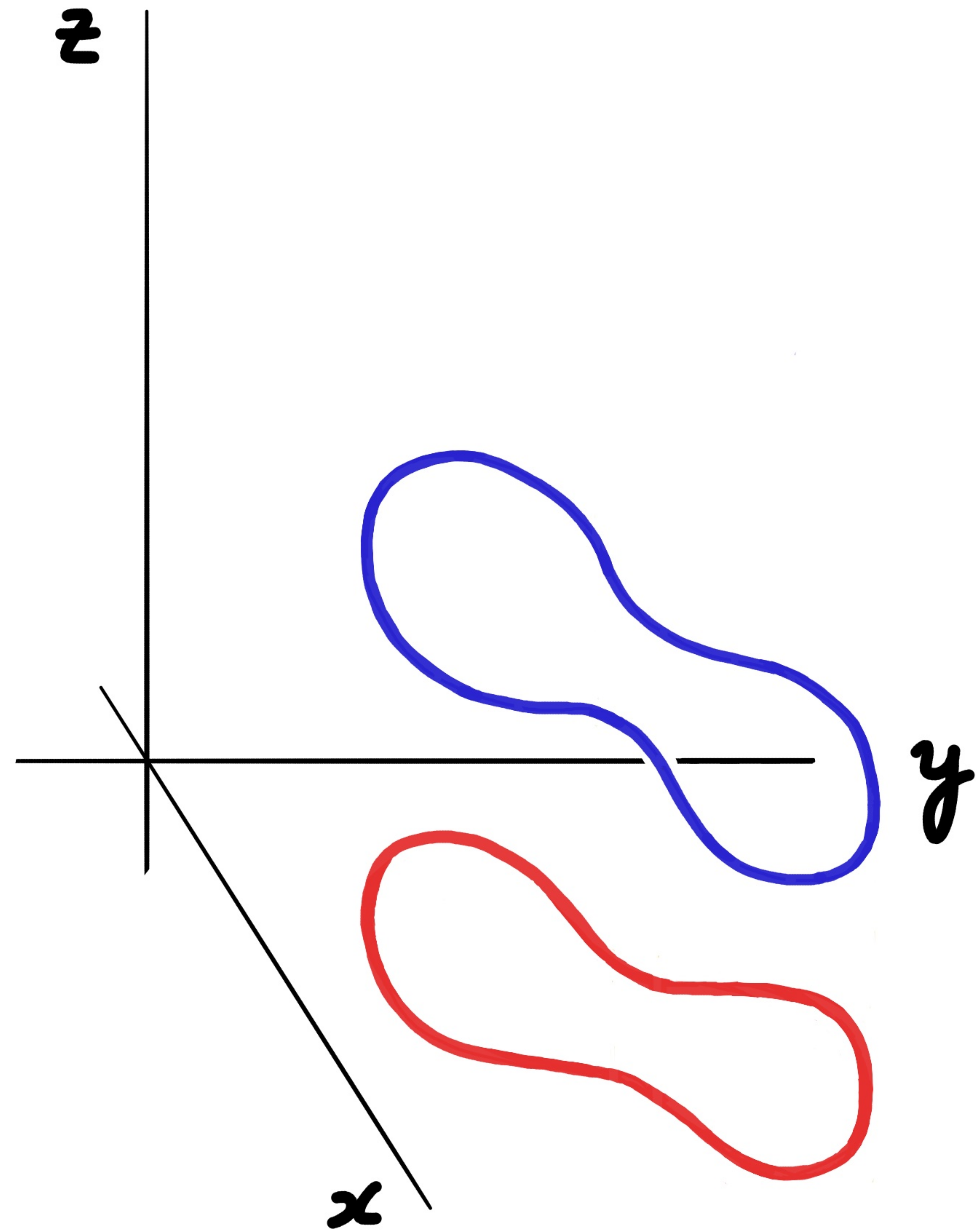

t
$\vec{r}(t)$
y
x

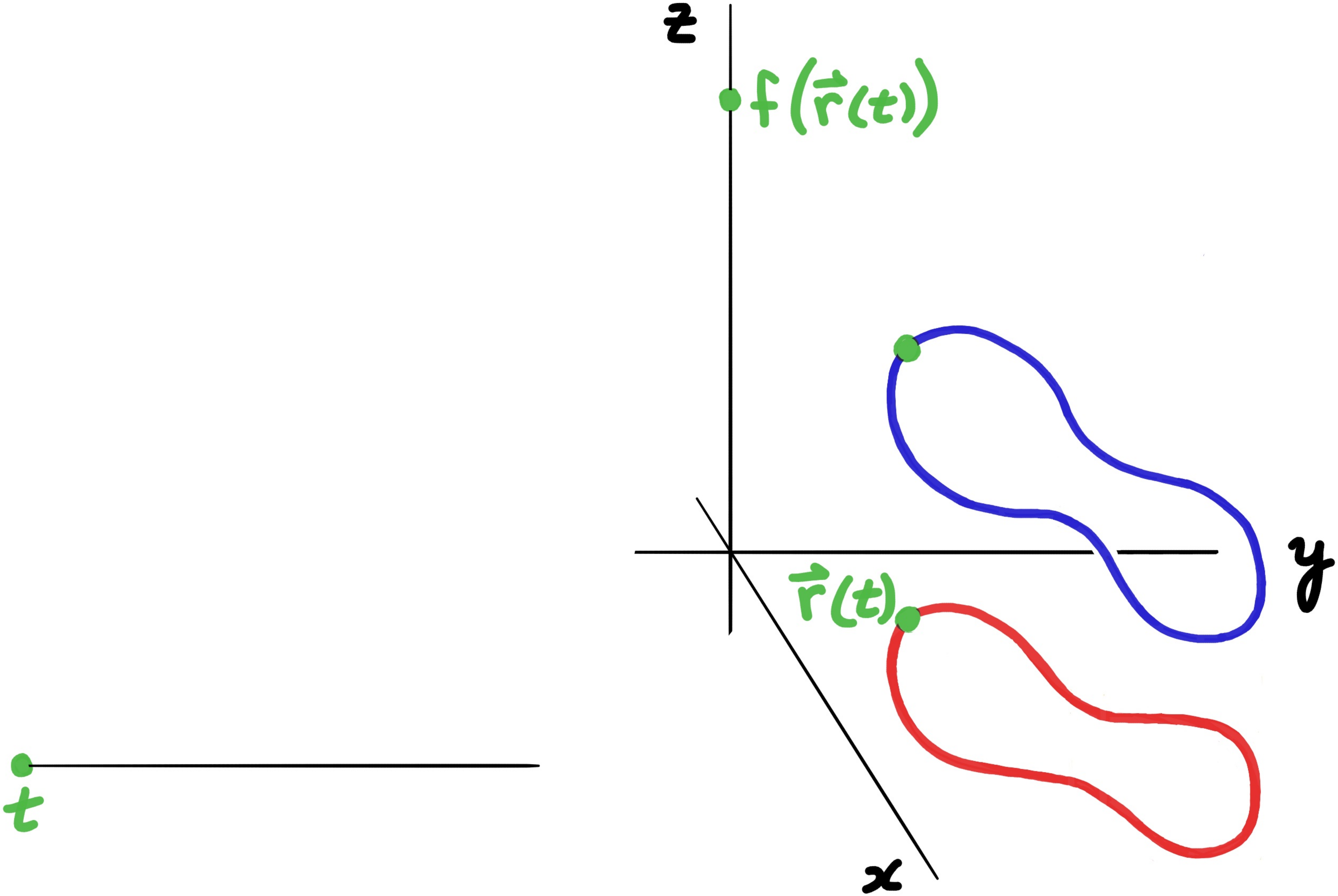


$$\frac{4}{3}x^3 - x + y^2 + 3y$$

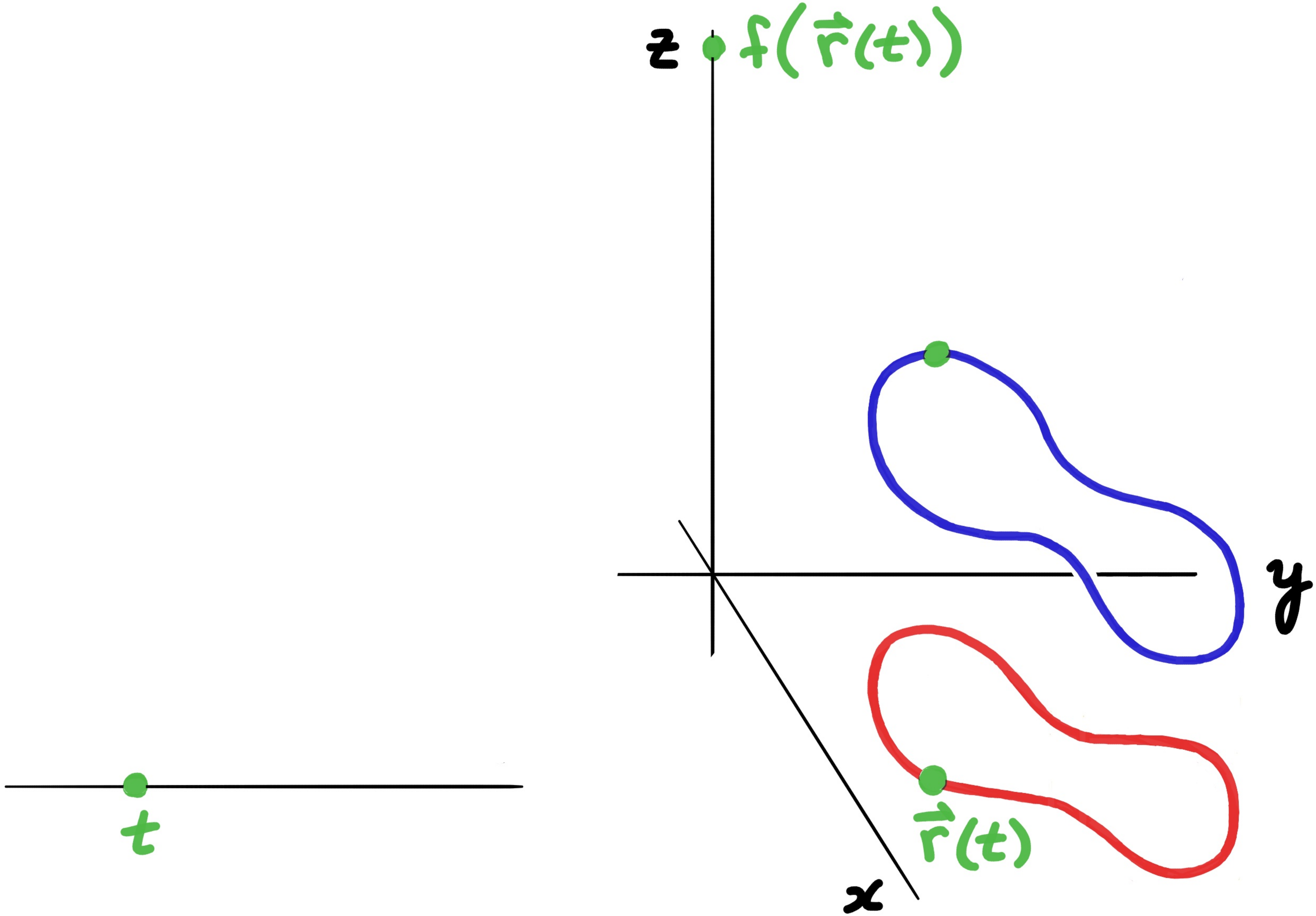

z
f(x,y)
y
x


z
f(x,y)
y
S
x


z
y
x

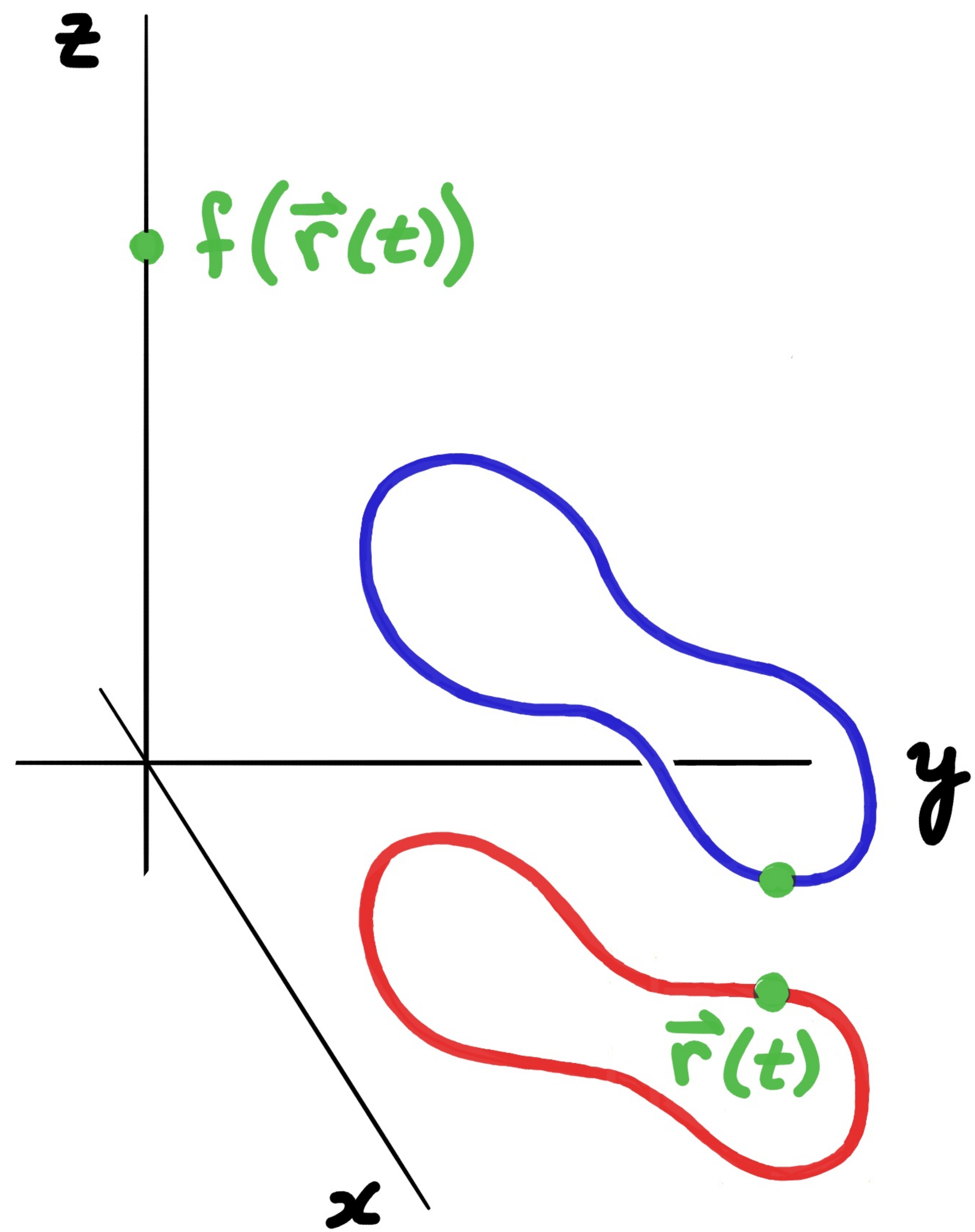


z
$f(\vec{r}(t))$
y
$\vec{r}(t)$
x
t

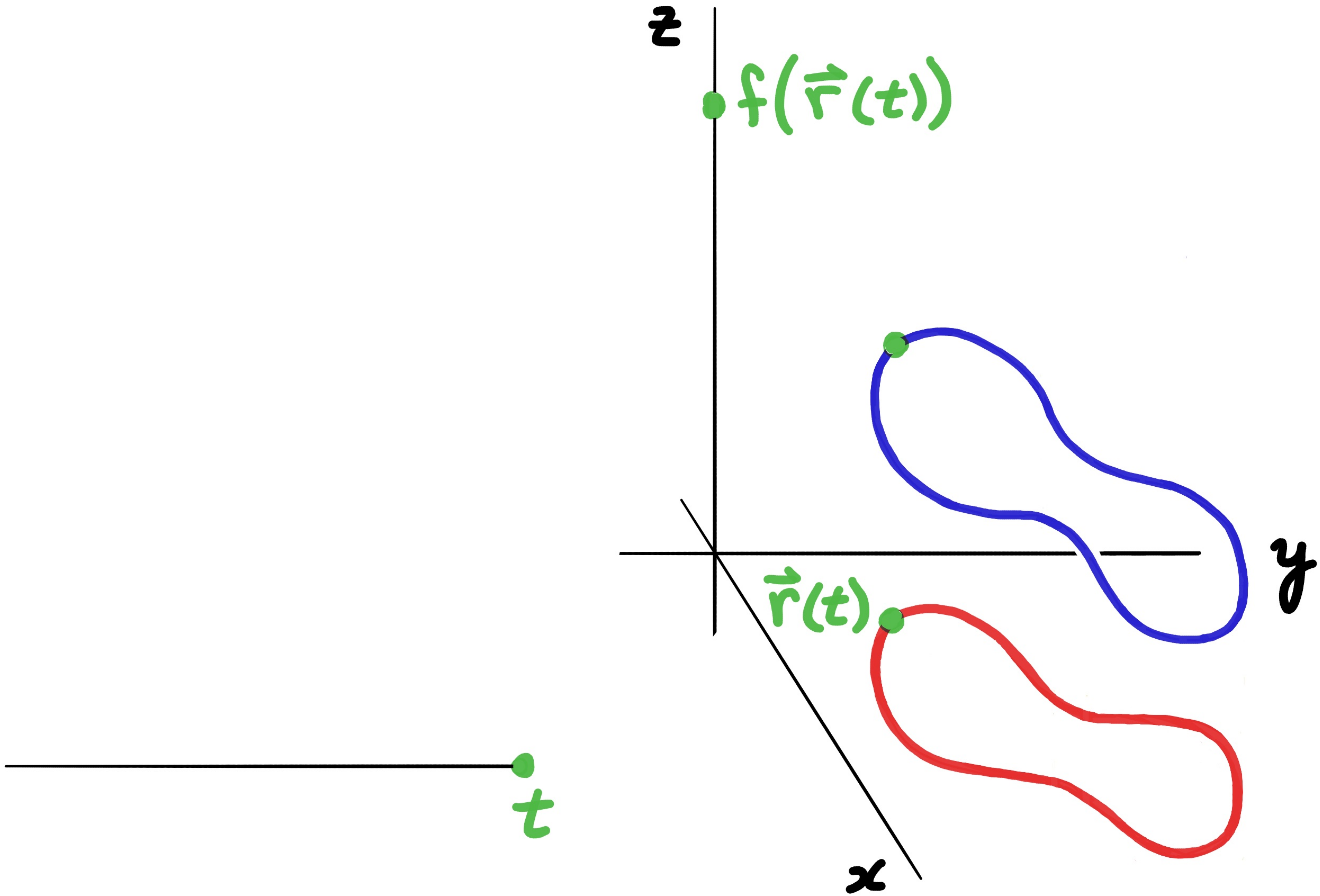


t
z
$f(\vec{r}(t))$
y
x
$\vec{r}(t)$

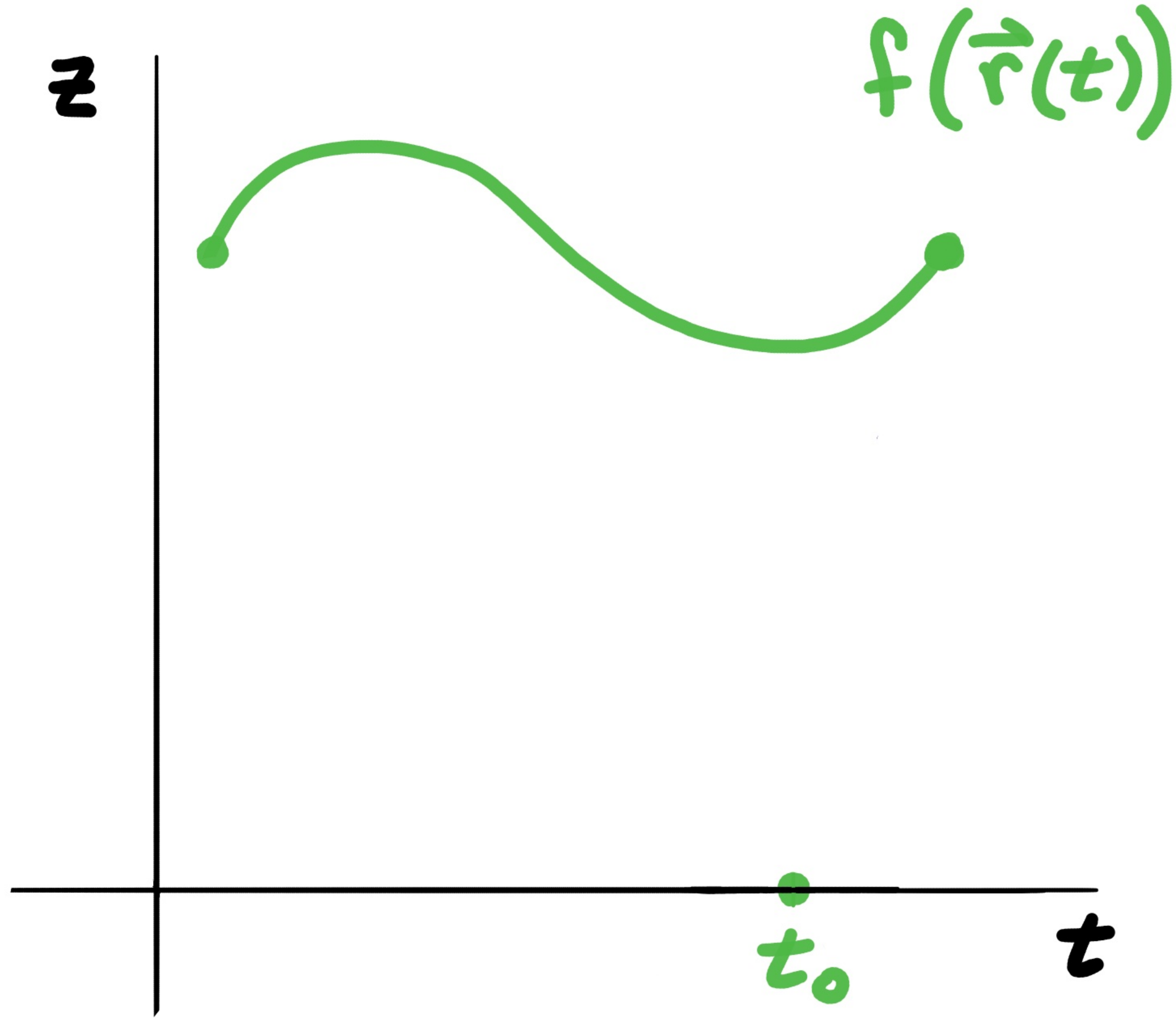
t

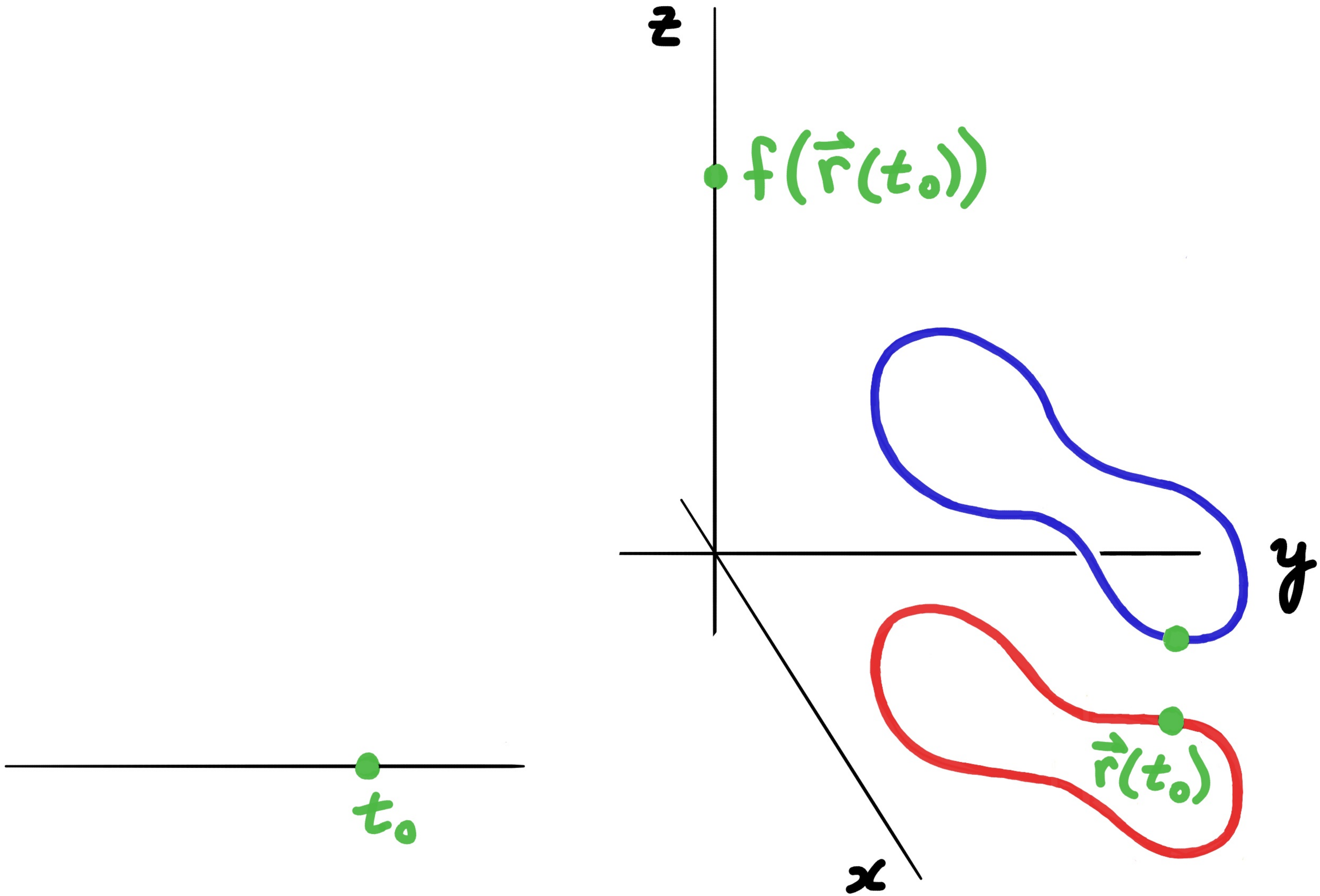


z
$f(\vec{r}(t))$
y
$\vec{r}(t)$
x

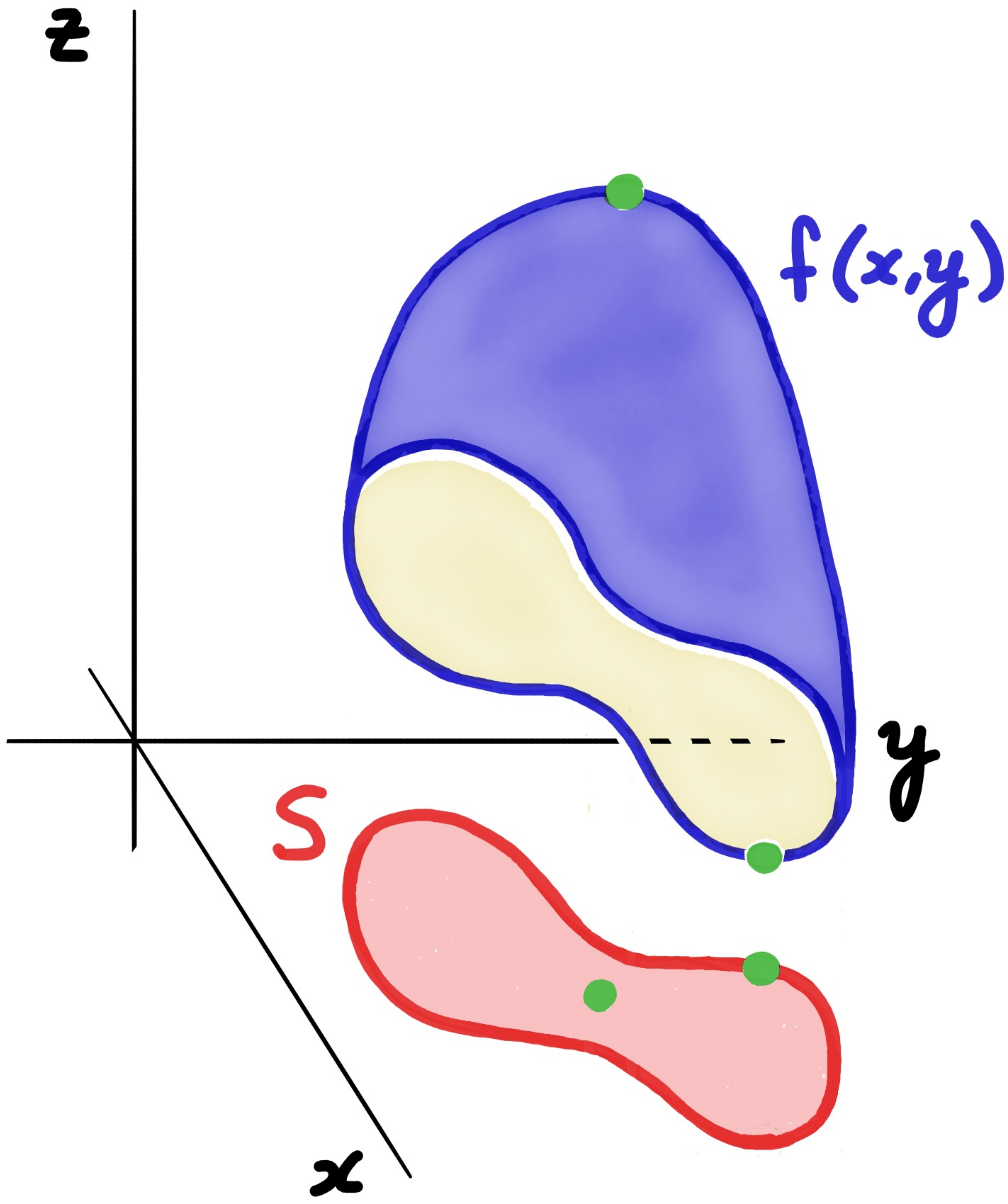


z
$f(\vec{r}(t))$
y
$\vec{r}(t)$
x
t

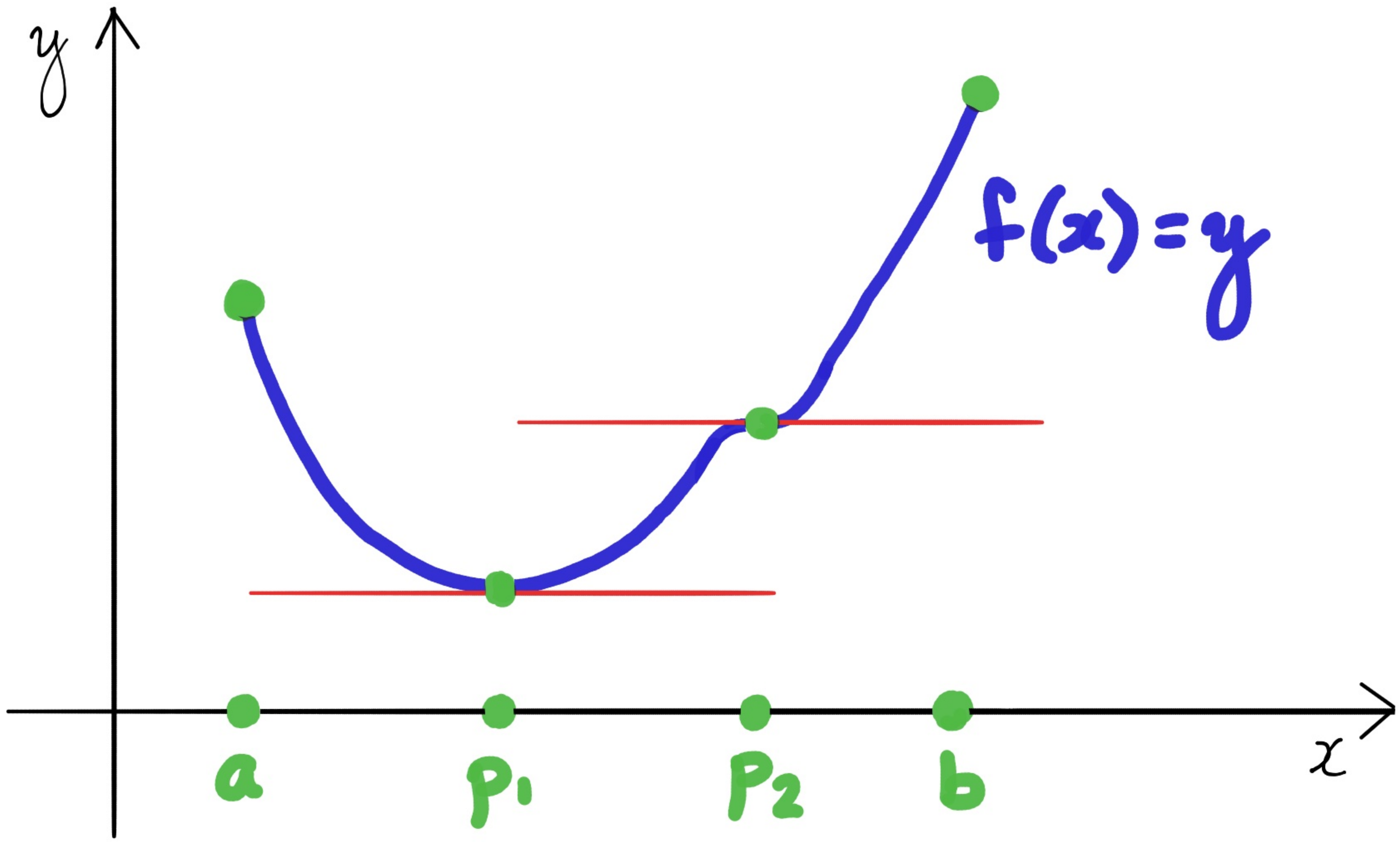

z
$f(\vec{r}(t))$
t_0
t



t_0
z
$f(\vec{r}(t_0))$
y
$\vec{r}(t_0)$
x


z
f(x,y)
y
S
x

max/min of $f:\mathbb{R}\to\mathbb{R}$ on $[a,b]$



critical points: a, b, p_1, p_2

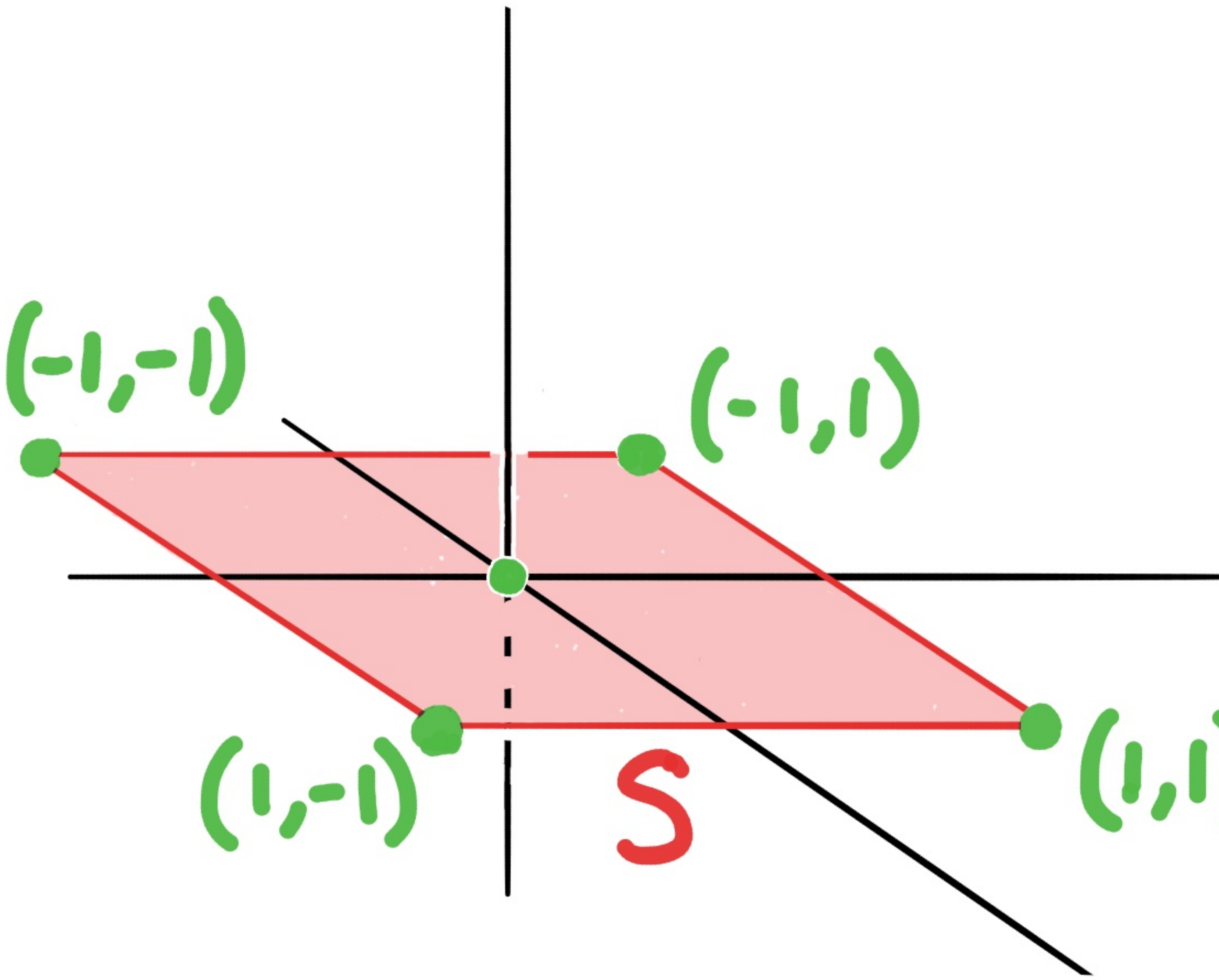
max/min will be among $f(a), f(b), f(p_1)$, and $f(p_2)$.

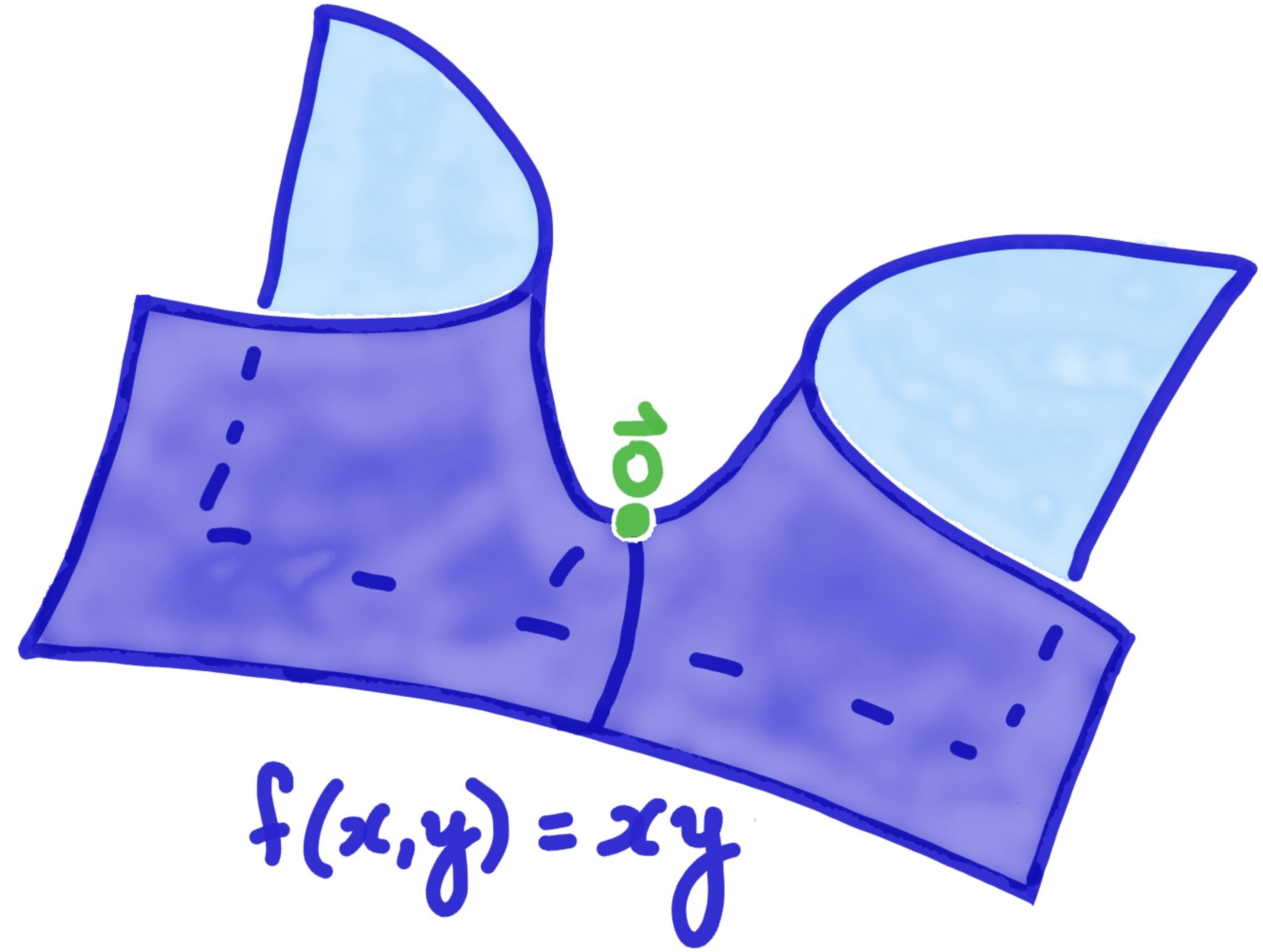
Problems:

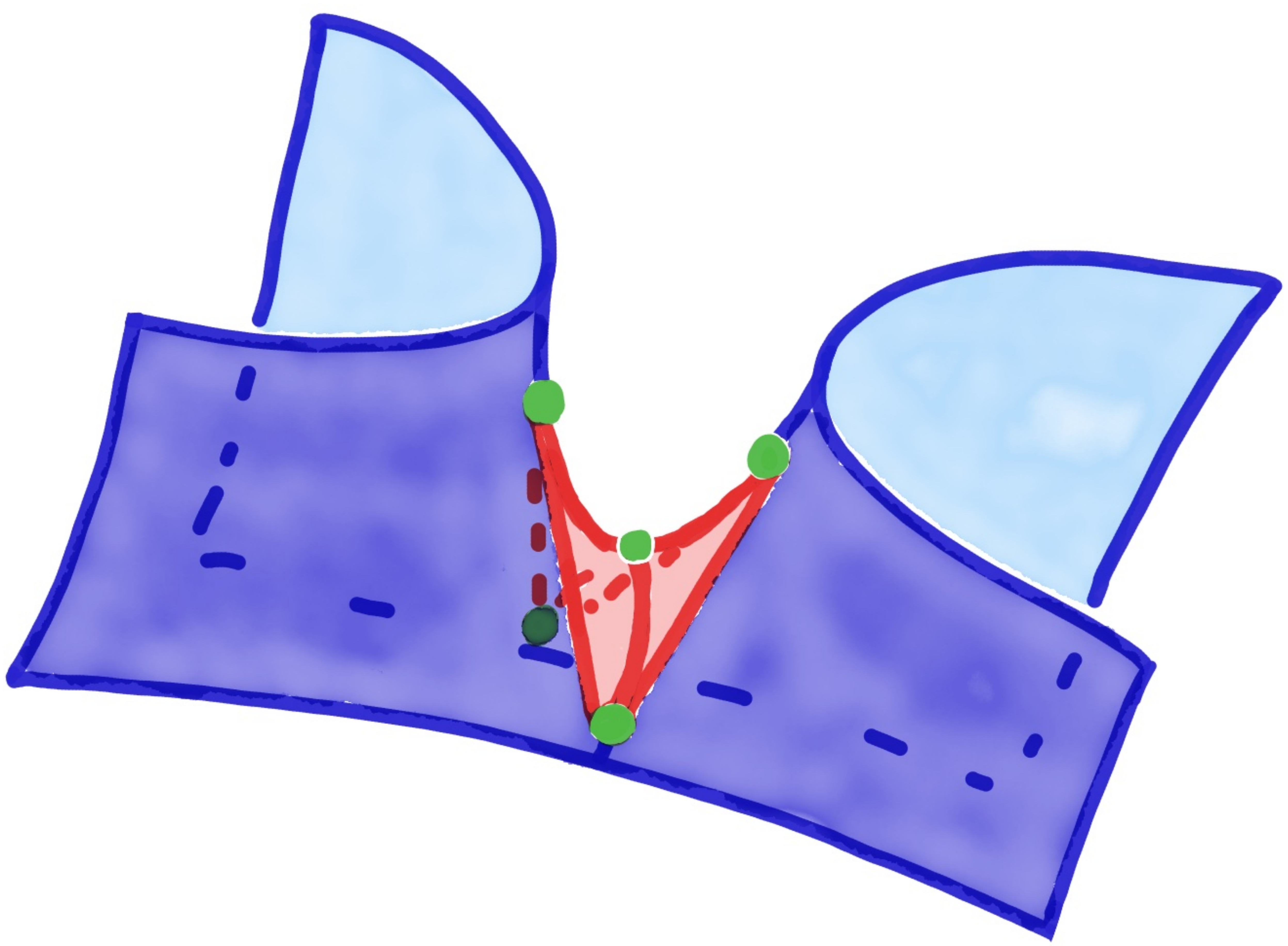
① $f(x,y) = \frac{4}{3}x^3 - x + y^2 + 3y$.
Find all critical points. Indicate whether each critical point gives a local max, local min, or a saddle point.

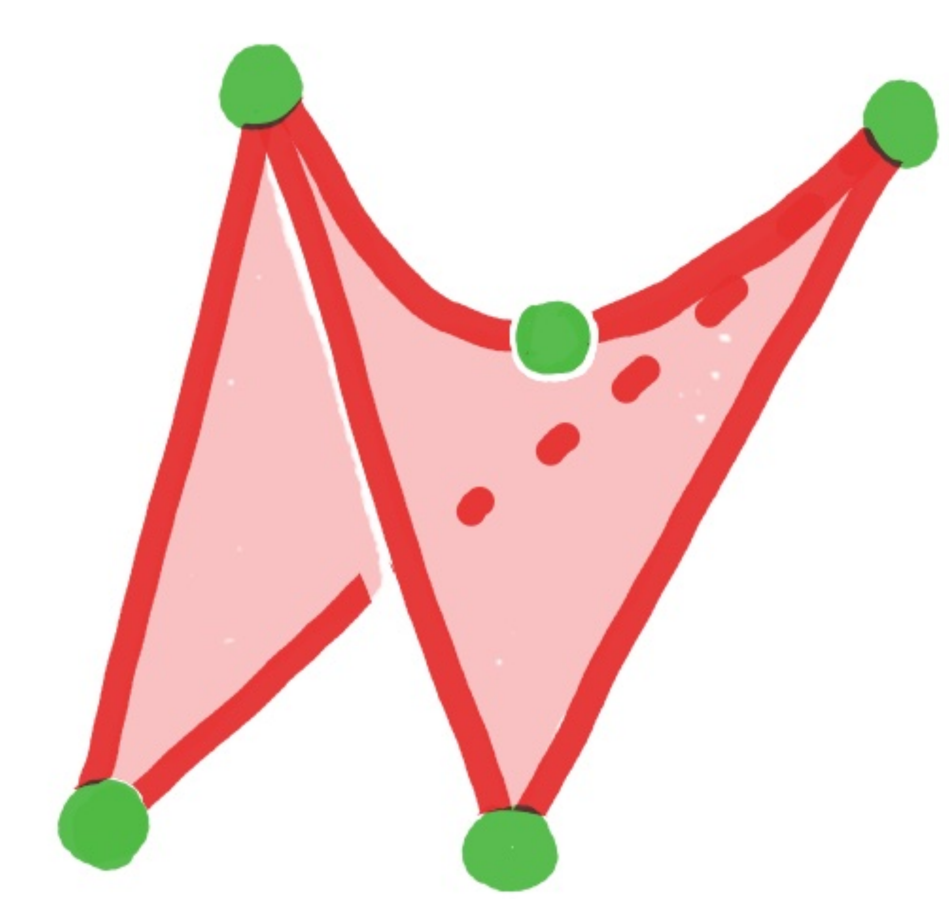
② Find max and min for $f(x,y) = xy$ on the domain $S = \{(x,y) : -1 \le x \le 1, -1 \le y \le 1\}$.

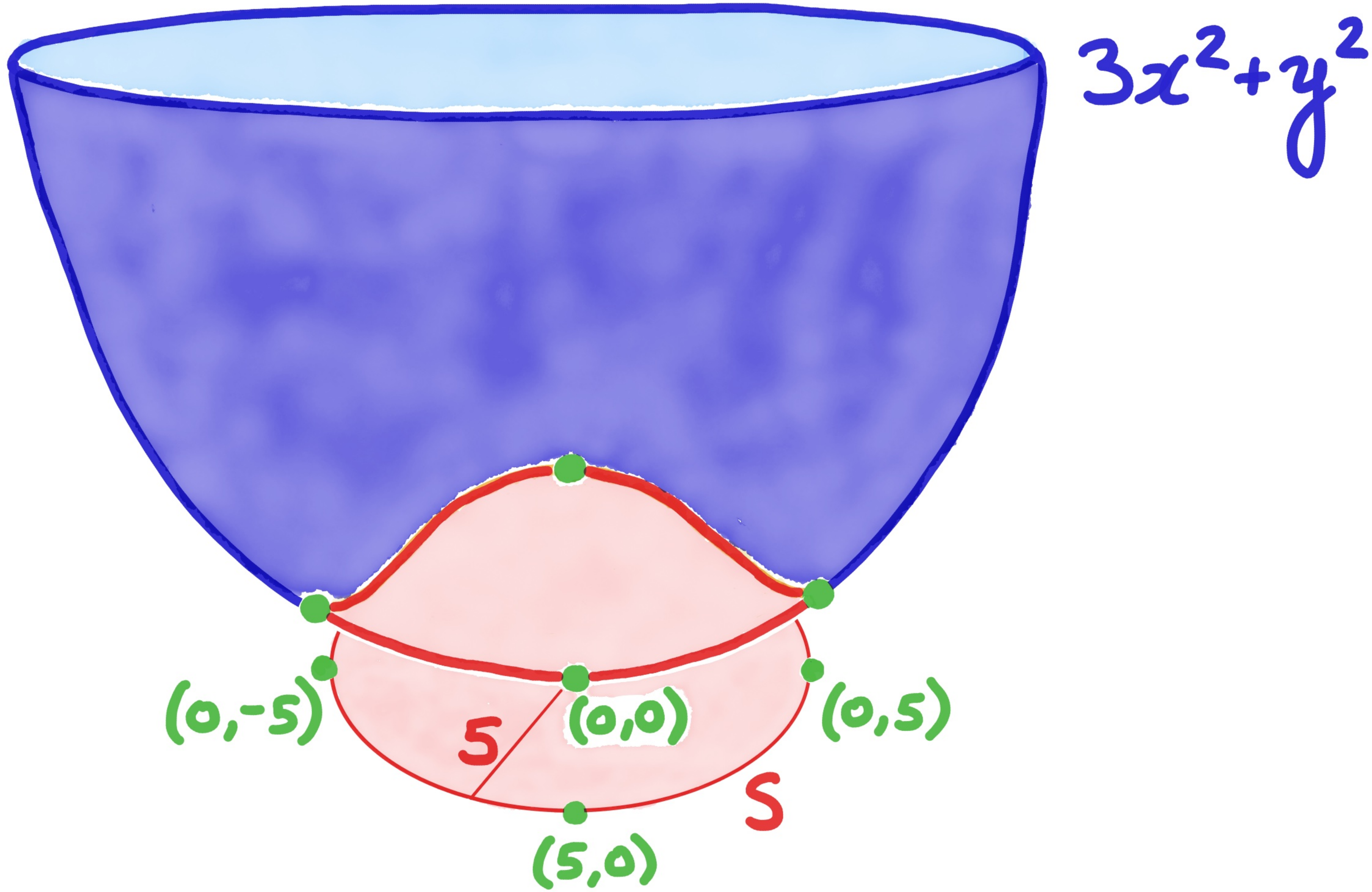
③ Find max and min for $f(x,y) = 3x^2 + y^2$ on the domain $S = \{(x,y) : x^2 + y^2 \le 25\}$.


(-1,-1)
(-1,1)
(1,-1)
(1,1)
S


$\vec{0}$
$f(x,y) = xy$







$3x^2+y^2$
(0,-5)
(0,0)
(0,5)
(5,0)
5
5