One

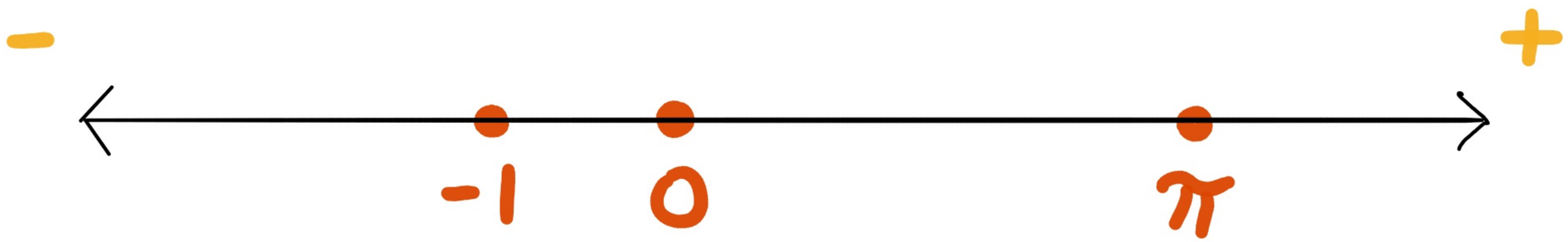
Ⅰ Line

Ⅱ Plane

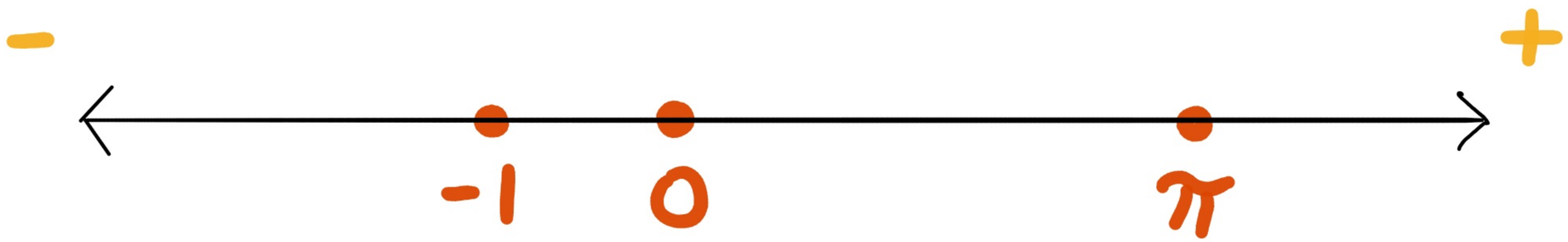
Ⅲ 3-space

㊣ Line

$\mathbb{R}$ "the real line"

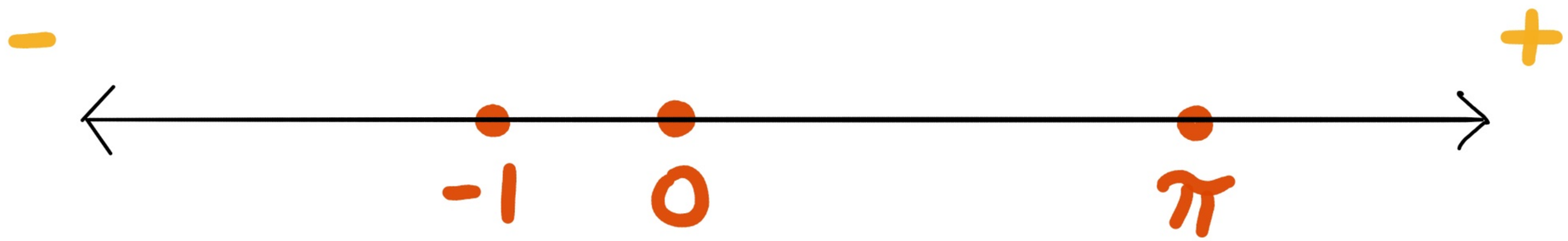


$\mathbb{R}$ "the real line"



$-1 \in \mathbb{R}$ $0 \in \mathbb{R}$ $\pi \in \mathbb{R}$

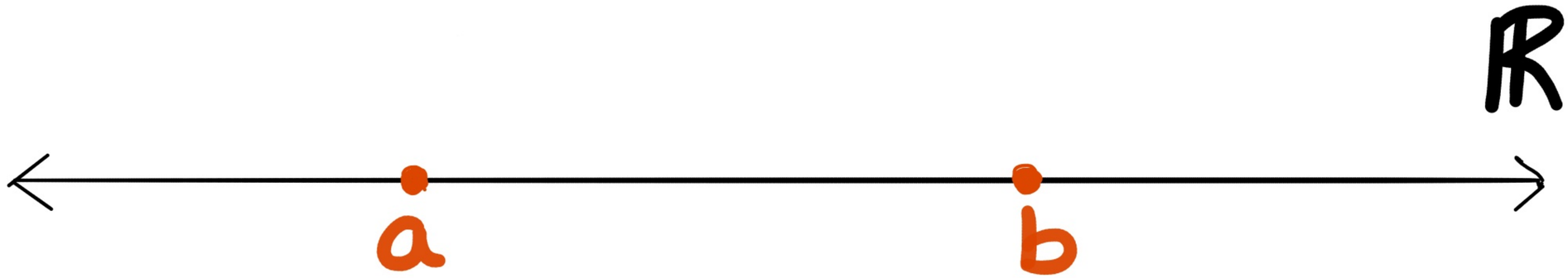
$\mathbb{R}$ "the real line"



$-1 \in \mathbb{R}$ $0 \in \mathbb{R}$ $\pi \in \mathbb{R}$

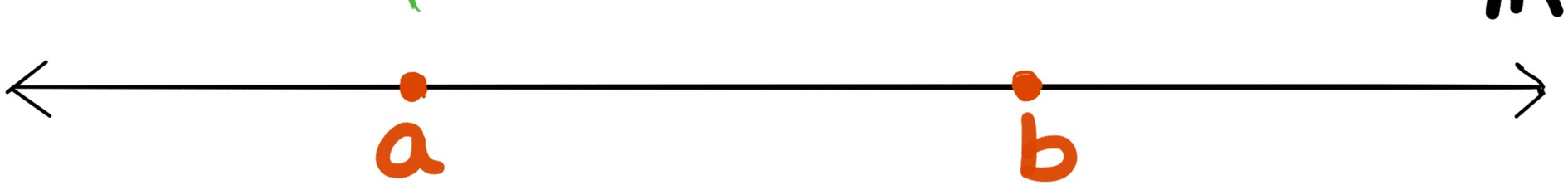
$-1, 0, \pi \in \mathbb{R}$

Distance between points in $\mathbb{R}$



Distance between points in $\mathbb{R}$

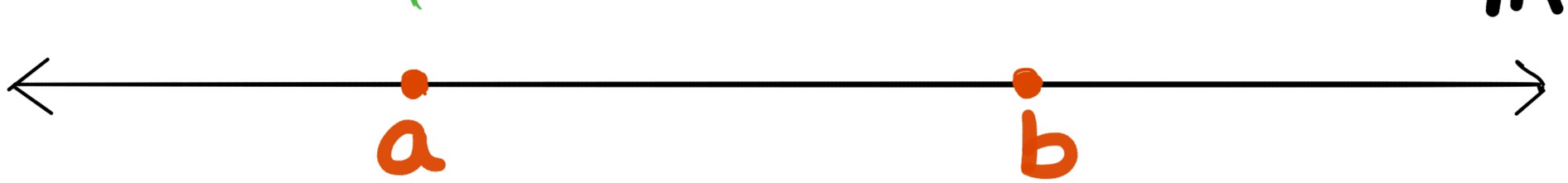
$|a-b|$



$\mathbb{R}$

Distance between points in $\mathbb{R}$

$|a-b|$

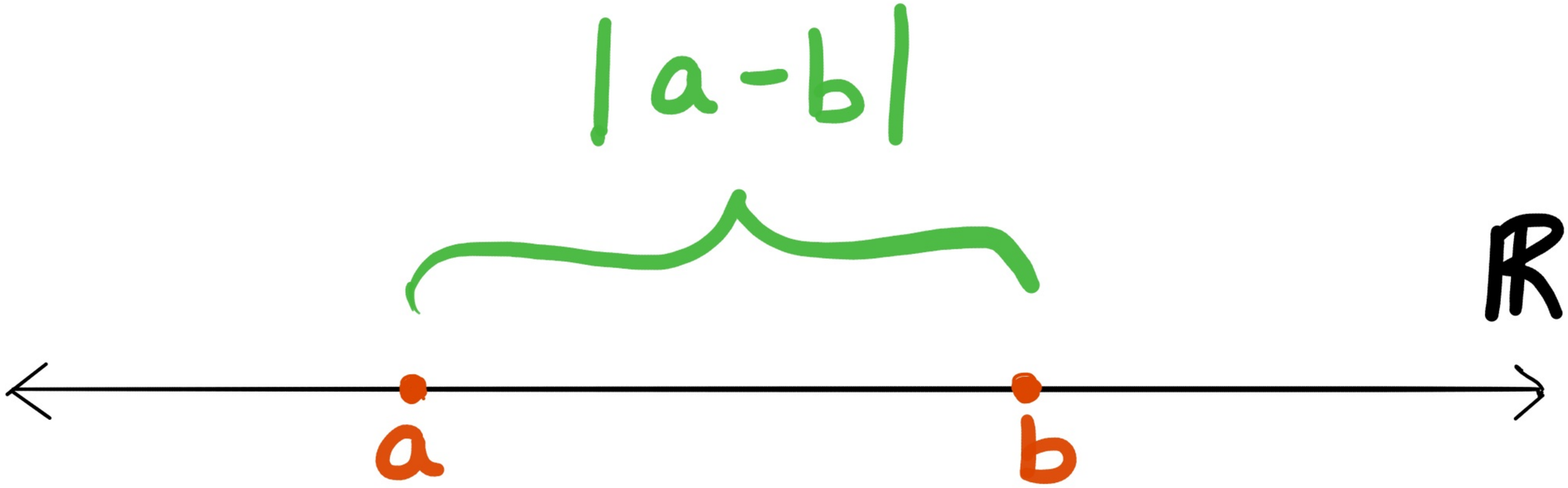


$\mathbb{R}$

Distance between:

- 5 and 3 is
- 2 and -7 is
- 3 and 5 is

Distance between points in $\mathbb{R}$



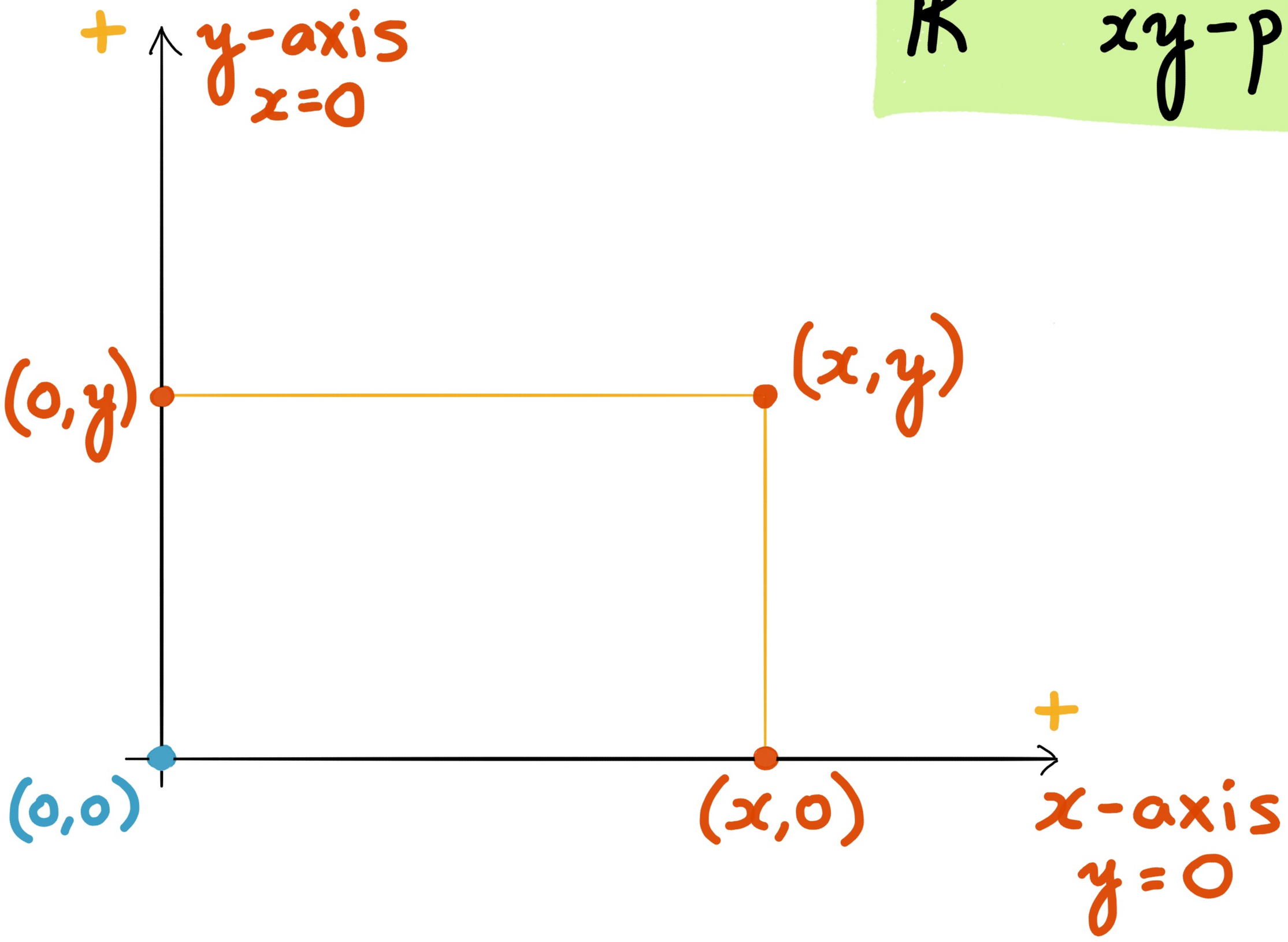
$\mathbb{R}$

Distance between:

- 5 and 3 is $|5-3| = |2| = 2$.
- 2 and -7 is $|2-(-7)| = |2+7| = 9$.
- 3 and 5 is $|3-5| = |-2| = 2$.

Ⅱ Plane

$\mathbb{R}^2$ "xy-plane"



$\mathbb{R}^2$ "xy-plane"

+ y-axis
x=0

(0,y)

(x,y)

$(2,3) \in \mathbb{R}^2$

$(-1,\sqrt{2}) \in \mathbb{R}^2$

(0,0)

(x,0)

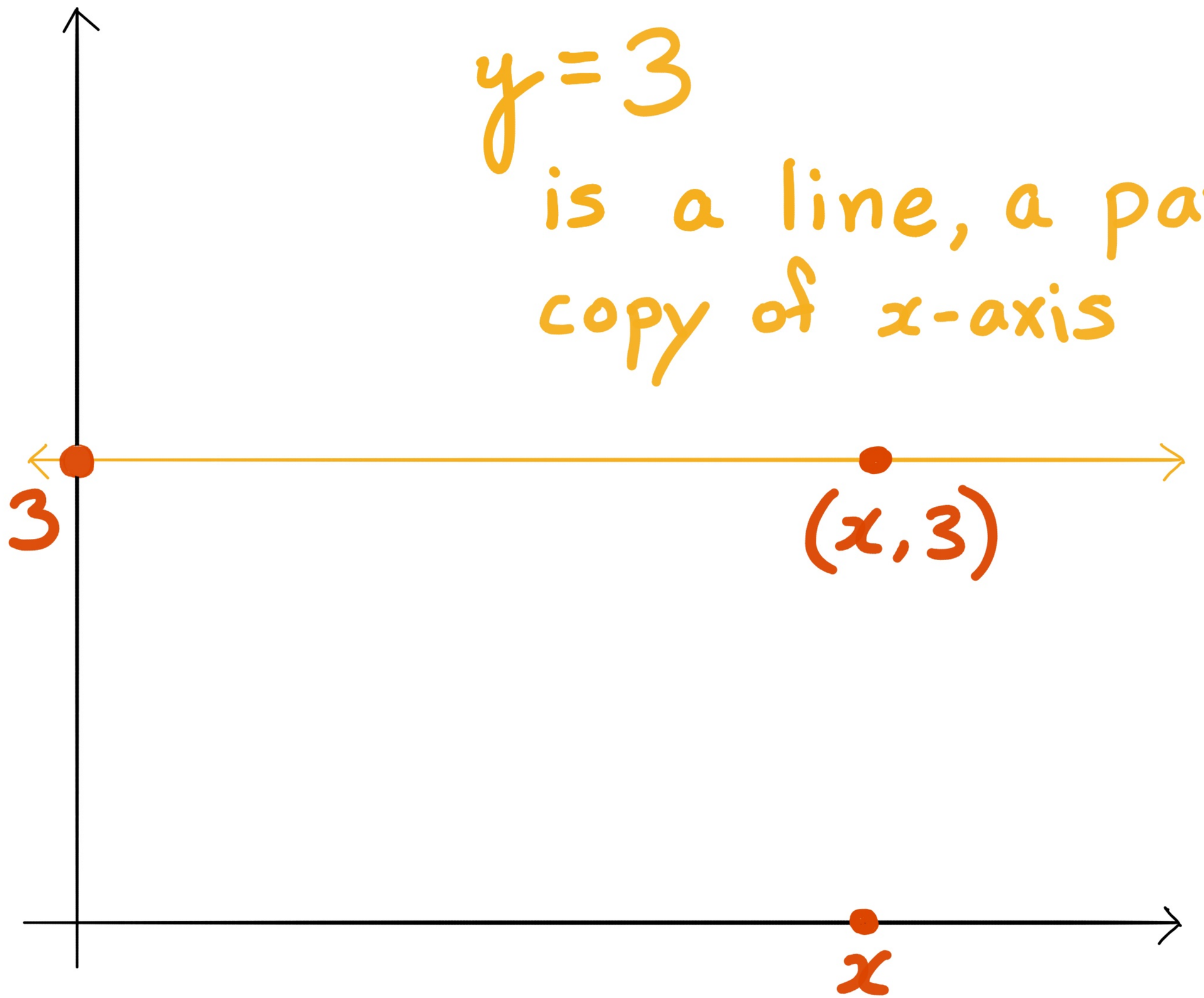
+

x-axis
y=0

$\mathbb{R}^2$

$y=3$

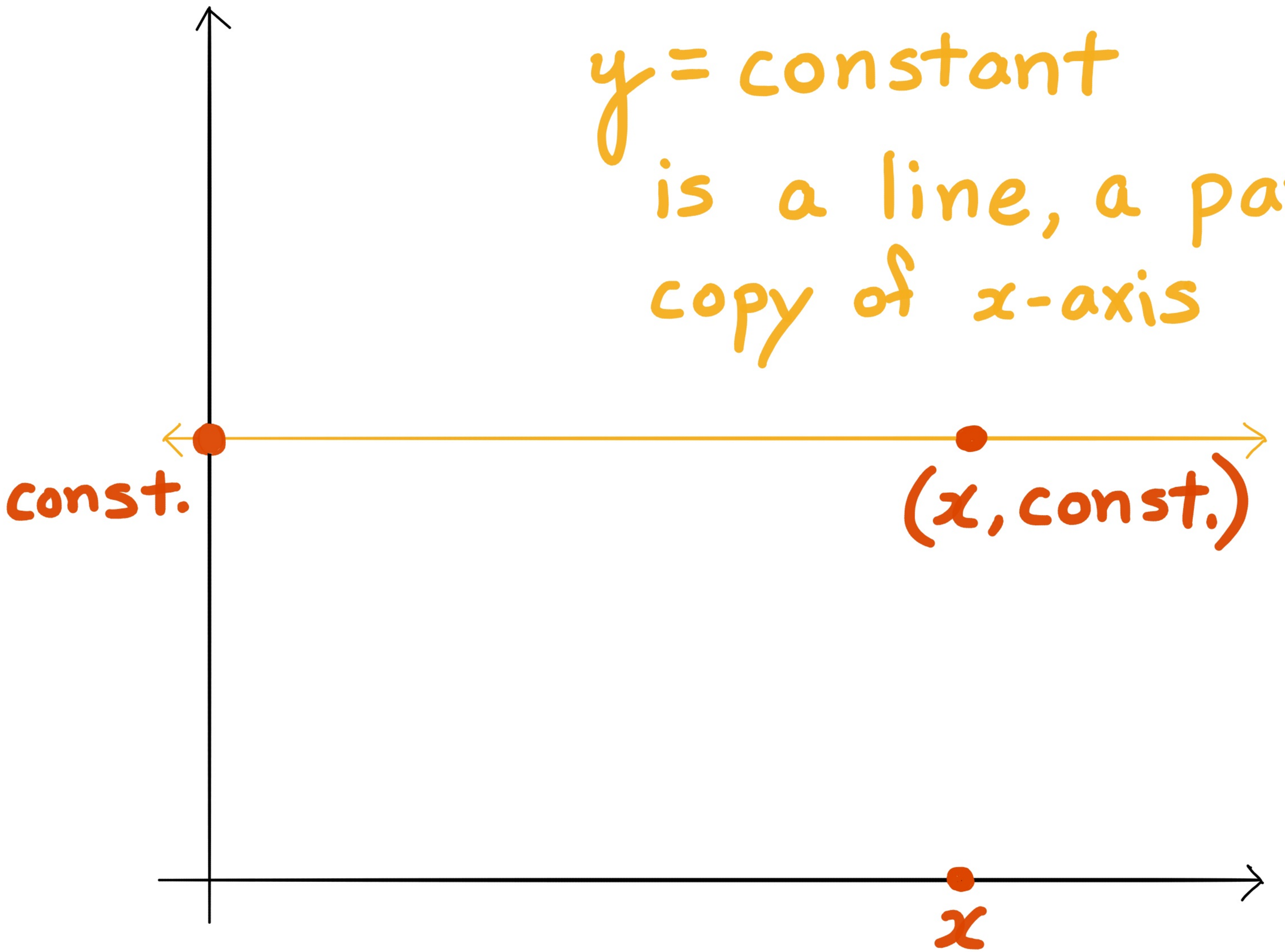
is a line, a parallel copy of x-axis



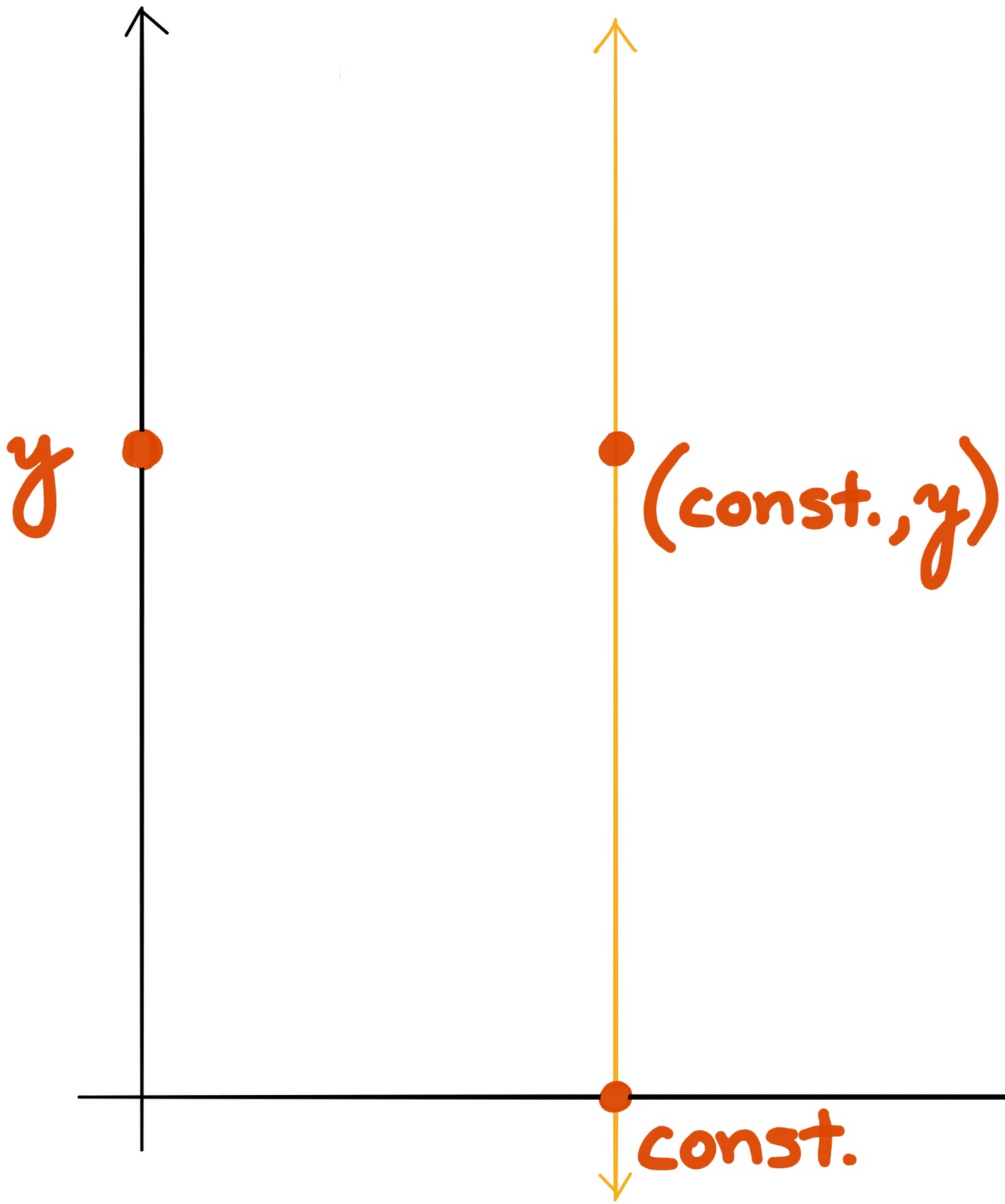
$\mathbb{R}^2$

y = constant

is a line, a parallel copy of x-axis



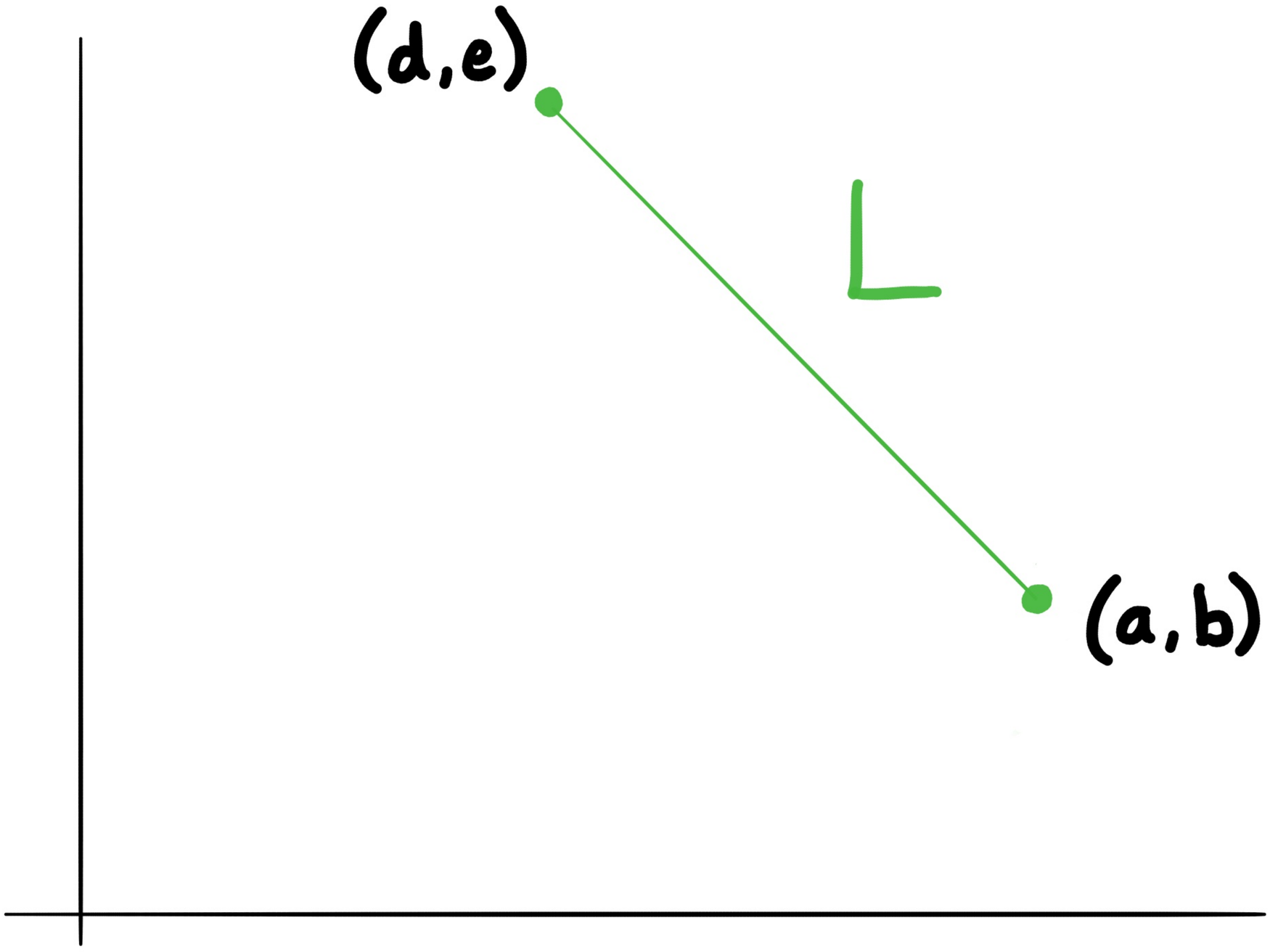
$\mathbb{R}^2$



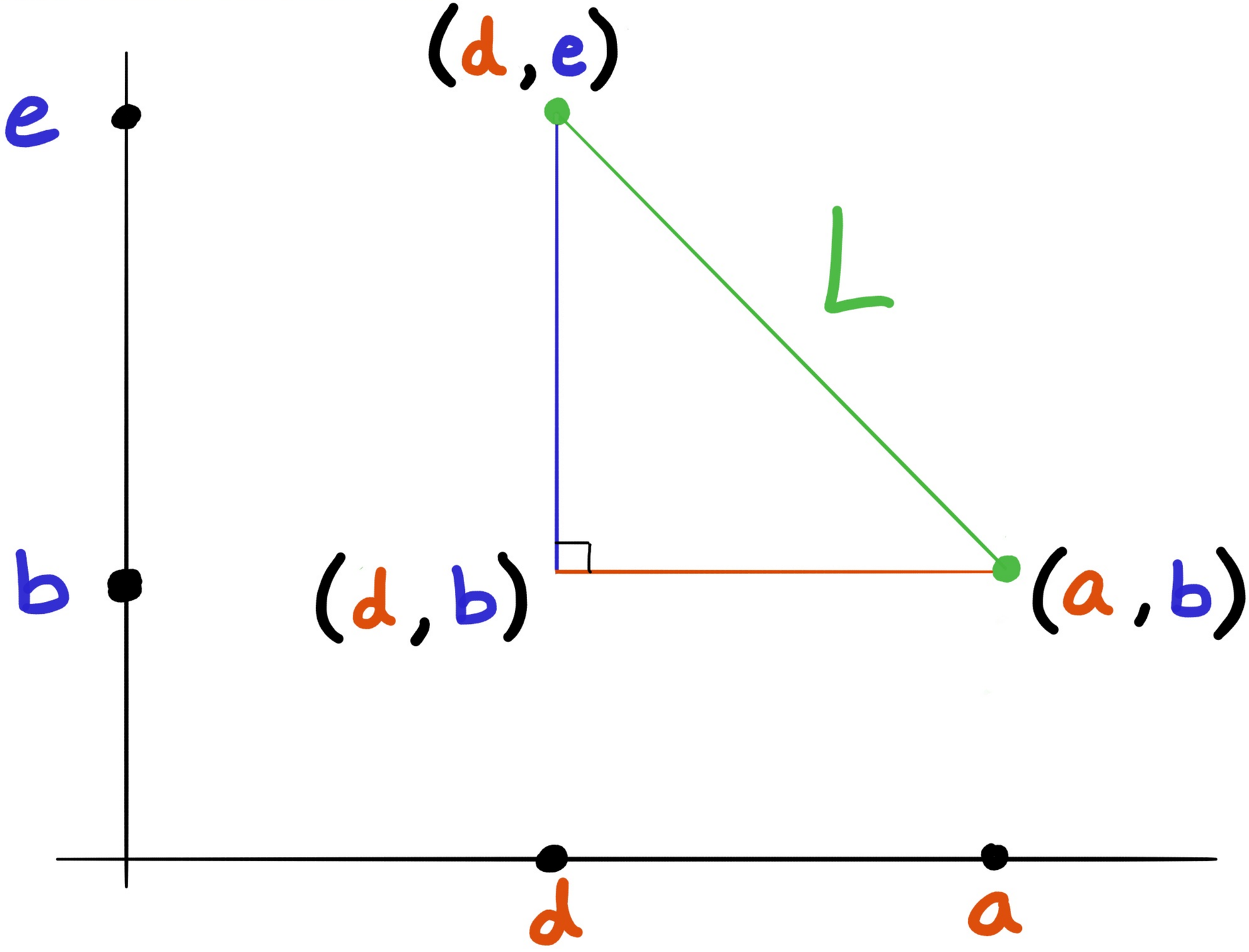
$x = \text{const.}$

is a line

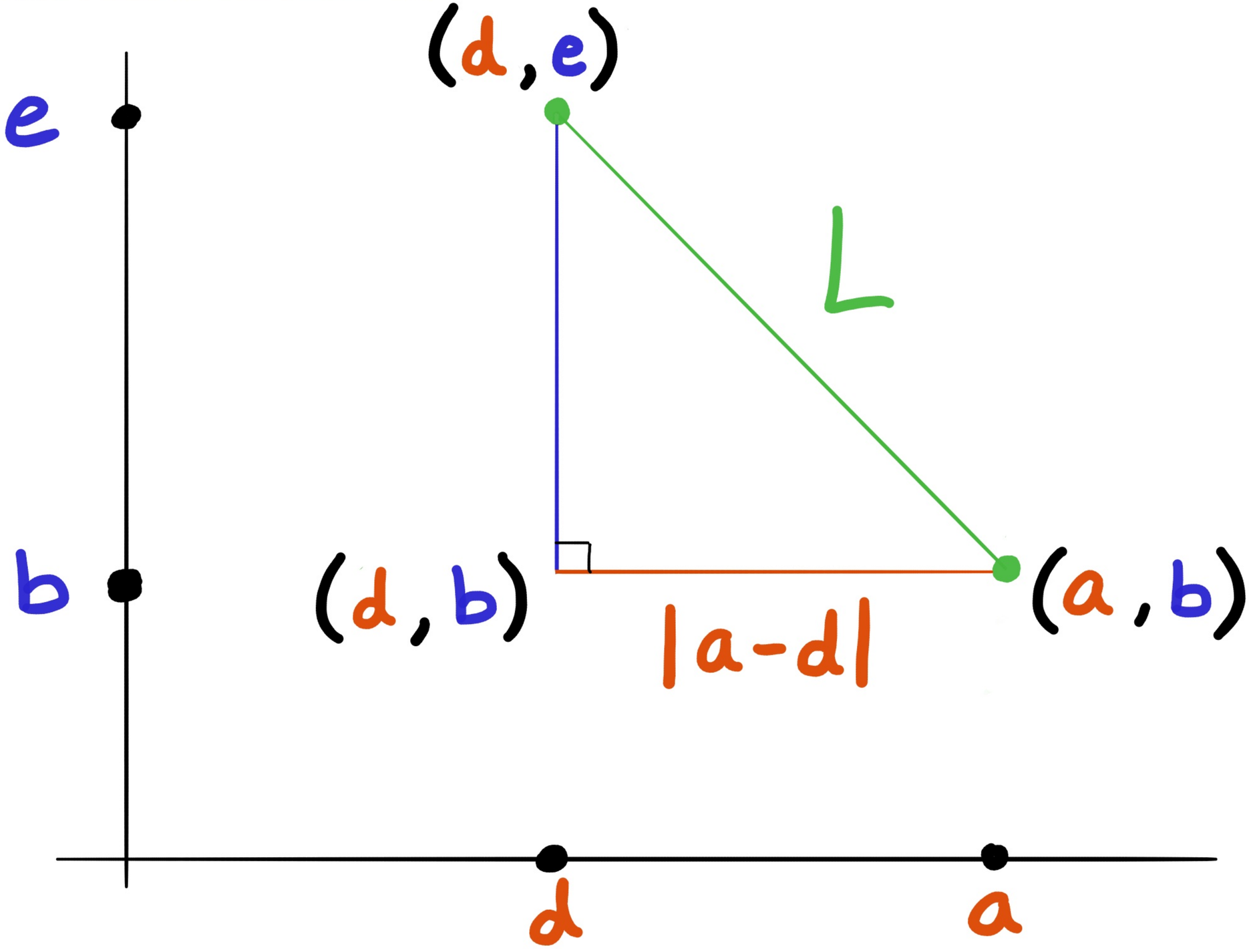
Distance between $(a,b),(d,e) \in \mathbb{R}^2$



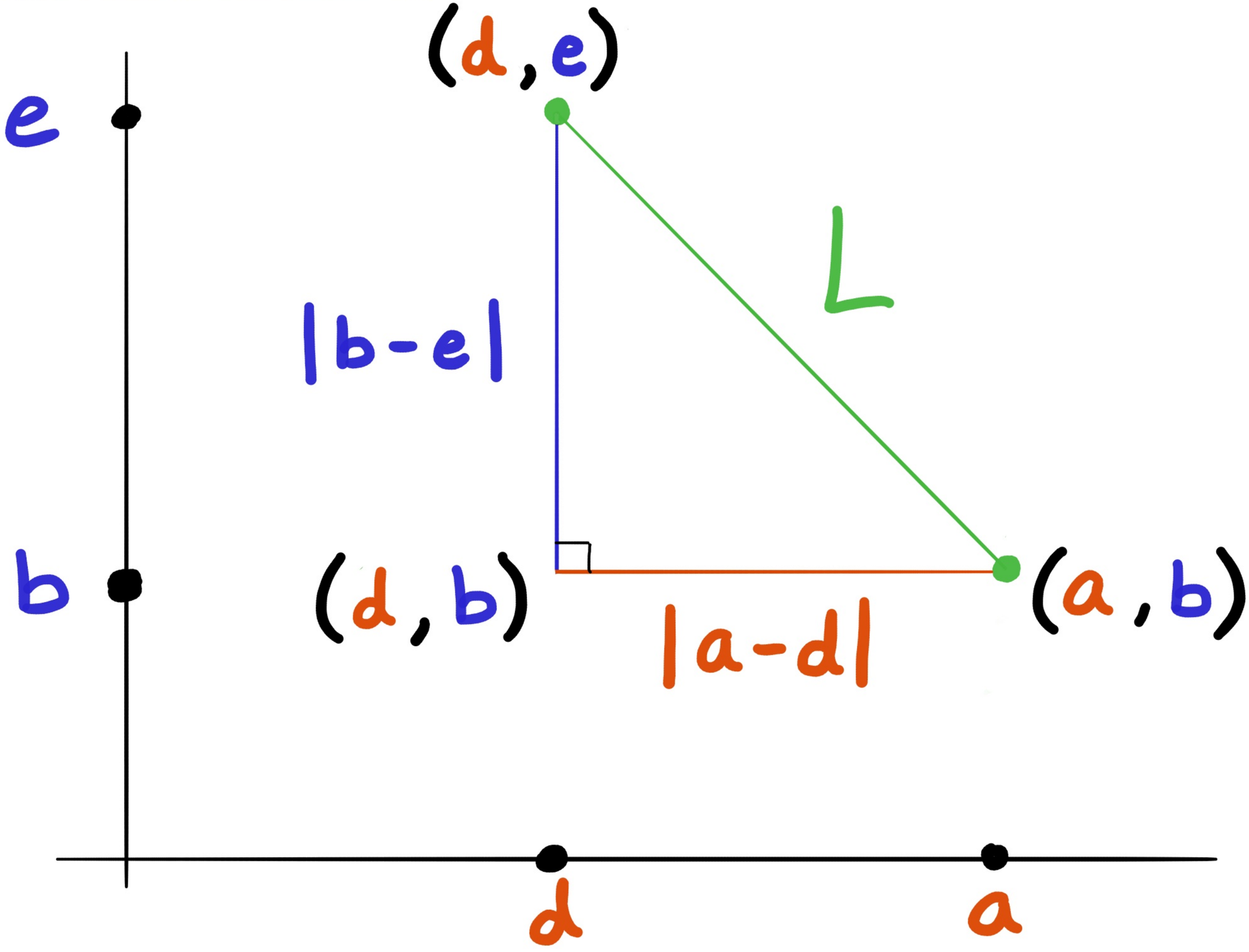
Distance between $(a,b),(d,e)\in\mathbb{R}^2$



Distance between $(a,b),(d,e)\in\mathbb{R}^2$



Distance between $(a,b), (d,e) \in \mathbb{R}^2$



Using Pythagorean Theorem, distance between $(a,b),(d,e) \in \mathbb{R}^2$ is

$$L = \sqrt{|a-d|^2 + |b-e|^2}$$

Example: Distance between (2,7) and (5,3) is

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Example: Distance between $(2,7)$ and $(5,3)$ is $\sqrt{3^2+4^2}$

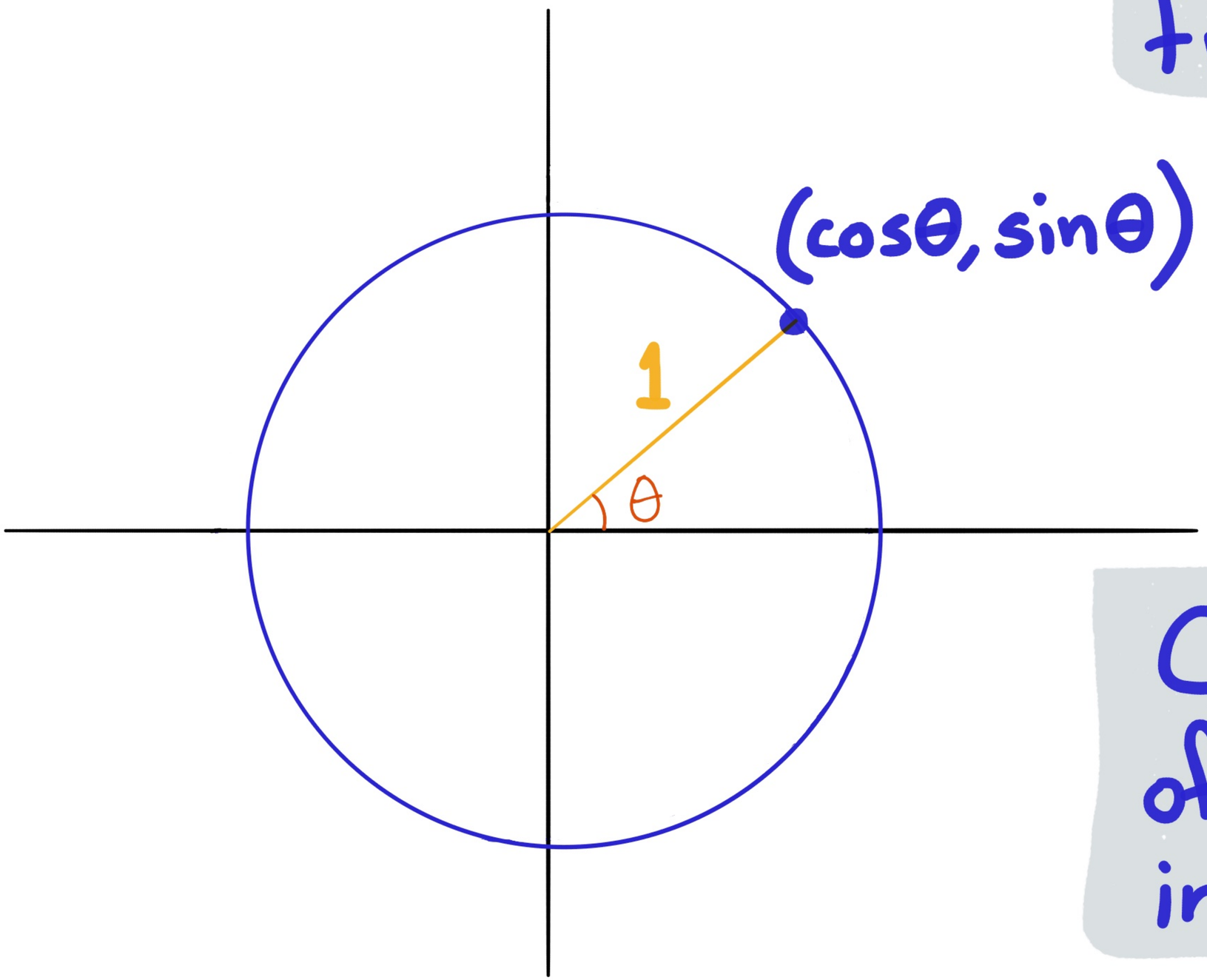
Using Pythagorean Theorem, distance between $(a,b),(d,e) \in \mathbb{R}^2$ is

$$L = \sqrt{|a-d|^2 + |b-e|^2}$$

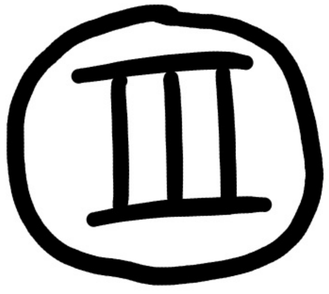
Example: Distance between $(2,7)$ and $(5,3)$ is $\sqrt{3^2+4^2} = \sqrt{9+16} = 5$.

"unit circle"

Points in $\mathbb{R}^2$ distance 1 from (0,0)

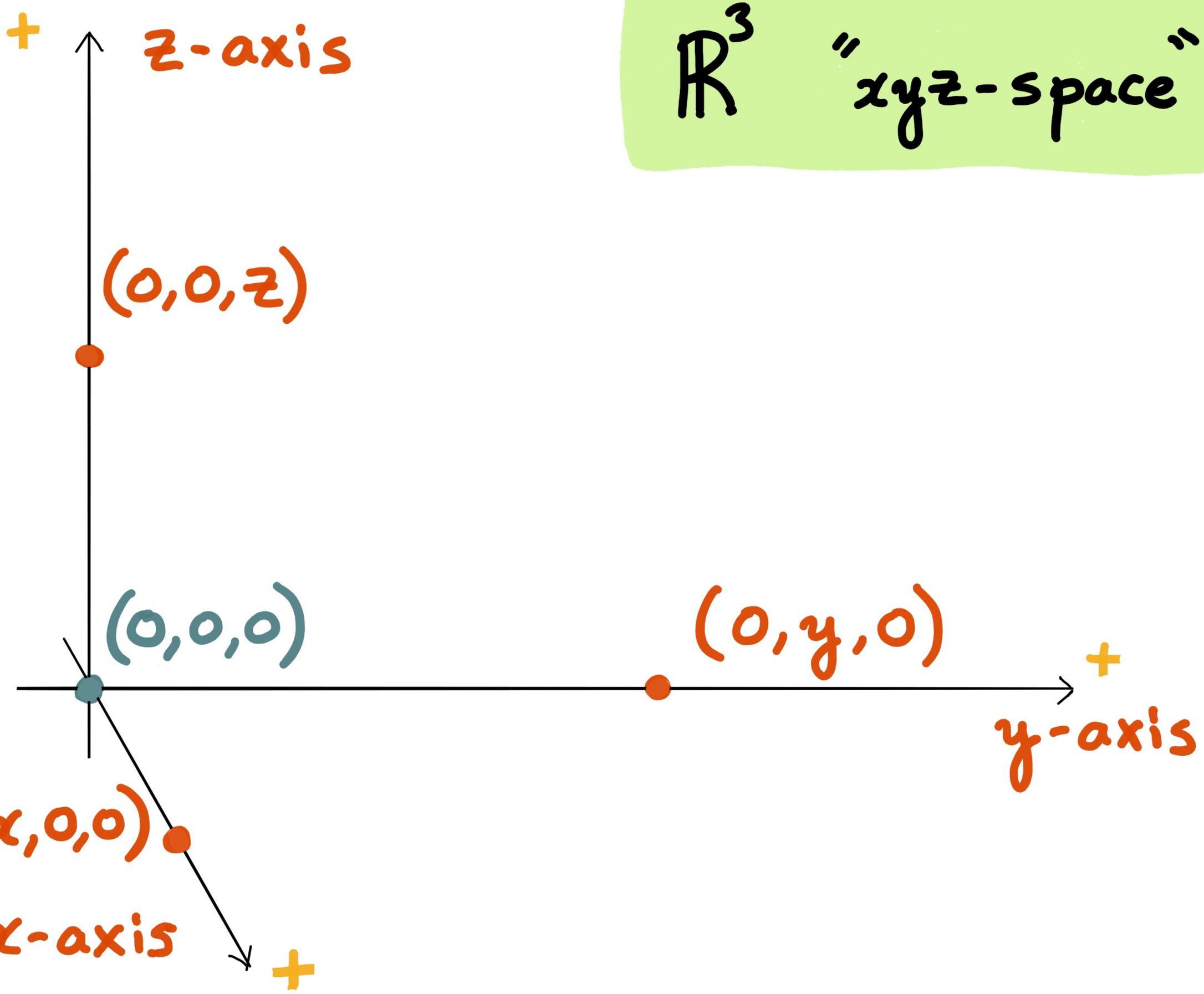


Circle's worth of directions in $\mathbb{R}^2$

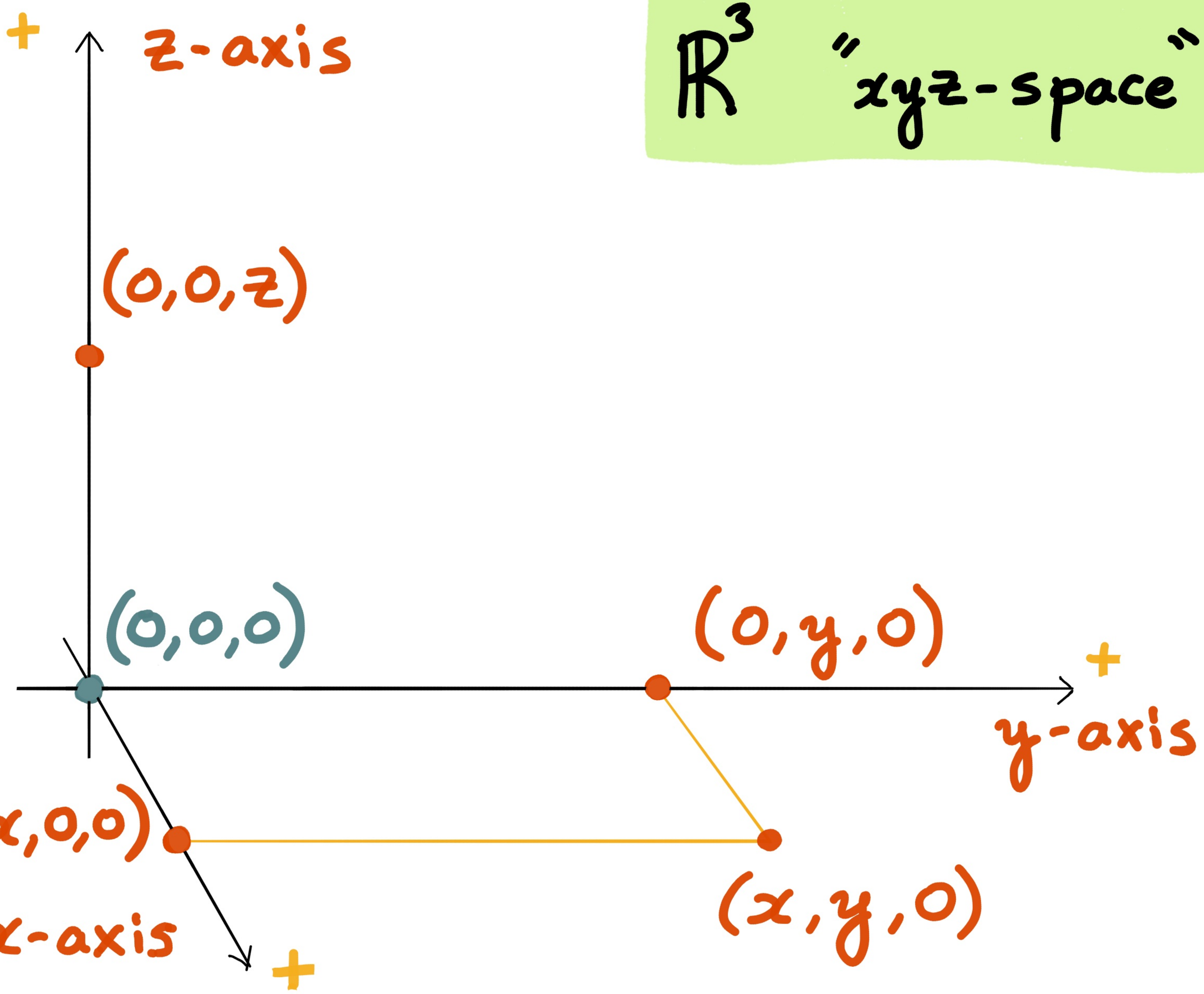


3-space

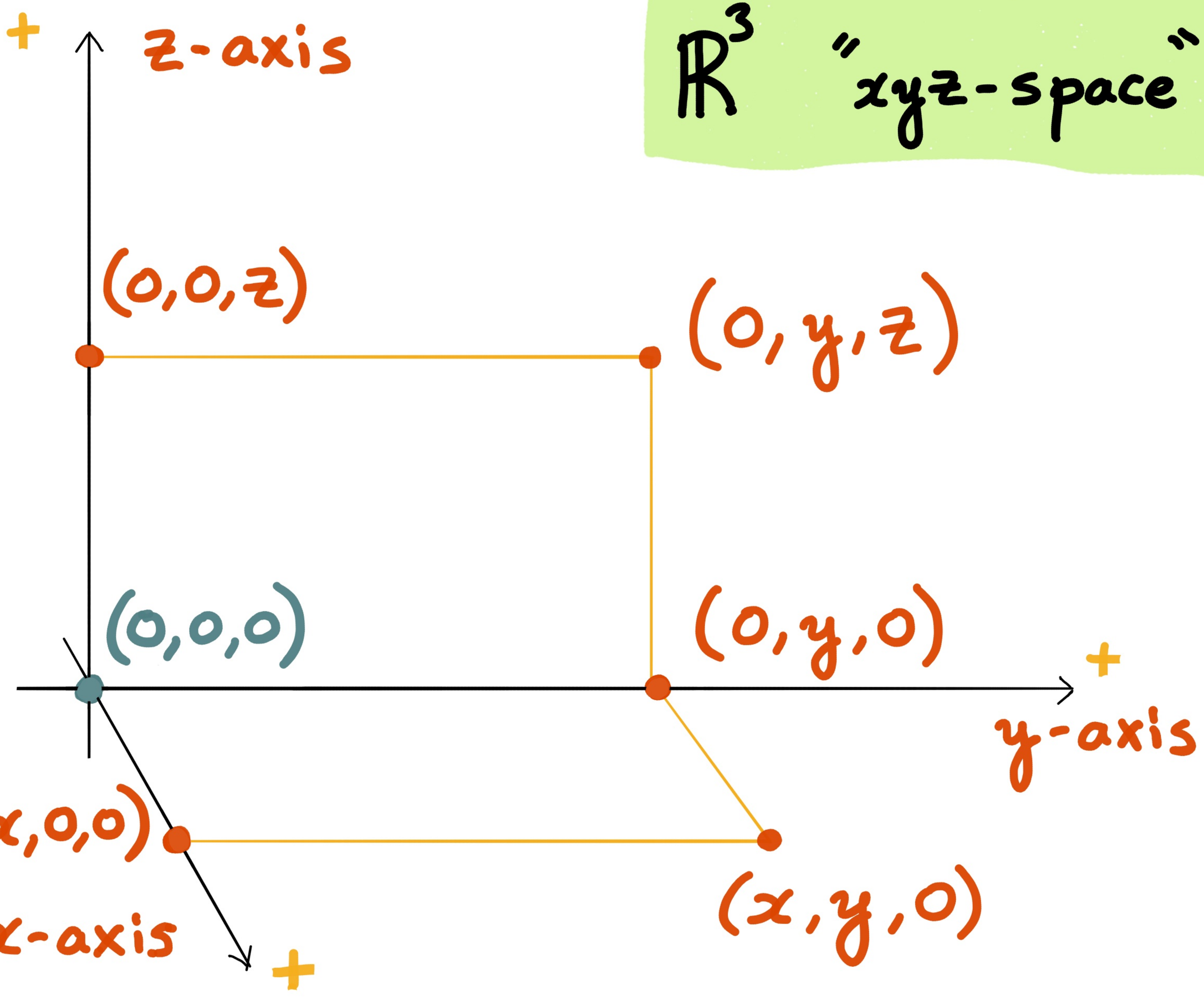
$\mathbb{R}^3$ "xyz-space"



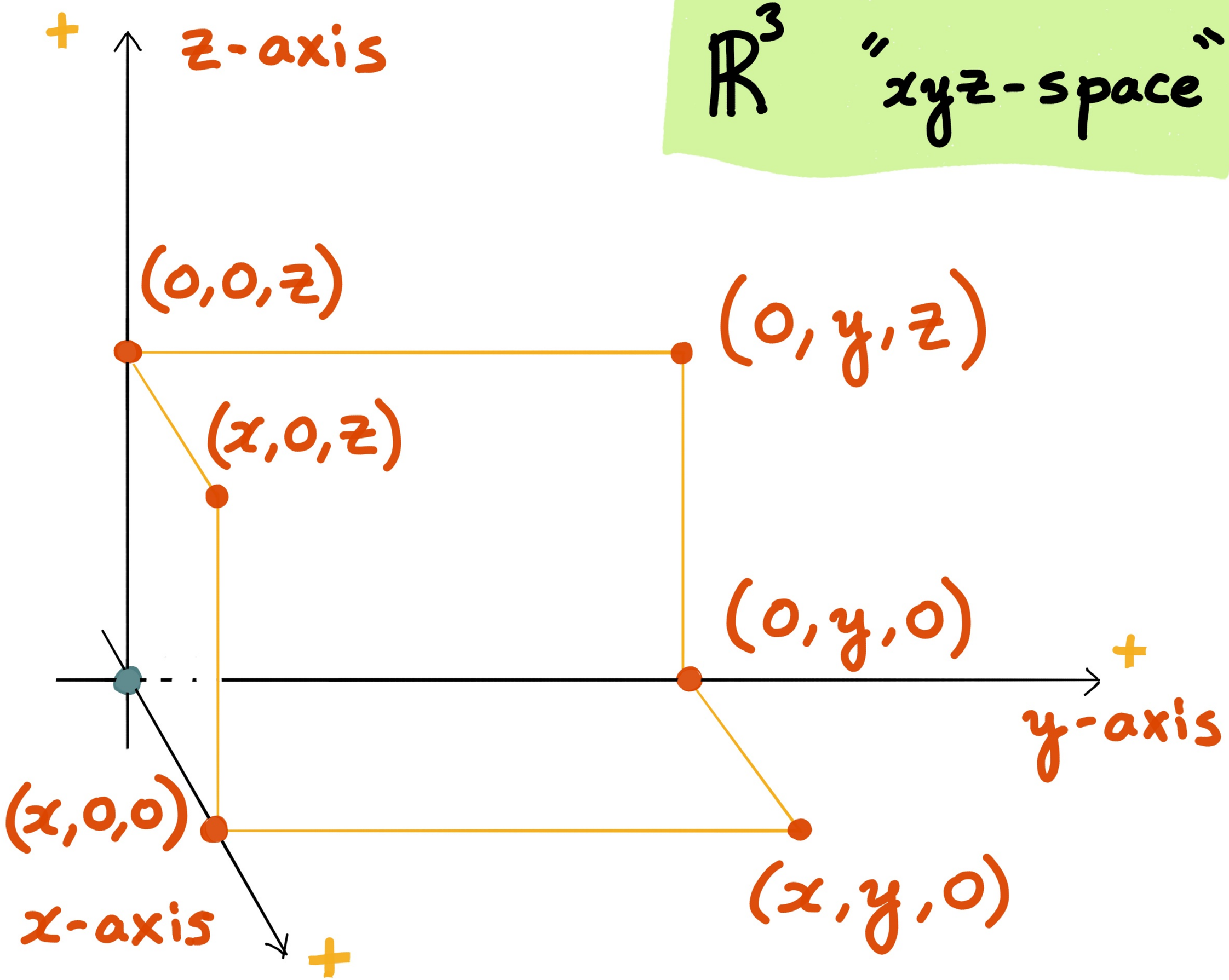
$\mathbb{R}^3$ "xyz-space"



$\mathbb{R}^3$ "xyz-space"



$\mathbb{R}^3$ "xyz-space"



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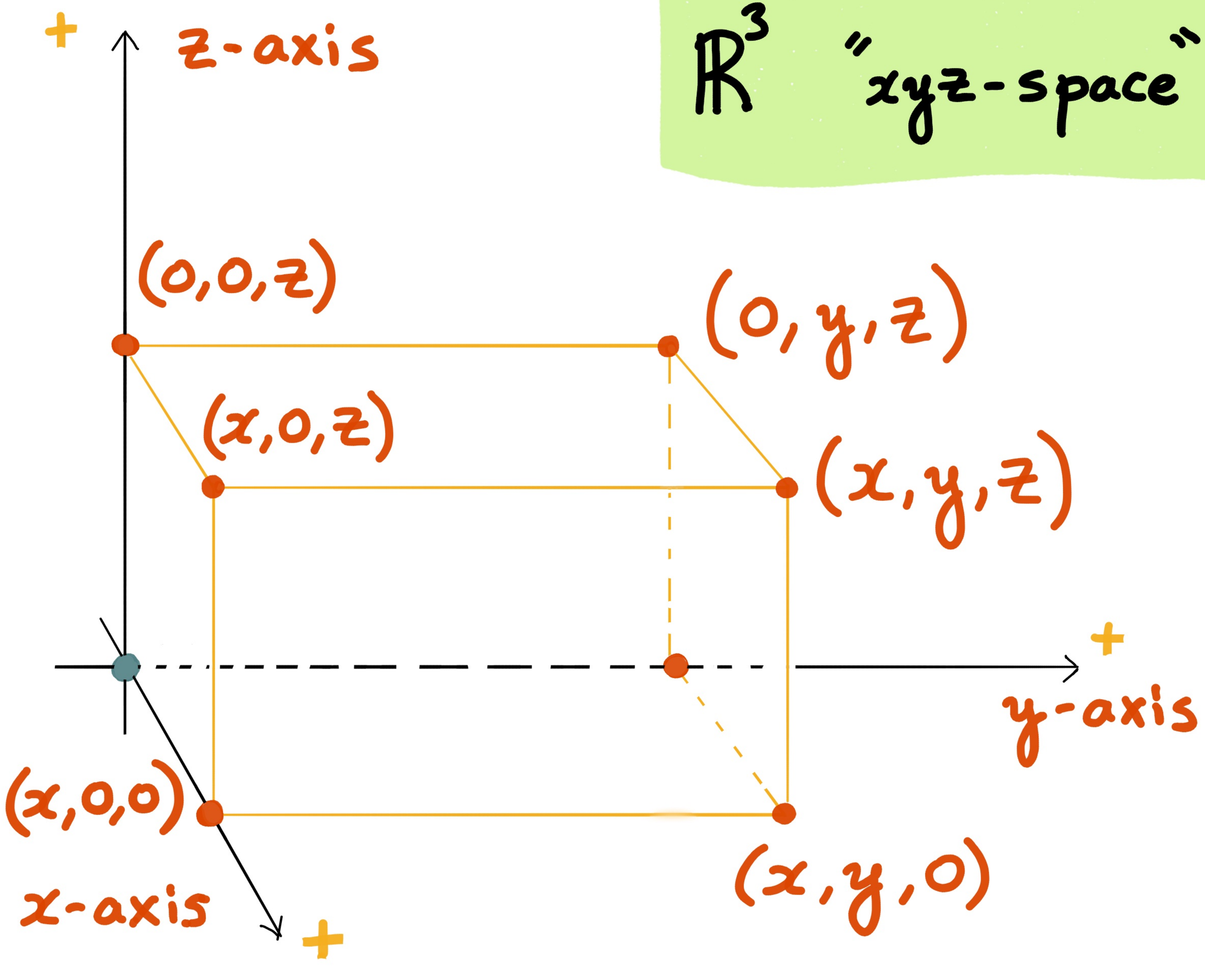
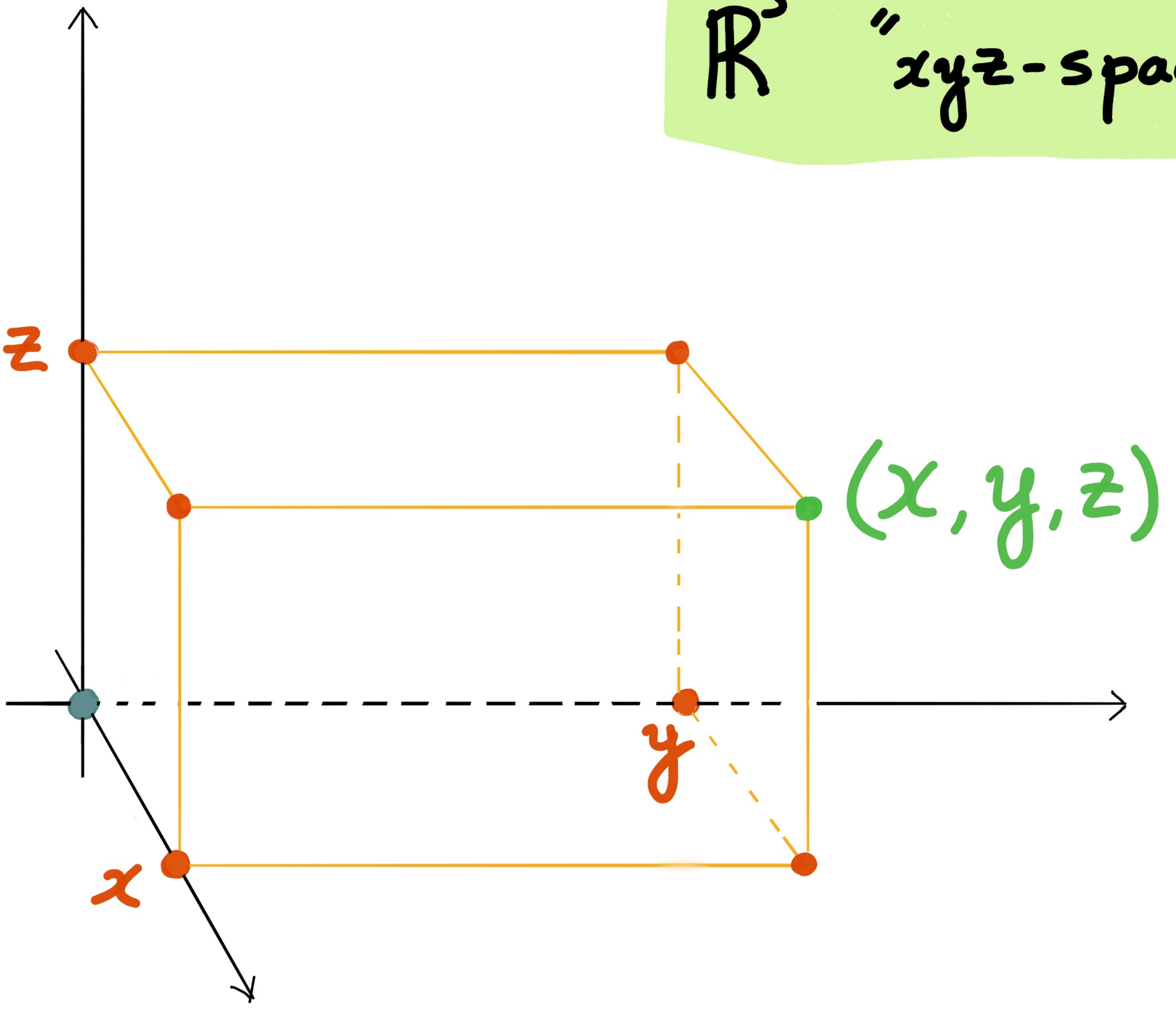
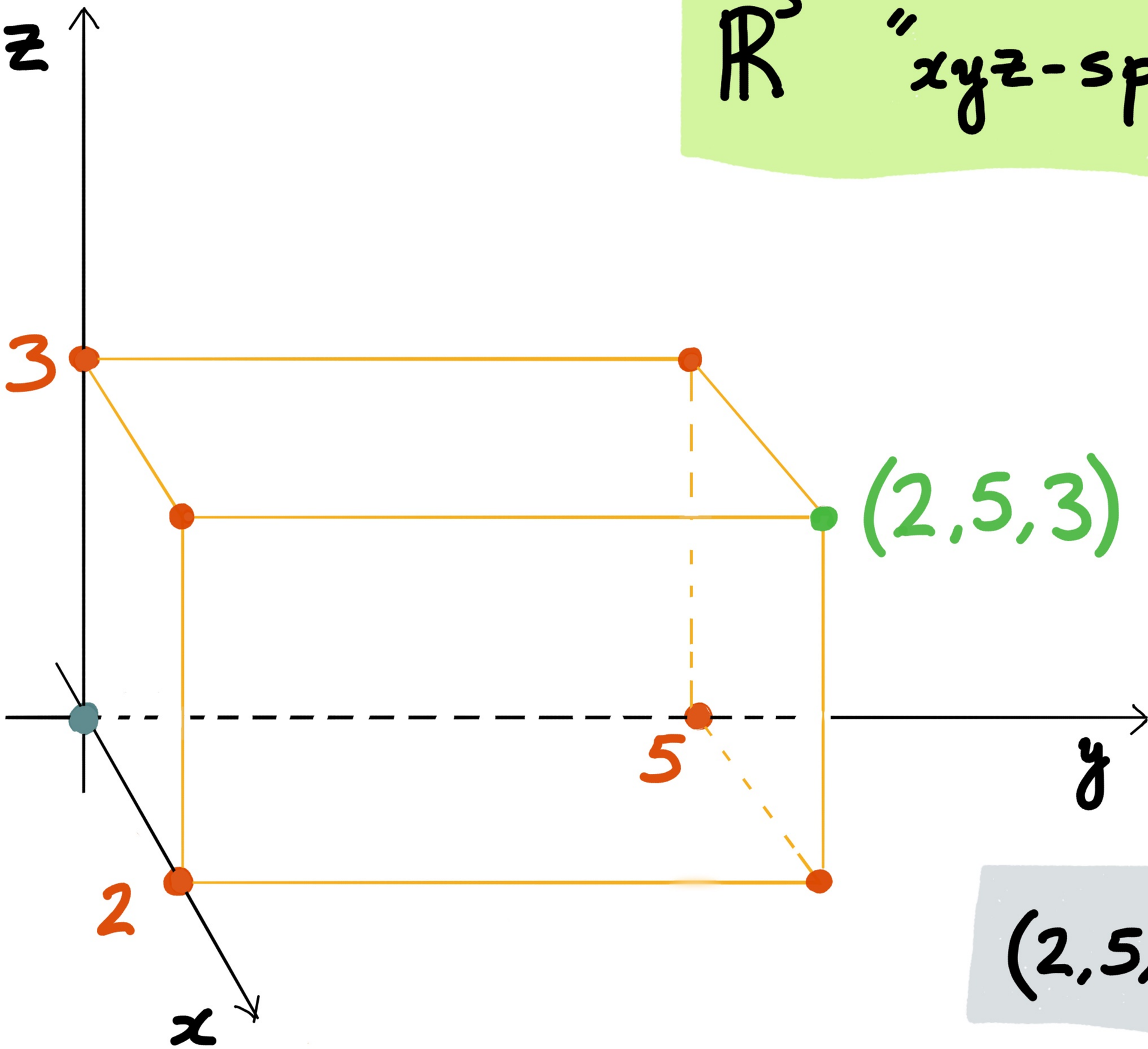


$\mathbb{R}^3$ "xyz-space"



$\mathbb{R}^3$ "xyz-space"



$(2,5,3) \in \mathbb{R}^3$

$\mathbb{R}^3$

z

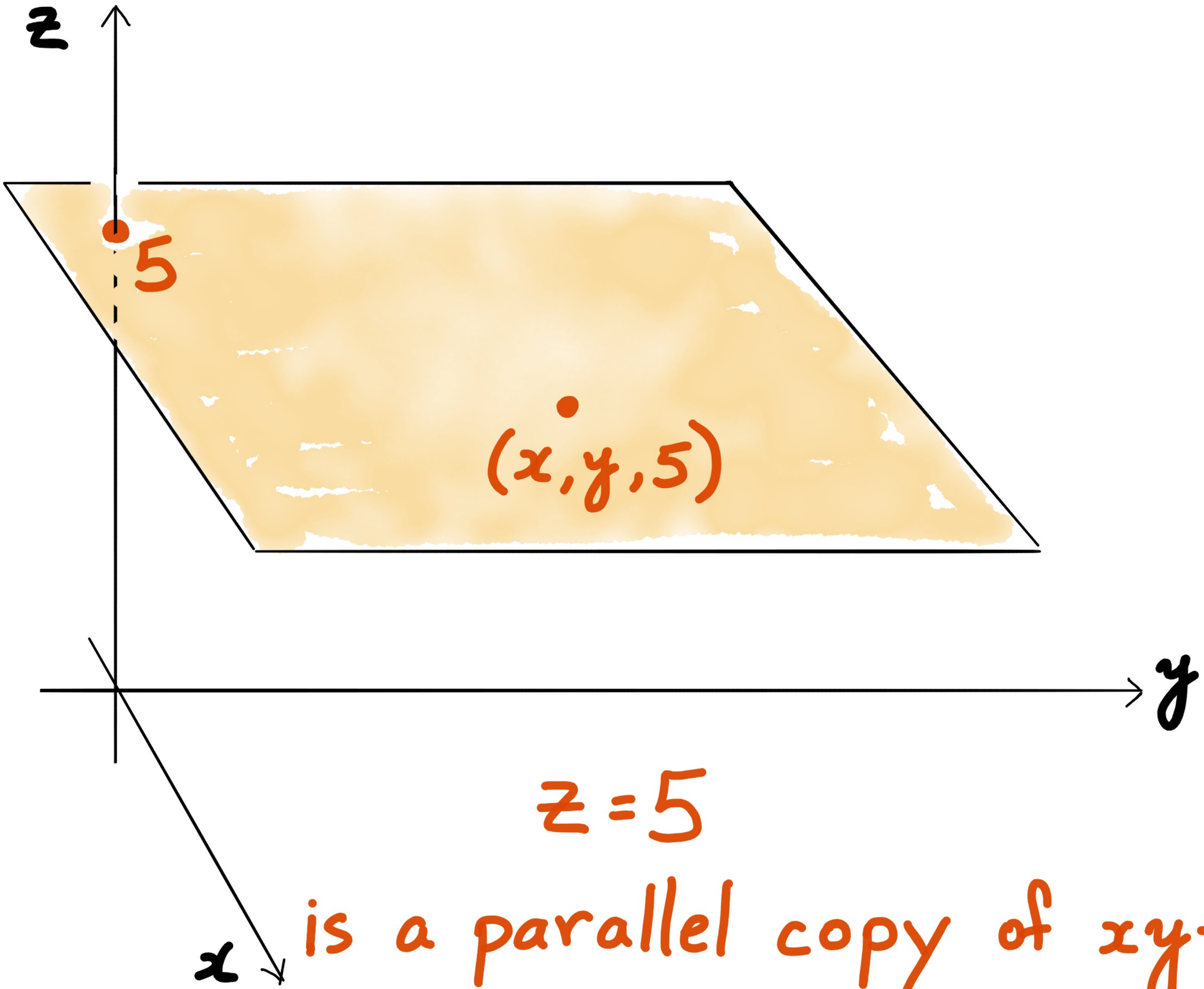
$z = 0$, "xy-plane"

y

x

$(x, y, 0)$

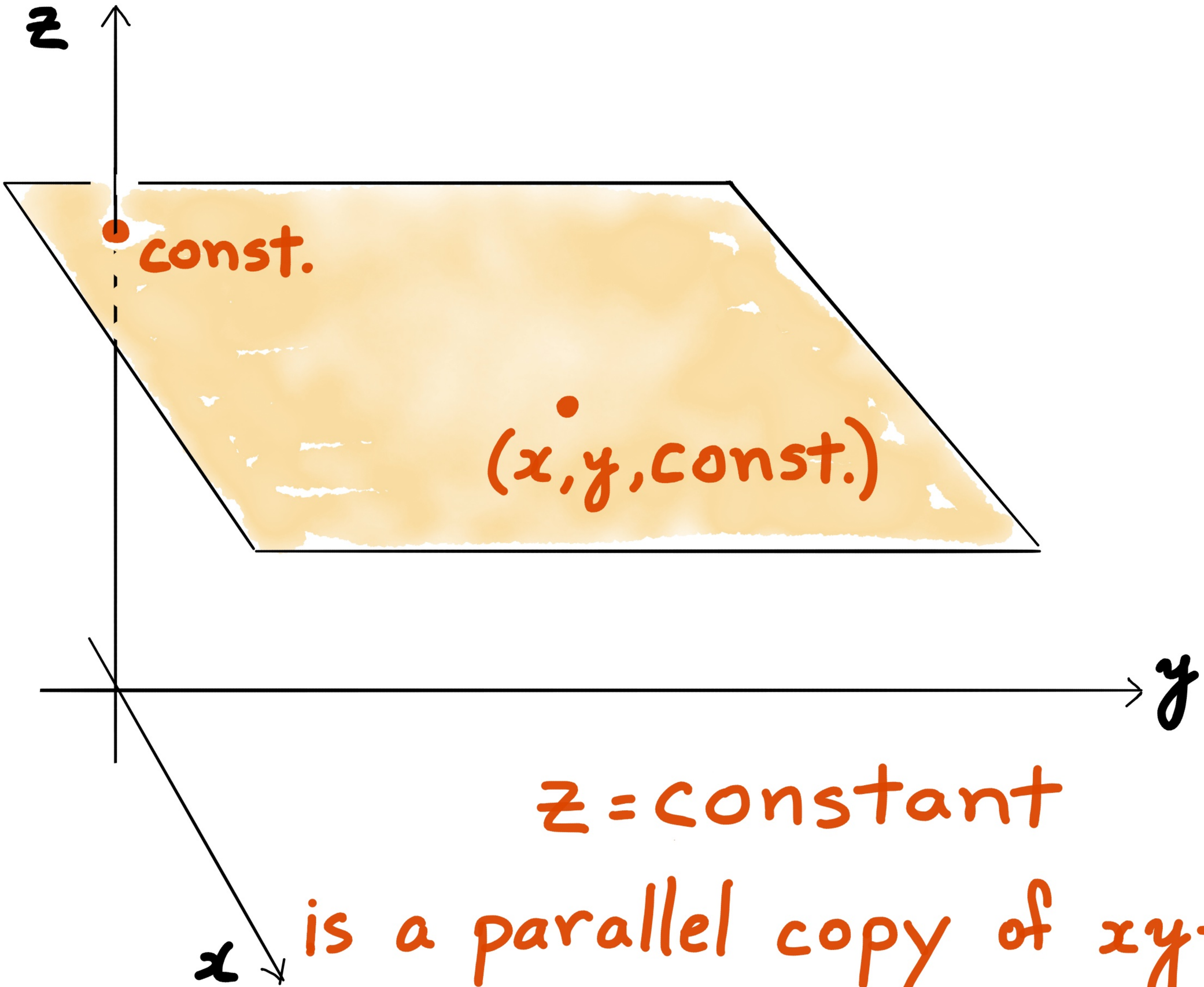
$\mathbb{R}^3$



$z = 5$

is a parallel copy of xy-plane

$\mathbb{R}^3$



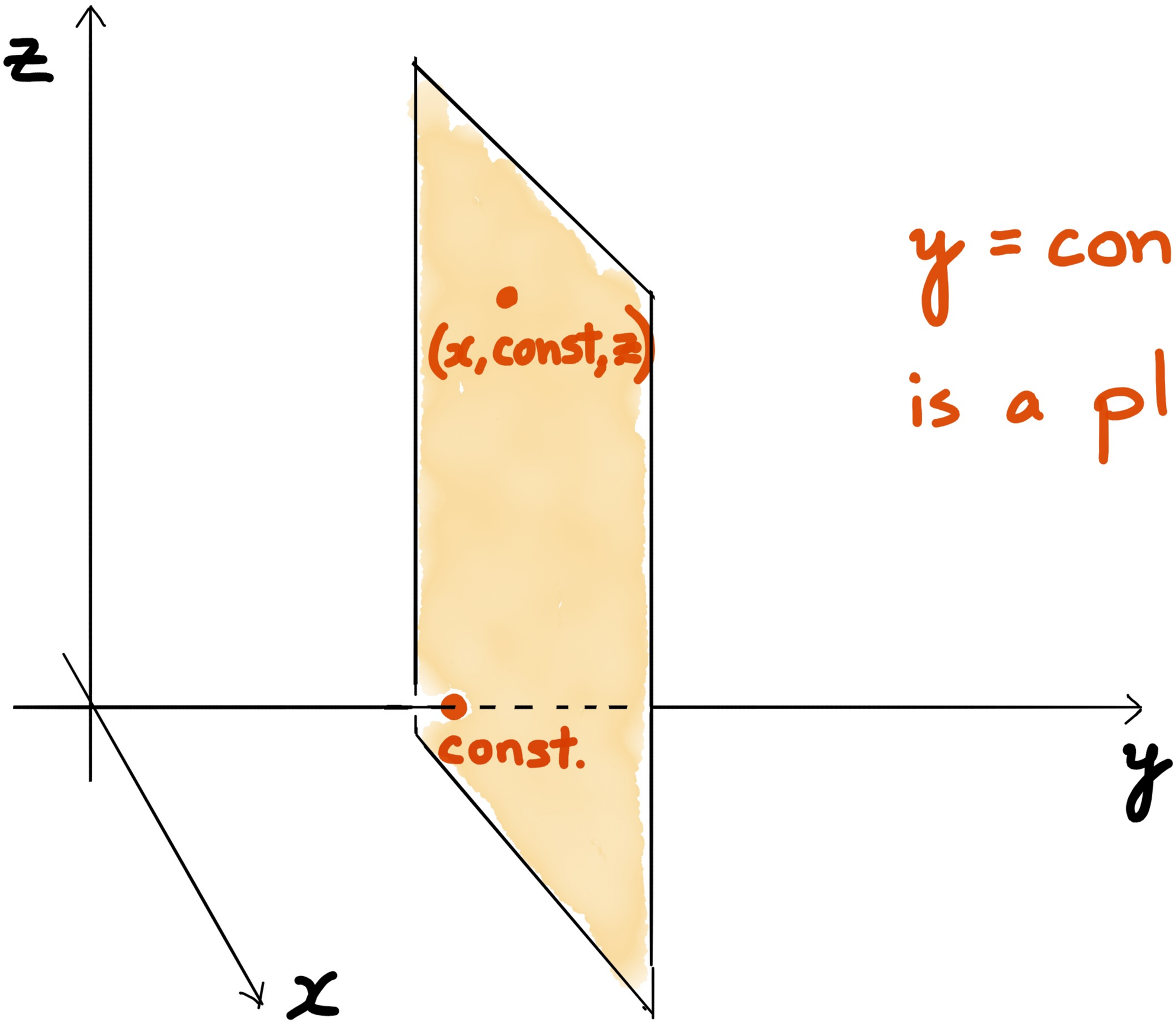
$z = \text{constant}$

is a parallel copy of xy-plane

$\mathbb{R}^3$

y = constant

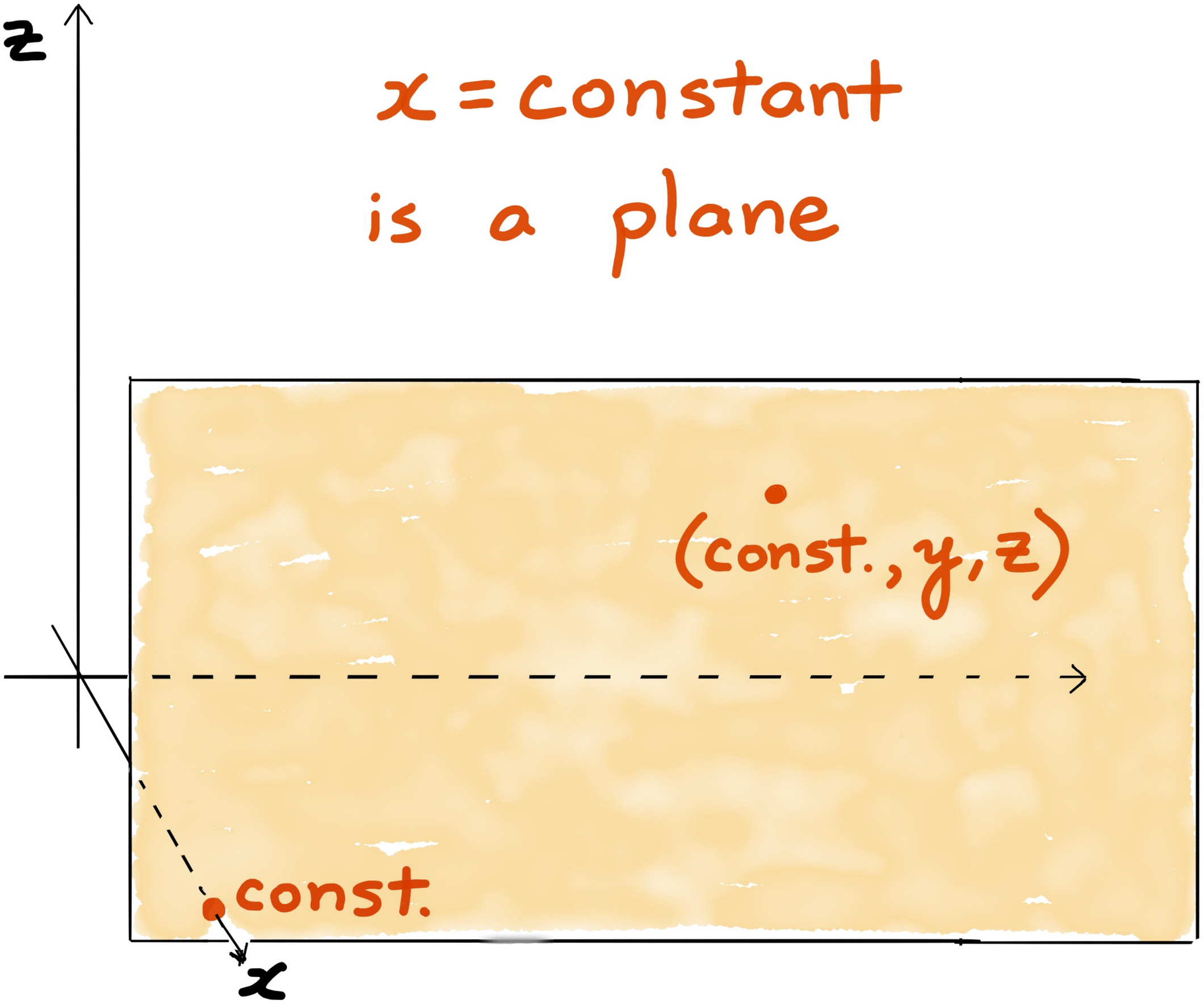
is a plane



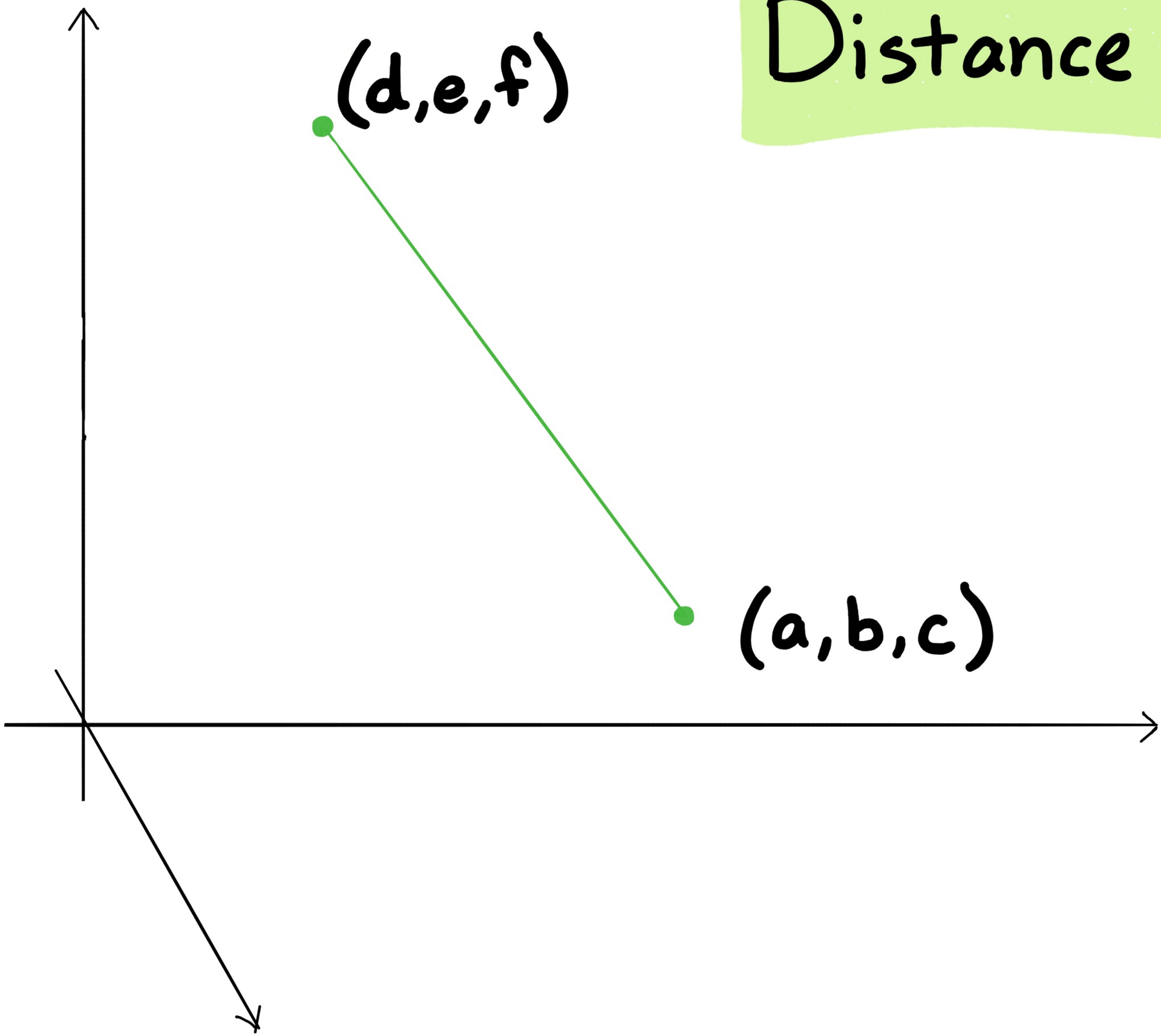
$\mathbb{R}^3$

$x = \text{constant}$

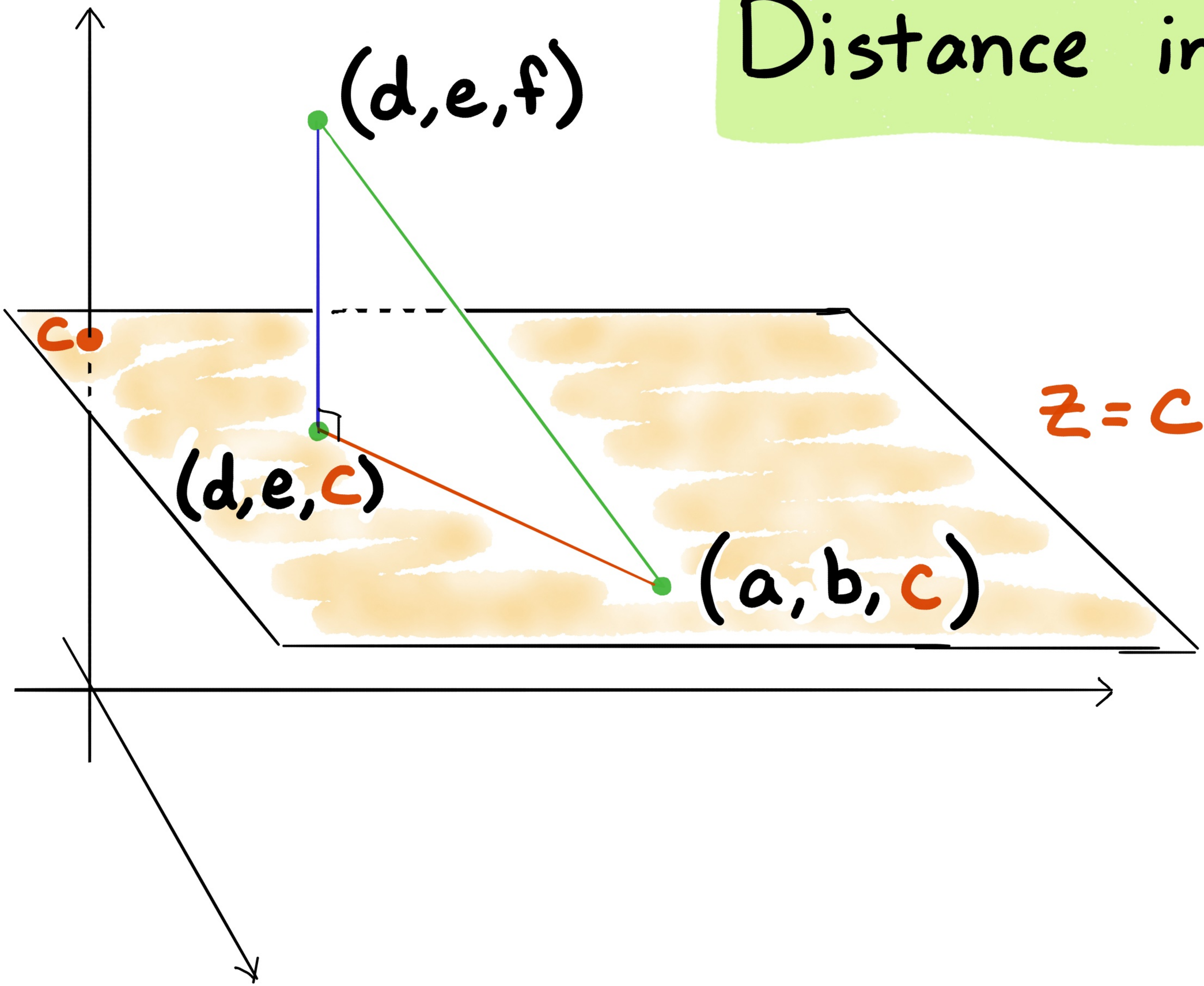
is a plane



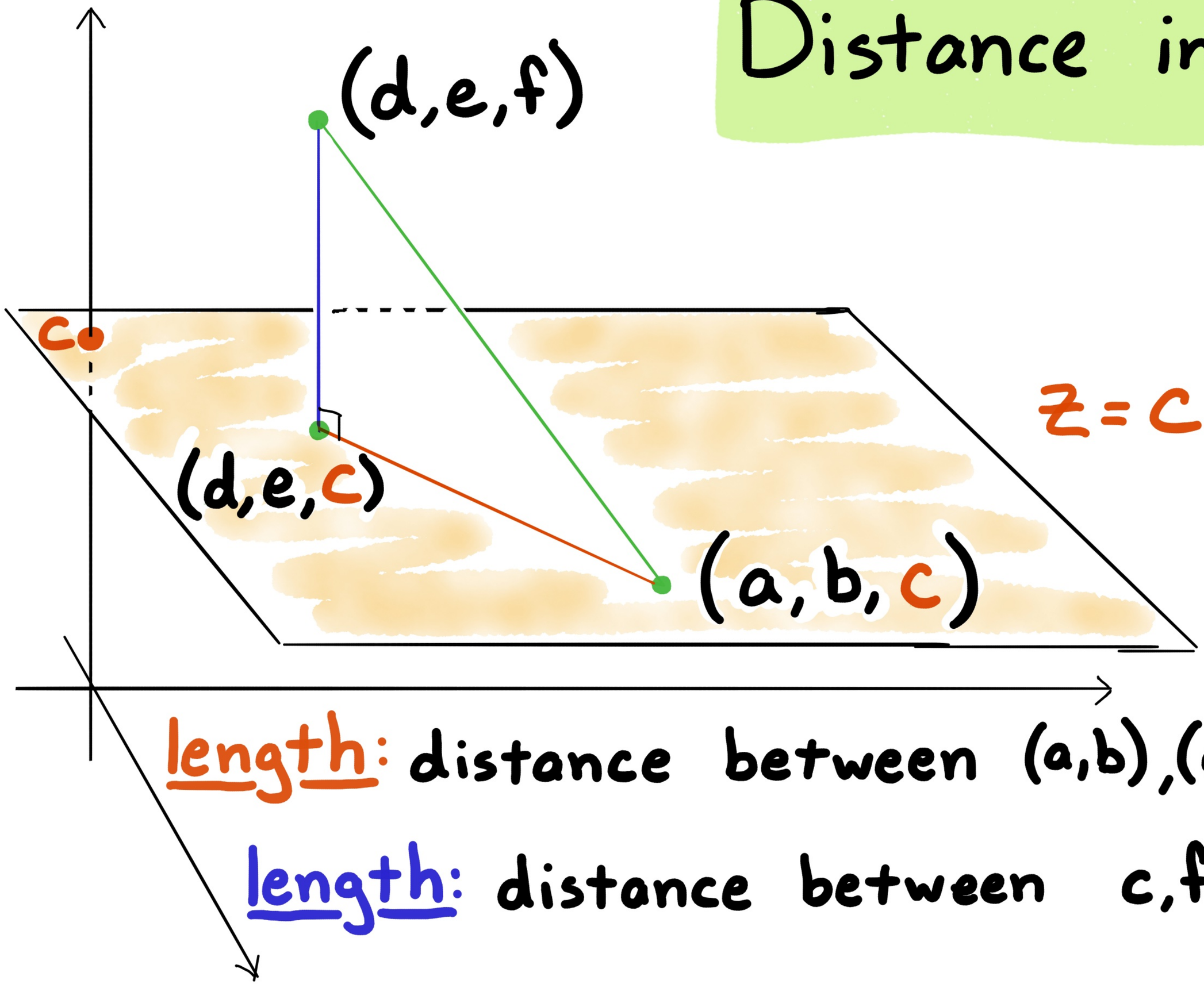
Distance in $\mathbb{R}^3$



Distance in $\mathbb{R}^3$



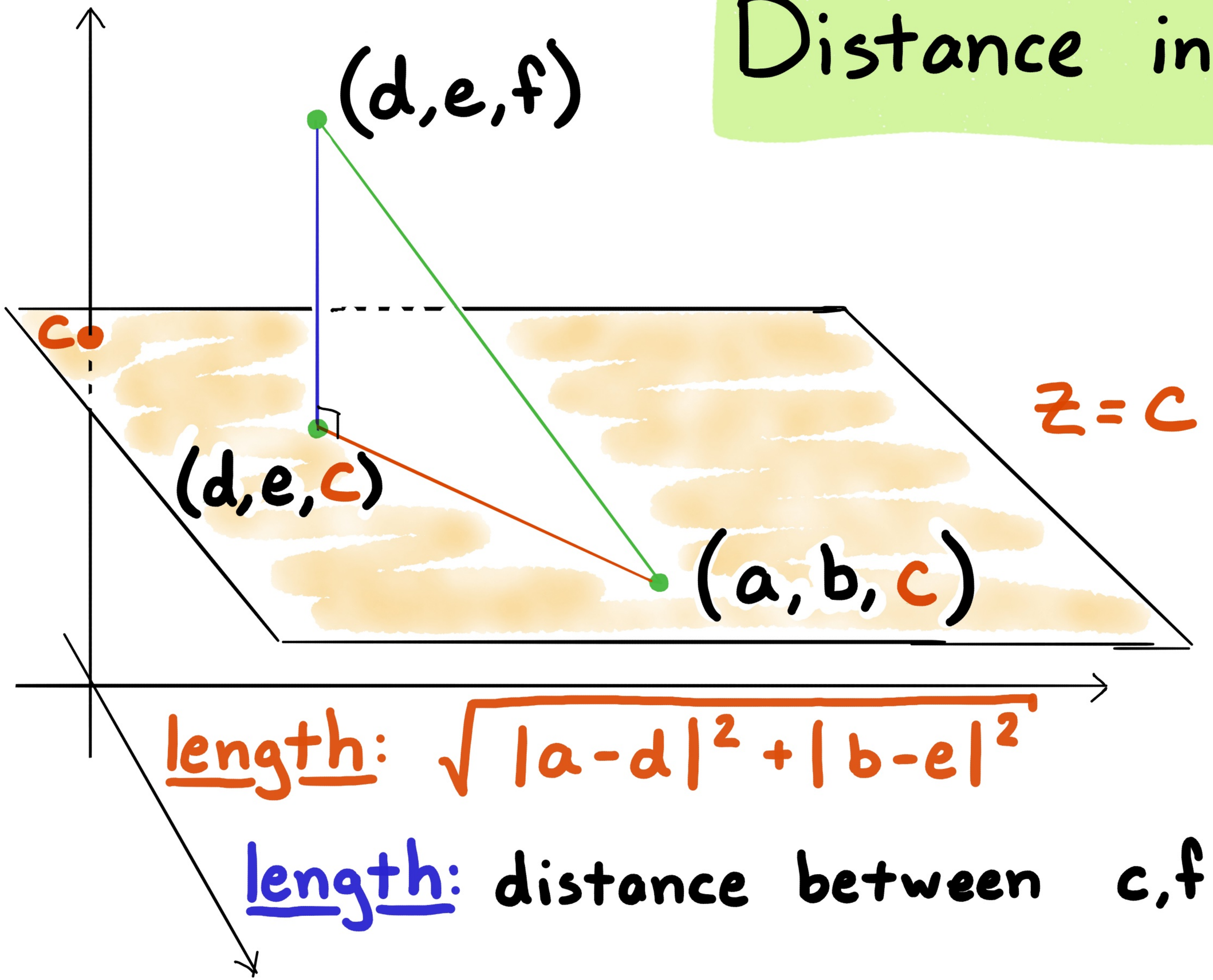
Distance in $\mathbb{R}^3$



length: distance between $(a,b), (d,e) \in \mathbb{R}^2$.

length: distance between $c, f \in \mathbb{R}$.

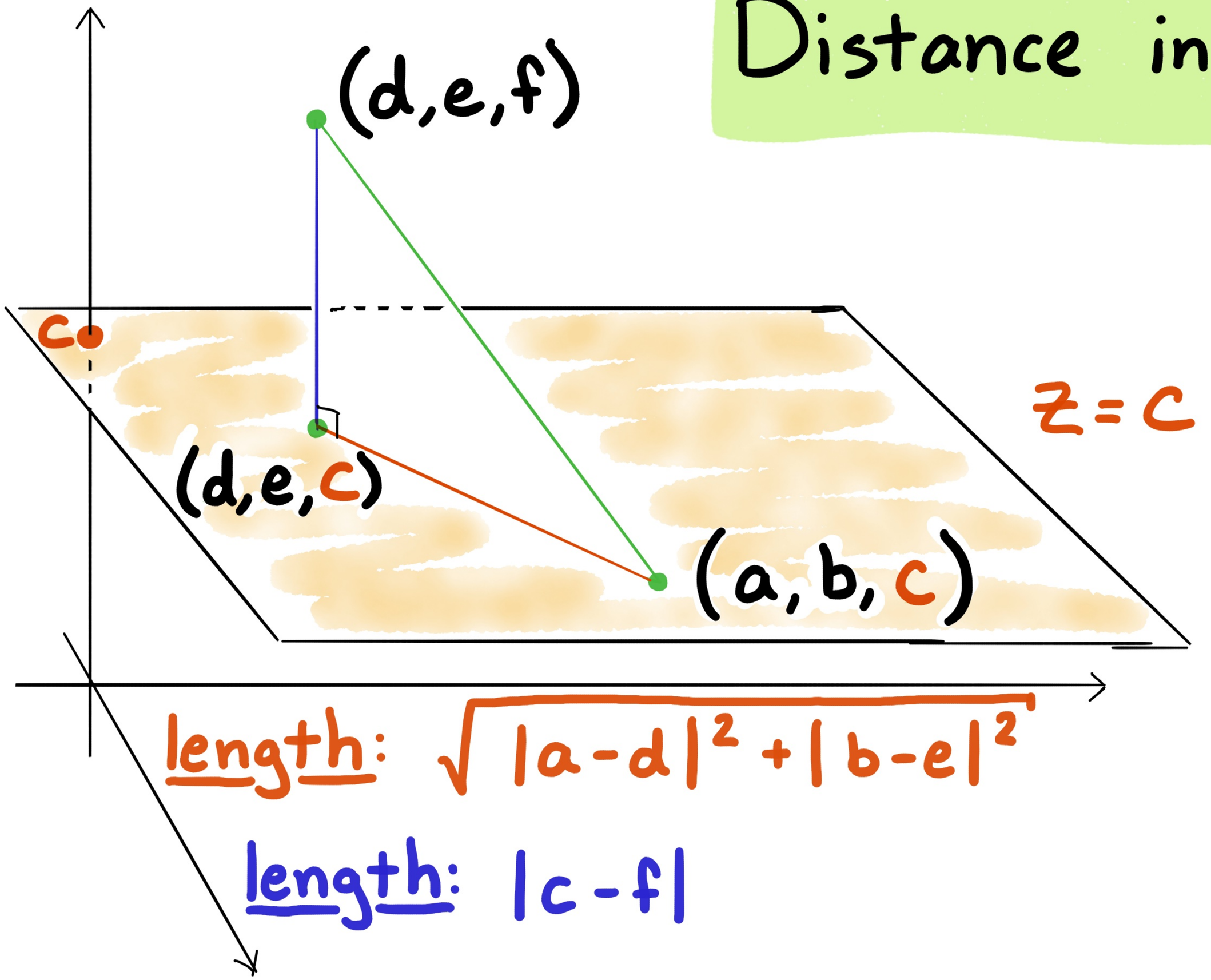
Distance in $\mathbb{R}^3$



length: $\sqrt{|a-d|^2+|b-e|^2}$

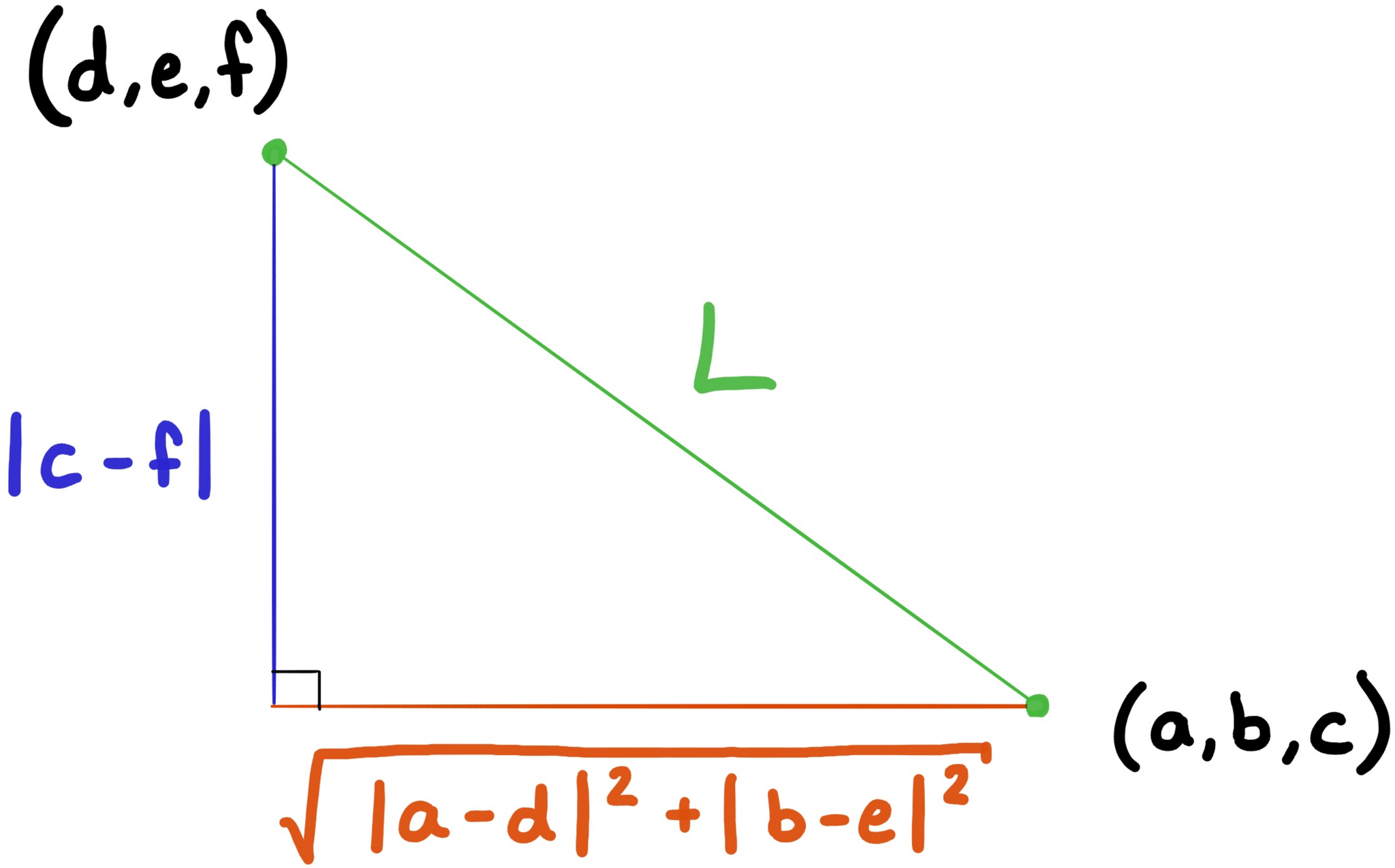
length: distance between $c, f \in \mathbb{R}$.

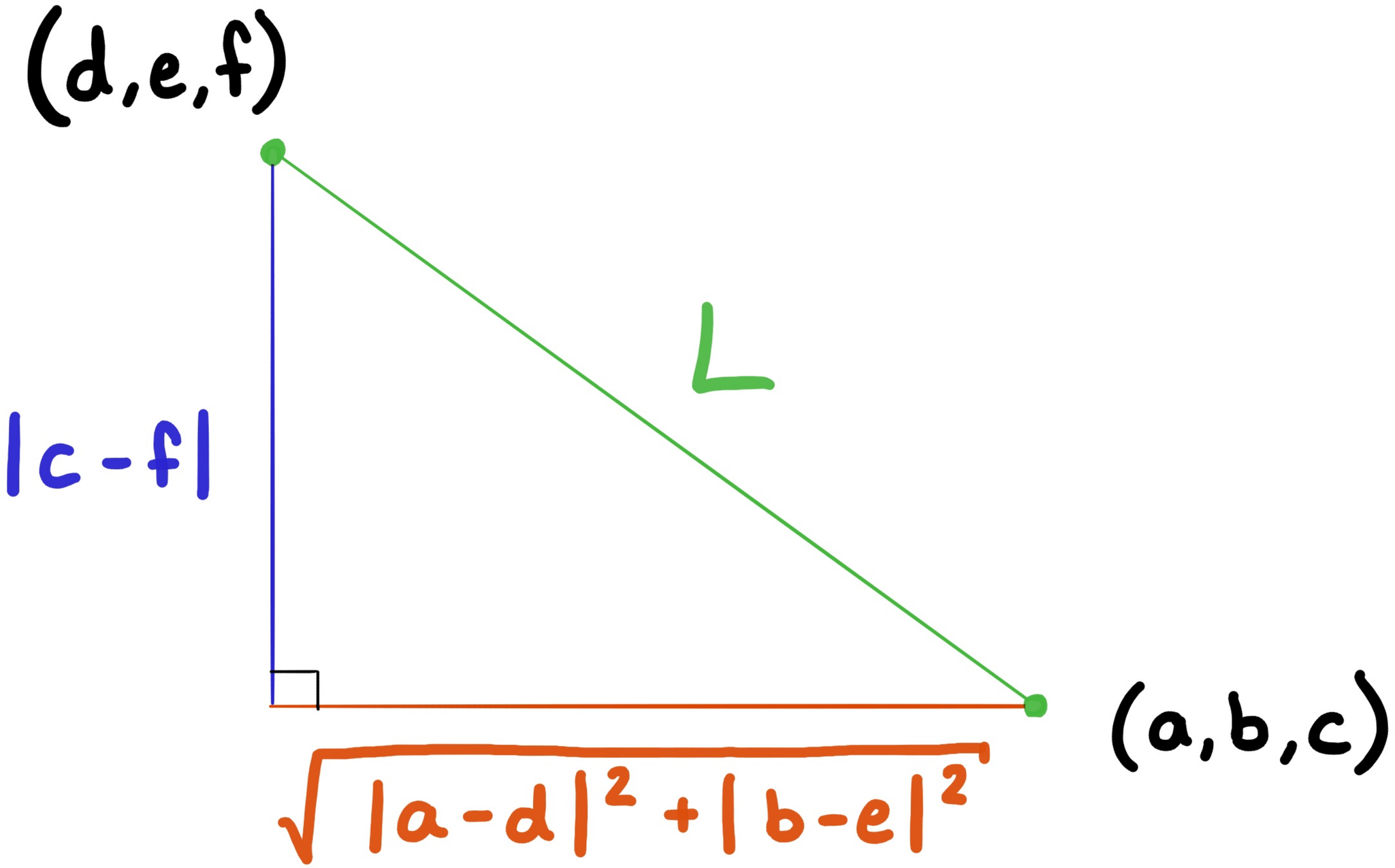
Distance in $\mathbb{R}^3$



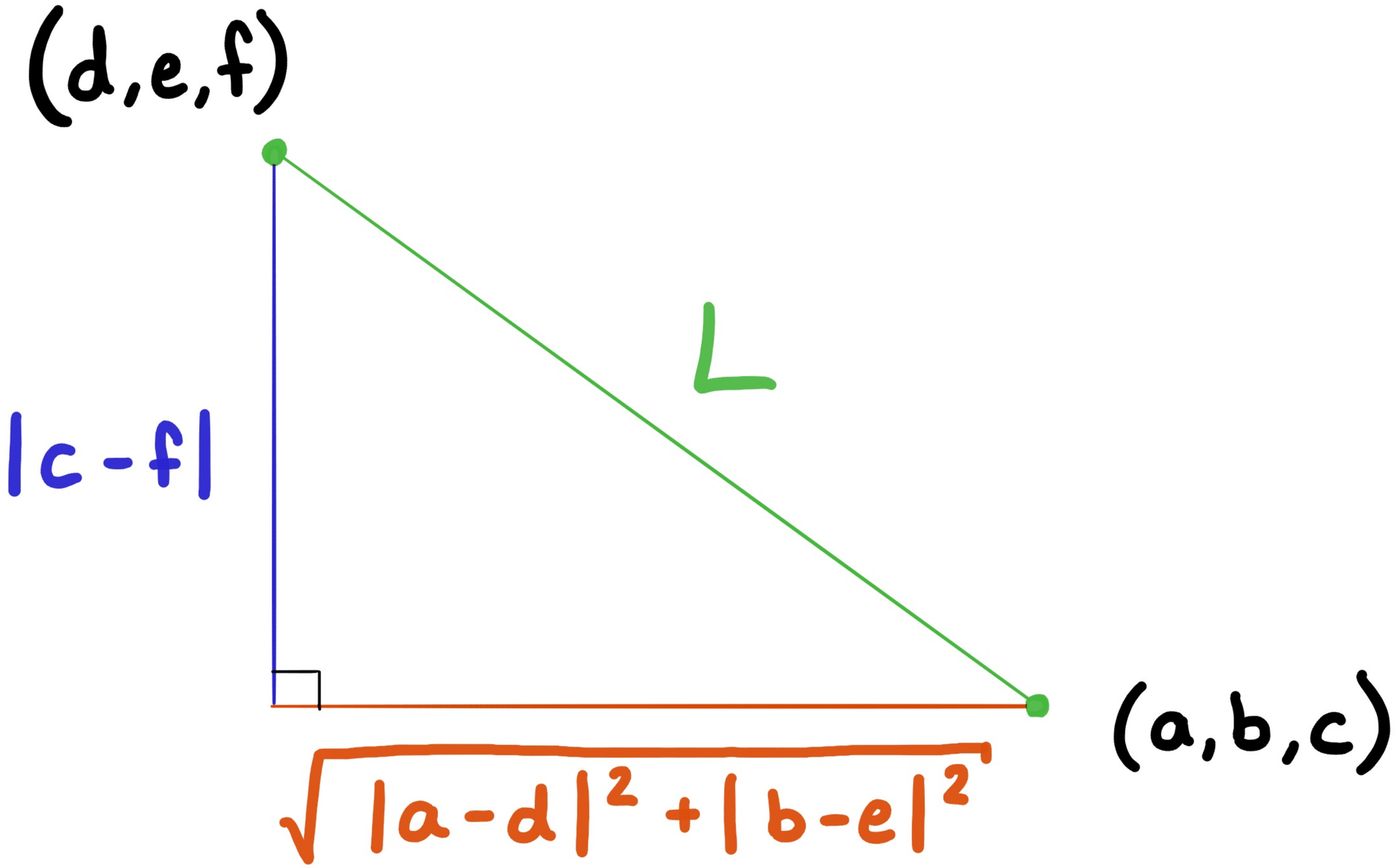
length: $\sqrt{|a-d|^2+|b-e|^2}$

length: $|c-f|$


(d,e,f)
L
|c-f|
(a,b,c)
√|a-d|² + |b-e|²



$$L = \sqrt{\sqrt{|a-d|^2+|b-e|^2}^2 + |c-f|^2}$$



$$L = \sqrt{|a-d|^2 + |b-e|^2 + |c-f|^2}$$

Distance between (a,b,c) and (d,e,f) in $\mathbb{R}^3$ is

$$\sqrt{|a-d|^2+|b-e|^2+|c-f|^2}$$

Distance between (a,b,c) and (d,e,f) in $\mathbb{R}^3$ is

$$\sqrt{|a-d|^2+|b-e|^2+|c-f|^2}$$

Example: Distance between $(-2,6,-1)$ and $(-3,6,1)$ is

Distance between (a,b,c) and (d,e,f) in $\mathbb{R}^3$ is

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Example: Distance between $(-2,6,-1)$ and $(-3,6,1)$ is 1 0 2

Distance between (a,b,c) and (d,e,f) in $\mathbb{R}^3$ is

$$\sqrt{|a-d|^2+|b-e|^2+|c-f|^2}$$

Example: Distance between $(-2,6,-1)$ and $(-3,6,1)$ is $\sqrt{1^2+0^2+2^2}$

Distance between (a,b,c) and (d,e,f) in $\mathbb{R}^3$ is

$$\sqrt{|a-d|^2+|b-e|^2+|c-f|^2}$$

Example: Distance between $(-2,6,-1)$ and $(-3,6,1)$ is $\sqrt{1^2+0^2+2^2}=\sqrt{5}$.

Example: In $\mathbb{R}^5$, the distance between the two points

$(3, 0, -2, 1, 7)$ and

$(6, -1, -4, 0, 8)$ is

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$(3, 0, -2, 1, 7)$ and

$(6, -1, -4, 0, 8)$ is

$3 \quad 1 \quad 2 \quad 1 \quad 1$

Example: In $\mathbb{R}^5$, the distance between the two points

$$(3, 0, -2, 1, 7) \quad \text{and}$$

$$(6, -1, -4, 0, 8) \quad \text{is}$$

$$\sqrt{3^2 + 1^2 + 2^2 + 1^2 + 1^2}$$

Example: In $\mathbb{R}^5$, the distance between the two points

$(3, 0, -2, 1, 7)$ and

$(6, -1, -4, 0, 8)$ is

$$\sqrt{3^2 + 1^2 + 2^2 + 1^2 + 1^2} = \sqrt{9 + 1 + 4 + 1 + 1}$$

$$= \sqrt{16}$$

$$= 4.$$