§4

$f: \mathbb{R}^n \to \mathbb{R}^m$ means that f is a function that takes inputs from $\mathbb{R}^n$ and gives outputs to $\mathbb{R}^m$.

Any function $\vec{f}: \mathbb{R} \to \mathbb{R}^n$ can be expressed as

$$\vec{f}(t) = \langle f_1(t), f_2(t), f_3(t) \rangle$$

where $f_i: \mathbb{R} \to \mathbb{R}$ for each i.

(Similar if $\mathbb{R}^3$ is replaced with $\mathbb{R}^n$.)

$$\vec{f}\,'(t) = \langle f_1'(t), f_2'(t), f_3'(t) \rangle$$

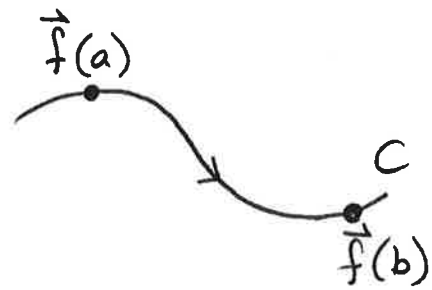
$$\vec{f}\,'(t) = \lim_{h \to 0} \frac{\vec{f}(t+h) - \vec{f}(t)}{h}$$

For $h \approx 0$, $\vec{f}(t+h) - \vec{f}(t) \approx h\vec{f}\,'(t)$

A <u>smooth parametrized curve</u> C is a collection of points in $\mathbb{R}^n$ that is the range of some one-to-one $\vec{f}: \mathbb{R} \to \mathbb{R}^n$ with $\vec{f}\,'(t)$ continuous and nonzero for all t.

If C is a smooth curve parametrized by $\vec{f}: \mathbb{R} \to \mathbb{R}^n$, then the <u>arclength</u> of C from $\vec{f}(a)$ to $\vec{f}(b)$ is

$$\int_a^b \|\vec{f}'(t)\|\,dt$$



Arc length for C doesn't depend on the parametrization $\vec{f}: \mathbb{R} \to \mathbb{R}^n$ used.