§33

Recall, if $\vec{F}: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, then $\operatorname{div}\vec{F}: \mathbb{R}^3 \longrightarrow \mathbb{R}$

is $\operatorname{div}\vec{F} = \frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3}$.

Gauss's Divergence Theorem

- Σ a (possibly lumpy) sphere in $\mathbb{R}^3$.
- R the region bounded by Σ.
- $\vec{F}: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ a vector field.
- $\vec{n}$ the outer unit normal to Σ.

$$\iint_{\Sigma} \vec{F} \cdot \vec{n} \, dS = \iiint_{R} \operatorname{div}\vec{F} \, dV$$