§3

dot product

$$\langle a,b,c\rangle \cdot \langle x,y,z\rangle = ax+by+cz$$
$$(a\vec{i}+b\vec{j}+c\vec{k})\cdot(x\vec{i}+y\vec{j}+z\vec{k}) = ax+by+cz$$

$$\langle a,b\rangle \cdot \langle x,y\rangle = ax+by$$
$$(a\vec{i}+b\vec{j})\cdot(x\vec{i}+y\vec{j}) = ax+by$$

For $\vec{v},\vec{w}\in\mathbb{R}^n$,

$$\vec{v}\cdot\vec{w}=0 \quad \text{same as} \quad \vec{v}\perp\vec{w}.$$

Generally,

$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\,\|\vec{w}\|\cos\theta$$

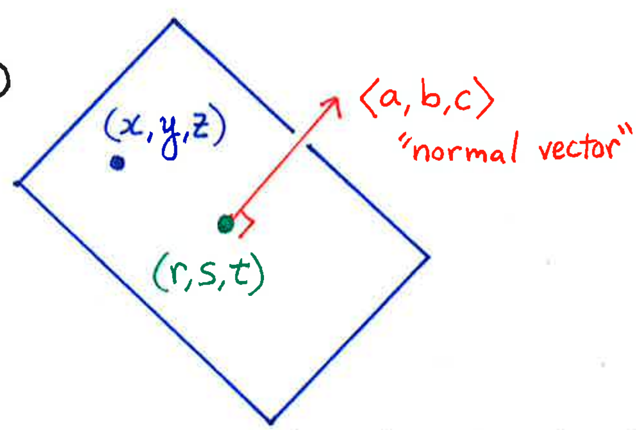
where θ is the angle between $\vec{v}$ and $\vec{w}$.

The collection of $(x,y,z)\in\mathbb{R}^3$ satisfying

$$a(x-r)+b(y-s)+c(z-t)=0$$

is a plane containing (r,s,t) and perpendicular to $\langle a,b,c\rangle$.

$\langle a,b,c\rangle$ is the <u>normal vector</u> for the plane.



$$0 = a(x-r)+b(y-s)+c(z-t)$$

If $(a,b,c) \neq (0,0,0)$,

then the collection of $(x,y,z) \in \mathbb{R}^3$ satisfying

$$ax + by + cz = d$$

is a plane.

Sometimes we write vectors as columns.

$$(v_1, v_2, v_3) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \qquad (a,b) = \begin{pmatrix} a \\ b \end{pmatrix}$$

Dot product:

$$(a,b)\begin{pmatrix} x \\ y \end{pmatrix} = ax + by$$

$$(a,b,c)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz$$