§28

If $\vec{F} = (F_1, \ldots, F_n)$ is a vector field, and if $\vec{F} = \nabla f$ for an unknown f, then we find f as follows:

① For each $1 \le i \le n$, find the antiderivative of F_i with respect to x_i.

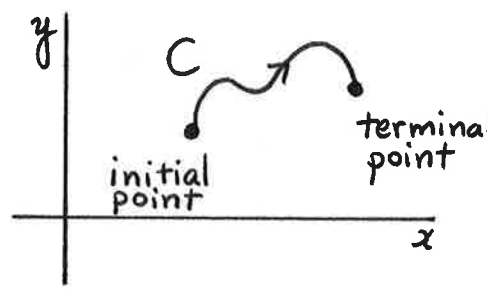
② Write the summands from ① for $i=1$. Cross out repeats for $i>1$.

③ Add the remaining summands for $i=2$ to what was written in ②. Cross out repeats for $i>2$.

④ Continue for remaining i.

⑤ Add a constant, C.

A smooth parametrized curve C in $\mathbb{R}^2$ is described by equations $x = x(t)$, $y = y(t)$, and $a \leq t \leq b$; where $a, b \in \mathbb{R}$; and $x'(t)$ and $y'(t)$ are not both 0 for the same value of t.



If C is a smooth parametrized curve, then $-C$ means the same curve except travelled in the opposite direction.

If C_1 and C_2 are smooth parametrized curves, then $C_1 \cup C_2$ means travel along C_1 and then along C_2.

A curve is <u>simple</u> if it doesn't intersect itself.

A curve is <u>closed</u> if its initial point is also its terminal point.

A <u>simple closed</u> curve is something of a floppy circle.

To parametrize a straight line between $(a,b), (c,d) \in \mathbb{R}^2$, you can write $0 \le t \le 1$, $x = a + (c-a)t$, and $y = b + (d-b)t$.

To parametrize the graph of $y = f(x)$ between $(a, f(a))$ and $(b, f(b))$, you can write $a \le t \le b$, $x = t$, and $y = f(t)$.

To parametrize $x = f(y)$ from $(f(a), a)$ to $(f(b), b)$, you can write $a \le t \le b$, $x = f(t)$, and $y = t$.

If $\vec{F} : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, $\vec{F} = (F_1, F_2, F_3)$, is a vector field, then we let $\operatorname{curl} \vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field

$$\operatorname{curl} \vec{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right).$$