§27

A vector field is a function $\vec{F} : \mathbb{R}^n \to \mathbb{R}^n$, $\vec{F}(x_1,\dots,x_n) = (F_1(x_1,\dots,x_n),\dots,F_n(x_1,\dots,x_n))$, where the $F_i : \mathbb{R}^n \to \mathbb{R}$ are called coordinate functions.

If $f : \mathbb{R}^n \to \mathbb{R}$ is a function, then $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$ is a vector field, where $\nabla f(x_1,\dots,x_n) = \left(\frac{\partial f}{\partial x_1},\dots,\frac{\partial f}{\partial x_n}\right)$ is the gradient of f.

If $\vec{F} = \nabla f$ for some function $f : \mathbb{R}^n \to \mathbb{R}$, then we say that $\vec{F}$ is a conservative vector field with potential function f.

Central Theorem for Unit 4:

A vector field $\vec{F} : \mathbb{R}^n \to \mathbb{R}^n$ is conservative exactly when $\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}$ for all $1 \le i,j \le n$.

If $\vec{F} : \mathbb{R}^n \to \mathbb{R}^n$ is a vector field, the divergence of $\vec{F}$ is the function $\operatorname{div}\vec{F} : \mathbb{R}^n \to \mathbb{R}$ where $\operatorname{div}\vec{F} = \sum_{i=1}^{n} \frac{\partial F_i}{\partial x_i}$.