

§ 25

Change of variables:

(*) $f(x,y)$ is continuous.

(*) $G: \underset{u,v}{\mathbb{R}^2} \longrightarrow \underset{x,y}{\mathbb{R}^2}$ is C^1 and one-to-one.

$$G(u,v) = (x(u,v), y(u,v))$$

(*) S is a region in the uv -plane, R is a region in the xy -plane, and $G(S) = R$.

Then,

$$\iint_R f(x,y) dx dy = \iint_S f(x(u,v), y(u,v)) |\det(D_{(u,v)}G)| du dv.$$

$$x \mapsto x(u,v)$$

$$y \mapsto y(u,v)$$

$$dx dy \mapsto du dv$$

$$R \mapsto S$$

Insert $|\det(D_{(u,v)}G)|$.