§24

Euclidean triple integrals

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(x,y,z)\,dx\,dy\,dz$$

$$dV \longmapsto dx\,dy\,dz$$

Cylindrical triple integrals

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(r\cos\theta, r\sin\theta, z)\; r\; dr\,d\theta\,dz$$

$$x \longmapsto r\cos\theta$$
$$y \longmapsto r\sin\theta$$

$$dV \longmapsto dr\,d\theta\,dz$$

Insert r.

Spherical triple integrals

$$\iiint_R f(x,y,z)\,dV = \iiint_R f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\theta)\,\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

$$x \longmapsto \rho\sin\phi\cos\theta$$
$$y \longmapsto \rho\sin\phi\sin\theta$$
$$z \longmapsto \rho\cos\theta$$

$$dV \longmapsto d\rho\,d\theta\,d\phi$$

Insert $\rho^2\sin\phi$.