§23

x_1 and x_2 are constants.

$y_1(x)$ and $y_2(x)$ are functions of x.

$z_1(x,y)$ and $z_2(x,y)$ are functions of x and y.

$$R = \{(x,y,z) : x_1 \le x \le x_2 \,,\; y_1(x) \le y \le y_2(x) \,,\; z_1(x,y) \le z \le z_2(x,y)\}.$$

Then,

$$\iiint_R f(x,y,z)\, dV = \int_{x_1}^{x_2} \int_{y_1(x)}^{y_2(x)} \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\, dz\, dy\, dx .$$

Volume of a solid, R, in $\mathbb{R}^3$ is

$$\text{Volume}(R) = \iiint_R dV .$$