§21

If $R = \{(r,\theta) : a \le r \le b,\ \alpha \le \theta \le \beta\}$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_\alpha^\beta f(r\cos\theta, r\sin\theta)\, r\, d\theta\, dr$$

$$= \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

If $R = \{(r,\theta) : a \le r \le b,\ \theta_1(r) \le \theta \le \theta_2(r)\}$, then

$$\iint_R f(x,y)\,dA = \int_a^b \int_{\theta_1(r)}^{\theta_2(r)} f(r\cos\theta, r\sin\theta)\, r\, d\theta\, dr$$

If $R = \{(r,\theta) : \alpha \le \theta \le \beta,\ r_1(\theta) \le r \le r_2(\theta)\}$, then

$$\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

If R is a region in $\mathbb{R}^2$, then

$$\text{Area}(R) = \iint_R 1\,dA, \text{ or more simply, } \iint_R dA.$$