§16

- $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$
- S a domain for f, a region in $\mathbb{R}^2$.
- $p \in S$ is a <u>critical point</u> if
  - (i) $\nabla f(p) = \vec{0}$, or
  - (ii) p is a boundary point of S.

<u>Theorem:</u> If S is bounded and contains all its boundary points, then f attains a (global) max and min on S.

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

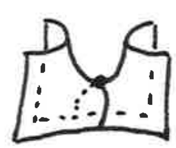
$$\nabla f(p) = \left(\left.\frac{\partial f}{\partial x}\right|_p, \left.\frac{\partial f}{\partial y}\right|_p\right)$$

curvature at p: $\begin{vmatrix} \left.\frac{\partial^2 f}{\partial x^2}\right|_p & \left.\frac{\partial^2 f}{\partial x \partial y}\right|_p \\ \left.\frac{\partial^2 f}{\partial x \partial y}\right|_p & \left.\frac{\partial^2 f}{\partial y^2}\right|_p \end{vmatrix}$

concavity at p: $\begin{cases} \left.\frac{\partial^2 f}{\partial x^2}\right|_p > 0 & \text{"up"} \\ \left.\frac{\partial^2 f}{\partial x^2}\right|_p < 0 & \text{"down"} \end{cases}$

<u>local max/min on "interior" of S (S but not its boundary)</u>

Find all $p \in S$ that are not boundary points and such that $\nabla f(p) = \vec{0}$. Then check curvature.

- zero $\longrightarrow$???
- negative $\longrightarrow$  saddle point. Neither local min. nor local max.
- positive $\longrightarrow$ check concavity
  - up $\longrightarrow$  local min.
  - down $\longrightarrow$  local max.

<u>local max/min on boundary of S</u>

This is a one-variable problem.

- Find critical points where derivative is 0, or boundary points of intervals.
- Check them.

<u>(global) max/min</u>

Select from local max/min in interior and boundary.