§15

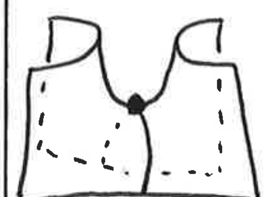
If S is a level surface in $\mathbb{R}^3$ given as $F(x,y,z) = \text{const.}$, and if $(x_0,y_0,z_0) \in \mathbb{R}^3$ is a point on S, then the <u>tangent plane</u> of S at (x_0,y_0,z_0) is given by the equation

$$\left.\frac{\partial F}{\partial x}\right|_{(x_0,y_0,z_0)}(x-x_0) + \left.\frac{\partial F}{\partial y}\right|_{(x_0,y_0,z_0)}(y-y_0) + \left.\frac{\partial F}{\partial z}\right|_{(x_0,y_0,z_0)}(z-z_0) = 0$$

- The tangent plane of the graph of $f: \mathbb{R}^2 \to \mathbb{R}$ is horizontal at $p \in \mathbb{R}^2$ if $\nabla f(p) = \vec{0}$.

  - When $\nabla f(p) = \vec{0}$, the curvature of the graph is $\begin{vmatrix} \left.\frac{\partial^2 f}{\partial x^2}\right|_p & \left.\frac{\partial^2 f}{\partial x \partial y}\right|_p \\ \left.\frac{\partial^2 f}{\partial x \partial y}\right|_p & \left.\frac{\partial^2 f}{\partial y^2}\right|_p \end{vmatrix}$.

    - If curvature is positive, $\left.\frac{\partial^2 f}{\partial x^2}\right|_p > 0$ means concave up and $\left.\frac{\partial^2 f}{\partial x^2}\right|_p < 0$ means concave down.



negative curvature

positive curvature concave up

positive curvature concave down