§13

For $f: \mathbb{R}^n \to \mathbb{R}$, the <u>gradient</u> of f at p, $\nabla f(p)$, is a vector in the domain of f.

For $f(x,y)$,

$$\nabla f(p) = \left(\frac{\partial}{\partial x}\Big|_p , \frac{\partial}{\partial y}\Big|_p \right)$$

For $f(x,y,z)$,

$$\nabla f(p) = \left(\frac{\partial}{\partial x}\Big|_p , \frac{\partial}{\partial y}\Big|_p , \frac{\partial}{\partial z}\Big|_p \right).$$

$$\boxed{D_p f(\vec{v}) = \nabla f(p) \cdot \vec{v} = \vec{v} \cdot \nabla f(p)}$$

<u>Proposition:</u>

Suppose $f: \mathbb{R}^n \to \mathbb{R}$, $p \in \mathbb{R}^n$, and $\nabla f(p) \neq \vec{0}$.
At p, f increases most rapidly in the direction of $\nabla f(p)$. The rate of increase, or slope, in that direction is $\|\nabla f(p)\|$.
At p, f decreases most rapidly in the direction of $-\nabla f(p)$. The rate of increase, or slope, in that direction is $-\|\nabla f(p)\|$.

Proposition:

Suppose $f: \mathbb{R}^n \to \mathbb{R}$ and $k \in \mathbb{R}$ is a constant.

Let S be the solutions in $\mathbb{R}^n$ of the equation $f(p) = k$.

If $p \in S$ and $\nabla f(p) \neq \vec{0}$, then $\nabla f(p)$ is orthogonal to S.