§12

All functions f below are of the form $f: \mathbb{R}^n \to \mathbb{R}$.

The <u>Jacobian matrix</u> of $f(x,y)$ at $p \in \mathbb{R}^2$ is

$$D_p f = \left(\frac{\partial f}{\partial x}\Big|_p , \frac{\partial f}{\partial y}\Big|_p \right)$$

The <u>Jacobian matrix</u> of $f(x,y,z)$ at $p \in \mathbb{R}^3$ is

$$D_p f = \left(\frac{\partial f}{\partial x}\Big|_p , \frac{\partial f}{\partial y}\Big|_p , \frac{\partial f}{\partial z}\Big|_p \right)$$

For $f: \mathbb{R}^n \to \mathbb{R}$, $p, \vec{v} \in \mathbb{R}^n$,

$$D_p f(\vec{v}) = \lim_{h \to 0} \frac{f(p + h\vec{v}) - f(p)}{h}.$$

$D_p f(\vec{v})$ is the <u>derivative of f at p with respect to $\vec{v}$.</u>

If $\|\vec{v}\| = 1$, then $D_p f(\vec{v})$ is the slope of the graph of f at p, in the direction of $\vec{v}$.

Generally, the slope of the graph of f at p in the direction of $\vec{v}$ equals

$$\frac{D_p f(\vec{v})}{\|\vec{v}\|}.$$

If $\|\vec{v}\| \neq 1$, then $D_pf(\vec{v})$ is the magnitude of $\vec{v}$ times the slope of the graph of f at p in the direction of $\vec{v}$:

$$D_pf(\vec{v}) = \|\vec{v}\| \, \frac{D_pf(\vec{v})}{\|\vec{v}\|}$$

If $\vec{v} \approx 0$,

$$f(p+\vec{v}) - f(p) \approx D_pf(\vec{v})$$

Deviation from the text:

The text only considers $D_pf(\vec{v})$ for $\|\vec{v}\| = 1$. If $\|\vec{v}\| \neq 1$, then in the text, the derivative of f at p in the direction of $\vec{v}$ means $D_pf\left(\frac{\vec{v}}{\|\vec{v}\|}\right)$, or equivalently, $\frac{D_pf(\vec{v})}{\|\vec{v}\|}$.

Also, the text writes $D_{\vec{v}}f(p)$ instead of $D_pf(\vec{v})$.