§11

For a function $f(x,y)$,

$$\frac{\partial f}{\partial x}(a,b) = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h}$$

$$\frac{\partial f}{\partial y}(a,b) = \lim_{h\to 0} \frac{f(a,b+h)-f(a,b)}{h}.$$

$\frac{\partial f}{\partial x}$ measures the rate of change in $f(x,y)$ if the x-variable is slightly increased.

To find $\frac{\partial f}{\partial x}$, treat x as the variable, constants as constants, and all other variables (y, or w, or v, etc.) as constants, and use the rules for differentiation that you know from previous calculus courses.

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{yx}. \qquad \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}.$$

If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is "nice", $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.