§ 10

For $f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$, $p \in \mathbb{R}^n$ and $L \in \mathbb{R}^m$, $\lim_{q \to p} f(q) = L$ means that for any $\varepsilon > 0$ there is some $\delta > 0$ such that if $0 < \|q - p\| < \delta$, then $\|f(q) - L\| < \varepsilon$.

If $g(p) = 0$, then

$\lim_{q \to p} \frac{f(q)}{g(q)}$ does not exist.

$f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is <u>continuous</u> if for every $p \in \mathbb{R}^n$,

$\lim_{q \to p} f(q) = f(p)$.

Functions that are continuous on their domain:

$h_1(x,y,z)=x$	e^x	$f\circ g$	
$h_2(x,y,z)=y$	$\sin x$	$f+g$	
$h_3(x,y,z)=z$	$\cos x$	$f-g$	If f and g are.
$h_4(x,y,z)=\text{const.}$	$\log x$	fg	
	$\sqrt{x}$	f/g	

A neighborhood of $p\in\mathbb{R}^n$ is the interior of (*) centered at p, where (*) is an interval if $n=1$, a circle if $n=2$, and a sphere if $n=3$.

$p\in\mathbb{R}^n$ is a boundary point of a subset S of $\mathbb{R}^n$ if every neighborhood of p contains points in, and not in, S.

S is open if it has none of its boundary points.

S is closed if it has all of its boundary points.