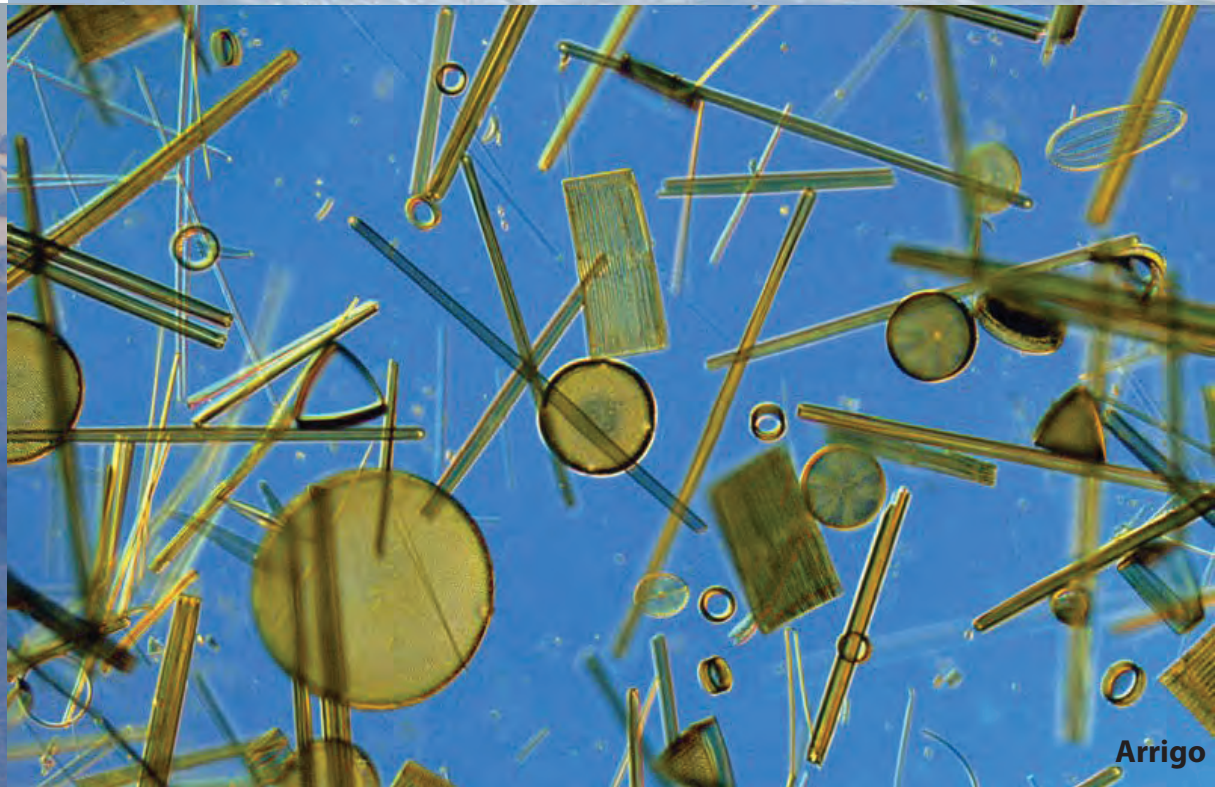


Microbial Ecology and the Physics of Sea Ice

Kenneth M. Golden
Department of Mathematics
University of Utah



Golden



Arrigo

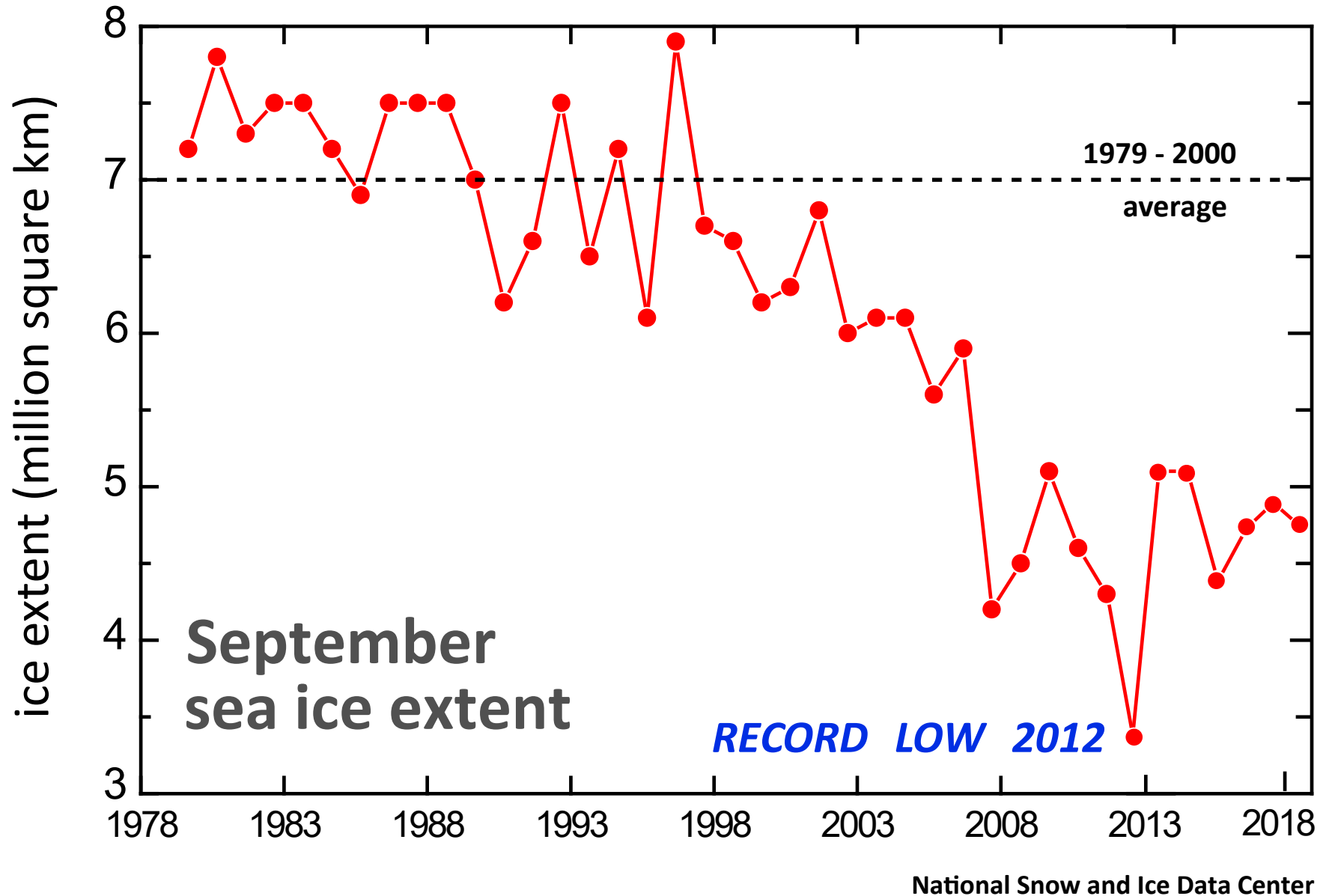
SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**



polar ice caps critical
to climate in reflecting
sunlight during summer

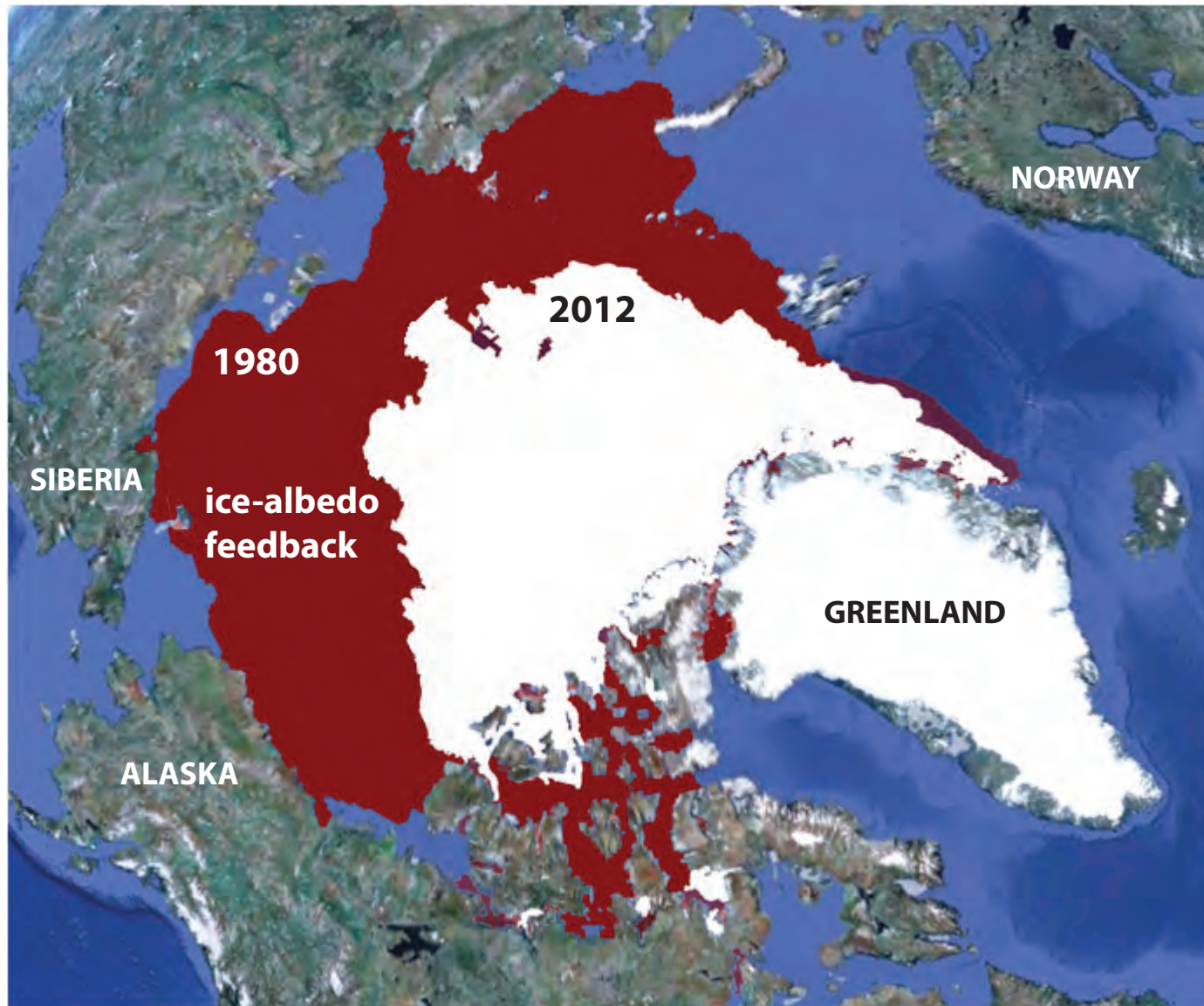
the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers

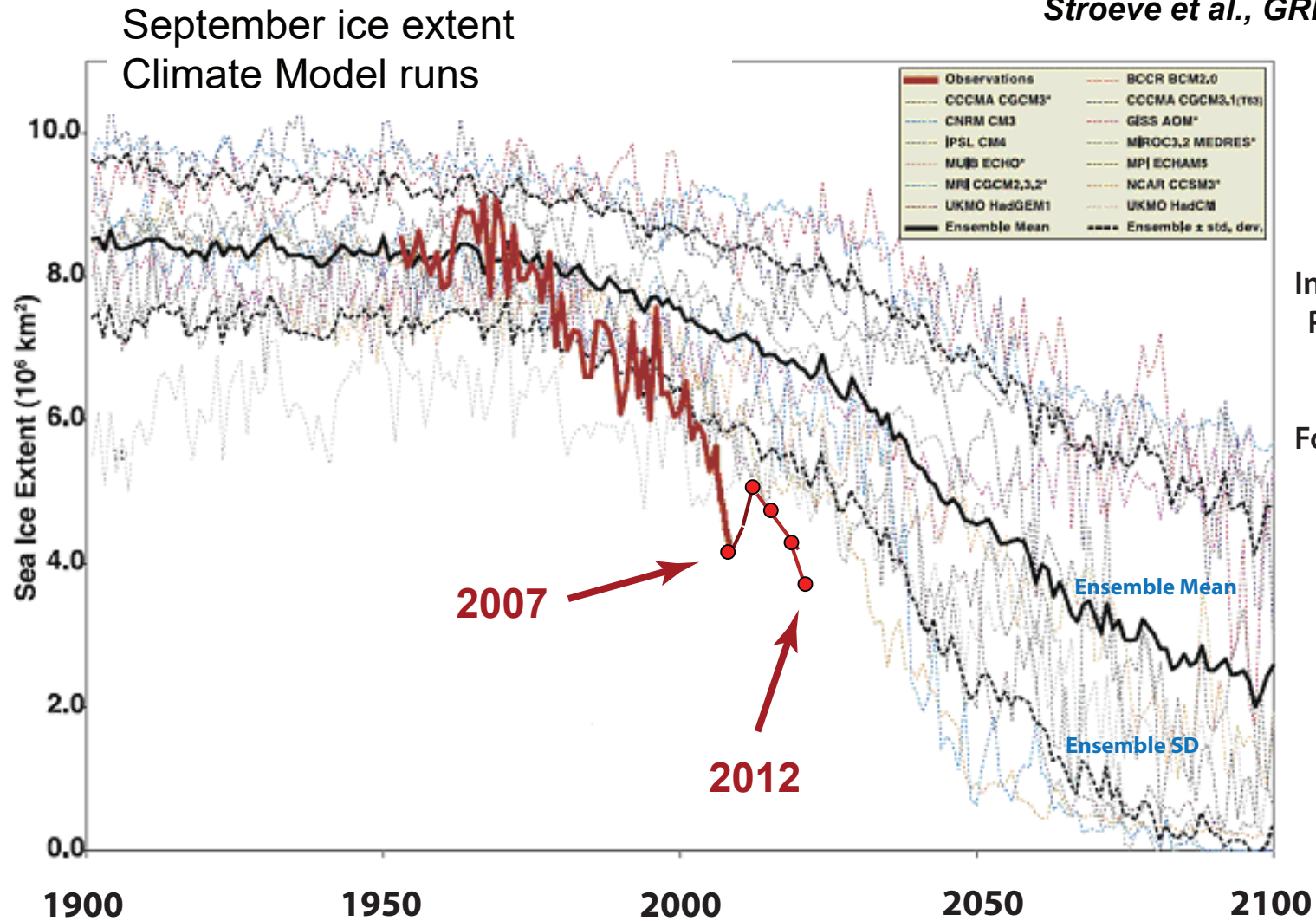
September 2012 -- 3.4 million square kilometers



Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



**IPCC AR4
Models**

Intergovernmental
Panel on Climate
Change (IPCC)

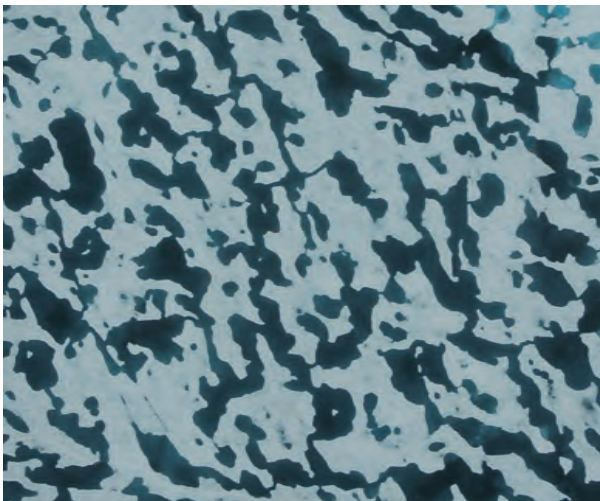
Fourth Assessment
AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice
simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

**For simulations with ponds
September ice volume is nearly 40% lower.**

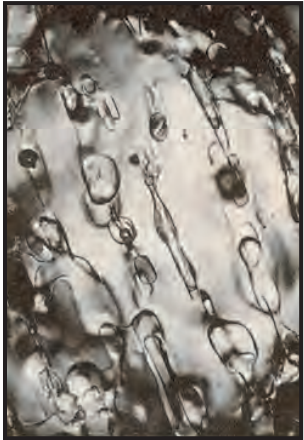
... and other sub-grid scale structures and processes

linkage of scales

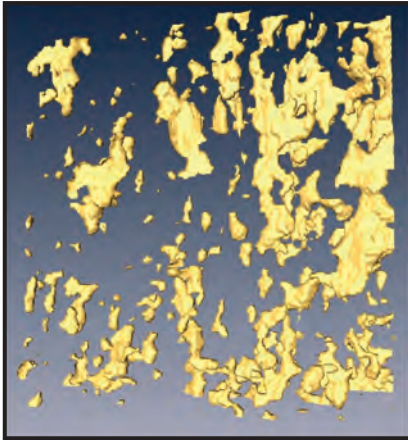
Sea Ice is a Multiscale Composite Material

sea ice microstructure

brine inclusions



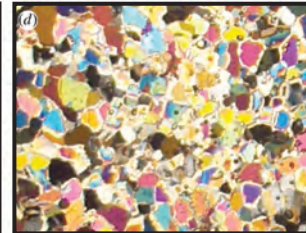
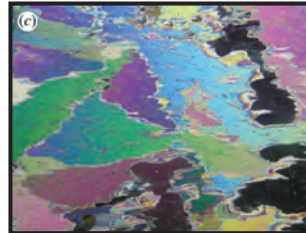
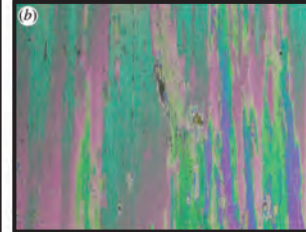
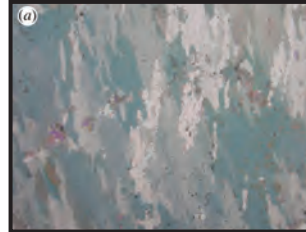
Weeks & Assur 1969



H. Eicken

Golden et al. GRL 2007

polycrystals

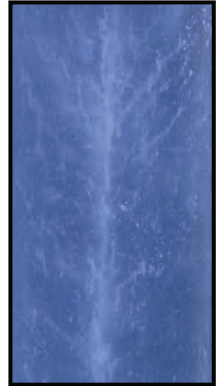


Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole



K. Golden

millimeters

centimeters

sea ice mesostructure

Arctic melt ponds



K. Frey

Antarctic pressure ridges



K. Golden

meters

sea ice macrostructure

sea ice floes



J. Weller

sea ice pack



NASA

kilometers

What is this talk about?

1. LIFE IN THE ICE

sea ice microphysics and fluid transport

2. LIFE UNDER THE ICE

melt ponds, under-ice light field, algal blooms

3. Species competition - resources depend on climate

Solving problems in physics and biology of sea ice drives advances in theory of composite materials and ecological systems.

Microbial life IN sea ice

sea ice microphysics

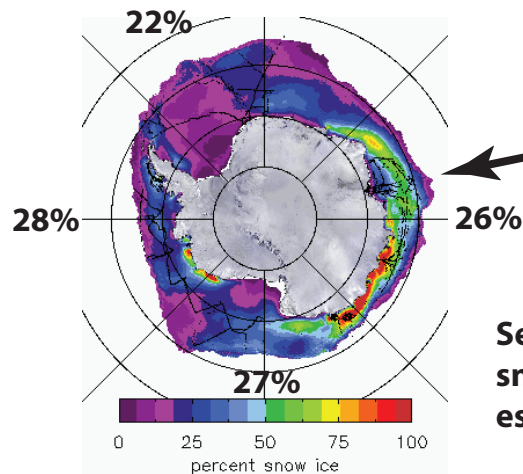
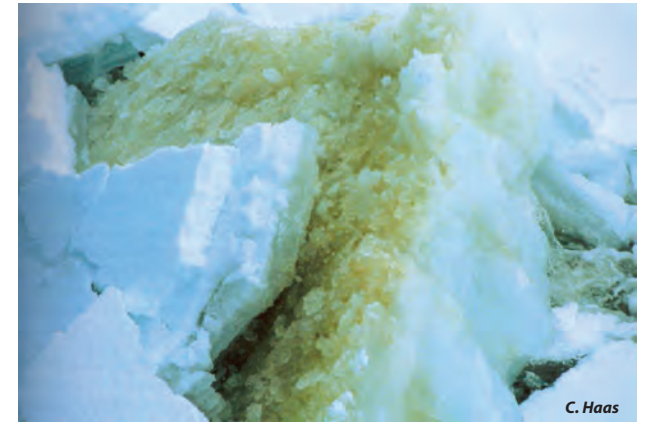
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

September
snow-ice
estimates

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO₂*

sea ice ecosystem



sea ice algae
support life in the polar oceans

fluid permeability k of a porous medium

porous
concrete



how much water
gets through the
sample per unit
time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

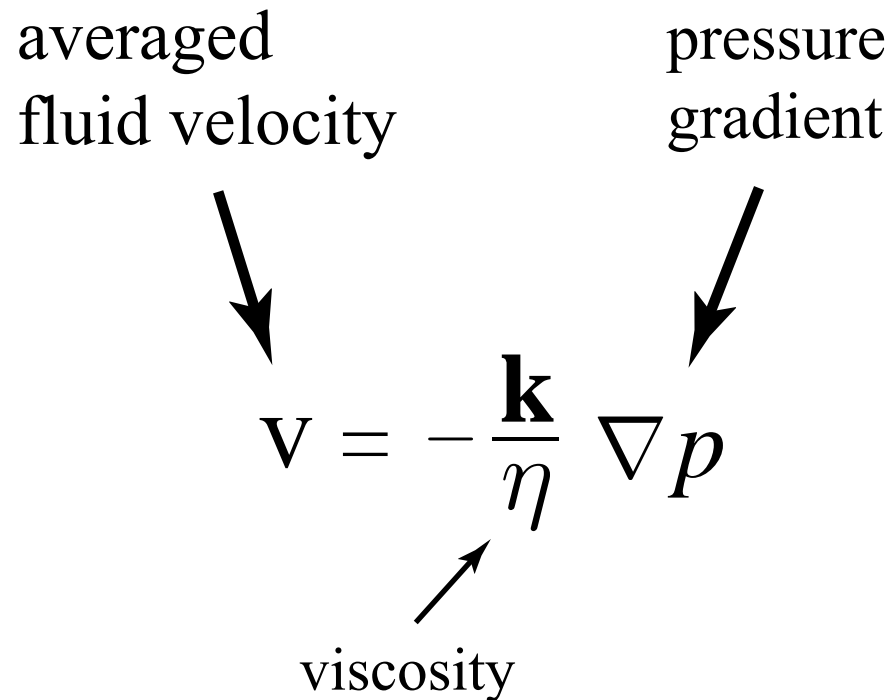
Darcy's Law for slow viscous flow in a porous medium

averaged
fluid velocity

pressure
gradient

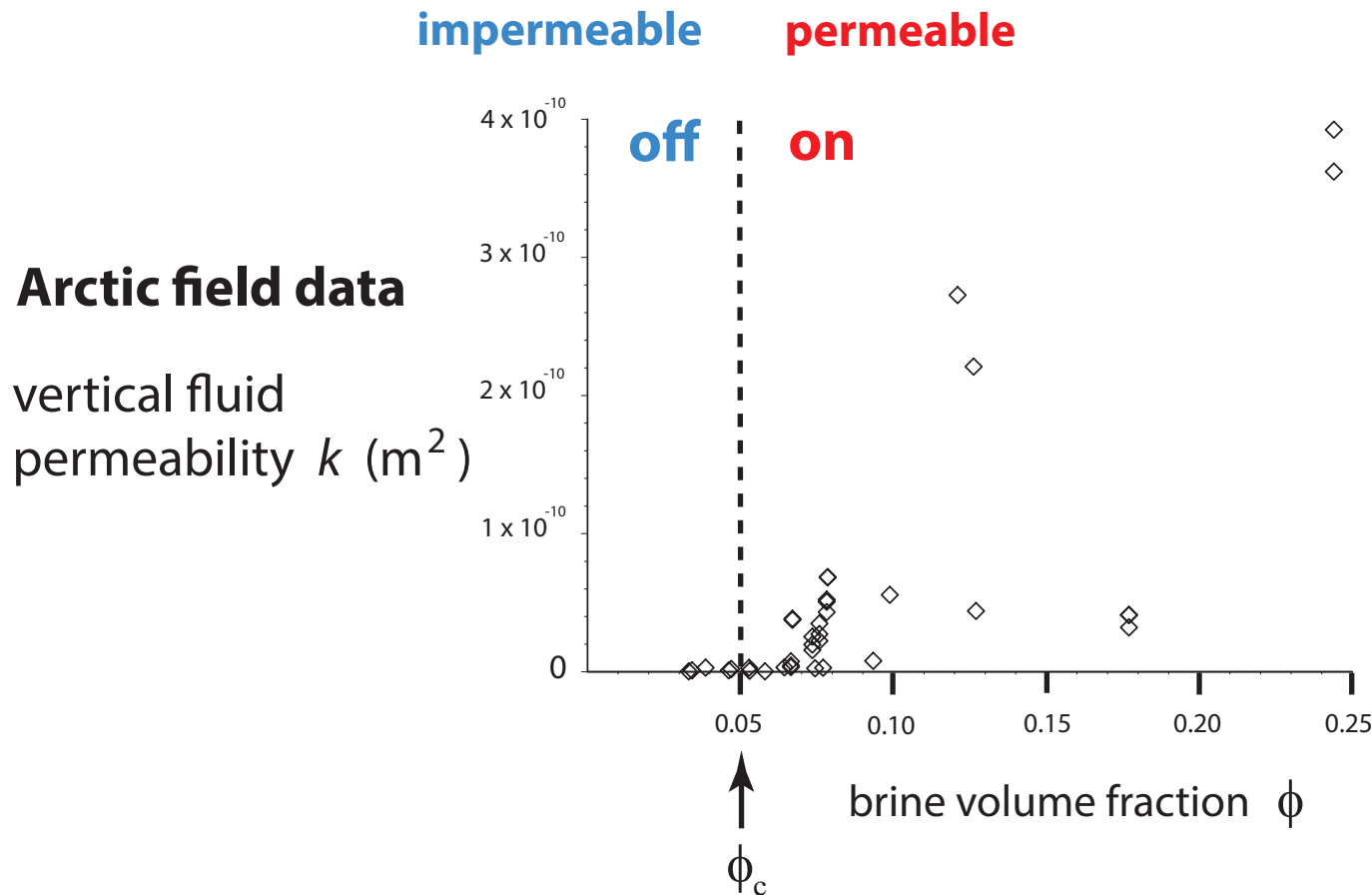
$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

The diagram shows the equation $\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$ centered on the slide. Three labels with arrows point to parts of the equation: 'averaged fluid velocity' points to \mathbf{v} , 'pressure gradient' points to ∇p , and 'viscosity' points to η .

\mathbf{k} = fluid permeability tensor

Critical behavior of fluid transport in sea ice



***“on - off” switch
for fluid flow***

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

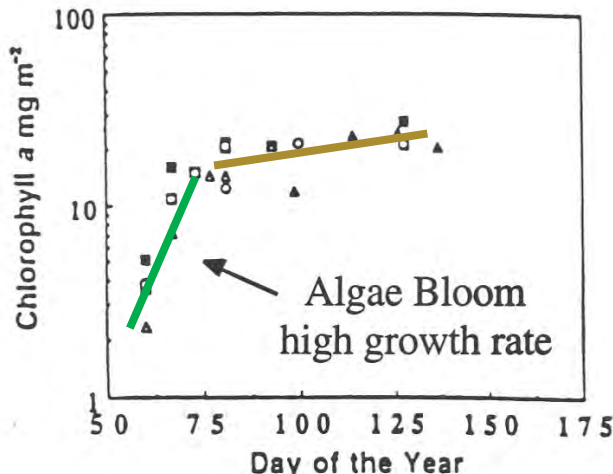
nutrient replenishment
controlled by ice permeability

biological activity turns on
or off according to
rule of fives

Golden, Ackley, Lytle **Science 1998**

Fritsen, Lytle, Ackley, Sullivan **Science 1994**

critical behavior of microbial activity

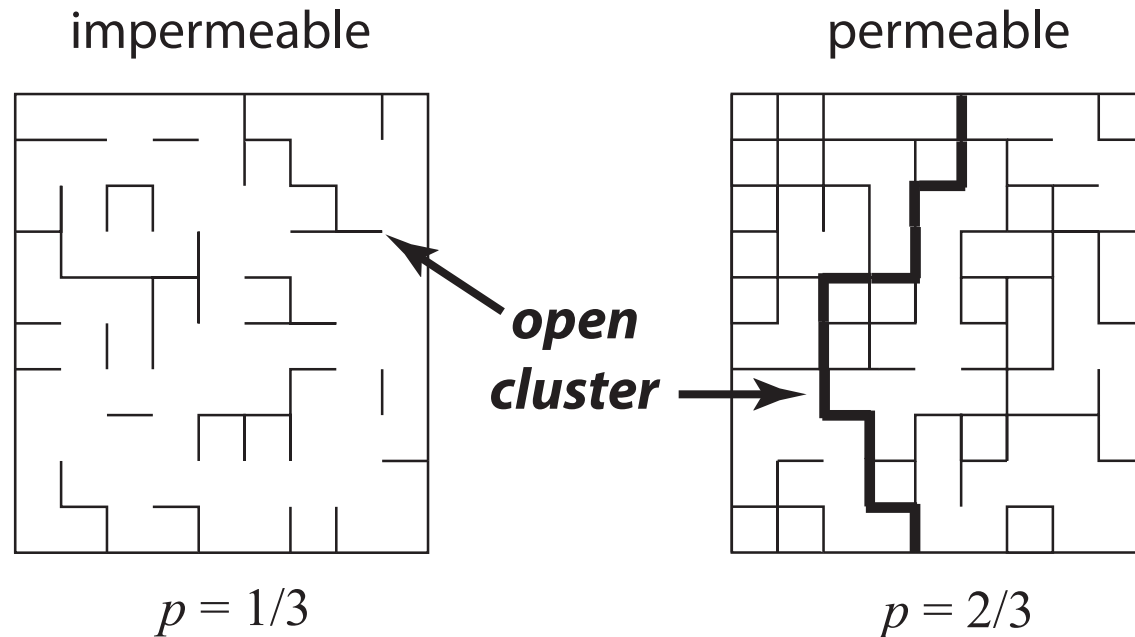


Convection-fueled algae bloom
Ice Station Weddell

Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open* with probability p
closed with probability $1-p$

percolation threshold

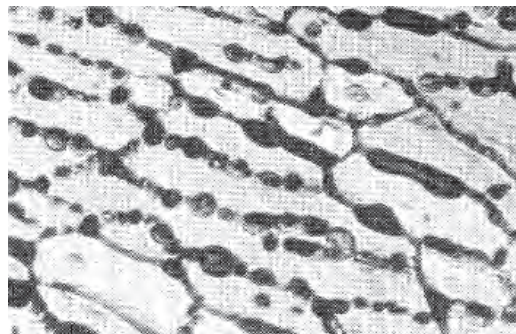
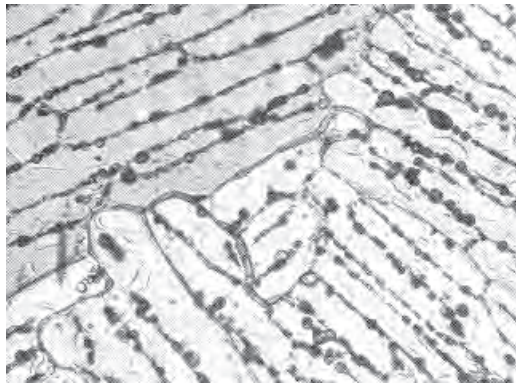
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest p for which there is an infinite open cluster

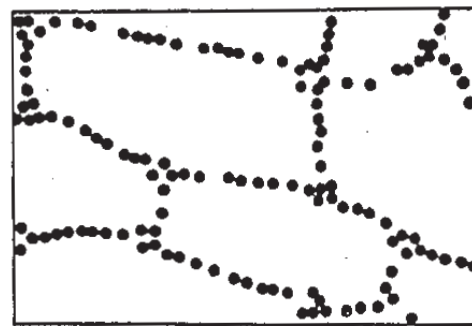
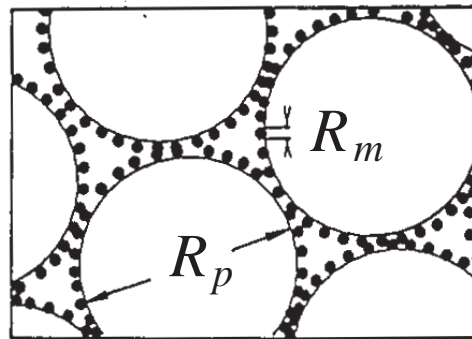
Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

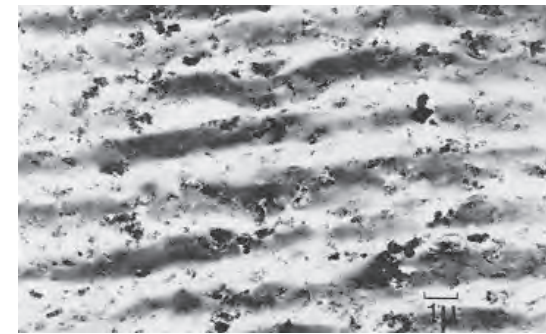
Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed
powder

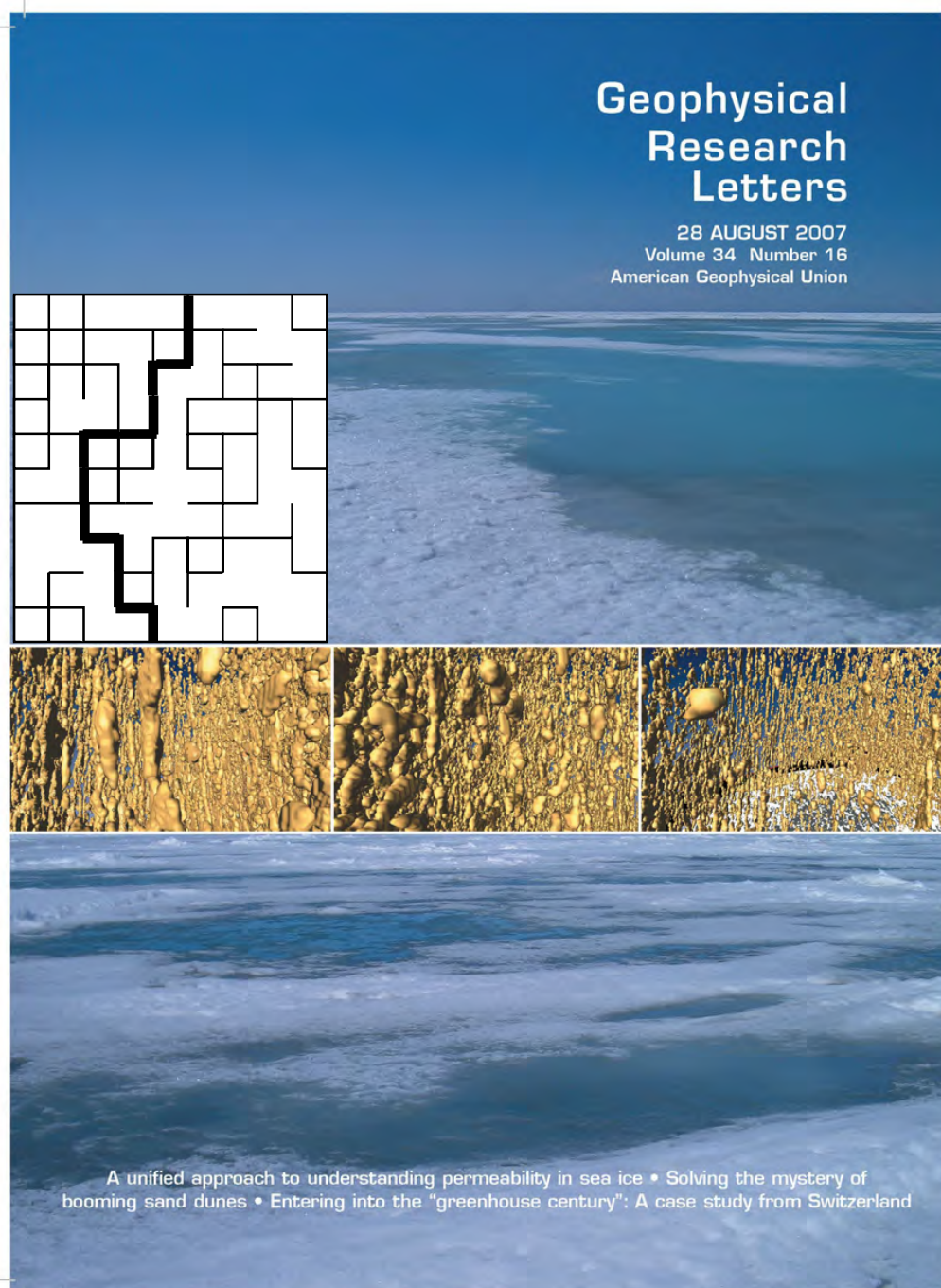


radar absorbing
composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



micro-scale
controls
macro-scale
processes

percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical
exponent
 t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

***hierarchical model
network model
rigorous bounds***

agree closely
with field data

***X-ray tomography for
brine inclusions***

***unprecedented look
at thermal evolution
of brine phase and
its connectivity***

confirms rule of fives

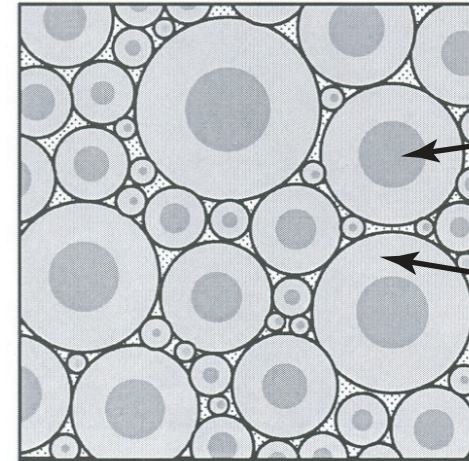
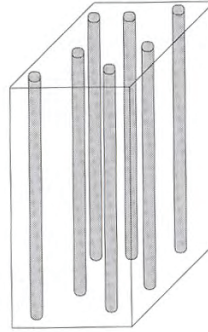
***Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009***

PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

vertical pipes
with appropriate radii
maximize k



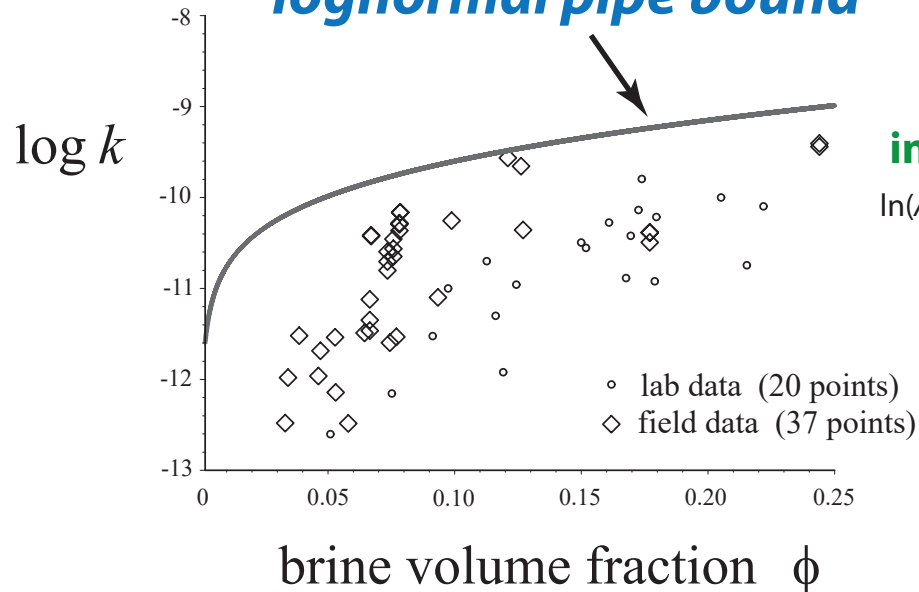
brine

ice

optimal coated
cylinder geometry

fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

lognormal pipe bound



Golden et al., Geophys. Res. Lett. 2007

$$k \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

$\ln(A)$ normally distributed, mean μ (increases with T) variance σ^2 (Gow and Perovich 96)

get bounds through variational analysis of **trapping constant** γ for diffusion process in pore space with absorbing BC

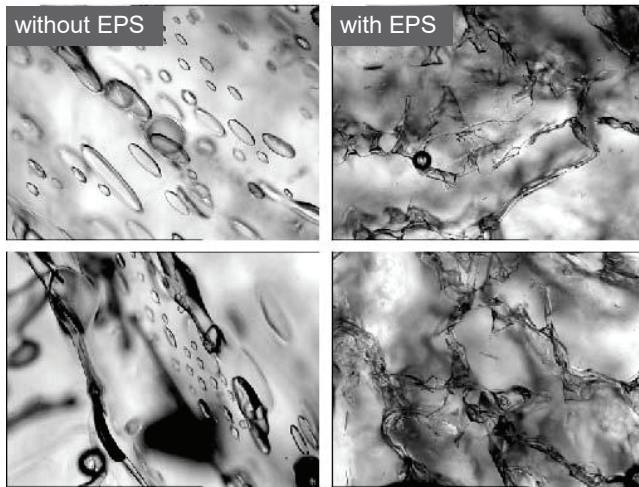
Torquato and Pham, PRL 2004

$$\mathbf{k} \leq \gamma^{-1} \mathbf{I} \quad \text{for any ergodic porous medium (Torquato 2002, 2004)}$$

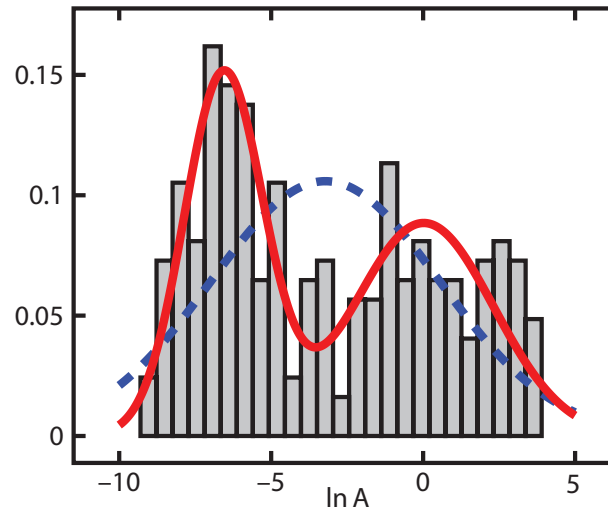
BACTERIAL FORAGING

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

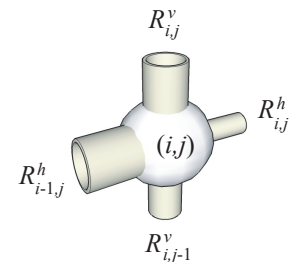
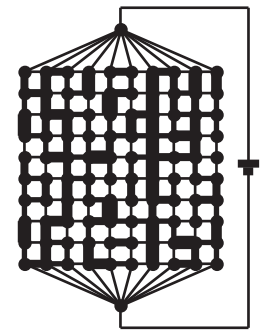
How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k .
- Rigorous bound on k for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

How does the biology affect the physics?

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice

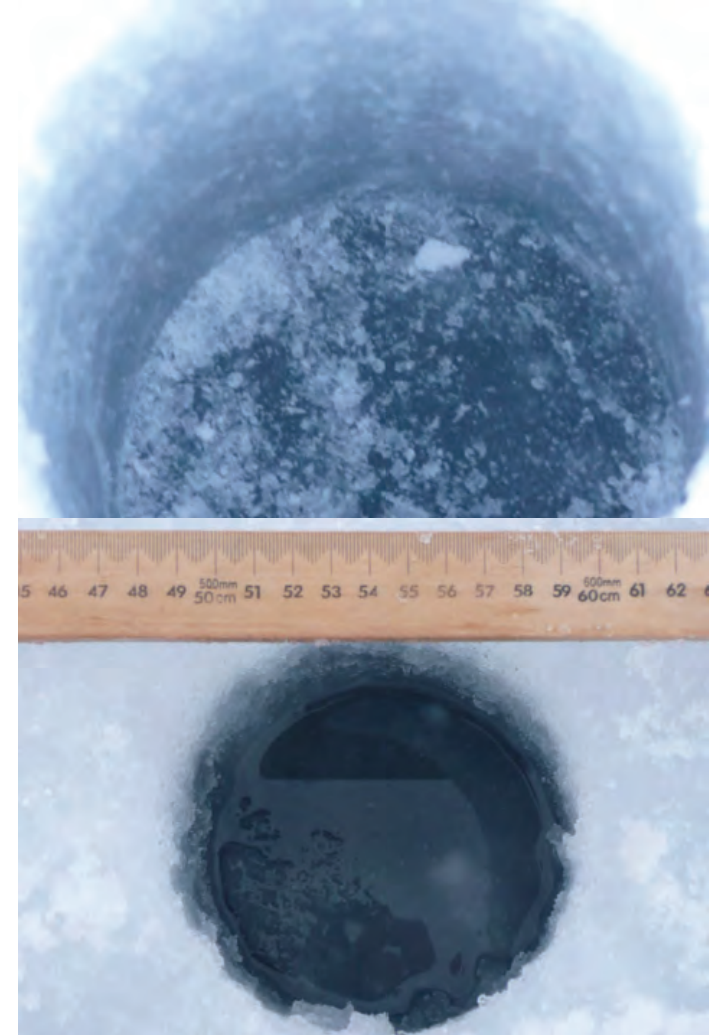
page 562

Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring
fluid permeability
of Antarctic sea ice***

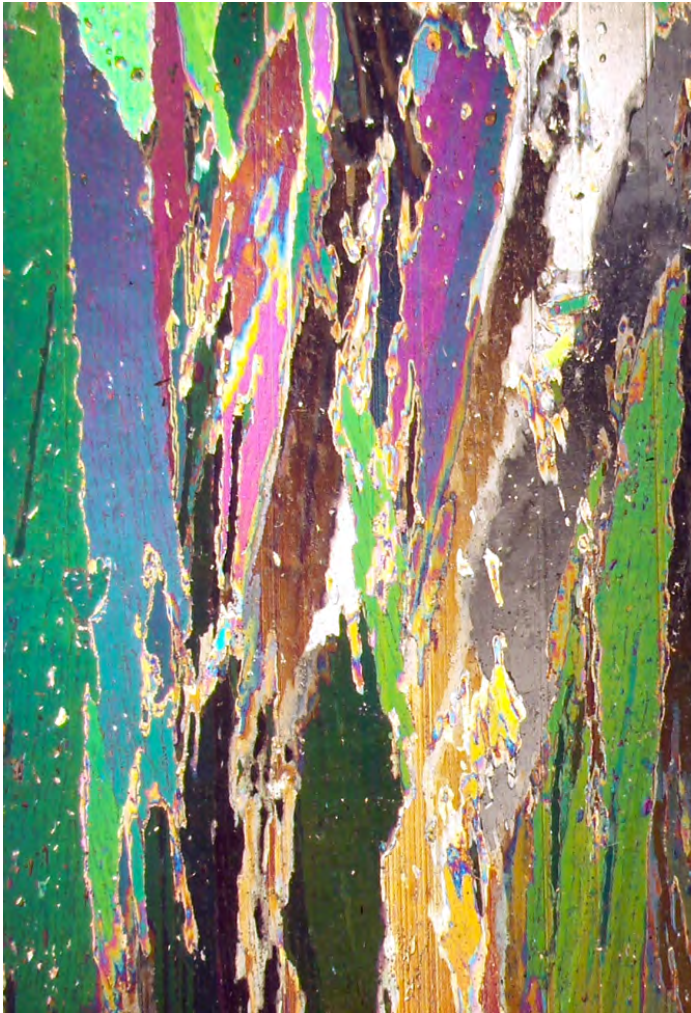
SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular

5%



10%



Golden, Sampson, Gully, Lubbers, Tison 2019

tracers flowing through inverted sea ice blocks



advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere

advection diffusion equation with a velocity field \vec{u}

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$



homogenize

$$\frac{\partial \bar{T}}{\partial t} = \kappa^* \Delta \bar{T}$$

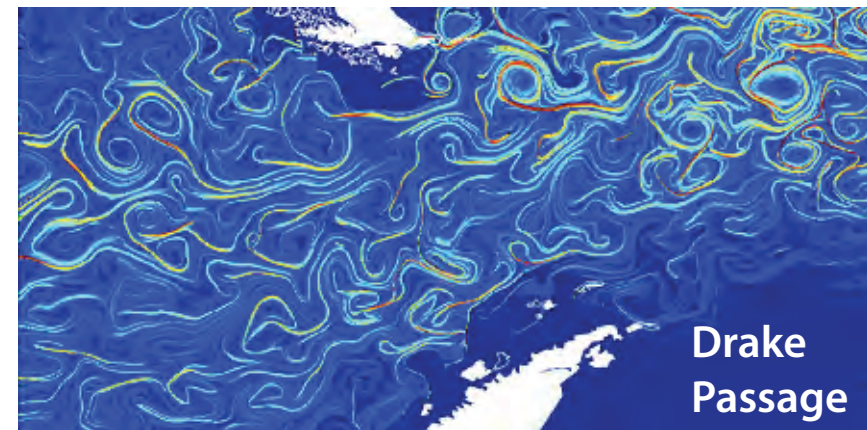
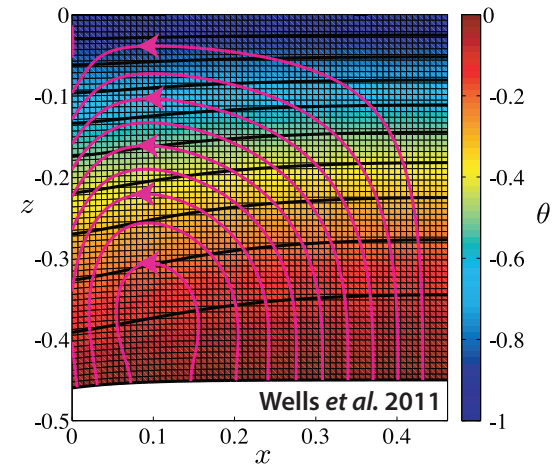
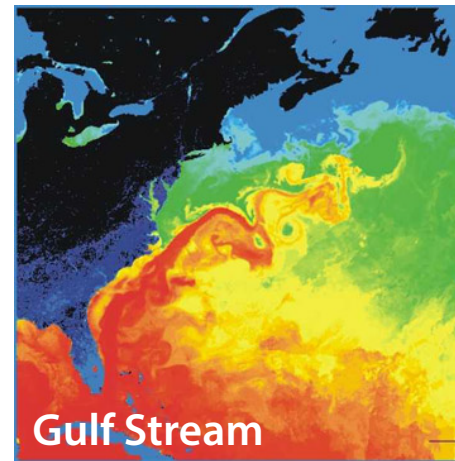
κ^* **effective diffusivity**

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2019



Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2019

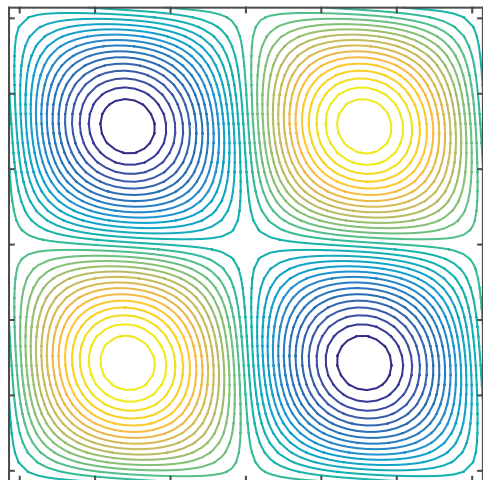
$$\kappa^* = \kappa \left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2} \right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H\Gamma$
- H = stream matrix , κ = local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla$, Δ is the Laplace operator
- $i\Gamma H\Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ -plane

separation of material properties and flow field
spectral measure calculations

Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Murphy, Zhu, Cherkaev, Strong, Golden 2019

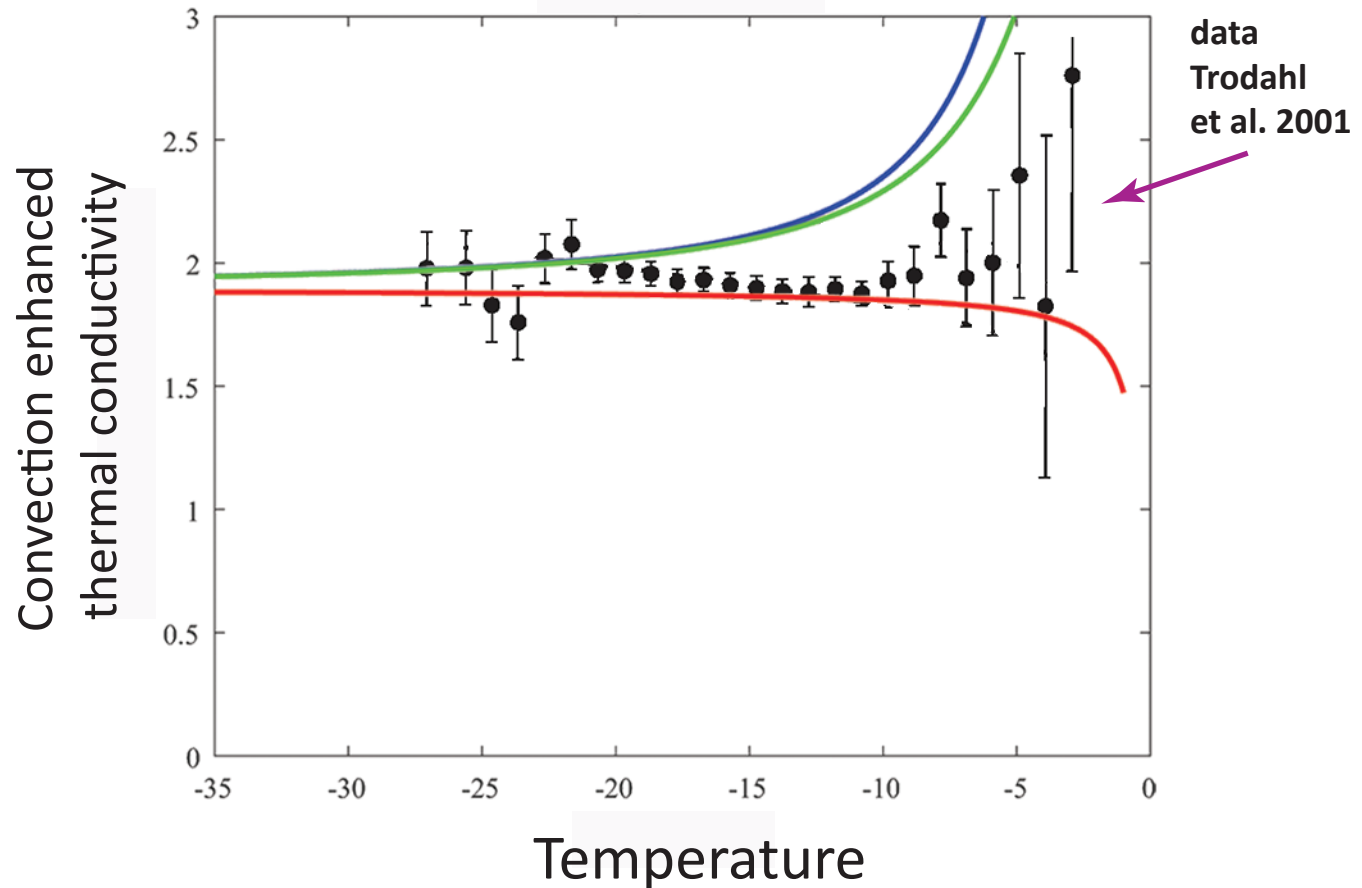


cat's eye flow model for
brine convection cells

similar bounds
for shear flows

**rigorous bounds assuming information
on flow field INSIDE inclusions**

Kraitzman, Cherkaev, Golden
SIAM J. Appl. Math (in revision), 2019



rigorous Padé bounds from Stieltjes integral +
analytical calculations of moments of measure



megafauna

Ice floe diffusion in winds and currents

Anomalous diffusion and sea ice dynamics

sub- and super-diffusive behavior of motion of sea ice floes as tracked by buoy data

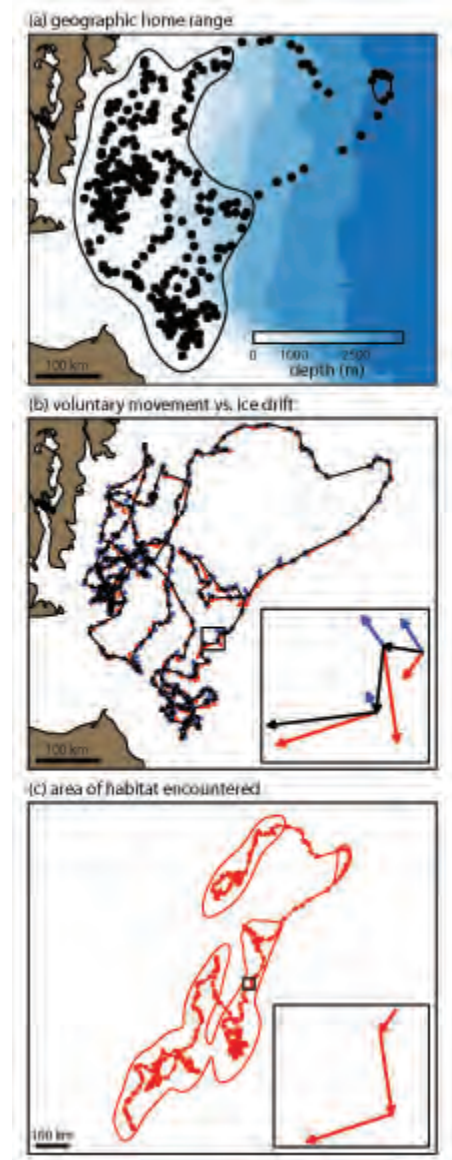
Jennifer Lukovich, Jennifer Hutchings, David Barber, *Ann. Glac.* 2015

Huy Dinh, Elena Cherkaev, Ken Golden, 2019

Home ranges in moving habitats: polar bears and sea ice

"diffusive" polar bear motion superimposed with drifting sea ice

Marie Auger-Méthé, Mark Lewis, Andrew Derocher, *Ecography*, 2016



Microbial life UNDER sea ice

melt ponds, algal blooms

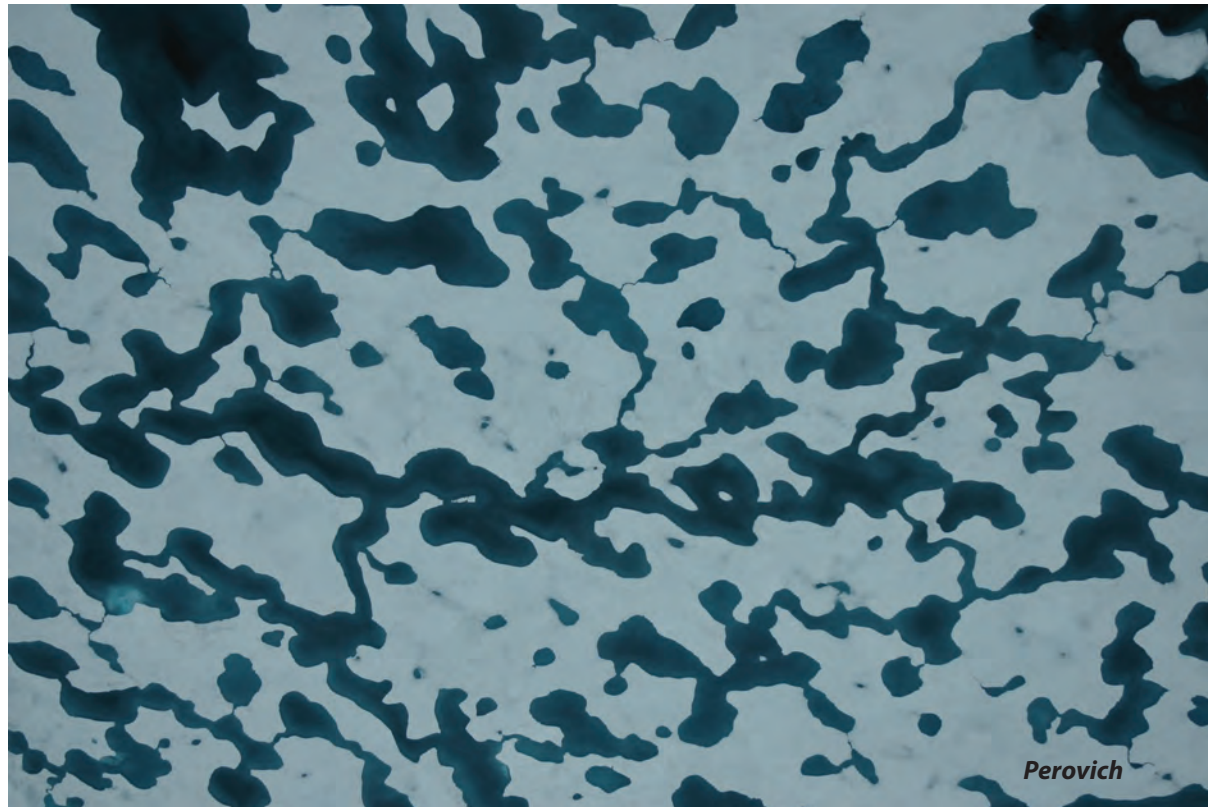
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



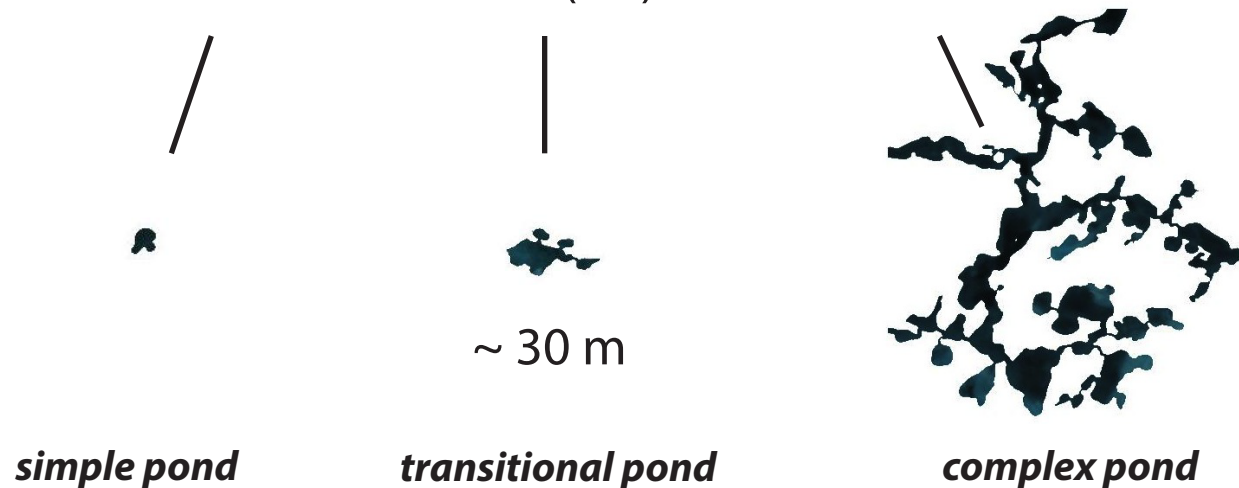
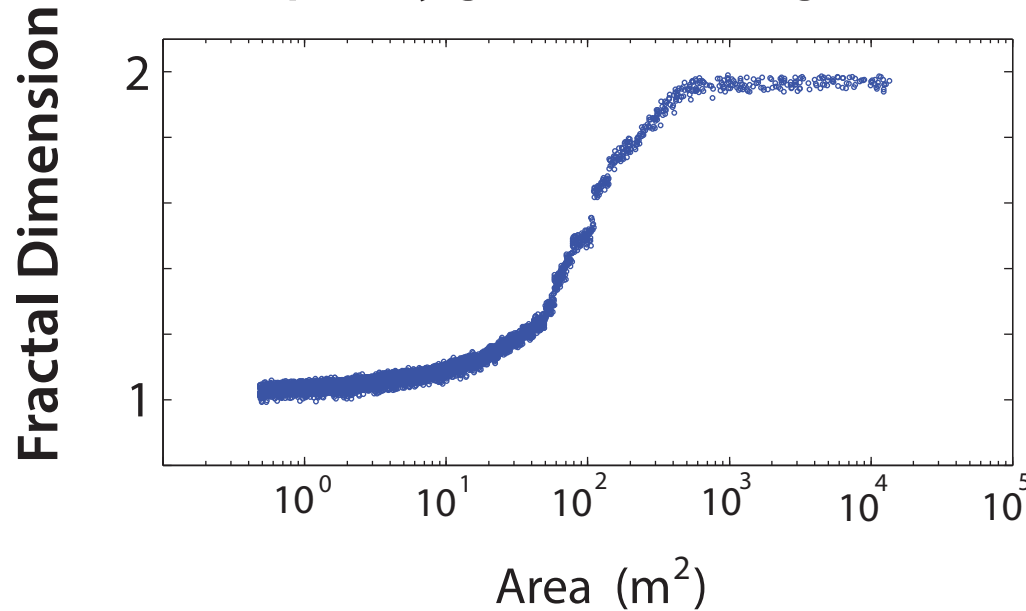
Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

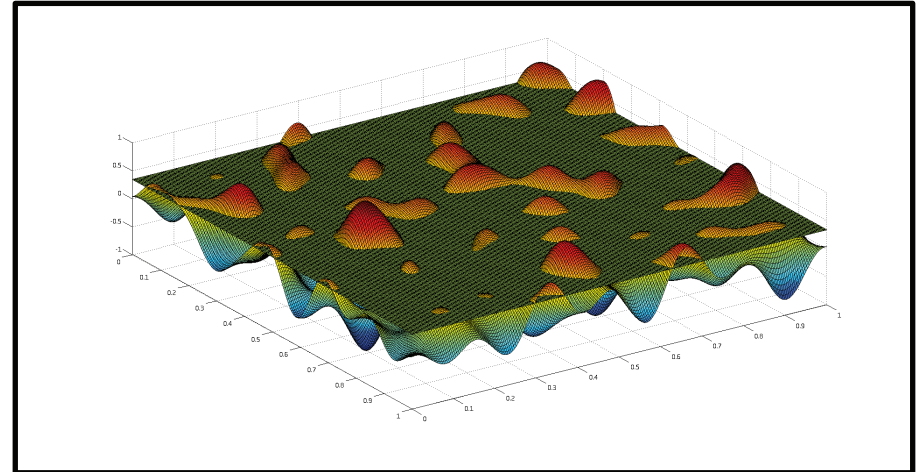
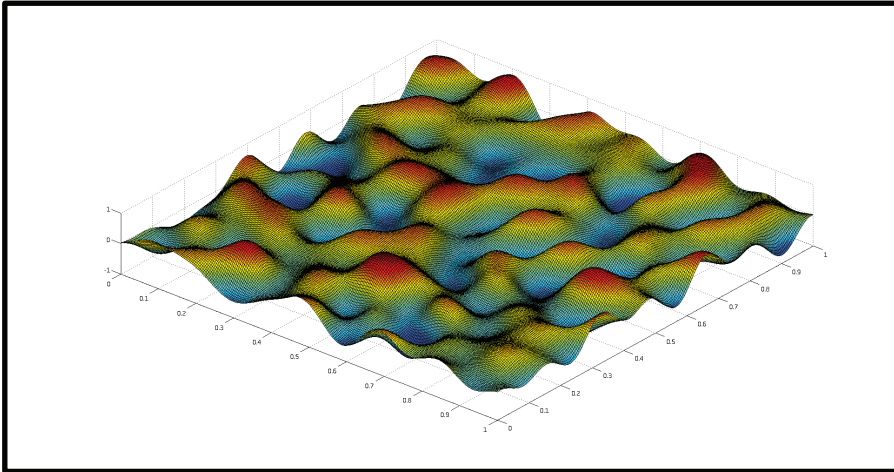
complexity grows with length scale



Continuum percolation model for melt pond evolution

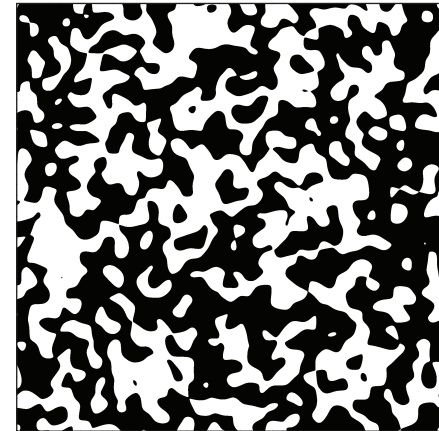
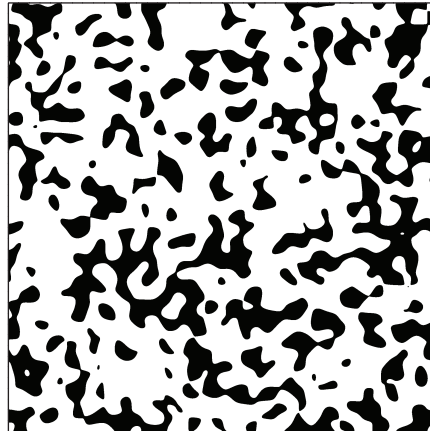
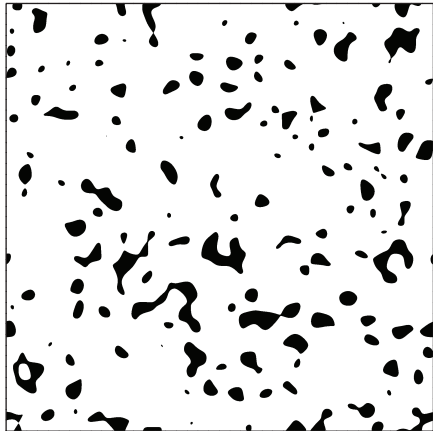
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

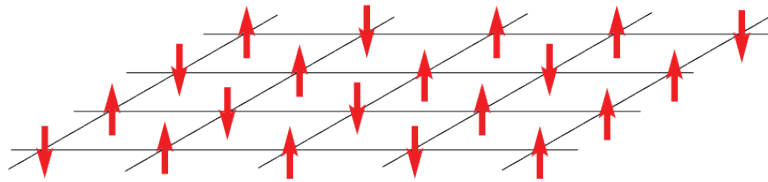


electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

Ising Model for a Ferromagnet



$$s_i = \begin{cases} +1 & \text{spin up} & \text{blue} \\ -1 & \text{spin down} & \text{white} \end{cases}$$

applied
magnetic
field



H

$$\mathcal{H} = -H \sum_i s_i - J \sum_{\langle i,j \rangle} s_i s_j$$

nearest neighbor Ising Hamiltonian

ferromagnetic interaction $J \geq 0$

magnetization

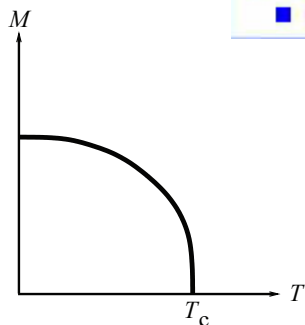
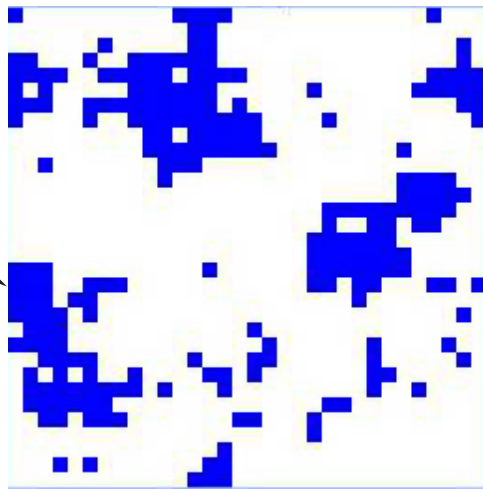
$$M(T, H) = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

homogenized parameter
like effective conductivity

Stieltjes integral representation for M

Baker, PRL 1968

**islands or
ponds of
like spins**



Curie point
critical temperature

Ising model for ferromagnets \longrightarrow Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

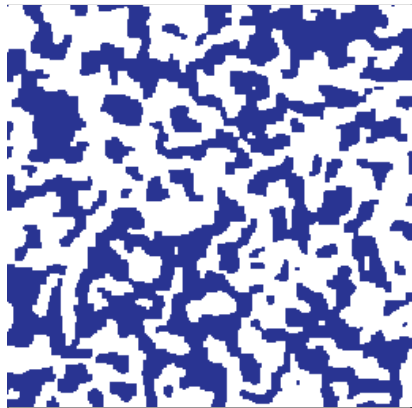
$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

random magnetic field
represents snow topography

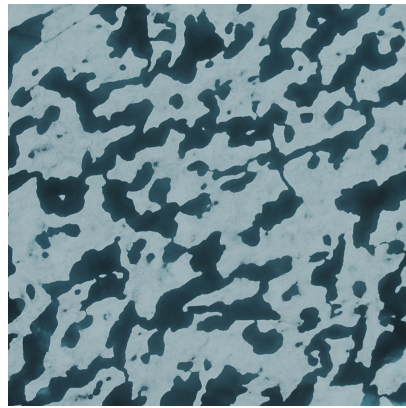
magnetization M pond coverage $\frac{(M+1)}{2}$
 \sim *albedo* only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

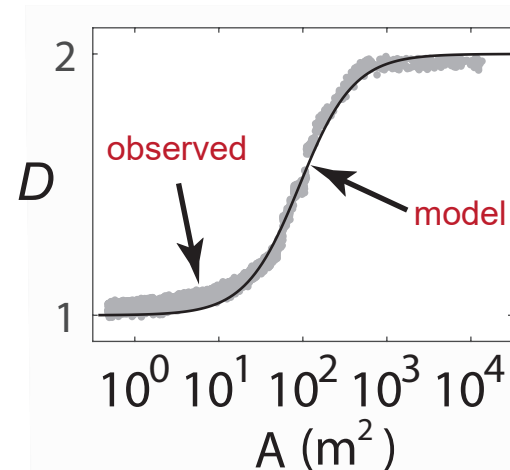
Order from Disorder



Ising
model



melt pond
photo (Perovich)



pond size
distribution exponent

observed -1.5
(Perovich, et al. 2002)

model -1.58

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



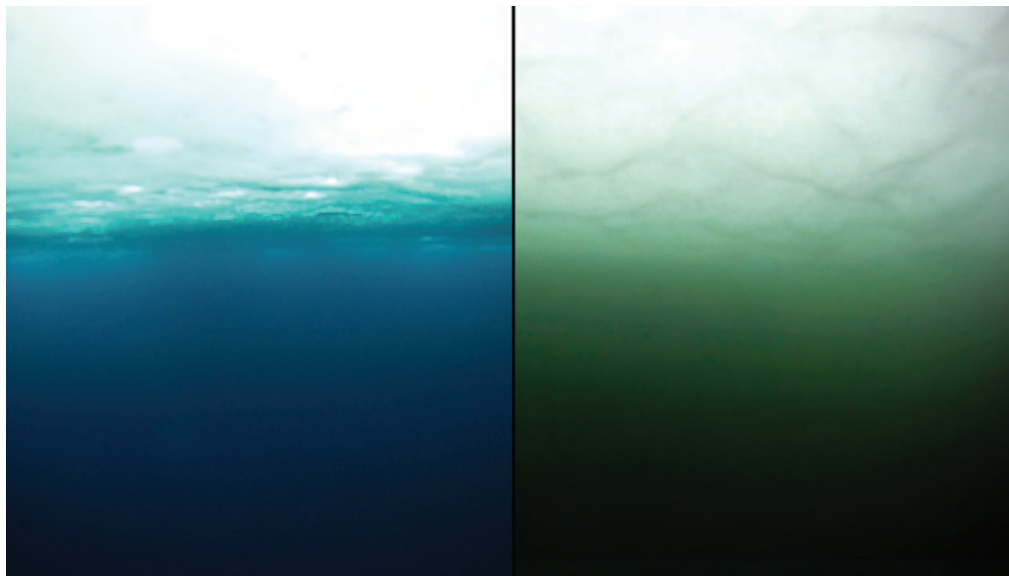
2011 massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light
through sea ice



no bloom

bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice
phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, *Science Advances*, 2017

The distribution of solar energy under
ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2019

(2015 AMS MRC @ Snowbird)

the view from underneath →



melt ponds are WINDOWS

light reaches the upper ocean

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

- Model for initiation of light-limited phytoplankton bloom (depth of mixed layer, ice thickness, melt pond area fraction, ...)
- Thinner summertime Arctic sea ice is increasingly covered in melt ponds, which permit more light penetration.
- Marked increase in light conditions conducive to sub-ice blooms.
- As little as 20 years ago, conditions for sub-ice blooms may have been uncommon; frequency has increased so that nearly 30% of the ice-covered Arctic Ocean in July permits sub-ice blooms.

Recent climate change may have significantly altered the ecology of the Arctic Ocean.

The distribution of solar energy under ponded first-year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, *in revision*, 2019

- Model for 3D light field under ponded sea ice.
- Distribution of solar energy at depth influenced by *shape and connectivity* of melt ponds, as well as area fraction.
- Aggregate properties of the sub-ice light field, such as a significant enhancement of available solar energy under the ice, are controlled by parameter closely related to pond fractal geometry.
- Model and analysis explain how melt pond geometry *homogenizes* under-ice light field, affecting habitability.

Pond geometry affects the ecology of the Arctic Ocean.

The Melt Pond Conundrum:

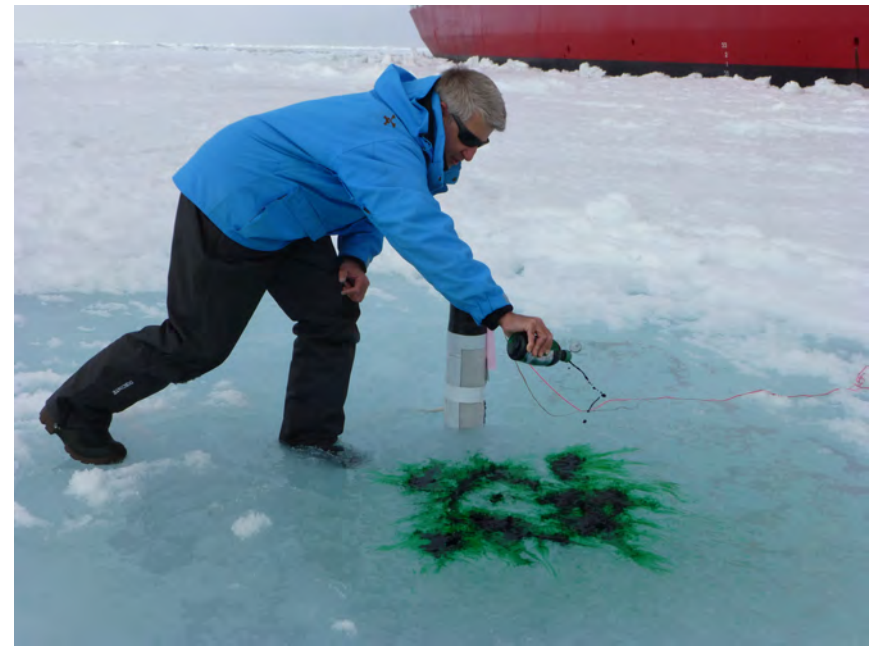
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

*2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy*



Large ecosystems in transition: bifurcations and mass extinction

I. Sudakov, S. Vakulenko, D. Kirievskaya, K. M. Golden, *Ecological Complexity*, 2017

model of multispecies populations competing for distributed resources

coupling climate and population dynamics via resources

feedback between species abundances and resources through temperature

resource competition model

Leon & Tumpson, *J. Theor. Biol.* 1975

Tilman, *Ecology*, 1977

Huisman & Weissing, *Nature*, 1999 (solved “plankton paradox”)

$$\frac{dx_i}{dt} = x_i(-r_i + \phi_i(v) - \sum_{j=1}^N \gamma_{ij} x_j), \quad 1 \leq i \leq N$$

$$\frac{dv}{dt} = D(S - v) - \sum_{j=1}^N c_j x_j \phi_j(v)$$

$$\phi_j(v) = \frac{a_j v}{K_j + v}, \quad a_j, K_j > 0$$

x_i species abundance

ϕ_i species growth rate

r_i species mortality rate

γ_{ij} describe competition (e.g. toxic compounds)
diagonals ~ self-regulation restricting abundances

S supply concentration of resource

v resource availability

D resource turnover rate

c_j determine how species share resource

K_j saturation constants

Extend model to M resources, whose supplies depend on temperature T , which depends on species abundances.

$$T = \bar{T} + \Delta T, \quad \Delta T = \sum_{k=1}^N \mu_{kj} x_j$$

$$S_k = \bar{S}_k(\bar{T}) + \Delta S_k + O(\Delta T^2)$$

$$\Delta S_k = \sum_{j=1}^N b_{kj}(\bar{T}) x_j, \quad k = 1, \dots, M$$

$$b_{kj} = \frac{dS_k(\bar{T})}{d\bar{T}} \mu_{kj} \begin{cases} > 0 & \text{positive feedback} \\ < 0 & \text{negative feedback} \end{cases}$$

reduction to Lotka-Volterra system

$|b_{ik}| \ll \gamma$ **weak climate coupling**

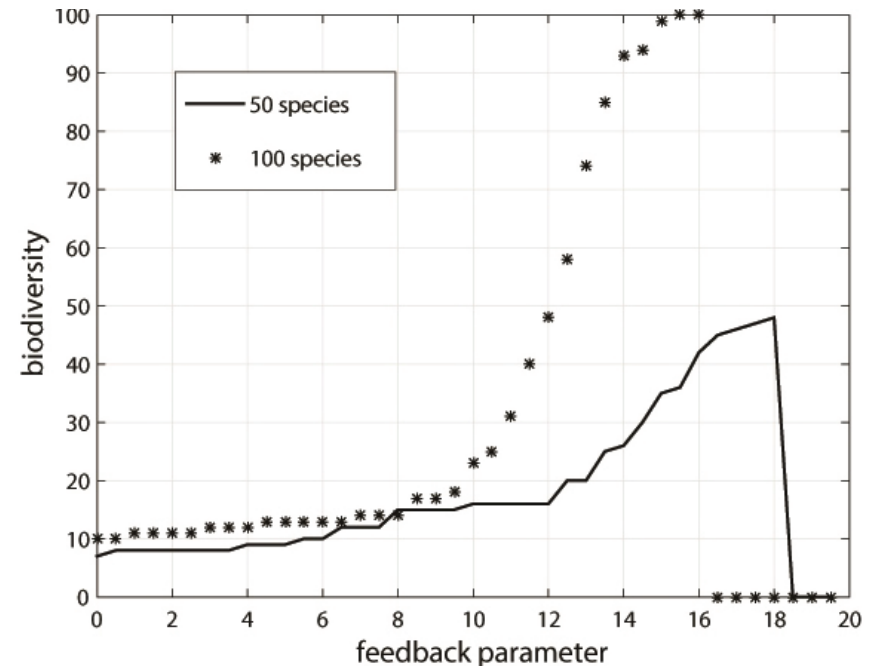
close to 'competitive systems' - exhibit no stable periodic or chaotic regimes: almost all trajectories converge to equilibria

$|b_{ik}| \gg \gamma$ **strong climate coupling**

$M = 1$, possible that all N species survive in equilibrium, or coexistence of many equilibria

$M = 2$, feedback coeffs have different signs, periodic sols

$M > 2$, system can produce time chaotic solutions



mass
extinctions

when the number of species increases, so does the likelihood of a sharp drop in species number as the climate changes and feedback processes grow stronger

The model exhibits coexistence of many species, yet also displays the possibility of catastrophic bifurcations, where all species become extinct under the influence of abiotic factors (strong climate coupling).

Conclusions

- 1. The physics of fluid transport in porous composites regulates microbial life inside sea ice.**
- 2. The geometry of melt ponds controls light in the upper ocean, and initiation of under-ice algal blooms.**
- 3. Resource competition models provide tools to study the complexity of these microbial communities.**

THANK YOU

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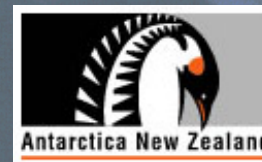
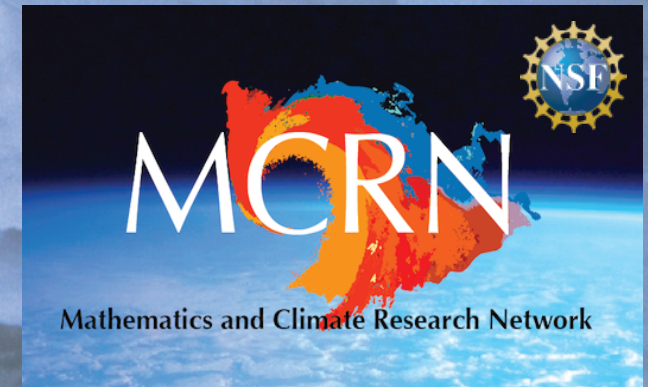
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Division of Polar Programs



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Happy Birthday Mike!