Name ______________________________

Student ID # ______________________________

Class (circle one) 9:40 10:45

Mathematics 1210
Fall 2009
K. M. Golden

FINAL EXAM

Thursday, December 17, 2009

Problem	Points	Score
1.	20	
2.	15	
3.	10	
4.	10	
5.	15	
6.	15	
7.	10	
8.	10	
9.	10	
10.	10	
11.	10	
12.	15	
	TOTAL	

(20 points) 1. Calculate the following. Be sure to show all of your work.

(a) $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$

(b) $\lim_{x \to 0} \dfrac{\int_0^x (\cos t - 1)\, dt}{x^3}$

(c) $\lim_{x \to 0} \dfrac{\sin(x^2)}{\pi x}$

(d) $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} 1, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$

(15 points) 2. A population of frogs in a swamp is found to grow at a rate proportional to the square root of the population size. The initial population P is 400 frogs, and 5 years later there are 900 of them. Write the differential equation for the frog population $P(t)$ with the two corresponding conditions, and find the particular solution which incorporates both conditions.

(10 points) 3. Use the differential to approximate $\sqrt[3]{28}$, showing all your work.

(10 points) 4. Suppose a ball is propelled upward from the ground $x_0 = 0$ with an initial velocity of $v_0 = 64$ feet/second. Its position $x(t)$ after t seconds is then given by $x(t) = -16t^2 + 64t$. When does the ball reach its maximum height? What is the maximum height reached by the ball?

(15 points) 5. Find the dimensions of the rectangle with maximum area inscribed in a circle of unit radius. Be sure to show all your work, and verify your result with the second derivative test.

(15 points) 6. Find the temperature profile $T(x)$ on the interval $[0, 2]$ satisfying the steady state heat equation

$$\frac{d^2T}{dx^2} = 0$$

and the boundary conditions $T(0) = -10$ and $T(2) = -2$.

(10 points) 7. For a given spring, the force required to keep it stretched x feet is given by $F = 9x$ pounds. How much work in ft·lb is done in stretching the spring 1 foot from its unstretched equilibrium state? Be sure to show all your work.

(10 points) 8. Consider the region bounded by the graph of $y(x) = \sin x$ and the $x-$axis between $x = 0$ and $x = 2\pi$. Revolve this region around the $x-$axis, and find the volume of the resulting solid.

(10 points) 9. Find the area of the finite region bounded by the graphs of $y = -x+2$ and $y = x^2$. Sketch the region.

(10 points) 10. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the $x-$axis and the upper half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(10 points) 11. Consider a circle of radius 5 centered at the origin, parameterized by $(x(t), y(t)) = (5\cos t, 5\sin t)$. Find the circumference of this circle by calculating the arc length of the curve parameterized by $(x(t), y(t))$ with $t \in [0, 2\pi]$.

(15 points) 12. Use the Theorem of Pappus to find the volume of the doughnut, or *torus*, obtained when the region bounded by the circle of radius 1, centered at the point $(3, 0)$, is revolved around the $y-$axis. The equation of the circle is $(x - 3)^2 + y^2 = 1$. (Hint: where is the centroid of the circular region?)