Mathematics 1210 Prof. Ken Golden

PRACTICE FINAL EXAM Fall 2009

Problems From Varberg, *et al.* (Chapter.Section.Number)

1. 5.1.6. Use the three step procedure (slice, approximate, integrate) to set up an integral or integrals for the area of the indicated region. The region lies above the parabola $y = x^2 - 2$ and below the line $y = x + 4$, and is enclosed by these two graphs.

2. 5.2.17. Find the volume of the solid generated by revolving about the x-axis the region bounded by the upper half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and the x-axis, and thus find the volume of a *prolate spheroid.* Here a and b are positive constants, with $a > b$.

3. 5.3.2. Find the volume of the solid generated when the region R bounded by the curves $y = x^2$, $x = 1$, $y = 0$ is revolved about the y-axis. Do this by performing the following steps: (a) Sketch the region R. (b) Show a typical rectangular slice properly labeled. (c) Write a formula for the approximate volume of the shell generated by this slice. (d) Set up the corresponding integral. (e) Evaluate this integral.

4. 5.4.10. Sketch the graph of the parametric equation

$$x = \sqrt{5}\sin 2t - 2 \qquad y = \sqrt{5}\cos 2t - \sqrt{3}; \qquad 0 \le t \le \frac{\pi}{4},$$

and find its length.

5. 5.4.21a. Find the length of the curve

$$y = \int_1^x \sqrt{u^3 - 1}\, du, \quad 1 \le x \le 2.$$

6. 5.5.6. For a certain type of nonlinear spring, the force required to keep the spring stretched a distance s is given by the formula $F = ks^{4/3}$. If the force required to keep it stretched 8 inches is 2 pounds, how much work is done in stretching this spring 27 inches?

7. 5.5.22. According to Coulomb's Law, two like electrical charges repel each other with a force that is inversely proportional to the square of the distance between them. If the force of repulsion is 10 dynes (1 dyne = 10^{-5} newton) when they are 2 centimeters apart, find the work done in bringing the charges from 5 centimeters apart to 1 centimeter apart.

8. 5.6.25. Use Pappus' Theorem to find the volume of the solid obtained when the region bounded by $y = x^3$, $y = 0$, and $x = 1$ is revolved about the y-axis (see Problem 11 for the centroid). Do the same problem by the method of cylindrical shells to check your answer.

9. Find the temperature profile $T(x)$ on the interval $[0, 1]$ satisfying the steady state heat equation

$$\frac{d^2T}{dx^2} = 0$$

and the boundary conditions $T(0) = 4$ and $T(1) = 2$.