Name _____

Student ID # _____

Class (circle one) 9:40 10:45

Mathematics 1210
Fall 2004
K. M. Golden

FINAL EXAM

Monday, December 13, 2004

Problem	Points	Score
1.	20	
2.	15	
3.	10	
4.	15	
5.	20	
6.	20	
7.	10	
8.	15	
9.	10	
10.	15	
	TOTAL	

(20 points) 1. Calculate the following. Be sure to show all of your work.

(a) $\dfrac{dm}{dv}$ where $m(v) = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

(b) $\lim\limits_{x \to 0} \dfrac{\displaystyle\int_0^x (\cos t \ - 1)\, dt}{x^3}$

(c) $\lim\limits_{x \to 0} \dfrac{\sin(\pi x)}{x}$

(d) $\lim\limits_{x \to +\infty} \dfrac{\sqrt[3]{8x^2 + 3x + 1}}{x - 5}$

(15 points) 2. A population of frogs in a swamp is found to grow at a rate proportional to the square root of the population size. The initial population P is 400 frogs, and 5 years later there are 900 of them. Write the differential equation for the frog population $P(t)$ with the two corresponding conditions, and find the particular solution which incorporates both conditions.

(10 points) 3. Use the differential to approximate $\sqrt{26}$.

(15 points) 4. Suppose a ball is thrown upward from the ground $x_0 = 0$ with an initial velocity of $v_0 = 32$ feet/second. Then its position $x(t)$ after t seconds is given by $x(t) = -16t^2 + 32t$.

(a) What is the maximum height reached by the ball?

(b) When does the ball hit the ground after falling back to earth?

(20 points) 5. Find the dimensions of the rectangle with maximum area inscribed in a circle of unit radius. Be sure to show all your work, and verify your result using the 1^{st} or 2^{nd} derivative test.

(20 points) 6. (a) For a given spring, the force required to keep it stretched x feet is given by $F = 9x$ pounds. How much work in ft·lb is done in stretching the spring 1 foot from its unstretched equilibrium state? Be sure to show all you work.

(b) Suppose there is a *positive* electrical charge at the origin $r = 0$ and an equal but opposite *negative* electrical charge 1 cm away at $r = 1$. If the Coulomb force of attraction on the negative charge is given by

$$F_{coul}(r) = -\frac{40}{r^2} \quad \text{dynes},$$

find how much work in dyne·cm you must do to move the negative charge from 1 cm away at $r = 1$ to 5 cm away at $r = 5$.

(10 points) 7. Find the area of the finite region bounded by the graphs of $y = -x + 2$ and $y = x^2$.

(15 points) 8. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the $x-$axis and the upper half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(10 points) 9. Find the arc length of the curve

$$y(x) = \int_0^x \sqrt{\sin^4 t - 1}\ dt, \quad 0 \leq x \leq \pi.$$

(15 points) 10. Use Pappus's Theorem to find the volume of the solid obtained when the region bounded by the graph of $y(x) = \sin x$ and the $x-$axis, for $0 \leq x \leq \pi$, is revolved about the $y-$axis.