Name _____

Student ID # _____

Class (circle one) 9:40 10:45

Math 1210
Fall 2006
K. M. Golden

EXAM III
Friday, November 17, 2006

Problem	Points	Score
1.	15	
2.	10	
3.	20	
4.	15	
5.	15	
6.	15	
7.	10	
	TOTAL	

(15 points) 1. Calculate the following. Be sure to show all of your work.

(a) $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$

(b) $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$

(10 points) 2. Solve the differential equation

$$\frac{dx}{dt} = -32t + 48$$

for the function $x(t)$ with the initial condition $x(0) = 0$.

(20 points) 3. A population of frogs in a swamp is found to grow at a rate proportional to the square root of the population size. The initial population P is 400 frogs, and 5 years later there are 900 of them.

(a) Write the differential equation for the frog population $P(t)$ with the two corresponding conditions.

(b) Solve this differential equation, that is, find the particular solution which incorporates both conditions.

(c) How long does it take for the frog population to quadruple (reach 1600) from its intitial value of 400?

(15 points) 4. Consider

$$f(x) = x + \frac{1}{x}\,.$$

Sketch the graph of $f(x)$, indicating any symmetry and asymptotes. Find where the function is increasing and decreasing, and where it is concave up and concave down. Indicate all local and global extrema, and any inflection points. Be sure to show all your work.

(15 points) 5. Find the dimensions of the rectangle with maximum area inscribed in a circle of unit radius. Be sure to show all your work, and verify your result with the second derivative test.

(15 points) 6. Calculate $\int_0^1 x\ dx$ from the **definition** of the integral – using a Riemann sum. Be sure to show all your work. Hint: use the fact that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\ .$$

Check your result using the **fundamental theorem of calculus**.

(10 points) 7. Calculate the following integrals:

(a) $\int_1^2 (3x^2 + x + 1)\ dx$

(b) $\int \sin^2 x\ \cos x\ dx$