Student ID # _____

Class (circle one) 9:40 10:45

Math 1210 Fall 2004 K. M. Golden

EXAM III

Friday, November 19, 2004

Problem	Points	Score
1.	20	
2.	20	
3.	10	
4.	10	
5.	20	
6.	20	
	TOTAL	

(20 points) - 1. Calculate the following. Be sure to show all of your work.

(a)
$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

(b)
$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

(c)
$$\lim_{x \to 0} \frac{\int_0^x \sin(t^2) dt}{x^3}$$

- (20 points)2. A population of frogs in a swamp is found to grow at a rate proportional to the square root of the population size. The initial population P is 400 frogs, and 5 years later there are 900 of them.
 - (a) Write the differential equation for the frog population P(t) with the two corresponding conditions.

(b) Solve this differential equation, that is, find the particular solution which incorporates both conditions.

(c) How long does it take for the frog population to quadruple (reach 1600) from its initial value of 400?

(10 points) 3. Determine whether or not $f(x) = \frac{1}{\sqrt{\sin x}}$ is Riemann integrable on the interval (0,1], and **fully** explain your result (you may want to do an explicit calculation to explain your result).

(10 points) 4. Suppose the velocity v(t) of a mass at the end of a spring is given by $v(t) = \cos t$. If the position x(t) of the mass and its velocity are related by $v(t) = \frac{dx}{dt}$, find x(t), given that its initial position is x(0) = 1.

(20 points) 5. Using the fact that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, calculate $\int_0^2 x^2 dx$ directly from the **definition** of the integral, i.e., using Riemann sums. *Check* your result using the **fundamental theorem of calculus**.

(20 points) 6. Calculate the following integrals:

(a)
$$\int_{1}^{3} (4x^3 - 2x^2 + x - 5) dx$$

(b)
$$\int \tan^2 x \, \sec^2 x \, dx$$

(c)
$$\int_0^1 \frac{u(u+1)^2}{\sqrt{u}} du$$

(d)
$$\int_0^\pi \sin^2 x \, dx$$