Mathematics 1210 PRACTICE EXAM II Fall 2004

1. Calculate the following:

   a. $\dfrac{d^2x}{dt^2}$, $x(t) = A\sin(\omega t - \phi)$

   b. $\dfrac{df}{dx}$, $f(x) = \left(\dfrac{x-2}{x-\pi}\right)^3$

   c. $\dfrac{dy}{dx}$, $\cos xy = y^2 + 2x$

   d. $\displaystyle\lim_{x\to 0} \frac{\sin x \tan x}{1-\cos x}$

   e. $\dfrac{df}{dx}$, $f(x) = \sin\sqrt{\dfrac{\tan x}{1+x^2}}$

   f. $\dfrac{df}{dx}$, $f(x) = x^2 \sin^2(x^3)$

2. Find all solutions to the differential equation for a mass on a spring,

$$\frac{d^2x}{dt^2} = -\omega^2 x(t),$$

   where $x(t)$ is the position as a function of time t, $\omega = \sqrt{k/m}$, k is the spring constant and m is the mass. (Hint: see problem 1(a) above.) Choose a set of parameters characterizing a particular solution, and graph it. Also find in this case for which times the position is maximized, and for which times the velocity is maximized.

3. A circular oil slick spreads so that its radius increases at the rate of 1.5 feet/second. How fast is the area of the enclosed oil increasing at the end of two hours?

4. The hands on a clock are of length 5 inches (minute hand) and 4 inches (hour hand). How fast is the distance between the tips of the hands changing at 3:00?

5. Approximate $\sqrt{66}$ and $\sin\left(\dfrac{\pi}{100}\right)$ using linear approximation (i.e., the differential). Also do problem #35 on page 155, section 3.10.

6. Use the differential to approximate the increase in volume of a spherical bubble as its radius increases from 3 to 3.025 inches.

7. Consider $f(x) = \begin{cases} |x|, x < 0, \\ \sin x, x \geq 0. \end{cases}$ Find all local maxima and minima of f, where f is increasing and decreasing, where f is concave up and concave down, and all inflection points. Does f have a global maximum or a global minimum? Sketch the graph of $f(x)$. Do the same for $f(x) = x^3 - 12x + 1$ and $f(x) = \dfrac{1}{1+x^2}$.

8. Consider $f(x) = 2\sin(x - \frac{\pi}{4})$ on the interval $[\frac{\pi}{2}, \frac{5\pi}{4}]$. Find where f is increasing and decreasing. Find the maximum and minimum values of f on the interval.

9. A rectangle has two corners on the x−axis and the other two on the parabola $y = 12 - x^2$, with $y \geq 0$. What are the dimensions of the rectangle of this type with maximum area?

10. Problem #29 on Snell's Law, page 186, section 4.4.