Climate Change and Mathematics of Sea Ice

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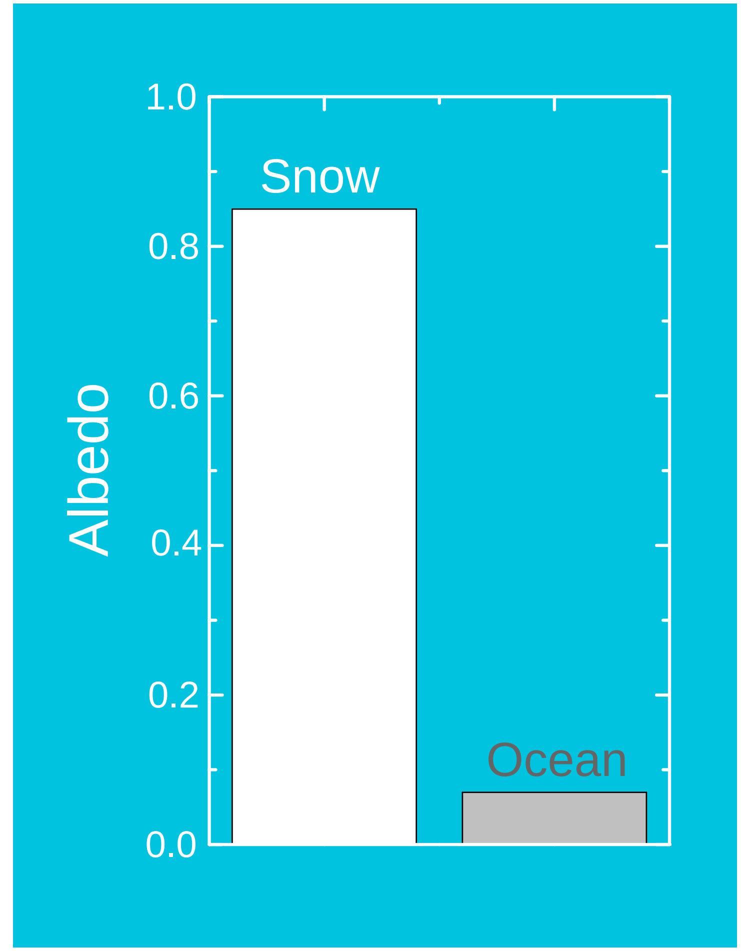
SEA ICE covers 7 - 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

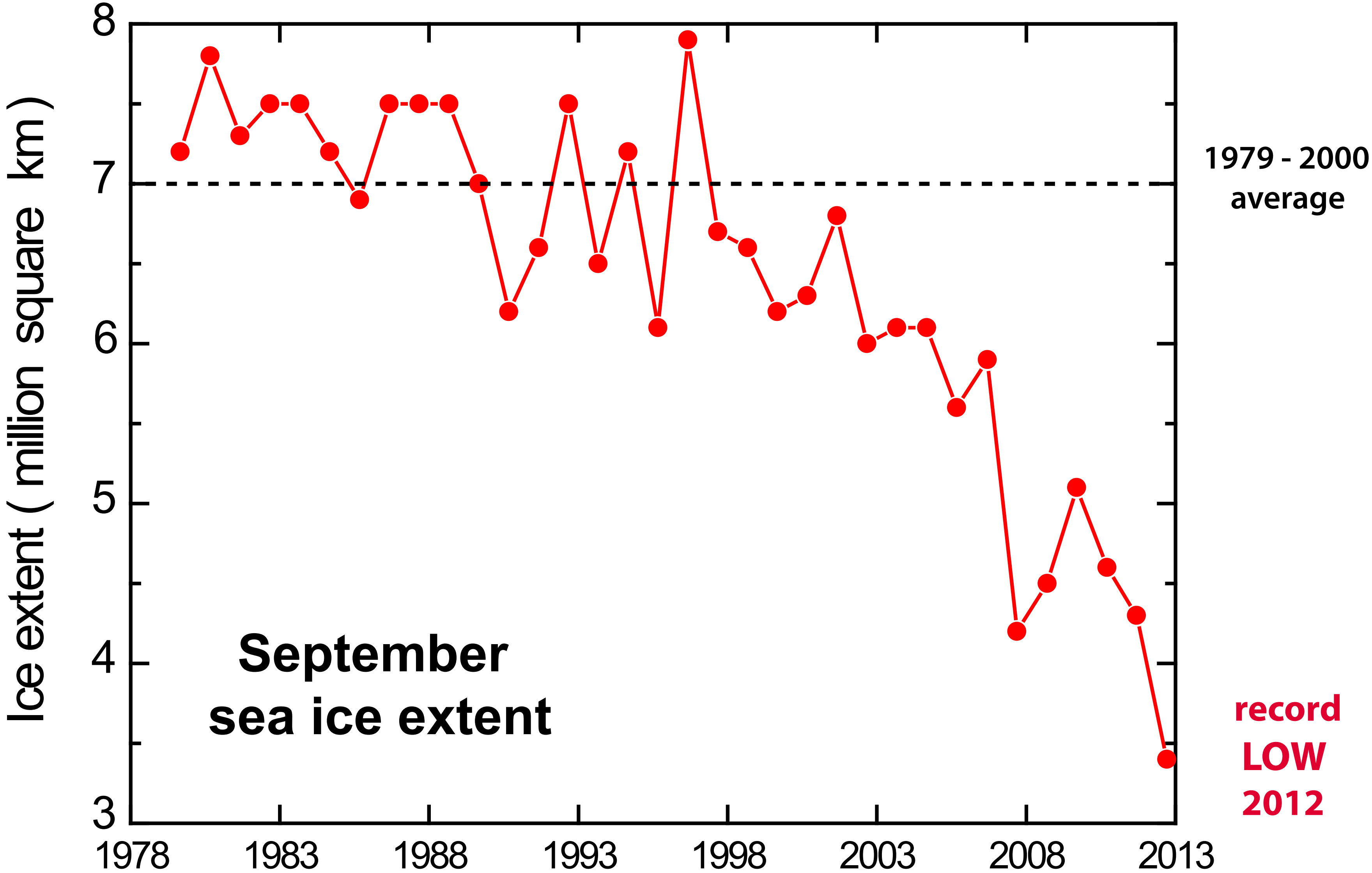
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

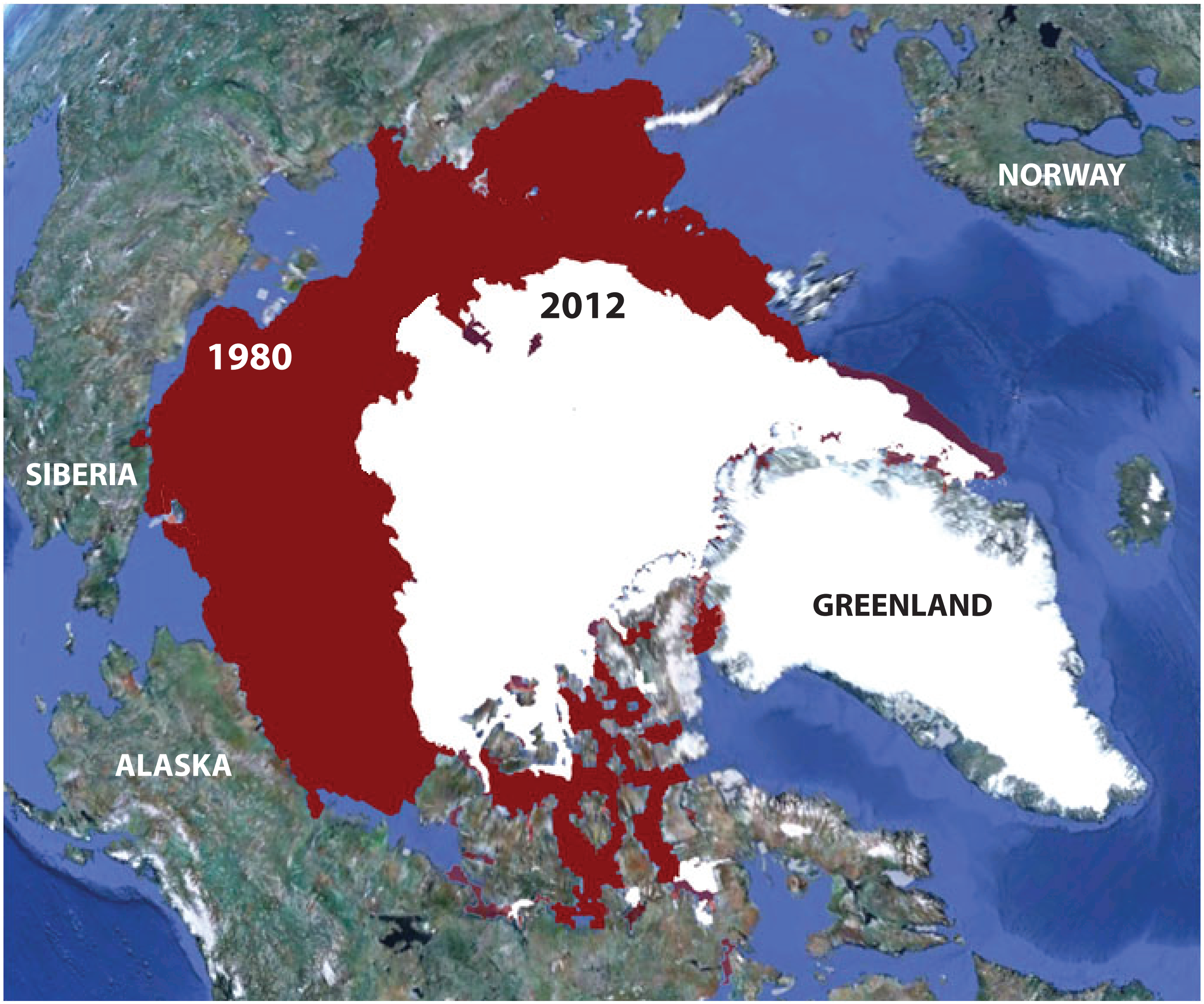
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

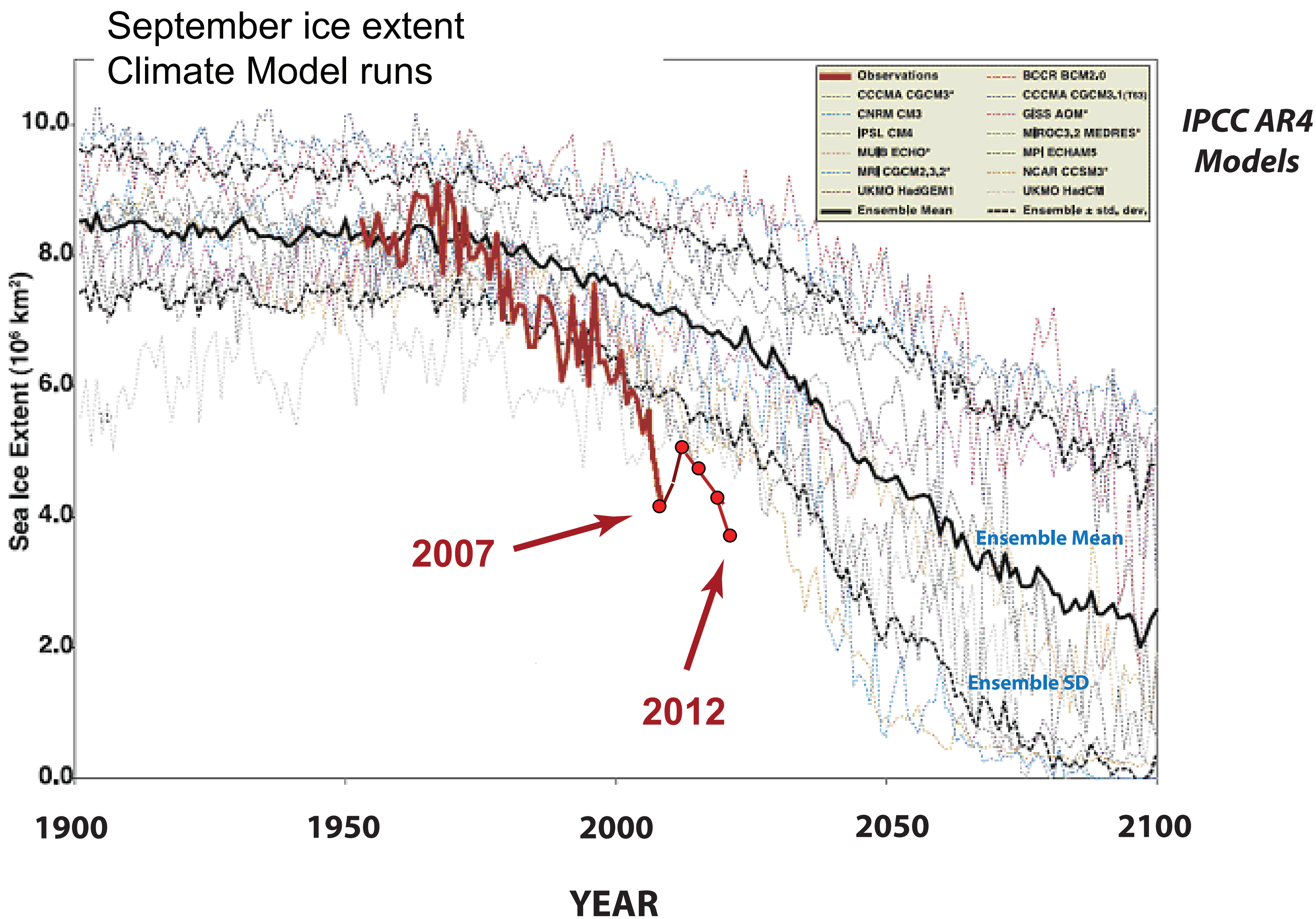
September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



Perovich

Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



Perovich

challenge

represent sea ice more rigorously in climate models

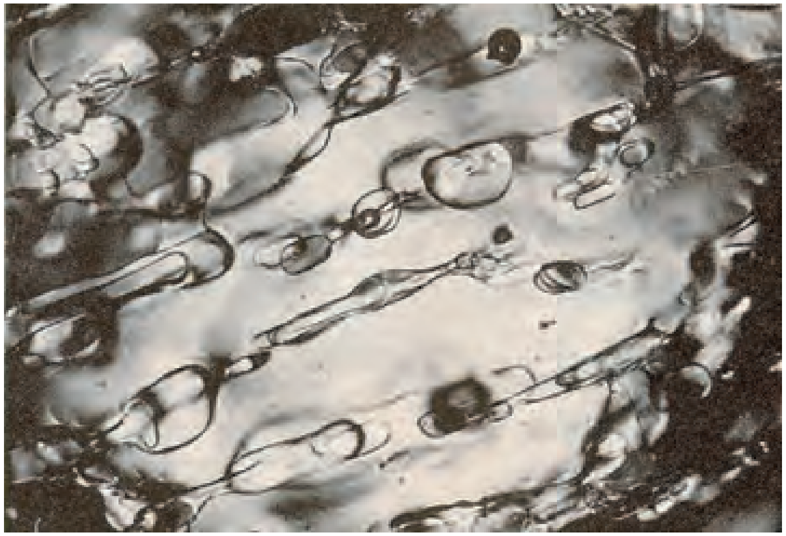
incorporate key processes

fundamental problem -- linkage of scales

sub-grid scale processes

sea ice is a multiscale composite

brine inclusions

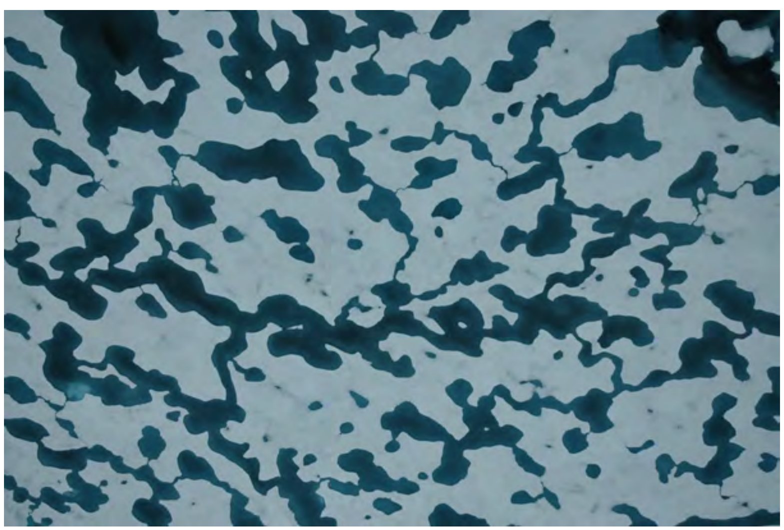


sub-millimeter

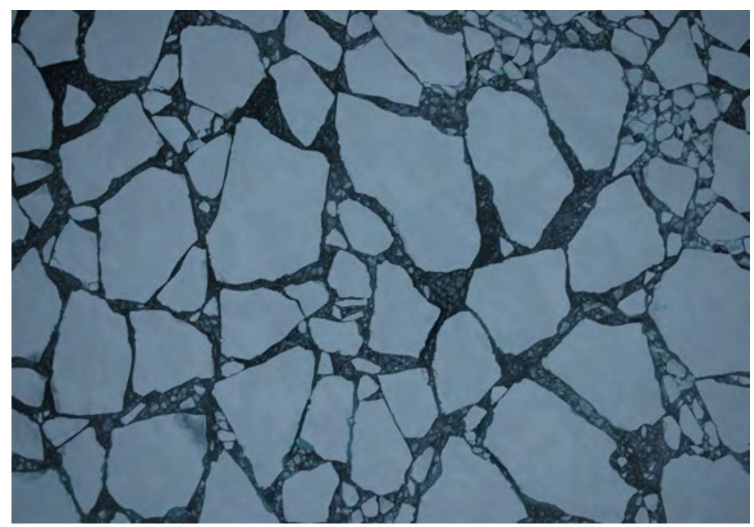
pancakes

centimeters

melt ponds



meters



ice floes

kilometers

What is this talk about?

Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.

1. ***Fluid flow through sea ice - percolation***

   homogenzation for composite materials

2. ***Electromagnetic monitoring of sea ice***

   homogenzation for larger scale structures

3. ***Arctic and Antarctic experiments***

4. ***Fractal geometry of Arctic melt ponds***

critical behavior ***linkage of scales***

cross-pollination

Develop rigorous representations of sea ice in climate models.

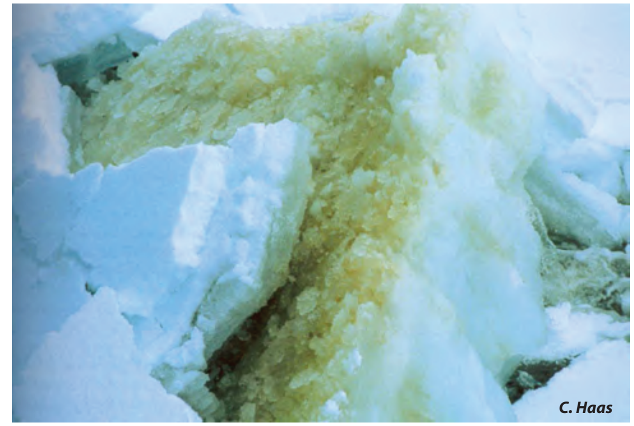
sea ice microphysics

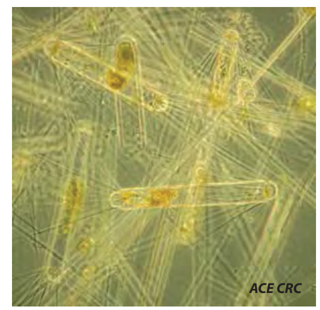
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

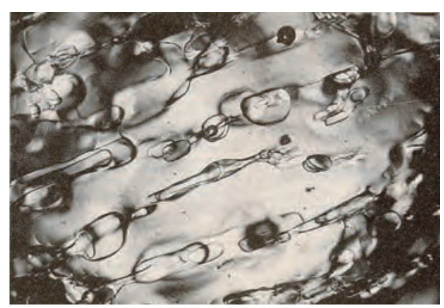
nutrient flux for algal communities





- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

linkage of scales



Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity → $\mathbf{v}$

pressure gradient → ∇p

$$\mathrm{v} = -\frac{\mathbf{k}}{\eta}\nabla p$$

viscosity → η

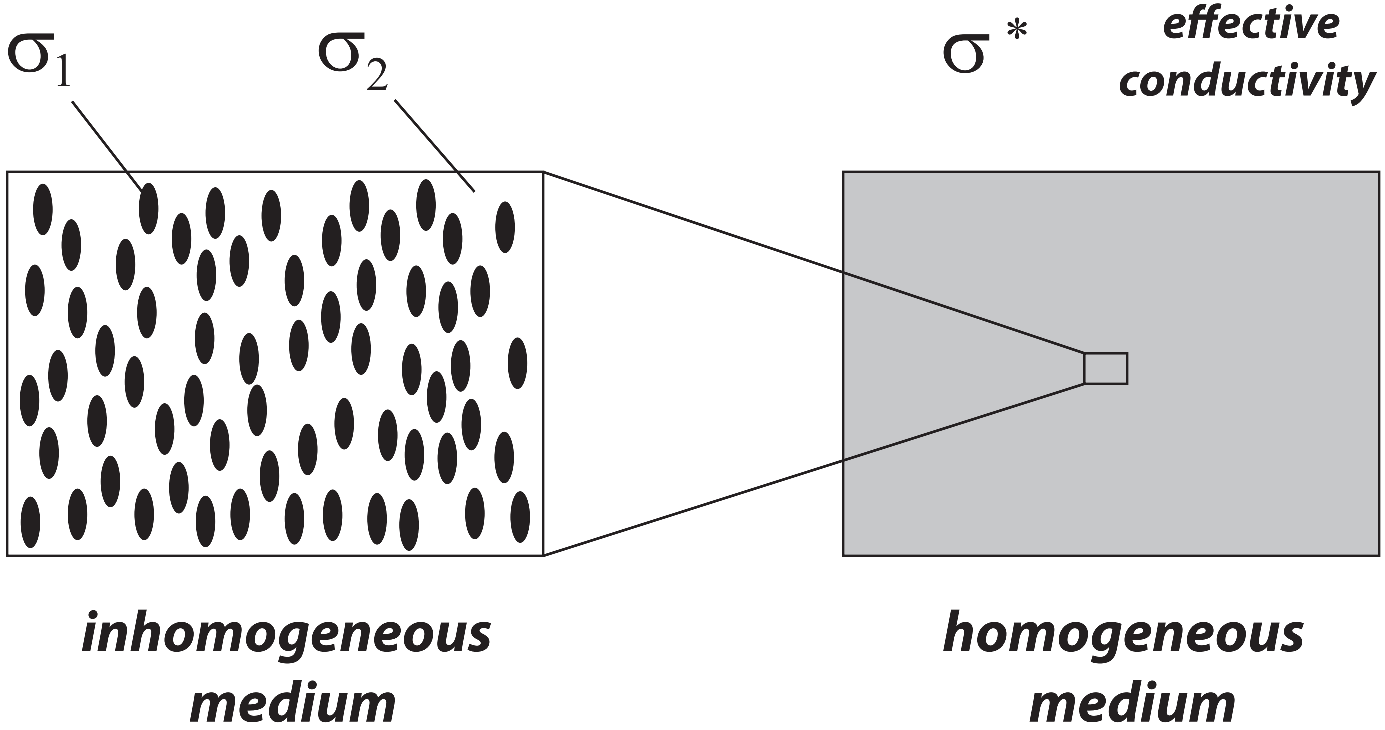
$\mathbf{k}$ = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

e.g. transport properties of composites - electrical conductivity, thermal conductivity, etc.

HOMOGENIZATION



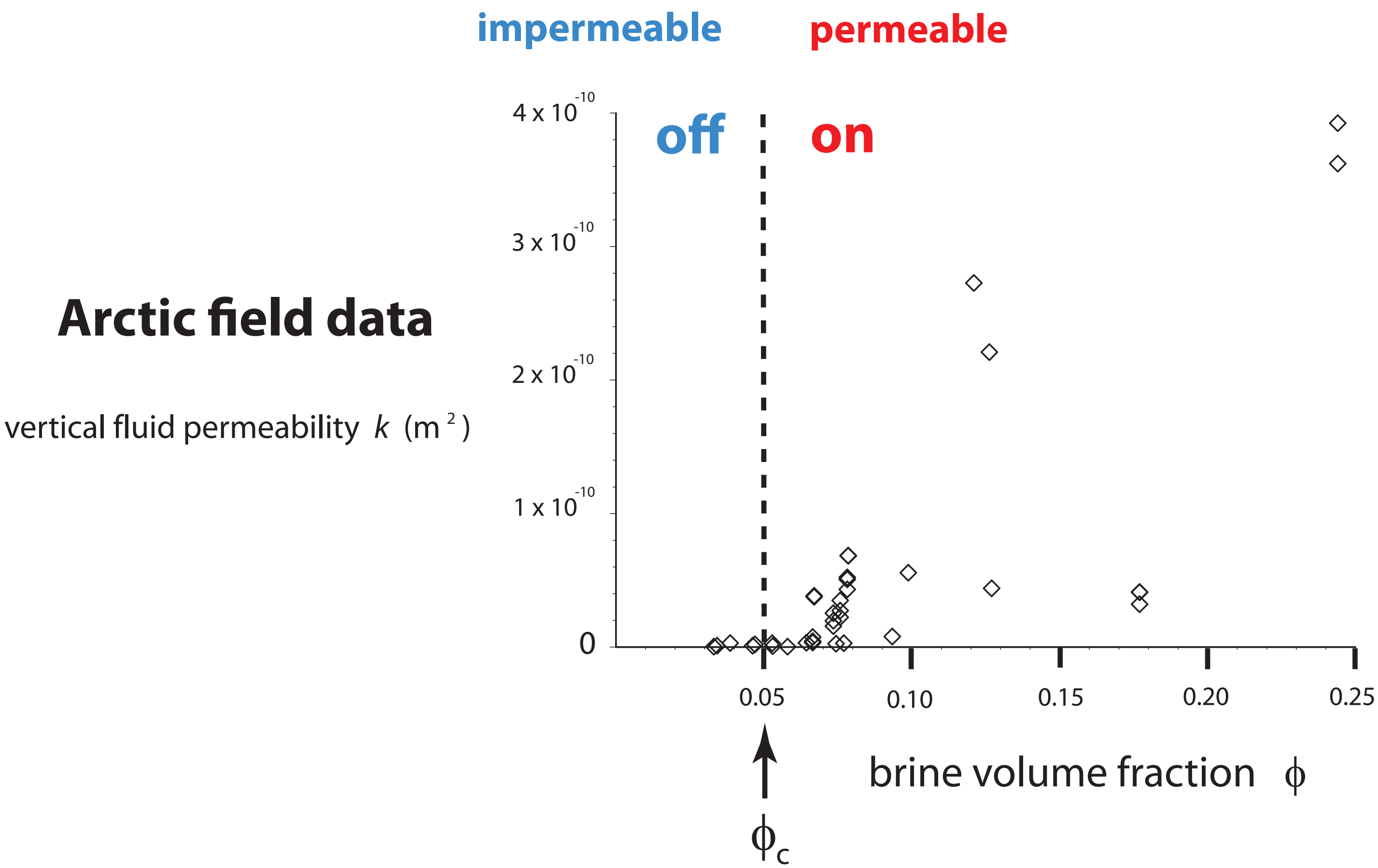
find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

Critical behavior of fluid transport in sea ice

Arctic field data



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

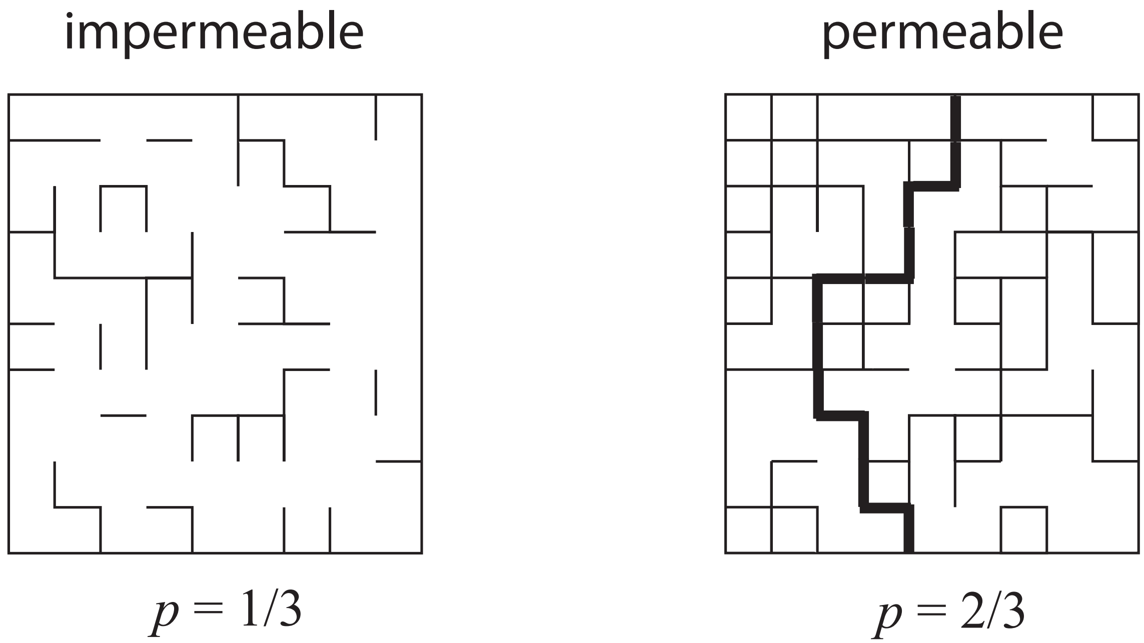
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

Why is the rule of fives true?

percolation theory

mathematical theory of connectedness



bond → *open* with probability p
closed with probability $1\text{-}p$

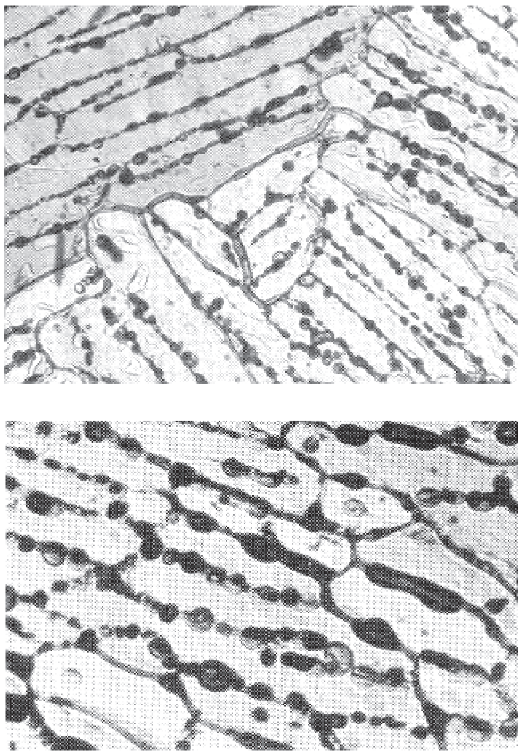
percolation threshold

$$p_c = 1/2 \quad \text{for} \quad d = 2$$

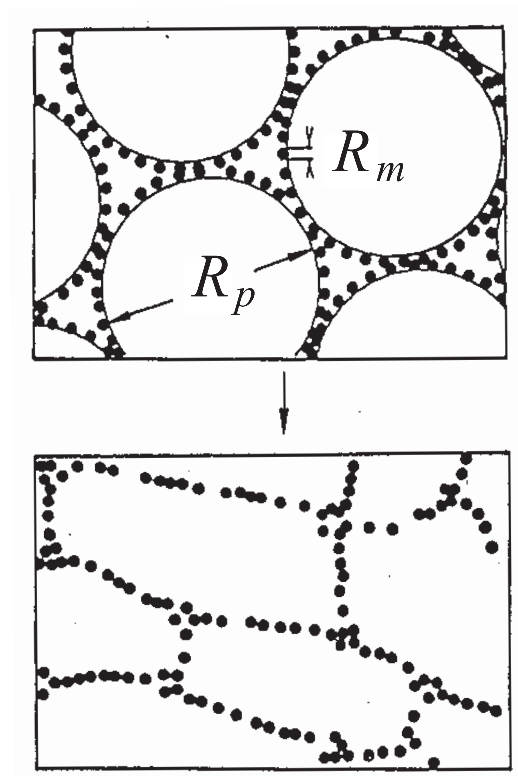
first appearance of infinite cluster ***"tipping point" for connectivity***

Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

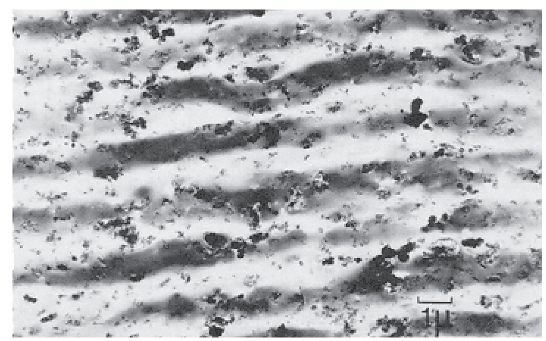
$\phi_c \approx 5\ \%$ Golden, Ackley, Lytle, *Science,* 1998



sea ice



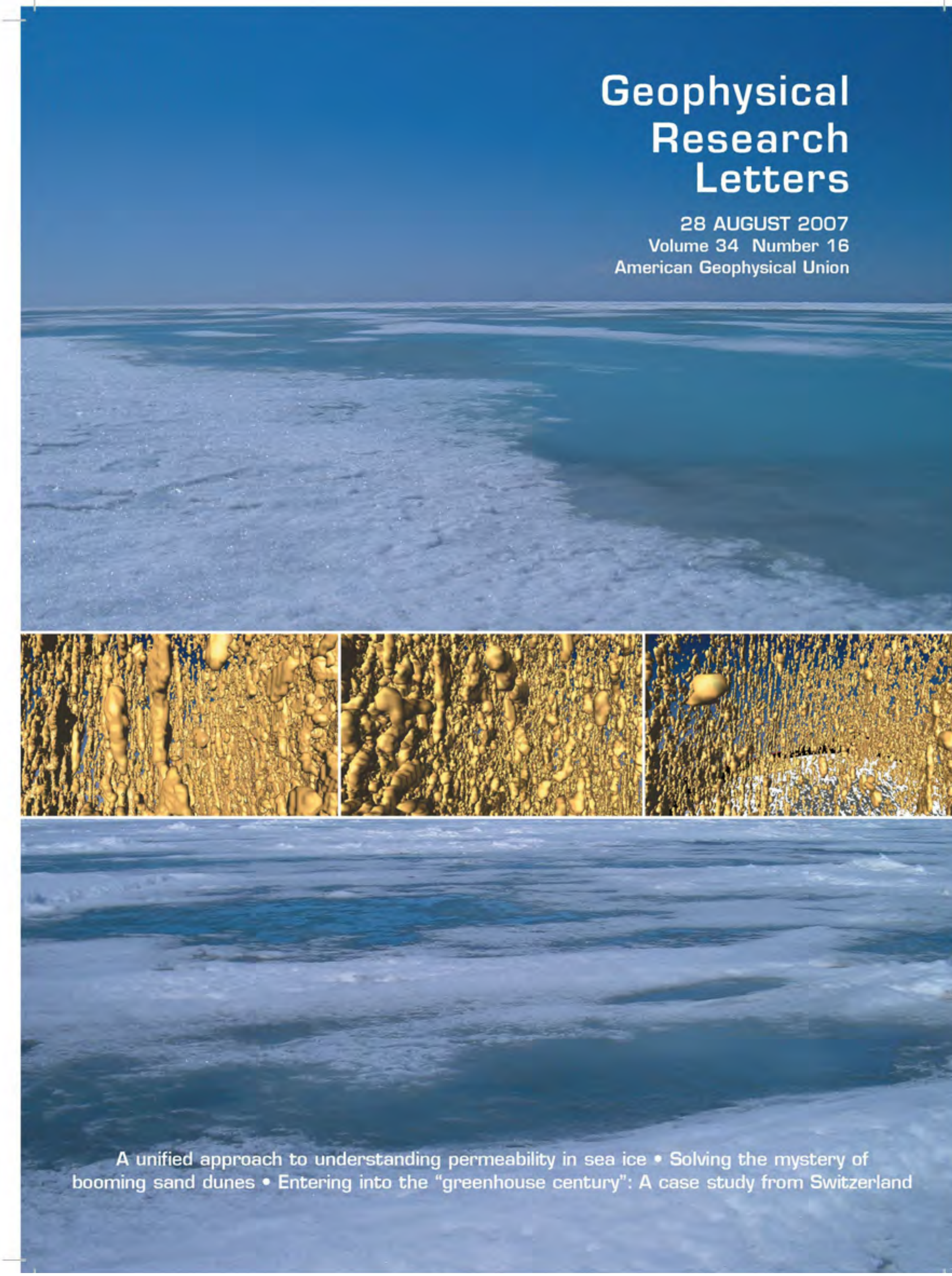
compressed powder



radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**



rigorous bounds
percolation theory
hierarchical model
network model

field data

micro-scale

controls

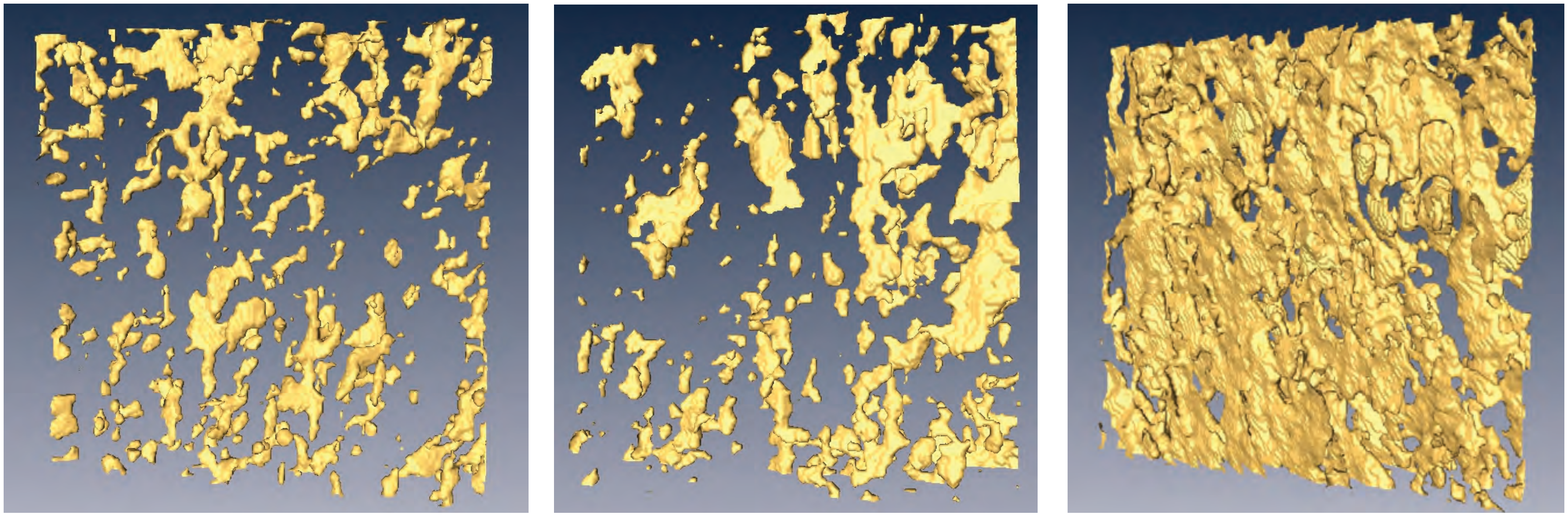
macro-scale processes

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

brine connectivity (over cm scale)

8 x 8 x 2 mm



-15 °C, $\phi = 0.033$ -6 °C, $\phi = 0.075$ -3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats → 3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

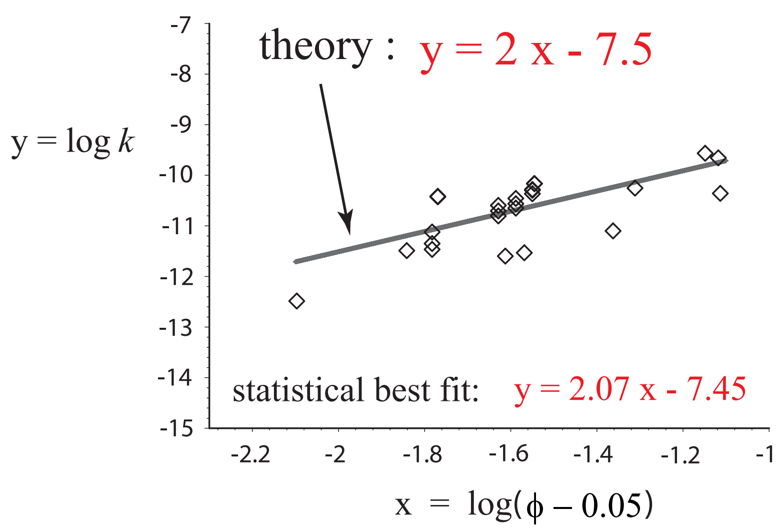
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



Remote sensing of sea ice

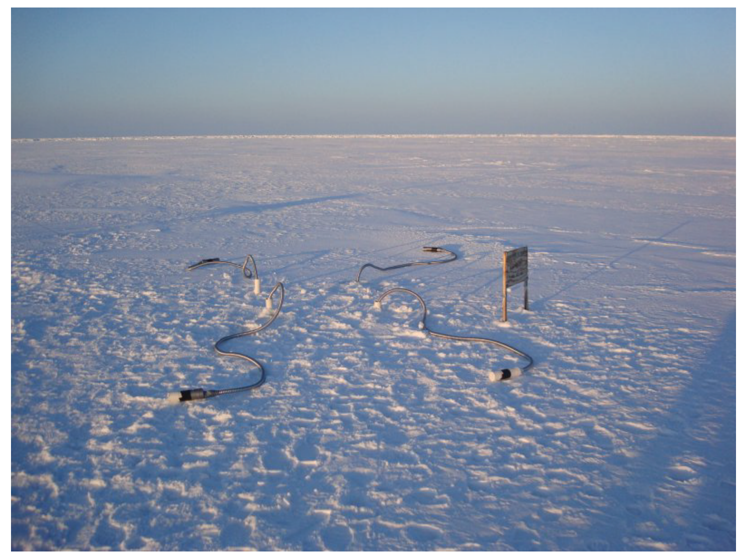
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

$$\varepsilon^*$$

effective complex permittivity (dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Theory of Effective Electromagnetic Behavior of Composites

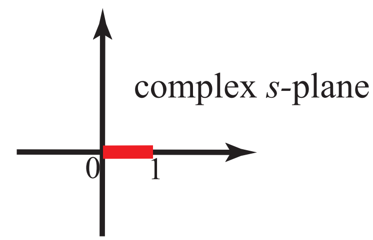
analytic continuation method

Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

composite geometry
(spectral measure μ) → ε^*

integral representations, rigorous bounds, approximations, etc.

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z} \qquad s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



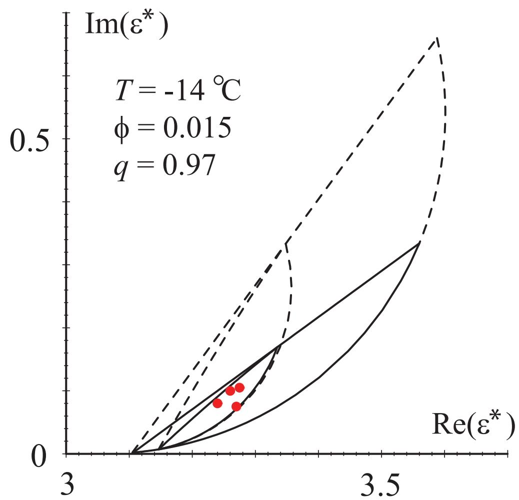
Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)
(McPhedran, McKenzie, and Milton, 1982)

ε^* → **composite geometry**
(spectral measure μ)

recover brine volume fraction, connectivity, etc.

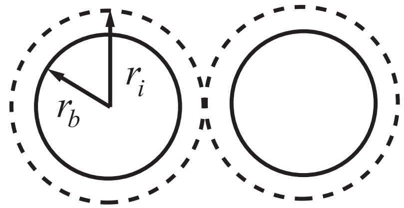
forward and inverse bounds for sea ice

forward bounds



4.75 GHz data

matrix particle

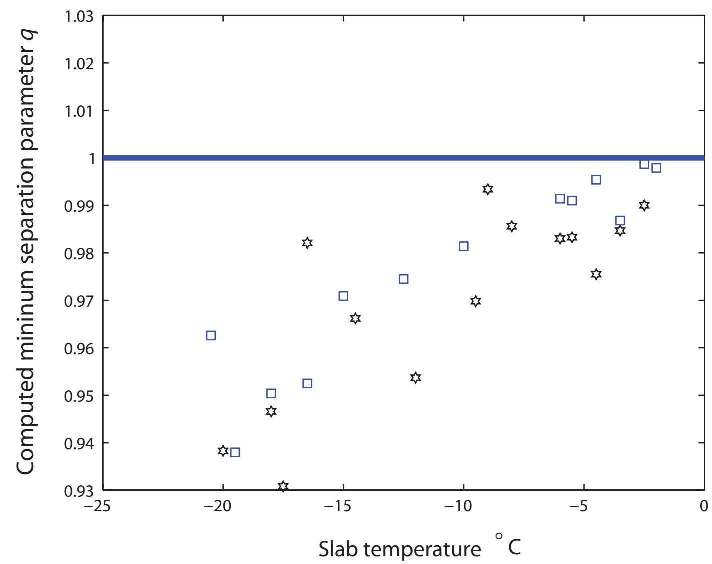


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

polycrystalline bounds two-scale homogenization

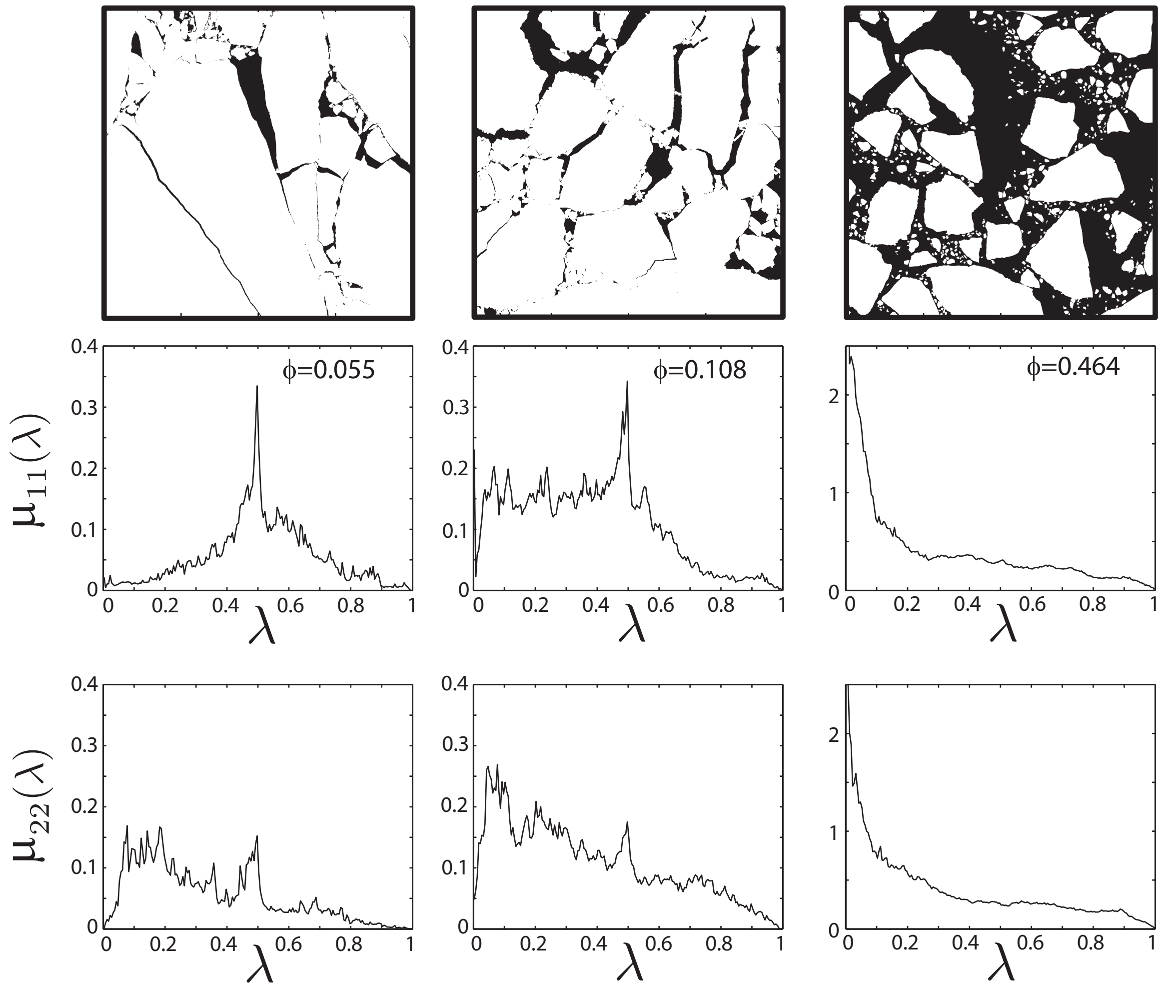
Gully, Lin, Cherkaev, Golden, 2013

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

spectral measures provide a path toward rigorously incorporating "composite microstructure" into calculations of effective behavior on larger scales

spectral measures for the Arctic sea ice pack



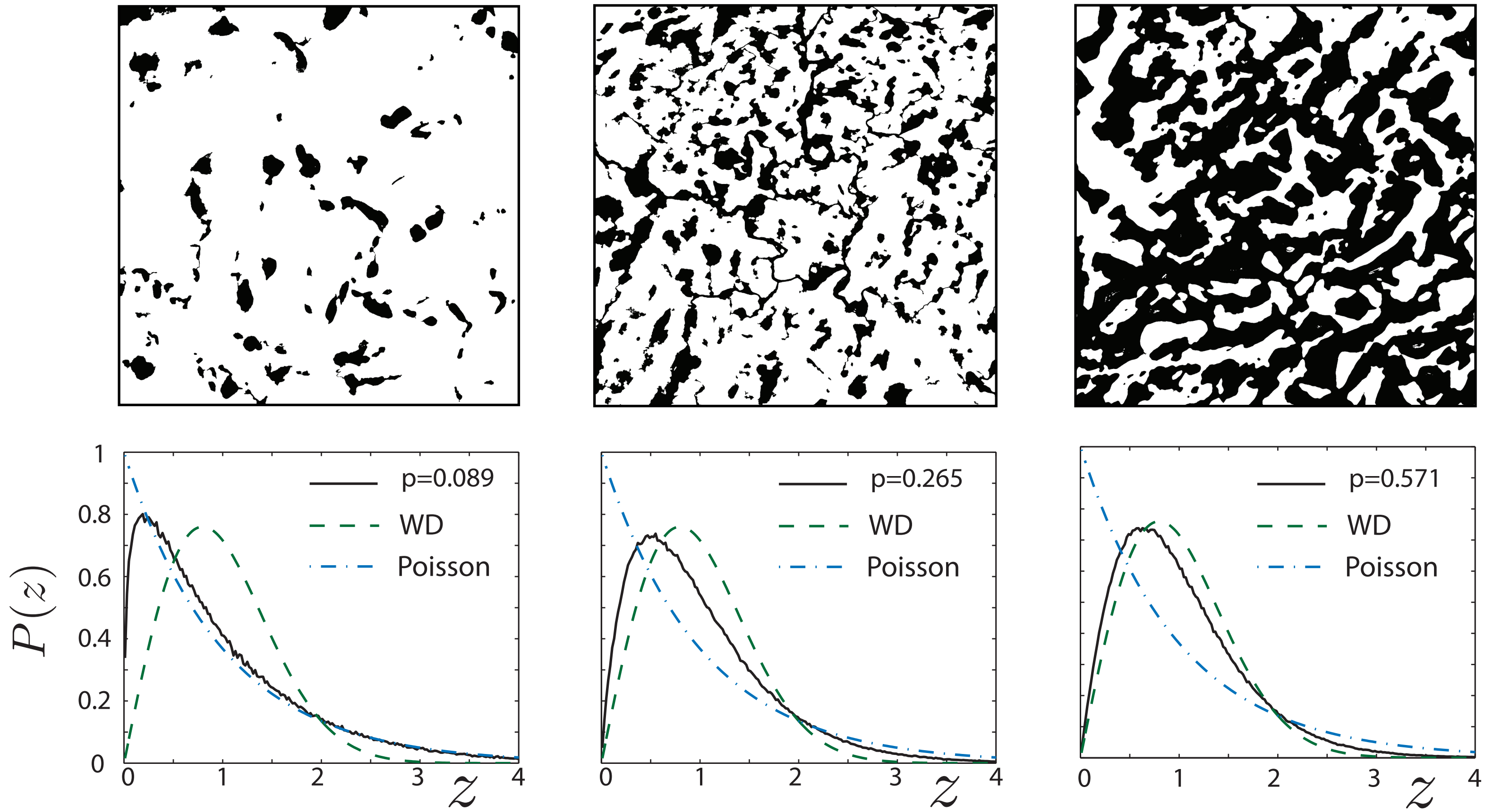
area under curve = φ = open water fraction

spectral gap closes as ocean phase becomes connected

N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden, 2013 *random matrix characterization of connectedness transition -- discretization of* $\chi\Gamma\chi$

Unfolded Eigenvalue Spacing Distribution

ARCTIC MELT PONDS



eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase becomes connected over large scales

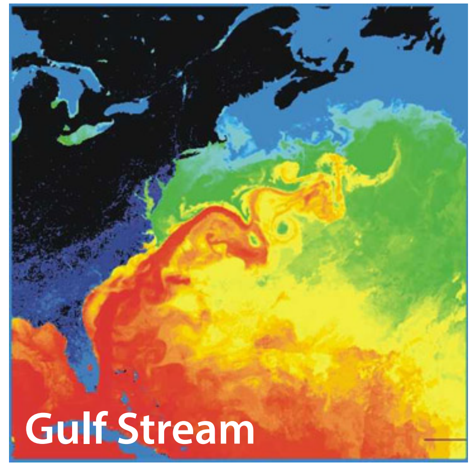
uncorrelated → *"level repulsion"*

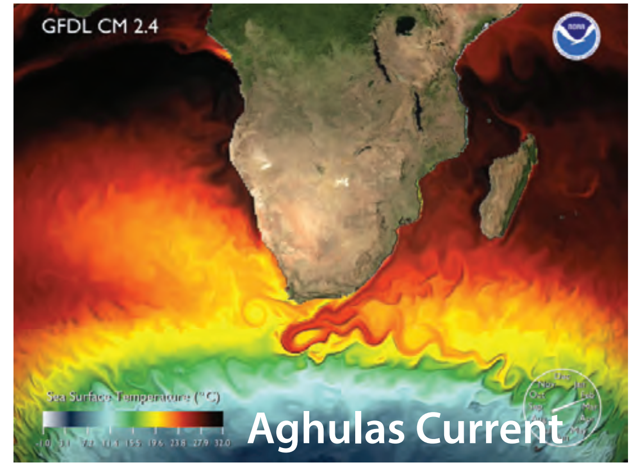
N. B. Murphy, K. M. Golden 2013

advection enhanced diffusion

effective diffusivity

tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere
salt and heat transport in ocean





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

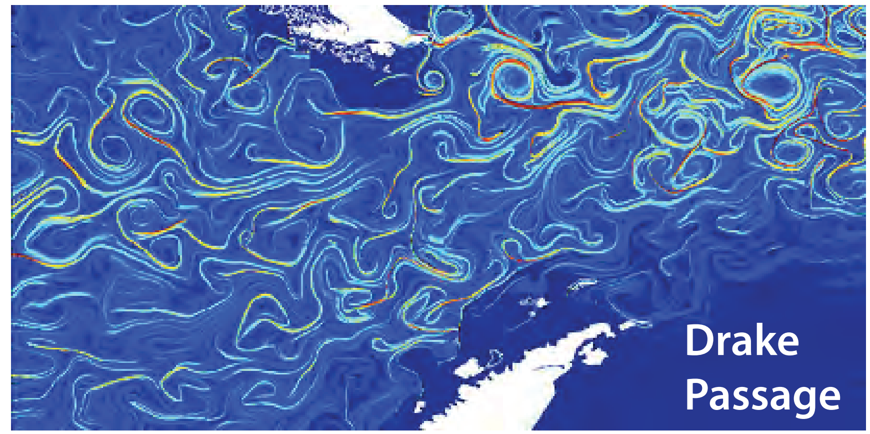
↓ ***homogenize***

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity***

Stieltjes integral for κ^* ***with spectral measure***

Avellaneda and Majda, PRL 89, CMP 91



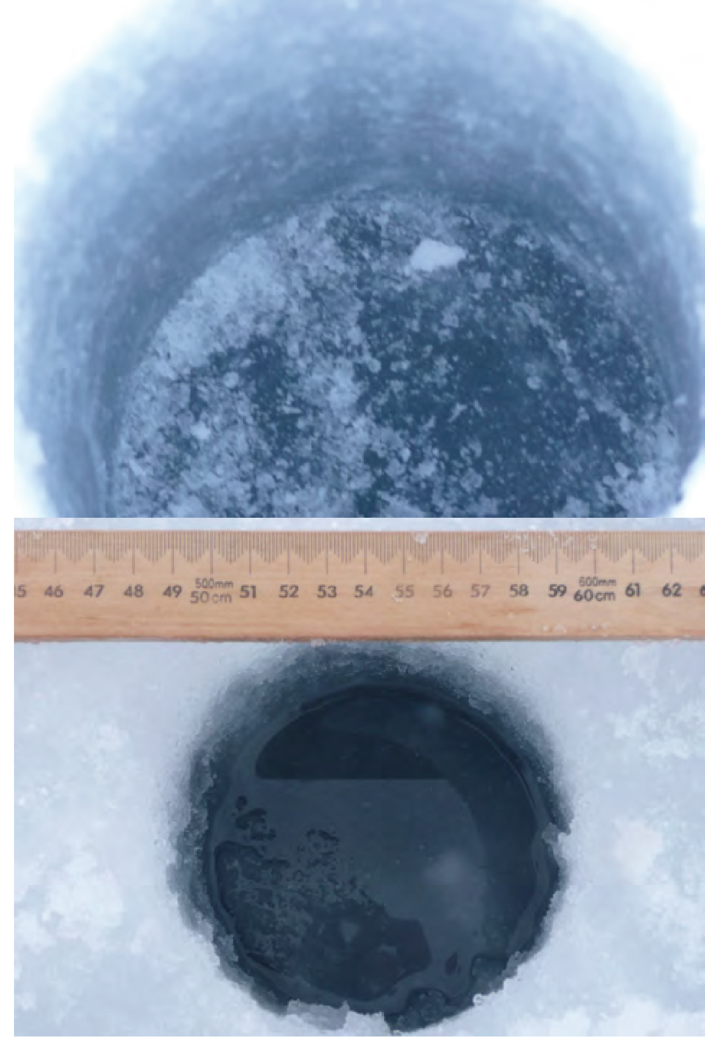
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*

photo by Jan Lieser



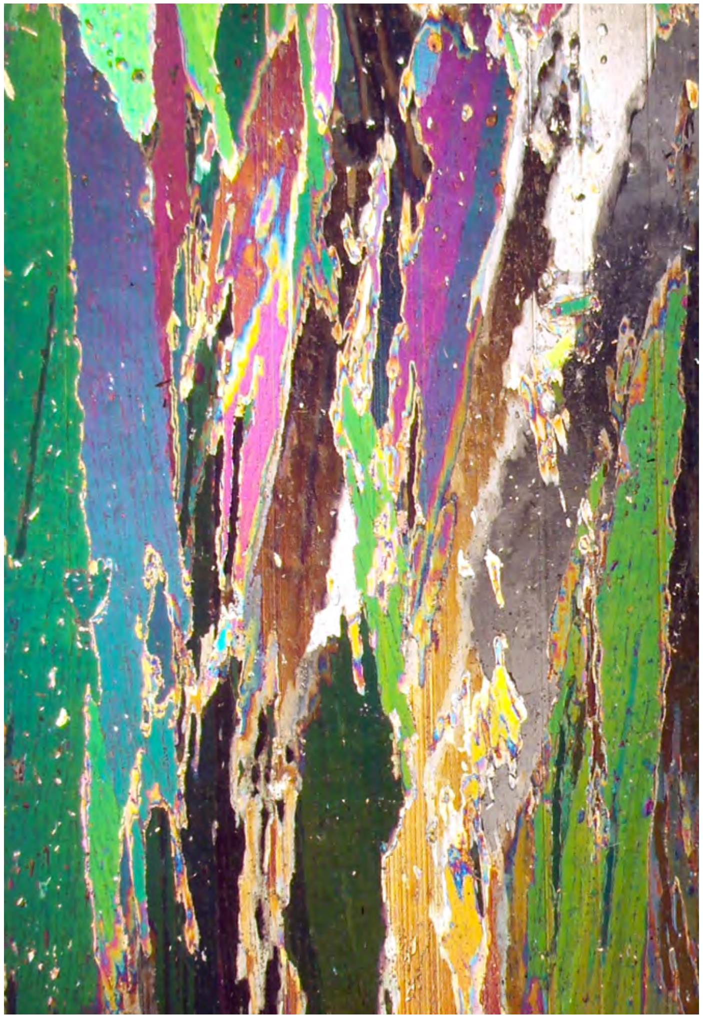
measuring fluid permeability of Antarctic sea ice

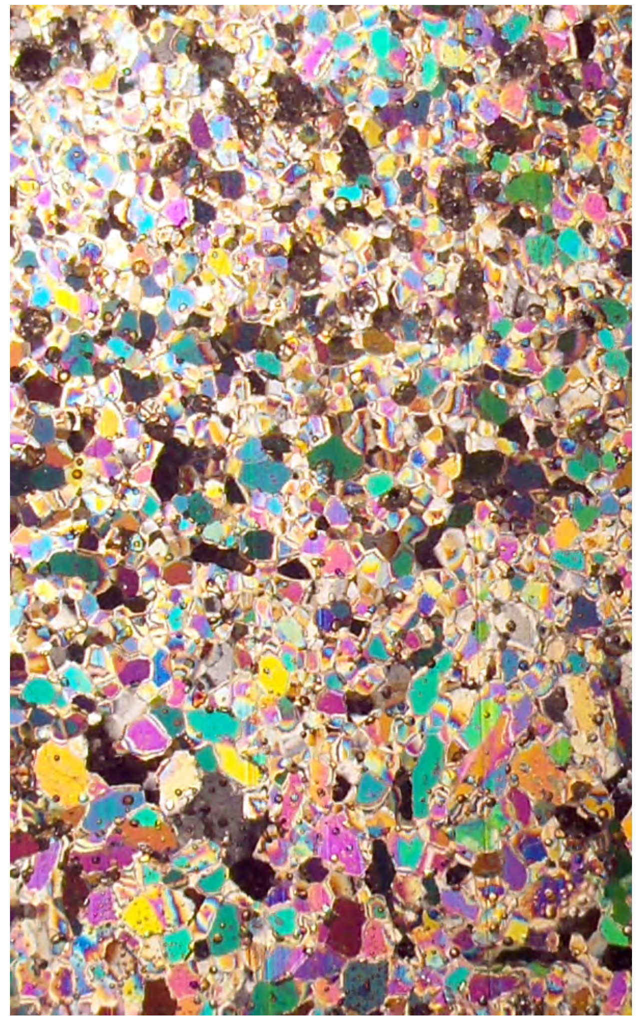
SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

granular





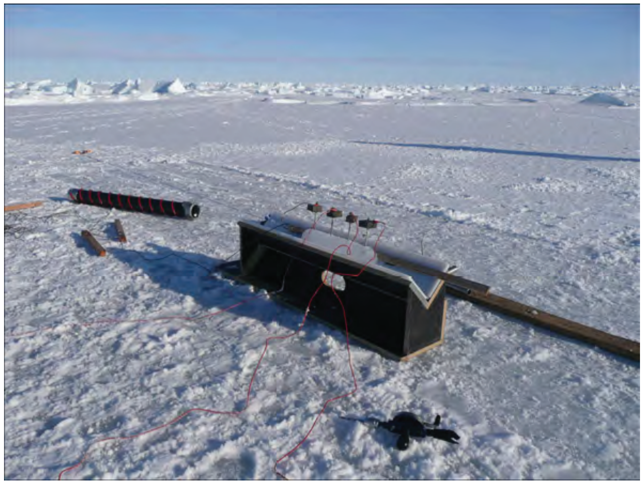
5%

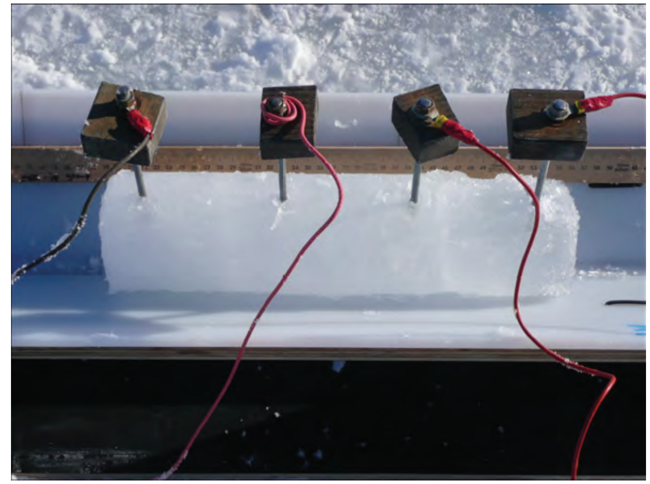
10%

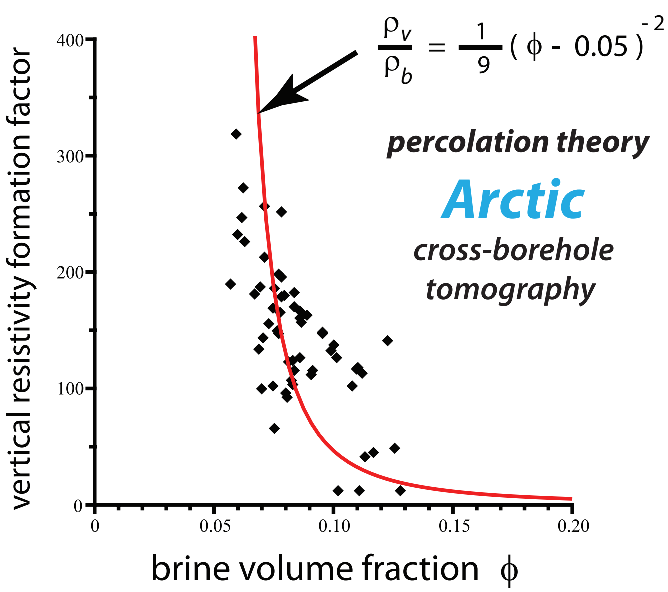
Golden, Gully, Lubbers, Sampson, Tison 2013

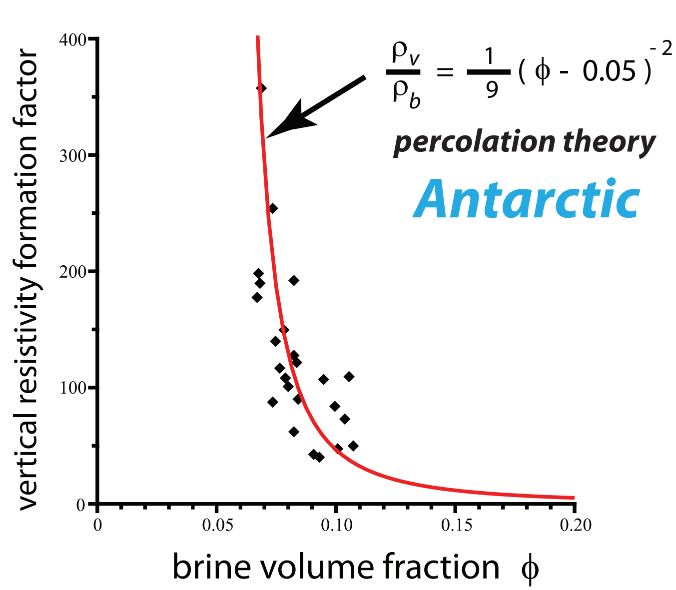
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow









cross-borehole tomography - electrical classification of sea ice layers

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2013

melt ponds on the surface of Arctic sea ice

Frey

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

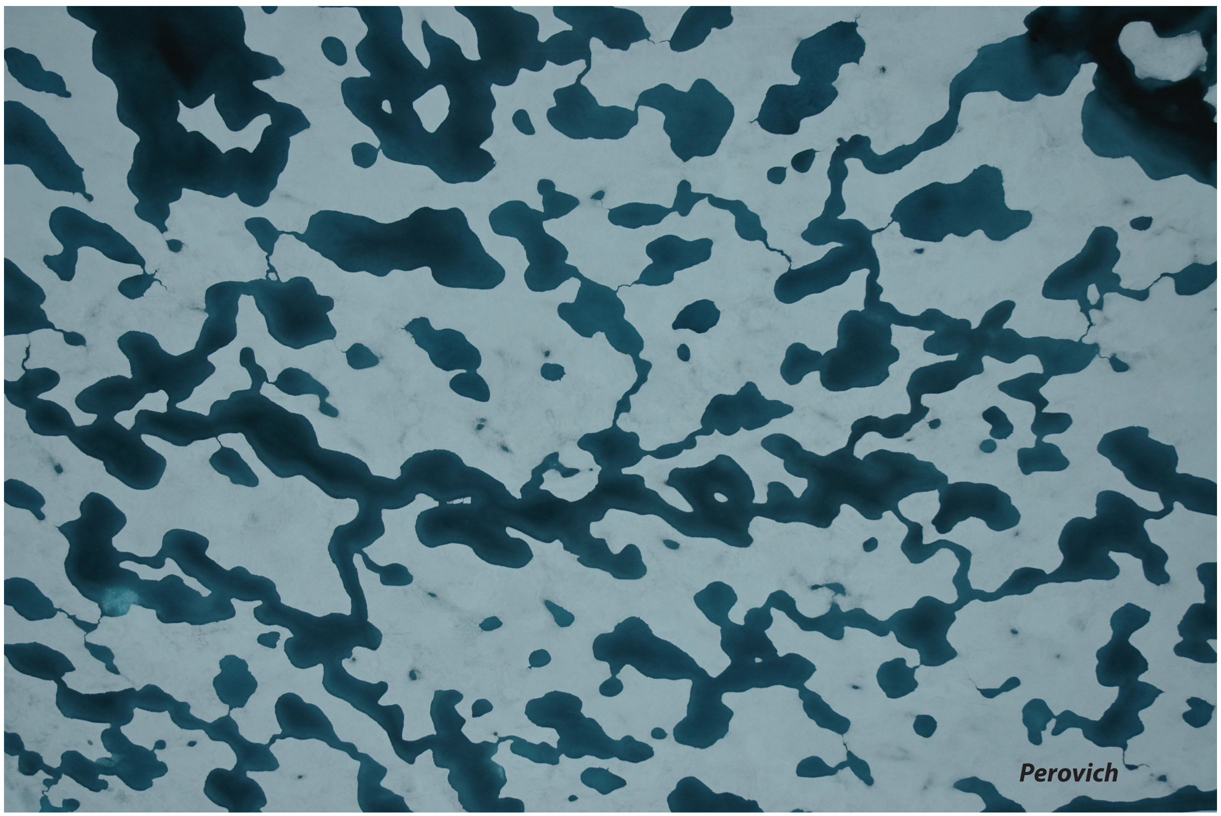
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

Flocco, Feltham, Hunke 2012

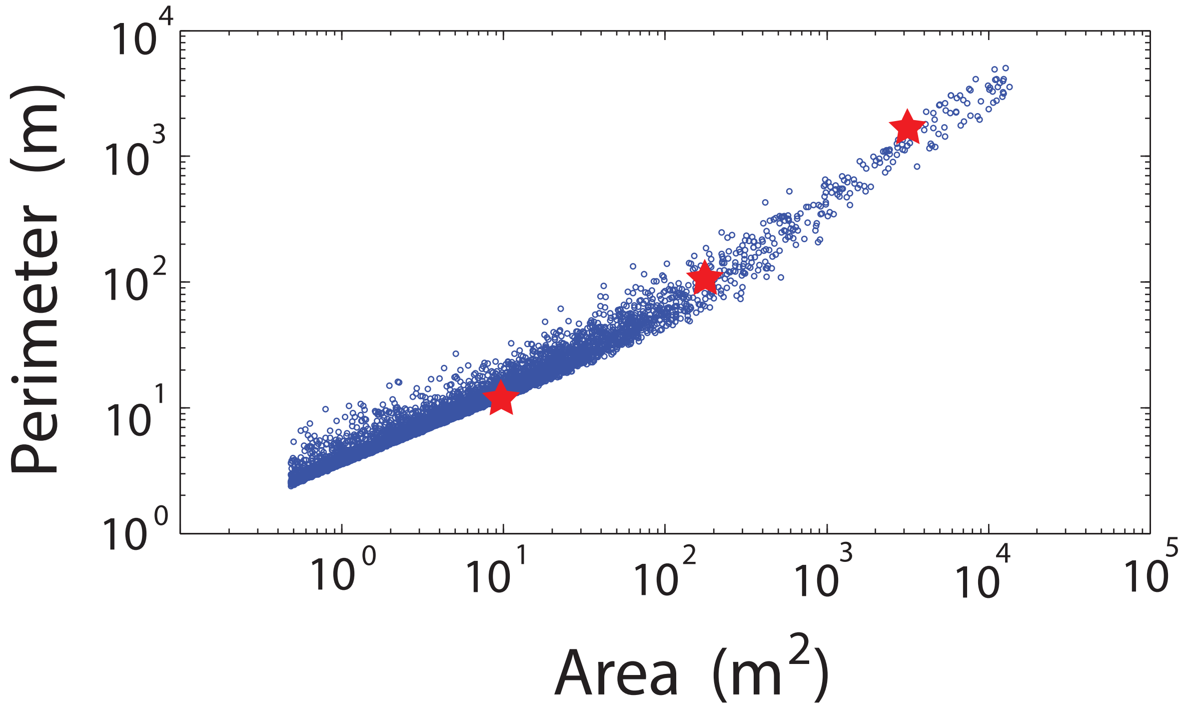


Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

The Cryosphere, 2012

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

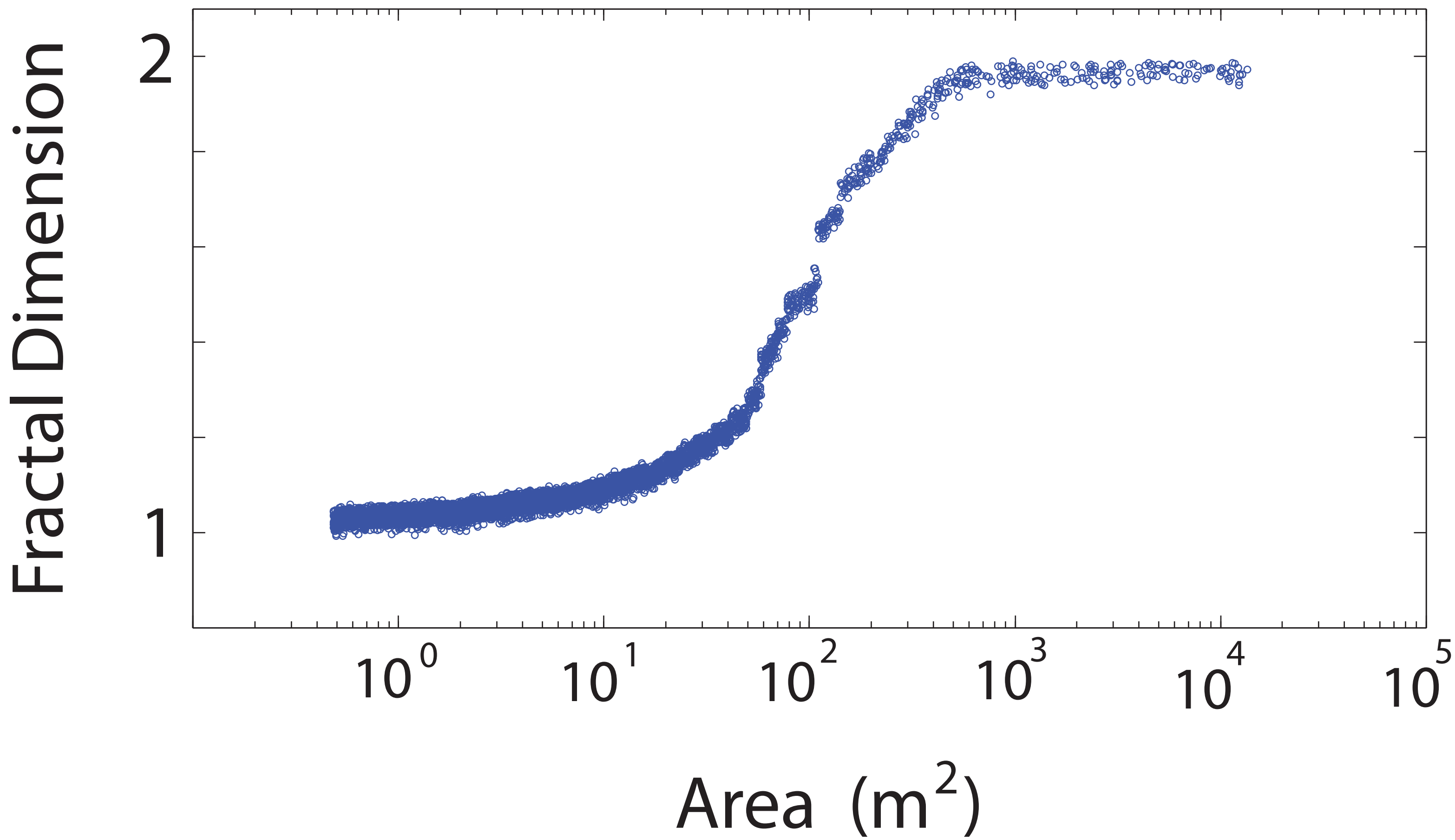


data from
5269 Arctic
melt ponds

simple pond ***transitional pond*** ***complex pond***

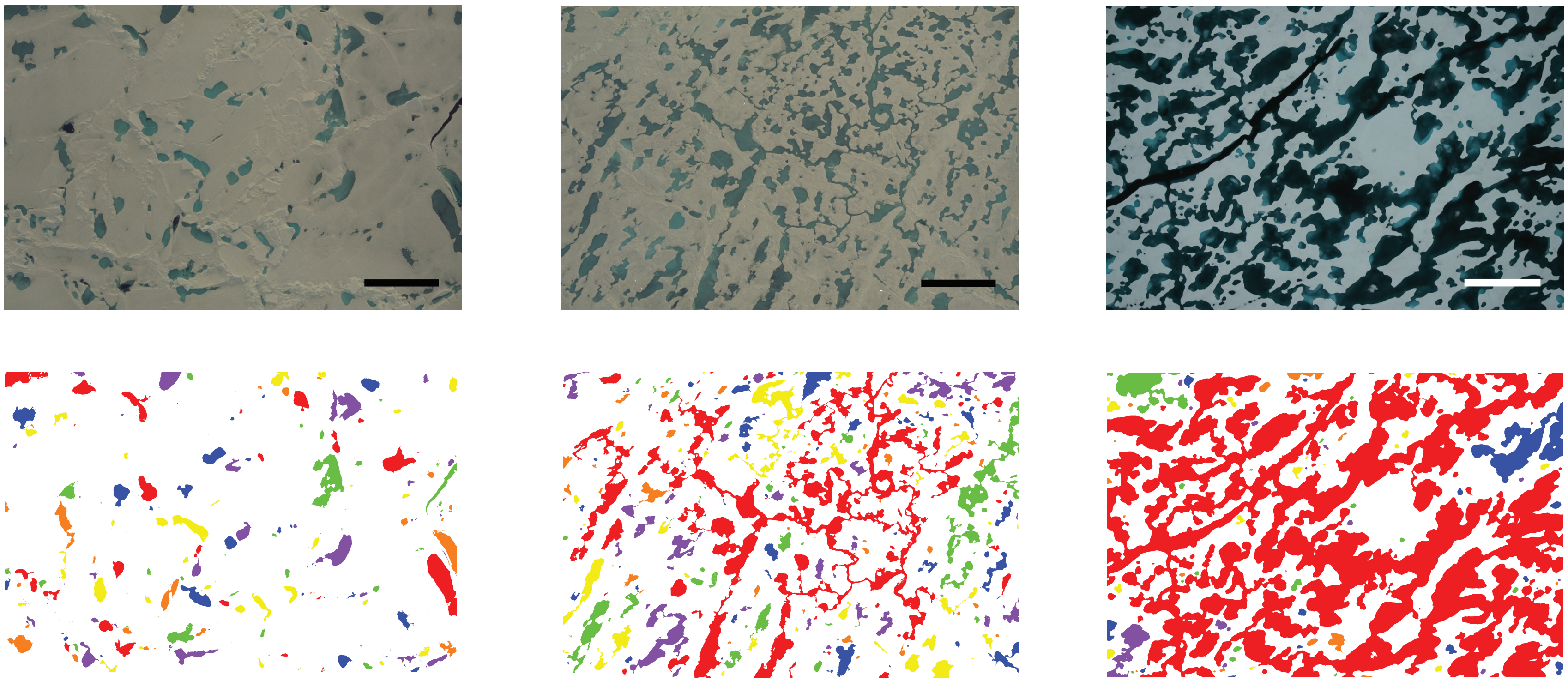
transition in the fractal dimension

complexity grows with length scale



compute “derivative” of area - perimeter data

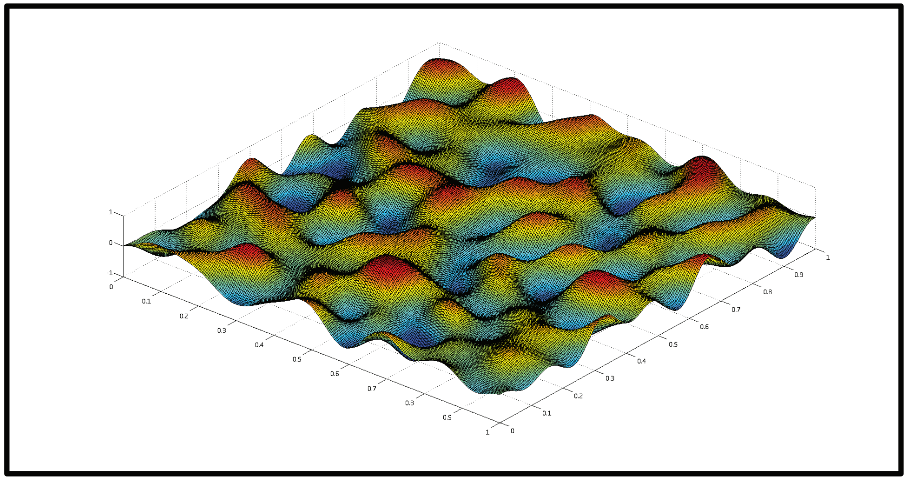
small simple ponds coalesce to form large connected structures with complex boundaries

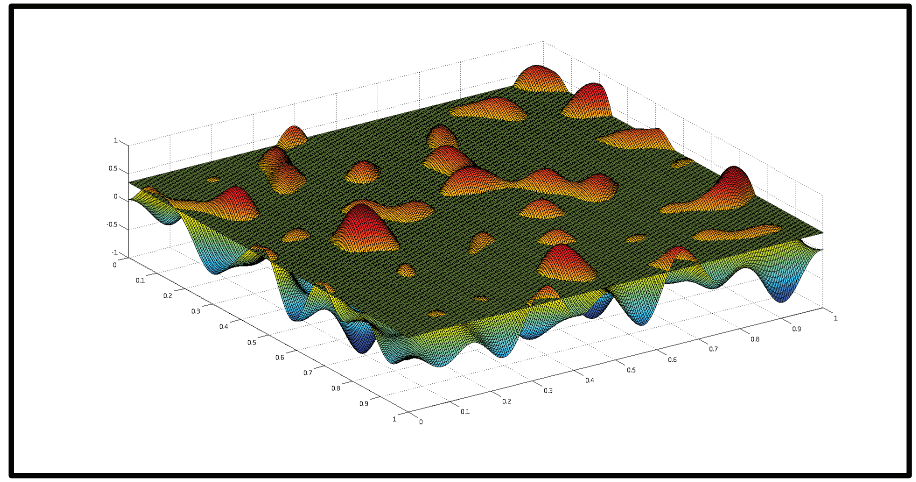


melt pond percolation

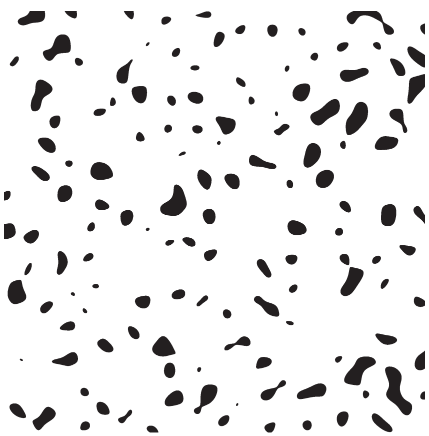
Continuum percolation model for melt pond evolution

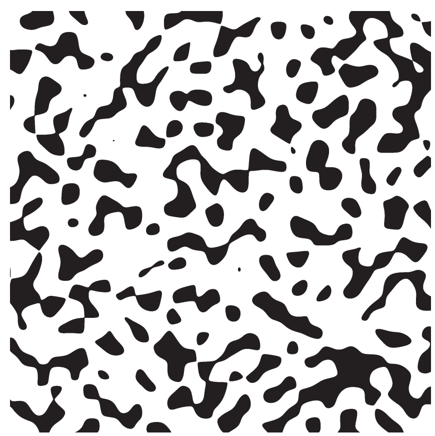
(Brady Bowen and Ken Golden, 2013)

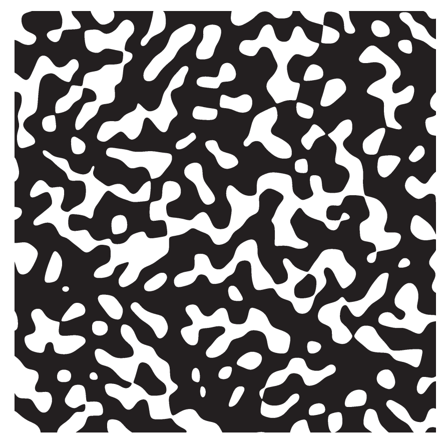




intersections of a plane with the surface define melt ponds



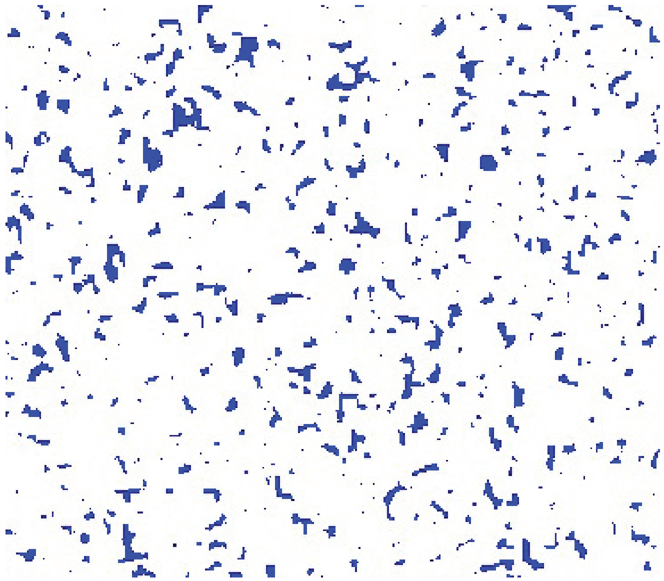




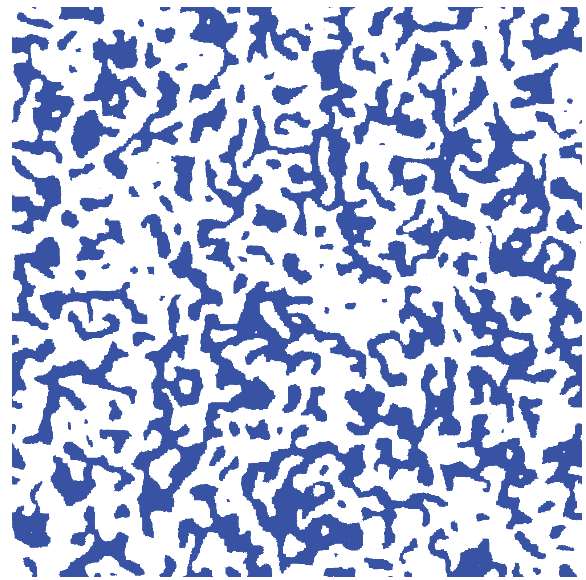
electronic transport in disordered media *diffusion in turbulent plasmas* *(Isichenko, Rev. Mod. Phys., 1992)*

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{ice} \\ \downarrow & -1 \quad \text{water} \end{cases} \qquad M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$



COLD



WARM

"melt ponds" are clusters of magnetic spins that align with the applied field

clusters exhibit transition in fractal dimension

Thekkedath, Alali, Strong, Golden
Sudakov, Ma, Golden

Conclusions

1. Summer Arctic sea ice is melting rapidly.

2. Fluid flow through sea ice mediates many processes of importance to understanding climate change and the response of polar ecosystems.

3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes, and provide a path toward rigorous representation of sea ice in climate models .

4. Field experiments are essential to developing relevant mathematics.

5. Our research will help to improve projections of climate change and the fate of the Earth sea ice packs.

THANK YOU

National Science Foundation

Division of Mathematical Sciences

Arctic Natural Sciences

Office of Polar Programs

CMG Program

(Collaboration in Mathematical Geosciences)

Office of Naval Research

Applied Computational Analysis Program

Arctic and Global Prediction Program

Department of the Navy ONR Science & Technology

MCRN Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

ace CRC ANTARCTIC CLIMATE & ECOSYSTEMS COOPERATIVE RESEARCH CENTRE

NSF

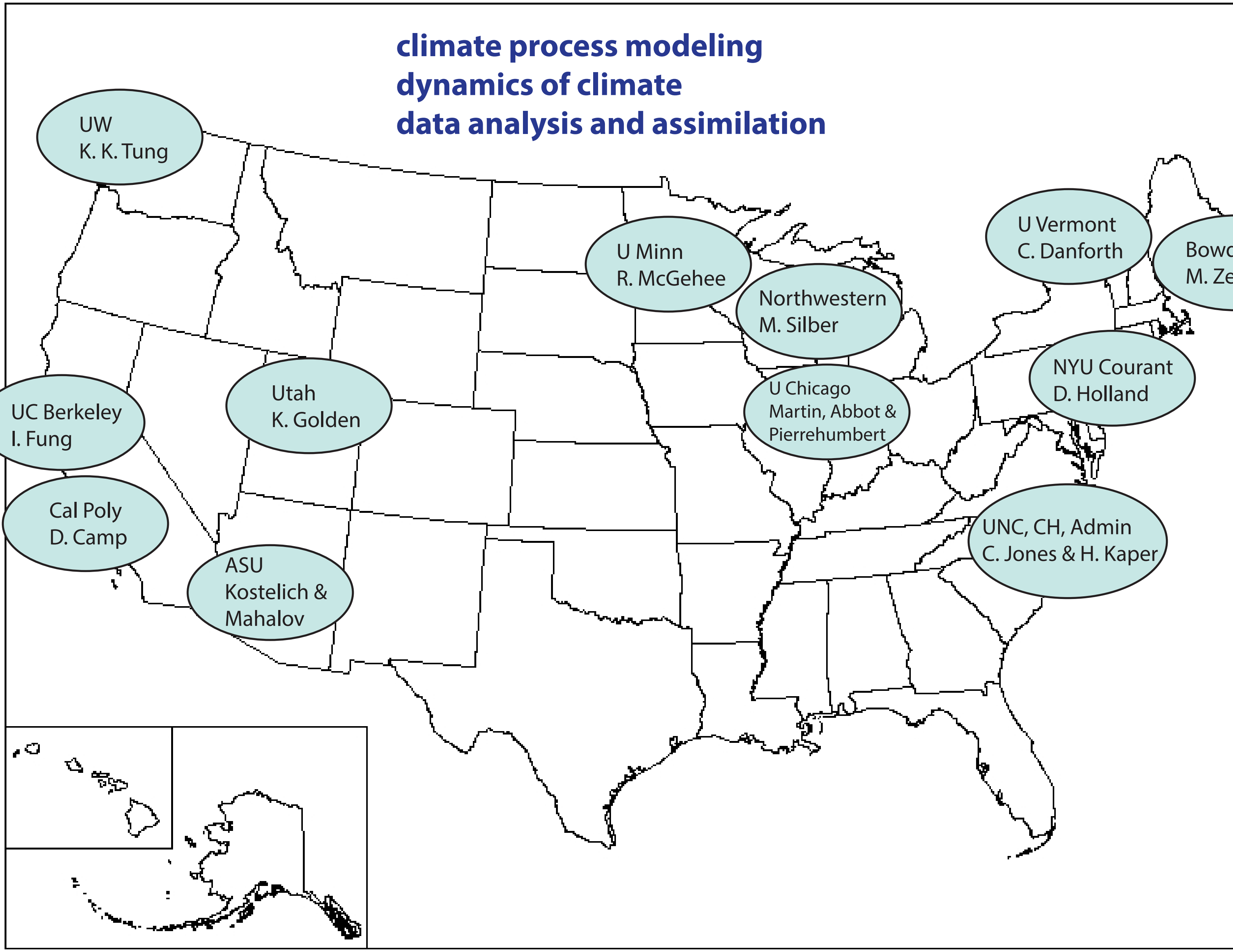
Antarctica New Zealand

UAF UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999

Mathematics and Climate Research Network (MCRN)



NSF DMS 2010-2015, Lorenz postdocs, grad, undergrad, polar expeditions

Jones, Golden, Kaper, Zeeman