

Critical Behavior of Electrical Transport in Sea Ice

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Fluid flow through sea ice mediates a broad range of geophysical and biological processes in the polar marine environment. For example, the evolution of melt ponds and summer ice albedo is constrained by drainage through porous sea ice¹, and it is believed that ice-albedo feedback has played a key role in the decline of summer Arctic sea ice². Fluid flow also facilitates snow-ice formation³, the evolution of the salt budget⁴, convection-enhanced thermal transport⁵, CO₂ exchange⁶, and biomass build-up sustained by nutrient fluxes^{4,7}. However, for brine volume fractions below about 5%, columnar sea ice is effectively impermeable to fluid flow, which controls these processes^{8–10}. In two different experiments conducted in the Arctic and Antarctic, we have found that this critical transition in fluid flow exhibits a strong, characteristic electrical signature. Sea ice conductivity data are accurately explained by percolation theory with a universal critical exponent of 2. The data also indicate marked changes in the conductivity profile with the onset of melt ponds. Our findings lay the foundation for electromagnetic monitoring of transport phenomena in sea ice, which can help track key transitions in the state of polar sea ice and improve projections of its fate and impact on ecosystems.

Polar sea ice is a key player in Earth’s climate system, and a leading indicator of climate change^{4,11}. It also hosts extensive microbial communities which sustain life in the polar oceans^{4,7}. While global climate models predict declines in sea ice area and thickness, they have significantly underestimated the recent loss in Arctic summer ice extent¹². We focus here on sea ice processes which must be better understood to represent the polar ice packs more realistically in climate models. In particular, we investigate the electrical behavior of sea ice associated with key transport phenomena and microstructural transitions.

While fluid flow is substantially restricted for brine volume fractions ϕ below about 5%, columnar sea ice is increasingly permeable for ϕ above 5%⁸. For a typical bulk salinity of 5 ppt, the critical porosity $\phi_c \approx 5\%$ corresponds to a temperature $T_c \approx -5^\circ \text{C}$, which is known as the *rule*

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of fives. This critical behavior of the fluid permeability results from a connectivity or percolation threshold in the brine microstructure^{8–10}. If the fluid transport properties of sea ice can be linked to its electrical properties, which is the aim of this paper, then new approaches can be brought to bear in monitoring the state of sea ice. For example, it could open the door to the development of sensors to enhance existing buoy networks, provide information on key ice processes, and improve integration with satellite data.

The electrical conductivity of sea ice has been studied over the past five decades^{13–18}. However, there have been no observations of critical behavior in electrical properties corresponding to the microstructural transition encapsulated in the rule of fives. Here we report on two types of experiments where electrical resistivity data clearly display critical behavior at the brine percolation threshold. The mathematical description we develop provides a rigorous link between fluid and electrical transport in sea ice, with both displaying the same type of universal critical behavior, thus laying the foundation for the techniques referred to above.

Sea ice is an anisotropic composite with vertically elongated brine inclusions and corresponding anisotropy in the effective fluid permeability and electrical conductivity tensors. Most methods for measuring sea ice conductivity involve indirect or inverse techniques, such as surface impedance tomography using a Wenner array of electrodes^{13, 15, 16, 18, 19}. Generally with these methods the vertical conductivity σ_v^* is inherently mixed with the horizontal components. Here we are most interested in σ_v^* due to its connection with vertical fluid flow.

During the Sea Ice Physics and Ecosystem Experiment (SIPEX) in September and October of 2007, we made *direct* measurements of σ_v^* in Antarctic pack ice by adapting a four probe Wenner array for use in cylindrical ice cores, as shown in Figure 1 a and b. The study area was located off the coast of East Antarctica, between 115° E and 130° E, and 64° S and 66° S. At 8 of the 15 ice stations along the cruise track of the Australian icebreaker *Aurora Australis*, we extracted vertical cores from thin first-year sea ice, with lengths ranging from 34 cm to 86 cm. We obtained 26 averaged data points from 67 raw measurements of the resistance between the inner probes.

We also measured temperature and salinity profiles for the cores, so that the data on σ_v^* could be viewed as a function of brine volume fraction ϕ . See Methods Summary.

In the Arctic, we used the technique of cross-borehole DC resistivity tomography^{18,20}, as shown in Figure 1 c and d. The ice is probed in its natural state, utilizing two or four vertical strings of electrodes frozen into the ice. It has been shown that this method can be used to derive the horizontal component of the anisotropic resistivity profile. Moreover, it has recently been demonstrated that the vertical component of σ^* can be obtained as well^{20,18}. If a minimum of four electrode strings are used, the geometric mean of the vertical and horizontal components of σ^* can be derived, along with the horizontal component¹⁸, yielding the vertical component.

Measurements of the temporal variation in the resistivity structure of first-year Arctic sea ice through spring warming have been made approximately 1 km off the coast of Barrow, Alaska at $71^\circ 21' 56.45''$ N, $156^\circ 32' 39.01''$ W. Electrode strings were installed in landfast first year ice in late January 2008. Cross-borehole measurements were made on 6 separate occasions between early April and mid June 2008, allowing both the horizontal and vertical components of the ice resistivity to be derived. A sea ice mass balance site and an ice core sampling program at the same location²¹ provided ice temperature and salinity data, allowing the variation in resistivity structure to be correlated with brine volume fraction ϕ .

Lattice and continuum percolation theories²² have been used to model a broad range of disordered materials where the connectedness of one phase dominates effective transport behavior. Consider the square ($d = 2$) or cubic ($d = 3$) network of bonds joining nearest neighbor sites on the integer lattice \mathbb{Z}^d . The bonds are assigned electrical conductivities $\sigma_0 > 0$ (open) or 0 (closed) with probabilities p and $1 - p$. Groups of connected open bonds are called open clusters, and the average cluster size grows as p increases. In this model there is a critical probability p_c , $0 < p_c < 1$, called the *percolation threshold*, where an infinite cluster of open bonds first appears. In $d = 2$, $p_c = \frac{1}{2}$, and in $d = 3$, $p_c \approx 0.25$. Typical configurations for the $d = 2$ square lattice above and below the threshold are shown in Figure 2 a and b.

Let $\sigma^*(p)$ be the effective conductivity of the network in the vertical direction²². For $p < p_c$, $\sigma^*(p) = 0$, as shown in Figure 2 c. For $p > p_c$ and near p_c , $\sigma^*(p)$ exhibits power law behavior,

$$\sigma^*(p) \sim \sigma_0(p - p_c)^t, \quad p \rightarrow p_c^+, \quad (1)$$

where t is the conductivity critical exponent. For lattices, t is believed to be universal, depending only on d . In $d = 2$, $t \approx 1.3$, and in $d = 3$, $t \approx 2.0$ ²². There is also a rigorous bound²³ that $1 \leq t \leq 2$ in $d = 2$ and $d = 3$. Since $\sigma^*(p) \rightarrow 0$ as $p \rightarrow p_c^+$, the effective resistivity $\rho^*(p) = 1/\sigma^*(p)$ diverges as $p \rightarrow p_c^+$, with a vertical asymptote at $p = p_c$, as shown in Figure 2 d. It should be remarked that for samples of two phase composites with finite component resistivities, like sea ice, the behavior only approximates the asymptote, and for $p < p_c$, ρ^* remains finite.

The fluid permeability $\kappa^*(p)$ corresponding to (1), where the open bonds are pipes of fluid conductivity $\kappa_0/\eta = r_0^2/8\eta$ and radius r_0 , behaves like $\kappa^*(p) \sim \kappa_0(p - p_c)^e$ as $p \rightarrow p_c^+$, with e the fluid permeability exponent and η the fluid viscosity. For lattices, it is believed²² that $e = t$. In the continuum, the exponents e and t can take non-universal values, and need not be equal, such as for the three dimensional Swiss cheese model^{24,22}. However, for lognormally distributed inclusions, as in sea ice, the behavior is *universal*^{9,25}. Thus for sea ice, $t = e \approx 2$.

In order to use percolation theory to quantitatively describe the vertical conductivity $\sigma_v^*(\phi)$, and to provide a link between fluid and electrical transport in sea ice, we recall our result⁹ for the vertical fluid permeability

$$k_v^*(\phi) \sim 3 (\phi - \phi_c)^2 \times 10^{-8} \text{ m}^2, \quad \phi \rightarrow \phi_c^+. \quad (2)$$

The scaling factor $k_0 = 3 \times 10^{-8}$ is estimated using critical path analysis^{22,26}. The effective behavior of media with a broad range of local conductances is dominated by a critical *bottleneck* conductance related to the minimal radius in a connected pathway of appropriate scale. To relate σ_v^* to k_v^* , we use the following relation from critical path analysis²⁶. With r_c denoting the critical radius for our centimeter scale electrical experiments, then

$$k_v^* = \frac{r_c^2}{8} \frac{\sigma_v^*}{\sigma_b}, \quad (3)$$

where σ_b is the conductivity of brine, which depends²⁷ on temperature T . By measuring the radii of vertical pathways in X-ray tomography images^{9,10}, we estimate a range in mm of $0.1 \leq r_c \leq 0.2$.

It is useful to consider the vertical conductivity formation factor $F = \sigma_v^*/\sigma_b$, which removes the dependence of the effective parameter on the changing conductivity of the brine, and depends only on the pore volume fraction and geometry. In view of (1) and (3), $F(\phi) \sim F_0 (\phi - \phi_c)^2$ as $\phi \rightarrow \phi_c^+$, where $F_0 = 8k_0/r_c^2$. The estimates for r_c yield a range for F_0 of $6 \leq F_0 \leq 24$.

In order to compare our conductivity measurements with percolation theory, we must exclude data below $\phi_c \approx 0.05$ ⁹, since the theory is only valid for $\phi > \phi_c$. It is more illustrative to display the data in terms of the reciprocal $1/F = \rho_v^*/\rho_b$, which is the vertical resistivity formation factor. In Figure 2 e and f we show the two data sets from the Antarctic and Arctic. By fixing the exponent $t = 2$ and the threshold value $\phi_c = 0.05$ in the above expression for $F(\phi)$, a statistical best fit of the data yields a value of $F_0 \approx 9$, which lies inside our predicted range, so that

$$F(\phi) \sim 9 (\phi - 0.05)^2, \quad \phi \rightarrow \phi_c^+. \quad (4)$$

We see that the data agree well with the theory, and that they both exhibit divergent behavior with a vertical asymptote at the percolation threshold. Moreover, in the logarithmic variables $x = \log(\phi - 0.05)$ and $y = \log F$, the line predicted by percolation theory in (4) is $y = 2x + \log F_0$, with $\log F_0 = 0.95$, $F_0 = 9$. Critical path analysis yields the bounds $0.8 \leq \log F_0 \leq 1.4$, and the statistical best fit for the Antarctic data in f is $y = 1.99x + 0.93$, where 0.93 lies inside these bounds. In logarithmic variables, the standard error of the regression is 0.38 for the Arctic data and 0.22 for the Antarctic data (that is, approximately 68% of the Antarctic data is within 0.22 of the regression line). The increased scatter in the Arctic data is not surprising given the substantial inverse computation required to obtain the formation factor data.

To model $\sigma_v^*(\phi)$ over all porosities, we consider features of the brine phase present over the full range — some degree of small-scale connectivity, and self-similarity. Hierarchical models of

spheres or other grains surrounded by smaller spheres, and so on, with brine in the pore spaces⁹, were used to model $k_v^*(\phi)$. The simplest model yields a result of $k_v^*(\phi) = k_0 \phi^3$. Via (3) we obtain an Archie's law result of $F(\phi) = F_0 \phi^3$. A statistical best fit of our Antarctic data yields a value of $F_0 \approx 16$, which is in the estimated range. In Figure 3 a, our Antarctic data is shown along with both theories, and in b, Arctic permeability data⁹ is shown with both theories.

Finally, in Figure 4 we show cross-borehole tomographic reconstructions of the vertical resistivity formation factor for Arctic sea ice. In a, the profile was obtained before the onset of melt pond formation. The ice is cold and electrically resistive. In b, the profile was obtained well after a melt pond had formed. The ice is warmer and significantly more conductive. By 16-17 June the ice had not only thinned but had ablated from the surface. The top 2 electrodes in each string were in air, and the third was in melt water, so that the top 0.3 m of the profile are blank. Further, the very high resistivity in the next 0.1-0.2 m likely results from fresh melt water percolating downwards, replacing the brine. This leads to values of the resistivity in this top layer roughly two to three times higher than before the formation of the melt pond. In the bottom 0.1 m or so, we also see highly conductive ice, likely due to high porosity and sea water infiltration.

It has been demonstrated in field experiments conducted in both the Arctic and Antarctic that sea ice exhibits critical behavior in its electrical transport properties at a percolation threshold. Such behavior provides the electrical signature of a key transition in fluid transport properties, known as the *rule of fives*, which determines whether or not fluid can flow through sea ice. This transition constrains a broad range of processes which are important in the geophysics and biology of the polar regions. The phenomenon is explained theoretically using percolation theory, which provides a universal power law describing the data from both poles, as well as a rigorous link between the fluid and electrical transport properties of sea ice. Our findings open the door to a new generation of techniques for *in situ* analysis and remote monitoring of transport processes which are critical to improving projections of the future trajectory of the polar ice packs.

Methods Summary

Traditional surface-based geophysical techniques for *in situ* measurements, such as the Wenner array used for surface impedance tomography, have been used to study the resistivity profile of sea ice, particularly its anisotropic structure^{13, 15, 16, 18, 19}. During SIPEX we adapted the Wenner array to directly measure the vertical component of the anisotropic conductivity tensor σ^* of sea ice. At 8 of the 15 ice stations along the cruise track of the Australian icebreaker *Aurora Australis*, we extracted vertical cores from thin first-year sea ice, with lengths ranging from 34 cm to 86 cm. Thermistor probes were inserted into small holes drilled every 5 cm. We used a Wenner electrode array along sections of the cores, connected to a YEW Earth Resistance Tester operating at 38 Hz, as indicated in Figure 1 a and b. This set-up yields the resistance along the axis of the cylindrical ice core between probes P1 and P2, corresponding to the vertical direction *in situ*, with $a = L = 10$ cm (or $a = L = 5$ cm in some cases). After the temperature and resistance measurements were taken, which took about 10 to 20 minutes, we cut each core into 10 cm sections which were later melted, so that we could obtain bulk salinity measurements for each section. The temperature and salinity measurements allowed us to calculate a brine volume fraction profile for each core²⁸.

Plate electrodes in contact with the ends of a cylinder generate parallel field lines which make measuring the conductivity of the cylinder material relatively straightforward, as illustrated in Figure 5 a. To assess the accuracy of our four probe method, the commercial package Comsol 3.5a was used to create a finite element model of cylindrical sea ice cores 0.09 m in diameter and 0.5 m in length. Four metal probes of 0.004 m in diameter and 0.09 m in length were inserted approximately 0.07 m into the core, similar to Figure 1 b. When the current is injected through the outer probes instead of parallel plates, as in Figure 5 b, the nearby field lines show significant curvature. However, in the boxed measurement region in Figure 5 b where the inner probes are located, the field lines are relatively straight, thus minimizing the error between the actual conductivity of the material and what is measured by the array. Numerical simulations show that if the outer probes are 5 cm or more from the inner measurement region, this error is less than 8.5%, and is less than 1.5% if the distance is 10 cm or more, as for much of our data.

When extracting a sea ice core to measure its properties, loss of brine is a principal concern. However, for our experiments we did not see any evidence of significant brine loss during the relatively short measurement periods, and with air temperatures ranging from about -6°C to -18°C (with most below -9°C). Moreover, the probes are inserted deep into the core, minimizing contact with potential brine surface films. Our numerical simulations and these observations establish the Wenner array as a viable field method for *direct* resistivity measurements.

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Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

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Author Contribution K. M. G. led the investigation, obtained the Antarctic conductivity data with A. G. and A. P. W., and wrote much of the paper. M. I., H. E., and K. A. J. developed the cross-borehole technique for sea ice and obtained the Arctic data. M. I. and K. A. J. developed the techniques to recover the vertical conductivity data, and analyzed the evolution of the electrical profile with melt pond onset. A. G. and C. S. analyzed the Antarctic data and developed the percolation theory that captures the data. J. L. carried out the mathematical and numerical analysis establishing the four probe Wenner array as an accurate technique to measure vertical conductivity. J. R. provided key insights into adapting the Wenner array for use with extracted cores. A. P. W. led the SIPEX expedition and provided the instrumentation and guidance for the electrical experiments.

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Figure Captions

Figure 1: **(a)** A Wenner electrode array is configured to measure the vertical conductivity of Antarctic sea ice, by inserting the four probes into an extracted ice core. **(b)** A current I is injected into the core through the outer electrodes C1 and C2. The potential difference ΔV resulting from the current flow is measured by the inner electrodes P1 and P2. The ratio $\Delta V/I$ is the resistance R in ohms. Here the electrode spacing is $L = 10$ cm and $a = 10$ cm. **(c)** A cross-borehole array is frozen into Arctic sea ice. The DC resistivity profile was tomographically reconstructed in the volume enclosed by the electrode strings. One of the strings, with 10 cm separation of the plates, is shown in **(d)**.

Figure 2: The two dimensional square bond lattice below its percolation or connectivity threshold $p_c = 1/2$ in **(a)**, and above in **(b)**. Below p_c , there is no bulk transport, and above p_c the effective conductivity takes off with power law behavior, as shown in **(c)**. In **(d)** the effective resistivity diverges as p approaches p_c from the right, with a vertical asymptote at $p = p_c$. We display the vertical resistivity formation factor data from the Antarctic in **(e)** and the Arctic in **(f)**, along with the same prediction from percolation theory in each. Both data and theory exhibit divergent behavior as ϕ approaches $\phi_c \approx 0.05$ from the right, with a vertical asymptote at $\phi = \phi_c$, electrically signaling the transition to relatively impermeable ice.

Figure 3: **(a)** Antarctic field data on the vertical formation factor $F = \sigma_v^*/\sigma_b$ is compared with the hierarchical model $F(\phi) = F_0\phi^3$. The prediction of percolation theory is also shown. **(b)** Comparison of Arctic fluid permeability data with the hierarchical model, along with percolation theory⁹. In both figures percolation theory captures the trend of the data in the percolation regime more closely than Archie's law.

Figure 4: Cross-borehole tomographic reconstructions of the vertical resistivity formation factor for Arctic sea ice before **(a)** and after **(b)** melt pond formation. The evolution of resistivity structure seen here is consistent with warming of the ice, thus increasing the fluid permeability and facilitating the infiltration of meltwater into the upper layer of sea ice from the surface.

Figure 5: Top-down view of current streamlines for a finite element model of a 3D sea ice core with measurements taken at 0.2 and 0.3 m. **(a)** Voltage is applied at the $x = 0$ and $x = 0.5$ boundaries, and the resulting streamlines are horizontal. **(b)** Voltage is applied at outer probes located at $x = 0.1$ and $x = 0.4$, causing the current lines to only be minimally curved in the region of interest, indicated by the dashed box.

Figures for Golden *et al.*, Critical behavior of electrical transport in sea ice, with brief captions. Full captions in manuscript file.

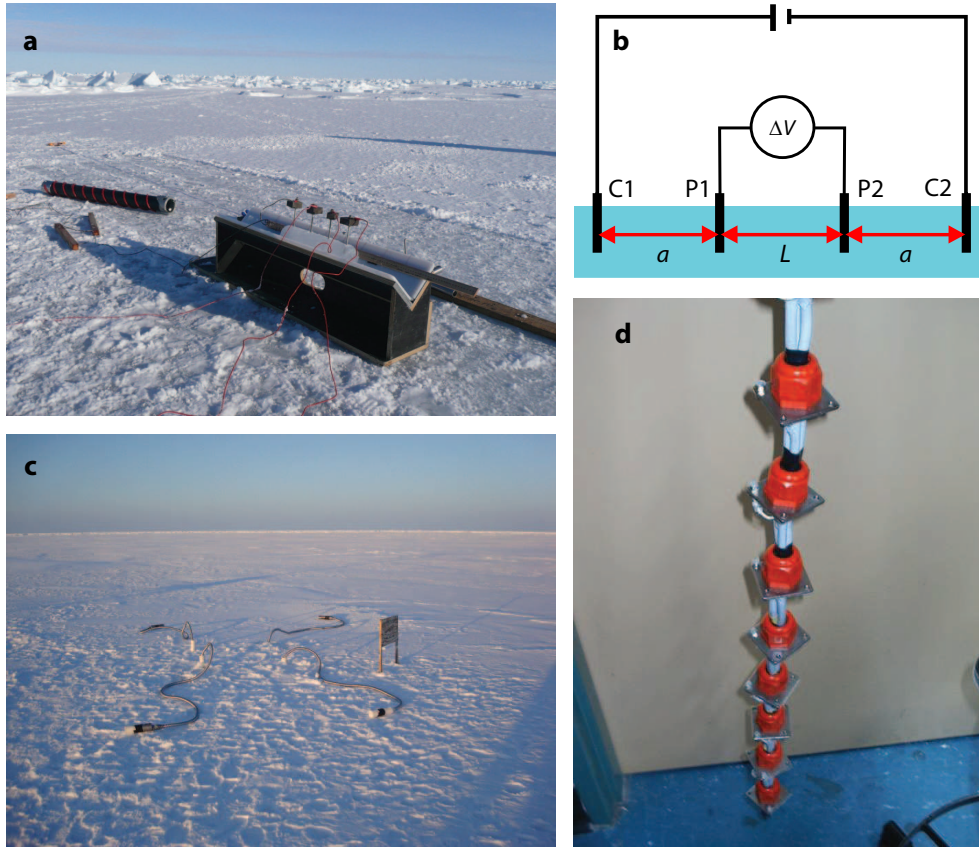


Figure 1: Direct measurements of the vertical conductivity of Antarctic sea ice in (a) and (b). Cross-borehole DC tomography for the conductivity of Arctic sea ice in (c) and (d).

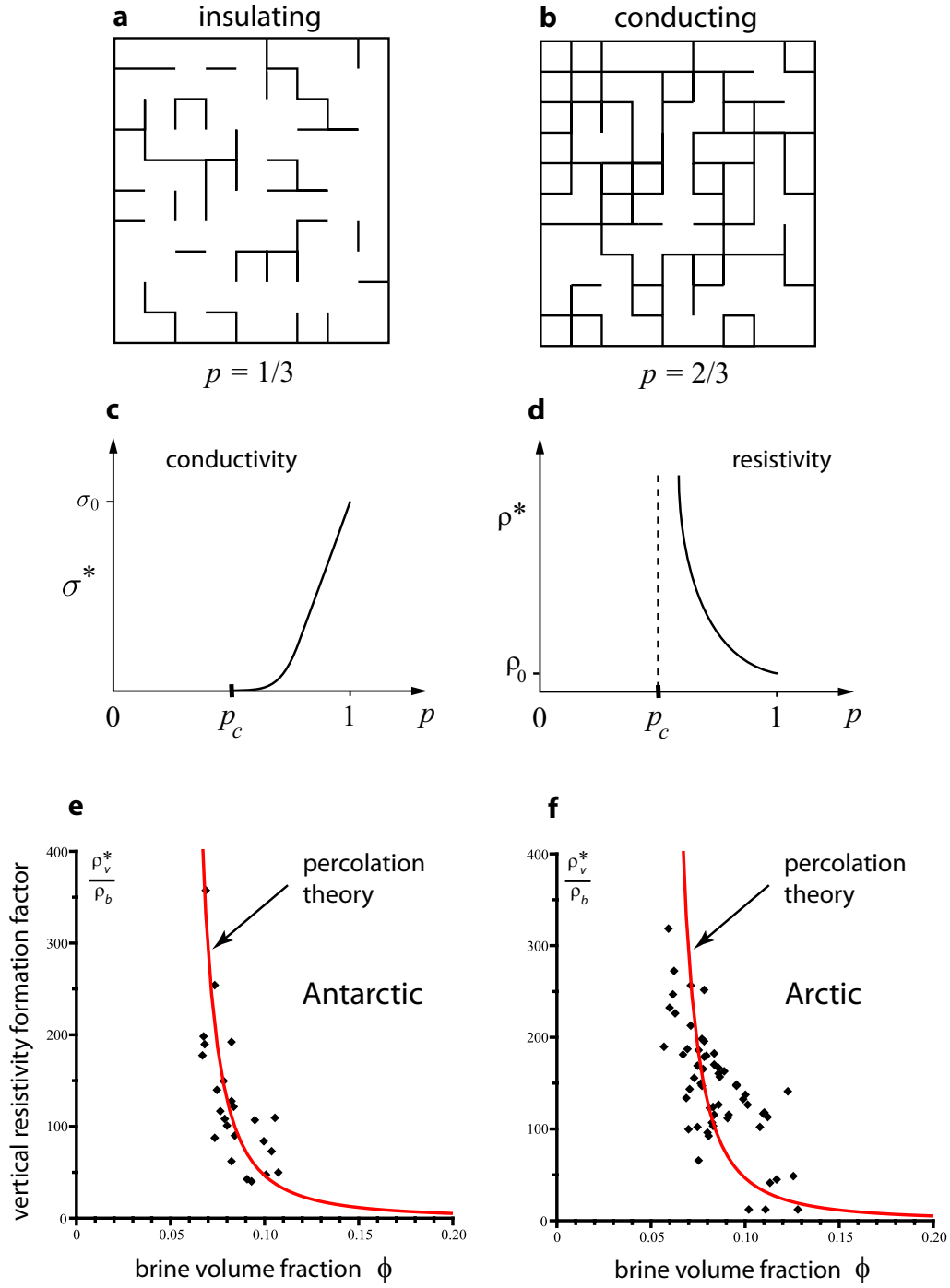


Figure 2: The lattice percolation model and critical behavior of its effective conductivity and resistivity in (a-d). Comparison of the resistivity data from the Antarctic (e) and the Arctic (f) with the same percolation theory prediction.

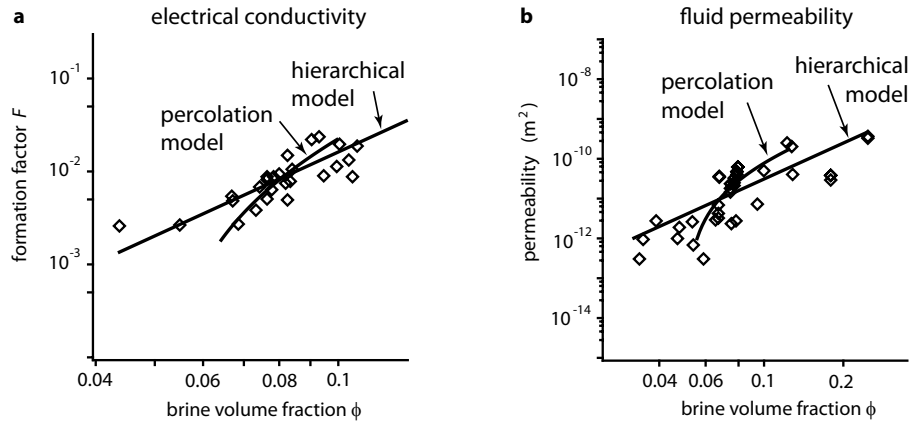


Figure 3: Conductivity data from the Antarctic in (a) and fluid permeability data from the Arctic in (b), compared with the predictions from percolation and hierarchical models.

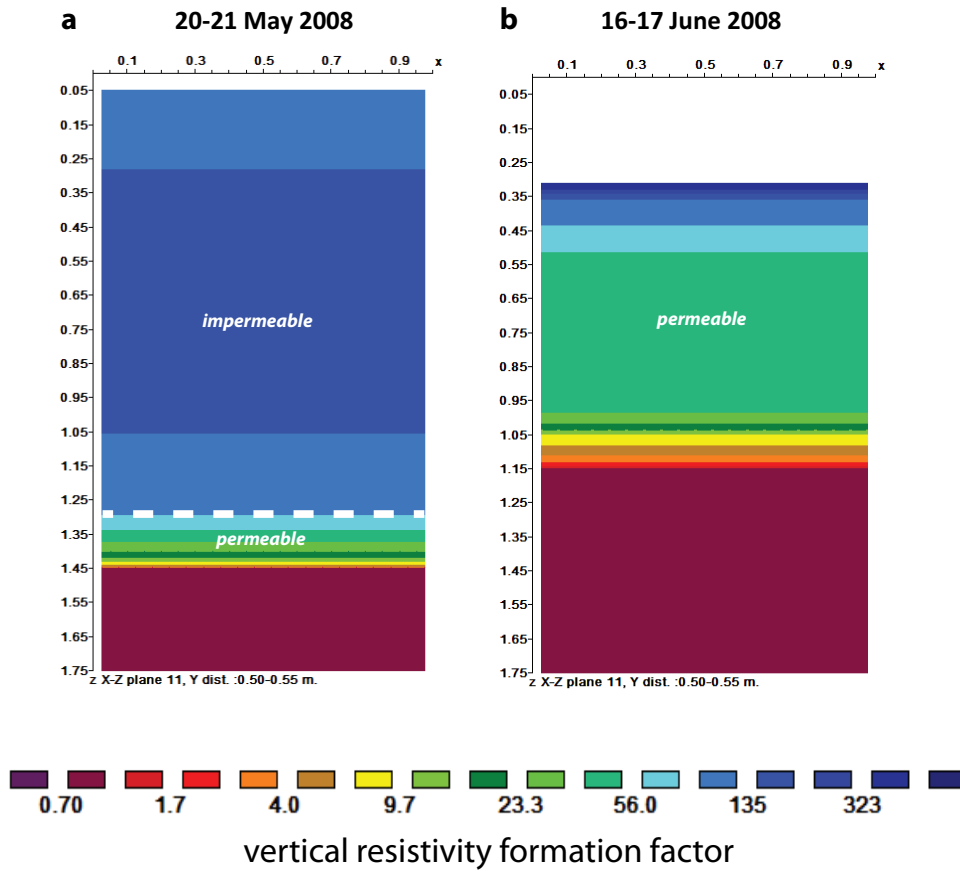


Figure 4: Change in electrical profile with the onset of surface ponding.

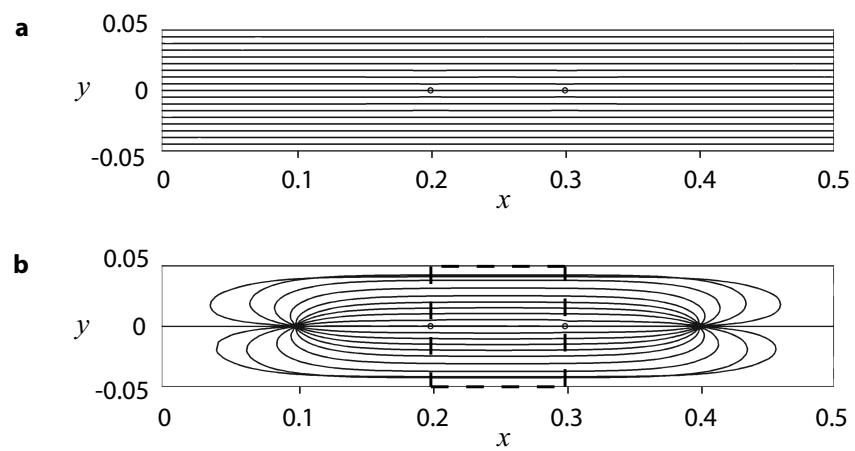


Figure 5: Comparison of field lines for a parallel plate configuration in (a) with those for a four probe Wenner array in (b).