

# Critical Behavior of Electrical Transport in Sea Ice

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**Fluid transport through sea ice mediates a broad range of geophysical and biological processes, such as melt pond evolution and nutrient replenishment for microbial communities. However, columnar sea ice is effectively impermeable to fluid flow for brine volume fractions below about 5%. In two different experiments conducted in the Arctic and Antarctic, we have found that this critical transition in fluid flow at the brine connectivity threshold displays a strong electrical signature. Sea ice conductivity data are accurately explained by percolation theory with a universal critical exponent of 2. Our findings lay the foundation for electromagnetic monitoring of transport phenomena in sea ice, which can help track key transitions in the state of polar sea ice and improve projections of its fate and impact on ecosystems.**

Polar sea ice is a key component of Earth’s climate system, and a leading indicator of climate change<sup>1,2</sup>. It also hosts extensive microbial communities which sustain life in the polar oceans<sup>1,3</sup>. While global climate models predict declines in sea ice area and thickness, they have significantly underestimated recent loss in Arctic summer ice extent<sup>4</sup>.

Improving projections of the fate of Earth’s sea ice cover and its importance for polar ecosystems depends on a better understanding of key processes and feedback mechanisms. For example, the evolution of melt ponds and summer ice albedo is constrained by drainage through porous sea ice<sup>5</sup>. It is believed that ice-albedo feedback has played a key role in the decline of summer Arctic sea ice<sup>6</sup>. Fluid flow through sea ice mediates the salt budget<sup>1</sup>, convection-enhanced thermal transport<sup>7</sup>, ocean-ice-atmosphere CO<sub>2</sub> exchanges<sup>8</sup>, and biomass build-up sustained by nutrient fluxes<sup>1,3</sup>. It also enables snow-ice formation, driven by sea water flooding and freezing at the ice surface, accounting for more than a quarter of the ice produced in the Southern Ocean<sup>9</sup>.

It has been observed that for brine volume fractions  $\phi$  below about 5%, columnar sea ice is effectively impermeable to vertical fluid flow, yet is increasingly permeable for  $\phi$  above 5%<sup>10</sup>. For a typical bulk salinity of 5 ppt, the critical porosity  $\phi_c \approx 5\%$  corresponds to a temperature  $T_c \approx -5^\circ \text{C}$ , which is known as the *rule of fives*. This critical behavior of the fluid permeability, which constrains the above processes, reflects a connectivity or percolation threshold in the brine microstructure<sup>10–12</sup>. If the fluid transport properties of sea ice can be linked to its electrical properties, which is the aim of this paper, then new approaches can be brought to bear in monitoring the state of sea ice. For example, it could open the door to the development of sensors to enhance existing buoy networks, provide information on key ice processes, and improve integration with satellite data.

The electrical conductivity of sea ice has been studied over the past five decades<sup>13–18</sup>. Success in relating the electrical transport properties to microstructural characteristics of sea ice, however, has been somewhat limited. In particular, there have been no observations

of critical behavior in electrical properties corresponding to the important microstructural transition expressed by the rule of fives. Here we report on two types of experiments conducted on sea ice in the polar regions. In both cases we have obtained extensive data on the electrical resistivity which clearly display critical behavior at the brine percolation threshold. Our mathematical description provides a rigorous link between fluid and electrical transport in sea ice, with both properties displaying the same type of universal critical behavior, thus laying the foundation for the types of techniques referred to above.

Sea ice is an anisotropic composite with a preference for vertical elongation and connectness in the brine inclusion microstructure, with a corresponding anisotropy in effective fluid and electrical transport coefficients. For processes of interest to climate and ecosystem studies such as the drainage of melt ponds, fluid flow in the vertical direction is most relevant; thus our interest here in the vertical electrical conductivity. Most methods for measuring sea ice conductivity involve indirect or inverse techniques, such as surface impedance tomography, where the vertical component of the conductivity is inherently mixed with the horizontal components. In the Antarctic, we made *direct* measurements of the vertical conductivity of sea ice using a four probe Wenner array inserted into extracted ice cores, as shown in Figure 1 (A) and (B). In the Arctic, we have used the emerging technique of cross-borehole DC tomography, utilizing four vertical strings of electrodes frozen into the ice<sup>18,19</sup>, as shown in Figure 1 (C) and (D). The vertical component of the resistivity is obtained from tomographic reconstructions of the horizontal and geometric mean resistivities. Although the two data sets were obtained in different types of sea ice and by quite different methods, they agree closely and are both accurately captured with the same critical exponent from lattice percolation theory used to predict fluid permeability<sup>11</sup>. Moreover, the Arctic data was taken in late spring and shows the transformation in electrical properties associated with onset of surface melt pond development.

The findings presented here also have implications for measuring ice thickness, a key

variable in tracking the response of the ice cover to changes in external forcing, such as global warming. Not only is thickness data important in comparing climate model predictions to observed behavior, but in specifying the initial conditions necessary for long-term numerical simulations. Key methods for obtaining such data depend on the interaction of electromagnetic (EM) fields with sea ice. For example, there has been significant interest in the development of EM induction devices<sup>20, 17</sup> mounted on ships, planes and helicopters. These techniques, and the interpretation of the data to obtain ice thickness, rely on knowledge of the electrical properties of sea ice, and how they vary with depth, temperature, salinity, and ice type. The results presented here shed significant light on such issues.

During the Sea Ice Physics and Ecosystem Experiment (SIPEX) in September and October of 2007, we measured the electrical conductivity of first-year Antarctic pack ice. The study area was located off the coast of East Antarctica, between 115° E and 130° E, and 64° S and 66° S. Traditional surface-based geophysical techniques for *in situ* measurements, such as the Wenner array used for surface impedance tomography, have been used to study the resistivity profile of sea ice, particularly its anisotropic structure<sup>13, 15, 16, 18, 21</sup>. During SIPEX we adapted the Wenner array to directly measure the vertical component of the anisotropic conductivity tensor  $\sigma^*$  of sea ice. At 8 of the 15 ice stations along the cruise track of the Australian icebreaker *Aurora Australis*, we extracted vertical cores from thin first-year sea ice, with lengths ranging from 34 cm to 86 cm. Thermistor probes were inserted into small holes drilled every 5 cm. We used a Wenner electrode array along sections of the cores, connected to a YEW Earth Resistance Tester operating at 38 Hz, as indicated in Figure 1 (A) and (B). This set-up yields the resistance along the axis of the cylindrical ice core between probes P1 and P2, corresponding to the vertical direction *in situ*. We obtained 26 averaged data points from 67 raw measurements with  $a = L = 10$  cm (or  $a = L = 5$  cm in some cases). After the temperature and resistance measurements were taken, which took about 10 to 20 minutes, we cut each core into 10 cm sections which were later melted,

so that we could obtain bulk salinity measurements for each section. The temperature and salinity measurements allowed us to calculate a brine volume fraction profile for each core<sup>22</sup>.

Plate electrodes in contact with the ends of a cylinder generate parallel field lines which make measuring the conductivity of the cylinder material relatively straightforward, as illustrated in Figure 2 (A). To assess the accuracy of our four probe method, the commercial package Comsol 3.5a was used to create a finite element model of cylindrical sea ice cores 0.09 m in diameter and 0.5 m in length. Four metal probes of 0.004 m in diameter and 0.09 m in length were inserted approximately 0.07 m into the core, similar to Figure 1 (B). When the current is injected through the outer probes instead of parallel plates, as in Figure 2 (B), the nearby field lines show significant curvature. However, in the boxed measurement region in Figure 2 (B) where the inner probes are located, the field lines are relatively straight, thus minimizing the error between the actual conductivity of the material and what is measured by the array. Numerical simulations show that if the outer probes are 5 cm or more from the inner measurement region, this error is less than 8.5%, and is less than 1.5% if the distance is 10 cm or more, as for much of our data.

When extracting a sea ice core to measure its properties, loss of brine is a principal concern. However, for our experiments we did not see any evidence of significant brine loss during the relatively short measurement periods, and with air temperatures ranging from about  $-6^{\circ}$  C to  $-18^{\circ}$  C (with most below  $-9^{\circ}$  C). Moreover, the probes are inserted deep into the core, minimizing contact with potential brine surface films. Our numerical simulations and these observations establish the Wenner array as a viable field method for *direct* resistivity measurements.

The technique of cross-borehole DC resistivity tomography<sup>18</sup>, where ice is probed in its natural state, utilizes vertical strings of electrodes frozen into the ice. It has been shown that this method can be used to derive the horizontal component of the anisotropic resistivity profile. Moreover, it has recently been demonstrated that the vertical component of  $\sigma^*$  can

be obtained as well<sup>19,18</sup>. If a minimum of four electrode strings are used, the geometric mean of the vertical and horizontal components of  $\sigma^*$  can be derived, along with the horizontal component<sup>18</sup>, yielding the vertical component.

Measurements of the temporal variation in the resistivity structure of first-year Arctic sea ice through spring warming have been made approximately 1 km off the coast of Barrow, Alaska at  $71^\circ 21' 56.45''$  N,  $156^\circ 32' 39.01''$  W. Electrode strings were installed in landfast first year ice in late January 2008. Cross-borehole measurements were made on 6 separate occasions between early April and mid June 2008, allowing both the horizontal and vertical components of the ice resistivity to be derived. A sea ice mass balance site and an ice core sampling program at the same location<sup>23</sup> provided ice temperature and salinity data, allowing the variation in resistivity structure to be correlated with brine volume fraction  $\phi$ .

Lattice and continuum percolation theories<sup>24</sup> have been used to model a broad range of disordered materials where the connectedness of one phase dominates effective behavior. Consider the square ( $d = 2$ ) or cubic ( $d = 3$ ) network of bonds joining nearest neighbor sites on the integer lattice  $\mathbb{Z}^d$ . The bonds are assigned electrical conductivities  $\sigma_0 > 0$  (open) or 0 (closed) with probabilities  $p$  and  $1 - p$ . Groups of connected open bonds are called open clusters, and the average cluster size grows as  $p$  increases. In this model there is a critical probability  $p_c$ ,  $0 < p_c < 1$ , called the *percolation threshold*, where an infinite cluster of open bonds first appears. In  $d = 2$ ,  $p_c = \frac{1}{2}$ , and in  $d = 3$ ,  $p_c \approx 0.25$ . Typical configurations for the  $d = 2$  square lattice above and below the threshold are shown in Figure 3.

Let  $\sigma^*(p)$  be the effective conductivity of this random resistor network in the vertical direction<sup>24</sup>. For  $p < p_c$ ,  $\sigma^*(p) = 0$ , as shown in Figure 3 (C). For  $p > p_c$  near the threshold,  $\sigma^*(p)$  exhibits power law behavior,

$$\sigma^*(p) \sim \sigma_0(p - p_c)^t, \quad p \rightarrow p_c^+, \quad (1)$$

where  $t$  is the conductivity critical exponent. For lattices,  $t$  is believed to be universal,

depending only on  $d$ . In  $d = 2$ ,  $t \approx 1.3$ , and in  $d = 3$ ,  $t \approx 2.0$ <sup>24</sup>. There is also a rigorous bound<sup>25</sup> that  $1 \leq t \leq 2$  in  $d = 2$  and  $d = 3$ . Since  $\sigma^*(p) \rightarrow 0$  as  $p \rightarrow p_c^+$ , the effective resistivity  $\rho^*(p) = 1/\sigma^*(p)$  diverges as  $p \rightarrow p_c^+$ , with a vertical asymptote at  $p = p_c$ , as shown in Figure 3 (D). It should be remarked that for finite samples of two phase composites with component resistivities  $\rho_0$  and  $\rho_1 \gg 1$  with  $\rho_0 \ll \rho_1$ , the behavior only approximates the asymptote, and for  $p < p_c$ ,  $\rho^*$  remains finite with  $\rho^* < \rho_1$  and  $\rho^* \rightarrow \rho_1$  as  $p \rightarrow 0^+$ .

The fluid permeability  $\kappa^*(p)$  corresponding to (1), where the open bonds are pipes of fluid conductivity  $\kappa_0/\eta = r_0^2/8\eta$  and radius  $r_0$ , behaves like  $\kappa^*(p) \sim \kappa_0(p - p_c)^e$  as  $p \rightarrow p_c^+$ , with  $e$  the fluid permeability exponent and  $\eta$  the fluid viscosity. For lattices, it is believed that  $e = t$ <sup>24</sup>. In the continuum, the permeability and conductivity exponents  $e$  and  $t$  can take non-universal values, and need not be equal, such as for the three dimensional Swiss cheese model<sup>26,24</sup>. However, for lognormally distributed inclusions, as in sea ice, the behavior is *universal*<sup>11,27</sup>. Thus for sea ice,  $t = e \approx 2$ .

In order to use percolation theory to quantitatively describe the vertical conductivity  $\sigma_v^*(\phi)$ , and to provide the first link between fluid and electrical transport in sea ice, we recall our result for the vertical fluid permeability  $k_v^*(\phi)$ <sup>11</sup>,

$$k_v^*(\phi) \sim 3 (\phi - \phi_c)^2 \times 10^{-8} \text{ m}^2, \quad \phi \rightarrow \phi_c^+. \quad (2)$$

The scaling factor  $k_0 = 3 \times 10^{-8}$  is estimated using critical path analysis<sup>24,28</sup>. The effective behavior of media with a broad range of local conductances is dominated by a critical *bottleneck* conductance related to the minimal radius in a connected pathway of appropriate scale. To relate  $\sigma_v^*$  to  $k_v^*$ , we use the following relation from critical path analysis<sup>28</sup>. With  $r_c$  denoting the critical radius for our centimeter scale electrical experiments, then

$$k_v^* = \frac{r_c^2}{8} \frac{\sigma_v^*}{\sigma_b}, \quad (3)$$

where  $\sigma_b$  is the conductivity of brine, which depends on temperature  $T$ <sup>29</sup>. By measuring

the radii of vertical pathways in X-ray tomography images<sup>11,12</sup>, we estimate  $r_c$  (mm) to be in the range  $0.1 \leq r_c \leq 0.2$ .

It is useful to consider the vertical conductivity formation factor  $F = \sigma_v^*/\sigma_b$ , which removes the dependence of the effective parameter on the changing conductivity of the brine, and depends only on the pore volume fraction and geometry. In view of (1) and (3),  $F(\phi) \sim F_0 (\phi - \phi_c)^2$  as  $\phi \rightarrow \phi_c^+$ , where  $F_0 = 8k_0/r_c^2$ . The estimates for  $r_c$  yield a range for  $F_0$  of  $6 \leq F_0 \leq 24$ .

In order to compare our conductivity measurements with percolation theory, we must exclude data below  $\phi_c \approx 0.05$ , as in<sup>11</sup>, since the theory is only valid for  $\phi > \phi_c$ . It is more illustrative to display the data in terms of the reciprocal  $1/F = \rho_v^*/\rho_b$ , which is the vertical resistivity formation factor. In Figure 3 (E) and (F) we show the two data sets from the Antarctic and Arctic. By fixing the exponent  $t = 2$  and the threshold value  $\phi_c = 0.05$  in the above expression for  $F(\phi)$ , a statistical best fit of the data yields a value of  $F_0 \approx 9$ , which lies inside our predicted range, so that

$$F(\phi) \sim 9 (\phi - 0.05)^2, \quad \phi \rightarrow \phi_c^+. \quad (4)$$

We see that the data agree well with the theory, and that they both exhibit divergent behavior with a vertical asymptote at the percolation threshold. Moreover, in the logarithmic variables  $x = \log(\phi - 0.05)$  and  $y = \log F$ , the line predicted by percolation theory in (4) is  $y = 2x + \log F_0$ , with  $\log F_0 = 0.95$ ,  $F_0 = 9$ . Critical path analysis yields the bounds  $0.8 \leq \log F_0 \leq 1.4$ , and the statistical best fit for the Antarctic data in (F) is  $y = 1.99x + 0.93$ , where 0.93 lies inside these bounds. In logarithmic variables, the standard error of the regression is 0.38 for the Arctic data and 0.22 for the Antarctic data (that is, approximately 68% of the Antarctic data is within 0.22 of the regression line). The increased scatter in the Arctic data is not surprising given the substantial inverse computation required to obtain the formation factor data from the cross-borehole measurements.



It is important to remark that since sea ice is a finite, real composite where the brine microstructure and its connectivity can be highly variable, particularly over small scales relevant to electrical measurements, we would not necessarily expect data for  $\phi < 5\%$  to be closely captured by extrapolation of percolation formulas to such situations. Nevertheless, we would expect the dominant feature of any data set on sea ice resistivity to be its rapid ascent as  $\phi$  approaches a threshold value  $\phi_c$  from the right.

To model  $\sigma_v^*(\phi)$  over all porosities, we consider features of the brine phase present over the full range – some degree of small-scale connectivity, and self-similarity. Hierarchical models of spheres or other grains surrounded by smaller spheres, and so on, with brine in the pore spaces, were used to model  $k_v^*(\phi)$  in<sup>11</sup>. The simplest model yields a result of  $k_v^*(\phi) = k_0 \phi^3$ . Via (3) we obtain an Archie’s law result of  $F(\phi) = F_0 \phi^3$ . A statistical best fit of our Antarctic data yields a value of  $F_0 \approx 16$ , which is again in the estimated range. In Figure 4 (A), our Antarctic conductivity data is shown along with both theories, and in (B), Arctic permeability data<sup>11</sup> is shown with both theories.

Finally, in Figure 5 we show cross-borehole tomographic reconstructions of the vertical resistivity formation factor for Arctic sea ice. In (A), the profile was obtained before the onset of melt pond formation. The ice is cold and electrically resistive. In (B), the profile was obtained well after a melt pond had formed. The ice is warmer and significantly more conductive. By 16-17 June the ice had not only thinned but had ablated from the surface. The top 2 electrodes in each string were in air, and the third was in melt water, so that the top 0.3 m of the profile are blank. Further, the very high resistivity in the next 0.1-0.2 m likely results from the fact that in this region fresh melt water had percolated downwards, replacing the brine. This leads to values of the resistivity in this top layer of the ice roughly two to three times higher than before the formation of the melt pond. In the bottom 0.1 m or so, we also see highly conductive ice, likely due to high porosity and sea water infiltration.

It has been demonstrated in field experiments conducted in both the Arctic and Antarc-

tic that sea ice exhibits critical behavior in its electrical transport properties at a percolation threshold. Such behavior provides the electrical signature of a key transition in fluid transport properties, known as the “rule of fives,” which determines whether or not fluid can flow through sea ice. This transition constrains a broad range of processes which are important in the geophysics and biology of the polar regions. The phenomenon is explained theoretically using percolation theory, which provides a universal power law describing the data from both poles, as well as the first rigorous link between the fluid and electrical transport properties of sea ice. Our findings open the door to a new generation of techniques for *in situ* analysis and remote monitoring of transport processes which are critical to improving projections of the future trajectory of the polar ice packs.

## References and Notes

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