Polar sea ice is a key player

in Earth's climate system, and

a leading indicator of climate change

It also hosts extensive microbial communities which sustain

life in the polar oceans

While global climate models predict declines in sea ice area and thickness,

they have significantly underestimated the recent loss

in Arctic summer ice extent We focus here on sea ice processes

which must be better understood

to represent the polar ice packs more

realistically in climate models.

In particular, we investigate the electrical behavior

of sea ice associated with key transport phenomena

and microstructural transitions.

While fluid flow is substantially restricted for

brine volume fractions $\phi$ below about 5\%, columnar sea ice

is increasingly permeable

for $\phi$ above 5\% For a typical bulk salinity

of 5 ppt, the critical porosity

$\phi\_c \approx 5\%$ corresponds to a temperature

$T\_c \approx -5^{\circ}$ C, which is known as the {\it rule of fives}.

This critical behavior of the fluid permeability

%which constrains the above processes,

results from a connectivity or percolation threshold

in the brine microstructure

If the fluid transport properties of sea ice

can be linked to its electrical properties,

which is the aim of this paper,

then new approaches can be brought to bear

in monitoring the state of sea ice.

For example, it could open the door to the development

of sensors to enhance existing buoy networks, provide

information on key ice processes, and improve

integration with satellite data.

The electrical conductivity of sea ice has been studied

over the past five decades

However, there have been no observations of critical behavior in

electrical properties corresponding to the

microstructural transition %articulated by

encapsulated in the rule of fives.

Here we report on two types of experiments where

electrical resistivity data clearly display

critical behavior at the brine percolation threshold.

The mathematical description we develop provides

a rigorous link between fluid and electrical

transport in sea ice, with both displaying

the same type of universal critical behavior,

thus laying the foundation for

the techniques referred to above.

Sea ice is an anisotropic composite with vertically

elongated brine inclusions and

corresponding anisotropy in the effective fluid

permeability and electrical conductivity tensors.

Most methods for measuring sea ice conductivity

involve indirect or inverse techniques, such as surface

impedance tomography using a Wenner array

of electrodes

Generally with these methods

the vertical conductivity

$\sigma^\*\_v$

is inherently mixed with the horizontal components.

Here we are most interested in $\sigma^\*\_v$ due to

its connection with vertical fluid flow.

During the

Sea Ice Physics and Ecosystem

Experiment (SIPEX) in September and October of 2007,

we made {\it direct}

measurements of $\sigma^\*\_v$

in Antarctic pack ice by adapting

a four probe Wenner array

for use in cylindrical ice cores,

as shown in Figure 1 a and b.

The study area was located off the coast of

East Antarctica, between

115$^{\circ}$ E and 130$^{\circ}$ E, and

64$^{\circ}$ S

and 66$^{\circ}$ S.

At 8 of the 15 ice stations along the cruise track

of the Australian icebreaker {\it Aurora Australis},

we extracted vertical cores from thin

first-year sea ice, with lengths ranging

from 34 cm to 86 cm. We obtained 26 averaged data

points from 67 raw measurements of the resistance

between the inner probes. We also measured

temperature and salinity profiles for the cores,

so that data on $\sigma^\*\_v$ can be viewed as a function

of brine volume fraction $\phi$.

See Methods Summary.

In the Arctic, we used the technique of

cross-borehole DC resistivity

tomography

as shown in Figure 1 c and d.

The ice is probed in its natural state,

utilizing two or four vertical strings of

electrodes frozen into the ice. It has been

shown that this method can be used to

derive the horizontal component of the

anisotropic resistivity profile.

Moreover, it has recently been demonstrated that the

vertical component of $\bsig^\*$ can be obtained

as well If a minimum of four electrode strings are used,

the geometric mean of the vertical and horizontal

components of $\bsig^\*$ can be derived, along with

the horizontal component yielding the vertical component.

Measurements of the temporal variation

in the resistivity structure of first-year Arctic

sea ice through spring warming have been made

approximately 1 km off the coast of

Barrow, Alaska at 71$^{\circ}$ 21$^\prime$

56.45$^{\prime \prime}$ N,

156$^{\circ}$ 32$^\prime$ 39.01$^{\prime \prime}$ W.

Electrode strings were installed in landfast

first year ice in late January 2008.

Cross-borehole measurements were made on

6 separate occasions between early April

and mid June 2008, allowing both the horizontal

and vertical components of the ice resistivity

to be derived.

A sea ice mass balance site and

an ice core sampling program

at the same

location provided ice temperature and

salinity data, allowing the variation in

resistivity structure to be correlated

with brine volume fraction $\phi$.

Lattice and continuum percolation theories

have been used to model a broad range of disordered

materials where the connectedness of one phase

dominates effective transport behavior.

Consider the square ($d=2$) or cubic ($d=3$) network of bonds

joining nearest neighbor sites on the integer lattice

${\mathbb{Z}}^d$.

The bonds are assigned electrical conductivities

$\sigma\_0 > 0$ (open) or 0 (closed) with probabilities $p$

and $1-p$.

Groups of connected open bonds

are called open clusters, and the average cluster size

grows as $p$ increases.

In this model

there is a critical probability $p\_c$, $0<p\_c<1$,

called the {\it percolation threshold},

where an infinite cluster of open bonds first appears.

In $d=2$, $p\_c = \frac{1}{2}$, and in $d=3$,

$p\_c \approx 0.25$.

Typical configurations for the $d=2$ square lattice

above and below the threshold

are shown in Figure 2 a and b.

Let $\sigma^\*(p)$ be the effective conductivity

of the network

in the vertical direction \cite{Stauffer-92}.

For $p < p\_c$, $\sigma^\*(p)=0$, as shown in

Figure 2 c. For $p>p\_c$ and near $p\_c$,

$\sigma^\*(p)$ exhibits power law behavior,

\sigma^\*(p

where $t$ is the conductivity critical exponent.

For lattices, %$p\_c$ depends on the type of lattice, while

$t$ is believed to be universal,

depending only on $d$.

In $d=2$, $t \approx 1.3$,

and in $d=3$,

$t \approx 2.0$ There is also a rigorous

bound that $1 \leq t \leq 2$

in $d=2$ and $d=3$.

Since $\sigma^\*(p) \rightarrow 0$ as $p \rightarrow p\_c^+$,

the effective resistivity $\rho^\*(p)=1/\sigma^\*(p)$ diverges

as $p \rightarrow p\_c^+$, with

a vertical asympote at $p=p\_c$, as shown in

Figure 2 d.

It should be remarked that for samples of

two phase composites with finite component resistivities,

like sea ice,

the behavior only approximates the asymptote, and for

$p < p\_c$, $\rho^\*$ remains finite.

The fluid permeability $\kappa^\*(p)$

corresponding to (\ref{critical\_conductivity}),

where the open bonds are pipes of fluid conductivity

$\kappa\_0/\eta=r\_0^2/8\eta$ and radius $r\_0$, behaves like

$\kappa^\*(p) \sim \kappa\_0(p-p\_c)^e$ as $p \rightarrow p\_c^+$,

with $e$ the fluid permeability exponent and $\eta$ the fluid viscosity.

For lattices, it is believed that $e=t$.

In the continuum,

the

exponents $e$ and $t$ can take

non-universal values, and need not be equal,

such as for the three dimensional Swiss cheese model

However, for

lognormally distributed inclusions, as in sea ice,

the behavior is {\it universal}

Thus for sea ice, $t=e \approx 2$.

In order to use percolation theory to quantitatively describe

the vertical conductivity $\sigma^\*\_v(\phi)$,

and to provide

a link between fluid and electrical transport in sea ice,

we recall our result

for the vertical fluid permeability

k\_v^\*(\phi) \; \sim \; 3 \; (\phi - \phi\_c)^2 \;

The scaling factor $k\_0=3 \times 10^{-8}$

is estimated using critical path analysis

The effective behavior of media with a broad

range of local conductances is dominated by a

critical {\it bottleneck} conductance related to

the minimal radius in a connected pathway

of appropriate scale.

To relate $\sigma^\*\_v$ to $k^\*\_v$, we use

the following relation

from critical path analysis With $r\_{c}$ denoting the critical radius for our

centimeter scale electrical

experiments, then

k\_v^\*

where $\sigma\_b$ is the conductivity of brine,

which depends on temperature $T$.

By measuring the radii of vertical pathways

in X-ray tomography

images we

estimate a range in mm of

$0.1 \leq r\_{c} \leq 0.2$.

It is useful to consider

the vertical conductivity formation factor

$F=\sigma^\*\_v/\sigma\_b$,

which removes the dependence of the effective parameter

on the changing conductivity of the brine,

and depends only on the pore

volume fraction and geometry.

In view of (\ref{critical\_conductivity})

and (\ref{cond\_perm\_relation}),

$F(\phi) \; \sim \; F\_0 \; (\phi - \phi\_c)^2$ as

$\phi \rightarrow \phi\_c^+$, where

$F\_0 = 8k\_0/r\_c^2$.

The estimates for $r\_{c}$ yield a range for

$F\_0$ of $6 \leq F\_0 \leq 24$.

In order to compare our conductivity

measurements with percolation theory, we must

exclude data below $\phi\_c \approx 0.05$

since the theory

is only valid for $\phi > \phi\_c$.

It is more illustrative to display the data in terms

of the reciprocal $1/F=\rho\_v^\*/\rho\_b$, which is the

vertical resistivity formation factor.

In Figure 2 e and f we show the two data

sets from the Antarctic and Arctic. By fixing

the exponent $t=2$ and the threshold value $\phi\_c = 0.05$

in the above expression for $F(\phi)$,

a statistical best fit of the data yields a value

of $F\_0\approx 9$, which lies inside our predicted range, so that

F(\phi)

We see that the data agree well with the theory, and that they both

exhibit divergent behavior with a vertical asymptote at the

percolation threshold. Moreover,

in the logarithmic variables

$x=\log{(\phi-0.05)}$ and $y=\log{F}$, the

line predicted by percolation theory in (\ref{perc\_formation2}) is

$y=2x+\log{F\_0}$, with $\log{F\_0}=0.95, F\_0=9$.

Critical path analysis yields the bounds

$0.8 \leq \log{F\_0} \leq 1.4$, and the statistical

best fit for the Antarctic data in f is $y=1.99x+0.93$,

where 0.93 lies inside these bounds.

In logarithmic variables, the standard error of the regression

is 0.38 for the Arctic data and 0.22 for the Antarctic

data (that is, approximately 68\% of the Antarctic data is within 0.22

of the regression line). The increased scatter in the Arctic data

is not surprising given the substantial inverse computation required to

obtain the formation factor data.

To model $\sigma\_v^\*(\phi)$ over all porosities,

we consider features

of the brine phase present over the full range $-$

some degree of small-scale connectivity,

and self-similarity.

Hierarchical models

of spheres or other grains surrounded by smaller spheres,

and so on, with brine in the pore spaces were used to model $k\_v^\*(\phi)$.

The simplest model yields a

result of $k\_v^\*(\phi)=k\_0 \, \phi^3$.

Via (\ref{cond\_perm\_relation}) we obtain an Archie's

law result of $F(\phi)=F\_0 \, \phi^3$. A statistical

best fit of our Antarctic data yields a value

of $F\_0 \approx 16$, which is in the

estimated range. In Figure 3 a,

our Antarctic data is shown along with both

theories, and in b, Arctic permeability

data is shown

with both theories.

Finally, in Figure 4 we show

cross-borehole tomographic reconstructions

of the vertical resistivity formation factor

for Arctic sea ice. In a, the

profile was obtained before the onset

of melt pond formation. The ice is cold and

electrically resistive. In b, the profile

was obtained well after a melt pond had formed.

The ice is warmer and significantly more conductive.

By 16-17 June the ice had not

only thinned but had ablated from the surface.

The top 2 electrodes in each string were in air,

and the third was in melt water, so that

the top 0.3 m of the profile are blank.

Further, the very high resistivity in the next

0.1-0.2 m likely results from

fresh melt water percolating

downwards, replacing the brine. This leads to

values of the resistivity in this top layer

roughly two to three times higher than

before the formation of the melt pond. In the bottom

0.1 m or so, we also see highly conductive ice,

likely due to high porosity and sea water infiltration.

It has been demonstrated in field experiments conducted

in both the Arctic and Antarctic that sea ice exhibits

critical behavior in its electrical transport properties

at a percolation threshold. Such behavior provides the

electrical signature of a key transition

in fluid transport properties, known as the

rule of fives}, which determines whether or

not fluid can flow through sea ice.

This transition constrains

a broad range of processes which are important in

the geophysics and biology of the polar regions.

The phenomenon is explained theoretically using

percolation theory, which provides a universal

power law describing the data from both poles,

as well as a rigorous link between the

fluid and electrical transport properties of sea ice.

Our findings open the door to a new generation of

techniques for {\it in situ} analysis and remote monitoring

of transport processes which are critical to

improving projections of the future trajectory

of the polar ice packs.