Mathematics and the Melting Polar Ice Caps

Using mathematics of composite materials to improve projections of climate change and the fate of Earth's sea ice packs

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Sea ice is an indicator and agent of climate change

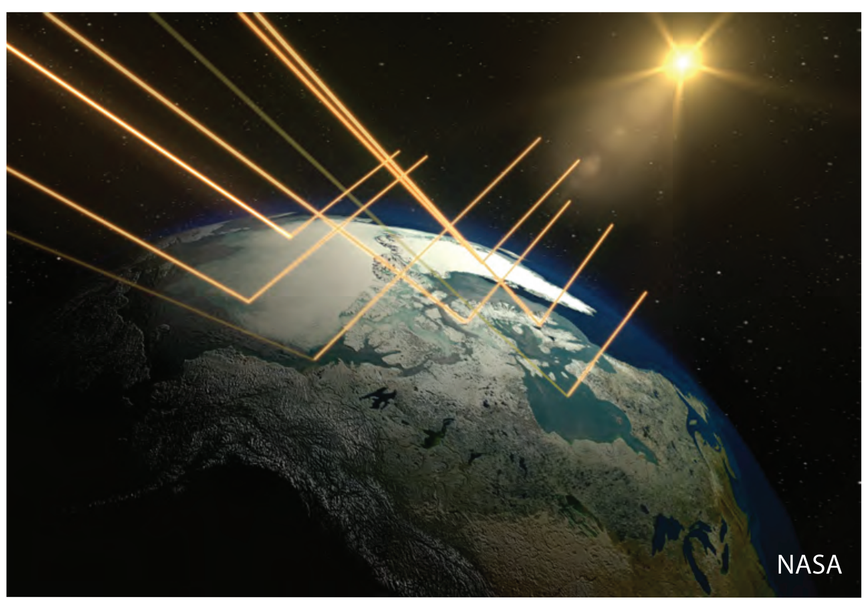
Sea ice covers 7-10% of Earth's ocean surface, and functions as the boundary between the atmosphere and the ocean, regulating the exchange of fluids, heat, and carbon dioxide.

Earth's refrigerator

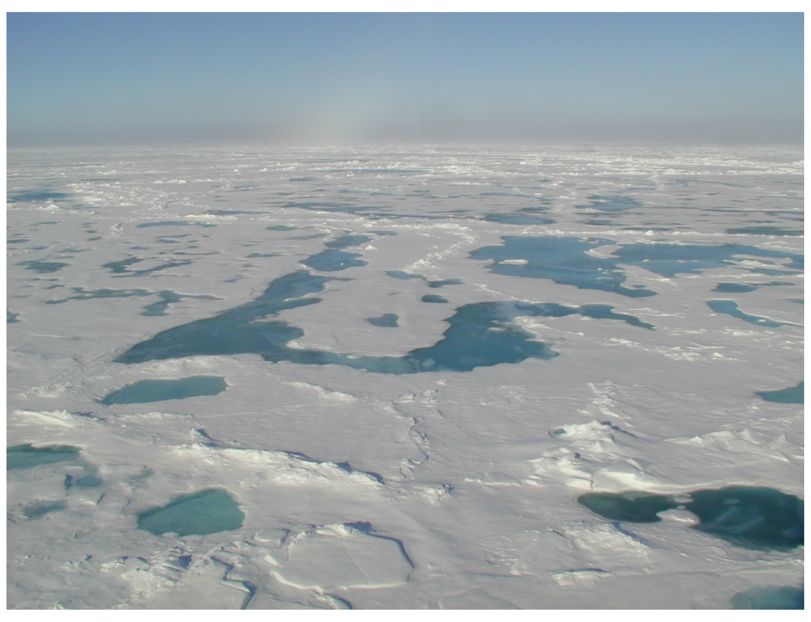
White snow and ice reflect sunlight.
Dark water and land absorb sunlight.

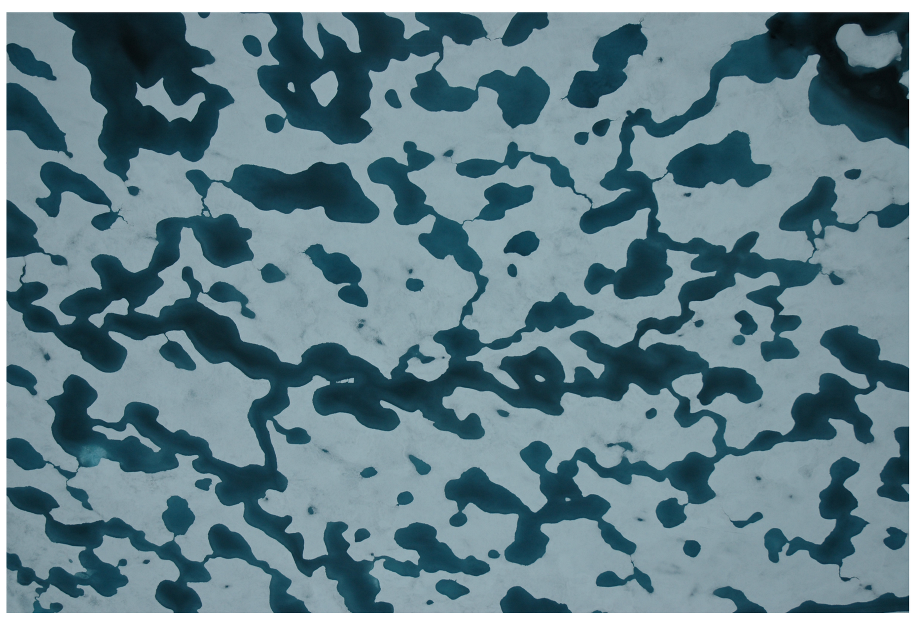
$$\textit{albedo} = \frac{\text{reflected radiation}}{\text{incident radiation}}$$

ice packs reflect solar radiation, shielding the ocean from solar heat absorption

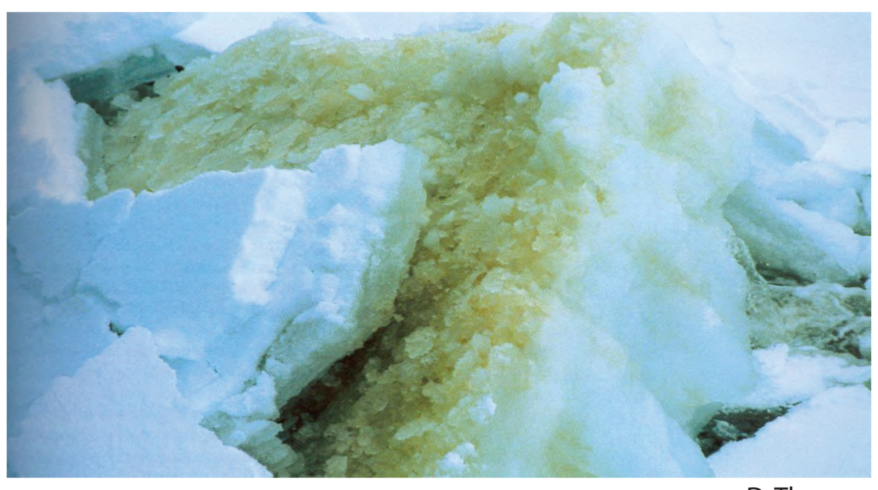


One of the big challenges in improving climate models is parameterizing ice pack albedo, which is determined by the evolution of melt ponds and ice floe configurations.





Sea ice hosts extensive algal communities that support polar ecosystems.



D. Thomas

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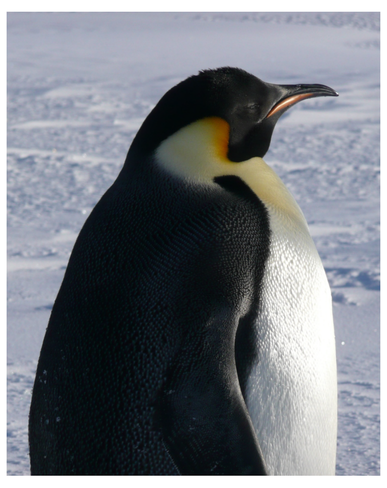
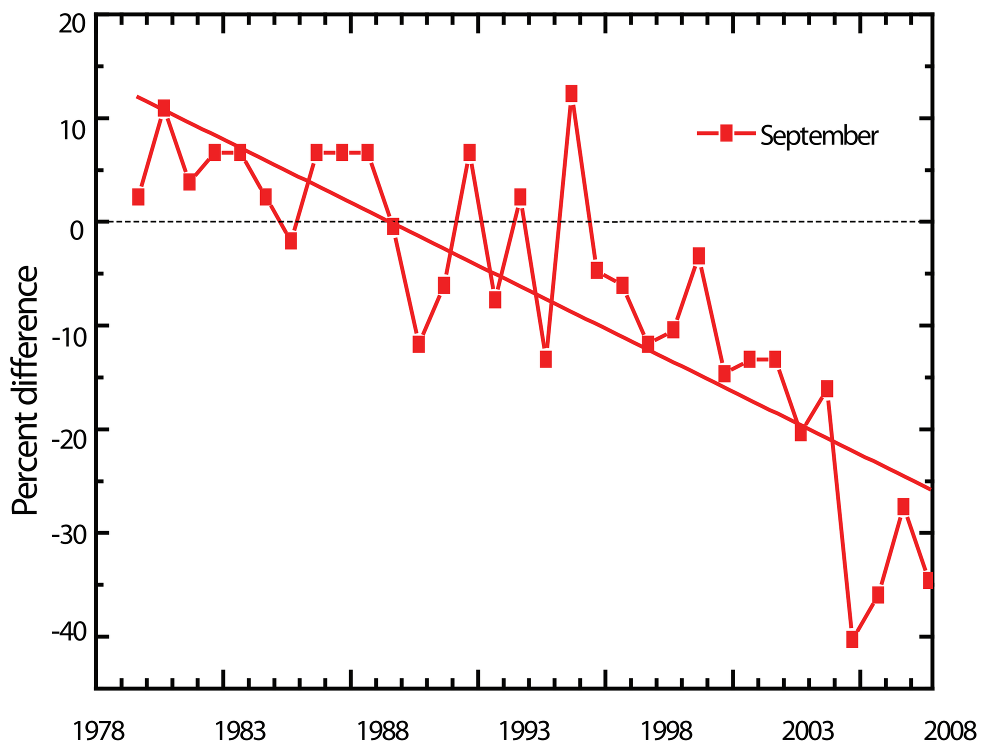


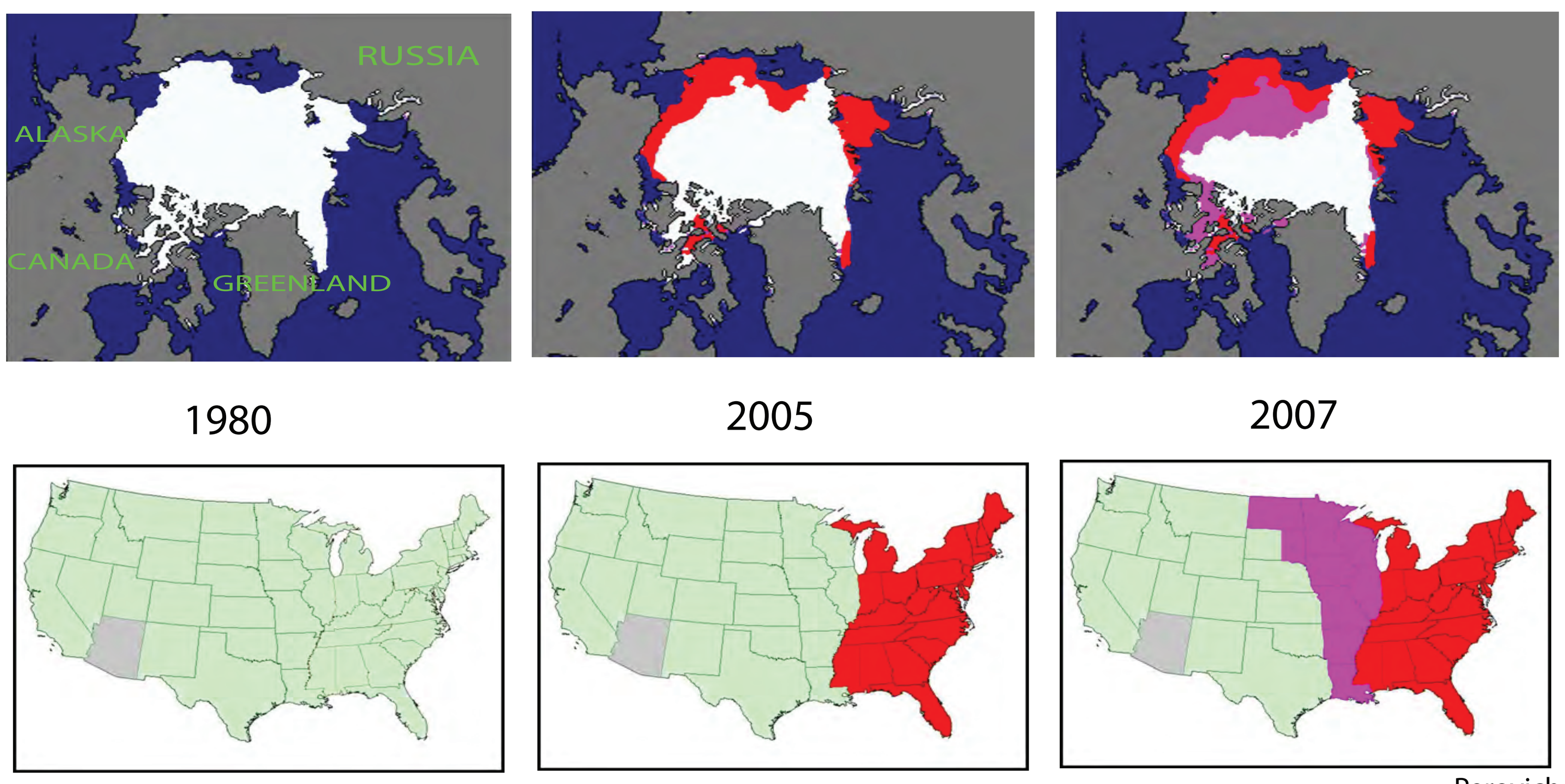
Arctic meltdown

Summer Arctic sea ice pack is declining, and thicker multiyear ice is being replaced by thinner first year ice.

Change in summer Arctic sea ice extent



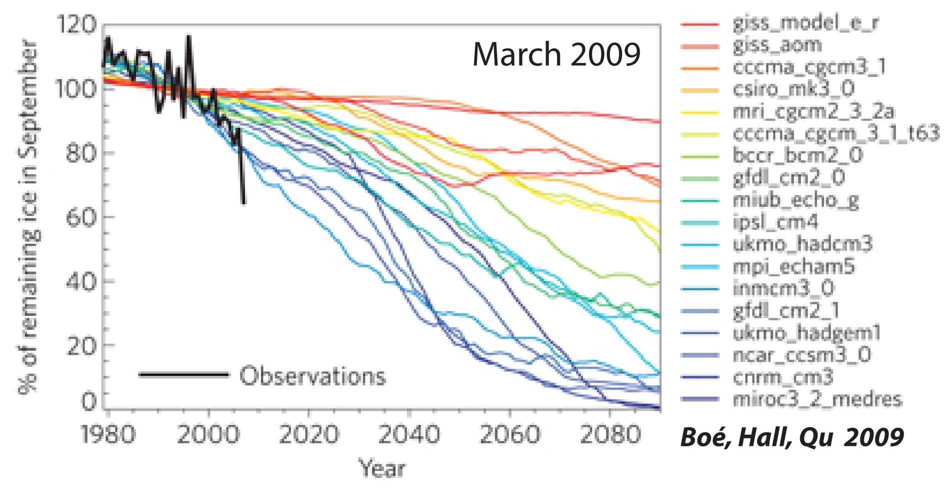
Perovich



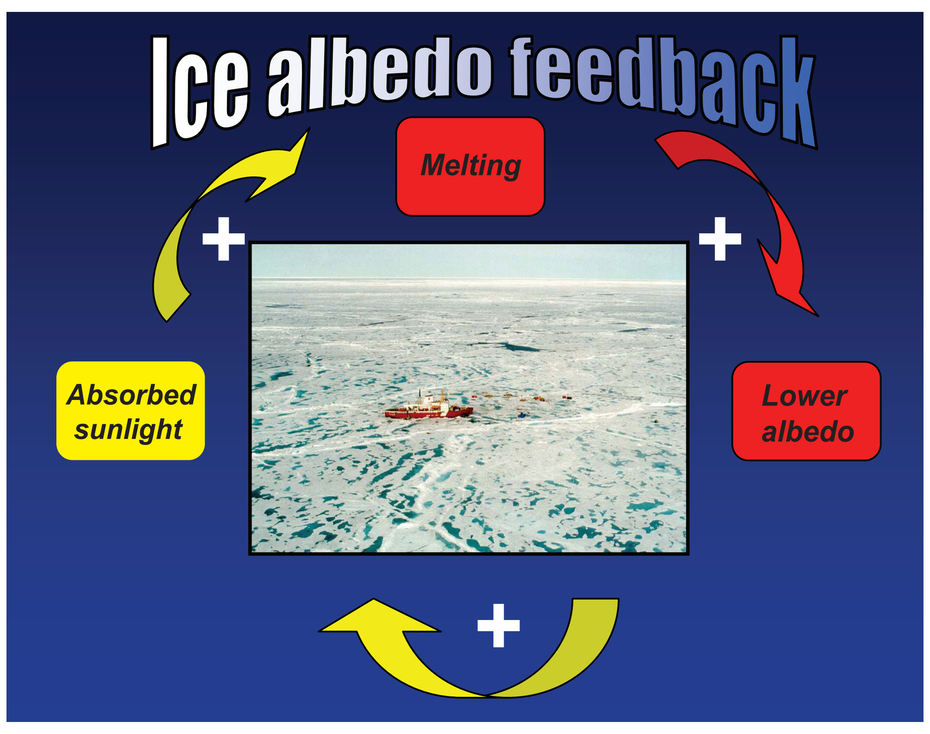
Perovich

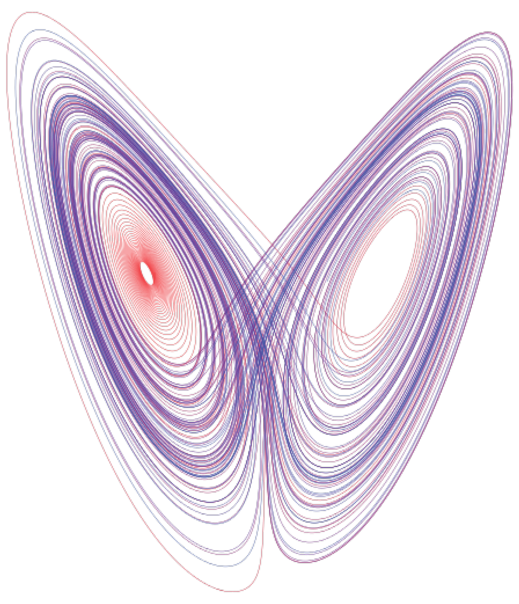
global climate models underestimate observed decline in summer Arctic sea ice extent

IPCC (Intergovernmental Panel on Climate Change) projections



Boé, Hall, Qu 2009



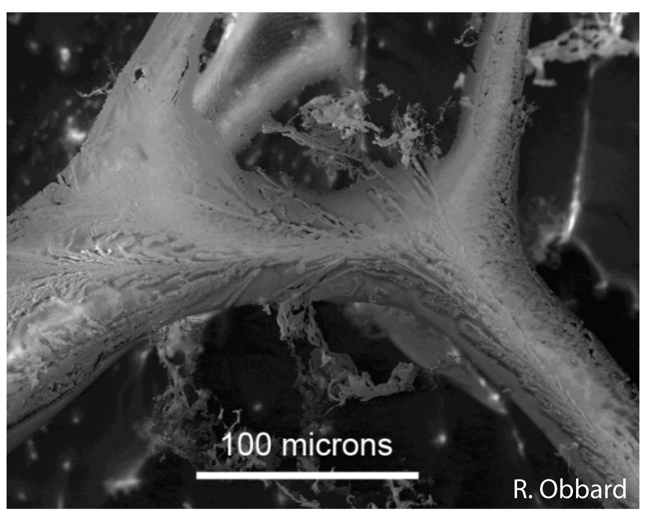


Have we passed a "tipping point" in a transition to an ice-free Arctic summer?

Sea ice is a composite material

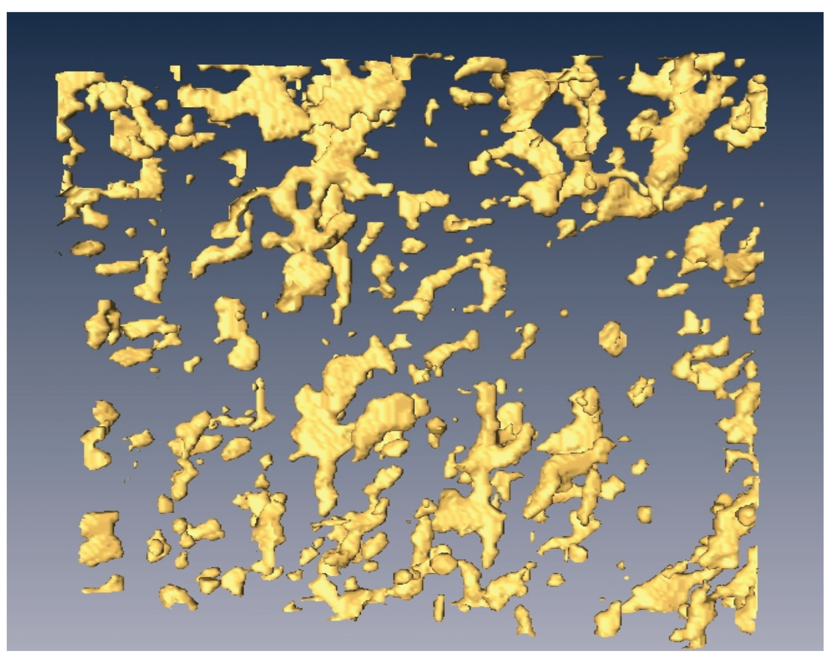
pure ice with brine inclusions

sub-millimeter scale fluid inclusions

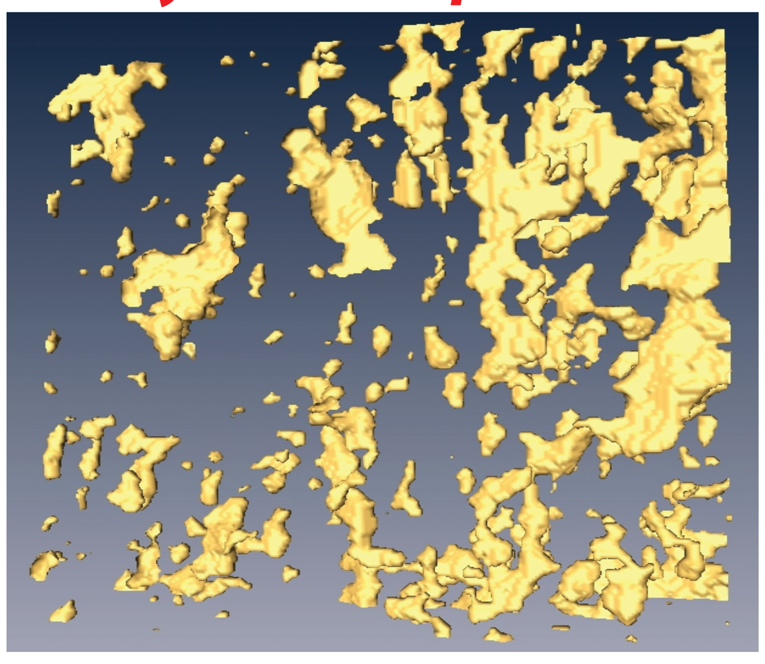


fluid can flow through connected brine channels

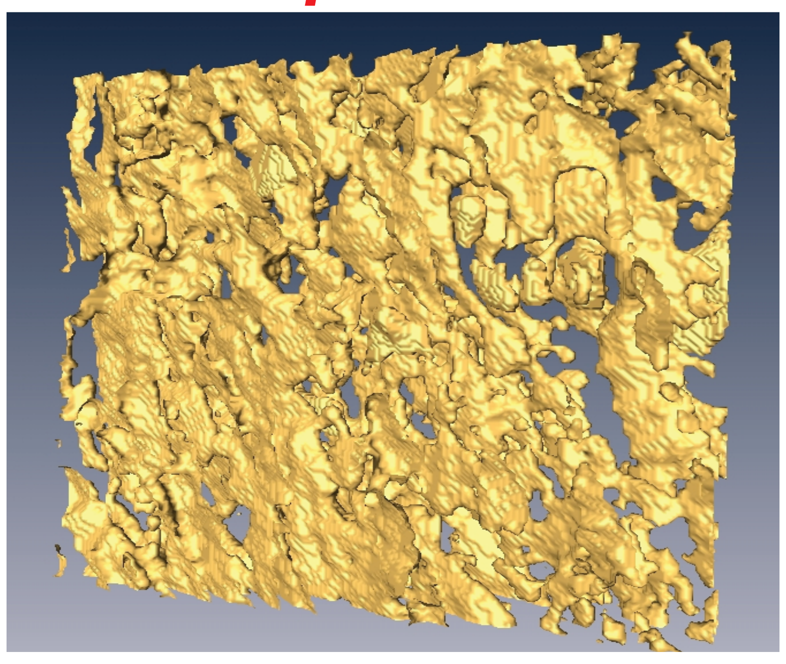
volume fraction and connectivity of brine phase increase as temperature rises



T = -15 °C, $\phi = 0.033$



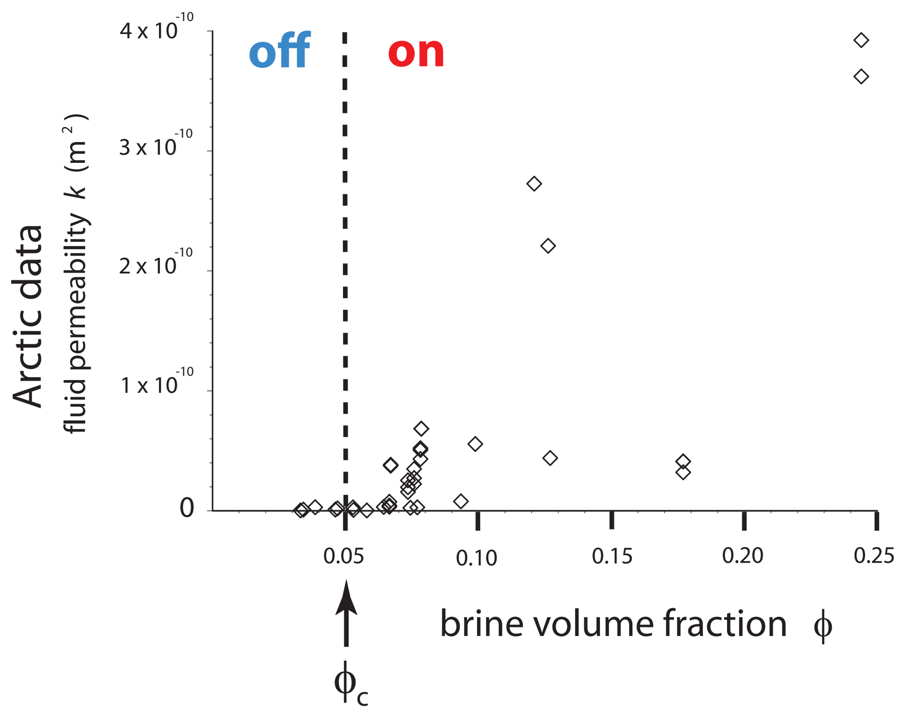
T = -6 °C, $\phi = 0.075$



T = -3 °C, $\phi = 0.143$

X-ray computed tomography

Golden et al., *Geophys. Res. Lett.*, 2007
Pringle et al., *J. Geophys. Res.* 2009



Critical behavior of fluid transport in sea ice

small changes in system parameters can induce large changes in behavior

on - off switch for fluid flow

RULE OF FIVES

Golden et al., *Science* 1998

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ\mathrm{C},\ S \approx 5$ ppt

on - off switch controls drainage and pooling of melt ponds, evolution of salinity profiles, snow-ice formation, nutrient replenishment for algae

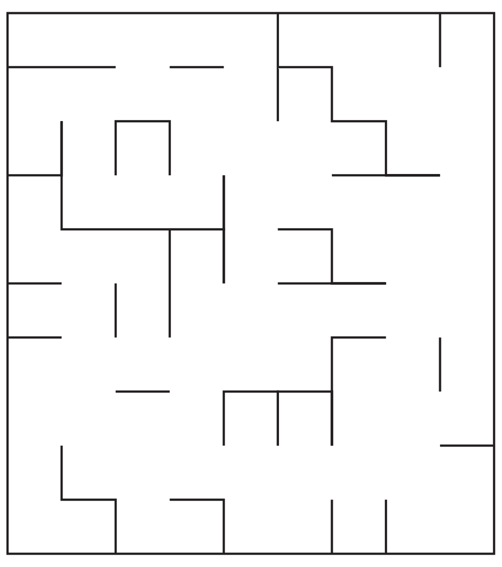
Goal: develop electromagnetic methods for monitoring fluid flow processes and the evolution of permeability and microstructure, to improve climate models

Modeling fluid and electrical properties of sea ice

percolation theory

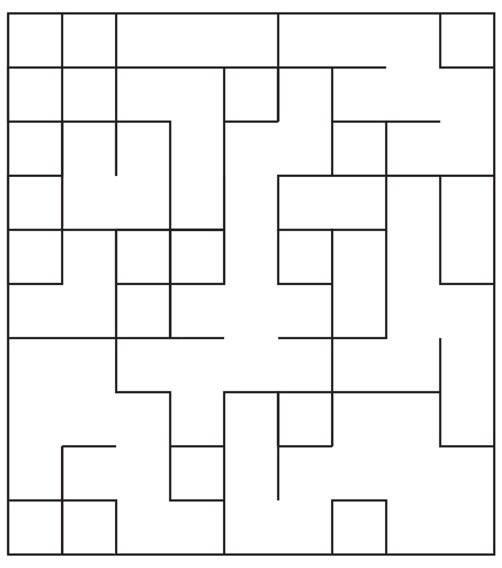
mathematical theory of connectedness

impermeable

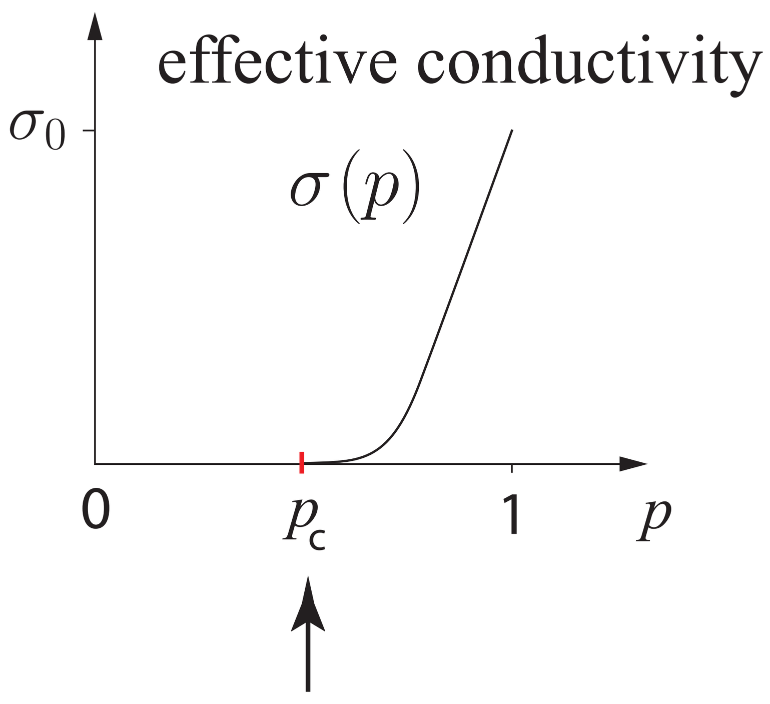


$p = 1/3$

permeable



$p = 2/3$



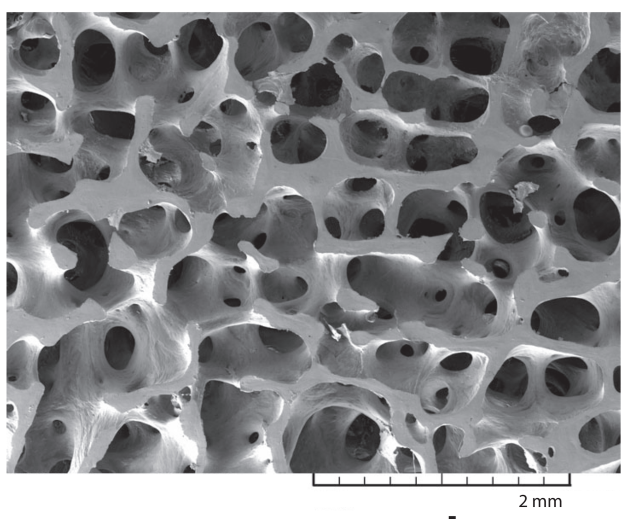
percolation threshold
$p_c = 1/2$ for $d = 2$

remote sensing

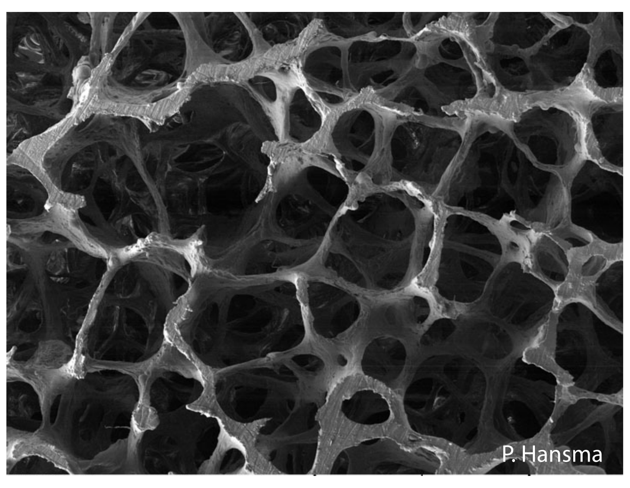
Sea ice is similar to biological and high-tech composites

use models of other composites to learn about sea ice, and vice versa

human bone

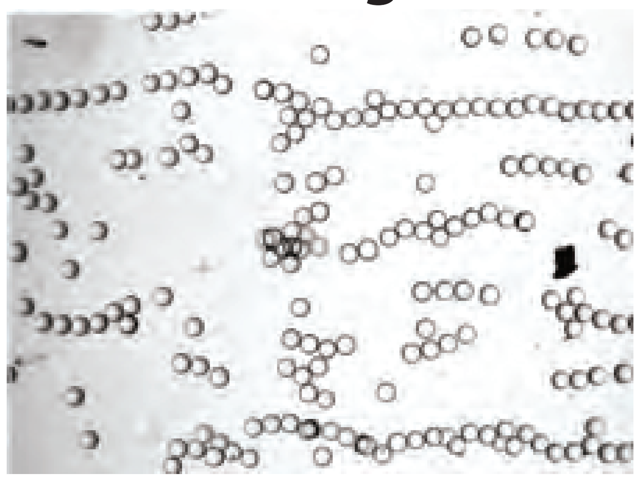


normal



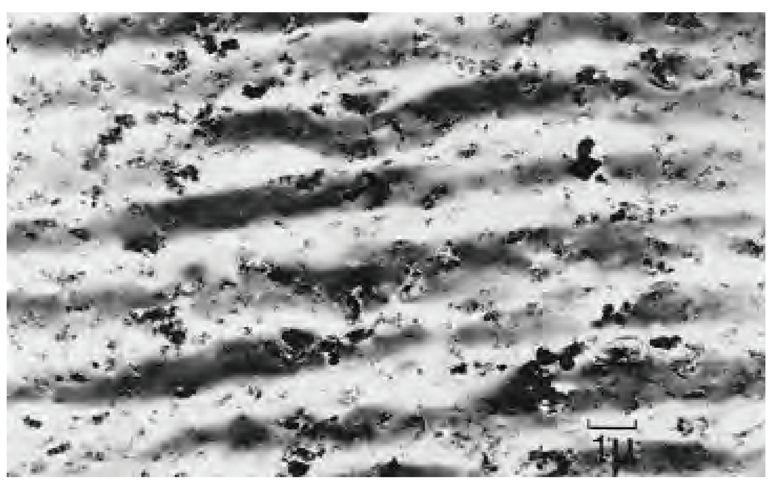
osteoporotic

electrorheological fluid



car industry, prosthetics

radar absorbing coatings

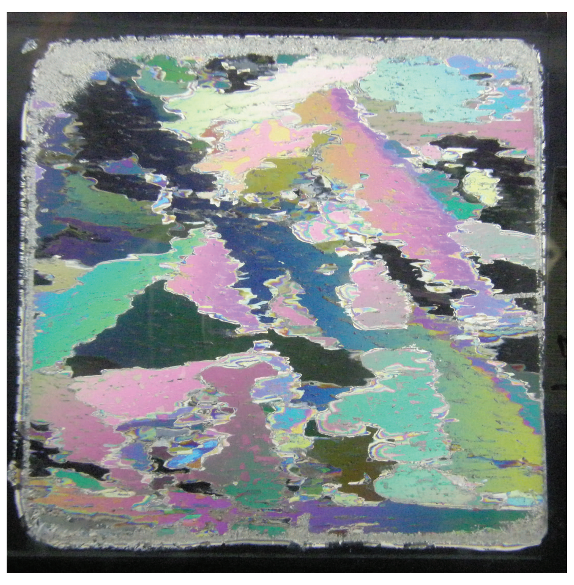
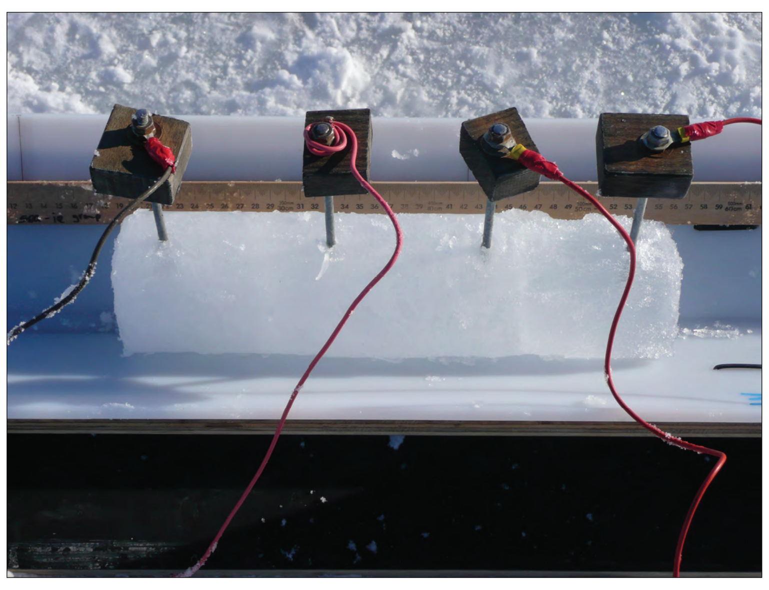


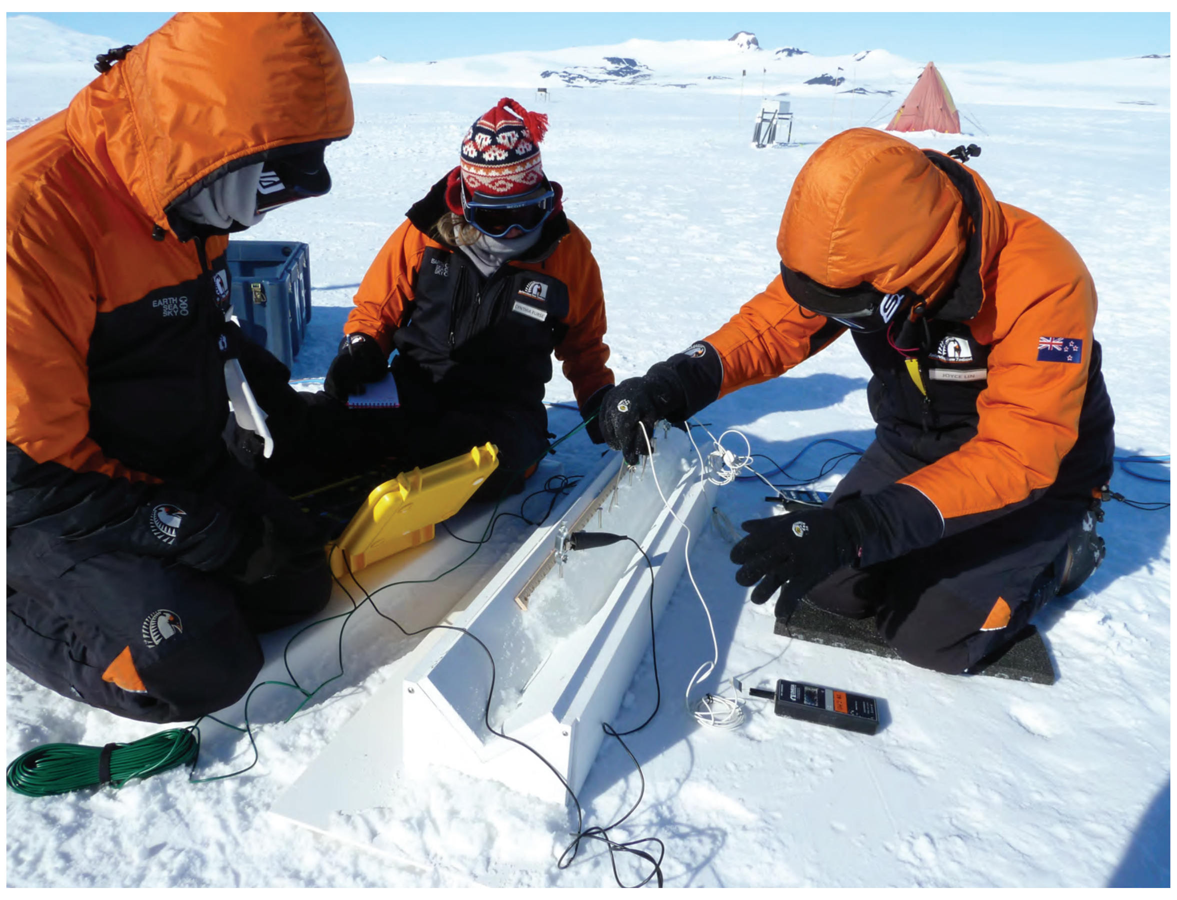
stealthy aircraft

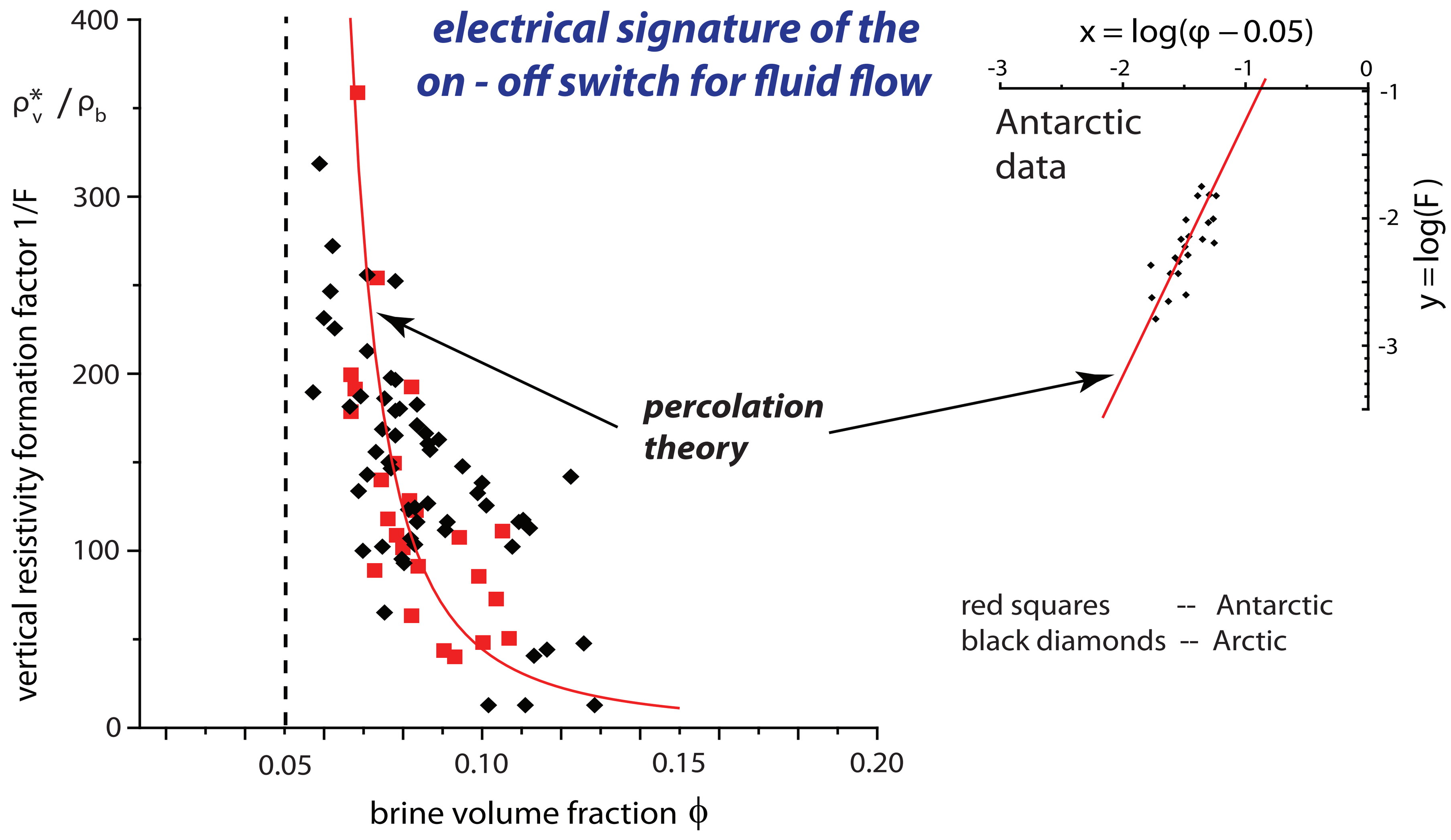
the math doesn't care if it's sea ice or bone!

Field measurements of Arctic and Antarctic sea ice







Findings and Conclusions

1. Brine flow through sea ice is a key to the geophysics and biology of polar regions.

2. Developed comprehensive theories of fluid permeability and electrical conductivity, using models from statistical physics and homogenization. Developed theory of enhanced thermal conductivity of sea ice in the presence of brine flow.

3. Developed methods to recover microstructural properties of sea ice from EM data.

4. Applied methods developed for sea ice to help monitor osteoporosis in bone.

5. Measured sea ice transport properties in the Arctic and Antarctic. Made the first permeability measurements in Antarctic pack ice.

6. Discovered that the boundaries of melt ponds exhibit a transition to complex space-filling curves with fractal dimension 2 above a critical length scale.

7. Sea ice processes such as melt pond evolution, snow-ice formation, nutrient fluxes can be modeled more realistically, and monitored electromagnetically. Results can help to predict how climate change may affect sea ice packs and polar ecosystems.

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