

On Thinning Ice: Modeling Sea Ice in a Warming Climate

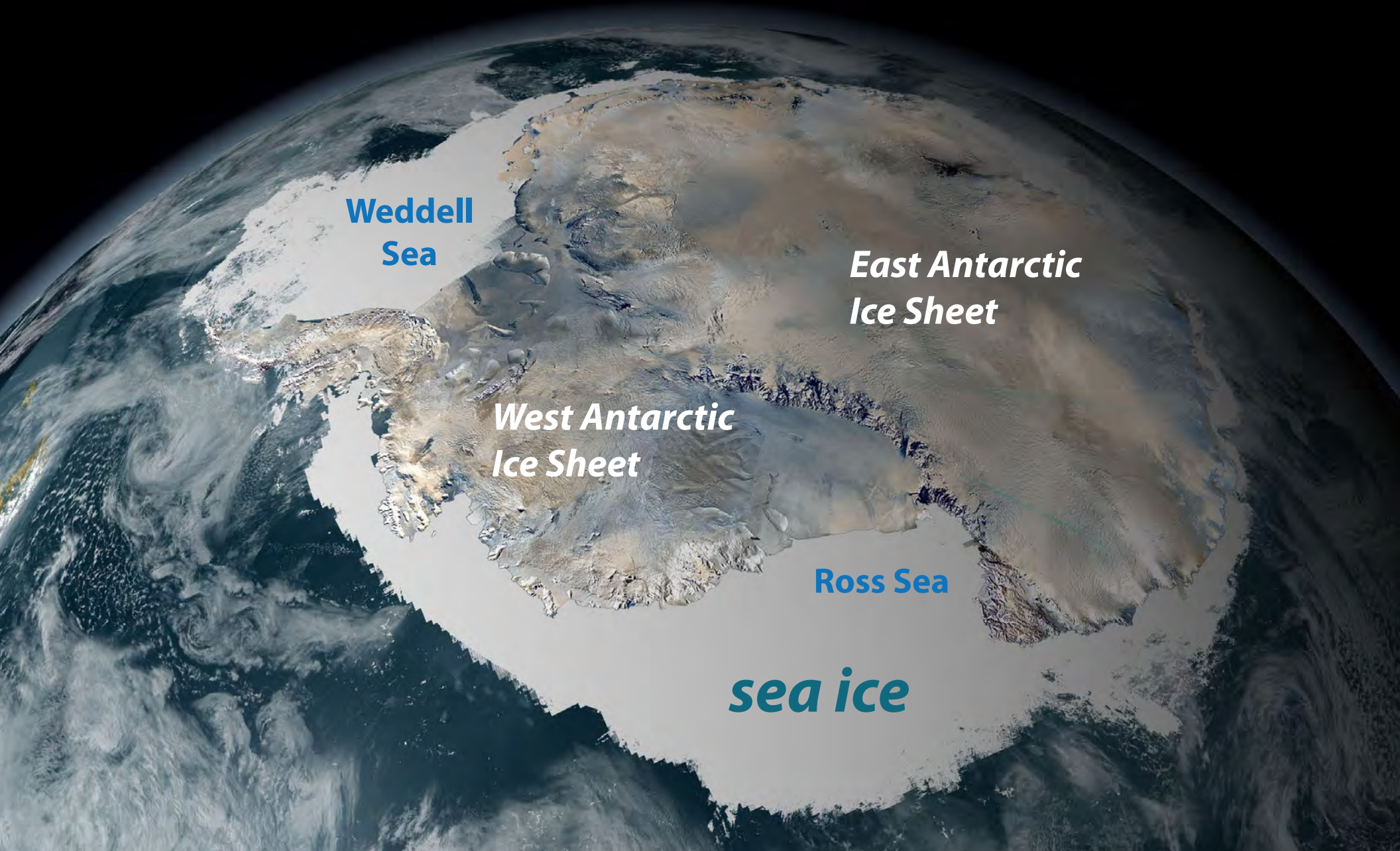
Kenneth M. Golden
Department of Mathematics, University of Utah



Math 2210 Fall 2021
December 8, 2021

ANTARCTICA

southern cryosphere



**Weddell
Sea**

***East Antarctic
Ice Sheet***

***West Antarctic
Ice Sheet***

Ross Sea

sea ice

SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**



polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice
reflect

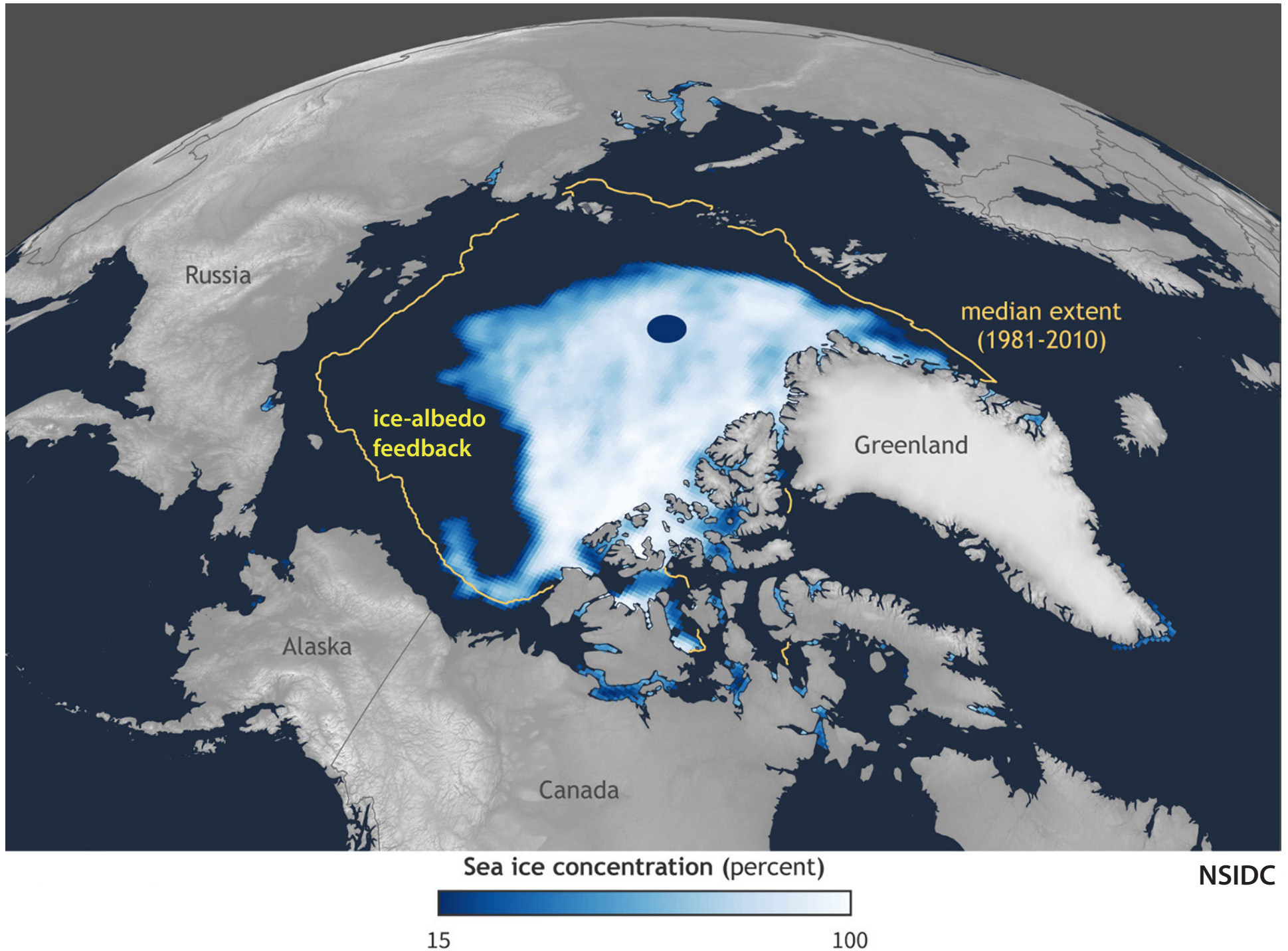


dark water and land
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Arctic sea ice extent

September 15, 2020



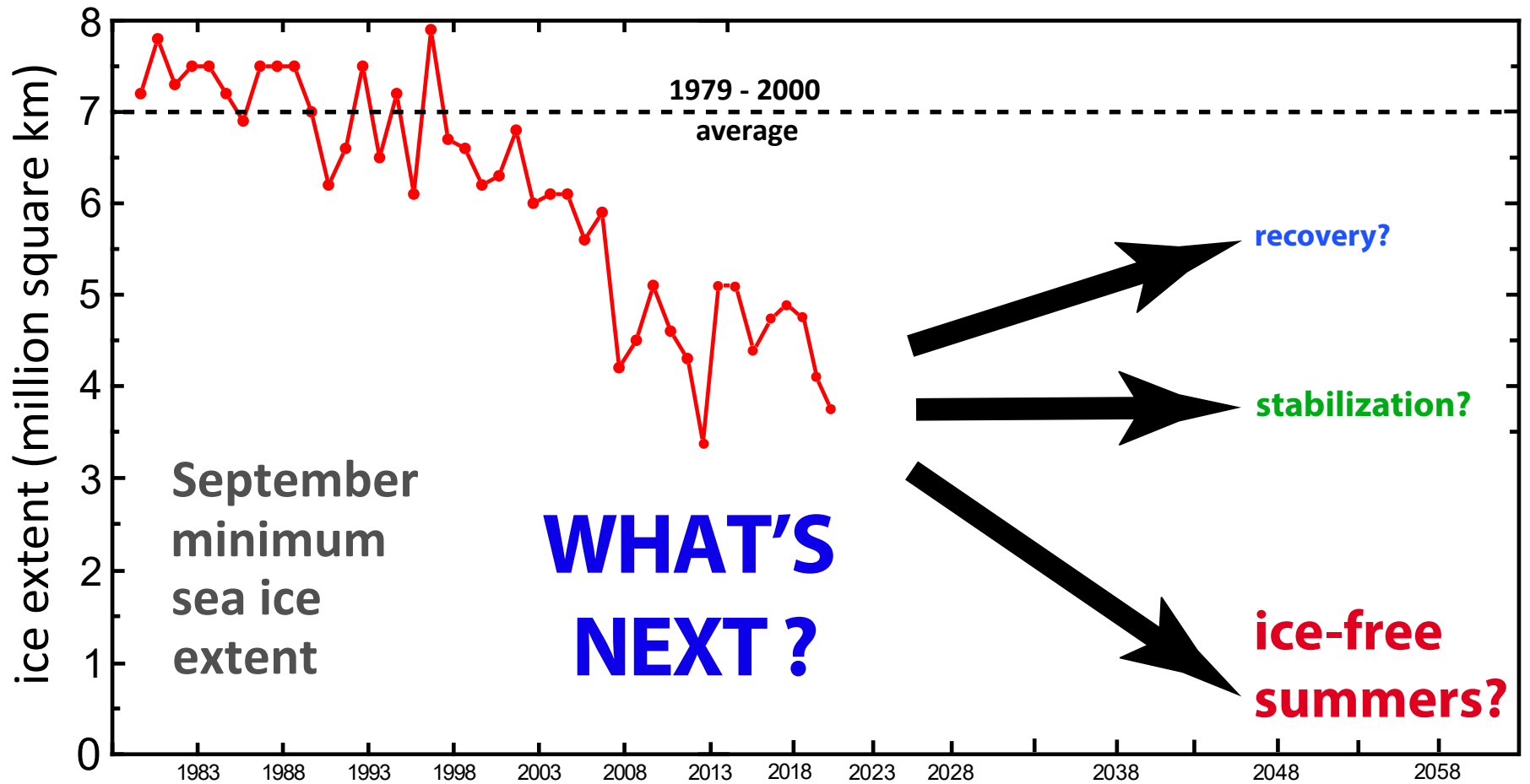


*recent losses
in comparison to
the United States*

Perovich



Predicting what may come next requires lots of math modeling.



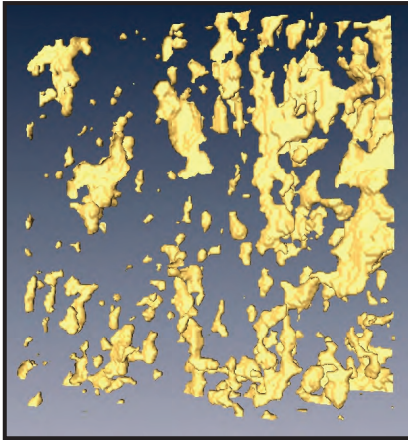
Sea Ice is a Multiscale Composite Material

microscale

brine inclusions

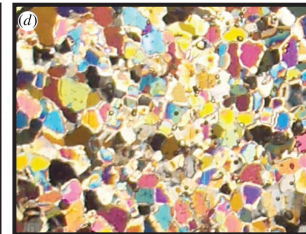
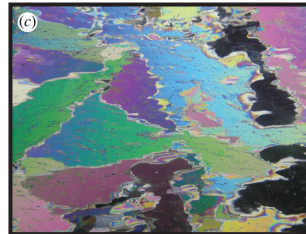
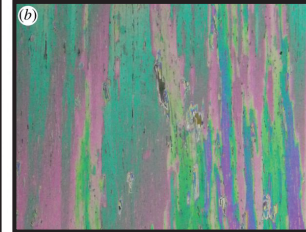


Weeks & Assur 1969



H. Eicken
Golden et al. GRL 2007

polycrystals

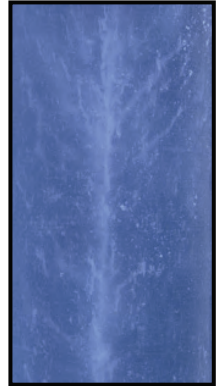


Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole



K. Golden

millimeters

centimeters

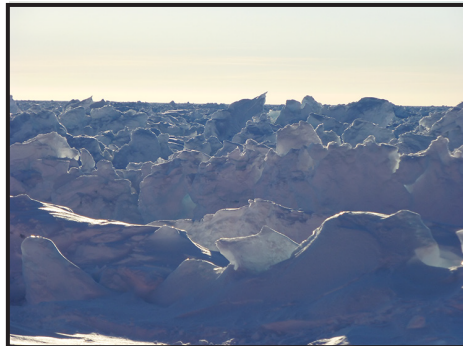
mesoscale

Arctic melt ponds



K. Frey

Antarctic pressure ridges



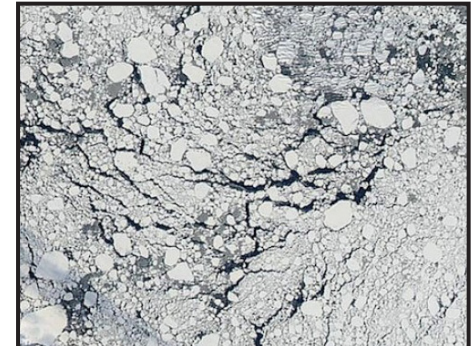
K. Golden

sea ice floes



J. Weller

sea ice pack



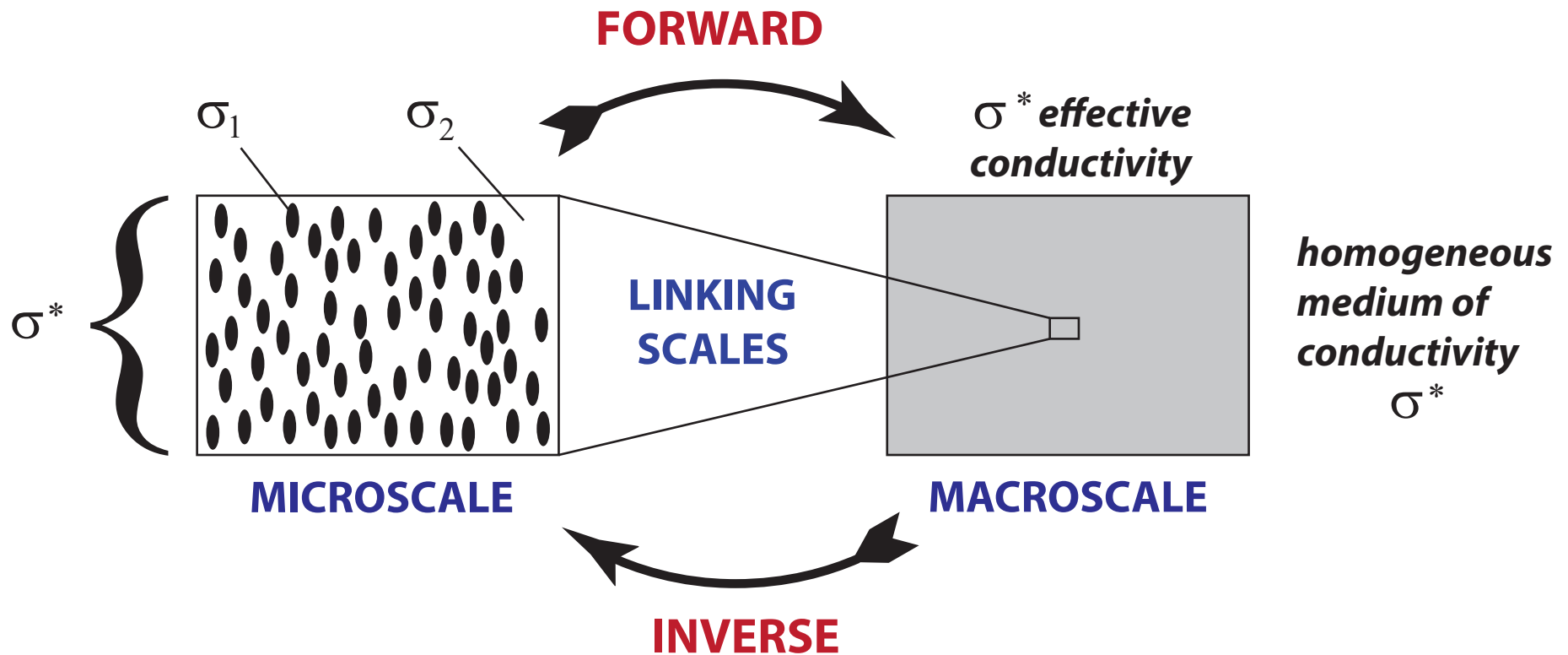
NASA

meters

kilometers

macroscale

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

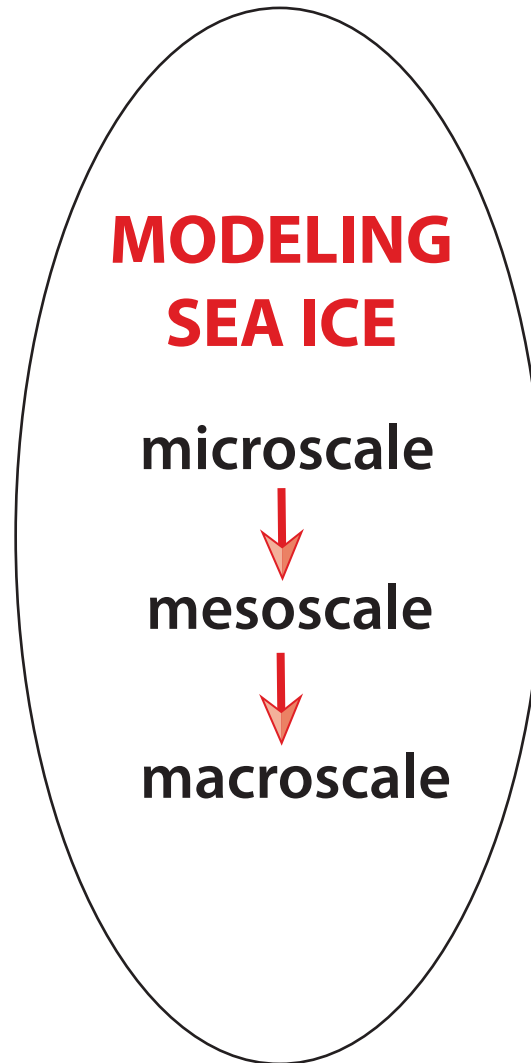
*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...



A tour of key sea ice processes on micro, meso, and macro scales.

What is our research about?

Using methods of **homogenization and statistical physics** to model sea ice effective behavior and advance representation of sea ice in climate models, process studies, ...

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
stealth technology

porous media,
oil extraction

statistical mechanics
of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations



MODELING SEA ICE

microscale

mesoscale

macroscale

Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

composites,
polycrystals

remote sensing

advection diffusion

biomedical imaging,
biomaterials, EPS

polar microbial ecology



What is our research about?

Inputs, Ingredients

COMPOSITE MATERIALS

electrical engineering,
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of ferromagnets

Anderson localization,
semiconductor physics

random matrix theory

differential equations

Cross pollination

magnets
radar absorbers
human bone
rat brains

**MODELING
SEA ICE**

Outputs, Impacts

CLIMATE MODELING

sea ice physics
& biology

composites,
polycrystals

remote sensing

advection diffusion

biomedical imaging,
biomaterials, EPS

polar microbial ecology

***Modeling sea ice drives advances in many
areas of science and engineering.***

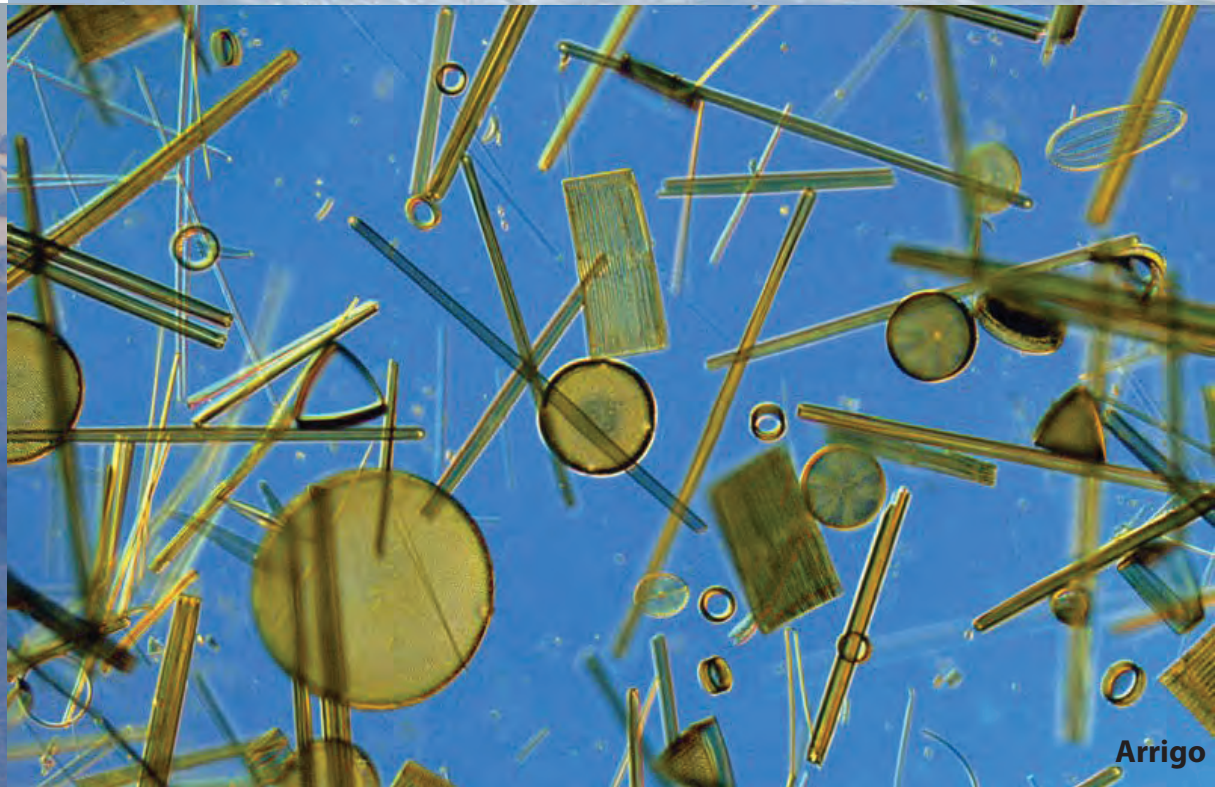
Microbial Ecology and the Physics of Sea Ice

How do the physical properties of sea ice affect microbial communities?

How does the presence of microbial life in sea ice affect its physical properties?



Golden



Arrigo

microscale

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



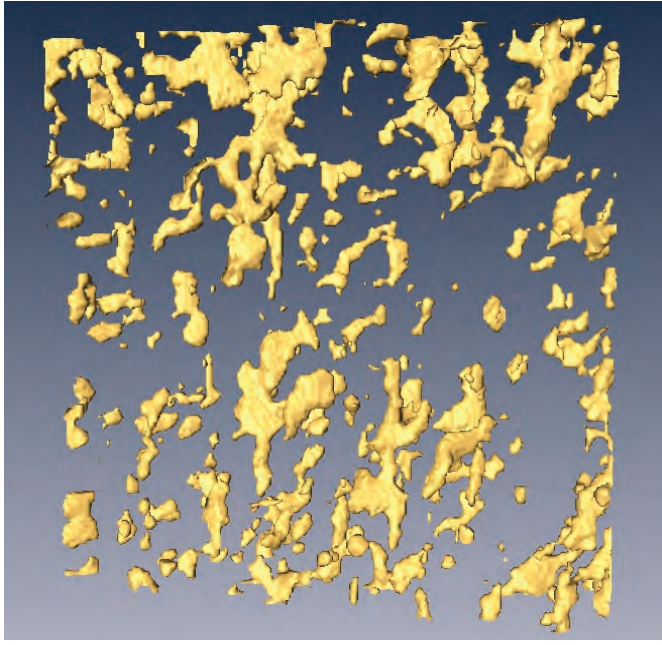
T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

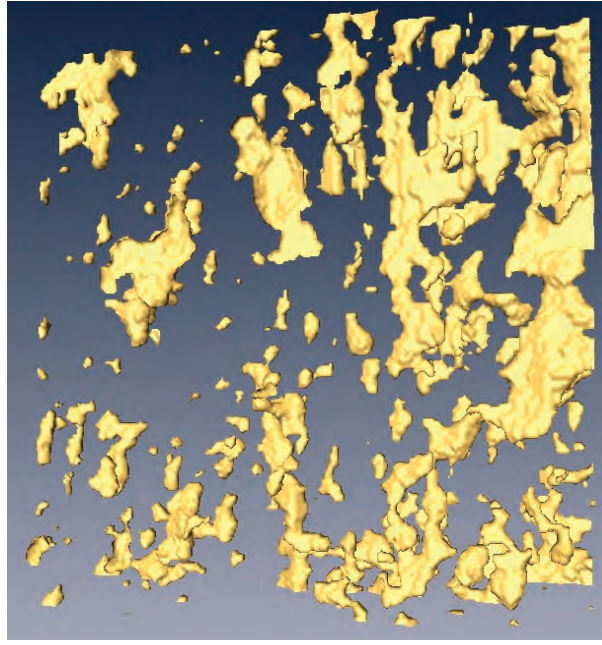
September
snow-ice
estimates

- evolution of salinity profiles
- ocean-ice-air exchanges of heat, CO_2

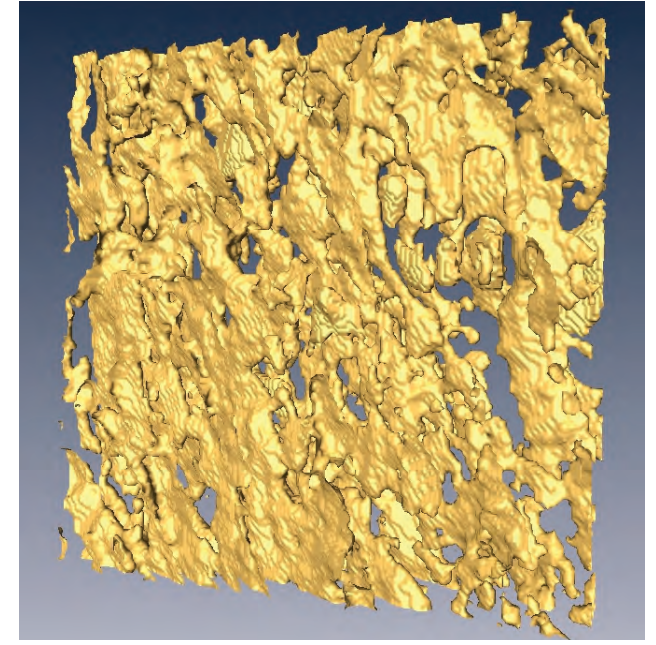
brine volume fraction and **connectivity** increase with temperature



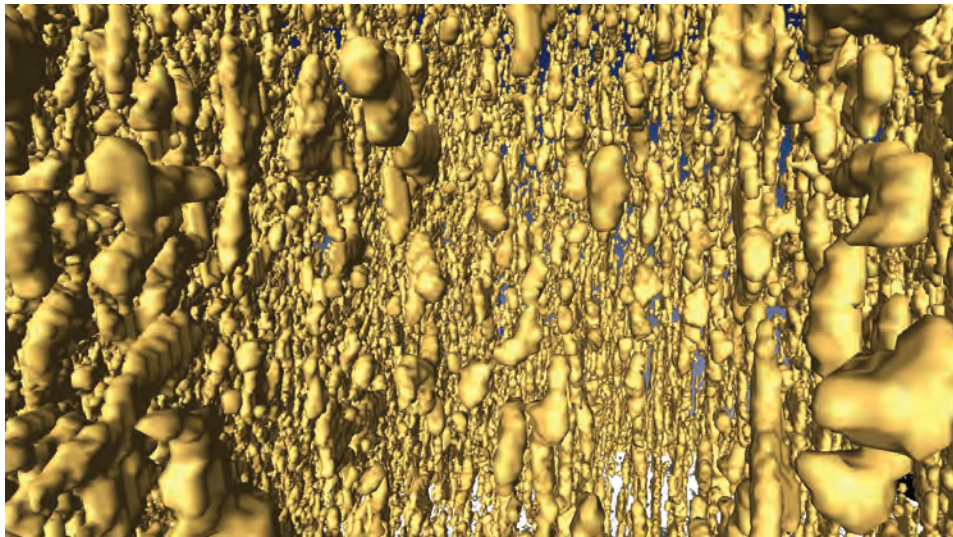
$T = -15\text{ }^{\circ}\text{C}$, $\phi = 0.033$



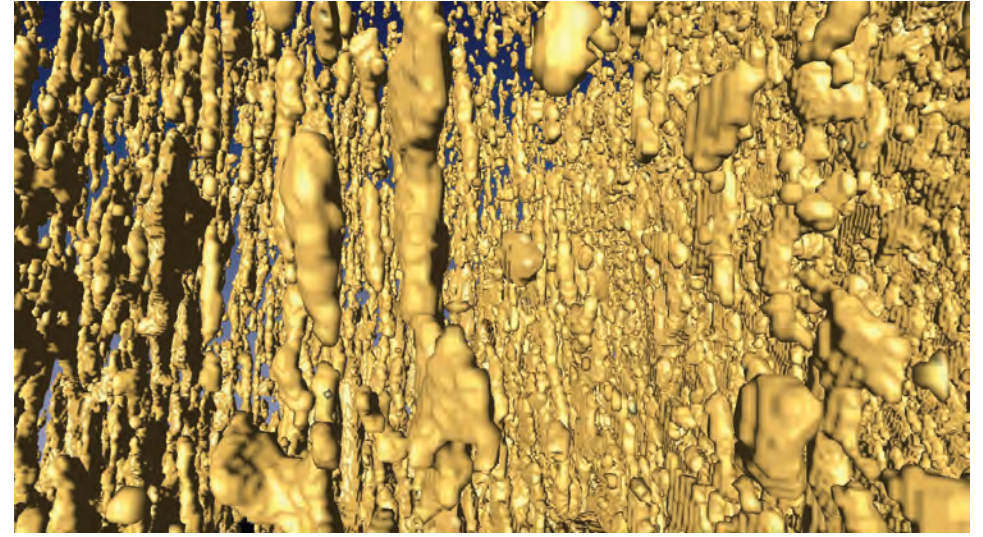
$T = -6\text{ }^{\circ}\text{C}$, $\phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}$, $\phi = 0.143$



$T = -8\text{ }^{\circ}\text{C}$, $\phi = 0.057$



$T = -4\text{ }^{\circ}\text{C}$, $\phi = 0.113$

X-ray tomography for brine in sea ice

Golden et al., *Geophysical Research Letters*, 2007

fluid permeability of a porous medium



how much water gets through the sample per unit time?

Darcy's Law

for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

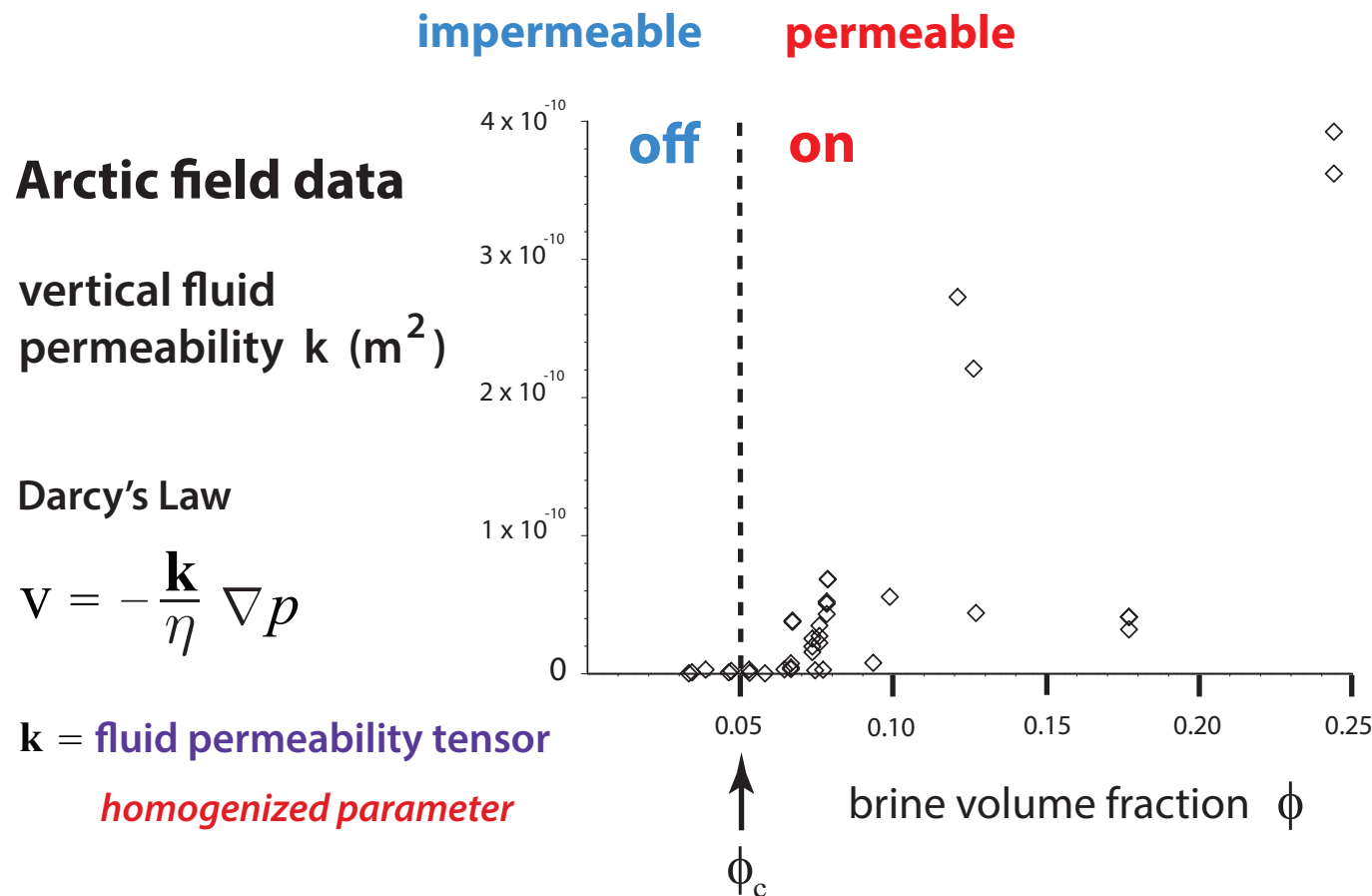
viscosity

\mathbf{k} = fluid permeability tensor

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Critical behavior of fluid transport in sea ice



***“on - off” switch
for fluid flow***

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

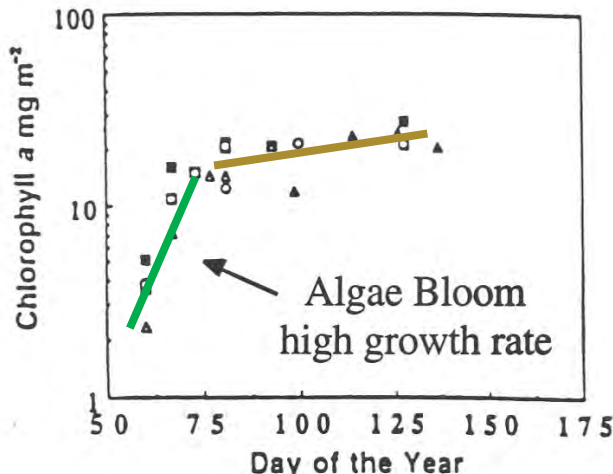
nutrient replenishment
controlled by ice permeability

biological activity turns on
or off according to
rule of fives

Golden, Ackley, Lytle Science 1998

Fritsen, Lytle, Ackley, Sullivan Science 1994

critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

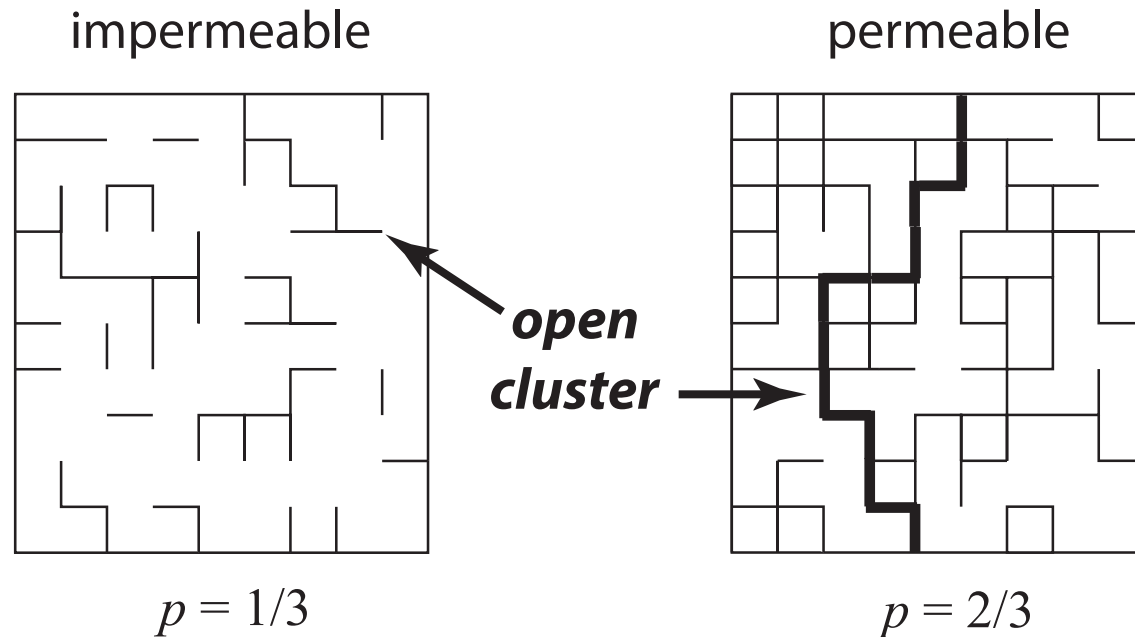
sea ice ecosystem



sea ice algae
support life in the polar oceans

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open* with probability p
closed with probability $1-p$

percolation threshold

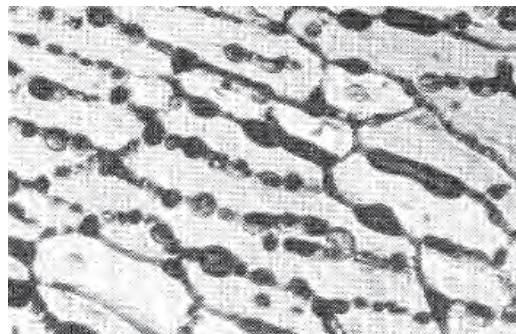
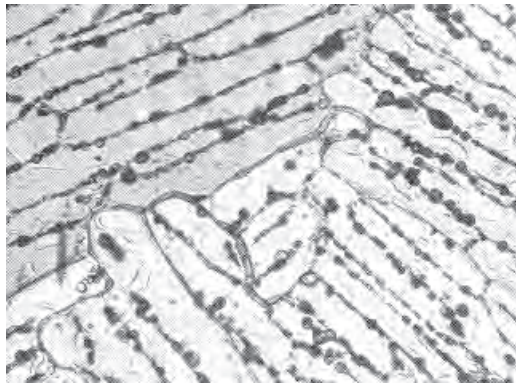
$$p_c = 1/2 \quad \text{for } d = 2$$

smallest p for which there is an infinite open cluster

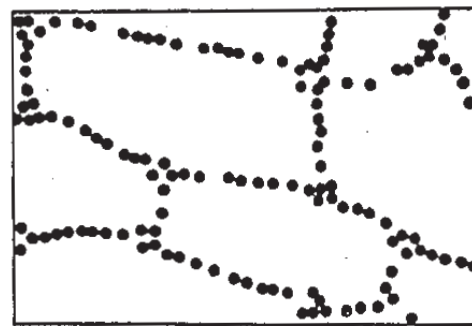
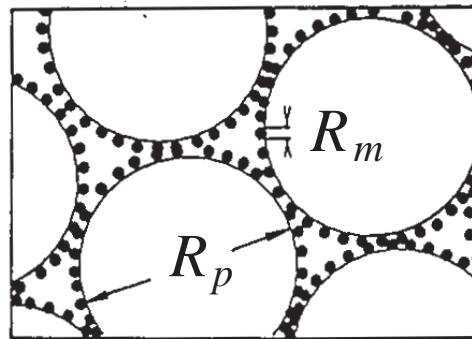
Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

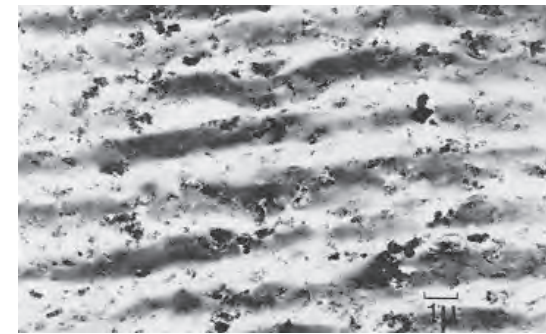
Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed
powder

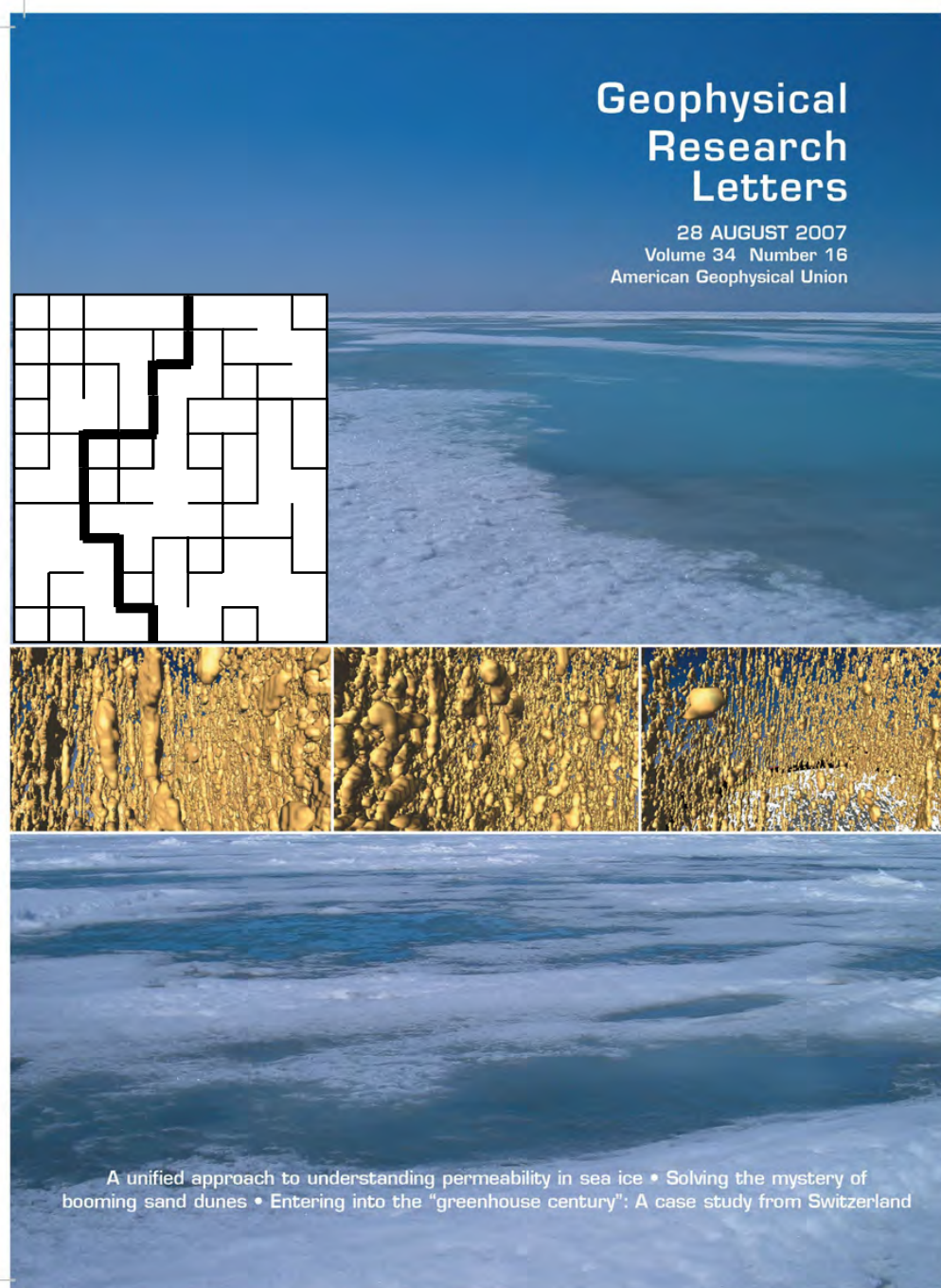


radar absorbing
composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



Geophysical
Research
Letters

28 AUGUST 2007
Volume 34 Number 16
American Geophysical Union

**percolation theory
for fluid permeability**

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical
exponent
t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

**critical path analysis
Kozlov and Golden, 1990s**

**hierarchical + network models
rigorous bounds**

**X-ray tomography for
brine inclusions**

confirms rule of fives

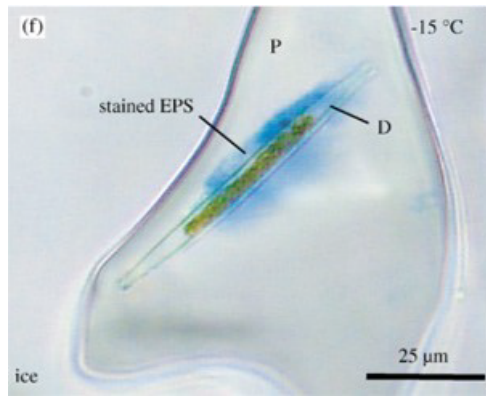
***Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009***

**theories agree closely
with field data**

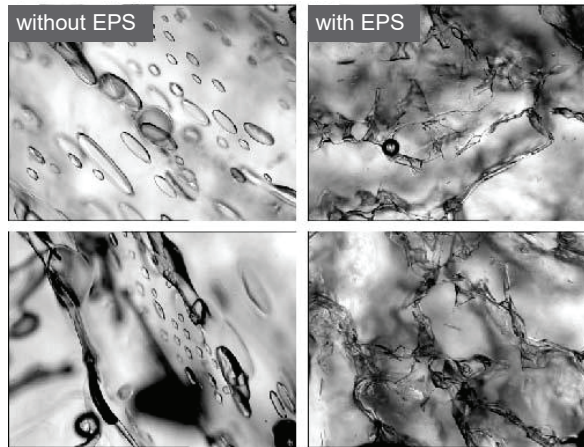
A unified approach to understanding permeability in sea ice • Solving the mystery of booming sand dunes • Entering into the "greenhouse century": A case study from Switzerland

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

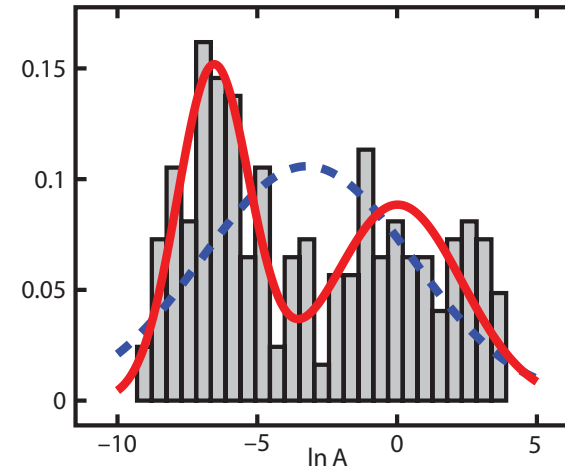
How does EPS affect fluid transport? How does the biology affect the physics?



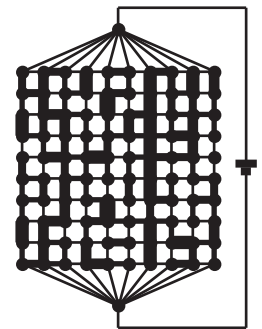
Krembs



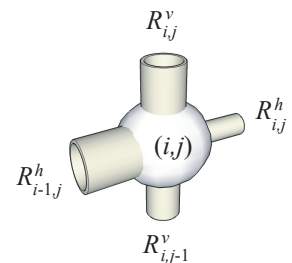
Krembs, Eicken, Deming, PNAS 2011



**RANDOM
PIPE
MODEL**



- 2D random pipe model with bimodal distribution of pipe radii
- Rigorous bound on permeability k ; results predict observed drop in k



Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice

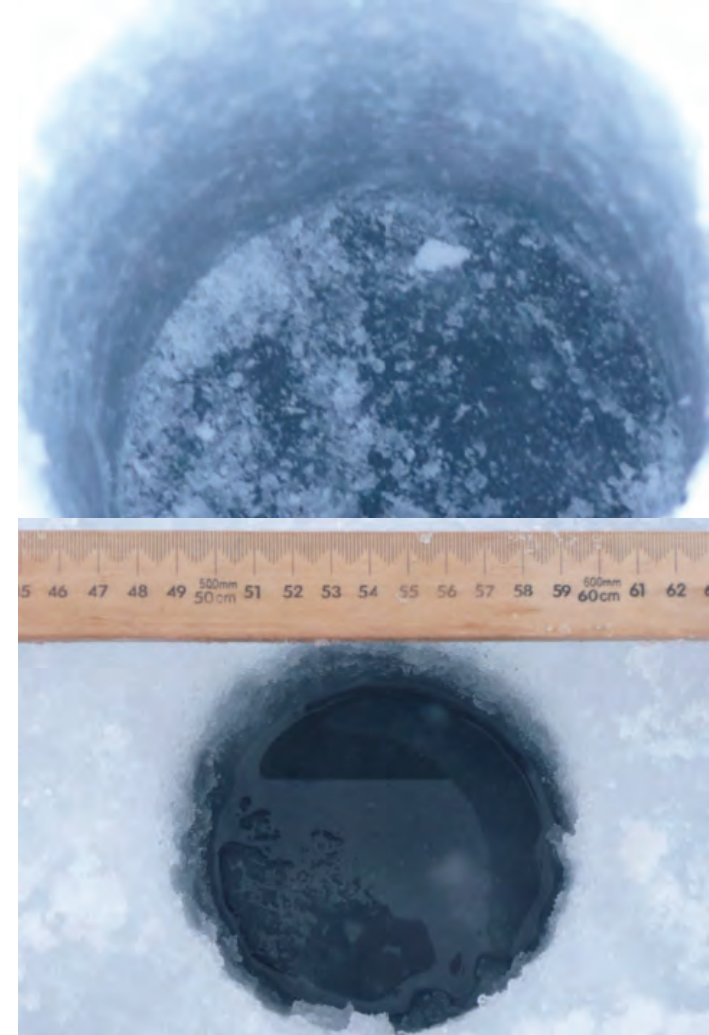
page 562

Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring
fluid permeability
of Antarctic sea ice***

SIPEX 2007

Remote sensing of sea ice



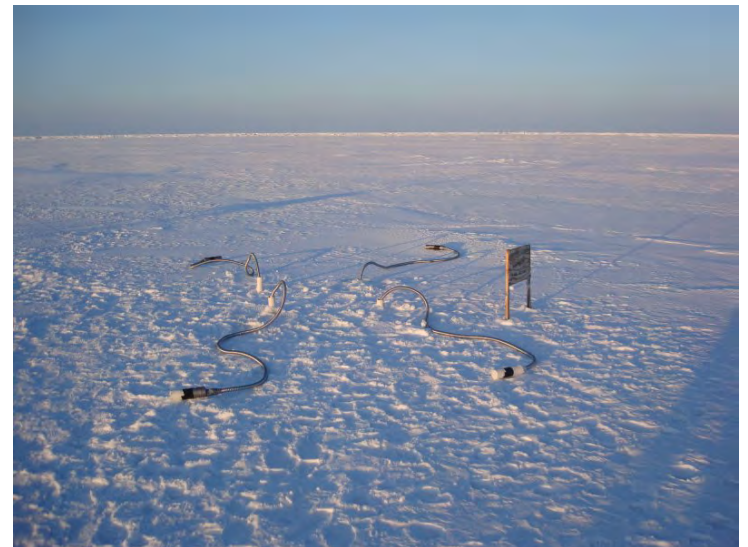
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

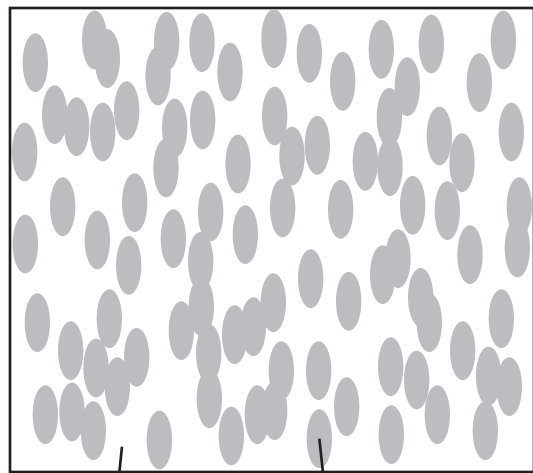
$$\epsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



ϵ_1

ϵ_2



ϵ^*

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1, p_2 = volume fractions of
the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2}, \text{ composite geometry} \right)$$

**What are the effective propagation characteristics
of an EM wave (radar, microwaves) in the medium?**

Analytic Continuation Method for Homogenization

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

separates geometry from parameters

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

← geometry

← material parameters

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

μ

- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n -point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla.$$

χ = characteristic function of the brine phase

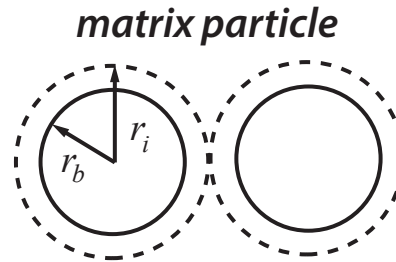
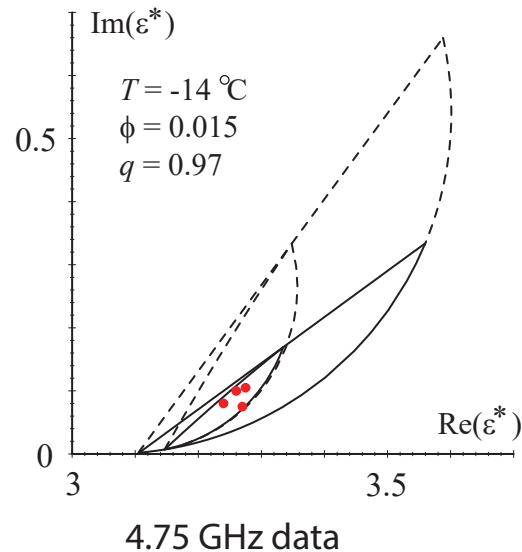
$$E = s (s + \Gamma\chi)^{-1} e_k$$

$\Gamma\chi$: microscale \rightarrow macroscale

$\Gamma\chi$ *links scales*

forward and inverse bounds on the complex permittivity of sea ice

forward bounds

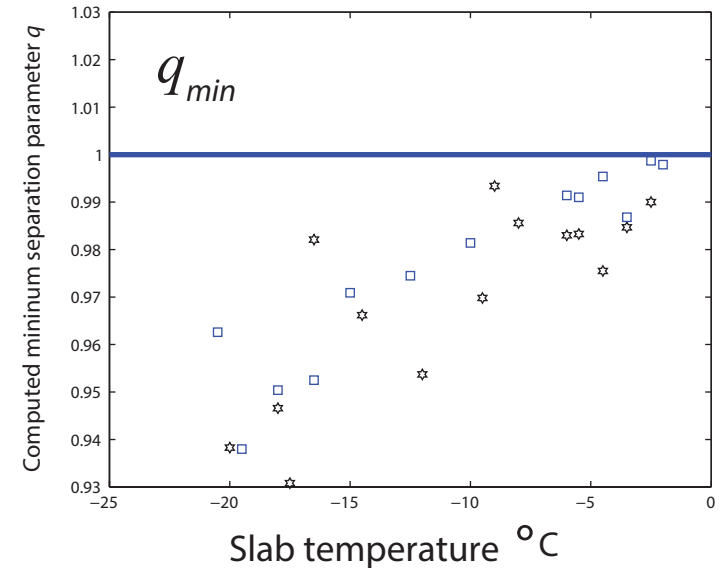


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Inverse Homogenization

Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001), McPhedran, McKenzie, Milton (1982), *Theory of Composites*, Milton (2002)

ϵ^* \longrightarrow composite geometry
(spectral measure μ)

inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ϵ^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p, q) -space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

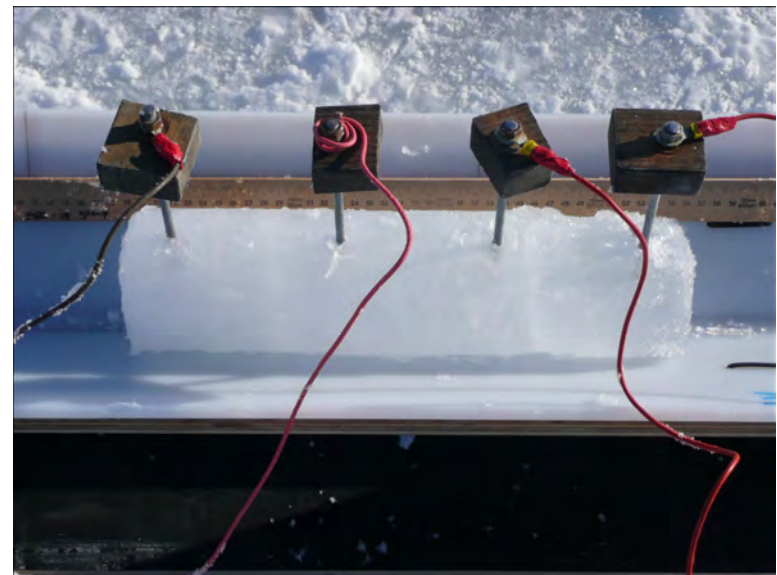
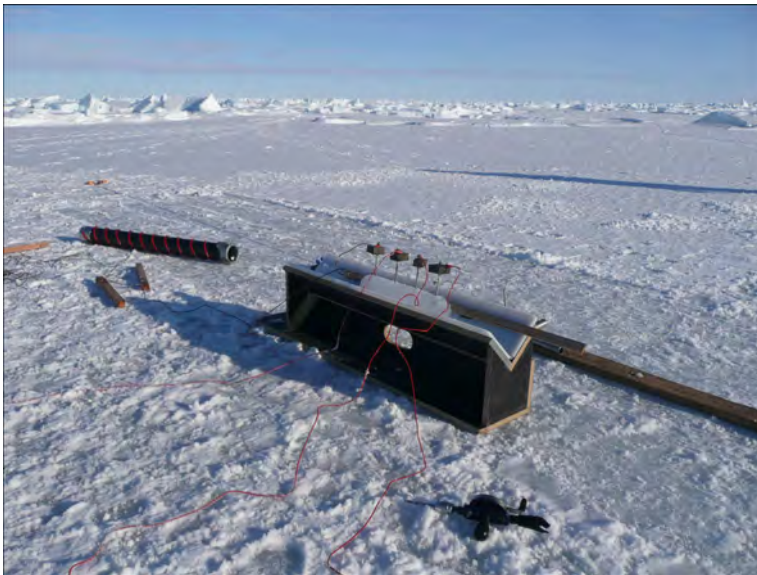
Measuring sea ice thickness



electrical measurements



Wenner array

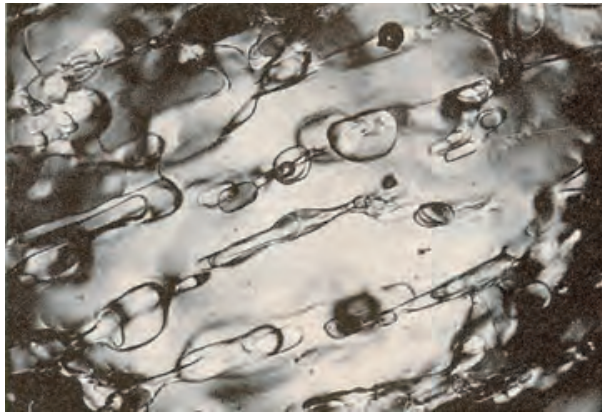


vertical conductivity

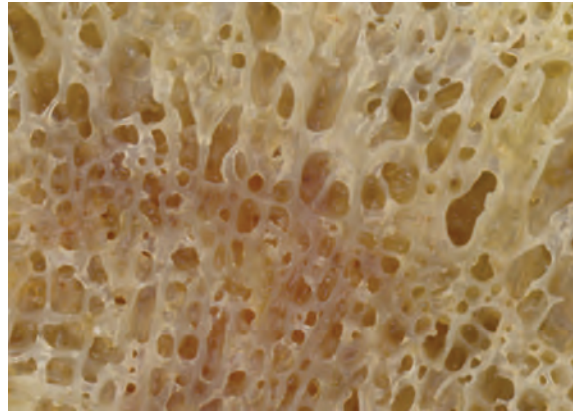
Zhu, Golden, Gully, Sampson *Physica B* 2010

Sampson, Golden, Gully, Worby *Deep Sea Research* 2011

SEA ICE

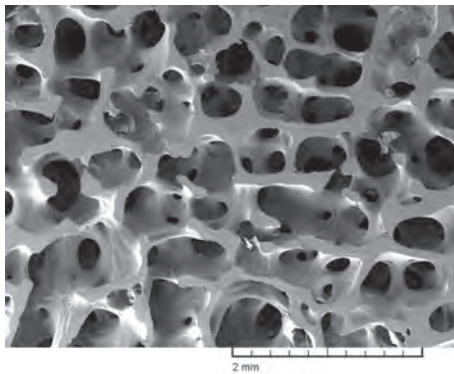


HUMAN BONE

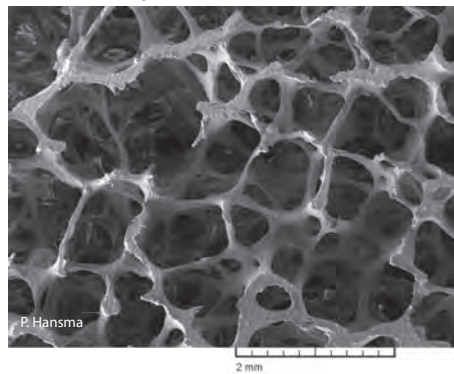


*spectral characterization
of porous microstructures
in human bone*

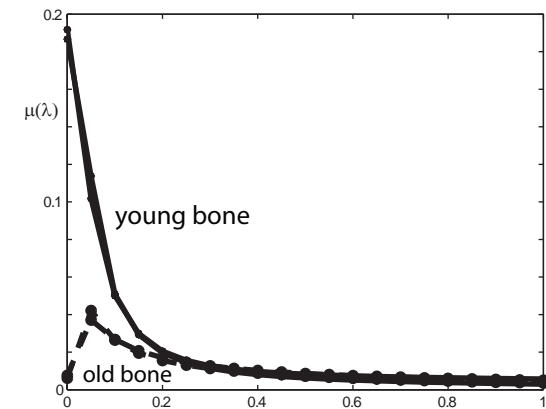
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures
from complex permittivity data



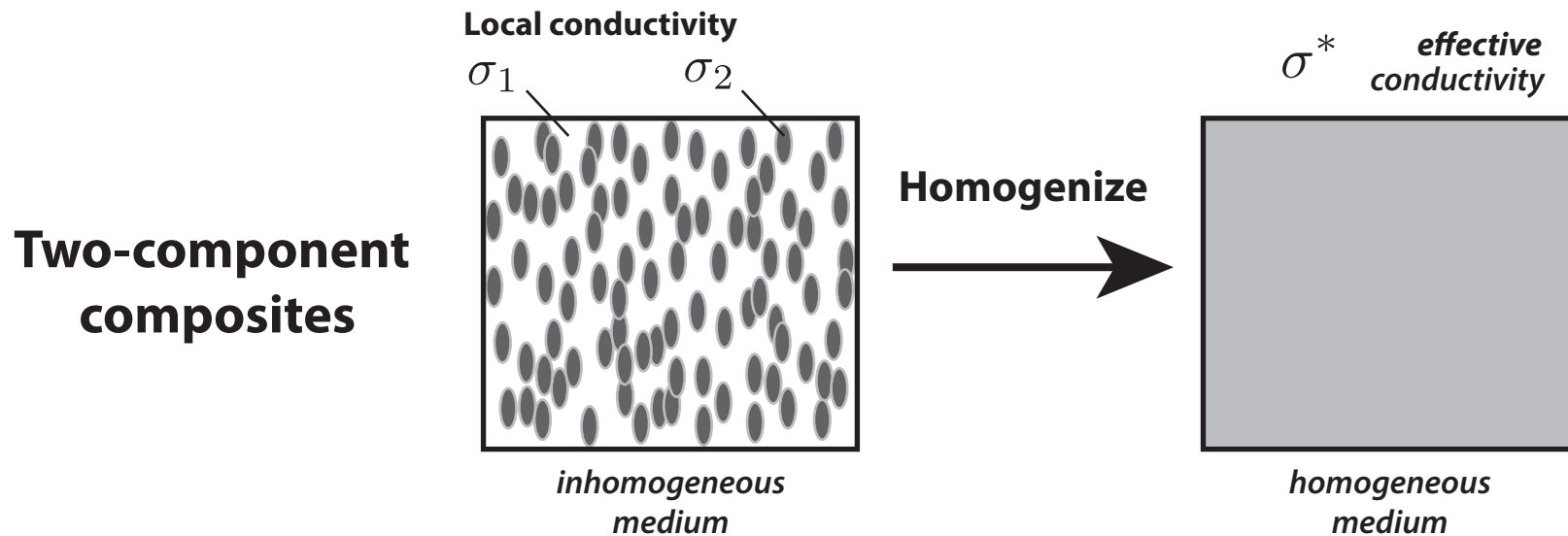
use regularized inversion scheme

*apply spectral measure analysis of brine connectivity and
spectral inversion to electromagnetic monitoring of osteoporosis*

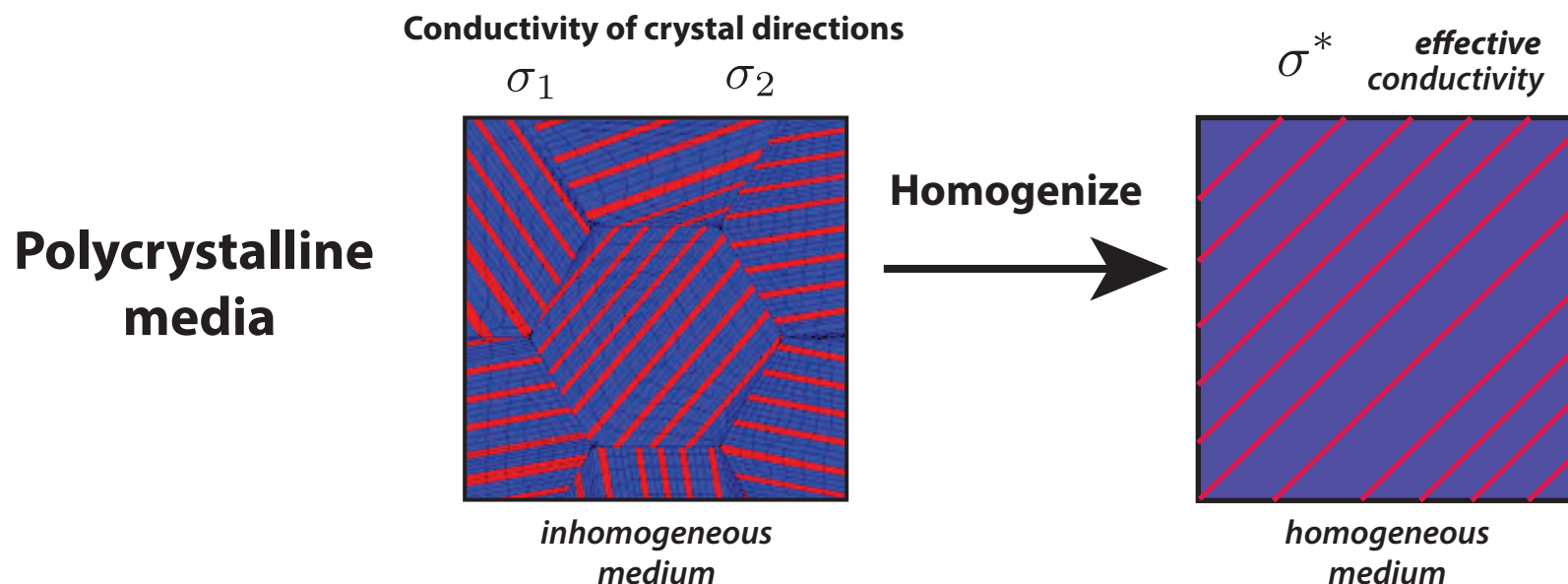
Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

Homogenization for polycrystalline materials



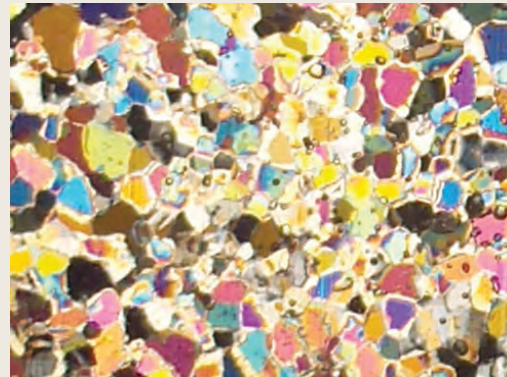
Find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium



Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**
Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
orientation statistics
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



PROCEEDINGS A

350 YEARS
OF SCIENTIFIC
PUBLISHING

An invited review
commemorating 350 years
of scientific publishing at the
Royal Society

A method to distinguish
between different types
of sea ice using remote
sensing techniques

A computer model to
determine how a human
should walk so as to expend
the least energy



THE
ROYAL
SOCIETY
PUBLISHING

higher threshold for fluid flow in granular sea ice

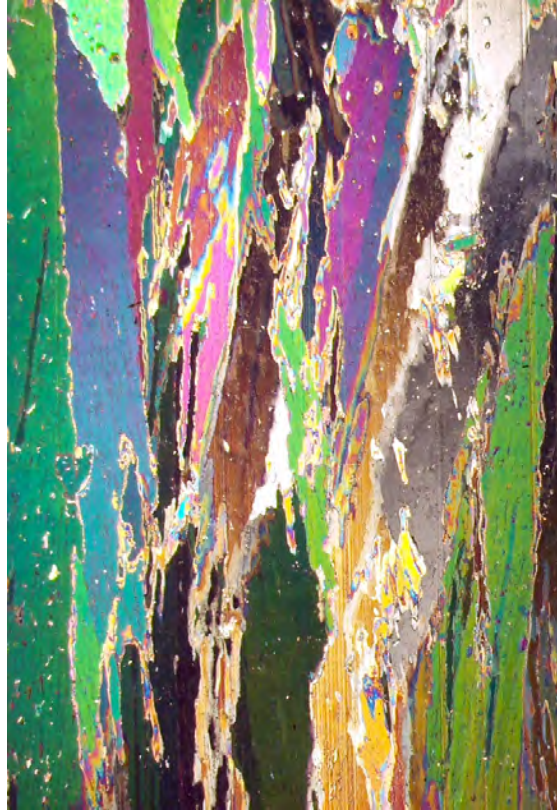
microscale details impact “mesoscale” processes

nutrient fluxes for microbes
melt pond drainage
snow-ice formation

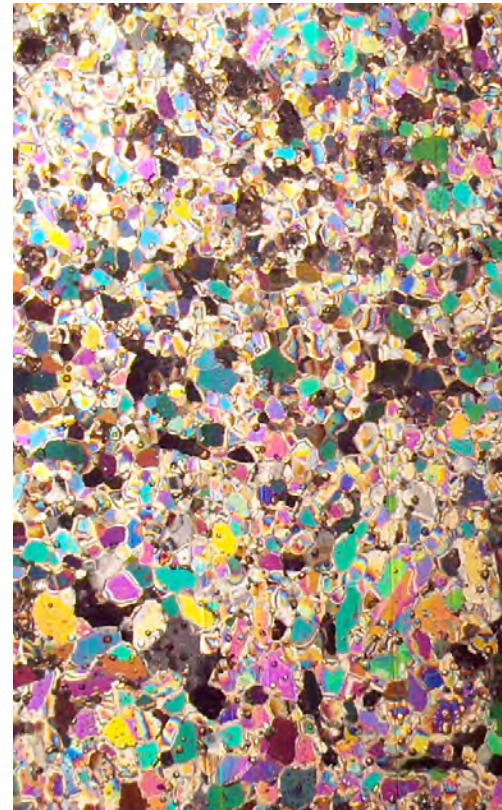
columnar

granular

5%



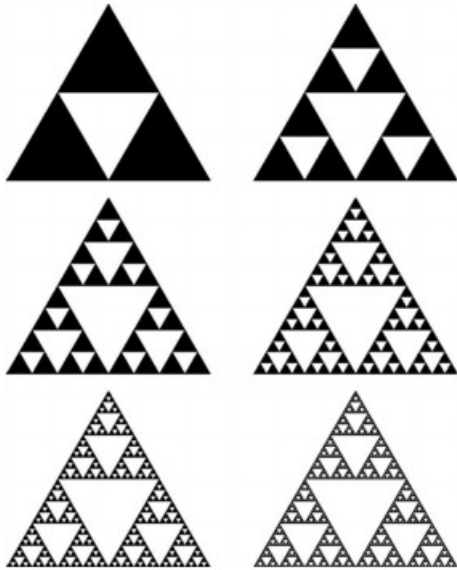
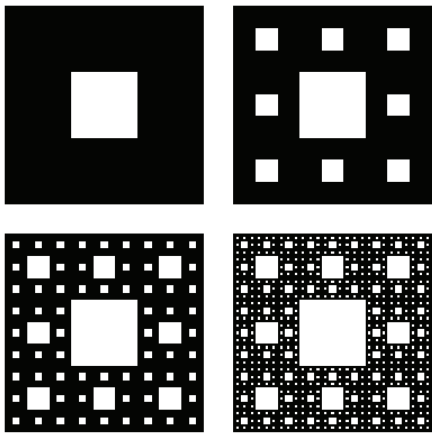
10%



Golden, Sampson, Gully, Lubbers, Tison 2021

electromagnetically distinguishing ice types
Kitsel Lusted, Elena Cherkaev, Ken Golden

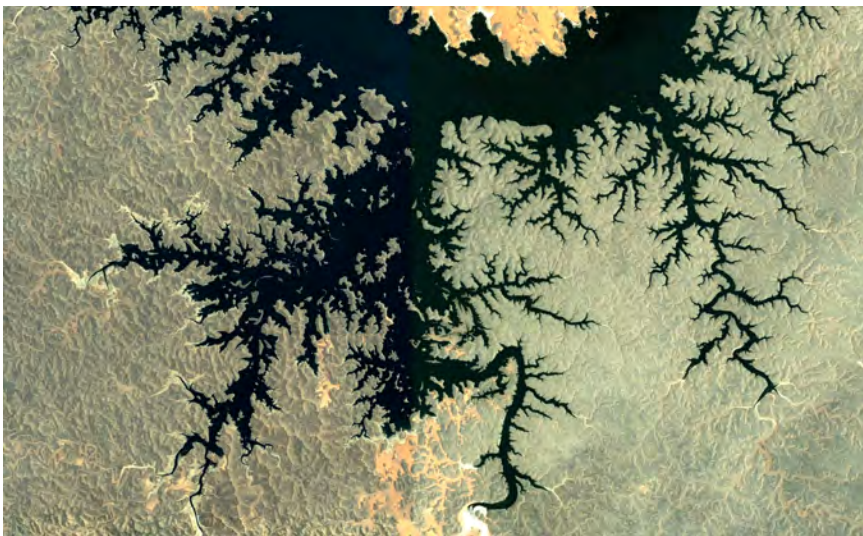
mesoscale



fractals

self-similar structure
non-integer dimension

$$D = \frac{\log 3}{\log 2} = 1.585...$$

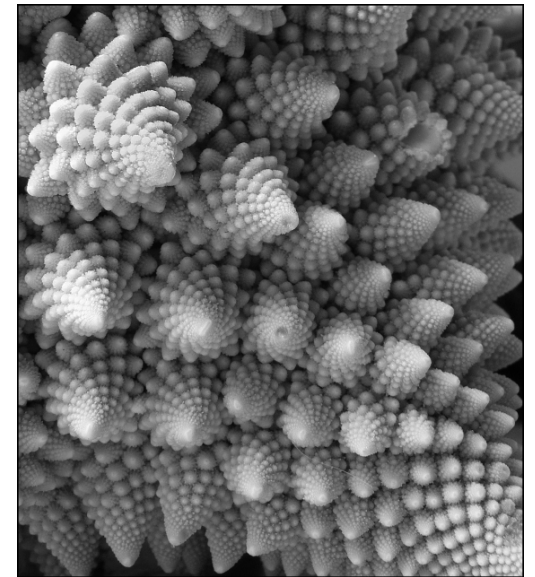
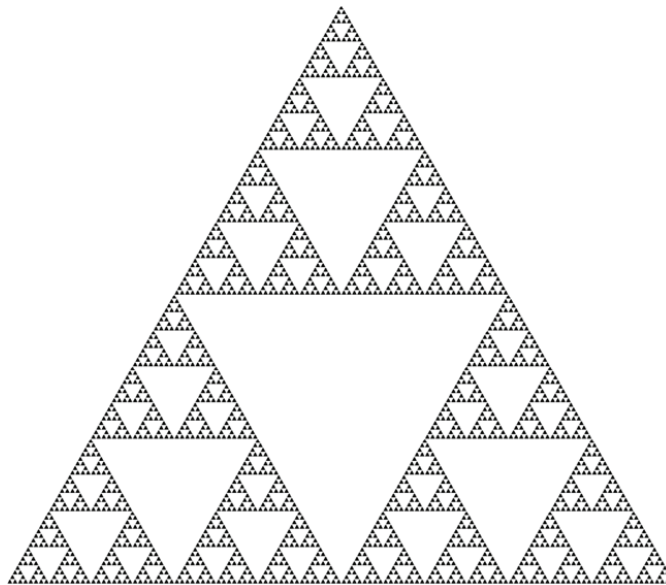
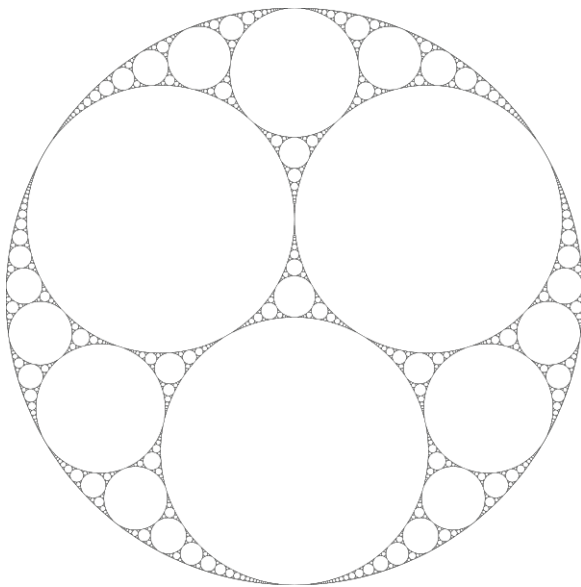


the sea ice pack is a *fractal*

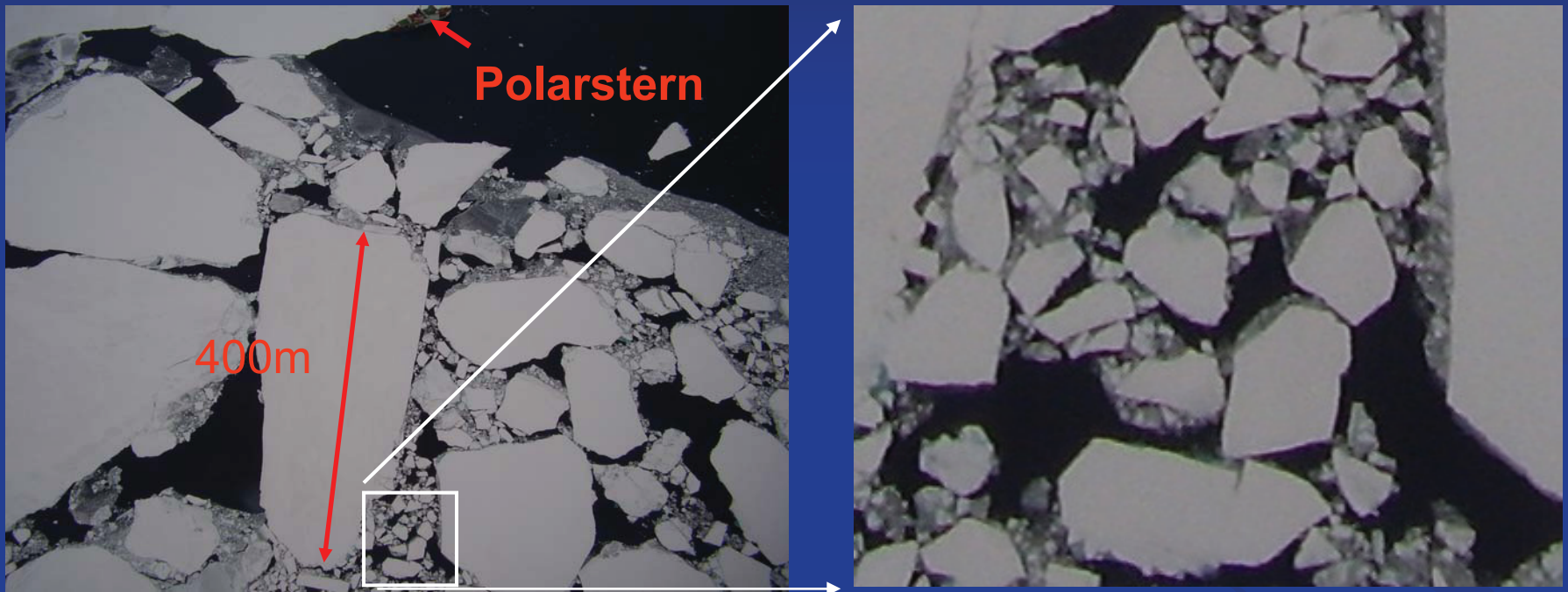
displaying self-similar structure on many scales

floe size distribution important in
dynamics (fracture), thermodynamics (melting)

bigger floes easier to break, smaller floes easier to melt



Self-similarity of sea ice floes



Weddell Sea, Antarctica
2006

Takenobu Toyota

wave propagation in the marginal ice zone (MIZ)

Stieltjes integral representation and bounds for
the complex viscoelasticity of the ice - ocean layer

Sampson, Murphy, Cherkaev, Golden 2021

first theory of key parameter
in wave-ice interactions only
fitted to wave data before

Analytic Continuation Method

Bergman (78) - Milton (79)
integral representation for ϵ^*

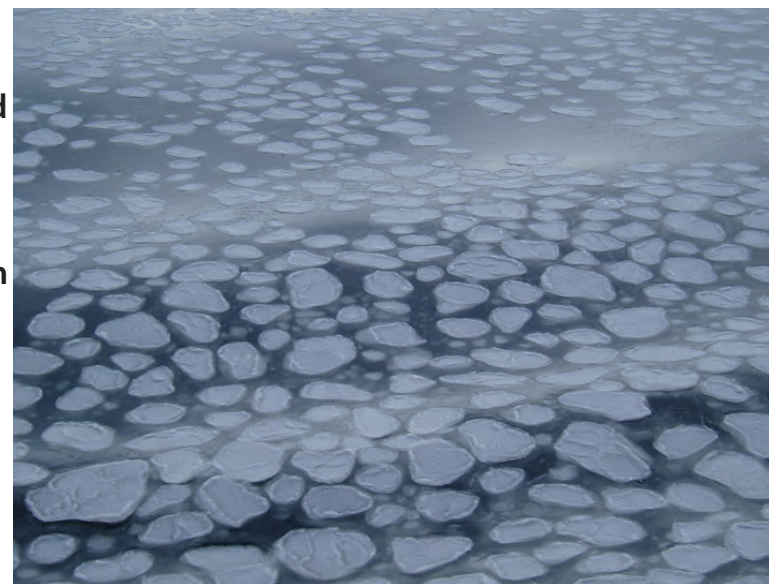
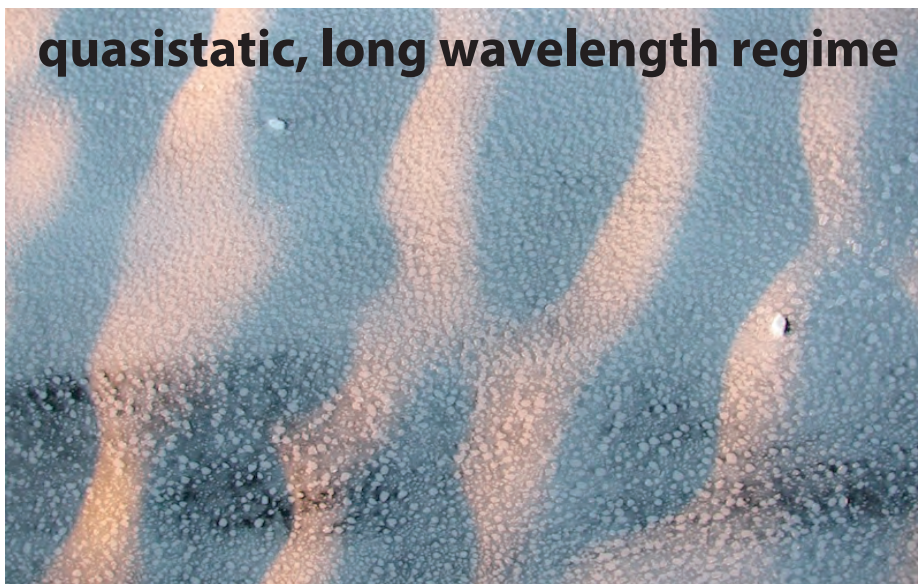
Golden and Papanicolaou (83)

Milton, Theory of Composites (02)

quasistatic, long wavelength regime

homogenized
parameter
depends on
sea ice
concentration
and ice floe
geometry

like EM waves



advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice
heat transport in sea ice with convection
sea ice floes in winds and ocean currents
tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere

advection diffusion equation with a velocity field \vec{u}

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$



homogenize

$$\frac{\partial \bar{T}}{\partial t} = \kappa^* \Delta \bar{T}$$

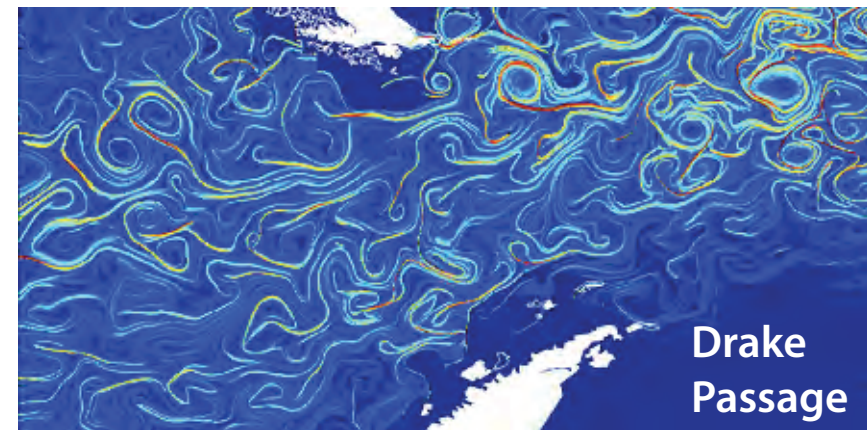
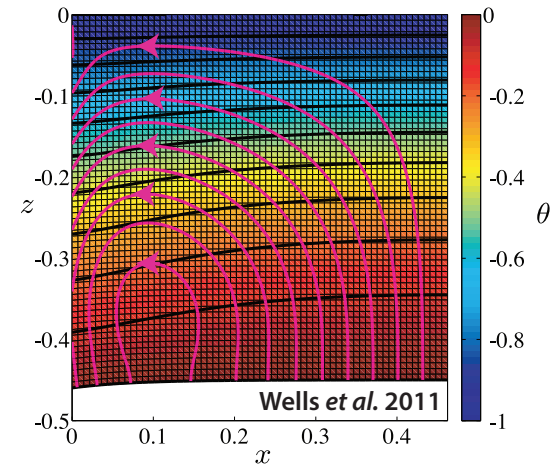
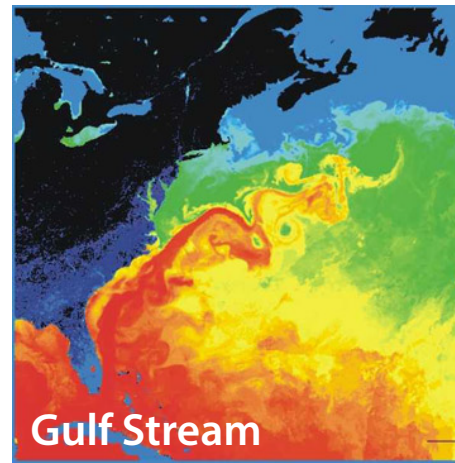
κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017

Murphy, Cherkaev, Zhu, Xin, Golden, *J. Math. Phys.* 2020



tracers flowing through inverted sea ice blocks



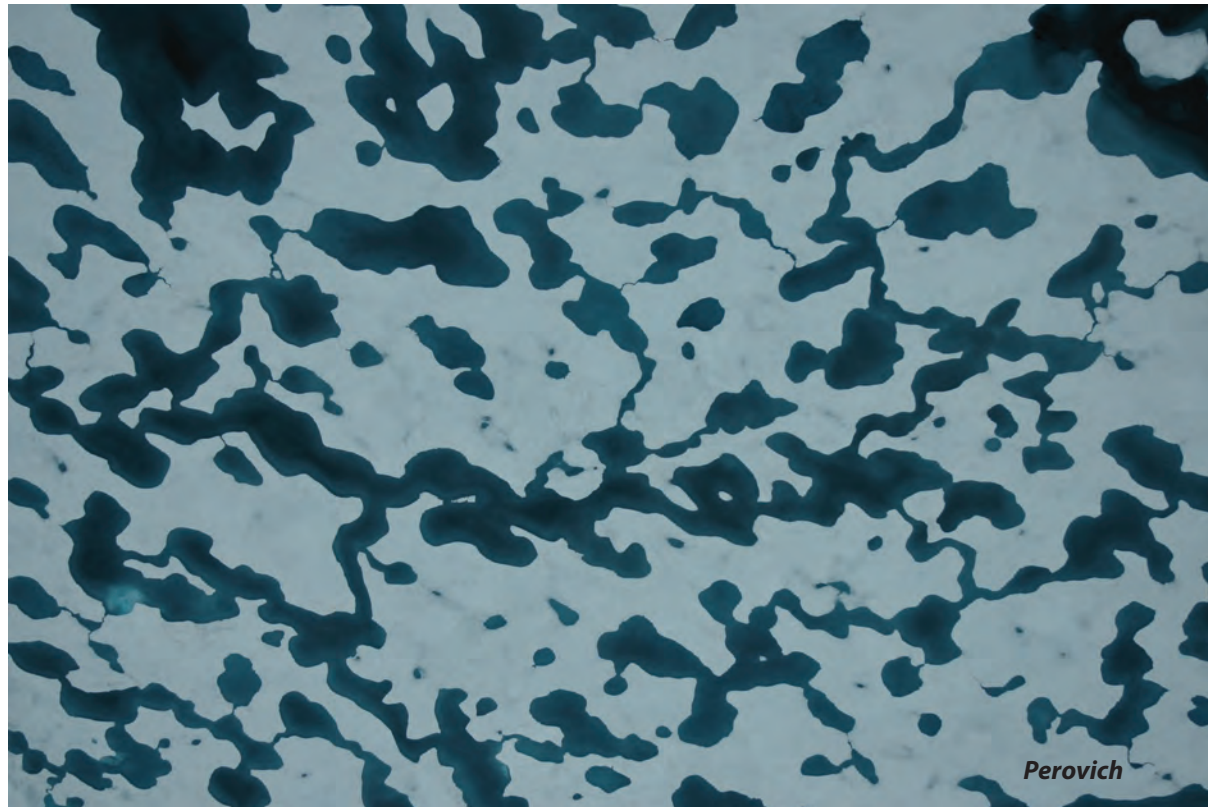
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

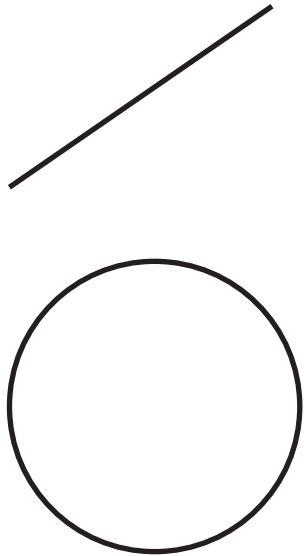
Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

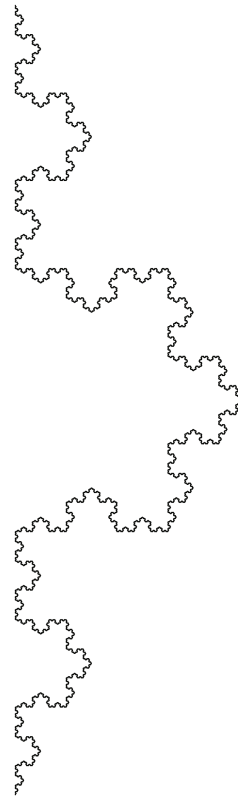
fractal curves in the plane

they wiggle so much that their dimension is >1



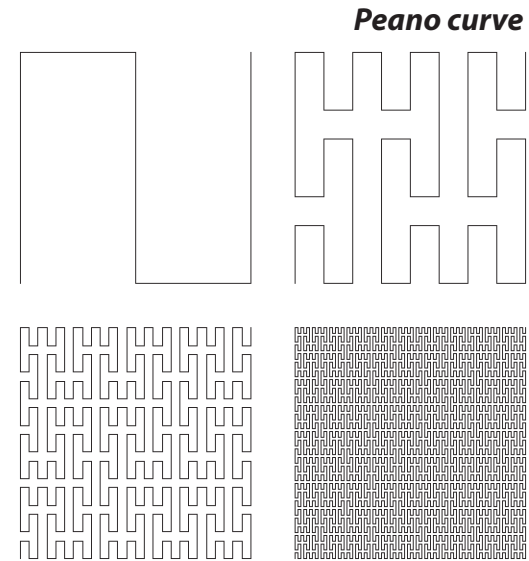
simple curves

$D = 1$

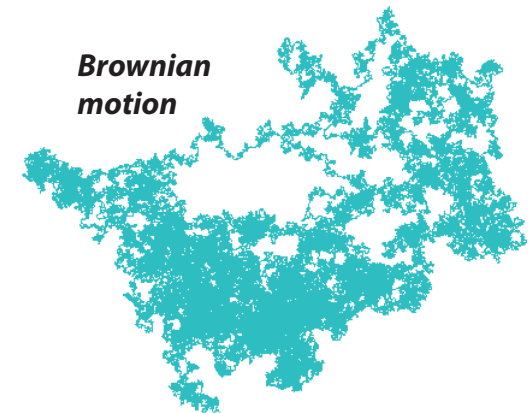


Koch snowflake

$D = 1.26$



Peano curve



Brownian motion

space filling curves

$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

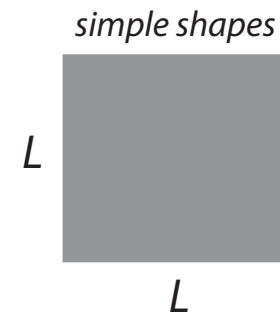
use **perimeter-area** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

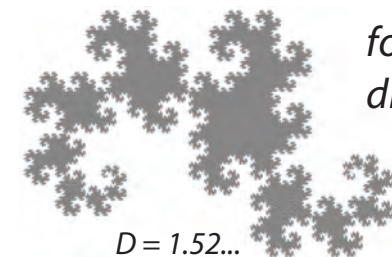


$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

$$P \sim \sqrt{A}^D$$



for fractals with dimension D

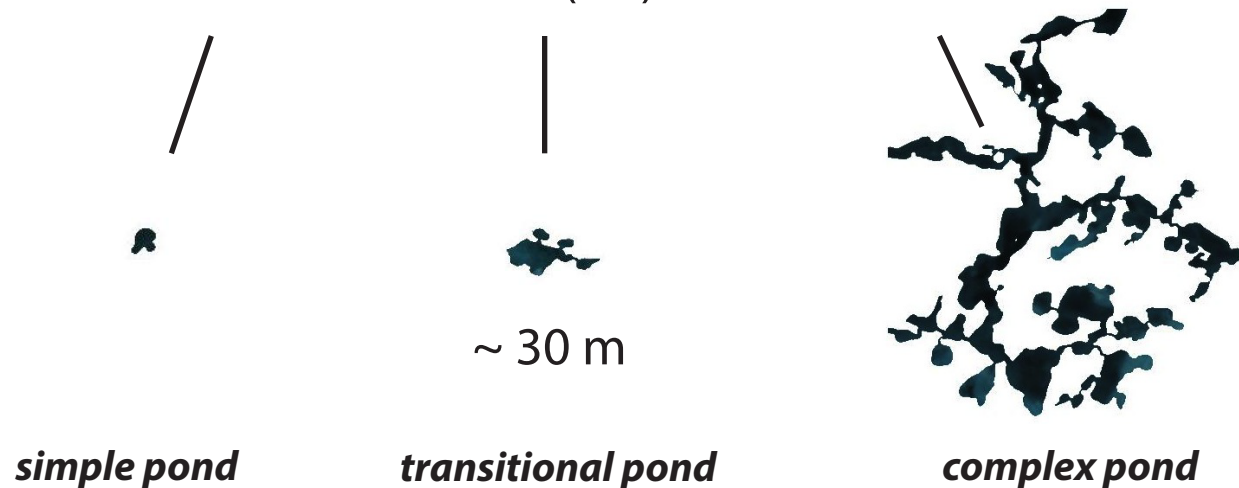
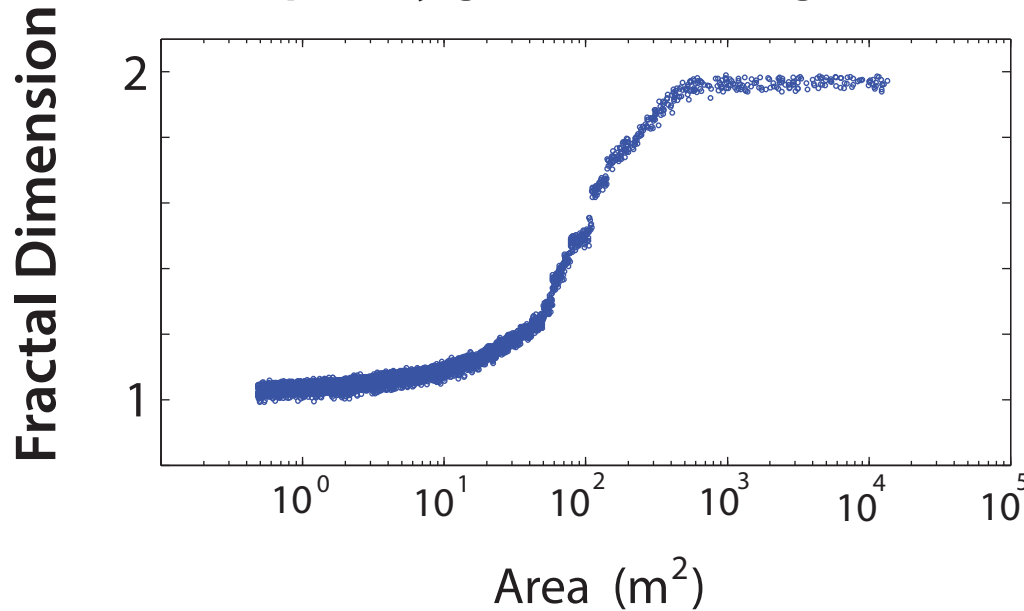
$D = 1.52...$

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

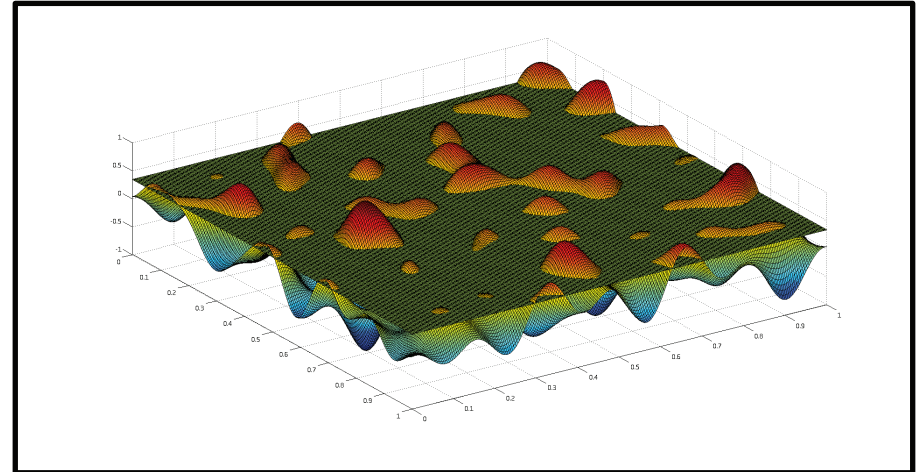
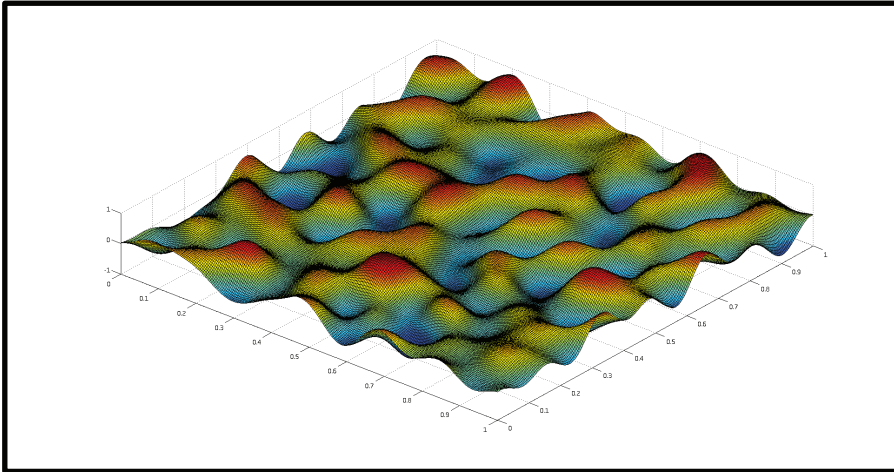
complexity grows with length scale



Continuum percolation model for melt pond evolution

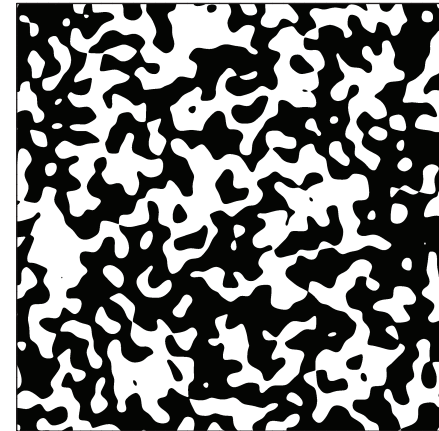
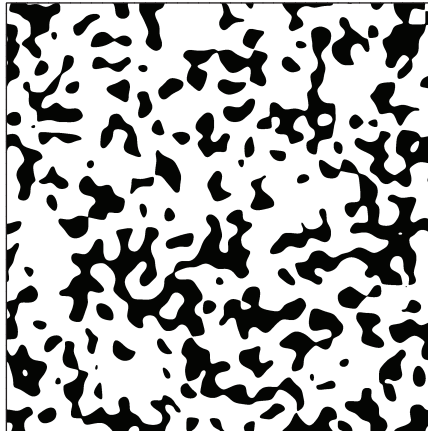
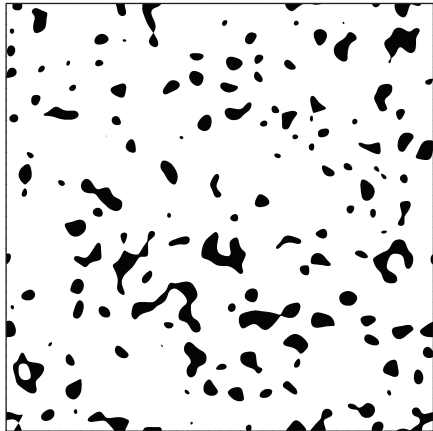
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

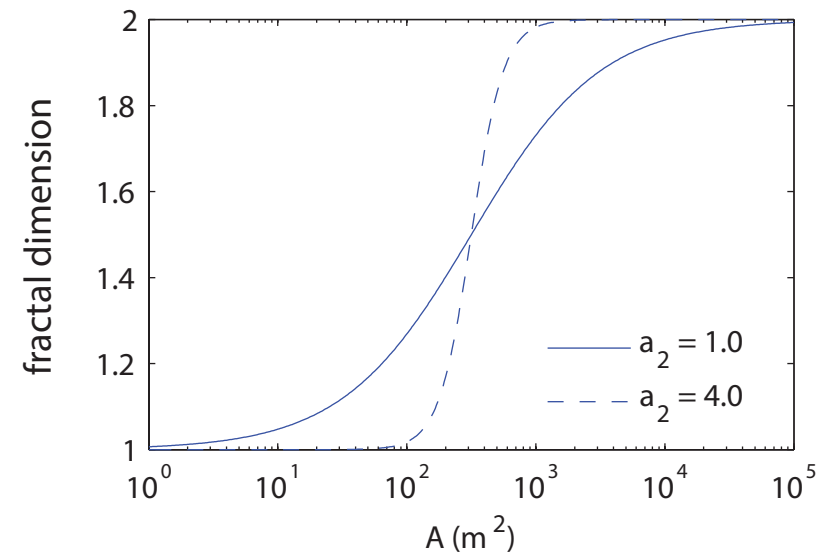
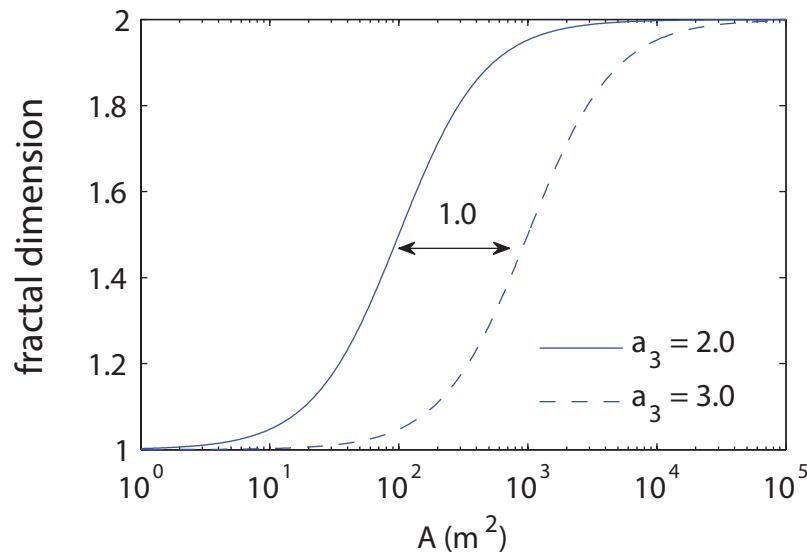


electronic transport in disordered media

diffusion in turbulent plasmas

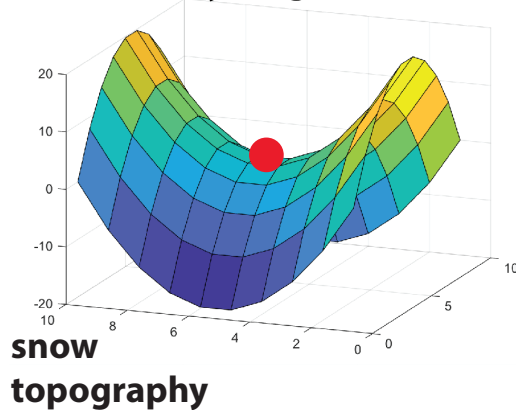
Isichenko, Rev. Mod. Phys., 1992

fractal dimension curves depend on statistical parameters defining random surface



Saddle Points, Morse Theory and the Fractal Geometry of Melt Ponds

Ryleigh Moore, Jacob Jones, Dane Gollero, Court Strong, Ken Golden 2021



As ponds coalesce at saddle points, fractal dimension proxy isoperimetric quotient $P^2/4\pi A$ jumps, driving the transition.



Ryleigh Moore
Department of Mathematics
University of Utah

Multidisciplinary drifting Observatory
for the Study of Arctic Climate (**MOSAiC**)

MOSAiC School aboard the icebreaker *RV Akademik Federov*

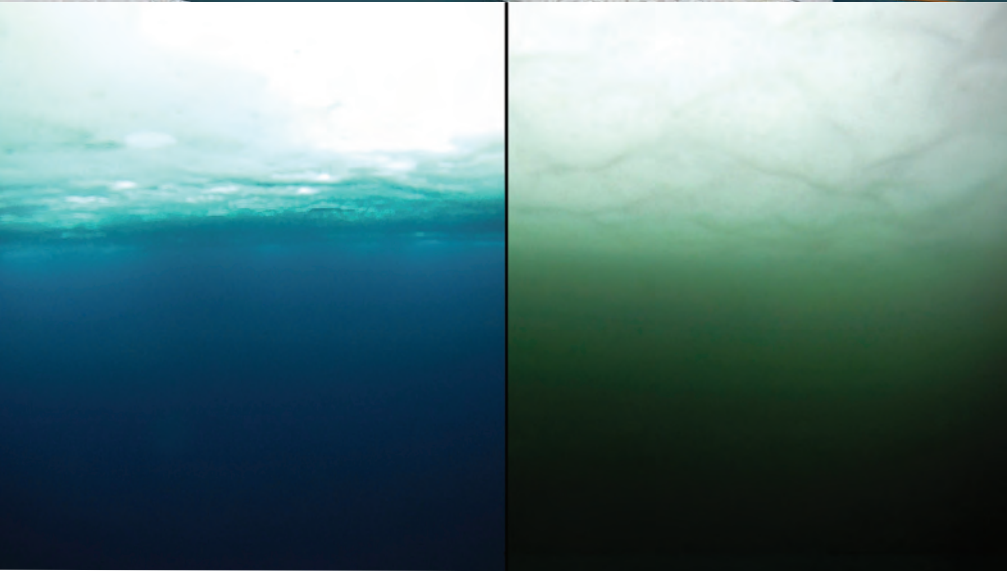
20 grad students from around the world
(**3 from U.S., 1 mathematician**)



Perovich

Melt ponds control transmittance of solar energy through sea ice, impacting upper ocean ecology.

WINDOWS



no bloom

bloom

massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances* 2017

The effect of melt pond geometry on the distribution of solar energy under first year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden
Geophys. Res. Lett. 2019

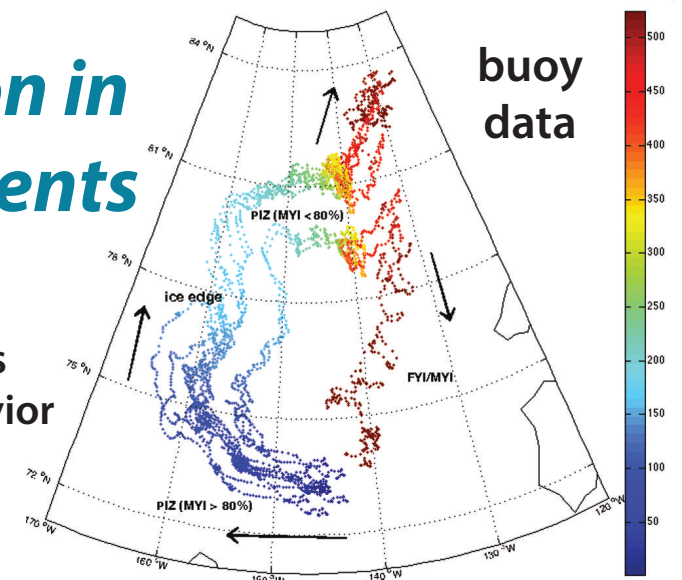
(2015 AMS MRC)

macroscale



Ice floe diffusion in winds and currents

on short time scales floes exhibit Brownian-like behavior



- Effective behavior is purely diffusive, sub-diffusive or super-diffusive depending on ice pack and advective conditions - **Hurst exponent**.

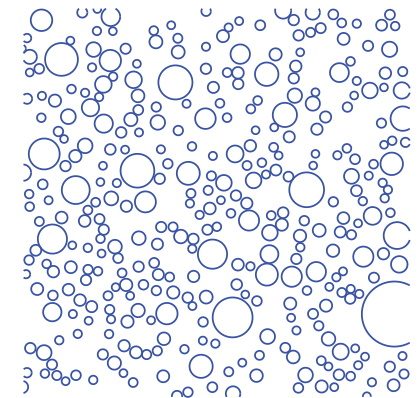
On sea-ice dynamical regimes in the Arctic Ocean

Jennifer Lukovich, Jennifer Hutchings, David Barber, *Ann. Glac.* 2015

Anomalous diffusion and sea ice dynamics

Huy Dinh, Ben Murphy, Elena Cherkaev, Ken Golden 2021

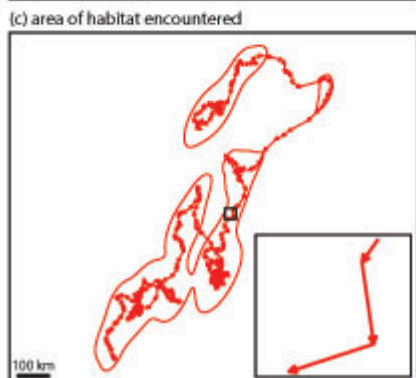
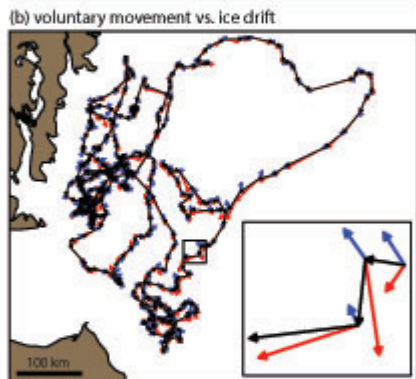
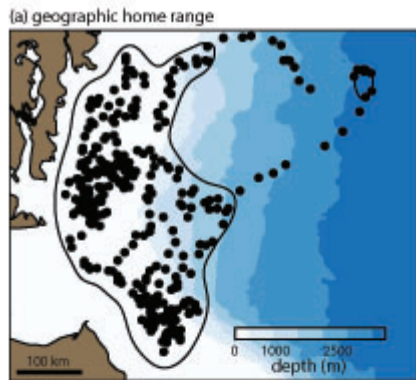
floe-scale model - crowding jamming, advective forcing

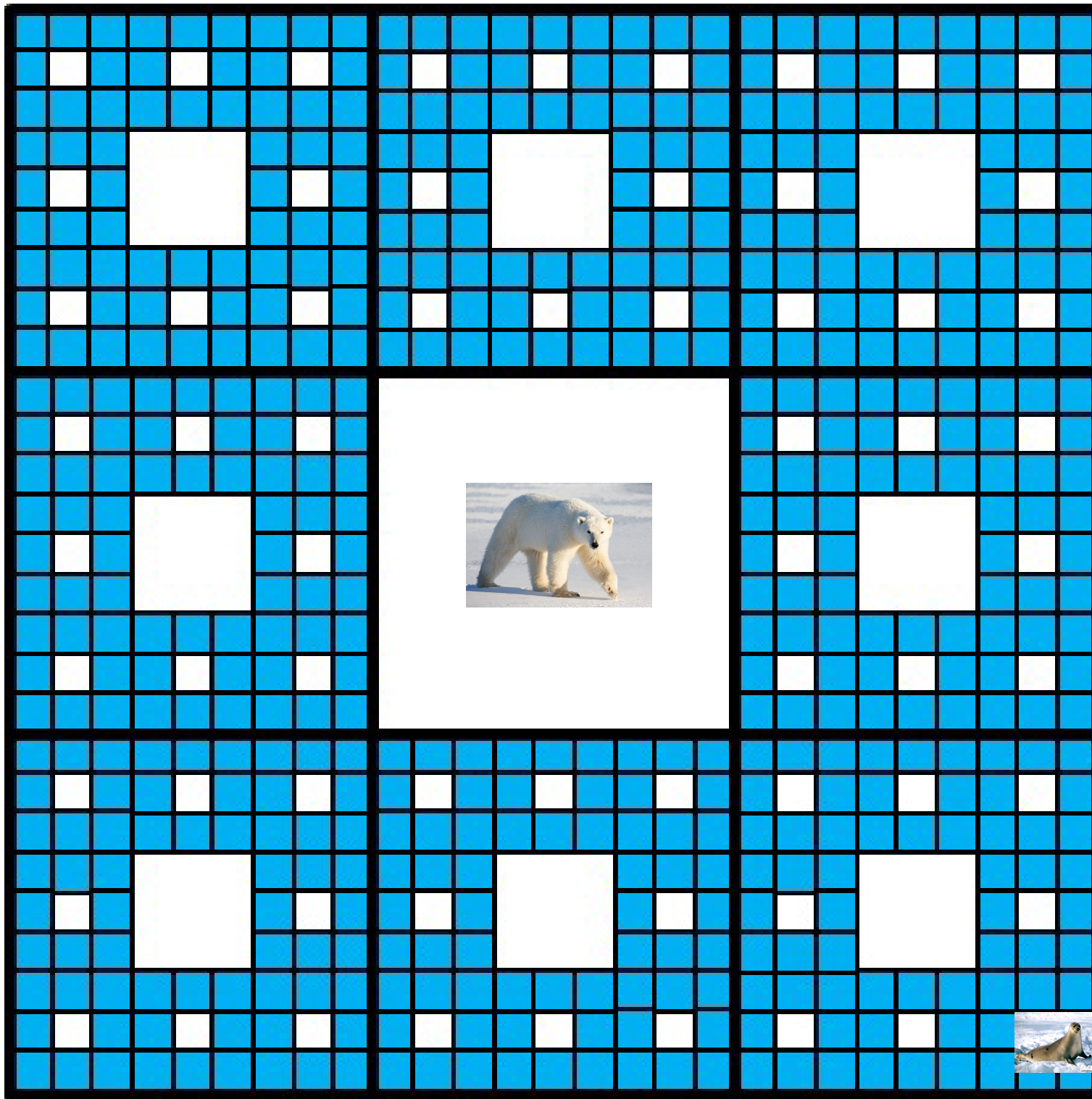


sea ice concentration = 0.3

Home ranges in moving habitats: polar bears and sea ice

Marie Auger-Méthé, Mark Lewis, Andrew Derocher, *Ecography*, 2016





polar bear foraging in a fractal icescape

Nicole Forrester
Jody Reimer
Ken Golden

It costs the polar bear
5 times the energy to
swim through water
than to walk on sea ice.

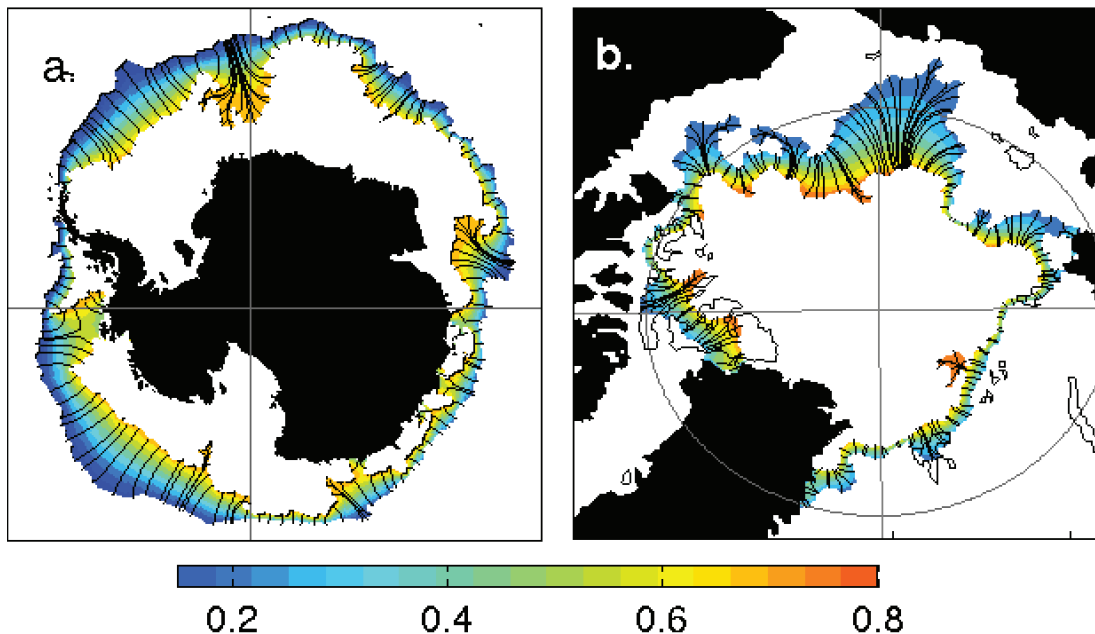
What pathway to a seal
5 km away minimizes
energy spent?



Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



MIZ WIDTH

fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012

Strong and Rigor, *GRL* 2013

transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

**How to objectively
measure the “width”
of this complex,
non-convex region?**

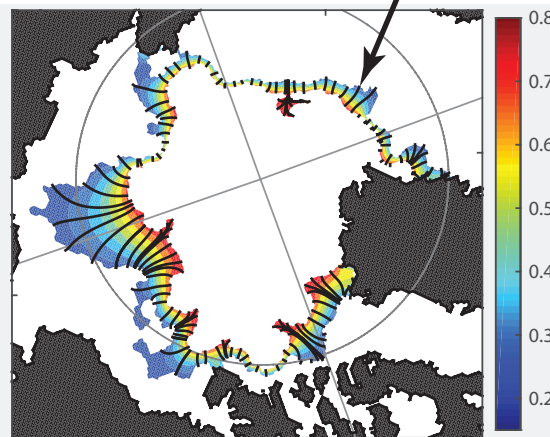
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

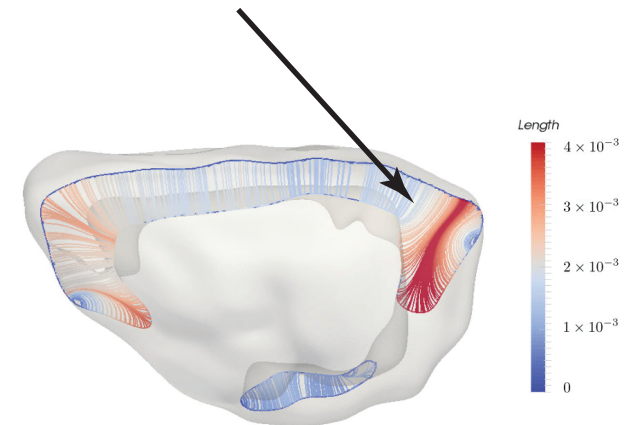
39% widening
1979 - 2012

“average” lengths of streamlines

streamlines of a solution
to Laplace’s equation



Arctic Marginal Ice Zone



**cross-section of the
cerebral cortex of a rodent brain**

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

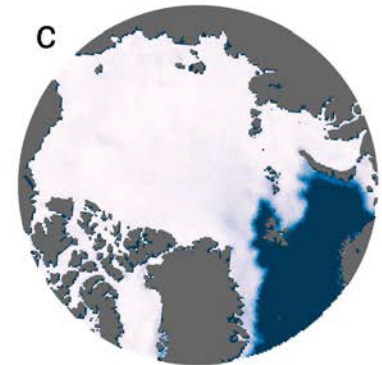
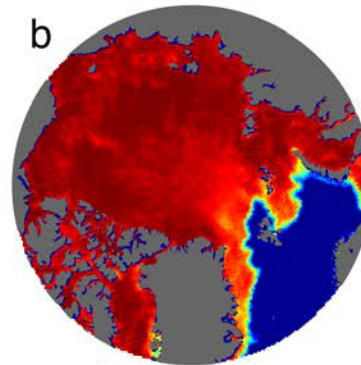
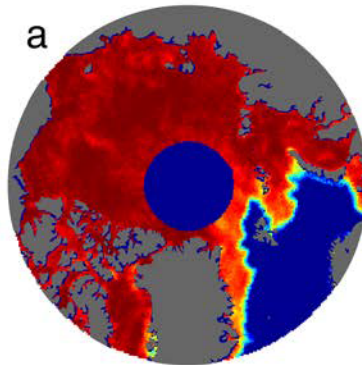
Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

Filling the polar data gap with partial differential equations

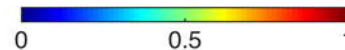
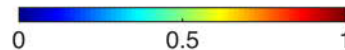
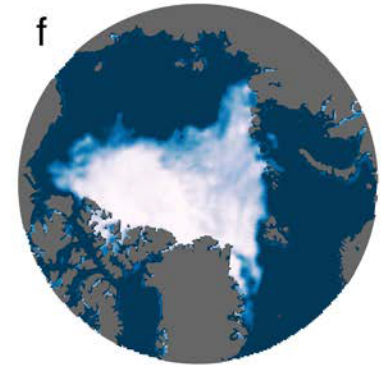
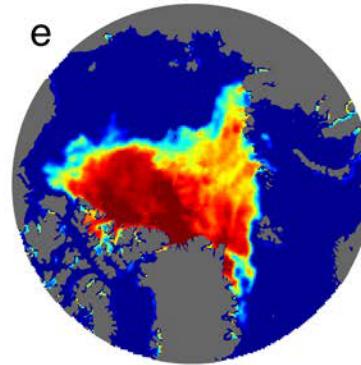
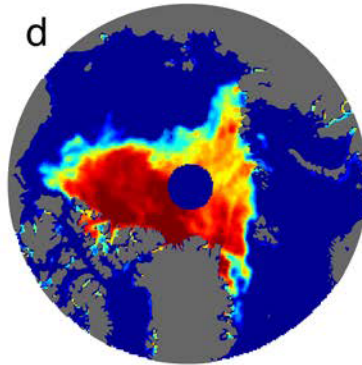
hole in satellite coverage
of sea ice concentration field

previously assumed
ice covered

Gap radius: 611 km
06 January 1985



Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

**fill = harmonic function with
learned stochastic term**

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

**NOAA/NSIDC Sea Ice Concentration CDR
product update will use our PDE method.**

University of Utah Sea Ice Modeling Group (2017-2021)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Adjunct Assistant Professor of Mathematics

Postdoctoral Researchers: Noa Kraitzman (now at ANU), Jody Reimer

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (now a sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier
Daniel Hallman

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich,
Dane Gollero, Samir Suthar, Anna Hyde,
Kitsel Lusted, Ruby Bowers, Kimball Johnston,
Jerry Zhang, Nash Ward, David Gluckman

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergraduates Kayla Stewart, Nicole Forrester

Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.
2. Mathematical methods developed for sea ice advance the theory of composites and other areas of science and engineering.
3. **Homogenization and statistical physics help *link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.
4. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
5. Field experiments are essential to developing relevant mathematics.
6. Our research is helping to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.



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Notices

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*The cover is based on "Modeling Sea Ice,"
page 1535.*

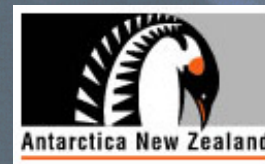
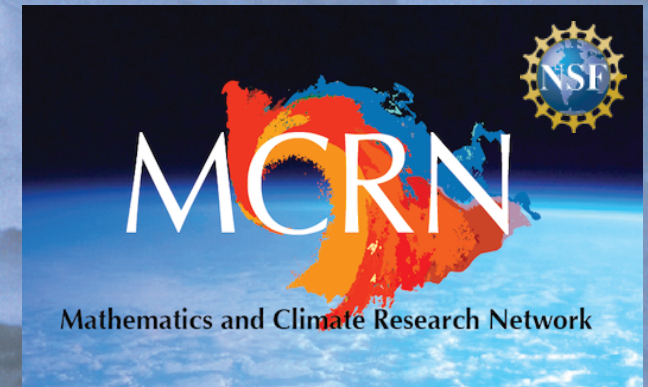
THANK YOU

Office of Naval Research

Applied and Computational Analysis Program
Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences
Division of Polar Programs



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship *Aurora Australis* without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the *Aurora Australis* in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The *Aurora Australis* is owned by P&O Australia and chartered by the Antarctic Div-

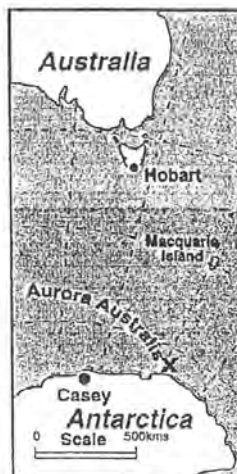
ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire.

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

THE CANBERRA TIMES

Thursday 23 July 1998

Page 4

Antarctic voyage stopped by fire

HOBART: An engine room fire has disabled the Australian icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected *Aurora Australis* would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The *Aurora*, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room"

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats"

Sydney Morning Herald
23 July, 1998

ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the *Aurora Australis* yesterday. The 54 people on board were forced on deck in the



Special Issue on the Mathematics of Planet Earth

Read about the application of mathematics and computational science to issues concerning invasive populations, Arctic sea ice, insect flight, and more in this Planet Earth **special issue!**

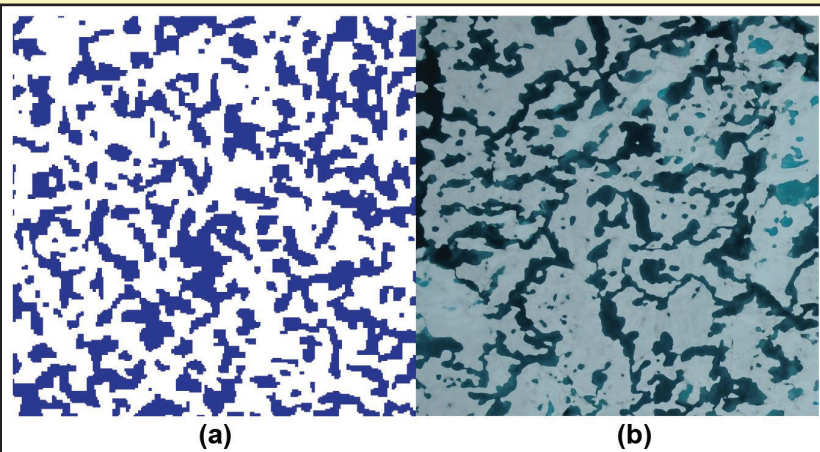


Figure 3. Comparison of real Arctic melt ponds with metastable equilibria in our melt pond Ising model. **3a.** Ising model simulation. **3b.** Real melt pond photo. Figure 3a courtesy of Yiping Ma, 3b courtesy of Donald Perovich.

Vast labyrinthine ponds on the surface of melting Arctic sea ice are key players in the polar climate system and upper ocean ecology. Researchers have adapted the Ising model, which was originally developed to understand magnetic materials, to study the geometry of meltwater's distribution over the sea ice surface. In an article on page 5, Kenneth Golden, Yiping Ma, Courtenay Strong, and Ivan Sudakov explore model predictions.

Controlling Invasive Populations in Rivers

By Yu Jin and Suzanne Lenhart

Flow regimes can change significantly over time and space and strongly impact all levels of river biodiversity, from the individual to the ecosystem. Invasive species in rivers—such as bighead and silver carp, as well as quagga and zebra mussels—continue to cause damage. Management of these species may include targeted adjustment of flow rates in rivers, based on recent research that examines the effects of river morphology and water flow on rivers' ecological statuses. While many previous methodologies rely on habitat suitability models or oversimplification of the hydrodynamics, few studies have focused on the integration of ecological dynamics into water flow assessments.

Earlier work yielded a hybrid modeling approach that directly links river hydrology with stream population models [3]. The hybrid model's hydrodynamic component is based on the water depth in a gradually varying river structure. The model derives the steady advective flow from this structure and relates it to flow features like water discharge, depth, velocity, cross-

sectional area, bottom roughness, bottom slope, and gravitational acceleration. This approach facilitates both theoretical understanding and the generation of quantitative predictions, thus providing a way for scientists to analyze the effects of river fluctuations on population processes.

When a population spreads longitudinally in a one-dimensional (1D) river with spatial heterogeneities in habitat and temporal fluctuations in discharge, the resulting hydrodynamic population model is

$$N_t = -A_t(x, t) \frac{N}{A(x, t)} + \frac{1}{A(x, t)} \left(D(x, t) A(x, t) N_x \right)_x - \frac{Q(t)}{A(x, t)} N_x + rN \left(1 - \frac{N}{K} \right)$$

$$\begin{aligned} N(0, t) &= 0 & \text{on } (0, T), x = 0, \\ N_x(L, t) &= 0 & \text{on } (0, T), x = L, \\ N(x, 0) &= N_0(x) & \text{on } (0, L), t = 0 \end{aligned} \quad (1)$$

See **Invasive Populations** on page 4

Modeling Resource Demands and Constraints for COVID-19 Intervention Strategies

By Erin C.S. Acquesta, Walt Beyeler, Pat Finley, Katherine Klise, Monear Makvandi, and Emma Stanislawski

As the world desperately attempts to control the spread of COVID-19, the need for a model that accounts for realistic trade-offs between time, resources, and corresponding epidemiological implications is apparent. Some early mathematical models of the outbreak compared trade-offs for non-pharmaceutical interventions [3], while others derived the necessary level of test coverage for case-based interventions [4] and demonstrated the value of prioritized testing for close contacts [7].

Isolated analyses provide valuable insights, but real-world intervention strategies are interconnected. Contact tracing is the lynchpin of infection control [6] and forms the basis of prioritized testing. Therefore, quantifying the effectiveness of contact tracing is crucial to understanding the real-life implications of disease control strategies.

Contact Tracing Demands

Contact tracers are skilled, culturally competent interviewers who apply their knowledge of disease and risk factors when notifying people who have come into contact with COVID-19-infected individuals. They also continue to monitor the situation after case investigations [1].

Case investigation consists of four steps:

1. Identify and notify cases
2. Interview cases
3. Locate and notify contacts
4. Monitor contacts.

Most health departments are implementing case investigation, contact identification, and quarantine to disrupt COVID-19 transmission. The timeliness of contact tracing is constrained by the length of the infectious period, the turn-around time for testing and result reporting, and the ability to successfully reach and interview patients and their contacts. The European Centre for Disease Prevention and Control approximates that contact tracers spend one to two hours conducting an interview [2]. Estimates regarding the timelines of other steps are limited to subject matter expert elicitation and can vary based on cases' access to phone service or willingness to participate in interviews.

Bounded Exponential

The fundamental structure of our model follows traditional susceptible-exposed-infected-recovered (SEIR) compartmental modeling [5]. We add an asymptomatic population A , a hospitalized population H , and disease-related deaths D , as well as corresponding quarantine states. We define the states $\{S_i, E_i, A_i, I_i, H, R, D\}_{i=0,1}$ for our compartments, such that $i=0$ and $i=1$

correspond to unquarantined and quarantined respectively. Rather than focus on the dynamics that are associated with the state transition diagram in Figure 1, we introduce a formulation for the real-time demands on contact tracers' time as a function of infection prevalence, while also respecting constraints on resources.

When the work that is required to investigate new cases and monitor existing contacts exceeds available resources, a backlog develops. To simulate this backlog, we introduce a new compartment C for tracking the dynamic states of cases:

$$\frac{dC}{dt} = [flow_{in}] - [flow_{out}].$$

Flow into the backlog compartment, represented by $[flow_{in}]$, reflects case identification that is associated with the following transitions in the model:

- The rate of random testing: $q_{rA}(t)A_0(t) \rightarrow A_1(t)$ and $q_{rI}(t)I_0(t) \rightarrow I_1(t)$
- Testing triggered by contact tracing: $q_{tA}(t)A_0(t) \rightarrow A_1(t)$, $q_{tI}(t)I_0(t) \rightarrow I_1(t)$, and $q_{tE}(t)E_i(t) \rightarrow \{A_i(t), I_i(t)\}$
- The population that was missed by the non-pharmaceutical interventions that require hospitalization: $\tau_{IH}(t)I_0(t) \rightarrow H(t)$.

Here, $q_{rs}(t)$ defines the time-dependent rate of random testing, $q_{ts}(t)$ signifies the time-dependent rate of testing that is triggered by contact tracing, and τ_{IH} is the inverse of the expected amount of time for which an infected individual is symptomatic before hospitalization. These terms collectively provide the simulated number of newly-identified positive COVID-19 cases. However, we also need the average number of contacts per case. We thus define function $\mathcal{K}(\kappa, T_s, \phi_\kappa)$ that depends on the average number of contacts a day (κ), the average number of days for which an individual is infectious before going into isolation (T_s), and the likelihood that the individual

See **COVID-19 Intervention** on page 3

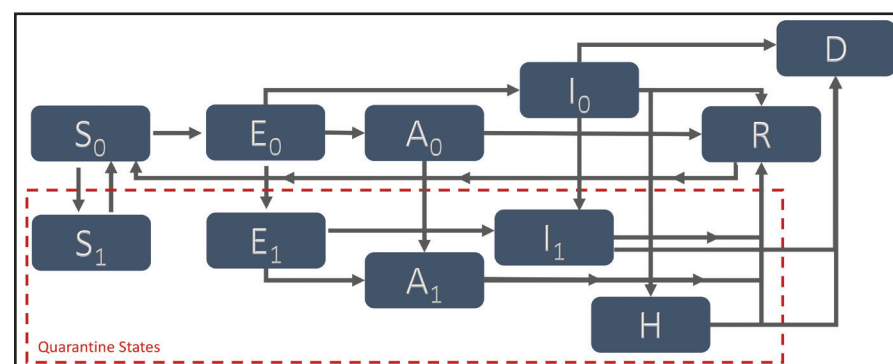


Figure 1. Disease state diagram for the compartmental infectious disease model. Figure courtesy of the authors.

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Modeling Sea Ice



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