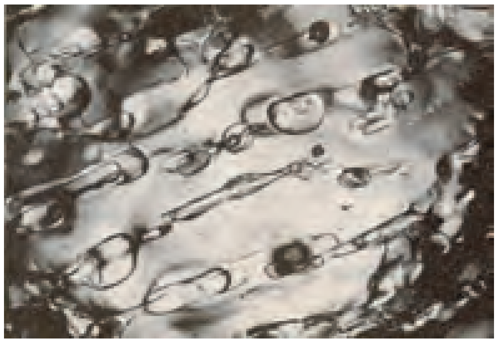
Linking Scales in Earth's Sea Ice System

Kenneth M. Golden
Department of Mathematics
University of Utah

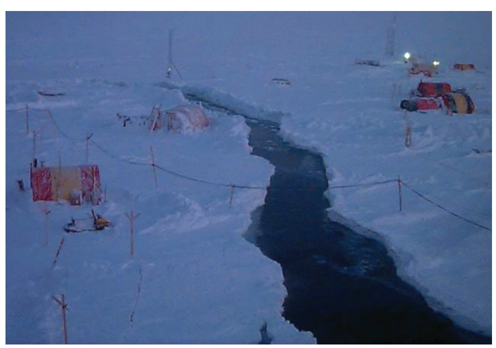
sea ice is a multiscale composite

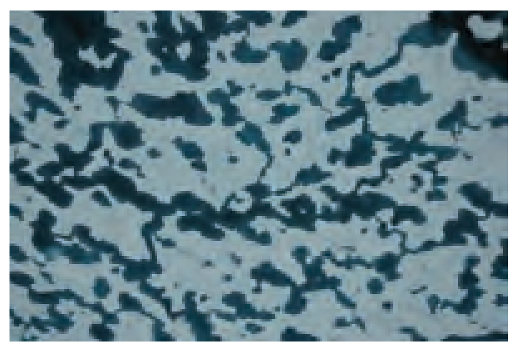


millimeters

centimeters

meters





meters

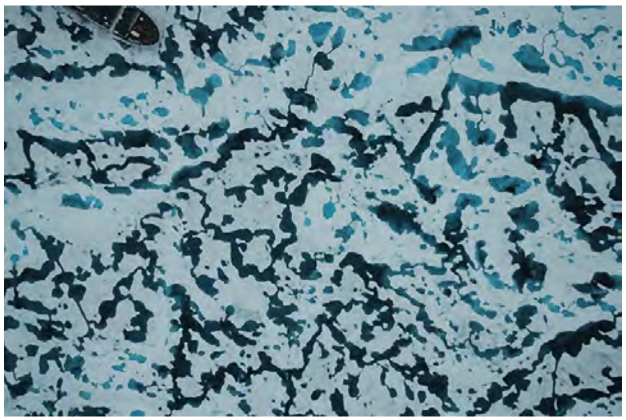
kilometers

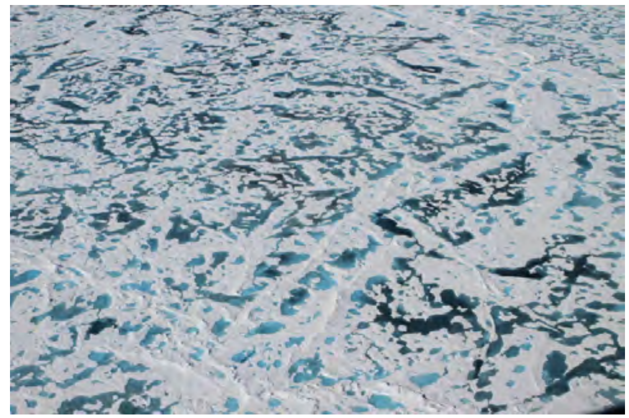
How do scales interact in the sea ice system?

basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds

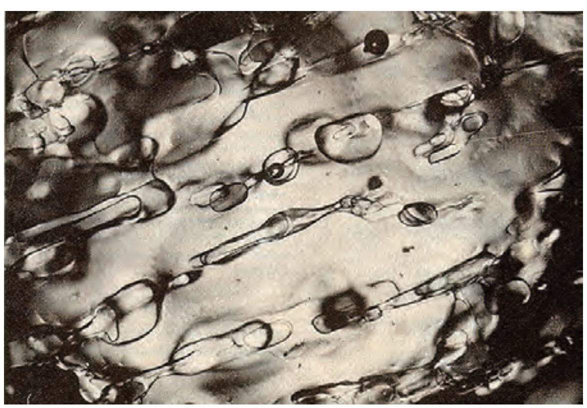




km
scale
melt
ponds

Linking Scales

mm
scale
brine
inclusions



meter
scale
snow
topography

What is this talk about? HOMOGENIZATION

Using methods of statistical physics and homogenization to* LINK SCALES *in the sea ice system ... rigorously compute effective behavior and improve climate models.

1. ***Sea ice microphysics and fluid transport***

2. ***Stieltjes integral representations for EM properties***

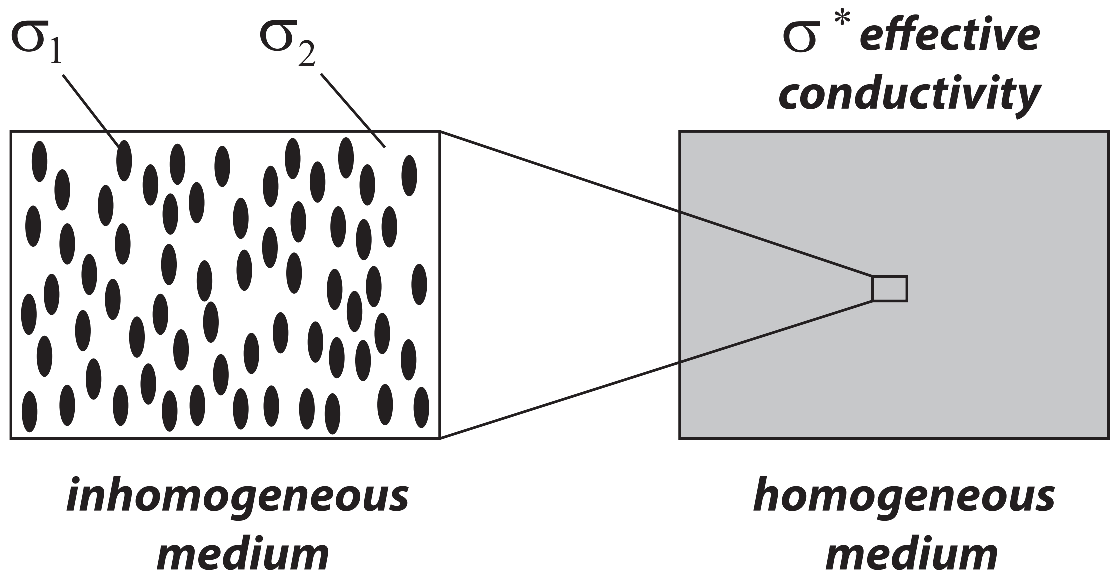
3. ***Extension to polycrystals, advection diffusion, waves in MIZ***

4. ***Fractal geometry of melt pond evolution***

cross - pollination

Solving problems in physics and biology of sea ice drives advances in theory of composite materials.

HOMOGENIZATION - Linking Scales in Composites



find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

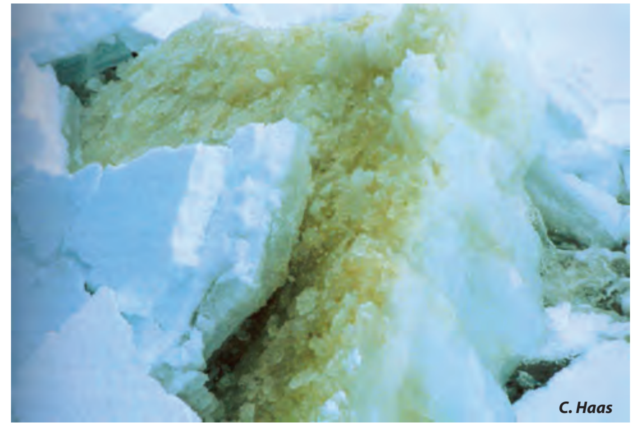
Wiener 1912 : arithmetic and harmonic mean ***bounds*** *on effective conductivity*
Hashin and Shtrikman 1962 : variational ***bounds*** *on effective conductivity*

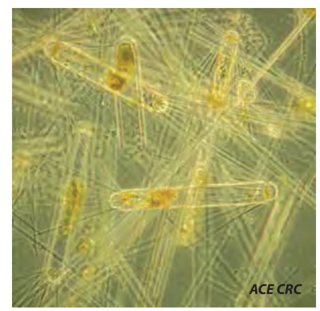
widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

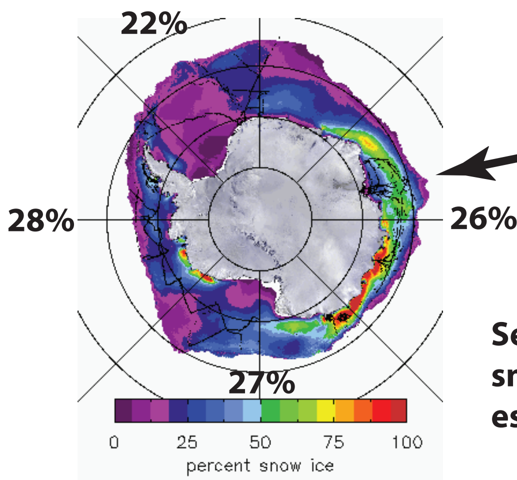
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities







T. Maksym and T. Markus, 2008

September snow-ice estimates

Antarctic surface flooding and snow-ice formation

- *evolution of salinity profiles*
- *ocean-ice-air exchanges of heat, CO_2*

fluid permeability of a porous medium

how much water gets through the sample per unit time?

Darcy's Law

for slow viscous flow in a porous medium

averaged fluid velocity — pressure gradient — viscosity

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta}\,\nabla p$$

$\mathbf{k}$ = fluid permeability tensor

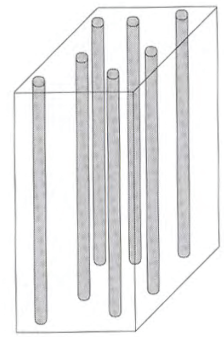
HOMOGENIZATION

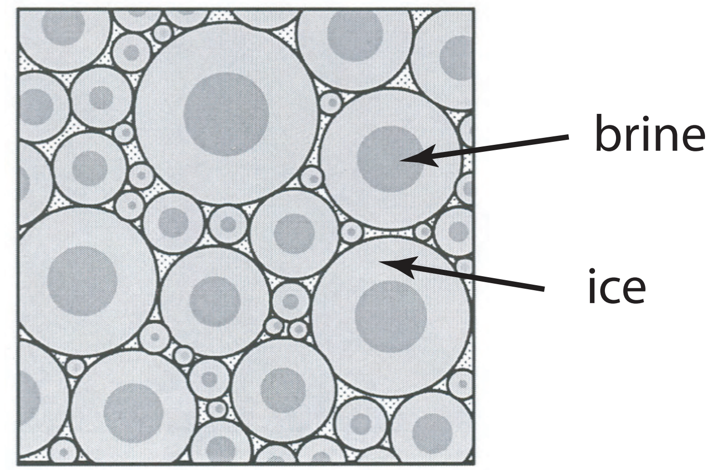
mathematics for analyzing effective behavior of heterogeneous systems

PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006
Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

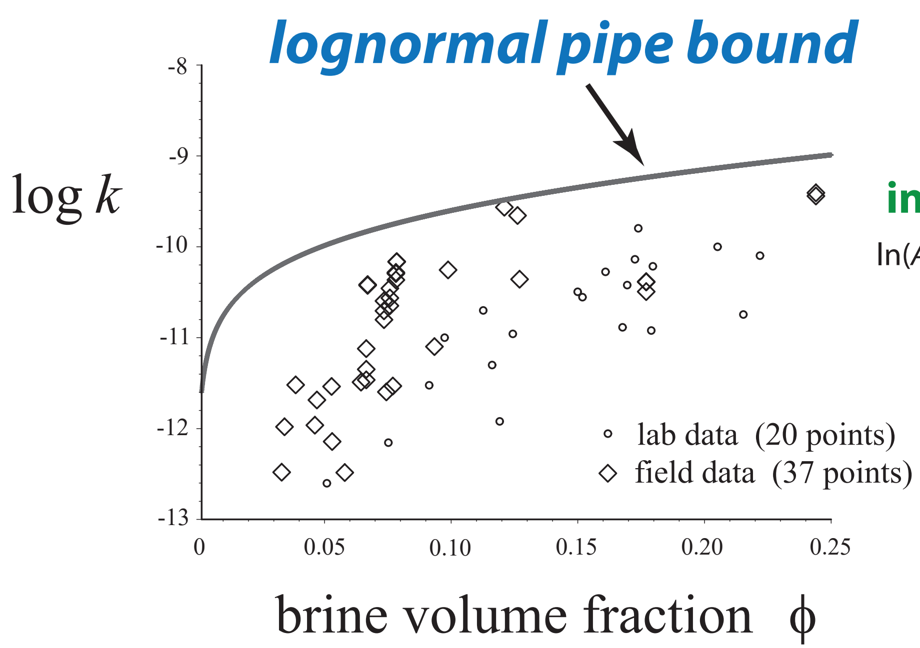
vertical pipes
with appropriate radii
maximize k





optimal coated cylinder geometry

fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)



Golden et al., Geophys. Res. Lett. 2007

$$k \;\leq\; \frac{\phi\langle R^4\rangle}{8\langle R^2\rangle} \;=\; \frac{\phi}{8}\langle R^2\rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

ln(A) normally distributed, mean μ (increases with T) variance σ^2 (Gow and Perovich 96)

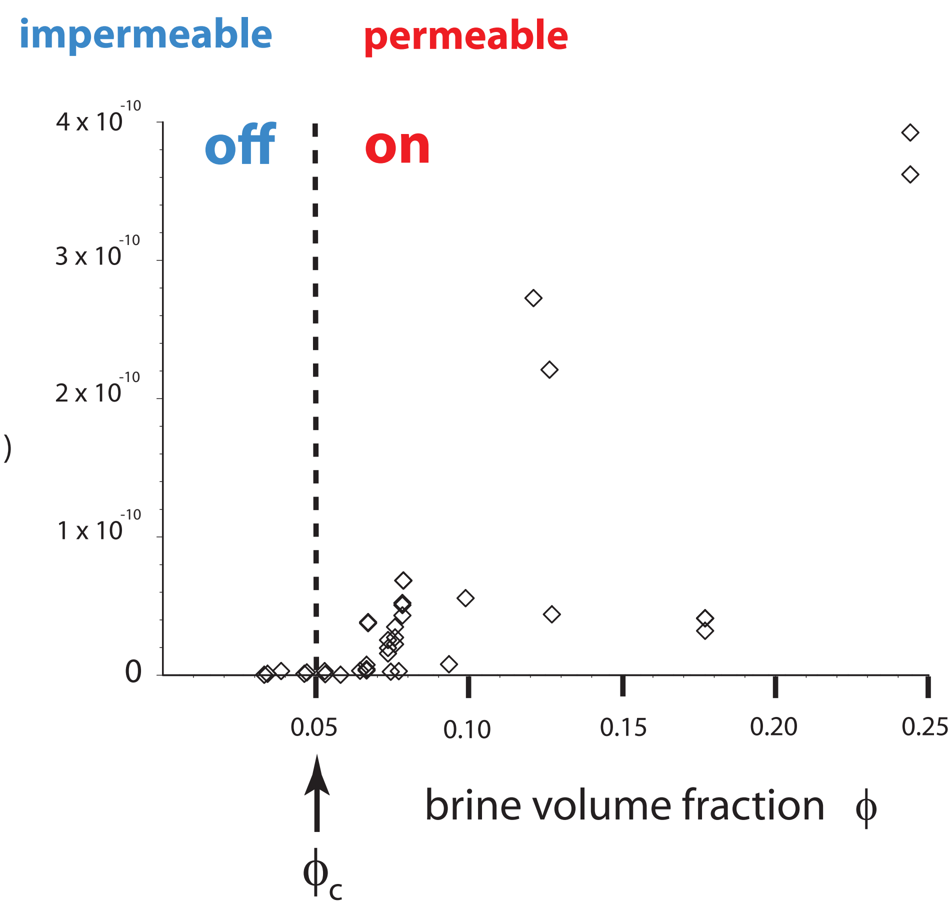
get bounds through variational analyis of ***trapping constant*** γ for diffusion process in pore space with absorbing BC

Torquato and Pham, *PRL* 2004

$$\mathbf{k} \;\leq\; \gamma^{-1}\mathbf{I}$$

for any ergodic porous medium (Torquato 2002, 2004)

Critical behavior of fluid transport in sea ice



Arctic field data

vertical fluid permeability k (m^2)

on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

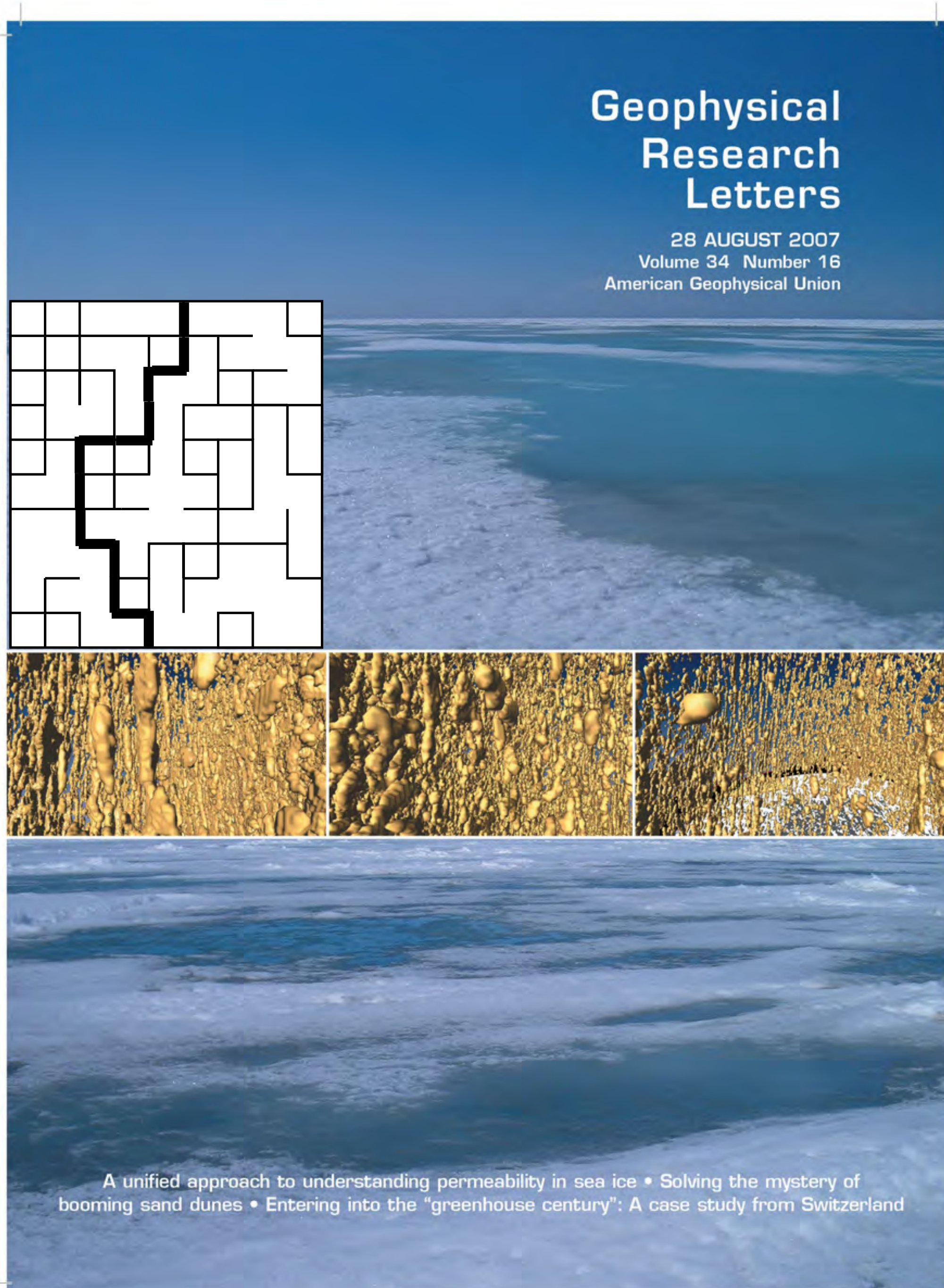
Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



micro-scale

controls

macro-scale

processes

percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8} \ \mathrm{m}^2$$

hierarchical model

network model

rigorous bounds

agree closely
with field data

X-ray tomography for
brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

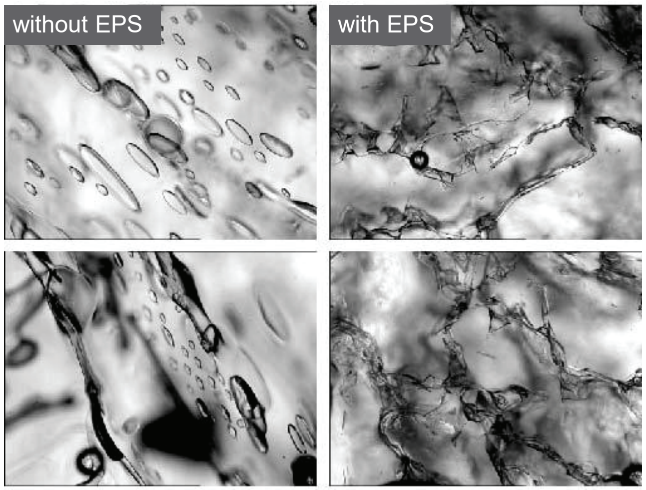
confirms rule of fives

Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009

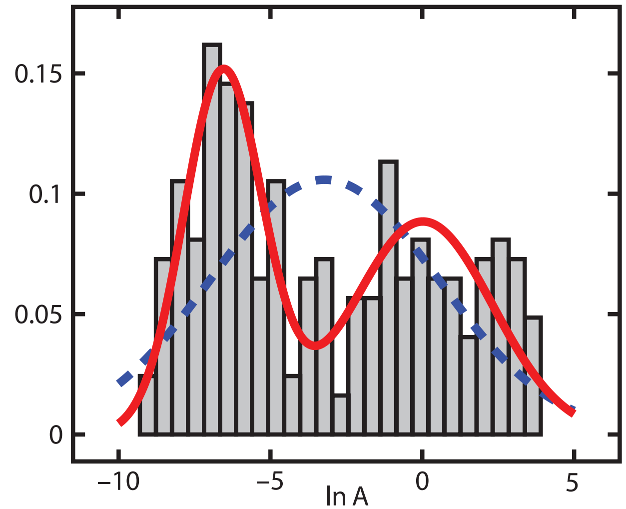
Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport?

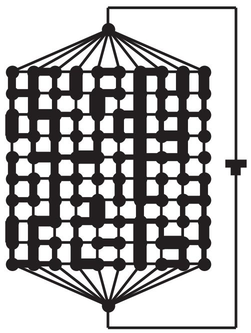
4:30 Today

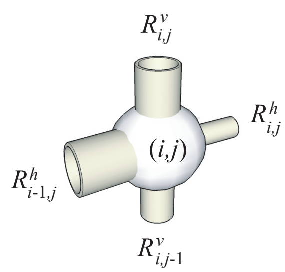


Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL





- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability *k*.
- **Rigorous bound on *k* for bimodal distribution of pore sizes**

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden, Eicken, Morris
Ann. Glac. 2006

How does the biology affect the physics?

Remote sensing of sea ice

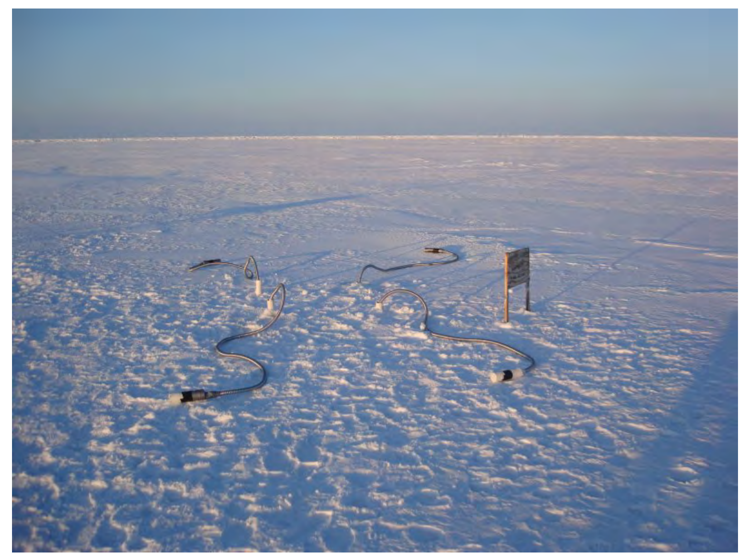
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

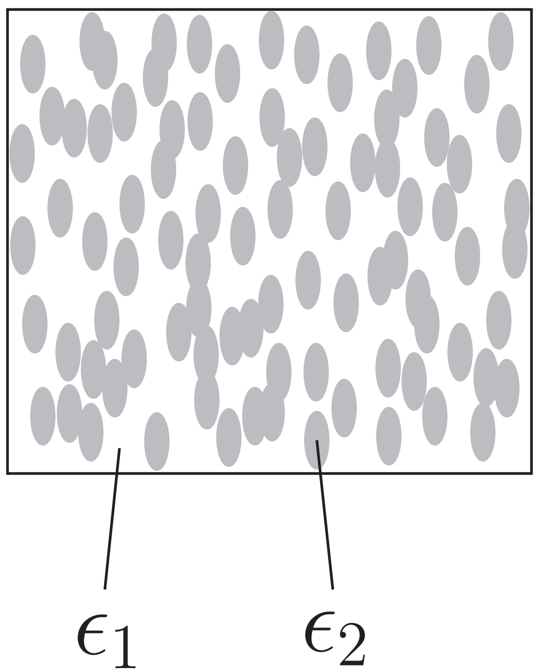
ε^*

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$$\epsilon^*$$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \text{ , composite geometry} \right)$$

Analytic Continuation Method

Bergman (1978), Milton (1979), Golden and Papanicolaou (1983), Theory of Composites, Milton (2002)

Stieltjes integral representation for homogenized parameter

$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

geometry (→ $d\mu(z)$)

material parameters (→ s)

separates geometry from parameters

μ —
- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

$\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$

$\chi =$ characteristic function of the brine phase

$E = (s + \Gamma\chi)^{-1} e_k$

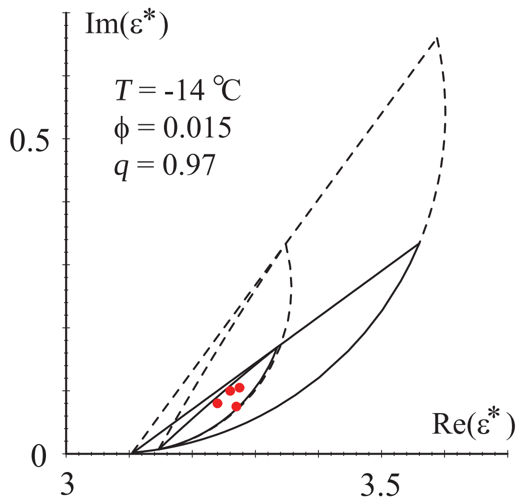
$\Gamma\chi$: microscale $\longrightarrow$ macroscale

$\Gamma\chi$ ***links scales***

Golden and Papanicolaou, Comm. Math. Phys. 1983

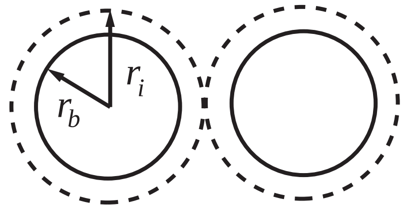
forward and inverse bounds on the complex permittivity of sea ice

forward bounds



4.75 GHz data

matrix particle

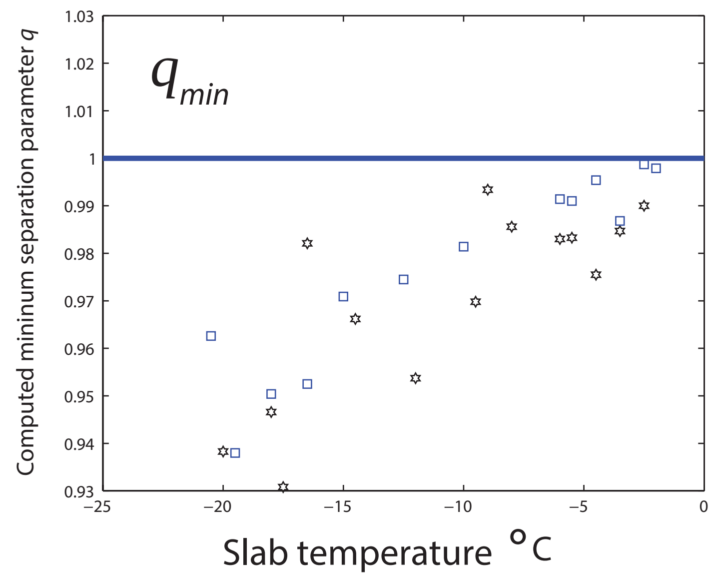


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

direct calculation of spectral measure

1. Discretization of composite microstructure gives lattice of 1's and 0's (random resistor network).

2. The fundamental operator $\chi\Gamma\chi$ becomes a random matrix depending only on the composite geometry.

3. Compute the eigenvalues λ_i and eigenvectors of $\chi\Gamma\chi$ with inner product weights α_i

$$\mu(\lambda) = \sum_i \alpha_i \, \delta(\lambda - \lambda_i)$$

Dirac point measure (Dirac delta)

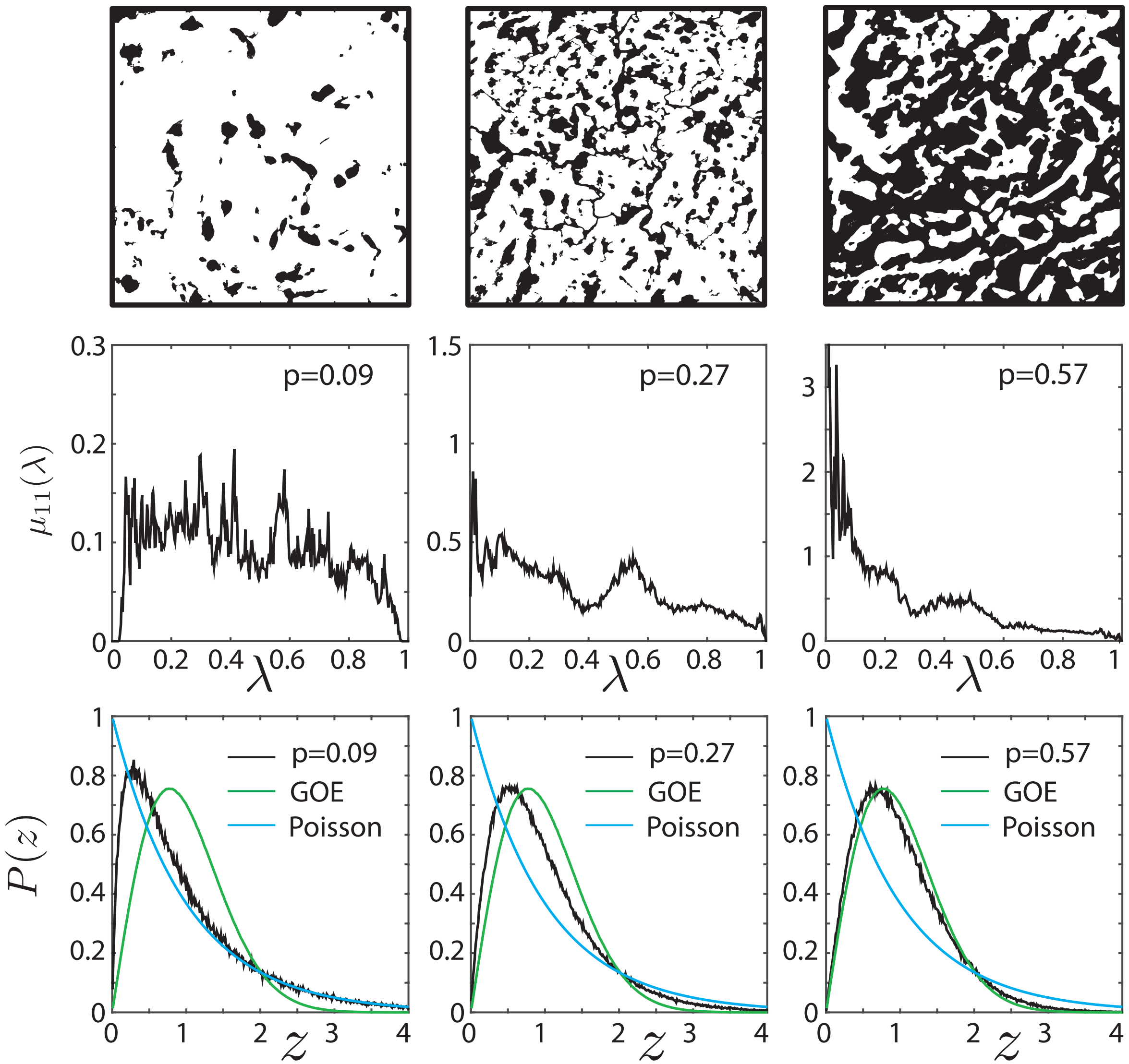
earlier studies of spectral measures

Day and Thorpe 1996
Helsing, McPhedran, Milton 2011

Spectral computations for Arctic melt ponds

spectral measures

eigenvalue spacing distributions



uncorrelated → **level repulsion**

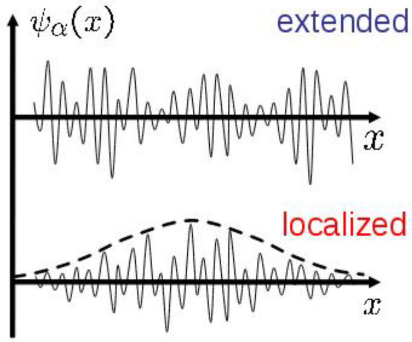
TRANSITION

Anderson Transition in Composites

Ben Murphy
Elena Cherkaev
Ken Golden

***Phys. Rev. Lett.* 2017**

eigenvalue statistics for transport tend toward the ***UNIVERSAL Wigner-Dyson distribution*** *as the "conducting" phase percolates*



metal / insulator transition

localization

Anderson 1958
Mott 1949
Shklovshii et al 1993
Evangelou 1992

Anderson transition in wave physics:
quantum, optics, acoustics, water waves, ...

we find a surprising analog

Anderson transition for classical transport in composites

Murphy, Cherkaev, Golden Phys. Rev. Lett. 2017

PERCOLATION TRANSITION → **transition to universal eigenvalue statistics (GOE) extended states, mobility edges**

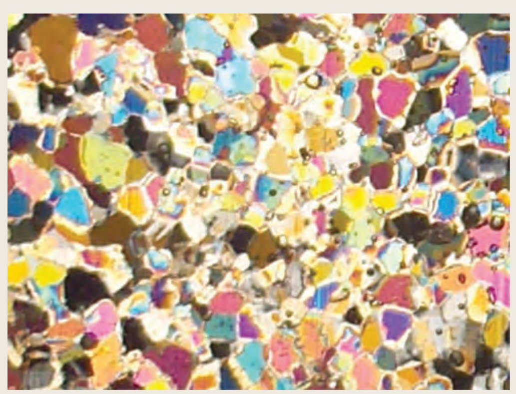
-- but without wave interference or scattering effects ! --

Bounds on the complex permittivity of polycrystalline materials by analytic continuation

Adam Gully, Joyce Lin,
Elena Cherkaev, Ken Golden

- **Stieltjes integral representation for effective complex permittivity**

  Milton (1981, 2002), Barabash and Stroud (1999), ...
- **Forward and inverse bounds**
- **Applied to sea ice using two-scale homogenization**
- **Inverse bounds give method for distinguishing ice types using remote sensing techniques**



Proc. Roy. Soc. A   8 Feb 2015

Proc. R. Soc. A | Volume 471 | Issue 2174 | 8 February 2015

ISSN 1364-5021 | Volume 471 | Issue 2174 | 8 February 2015

PROCEEDINGS A

350 YEARS OF SCIENTIFIC PUBLISHING

An invited review commemorating 350 years of scientific publishing at the Royal Society

A method to distinguish between different types of sea ice using remote sensing techniques

A computer model to determine how a human should walk so as to expend the least energy

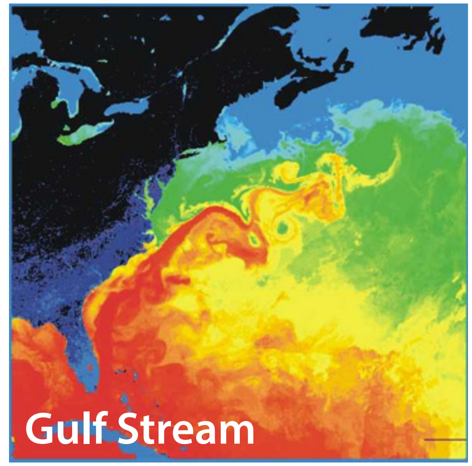
THE ROYAL SOCIETY PUBLISHING

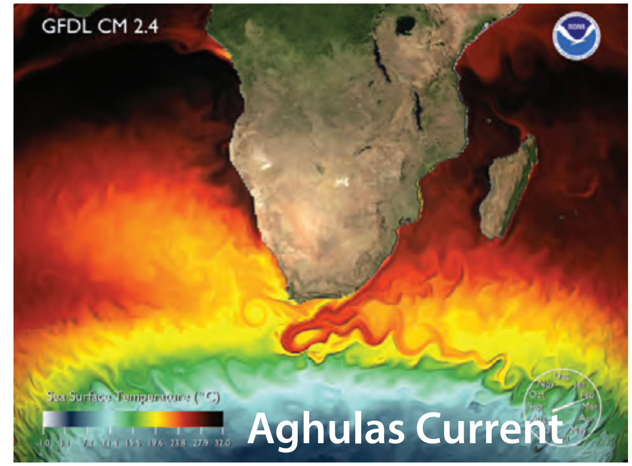
21/01/15 1:47 PM

advection enhanced diffusion

effective diffusivity

sea ice floes diffusing in ocean currents
diffusion of pollutants in atmosphere
salt and heat transport in ocean
heat transport in sea ice with convection





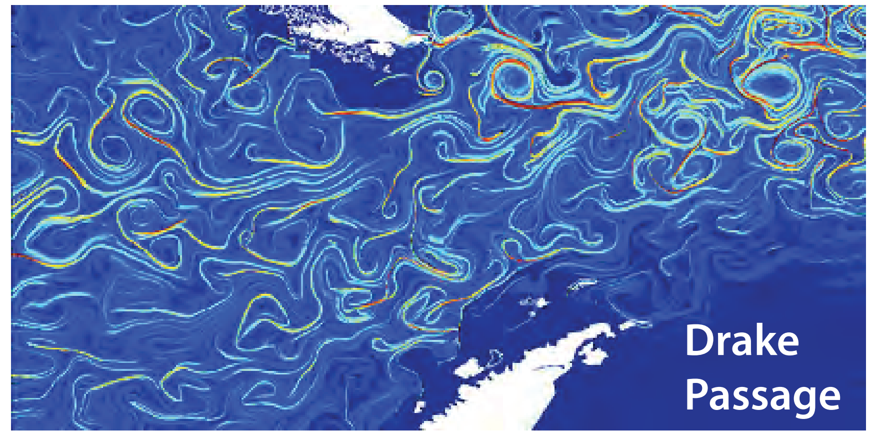
advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$



κ^* ***effective diffusivity***

Stieltjes integral for κ^* ***with spectral measure***

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, *Ann. Math. Sci. Appl.* 2017
Murphy, Cherkaev, Zhu, Xin, Golden, 2018

Stieltjes Integral Representation for Advection Diffusion

[Murphy, Cherkaev, Zhu, Xin & Golden 2018]

[Murphy, Cherkaev, Xin, Zhu & Golden 2017]

$$\kappa^* = \kappa \left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2} \right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}$$

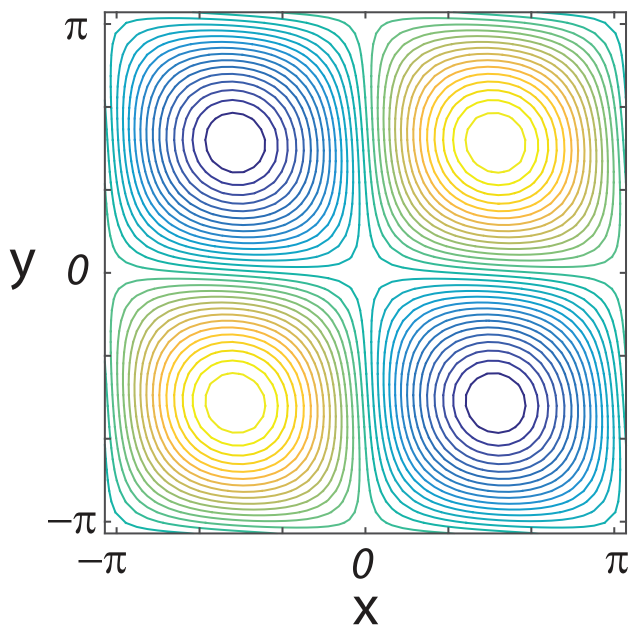
- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H \Gamma$
- $H =$ stream matrix , $\kappa =$ local diffusivity
- $\Gamma := -\nabla(-\Delta)^{-1}\nabla\cdot$, Δ is the Laplace operator
- $i\Gamma H \Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ-plane

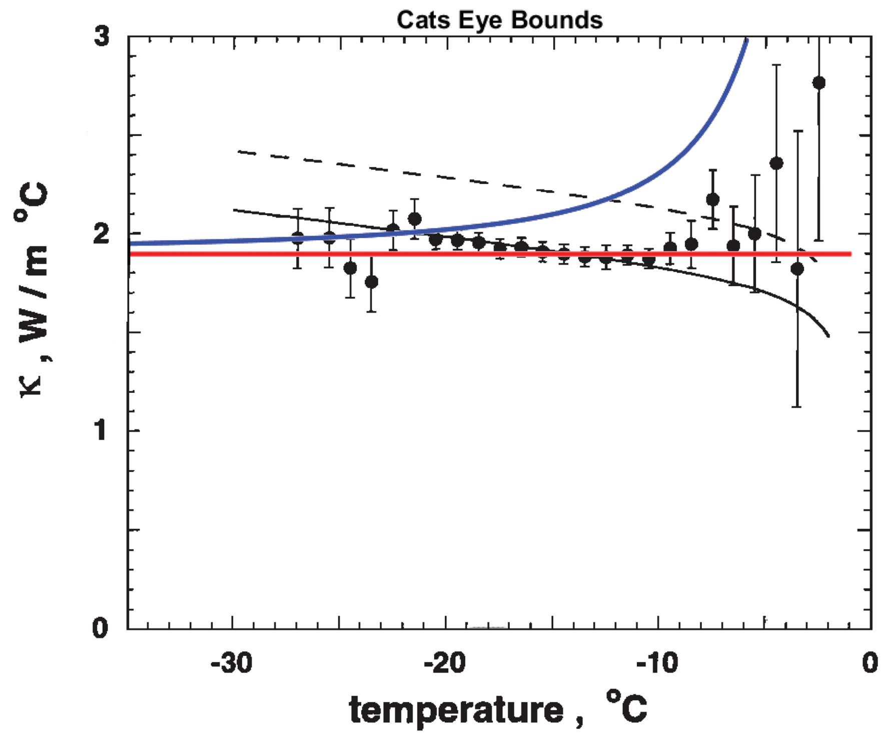
separation of material properties and flow field

spectral measure calculations

RIGOROUS BOUNDS on convection - enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Murphy, Zhu, Cherkaev, Golden 2018

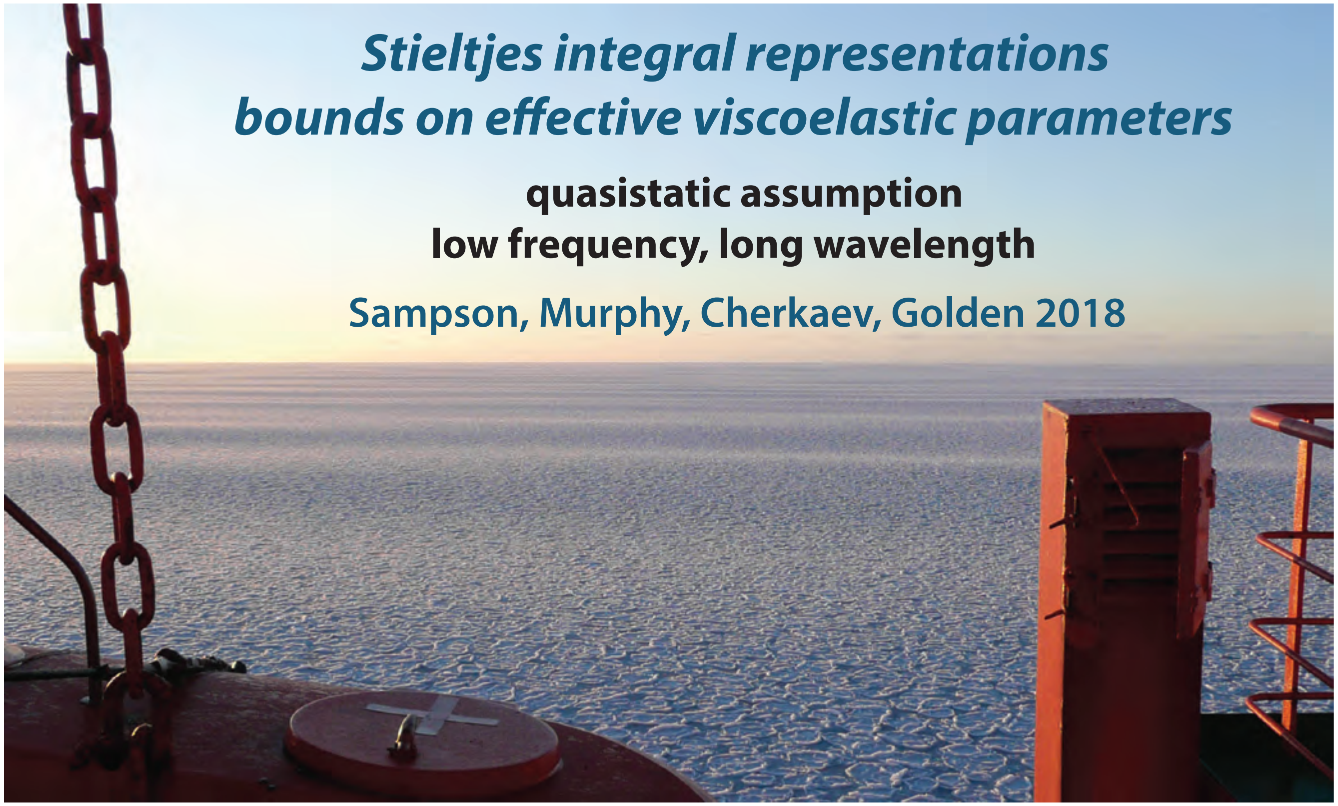




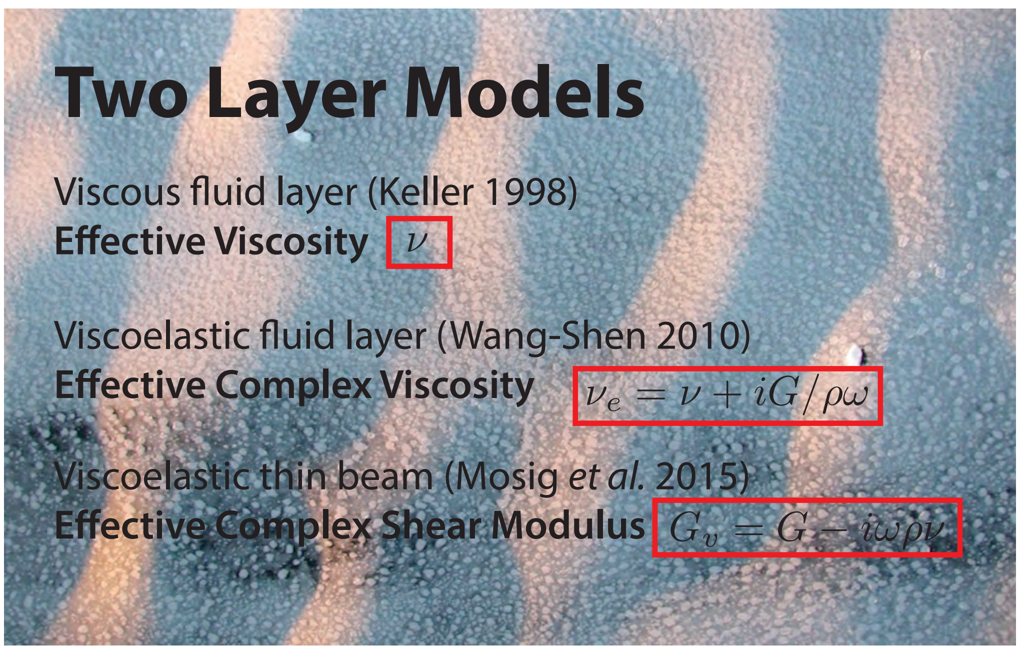
Friday 10:30

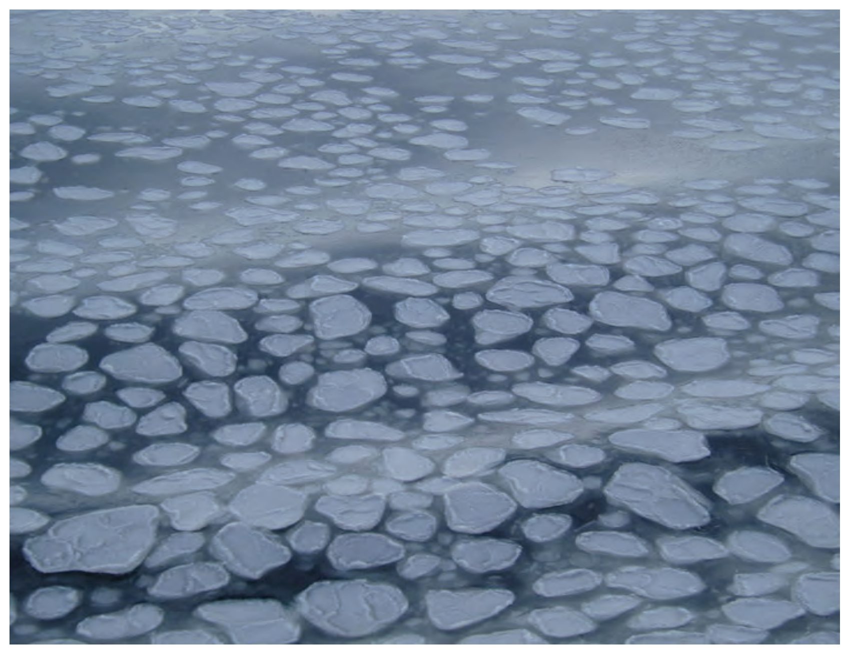
rigorous Padé bounds from Stieltjes integral + analytical calculations of moments of measure

wave propagation in the marginal ice zone



Friday 11:00



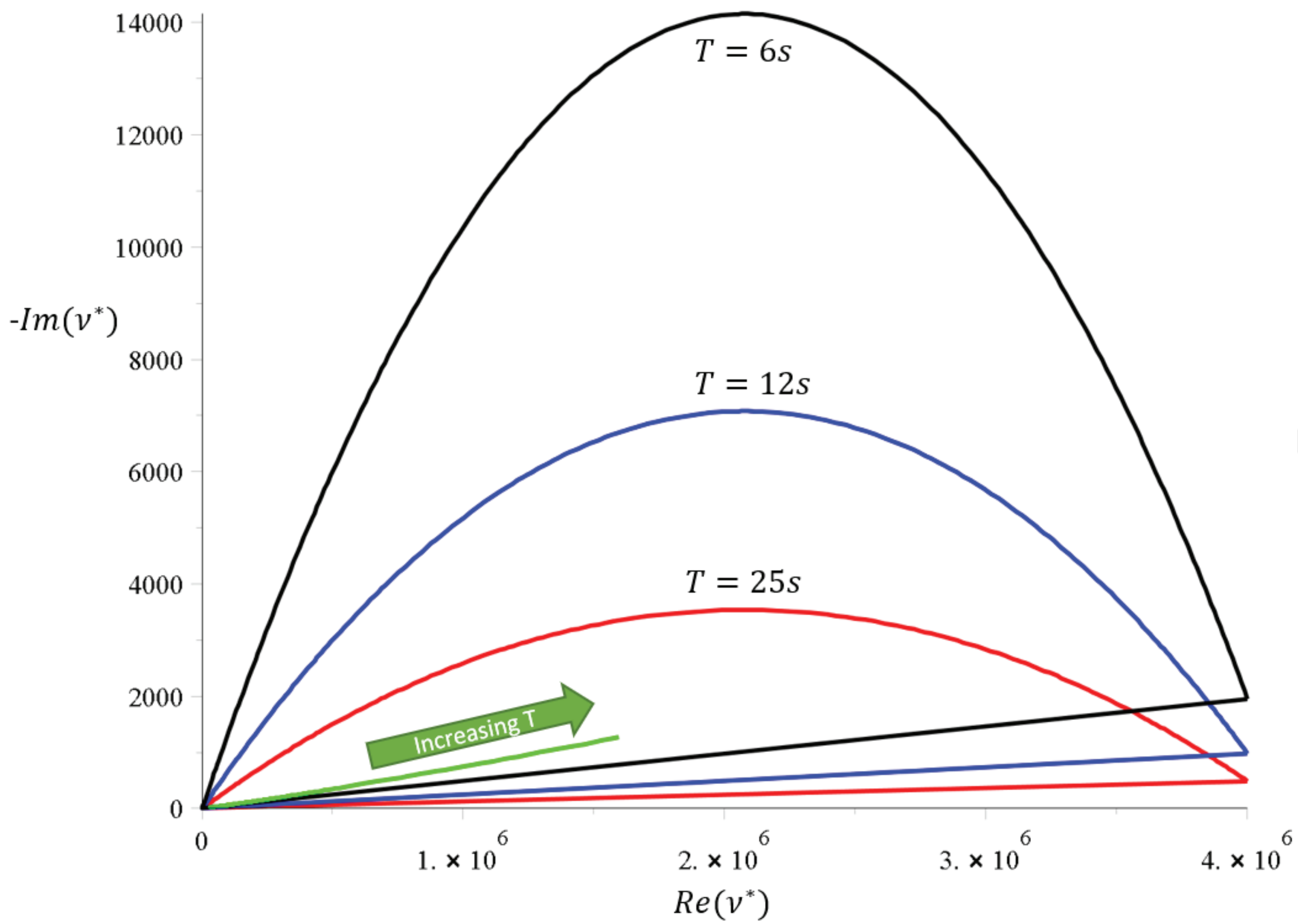


bounds on the effective complex viscoelasticity

complex elementary bounds (fixed area fraction of floes)

$\nu_1 = 10^7 + i\,4875$ **pancake ice**

$\nu_2 = 5 + i\,0.0975$ **slush / frazil**



Sampson, Murphy, Cherkaev, Golden 2018

Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

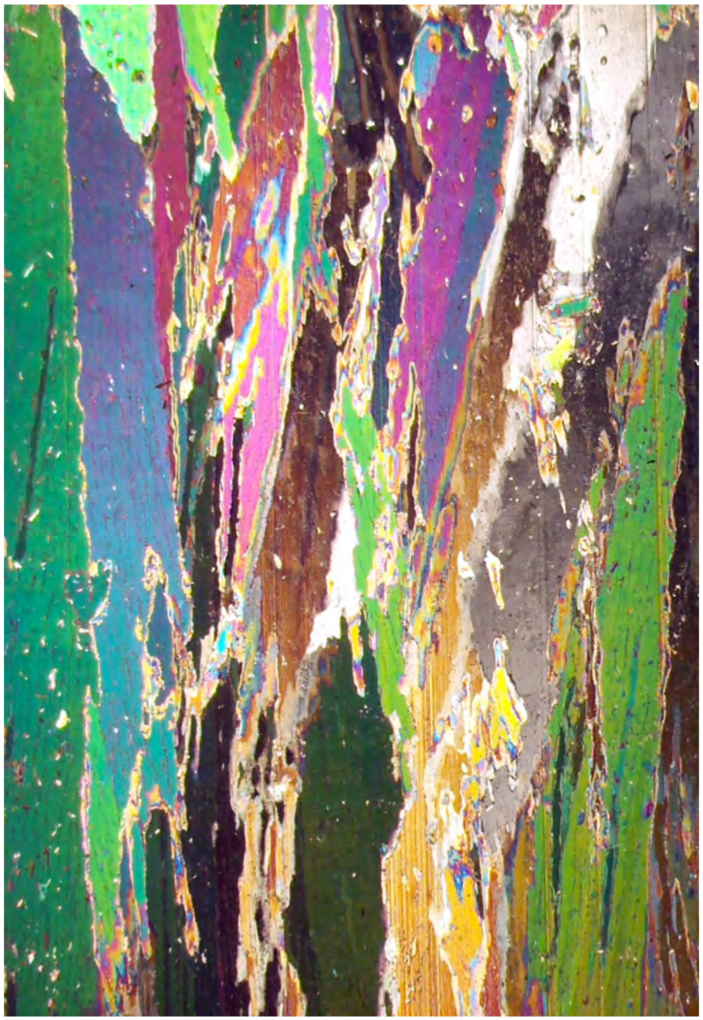
extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

higher threshold for fluid flow in Antarctic granular sea ice

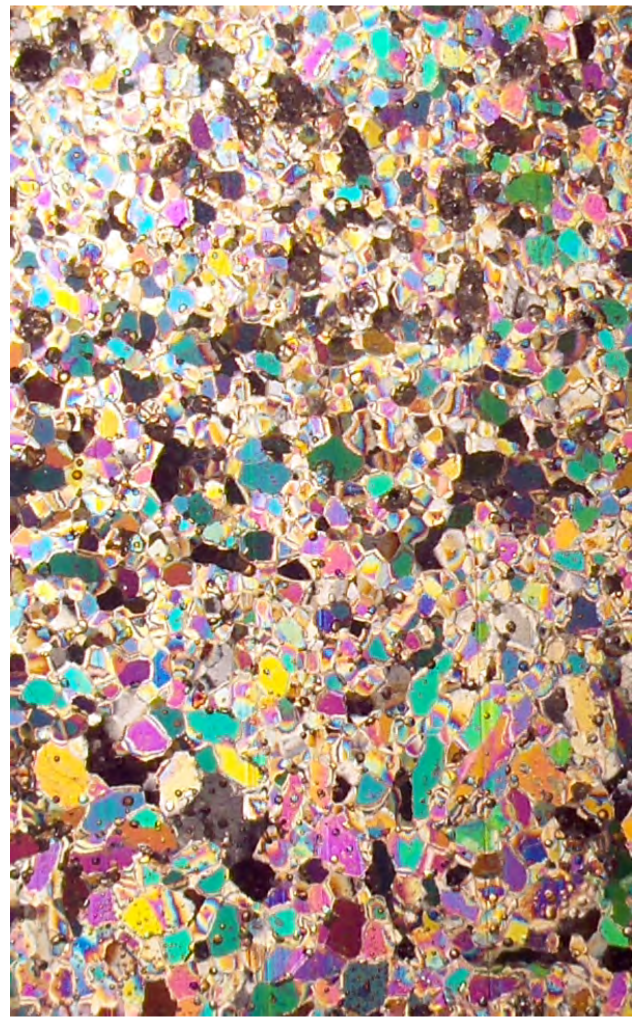
linkage of scales: details of microscale impact macroscale behavior

columnar



5%

granular



10%

Golden, Sampson, Gully, Lubbers, Tison 2018

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

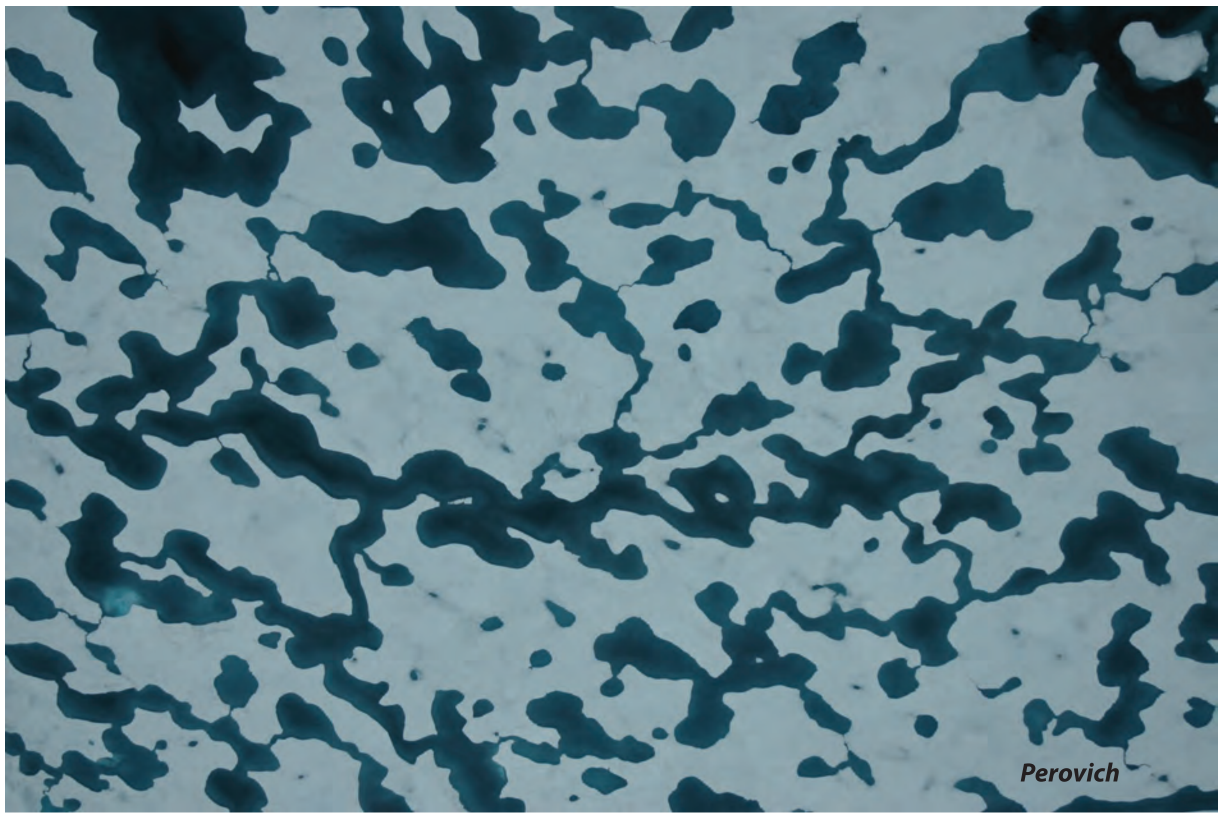
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

Flocco, Feltham, Hunke 2012



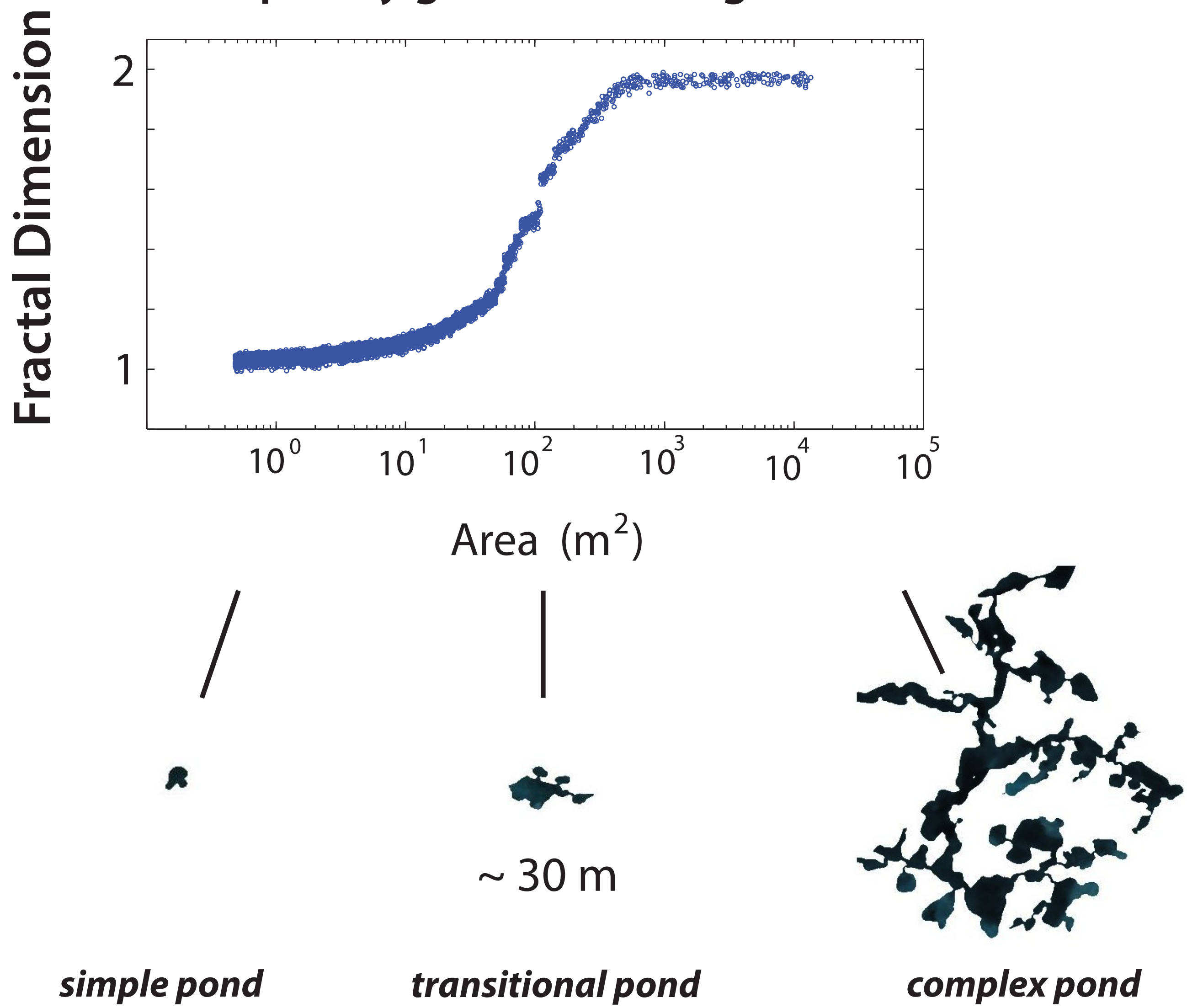
Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

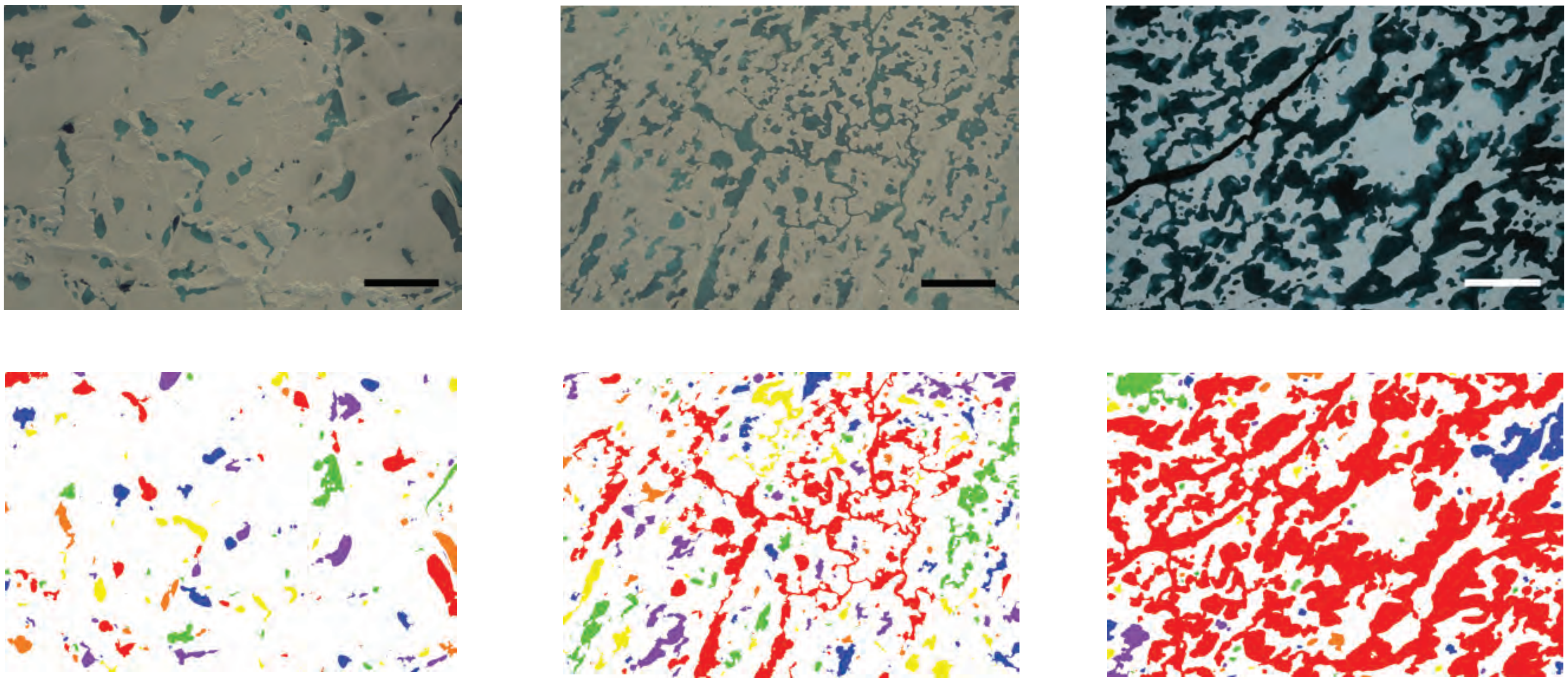
The Cryosphere, 2012

complexity grows with length scale



small simple ponds coalesce to form large connected structures with complex boundaries

Hohenegger, et al., The Cryosphere, 2012



melt pond percolation

results on percolation threshold, correlation length, cluster behavior

A. Cheng (Hillcrest HS), D. Webb (Skyline HS), R. Moore, C. Strong, K. M. Golden

Melt pond geometrical characteristics consistent with behavior of continuum percolation models:

1. Void model

5:30 today

P. Popovic, B. B. Cael, M. Silber, and D. S. Abbot, *Phys. Rev. Lett.* 2018.

disks of varying size which represent ice are placed randomly on the plane (possibly overlapping) with the voids between them representing the ponds

data on pond sizes, area fractions, correlations measured from helicopter photos incorporated into model

2. Random surface model

B. Bowen, C. Strong, K. M. Golden, *J. Fractal Geometry* 2018.

pond boundary = level set of random surface representing snow surface

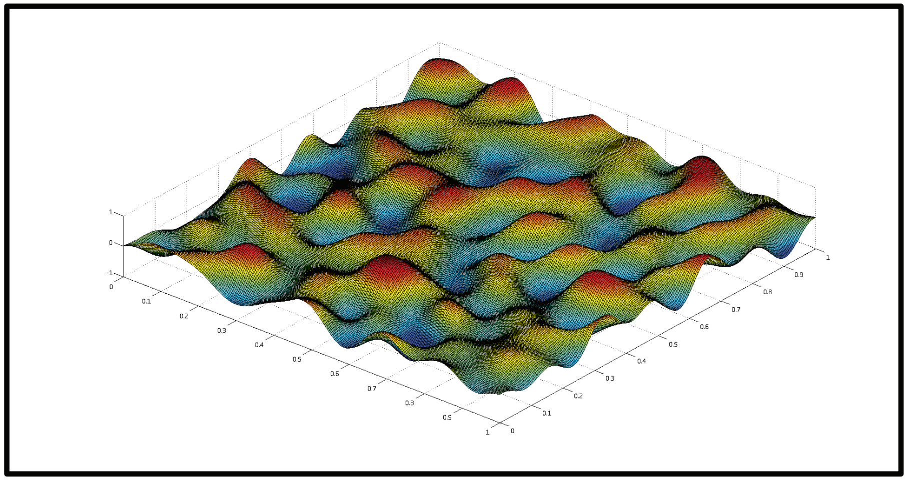
data on snow topography incorporated into model

These models reproduce observed fractal dimension *vs.* area, ...

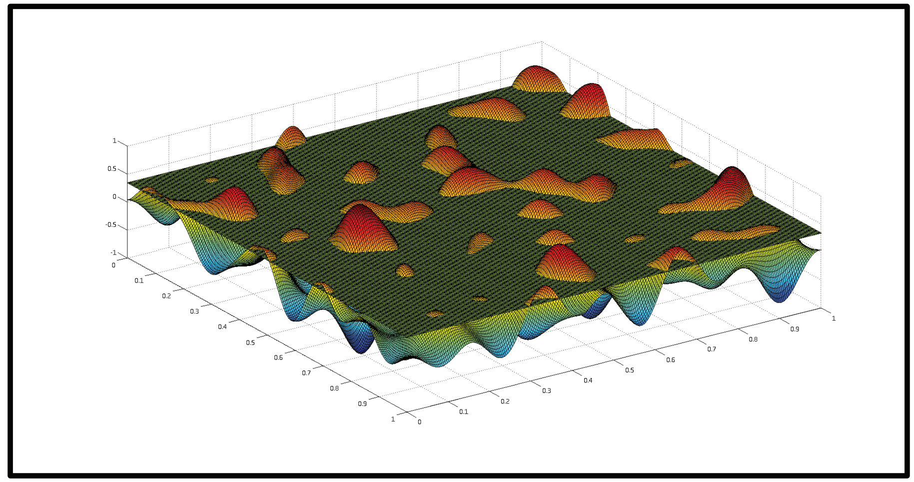
Continuum percolation model for melt pond evolution

level sets of random surfaces

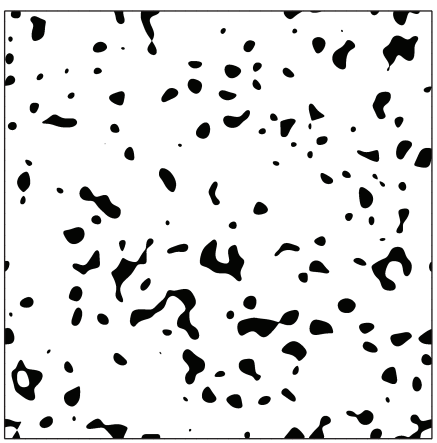
Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018

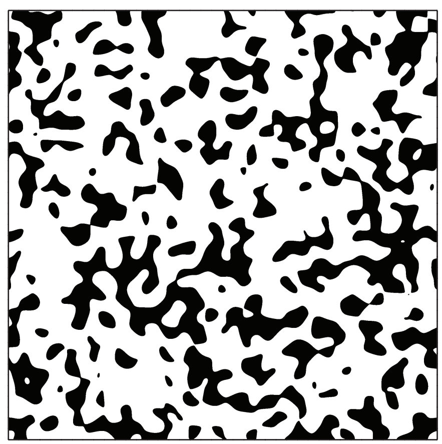


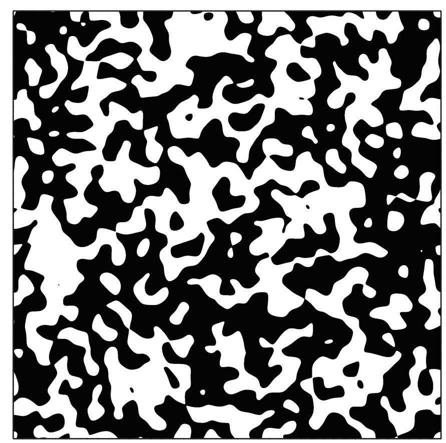
random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds





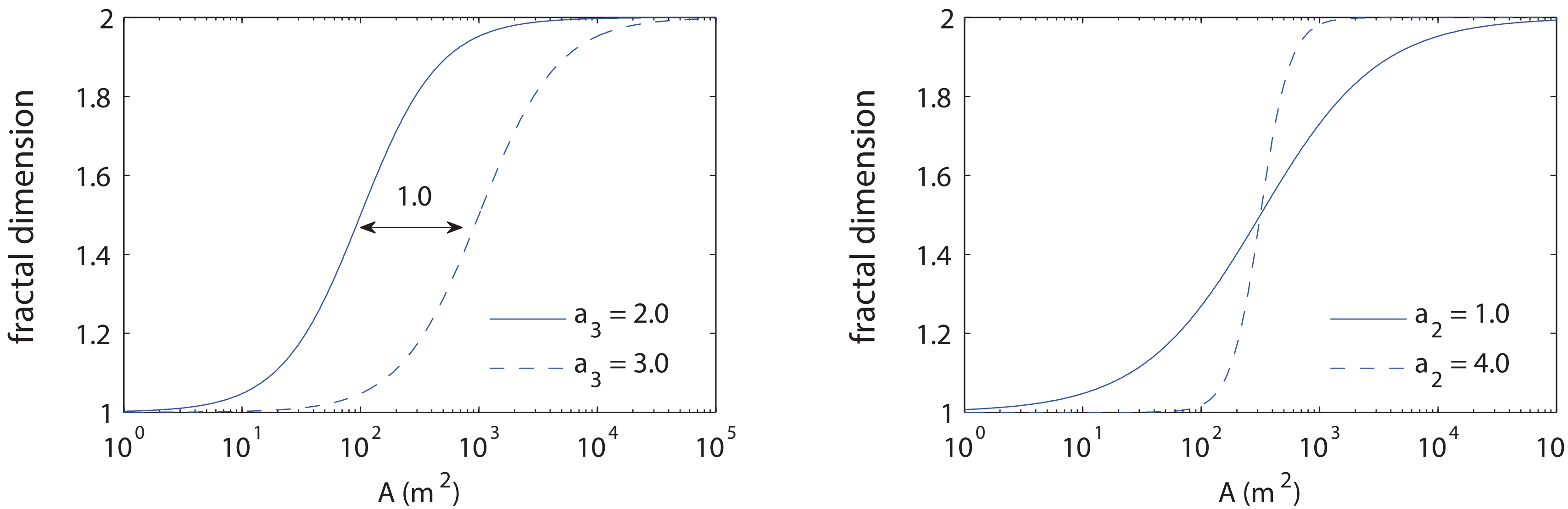


electronic transport in disordered media

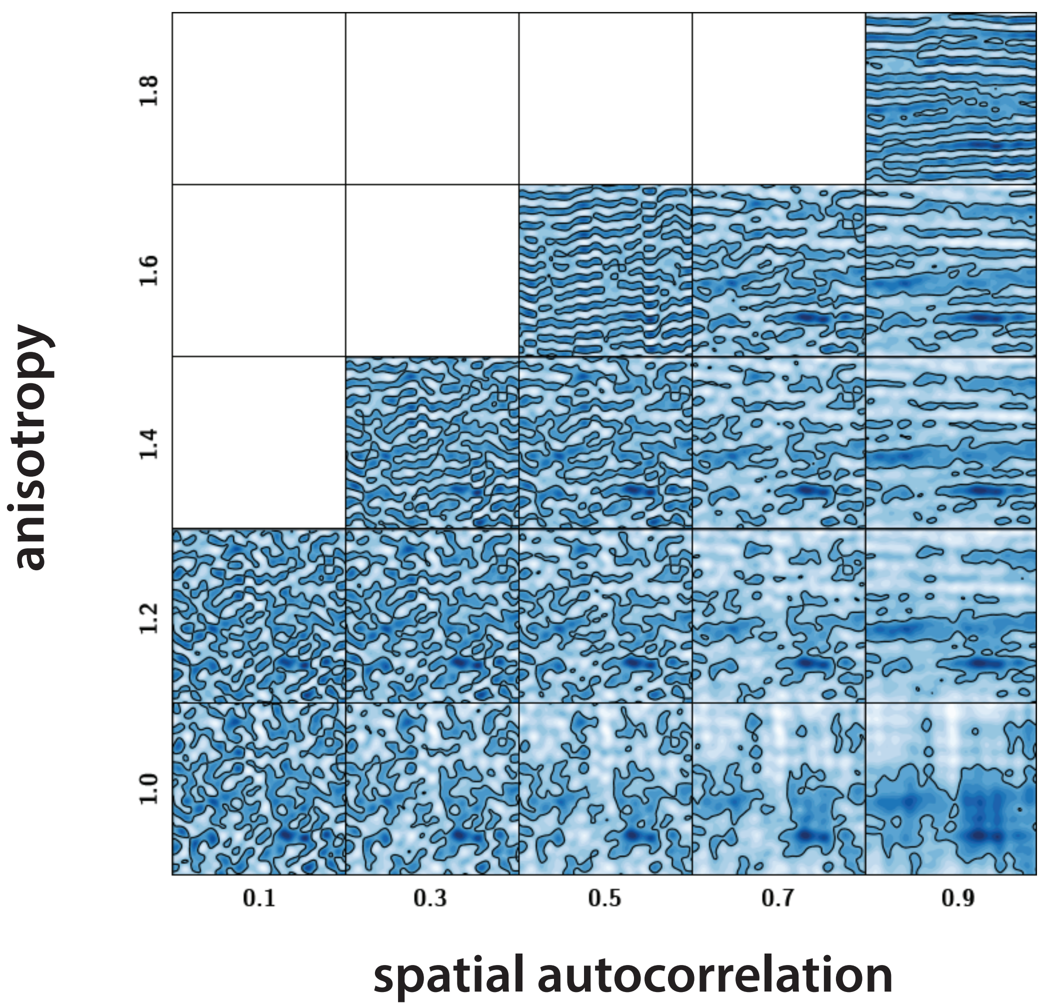
diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

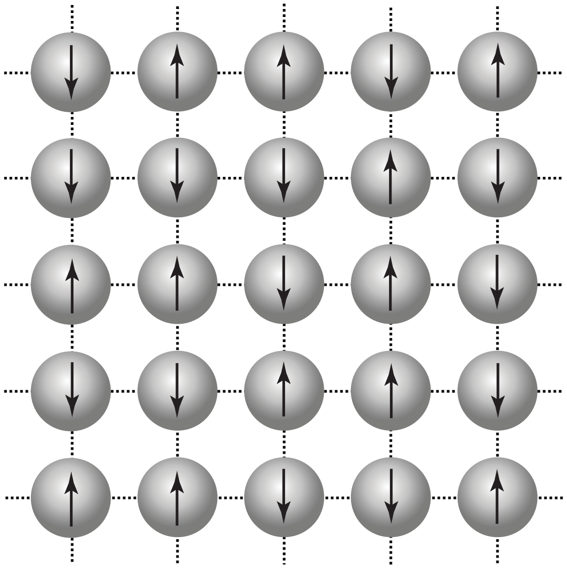
fractal dimension curves depend on statistical parameters defining random surface



Coefficients of Fourier surface chosen to produce topography with given autocorrelation and anisotropy



Ising Model for a Ferromagnet



$$s_i = \begin{cases} +1 & \text{spin up} \\ -1 & \text{spin down} \end{cases}$$

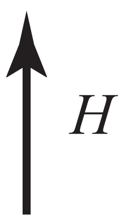
$$\mathcal{H}_\omega = -J \sum_{<i,j>} s_i s_j \; - \; H \sum_i s_i$$

nearest neighbor Ising Hamiltonian

for any configuration $\omega \in \Omega = \{-1, 1\}^N$ of the spins

$$J \geq 0$$

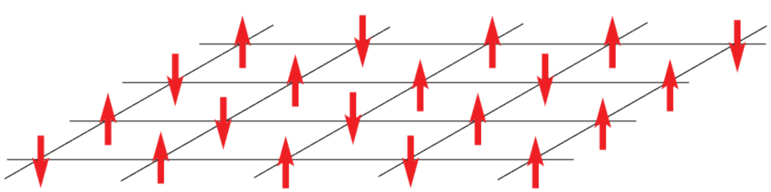
applied magnetic field



canonical partition function

$$Z_N(T, H) = \sum_{\omega \in \Omega} \exp(-\beta \mathcal{H}_\omega) = \exp(-\beta N f_N)$$

$$\beta = 1/kT$$



2–D Ising Model

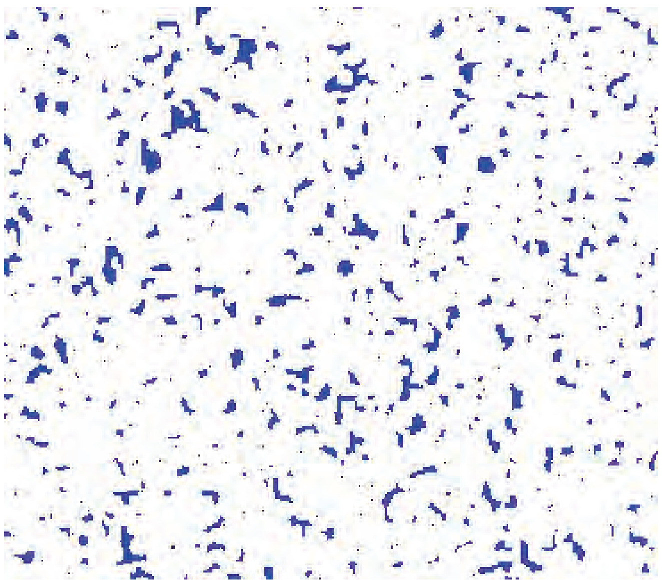
free energy per site

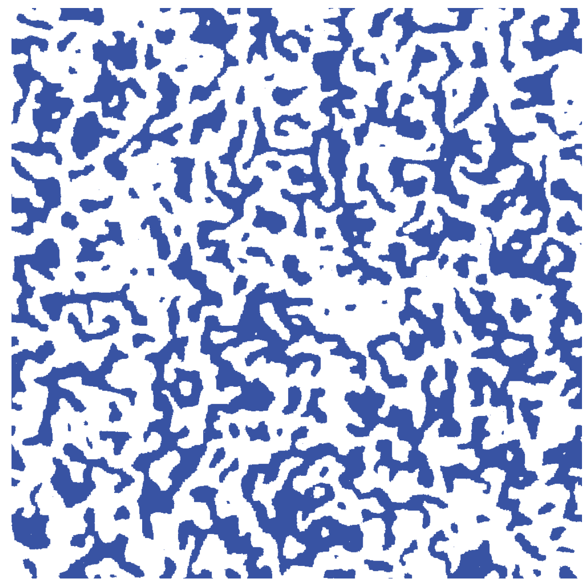
$$f_N(T, H) = \frac{-1}{\beta N} \log Z_N(T, H)$$

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{water} \quad \text{(spin up)} \\ \downarrow & -1 \quad \text{ice} \quad \text{(spin down)} \end{cases}$$

magnetization $M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$ **pond coverage** $\frac{(M+1)}{2}$





"melt ponds" are clusters of magnetic spins that align with the applied field

predictions of fractal transition, pond size exponent **Ma, Sudakov, Strong, Golden 2018**

Glauber Dynamics (Metropolis at T=0):

if spin flip lowers energy, accept
if spin flip raises energy, reject

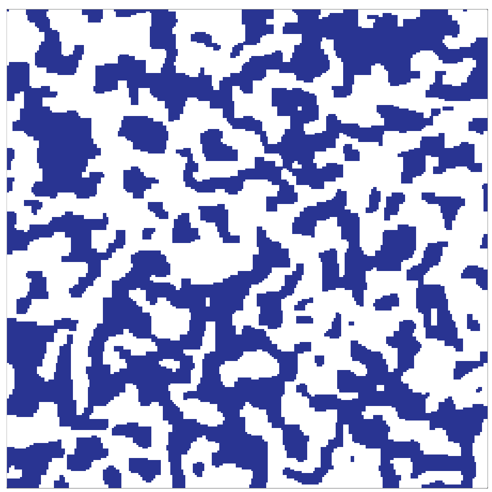
majority wins, water fills troughs

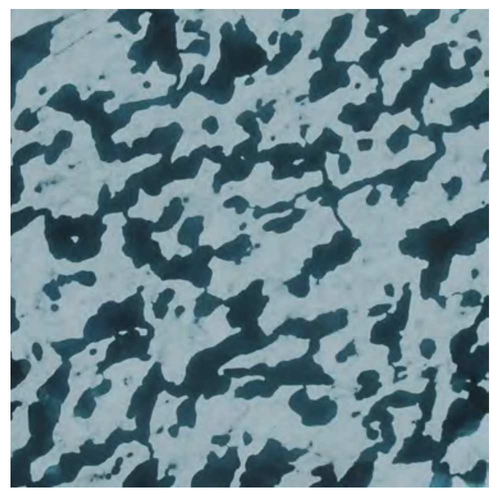
Metropolis algorithm: if lower accept
if raises accept with prob = Gibbs factor

Random initial configuration; as energy is minimized system "flows" toward metastable equilbrium

Order from Disorder

Ising model





melt pond photo

(Perovich)

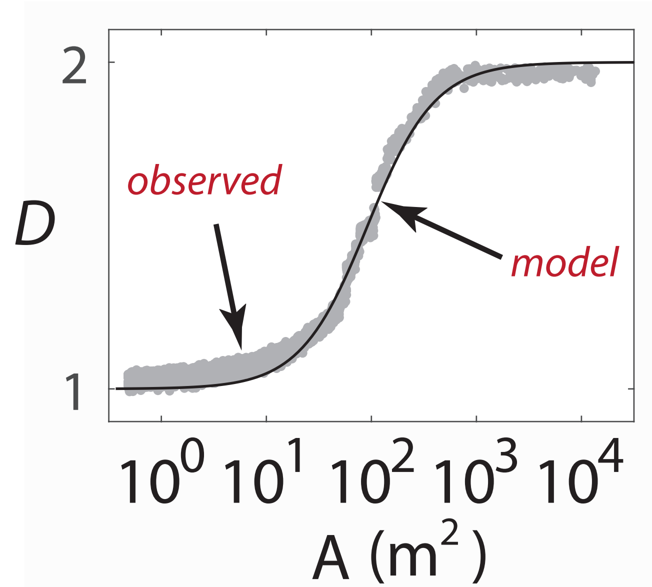
ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE)

Ising model results

Minimize Ising Hamiltonian energy

Random magnetic field represents snow topography; interaction term represents horizontal heat transfer.

Melt ponds —
metastable islands of like spins
in our random field Ising model.

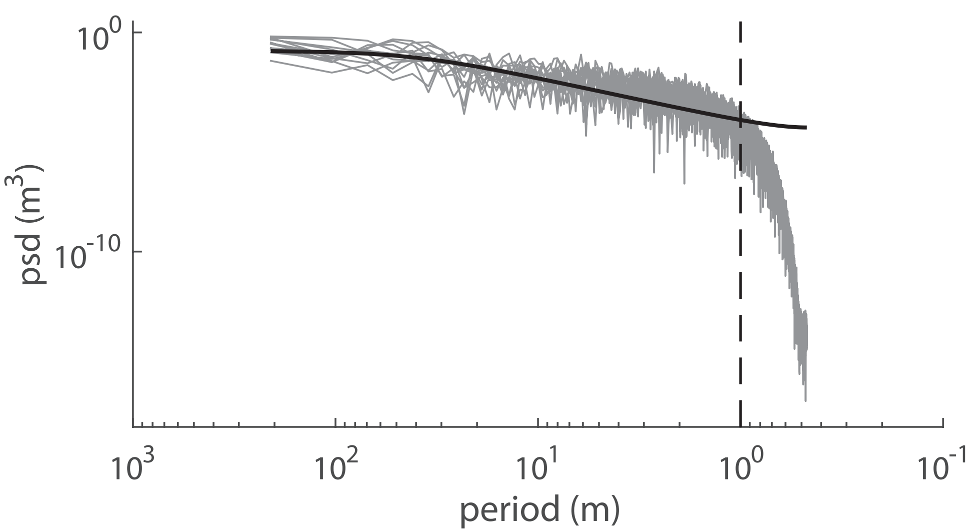


pond size distribution exponent

observed -1.5
(Perovich, et al 2002)

model -1.58

order out of disorder

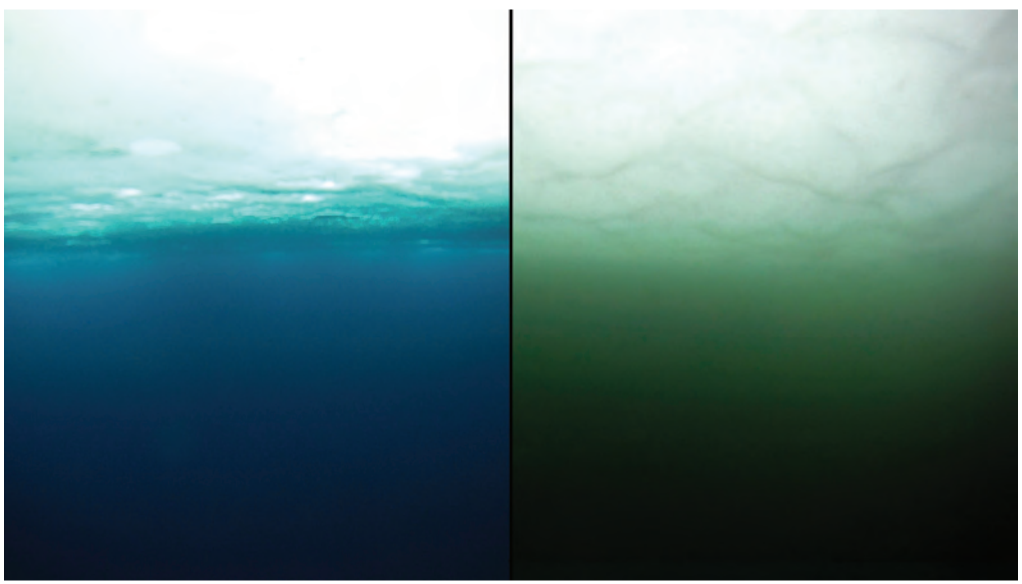


The lattice constant *a* must be small relative to the 10-20 m length scales prominent in sea ice and snow topography. We set *a*=1 m as the length scale above which important spatially correlated fluctuations occur in the power spectrum of snow topography.

2011 massive under-ice algal bloom

Arrigo et al., *Science* 2012

melt ponds act as *WINDOWS* allowing light through sea ice



no bloom bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, ***Science Advances***, 2017

The distribution of solar energy under ponded sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, 2018

(2015 AMS MRC)

The Melt Pond Conundrum:

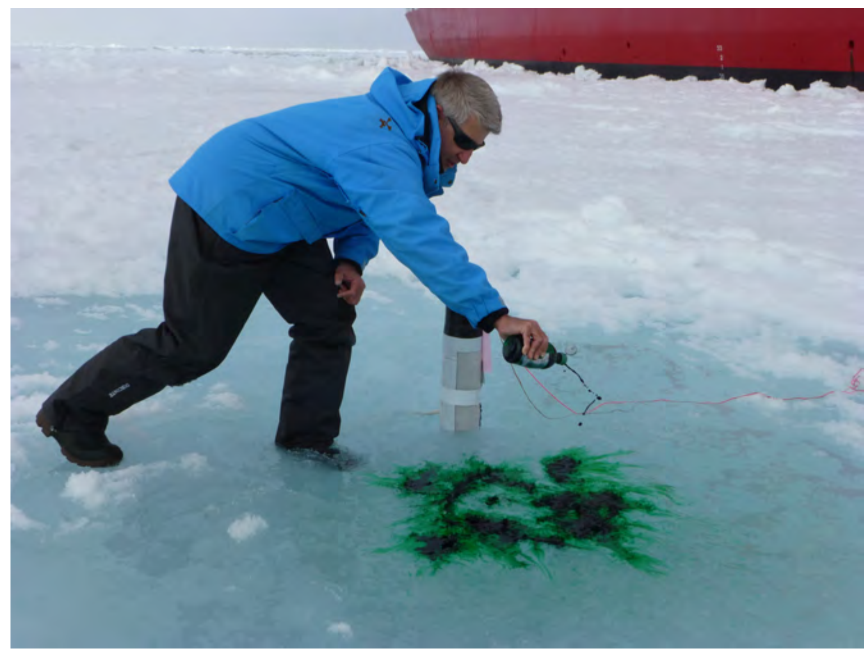
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy



Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.

2. Mathematical methods developed for sea ice advance the theory of composites in general.

2. Homogenization and statistical physics help ***link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.

3. Fluid flow through sea ice mediates melt pond evolution and many processes important to climate change and polar ecosystems.

5. Field experiments are essential to developing relevant mathematics.

6. Our research will help to improve projections of climate change, the fate of Earth's sea ice packs, and the ecosystems they support.

THANK YOU

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs

Office of Naval Research

Arctic and Global Prediction Program

Applied and Computational Analysis Program

Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

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Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999