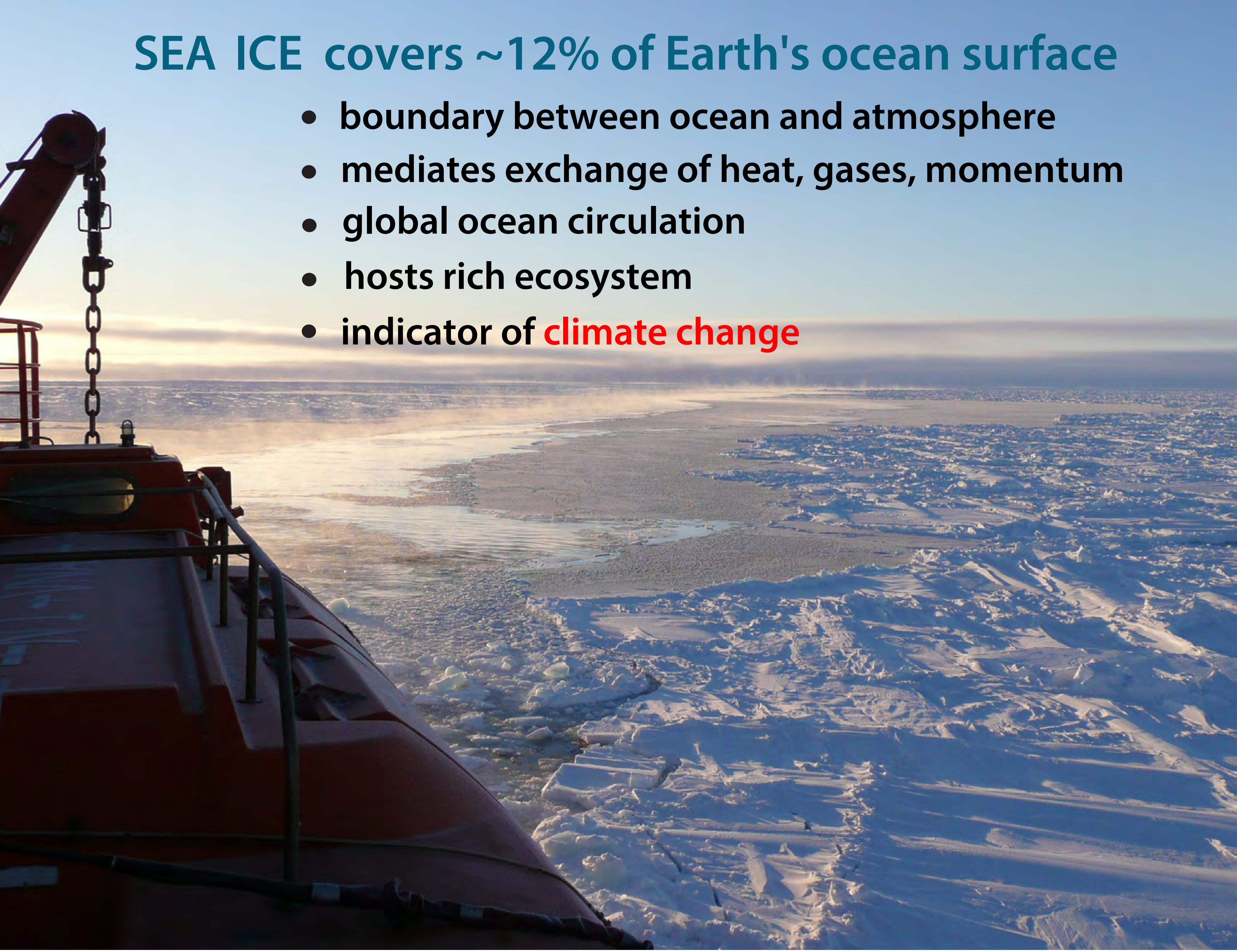
MODELING the MELT: what math tells us about disappearing polar sea ice and its ecosystems

Kenneth M. Golden
Department of Mathematics
University of Utah

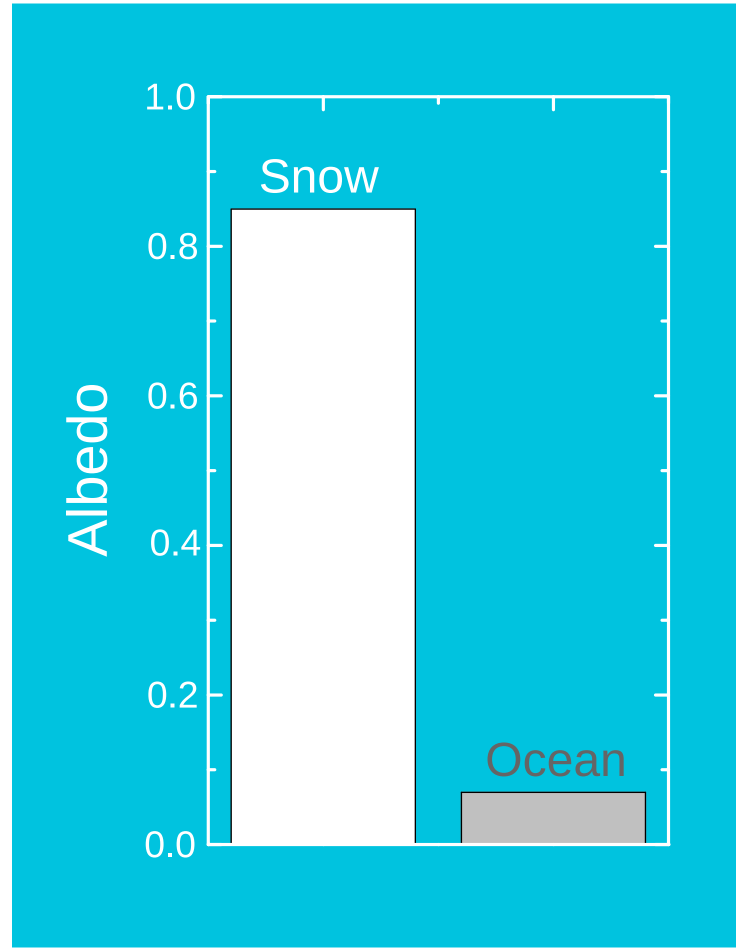
Mathematics Graduate Recruitment Weekend, March 2020

Frey


SEA ICE covers ~12% of Earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• hosts rich ecosystem
• indicator of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

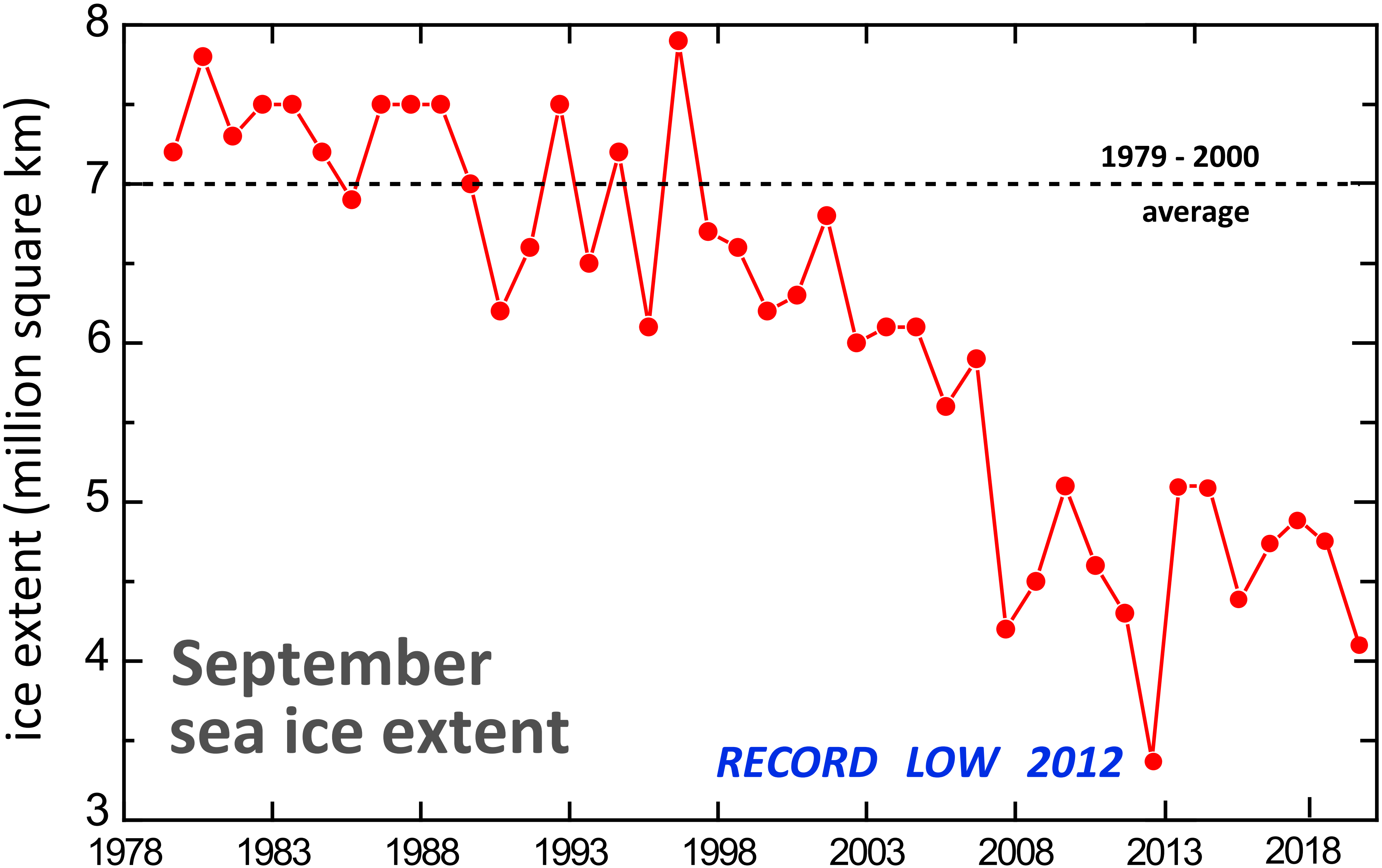
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

the summer Arctic sea ice pack is melting



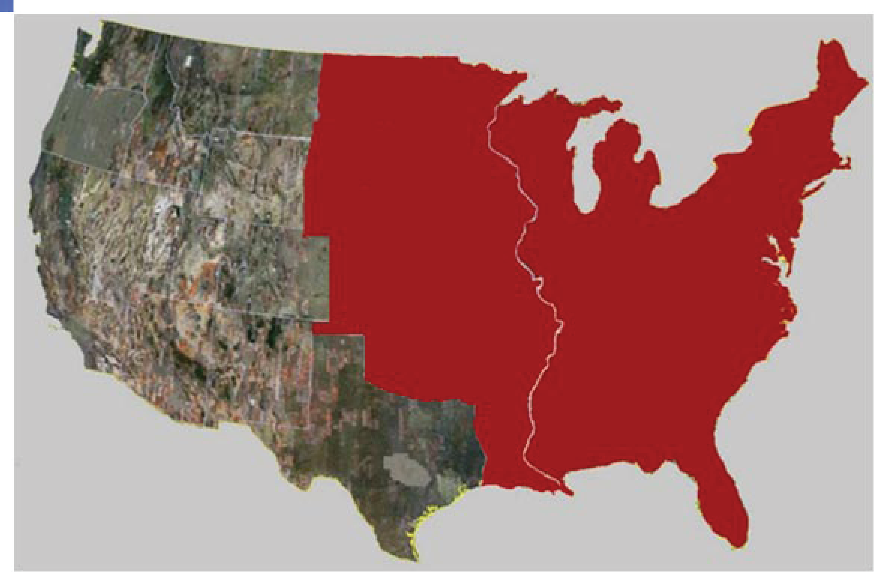
National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million km^2

September 2012 -- 3.4 million km^2

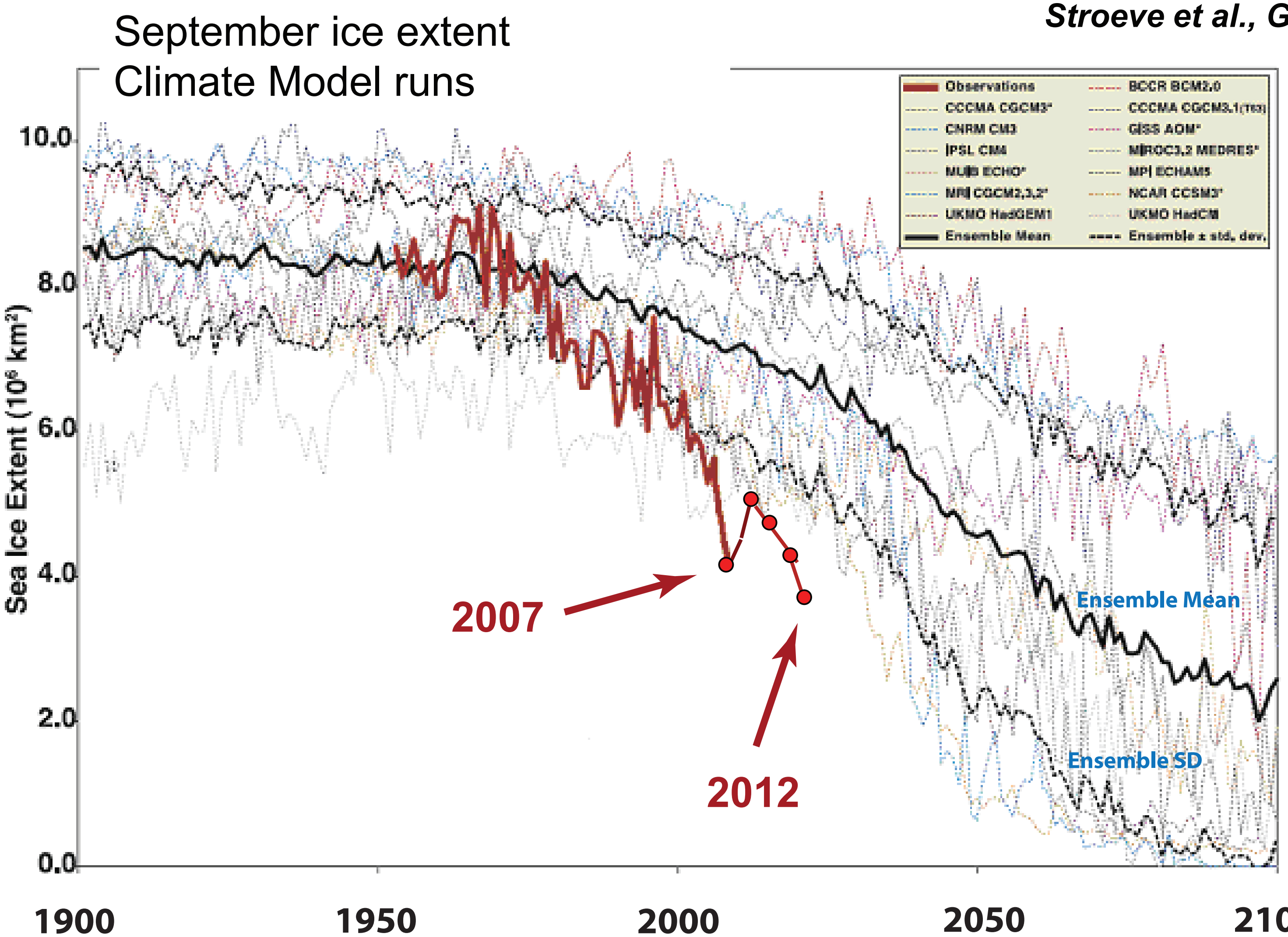
recent losses in comparison to the United States



Perovich

Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007
Stroeve et al., GRL, 2012



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

Fourth Assessment AR4, 2007

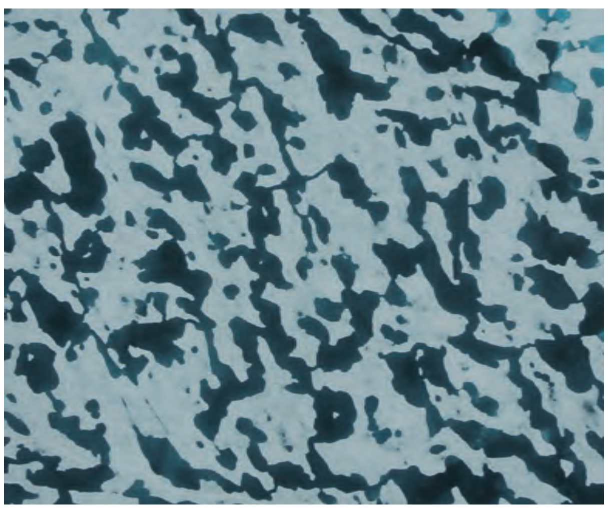
challenge

represent sea ice more realistically in climate models

account for key processes

such as melt pond evolution

How do patterns of dark and light evolve?



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes

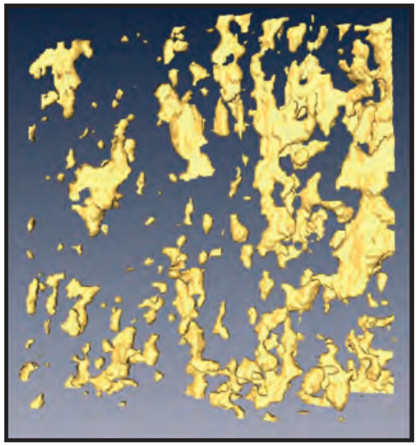
linkage of scales

Sea Ice is a Multiscale Composite Material

sea ice microstructure

brine inclusions

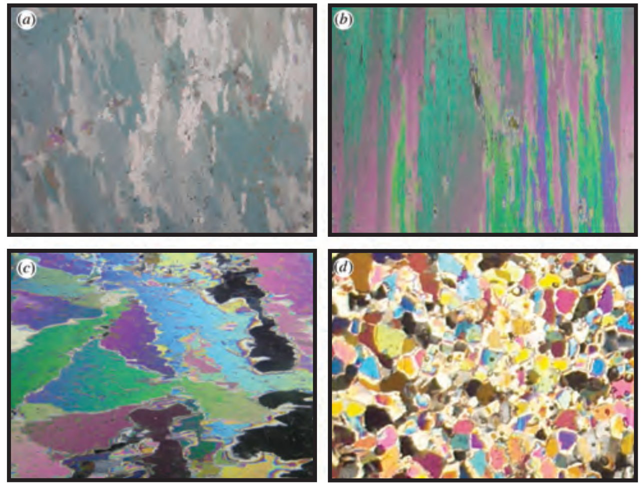
Weeks & Assur 1969



H. Eicken

Golden *et al. GRL* 2007

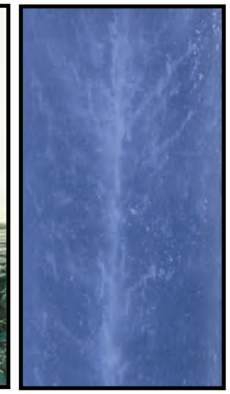
polycrystals



Gully *et al. Proc. Roy. Soc. A* 2015

brine channels

D. Cole



K. Golden

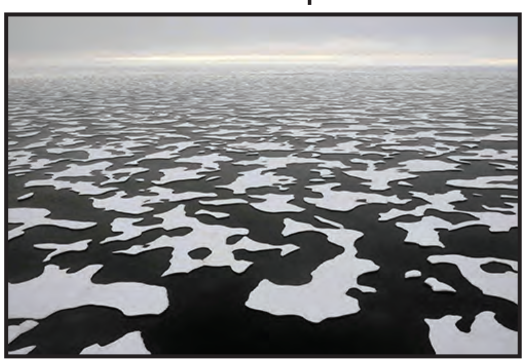
millimeters

centimeters

---

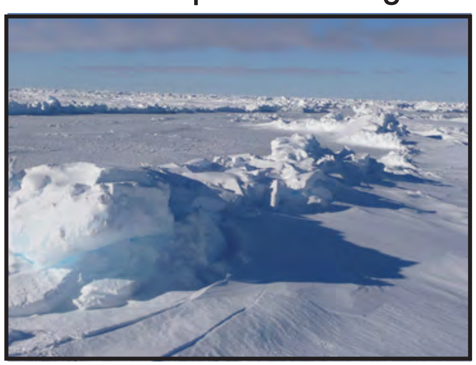
sea ice mesostructure

Arctic melt ponds



K. Frey

Antarctic pressure ridges



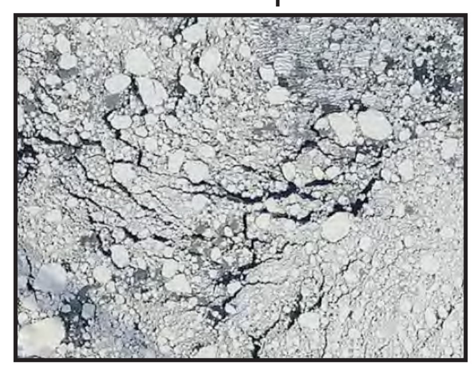
K. Golden

sea ice macrostructure

sea ice floes

J. Weller

sea ice pack



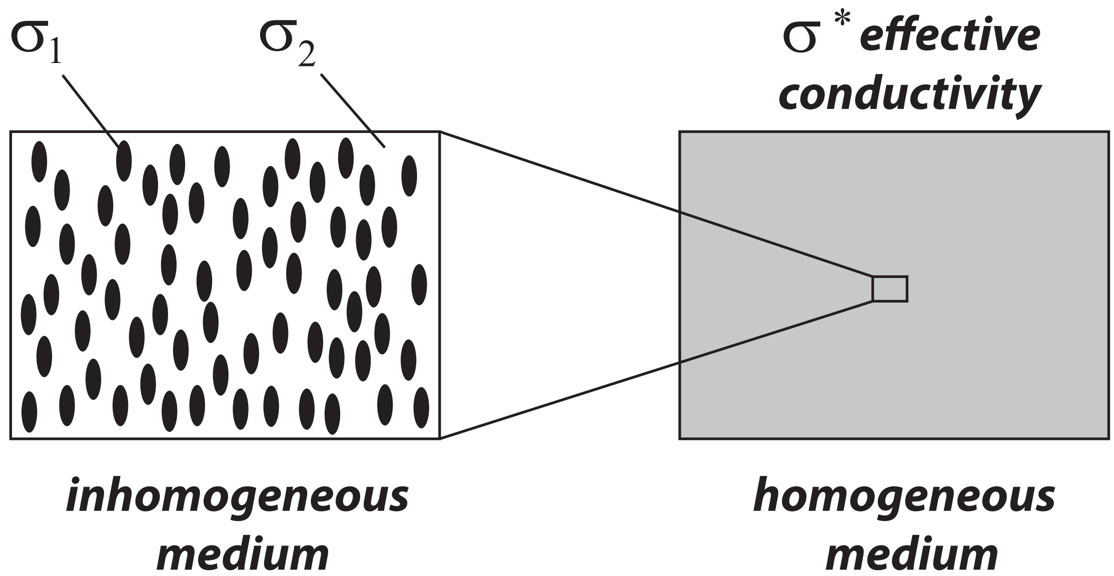
NASA

meters

kilometers

Forward and Inverse HOMOGENIZATION for Composites

LINKING SCALES



FORWARD

find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

INVERSE

find the microstructure which gives rise to observed or desired effective behavior

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

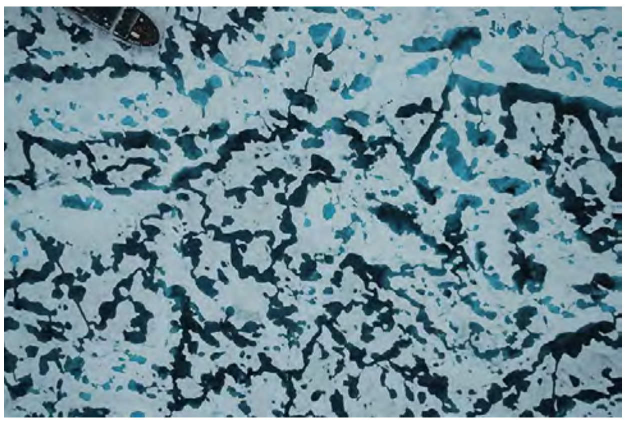
widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

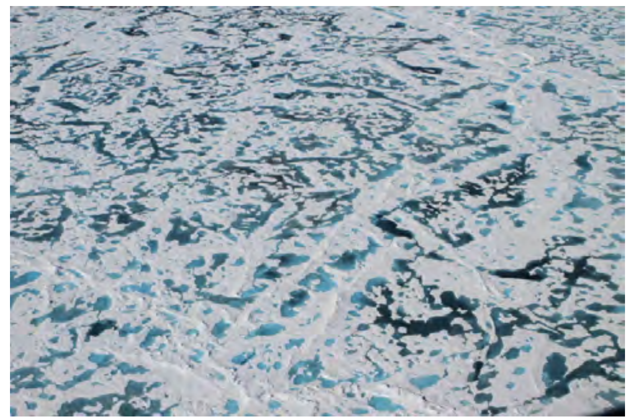
How do scales interact in the sea ice system?

basin scale -
grid scale
albedo

Linking Scales

km
scale
melt
ponds

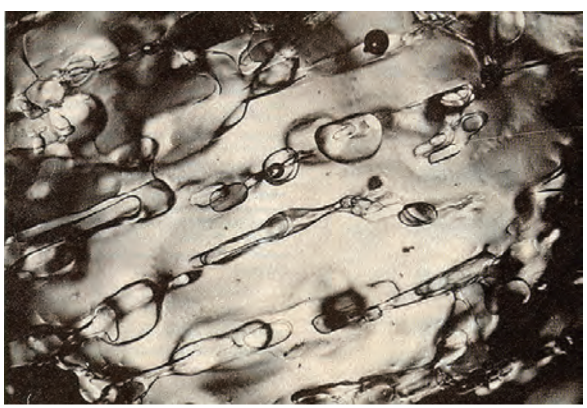




km
scale
melt
ponds

Linking Scales

mm
scale
brine
inclusions

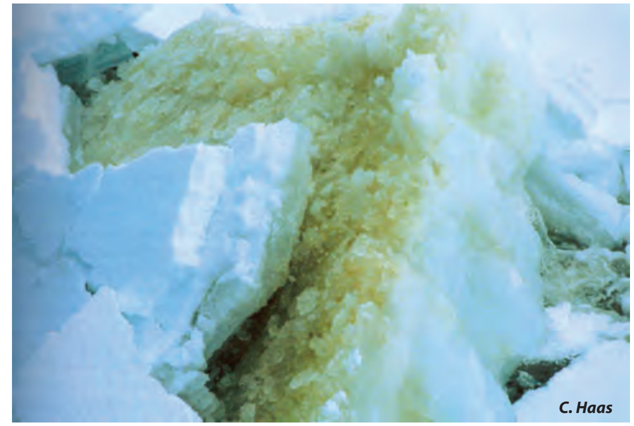


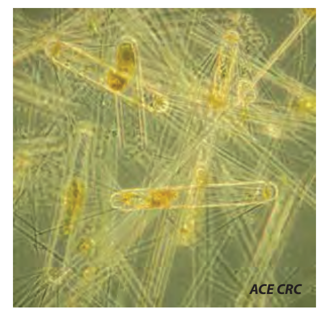
meter
scale
snow
topography

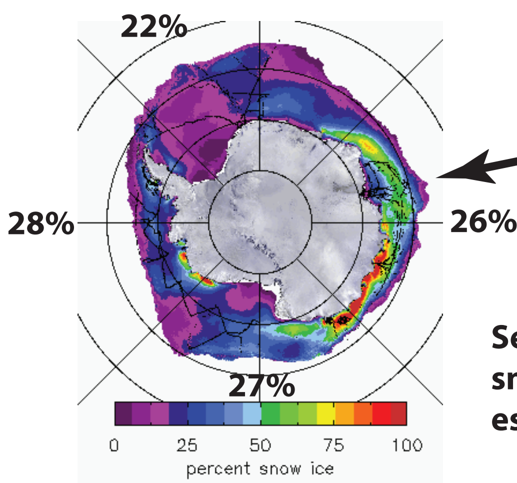
fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

nutrient flux for algal communities







T. Maksym and T. Markus, 2008

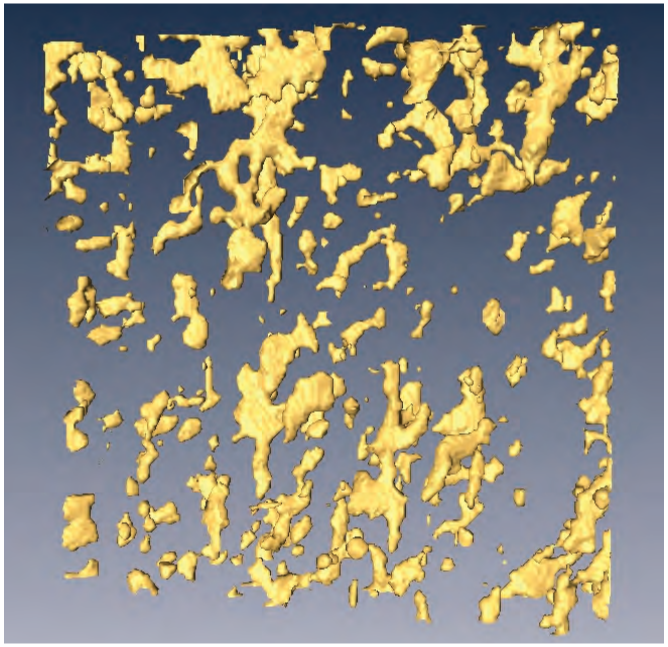
September snow-ice estimates

Antarctic surface flooding and snow-ice formation

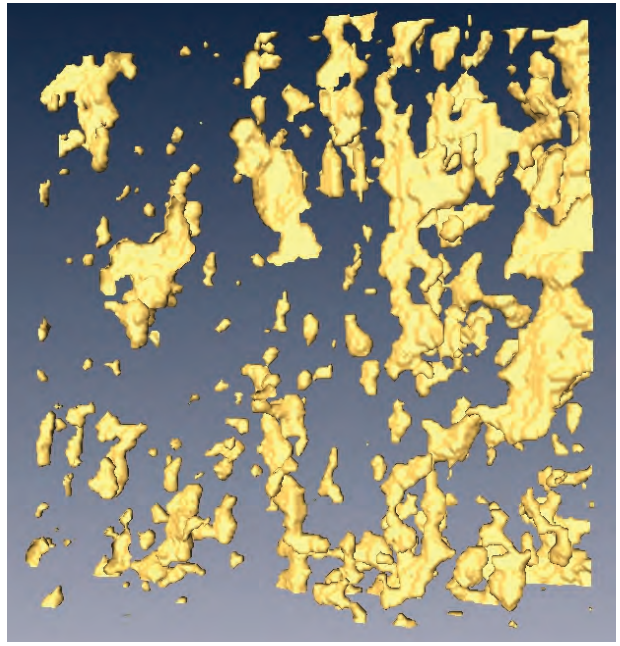
- evolution of salinity profiles

- ocean-ice-air exchanges of heat, CO_2

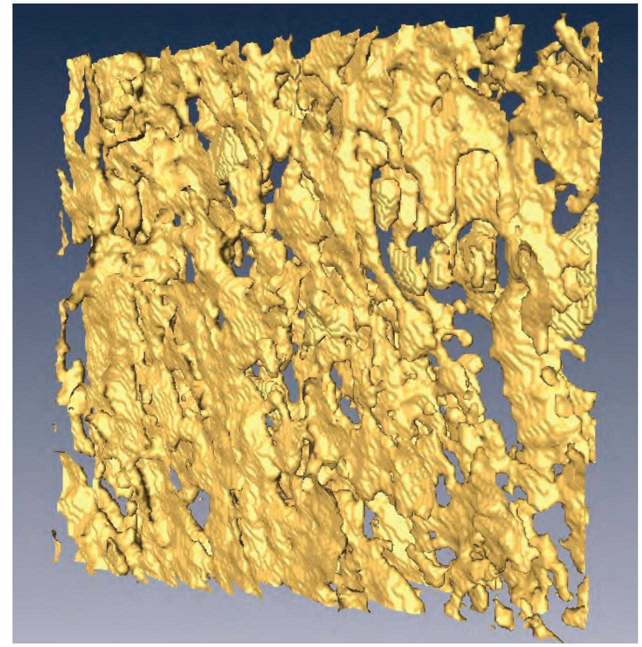
brine volume fraction and ***connectivity*** increase with temperature



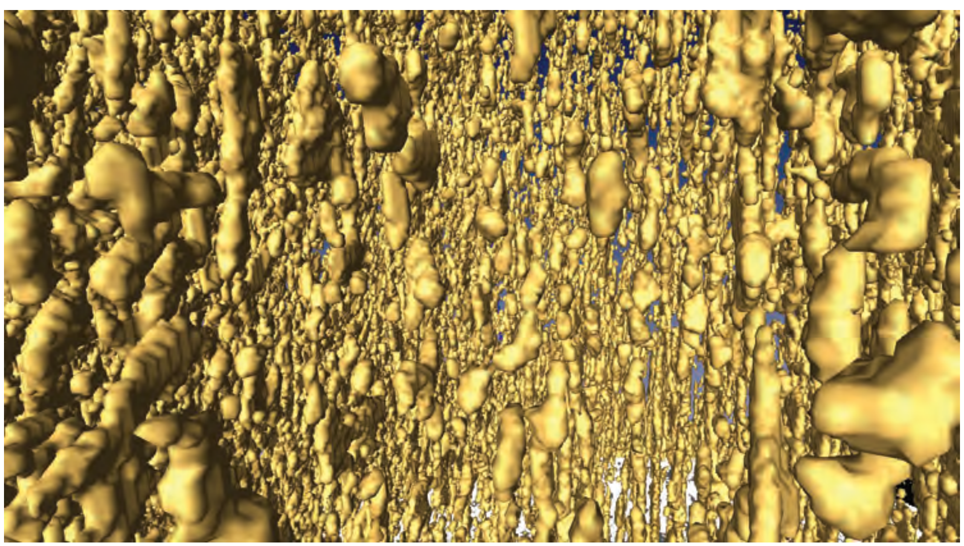
$T = -15\,^\circ C, \ \phi = 0.033$



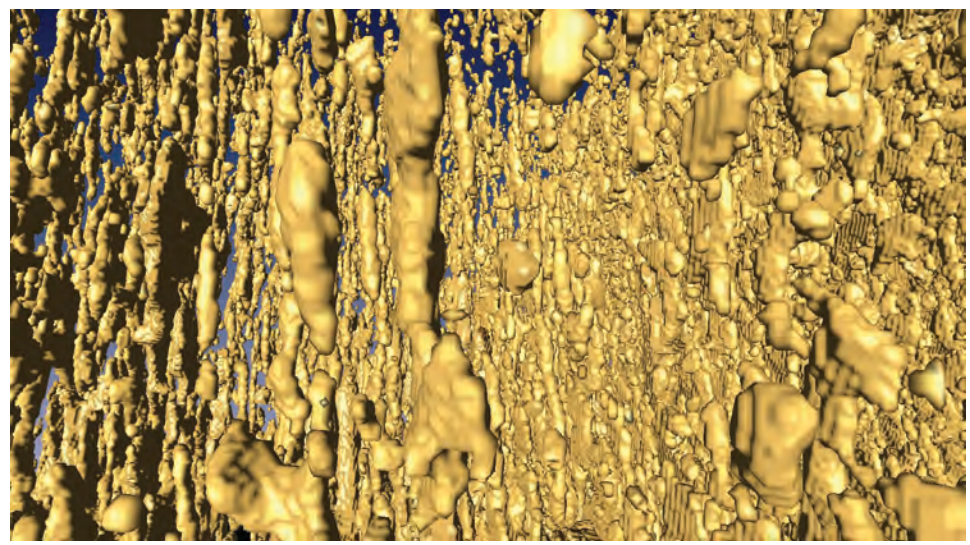
$T = -6\,^\circ C, \ \phi = 0.075$



$T = -3\,^\circ C, \ \phi = 0.143$



$T = -8^\circ C, \ \phi = 0.057$



$T = -4^\circ C, \ \phi = 0.113$

X-ray tomography for brine in sea ice

Golden *et al.*, *Geophysical Research Letters*, 2007

What is our research about?

Developing mathematical models of sea ice structures, processes and ecosystems.

Rigorously compute effective or collective behavior
multiscale homogenization
Improve climate models and projections of polar sea ice and the ecosystems they support

sea ice → physics of climate

sea ice → polar marine ecosystems

Solving problems in physics and biology of sea ice drives advances in mathematics of composite materials, transport phenomena, porous media, inverse problems, biophysics.

sea ice physical processes

homogenization for composites and statistical physics of phase transitions in complex materials

1. Sea ice microphysics and fluid transport, brine percolation

2. Integral representations and bounds for homogenized parameters:
   electromagnetic properties, polycrystalline media, thermal transport, advection diffusion, ocean waves in sea ice pack

3. Anomolous diffusion in sea ice dynamics

4. Low order PDE models of sea ice concentration field

5. Fractal geometry of melt ponds, level set method, Ising model

critical behavior

cross - pollination

sea ice biological processes

How do sea ice physical processes and material properties influence microbial communities?
How do sea ice microbes impact the physics?

1. Coupled ODE model for sea ice algae - nutrient - light dynamics

   BLOOMS

2. Extracellular Polymeric Substances (EPS) secreted by algae modify brine microstructure, impacting fluid permeablity

3. Light field under fractal ponds impacts upper ocean ecology

4. Nematodes in sea ice

5. Microbial habitability on the icy moons of Jupiter and Saturn

critical behavior *cross - pollination*

What kind of math do we use?

homogenization theory for partial differential equations

stochastic processes, advection diffusion

percolation theory, statistical mechanics

dynamical systems and bifurcation theory

functional analysis, complex analysis, spectral theory

random matrix theory

inverse problems

learning “hidden physics”

www.math.utah.edu/~golden/resources/grad_recruitment_2020/

two PDFs on sea ice physics and biology
3 minute movie on Antarctic expedition

THANK YOU

Office of Naval Research

Arctic and Global Prediction Program

Applied and Computational Analysis Program

National Science Foundation

Division of Mathematical Sciences

Division of Polar Programs

UNITED STATES COAST GUARD SEMPER PARATUS 1790

DEPARTMENT OF THE NAVY ONR Science & Technology

MCRN Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

ace CRC ANTARCTIC CLIMATE & ECOSYSTEMS COOPERATIVE RESEARCH CENTRE

NSF

Antarctica New Zealand

UAF UNIVERSITY OF ALASKA FAIRBANKS

THE UNIVERSITY OF UTAH

Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999