Mathematics of Composite Materials

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David Dobson

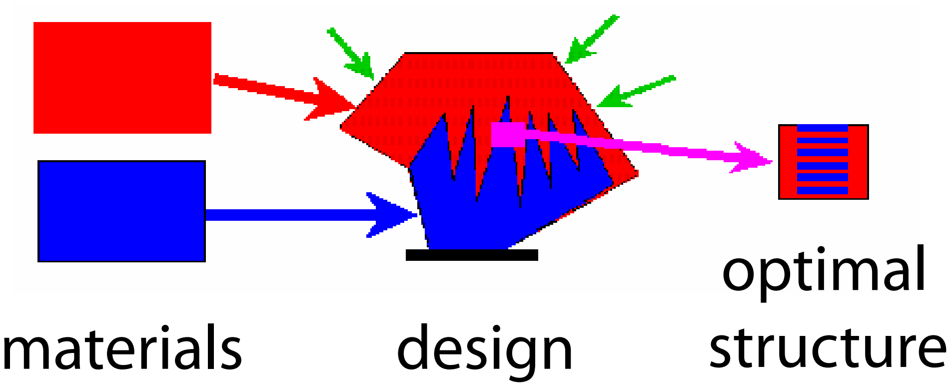
Kenneth Golden

Graeme Milton

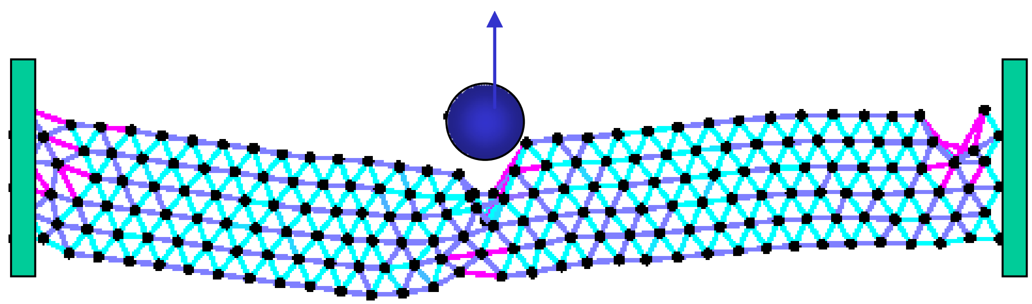
Optimal design of protective structures

Andrej Cherkaev

L. Slepyan, A. Balk, E. Cherkaev, L. Zhornitskaya, and V. Vinogradov



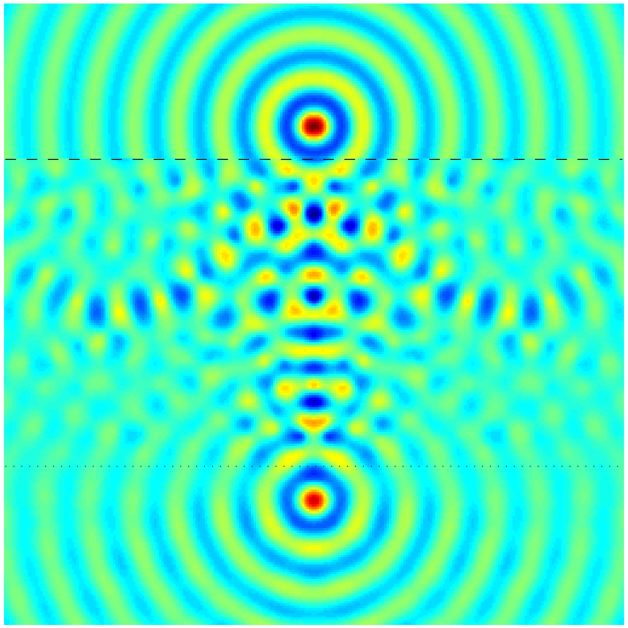
damage waves
dissipate projectile energy



protective structure rejects the projectile, and maintains structural integrity
(purple links are partially damaged)

Composite materials for focusing

David C. Dobson

Goal: design composite materials which focus electromagnetic waves (such as light).

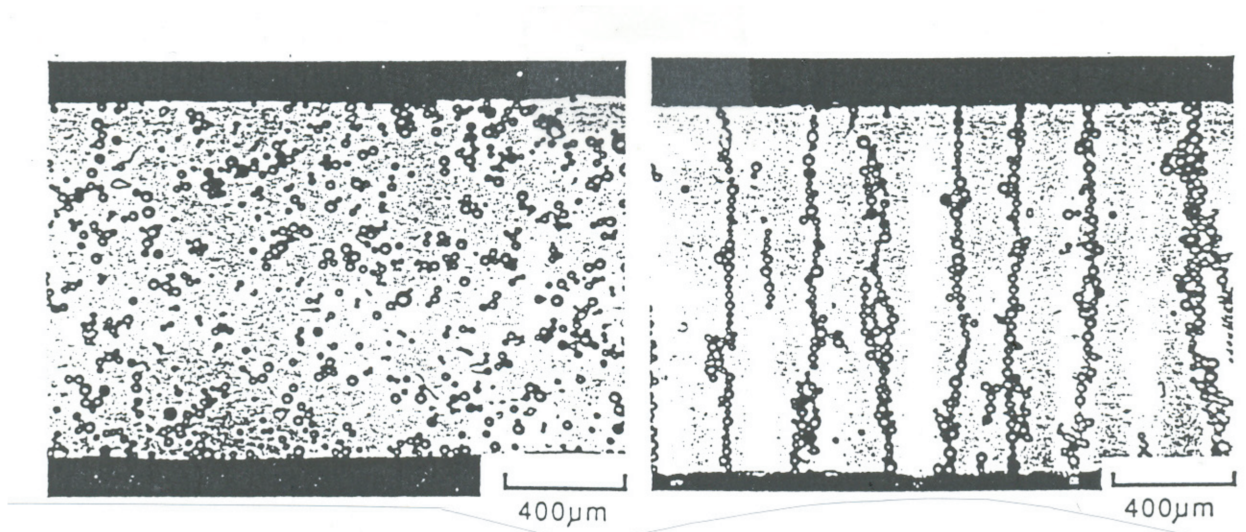
Designs require advanced mathematical optimization.

Possible applications: biomedicine (tumor irradiation),
optoelectronics (communications).

electrorheological (ER) fluids

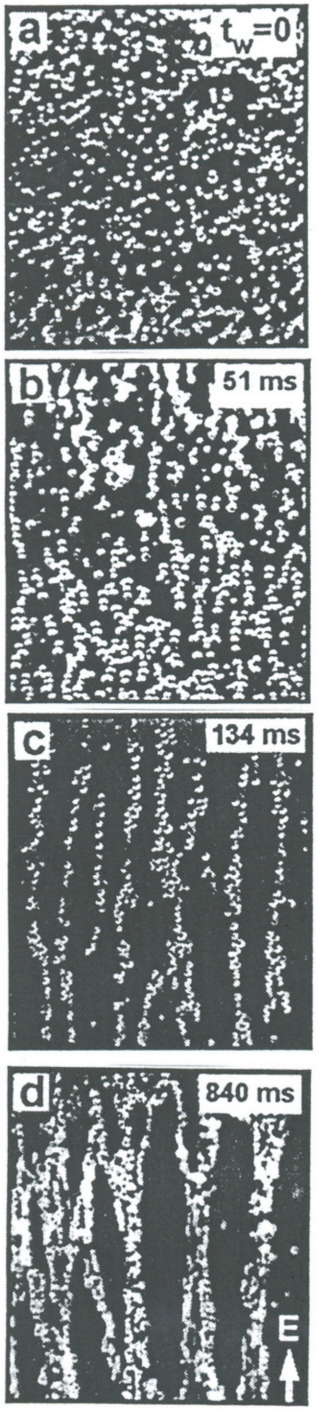
Dobson and Golden

suspension of dielectric spheres in oil

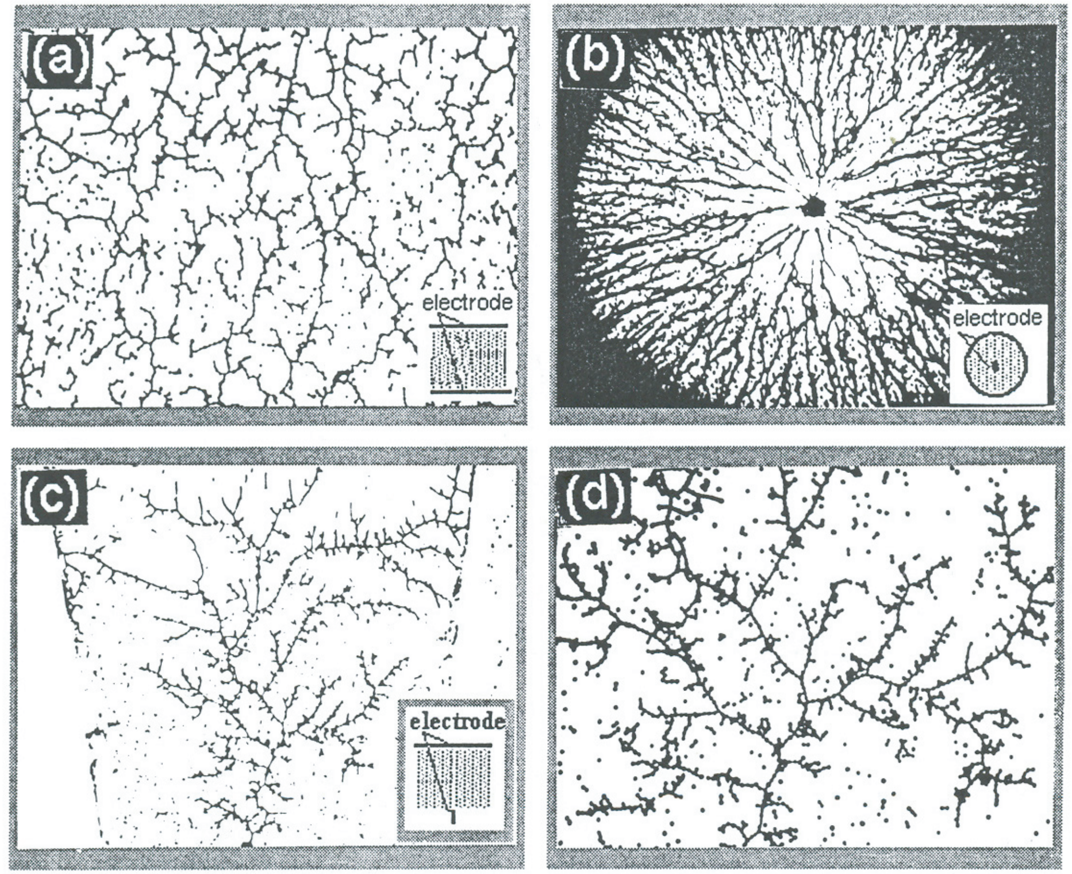


$E = 0$ $\qquad$ $E > E_c$

application of strong enough electric field induces
FAST fluid/solid transition

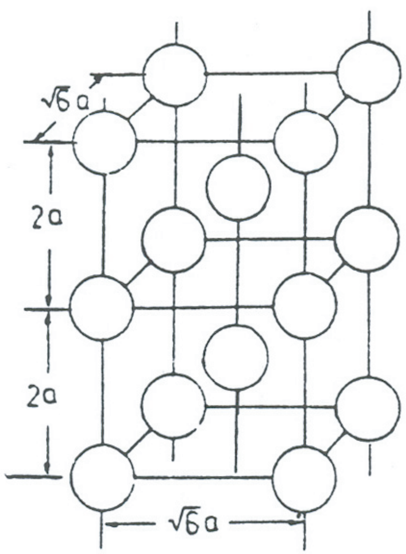


Wen, Zheng, Tu,
Rev. Sci. Instr. 1998



fractal net
ground states
for metal spheres

Wen, et. al.,
Phys. Fluids Lett. 1996



crystalline
ground state for
dielectric spheres

Tao, Sun, Phys. Rev. Lett. 1991

Applications of ER / MR fluids:

Automotive - brakes and clutches, shock absorption

fast, electrical control of fluid/solid transition
for quick momentum transfer

Prosthetics - Rheo Knee (H. Herr, MIT)

knee's microprocessor sends signals
to a joint filled with MR fluid

" ... if you put a magnetic field across the fluid
it thickens, so we can very rapidly change
the resistance of the knee so the knee can
be freely swinging or it can lock up ..."

Electromagnetic cloaking

Graeme Milton

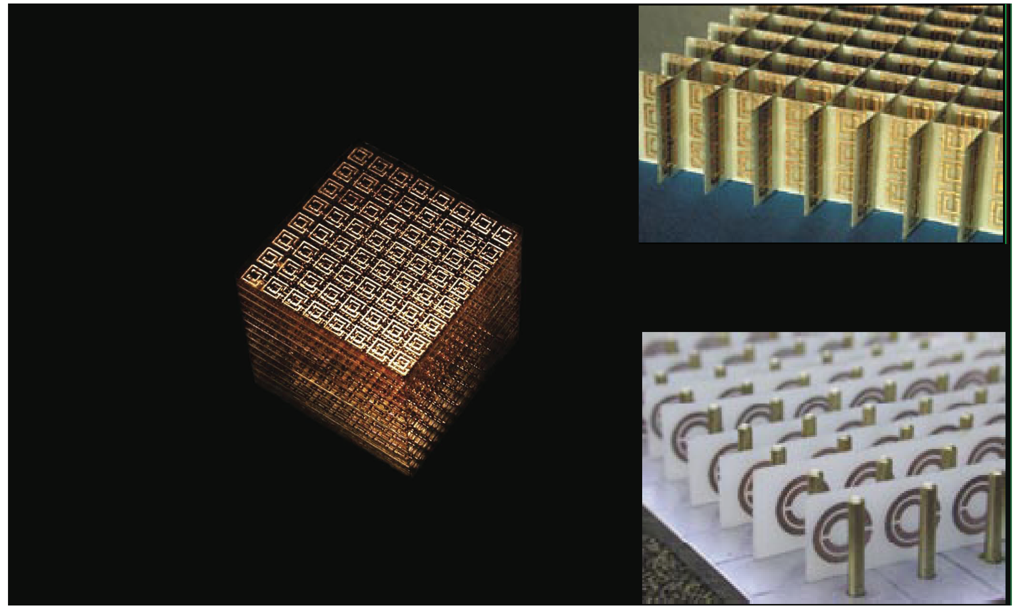
metamaterials

with

negative

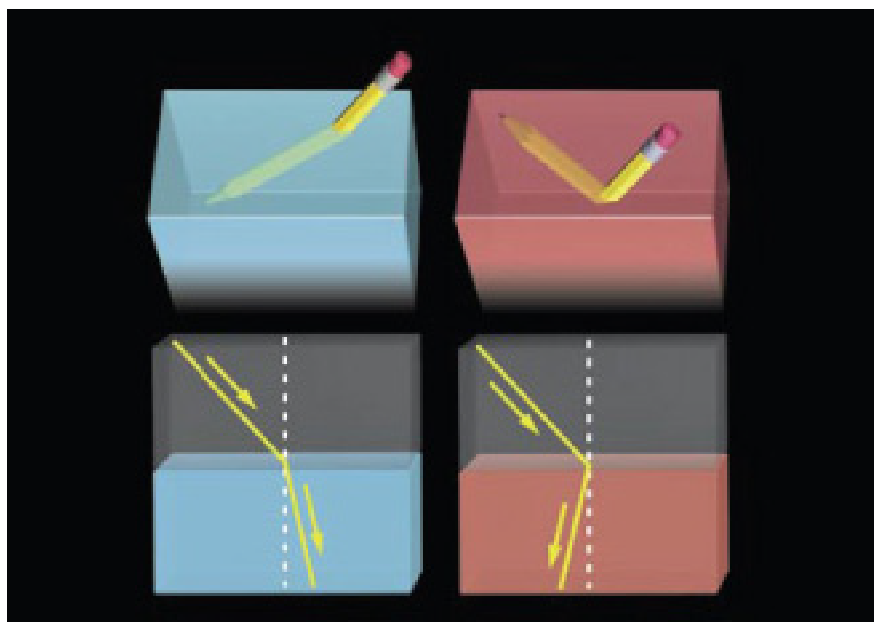
index of refraction

$$n = \pm\sqrt{\varepsilon\mu}$$



Pendry and Smith
Scientific American, 2006

positive
index
medium



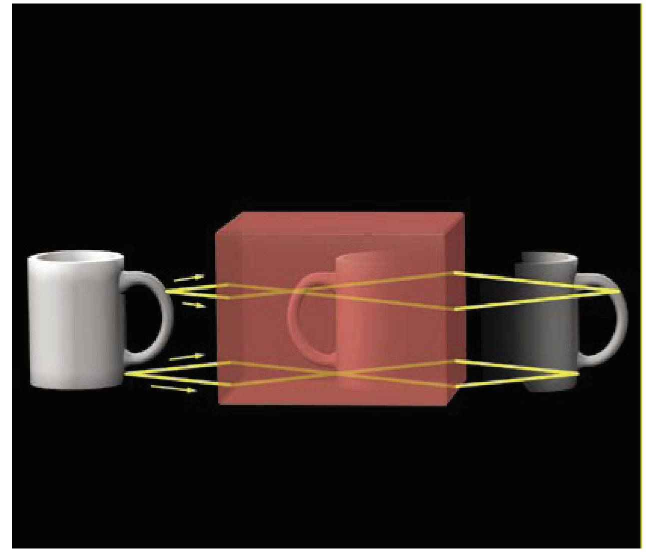
negative
index
medium

superlens

block of negative index metamaterial

finer resolution than wavelength

Pendry 2000



cloaking

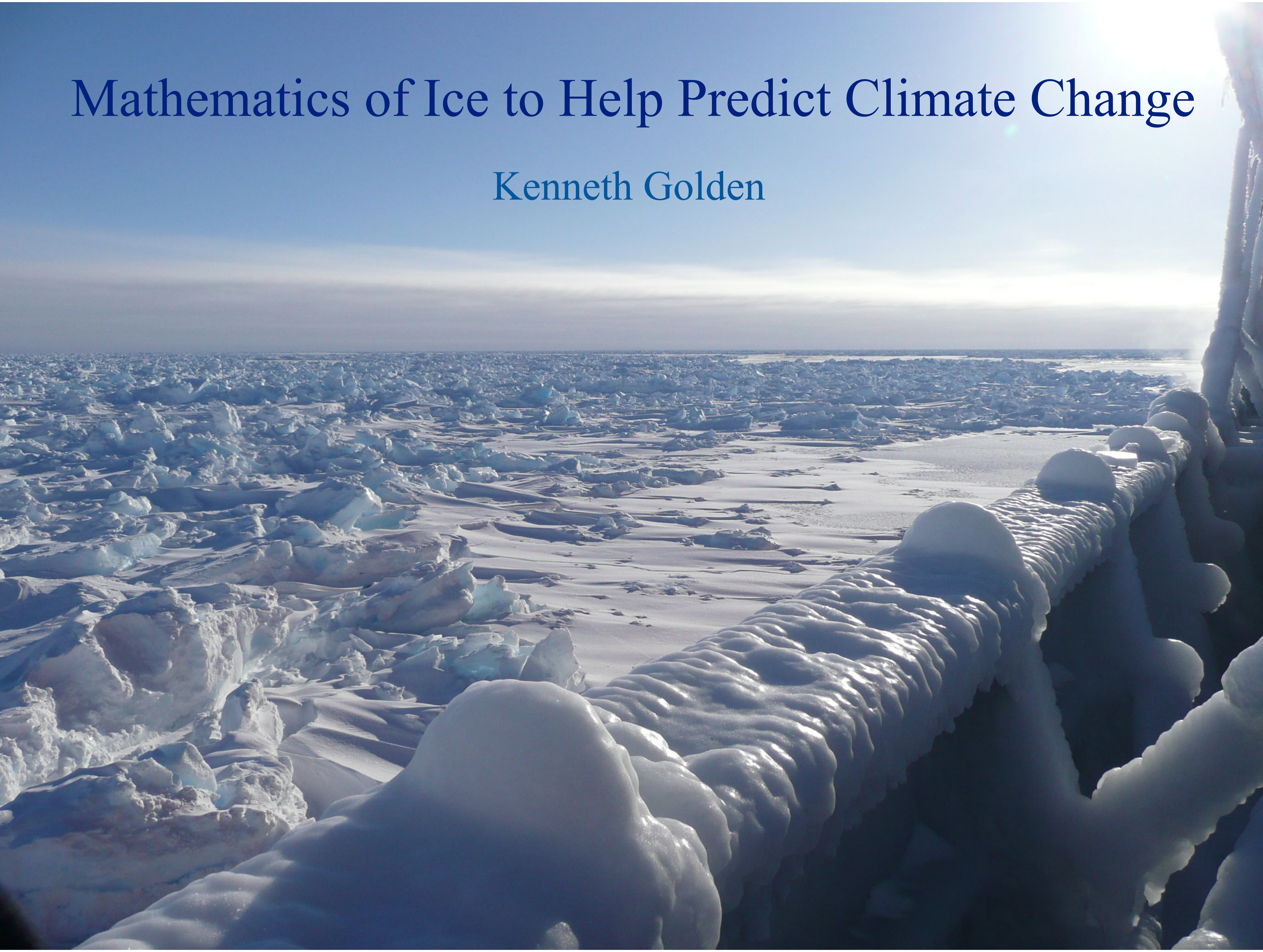
G. Milton and N. Nicorovici

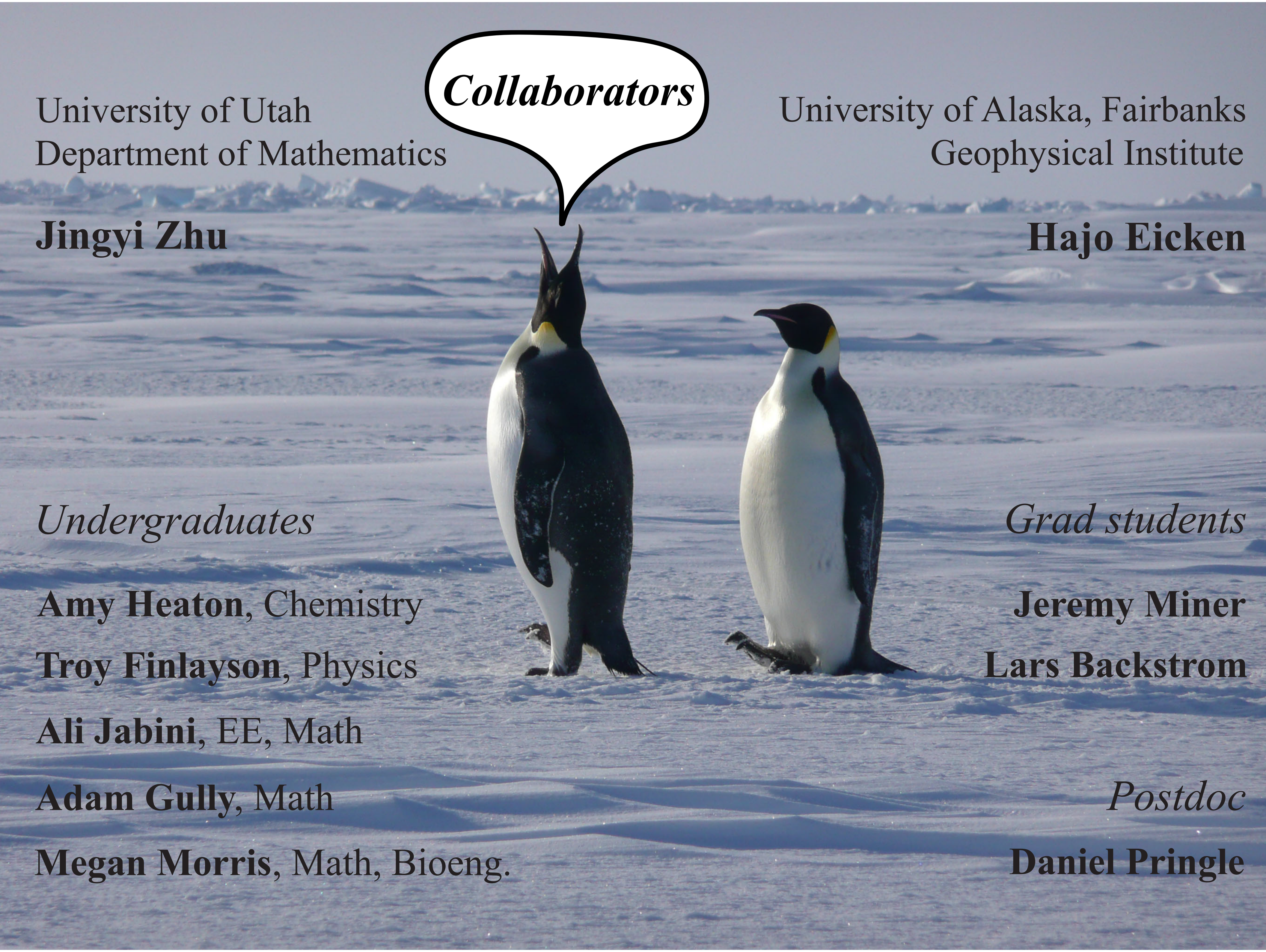
2006

Pendry, Smith

Mathematics of Ice to Help Predict Climate Change

Kenneth Golden





Collaborators

University of Utah
Department of Mathematics

Jingyi Zhu

Undergraduates

Amy Heaton, Chemistry

Troy Finlayson, Physics

Ali Jabini, EE, Math

Adam Gully, Math

Megan Morris, Math, Bioeng.

University of Alaska, Fairbanks
Geophysical Institute

Hajo Eicken

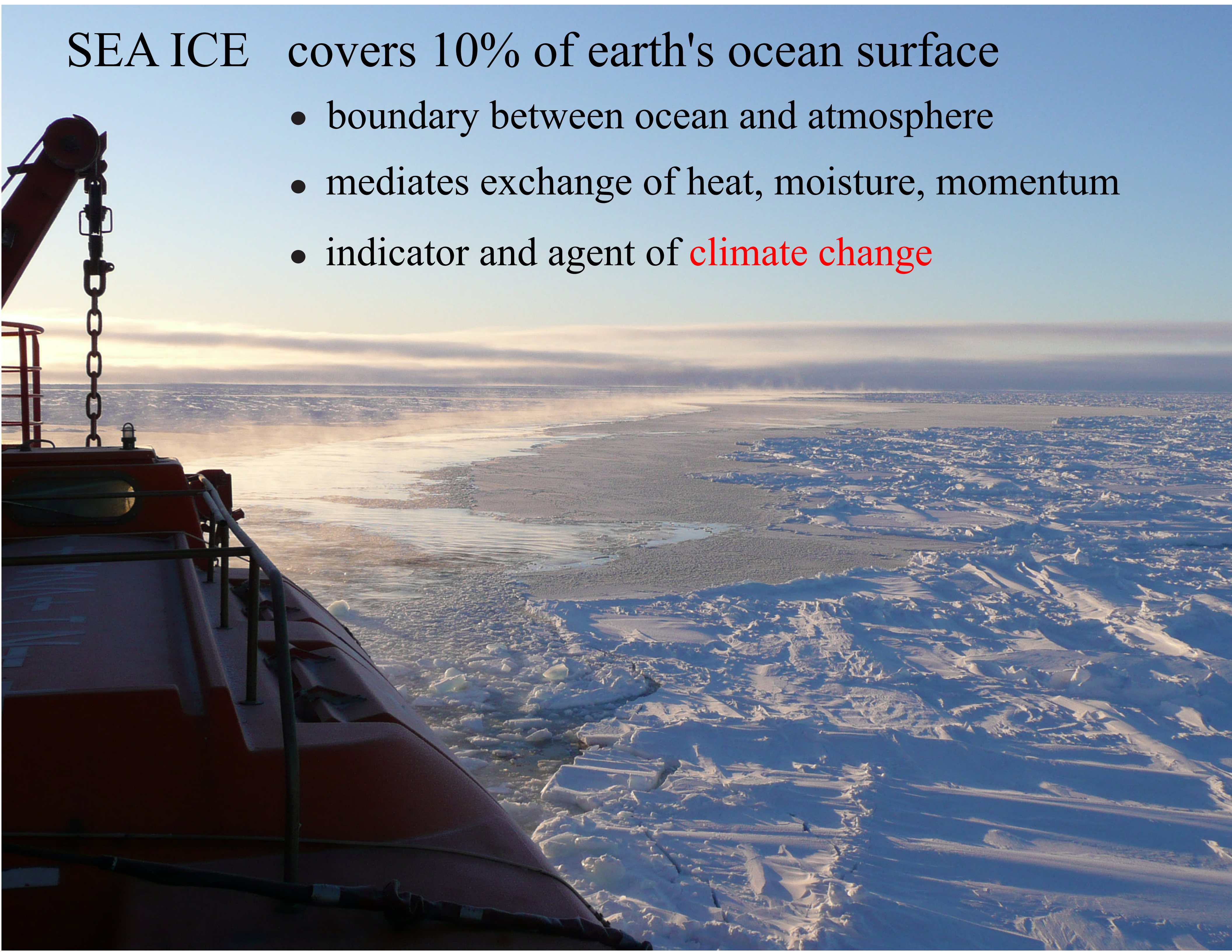
Grad students

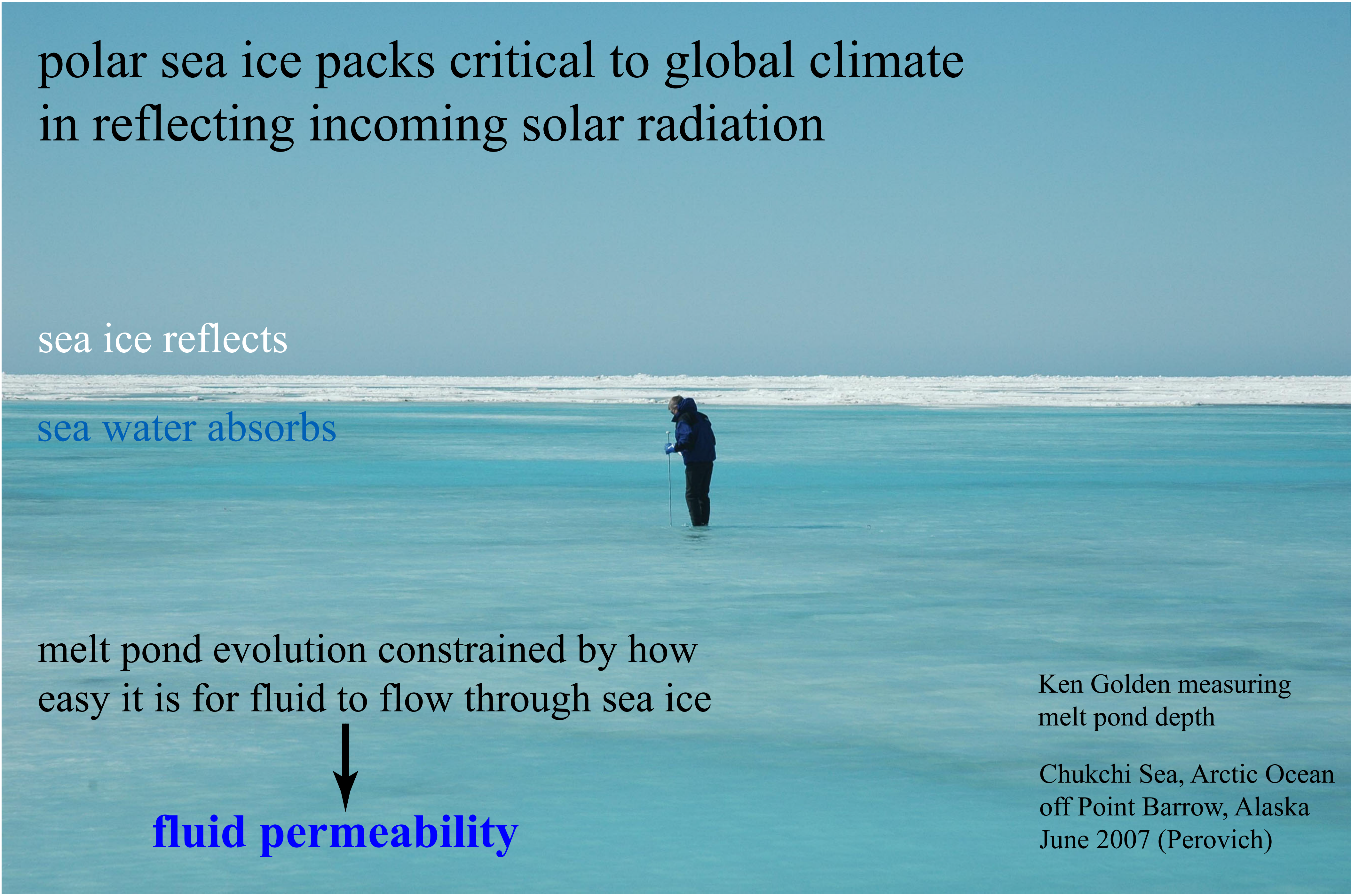
Jeremy Miner

Lars Backstrom

Postdoc

Daniel Pringle


SEA ICE covers 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, moisture, momentum
• indicator and agent of climate change


polar sea ice packs critical to global climate
in reflecting incoming solar radiation
sea ice reflects
sea water absorbs
melt pond evolution constrained by how
easy it is for fluid to flow through sea ice
fluid permeability
Ken Golden measuring
melt pond depth
Chukchi Sea, Arctic Ocean
off Point Barrow, Alaska
June 2007 (Perovich)

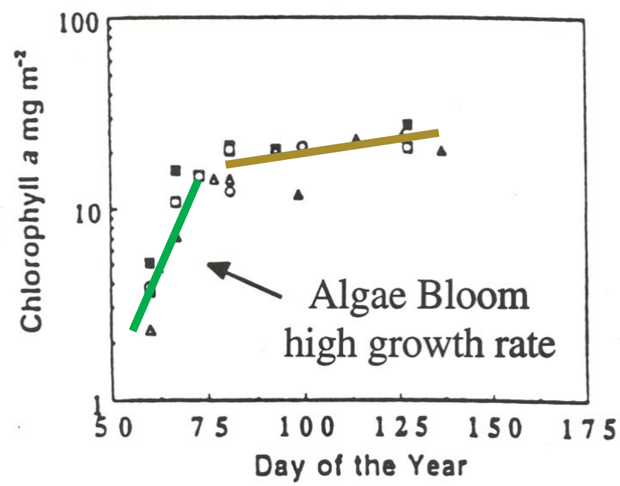
sea ice biology

robust algal and bacterial communities support rich *food webs* in polar oceans

nutrient replenishment controlled by ice permeability

Convection-fueled algae bloom
Ice Station Weddell

(Fritsen, Lytle, Ackley, Sullivan, *Science* 1994)



algae

krill

penguins

leopard seal

Unified approach to understanding fluid permeability in sea ice:

Thermal evolution of permeability and microstructure in sea ice, K. M. Golden, H. Eicken, A. L. Heaton, J. Miner, D. Pringle, and J. Zhu, *Geophysical Research Letters,* vol. 34, 2007 (+ cover).

1. Homogenization and Darcy's law

2. Rigorous bounds

3. Percolation theory

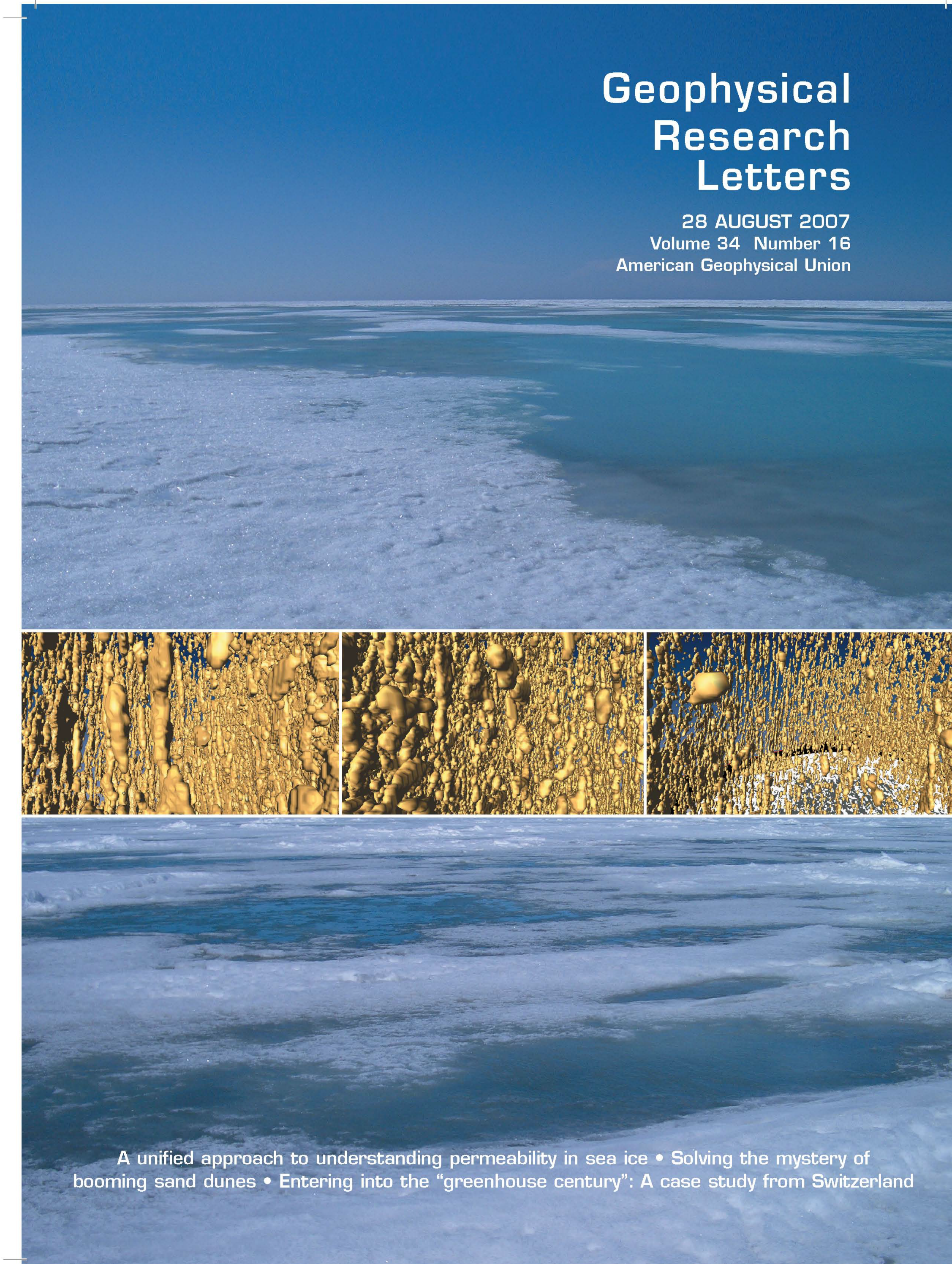
4. Hierarchical models

5. Network model

\+

X-ray CT imaging and pore analysis provide unprecedented look at temperature evolution of brine phase and its connectivity

Validated with lab and Arctic field data.



Arctic Ocean near Point Barrow, June 2007

micro-scale

brine inclusion tomography

controls

Arctic Ocean near Point Barrow, June 2004

macro-scale processes in global climate

Inverse Homogenization for Bone Porosity

Carlos Bonifasi-Lista, Elena Cherkaev, 2006

math we developed to recover brine porosity in sea ice adapted to estimate porosity of bone

normal   osteoporotic

Application*: *Monitoring osteoporosis

the math doesn't care if it's sea ice or bone

Summary

1. Brine flow through sea ice is a key to geophysics and biology of polar regions.

2. Comprehensive theory of fluid permeability, using mathematical techniques from solid state physics.

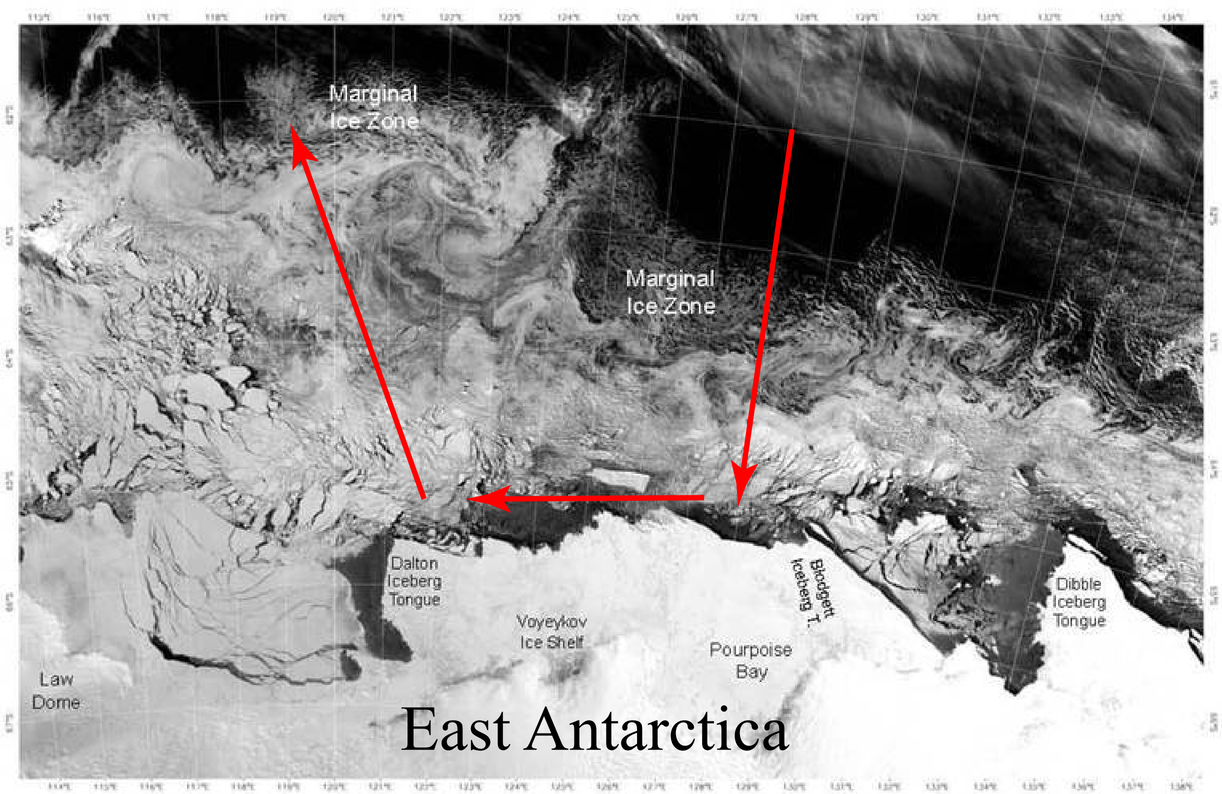
3. Sea ice processes such as melt pond evolution, snow-ice formation, nutrient flux can be modeled more realistically.

4. Results can help to predict how climate change may affect sea ice packs and polar ecosystems.

- 2007 Antarctic springtime (September-October) measured fluid and electrical flow properties

Sea Ice Physics and Ecosystem eXperiment (SIPEX)

4 September - 17 October 2007



Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

International Polar Year (IPY) 2007-2008

Measuring fluid and electrical properties during SIPEX

with Adam Gully

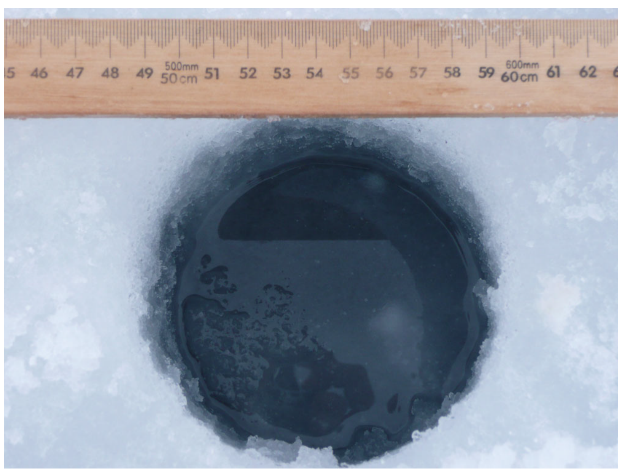
1. Fluid permeability -- first measurements in Antarctic pack ice

2. Horizontal DC conductivity - Wenner array
   surface impedance tomography: invert resistance data to get profile (w/ T. Worby and J. Reid)

3. Vertical DC conductivity - direct from cores

4. Tracer studies

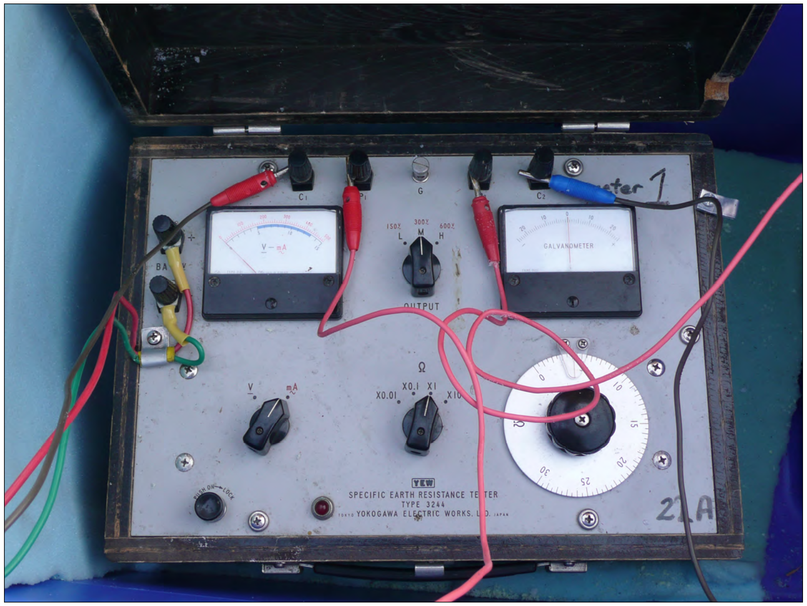
Measurements of fluid permeability



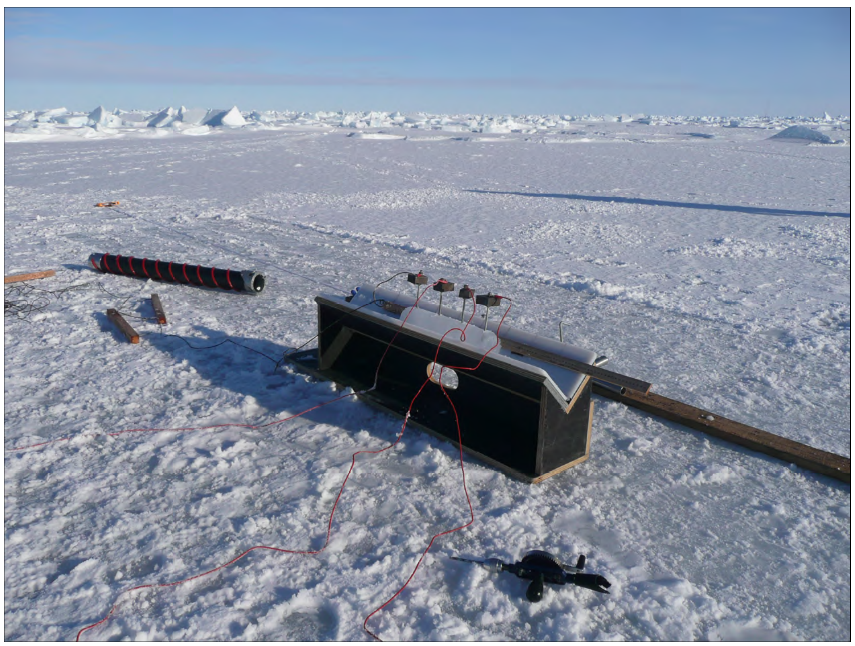
(Photo by Jan Lieser)

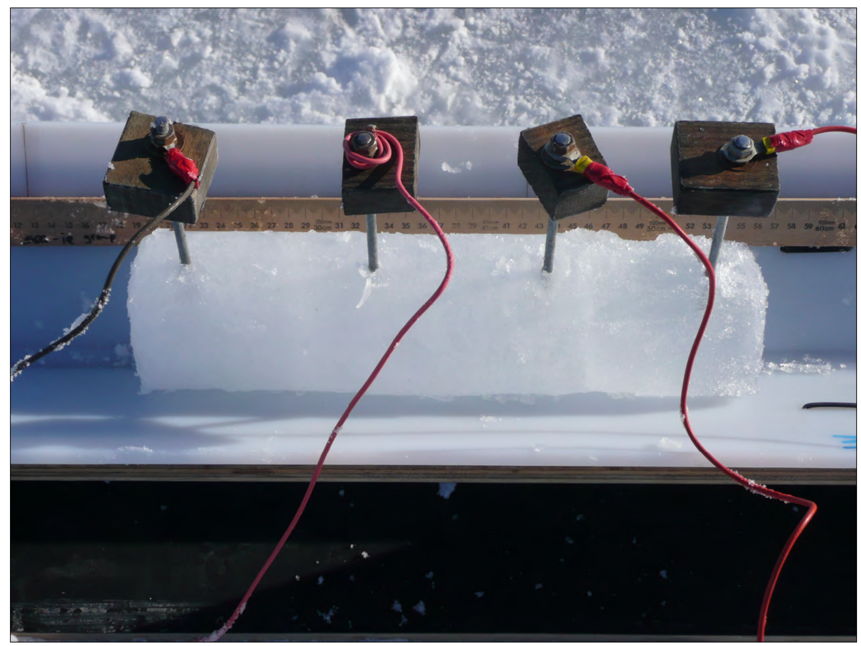
demonstrated Rule of Fives

electrical measurements



Wenner array





vertical conductivity

correlate with EM soundings of thickness

Kazu Tateyama
Adam Gully

Worbot (T. Worby)

tracers flowing through inverted sea ice blocks