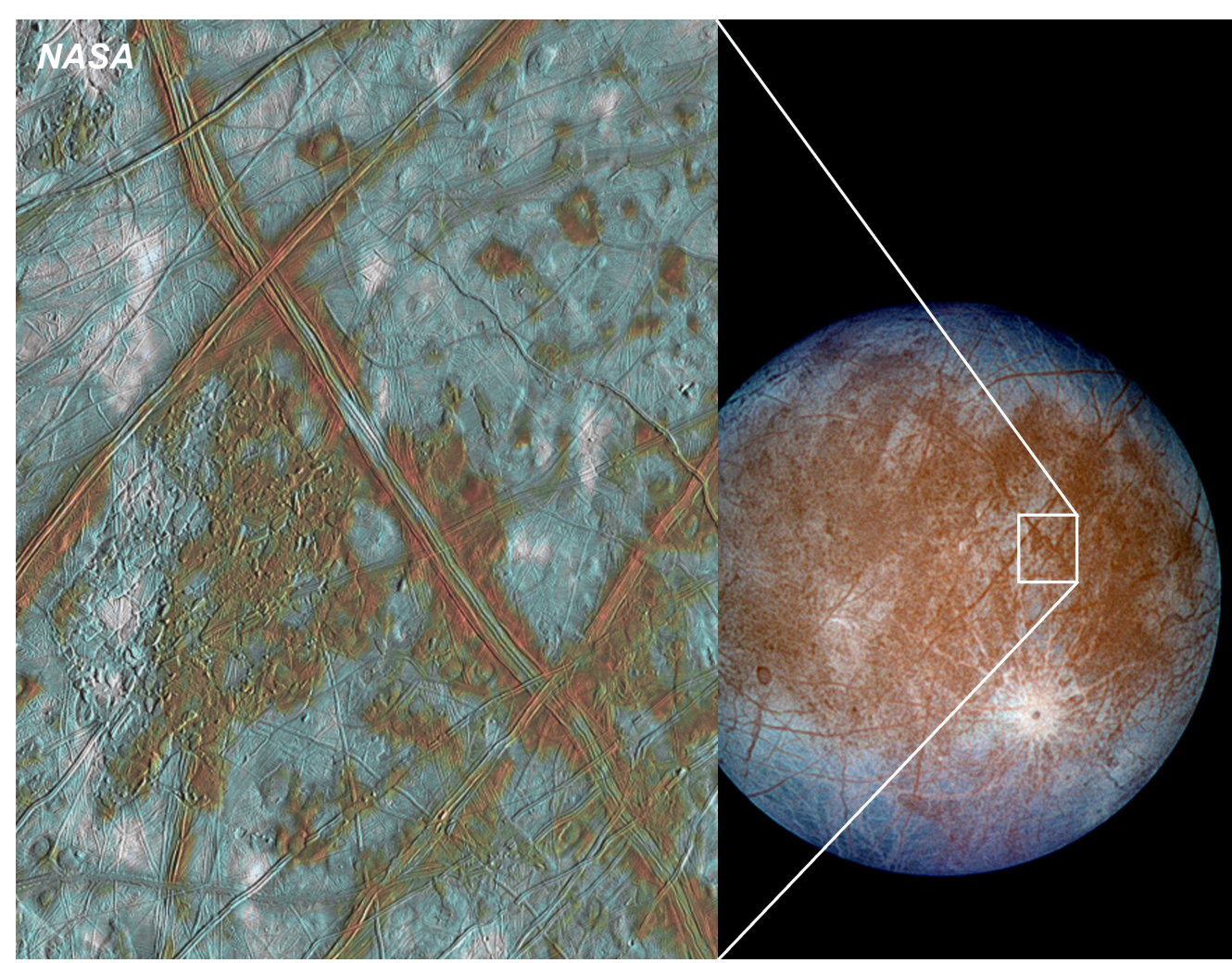


Mathematical Exploration of Life in Europa's Icy Shell

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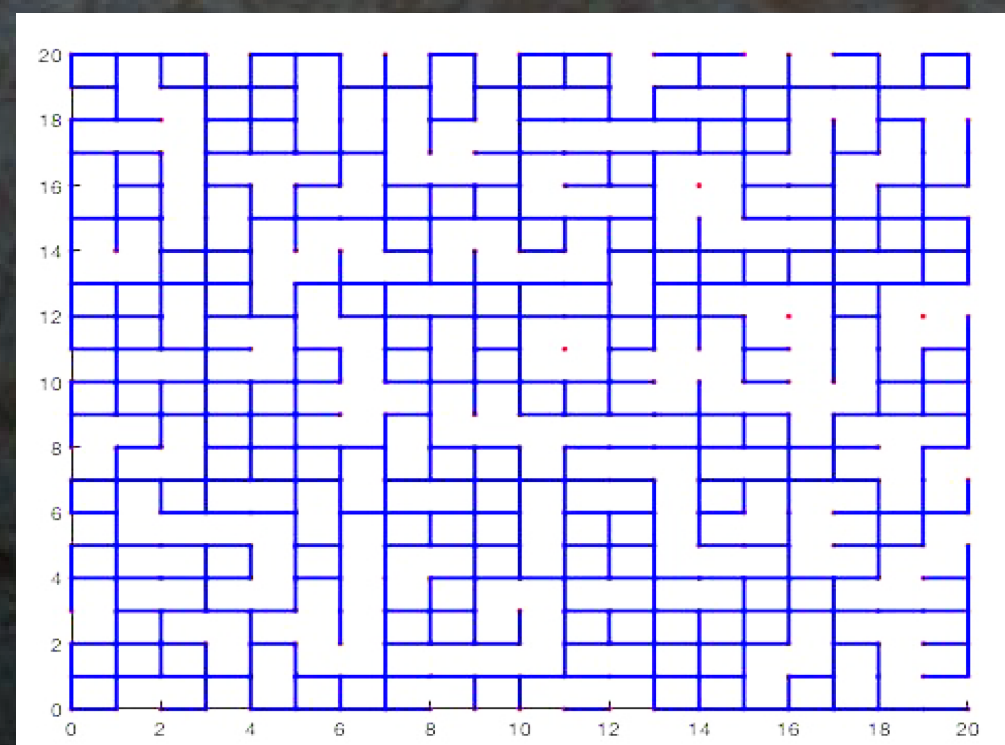
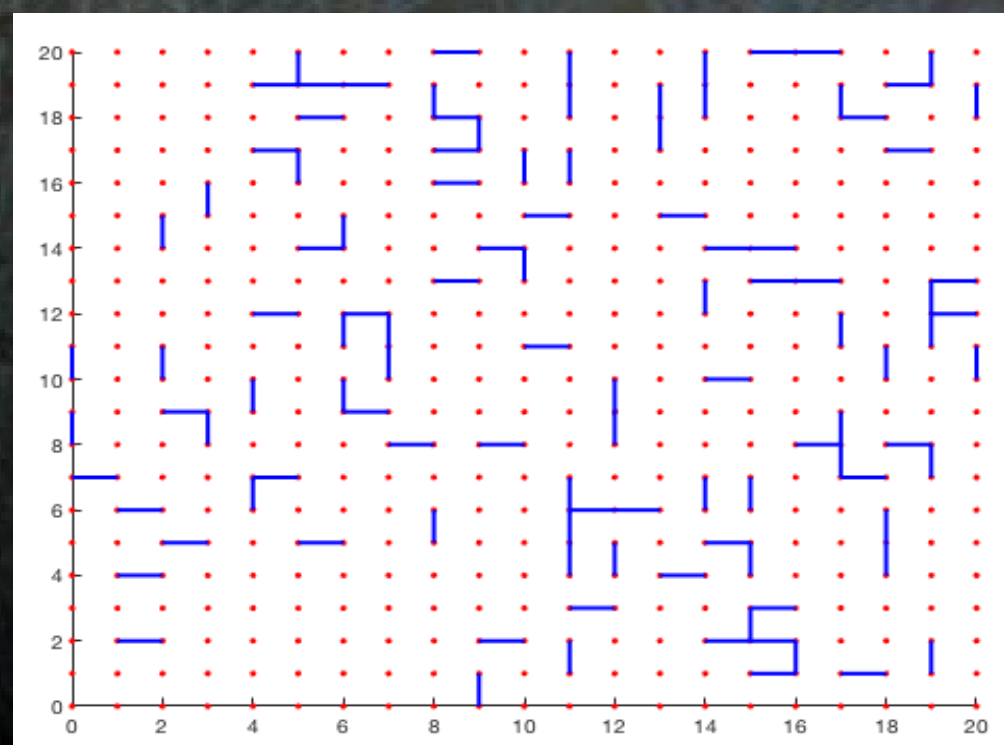


Introduction and Background

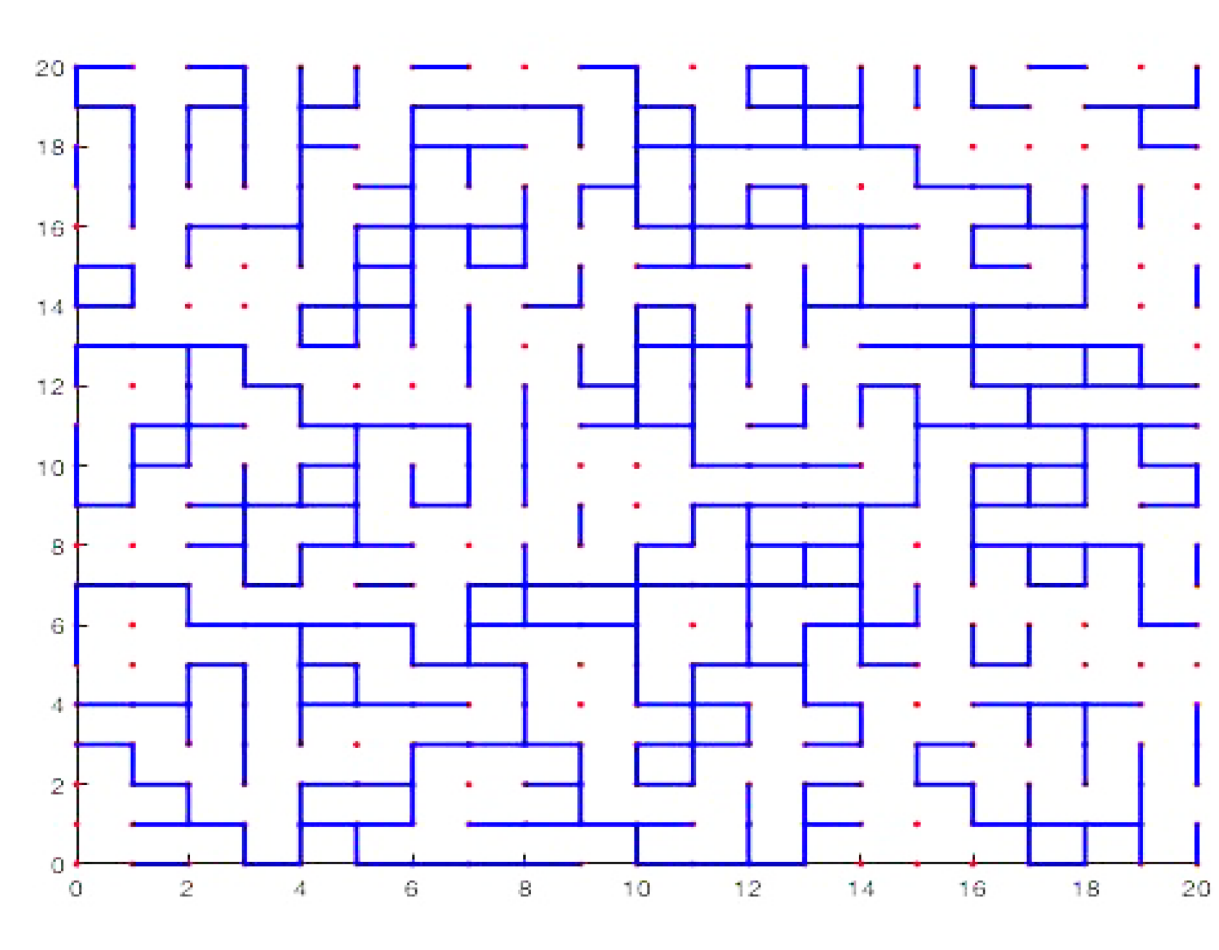


- Europa is one of Jupiter's icy moons, slightly smaller than Earth's.
- It is thought to be composed of a solid core surrounded by a large salt water ocean with an icy shell.
- Because it is one of the few bodies in our solar system with water, Europa is considered one of the best places in our solar system to search for simple extraterrestrial life.

- Sea ice on earth, which freezes at -1.8°C , is a composite material of an ice host containing brine inclusions.
- In warmer ice, inclusions connect, allowing nutrients to be transported through the sea ice.
- Extremophiles (such as the algae to the right) thrive in fluid inclusions of Earth's sea ice.
- Potential life on Europa could be similar to extremophiles in Earth's sea ice, living in the fluid inclusions of Europa's icy shell.
- **In warmer ice near the bottom of the ice layer, the inclusions coalesce to form channels where nutrients from the ocean interface below can be transported upwards, creating a potentially habitable layer.**



- Bond percolation models. Open bonds (blue, where fluid can flow) occur with probability p .
- The system percolates when there is a pathway of connected open bonds crossing the sample.
 - For low probabilities, the system will not percolate (upper left, $p = 0.1$), while for higher probabilities (upper right, $p = 0.7$) the system is likely to percolate.
 - The percolation threshold (below, $p = 0.5$) is the smallest probability for which the system is likely to percolate.



- Each bond represents an open channel in the ice through which water can flow.
- The percolation threshold for Earth's sea ice occurs at approximately -5°C .
- Beneath the depth of the percolation threshold, the inclusions connect and water is able to percolate through the ice, transporting nutrients, salt, and heat.

Methods and Results

To determine the thickness of the layer of Europa's ice shell with salty fluid inclusions, we use the heat equation where we assume a constant coefficient D of thermal conductivity:

$$D\nabla^2 U = \frac{\partial U}{\partial t}$$

One-dimensional form:

$$D \frac{\partial^2 U}{\partial x^2} = \frac{\partial U}{\partial t}$$

Assume temperature does not change as a function of time:

$$\frac{\partial}{\partial x} \left[D \frac{\partial U}{\partial x} \right] = 0$$

Then the temperature $U(x)$ is a linear function of depth x , D is the thermal conductivity of the ice, and the other variables are unknown constants.

$$U(x) = \frac{C}{D}x + \beta$$

Assumptions:

Composition of Europa's ice shell: Frozen salty water with 1% MgSO_4 concentration

Surface temperature ($x = 0$ km): -173.15°C (100 K)

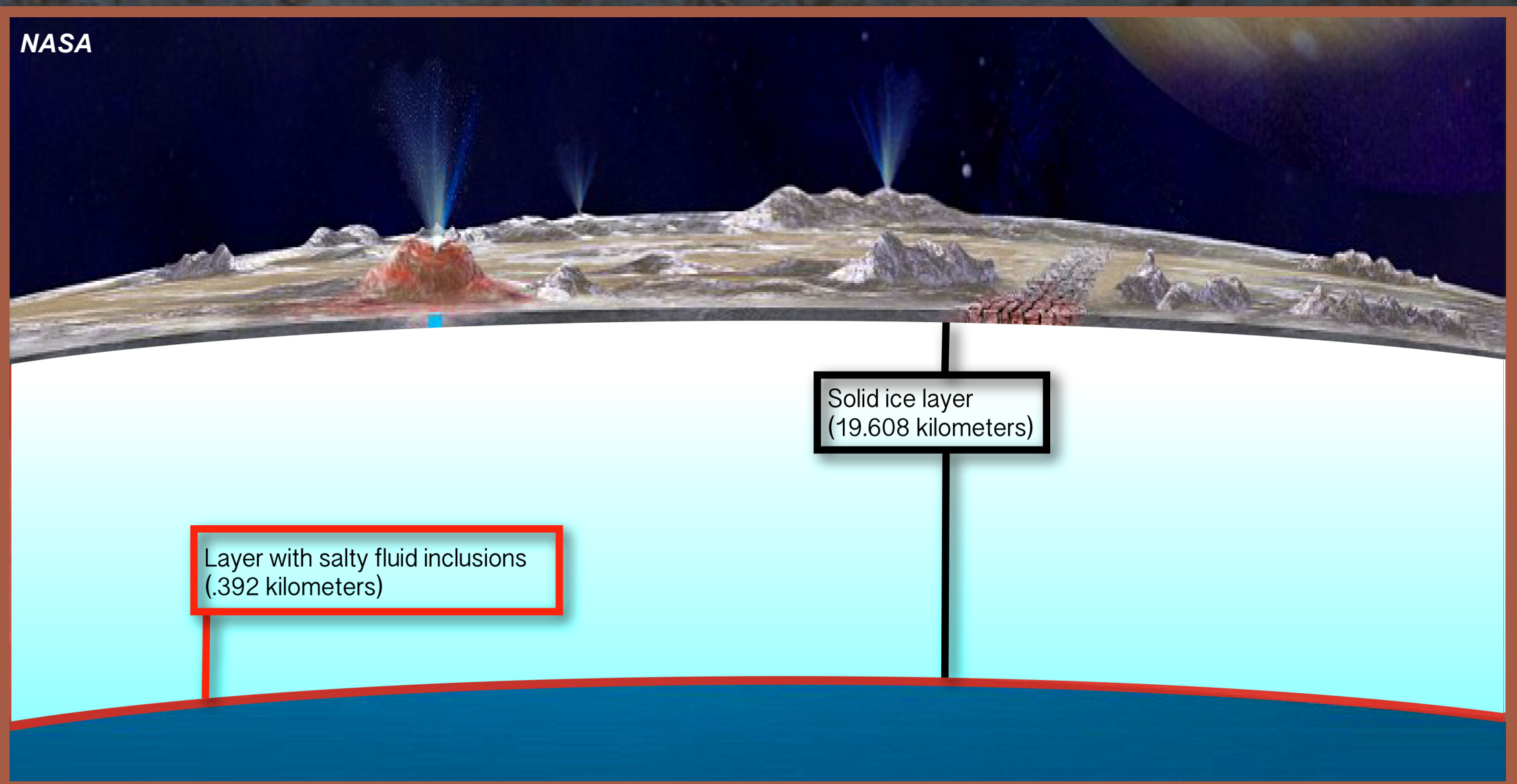
Total thickness of ice shell: $x = 20$ km

Melting point of magnesium sulfate and water solution: -0.31°C (272.84 K)

Thermal Conductivity: 2.3 W/mK

Using known initial conditions, we derive a linear model for temperature as a function of depth:

$$U(x) = 8.642x + 100$$



Estimated thickness of the layer of Europa's shell with salty fluid inclusions.

- The eutectic temperature of the water + MgSO_4 system is -3.7°C (269.45 K). At temperatures higher than the eutectic, fluid inclusions will exist in the ice.

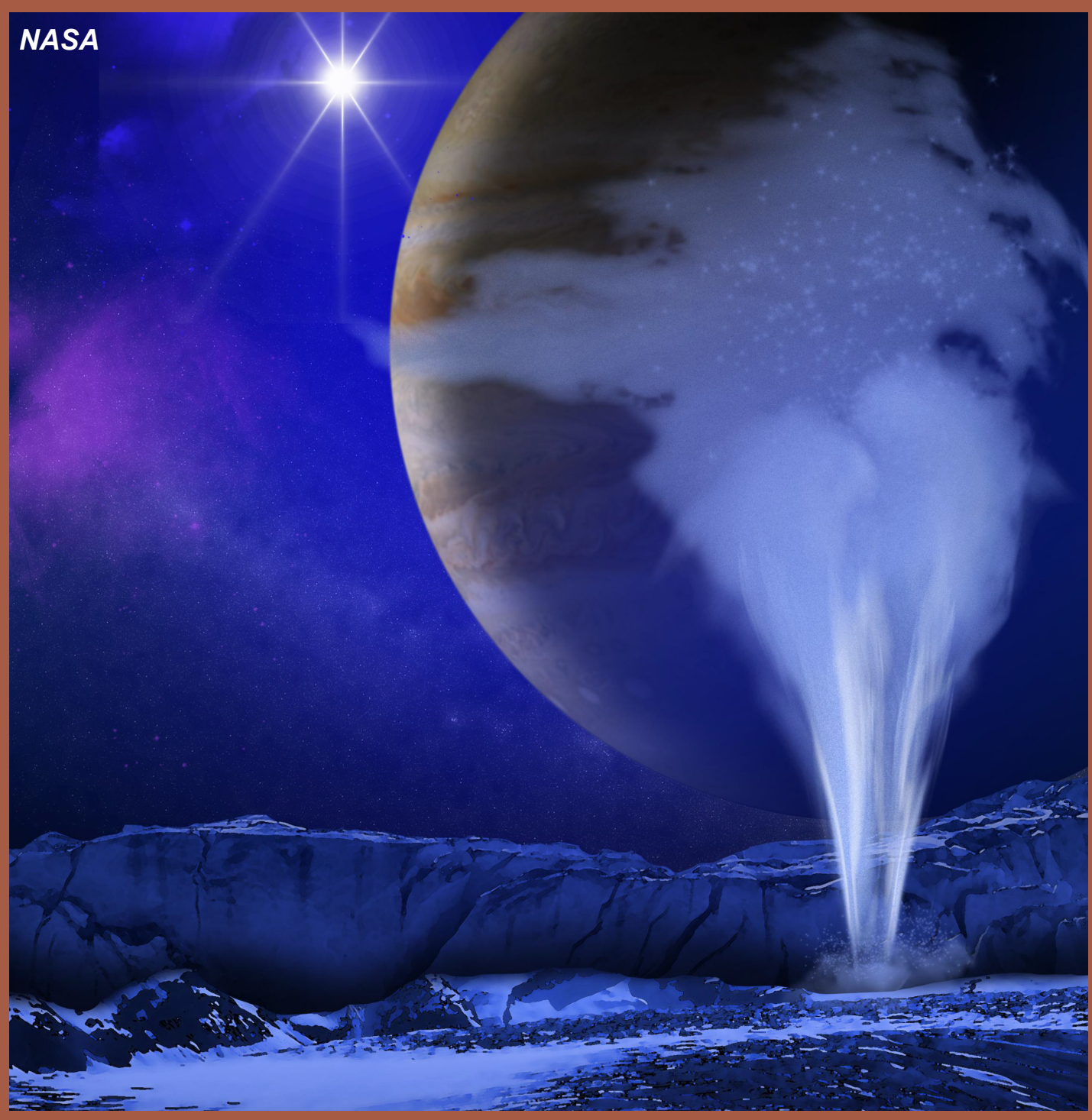
$$269.45 = 8.642x + 100$$

$$x = 19.608$$

- Depth of solid ice with no fluid inclusions on Europa: 19.608 km.
- **Thickness of layer with salty fluid inclusions and potential for life: 392 meters.**

Discussion and Future Work

- Because the eutectic point of the MgSO_4 and water system is so close to its melting point, the layer with fluid inclusions has a narrow temperature range.
- Europa's ice shell, however, is deep enough that this layer could still be hundreds of meters thick, including a percolating layer through which liquid from the nutrient-laden ocean below could be transported.
- The thickness of both layers would likely be highly sensitive to slight changes in temperature.
- Any potential life in Europa's ice shell must be able to survive in these extreme conditions with extreme variations.



- Currently, this model is limited by the amount of available knowledge of Europa.
- The temperature distribution in Europa's ice is unlikely to be linear and one-dimensional, as we assume in this model. Future models should use more complex versions of the heat equation to reflect the complex conditions on Europa.
- We would like to determine the percolation threshold of Europa's ice and approximate the thickness of its permeable layer.

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