MODELING *the* MELT:

what math tells us about the disappearing polar ice caps

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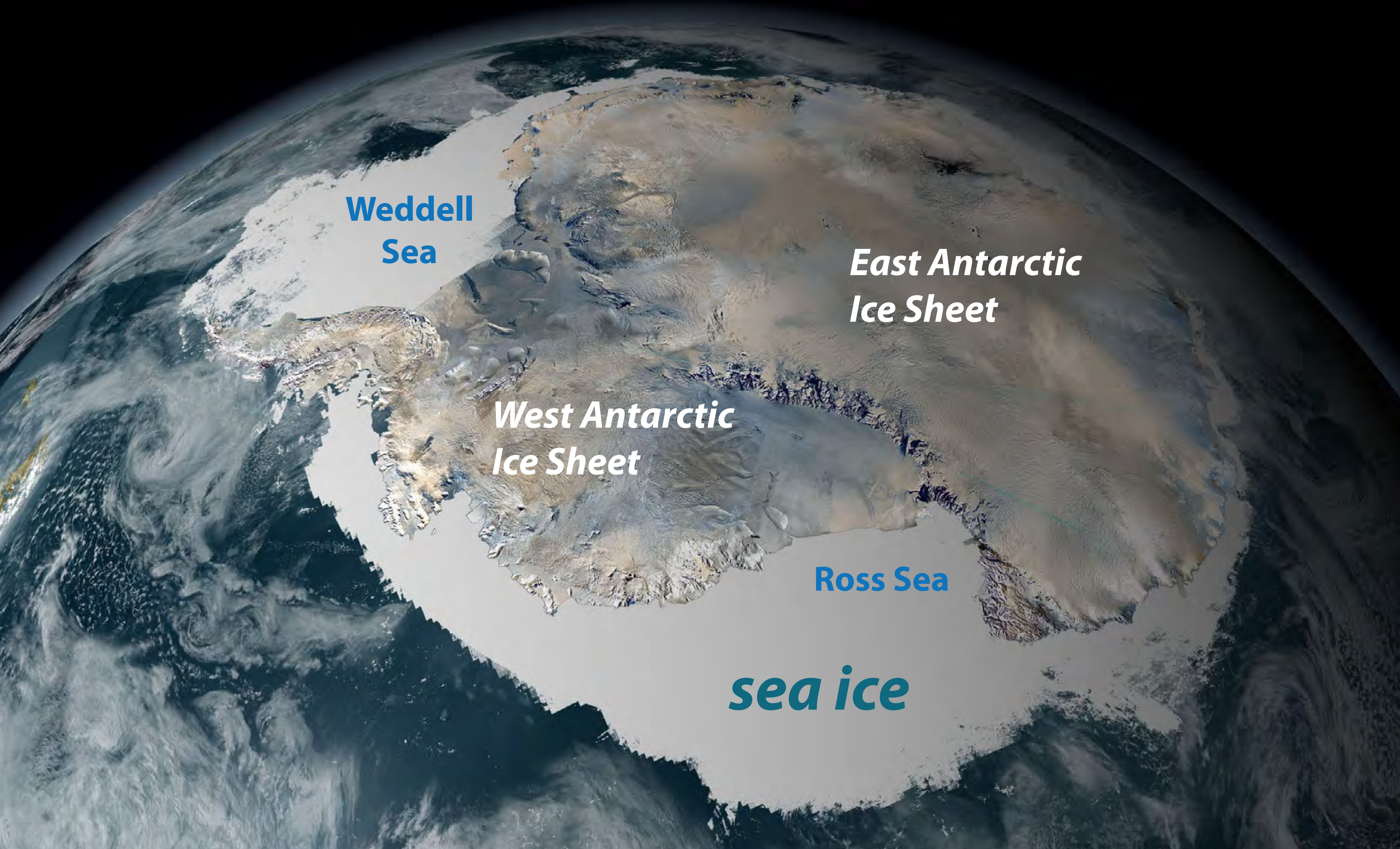
Public Lecture *CMDS13* *July 21, 2014*

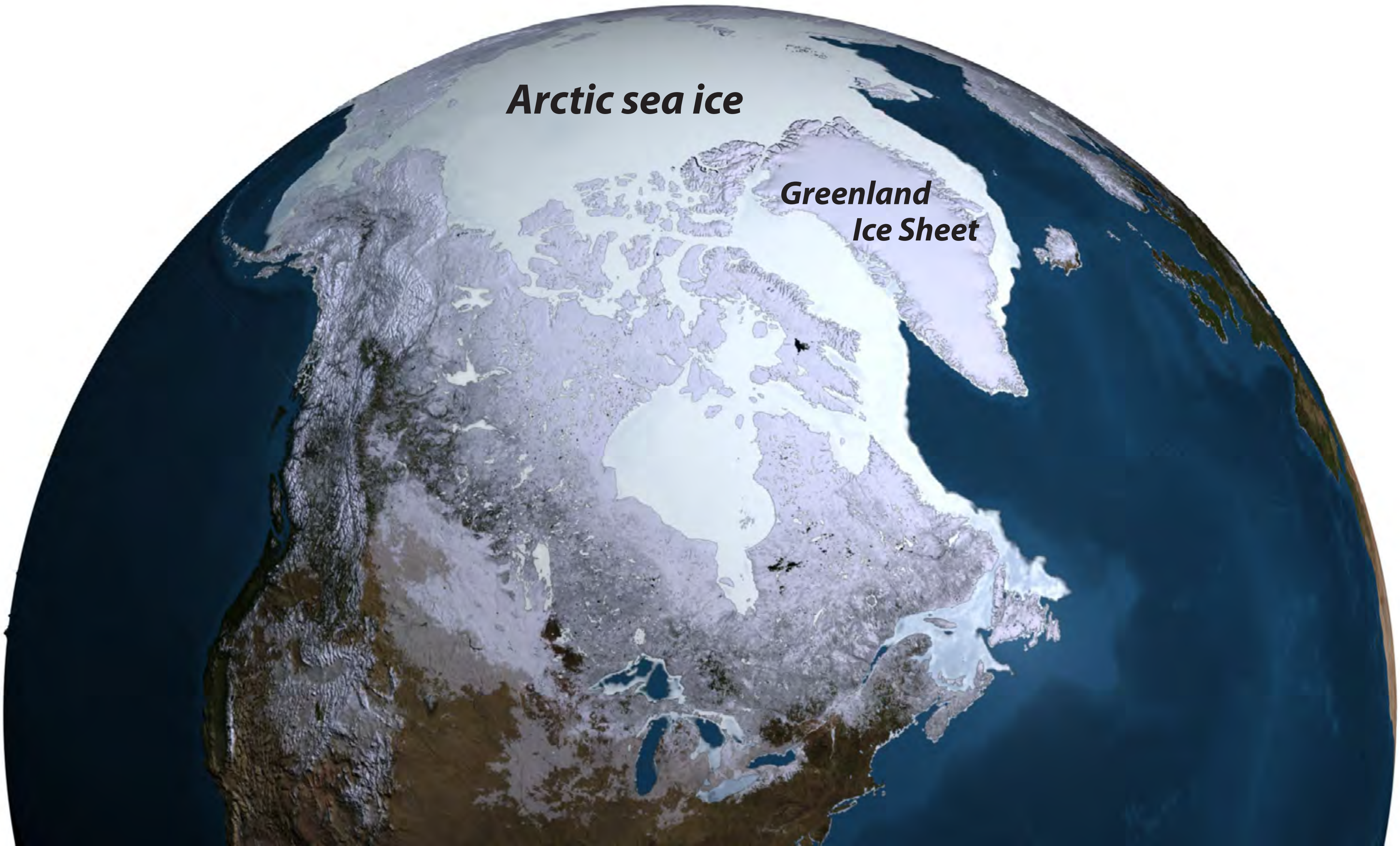
Frey

ANTARCTICA

southern cryosphere



northern cryosphere

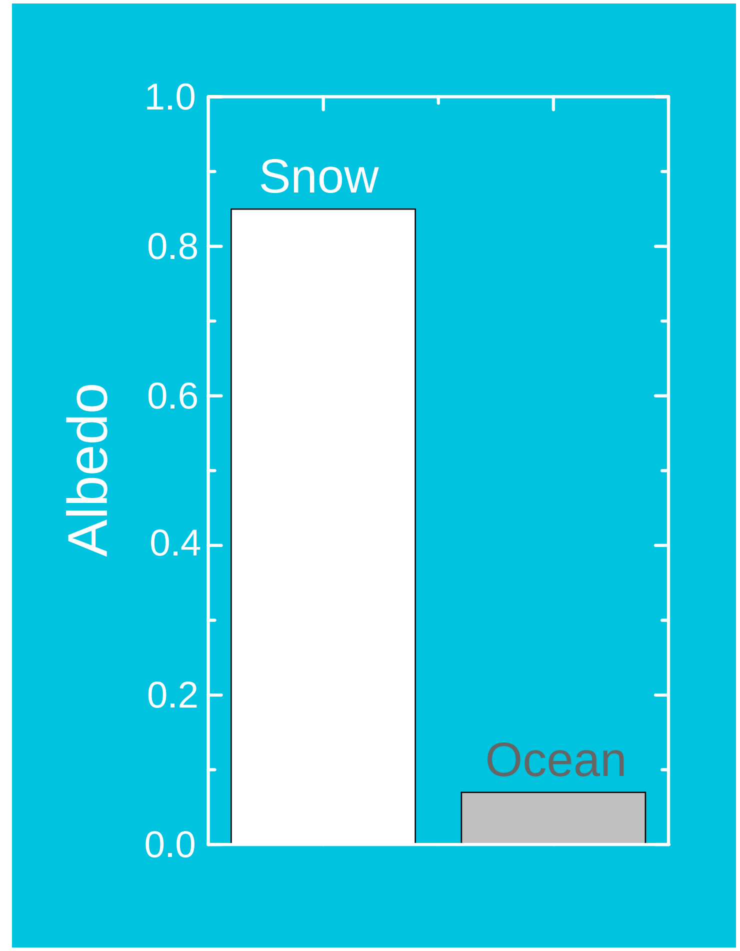


SEA ICE covers 7 - 10% of earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

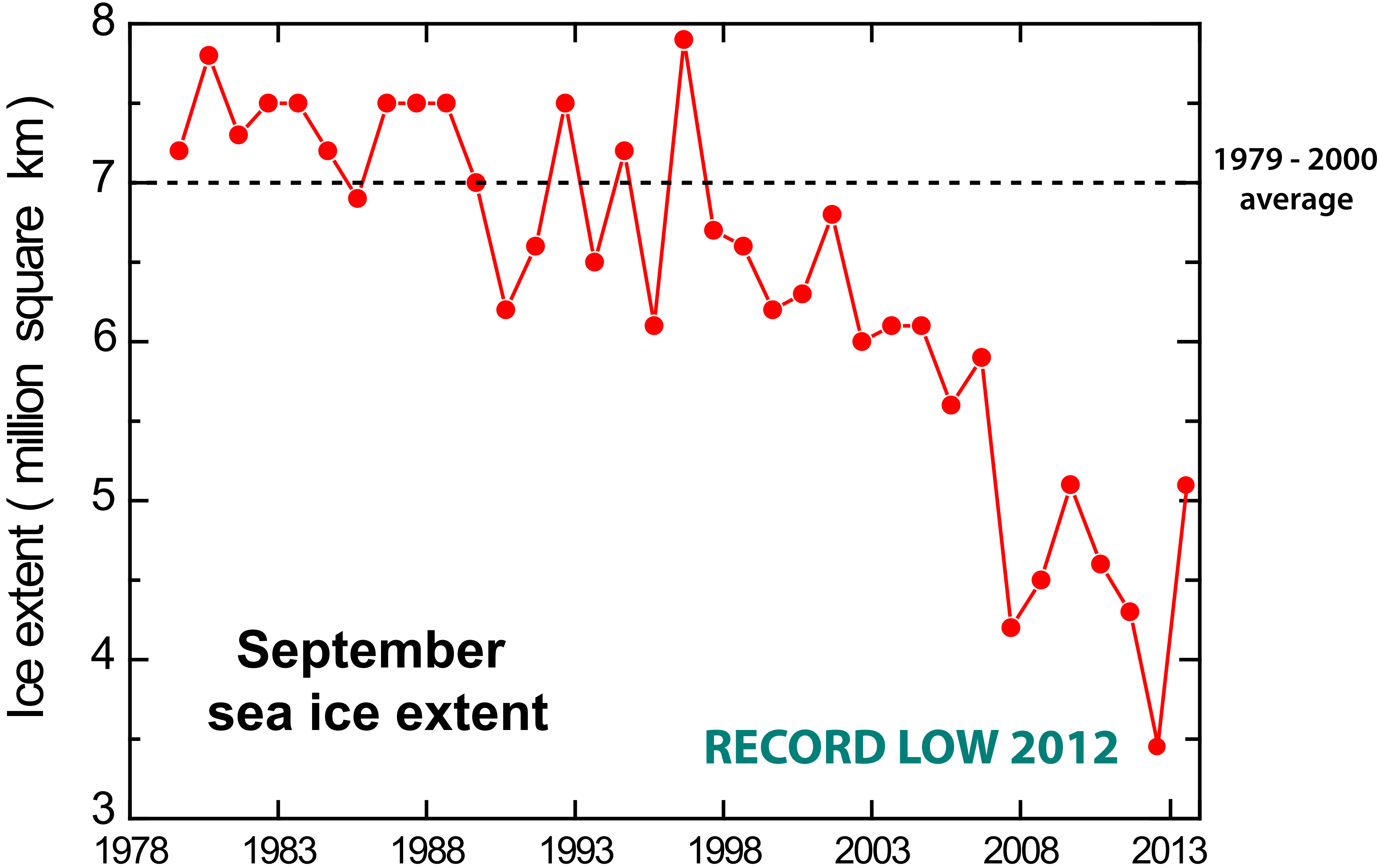
white snow and ice reflect



dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

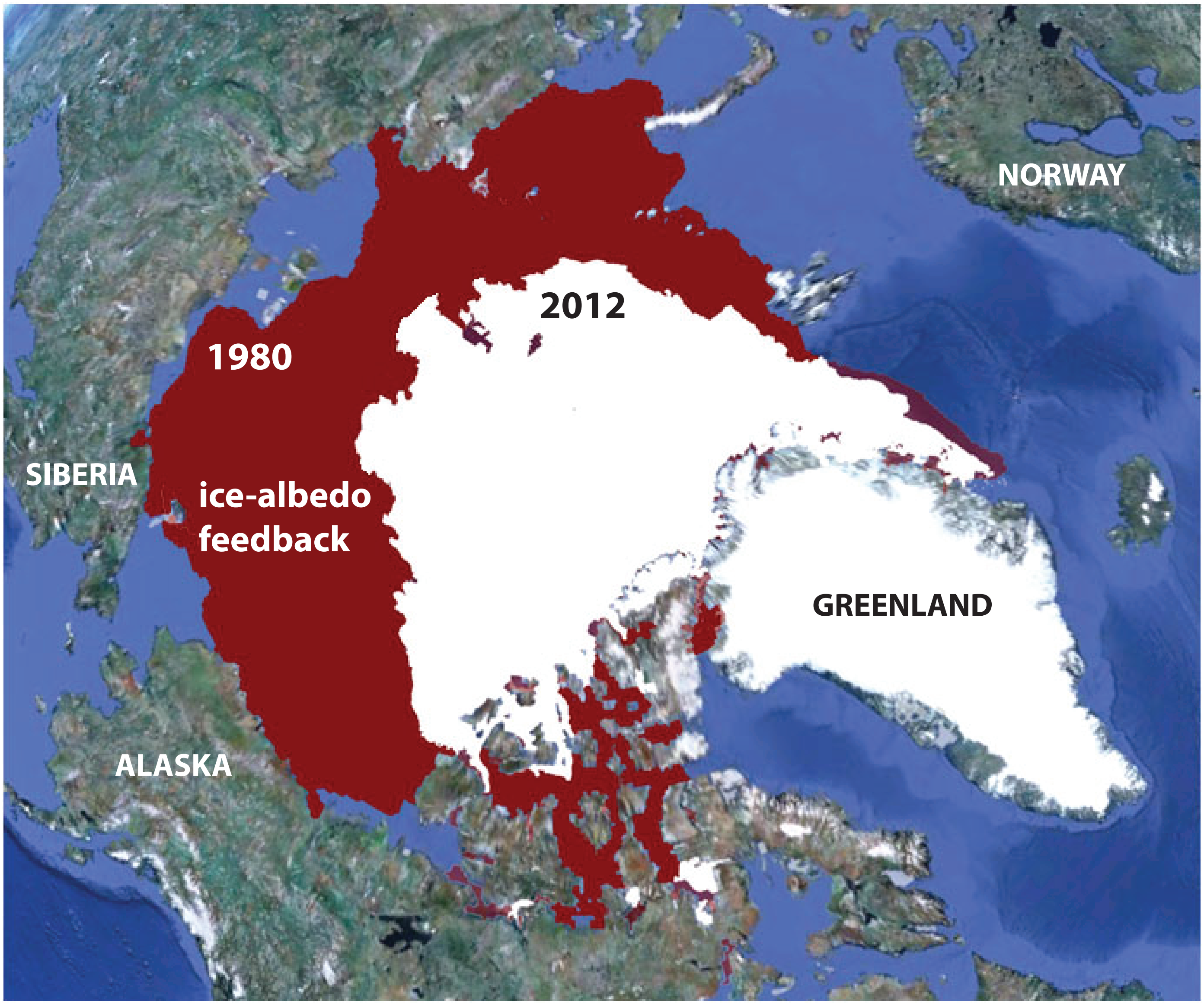
the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

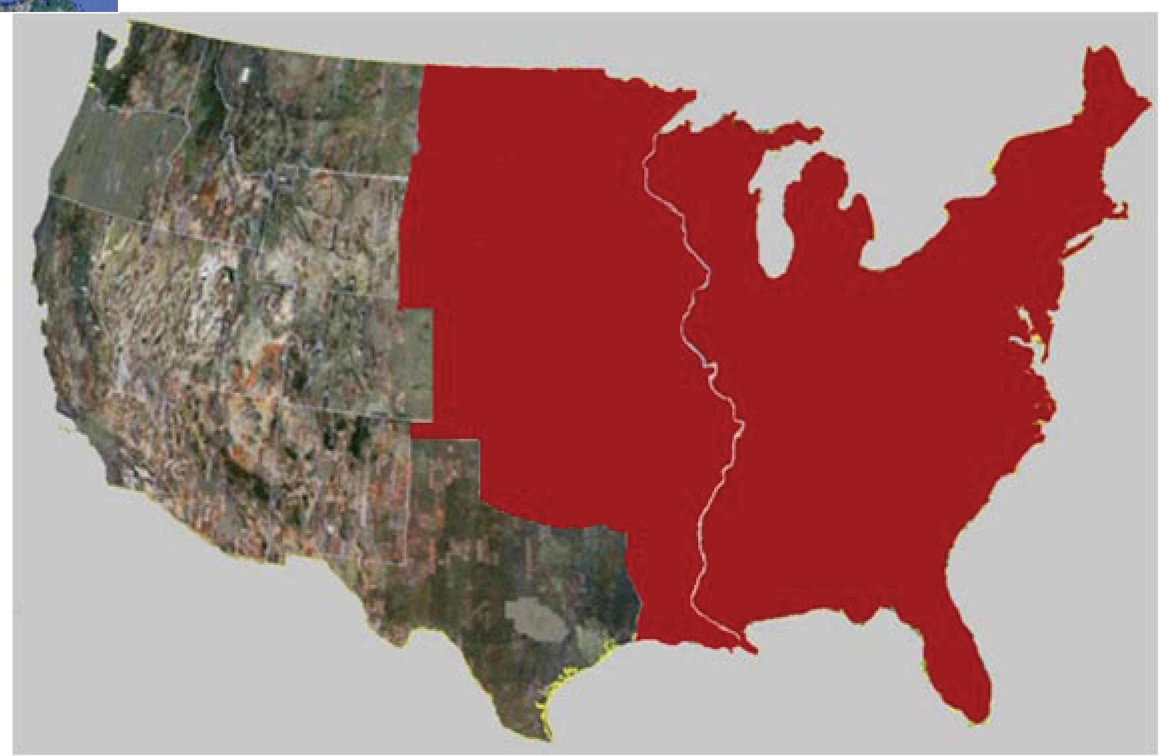
Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers
September 2012 -- 3.4 million square kilometers



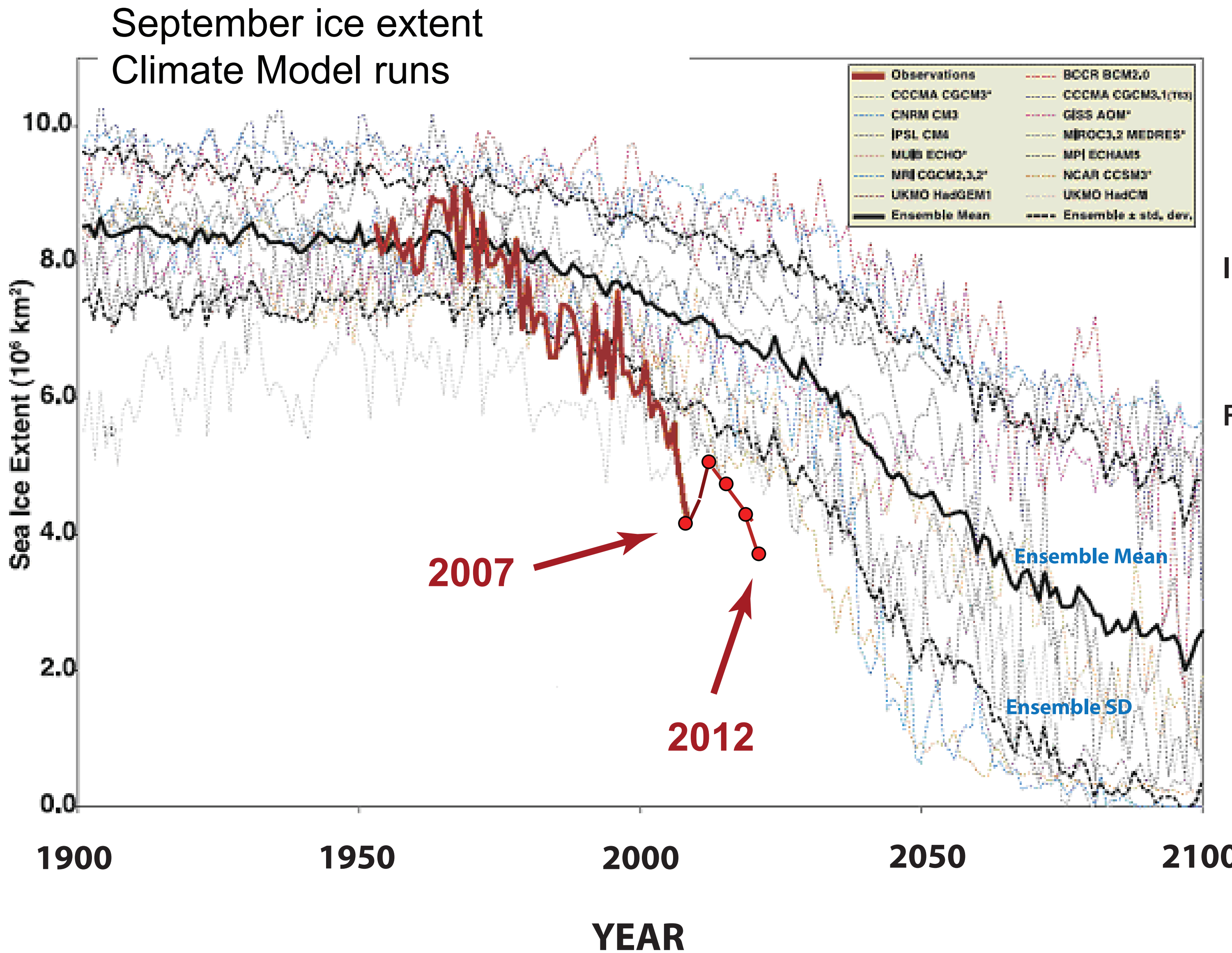
Perovich

recent losses in comparison to the United States



Arctic sea ice decline - faster than predicted by climate models

Stroeve et al., GRL, 2007



IPCC AR4 Models

Intergovernmental Panel on Climate Change (IPCC)

Fourth Assessment AR4, 2007

challenge

represent sea ice more rigorously in climate models

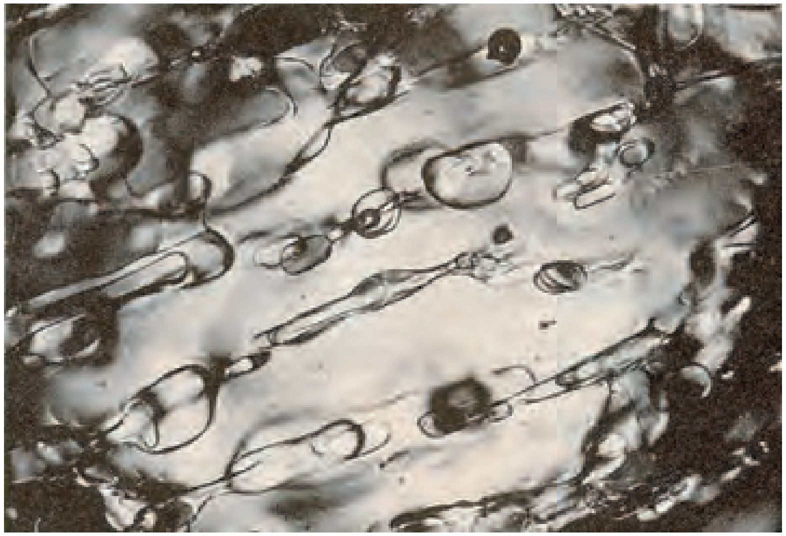
incorporate key processes

fundamental problem -- linkage of scales

sub-grid scale processes

sea ice is a multiscale composite

brine inclusions

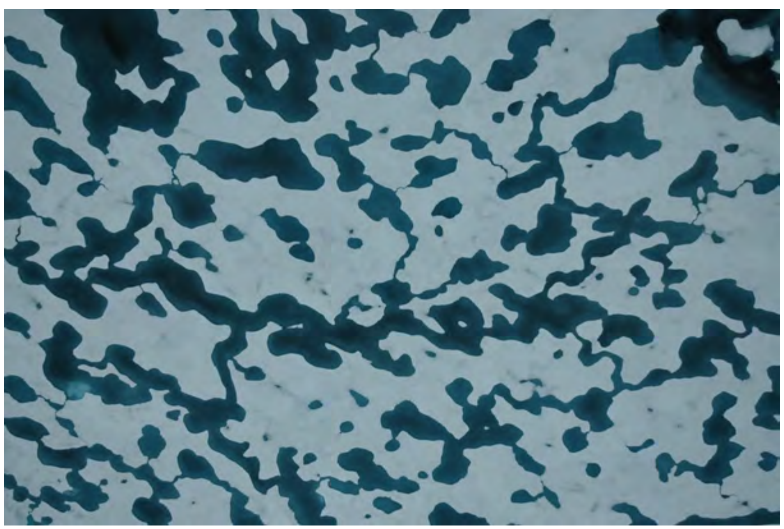


sub-millimeter

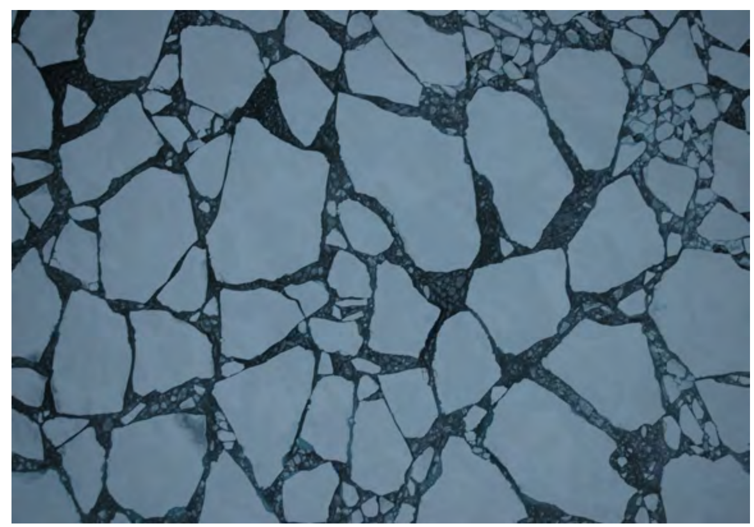
pancakes

centimeters

melt ponds



meters



ice floes

kilometers

What is this talk about?

Using the mathematics of composite materials, phase transitions and dynamical systems to study sea ice .... to improve projections of climate change and how polar ecosystems may respond

1. ***Opposite poles of climate modeling***
2. ***Fluid flow through sea ice - percolation***
3. ***Electromagnetic monitoring of sea ice***
4. ***Arctic and Antarctic experiments***
5. ***Fractal geometry of Arctic melt ponds***

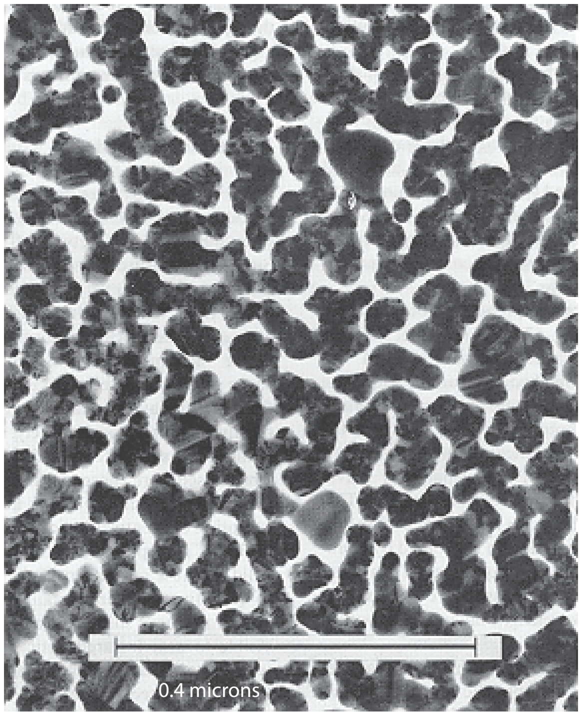
critical behavior ***linkage of scales***

cross-pollination

Develop rigorous representations of sea ice in climate models.

thin silver film

microns



(Davis, McKenzie, McPhedran, 1991)

Arctic melt ponds

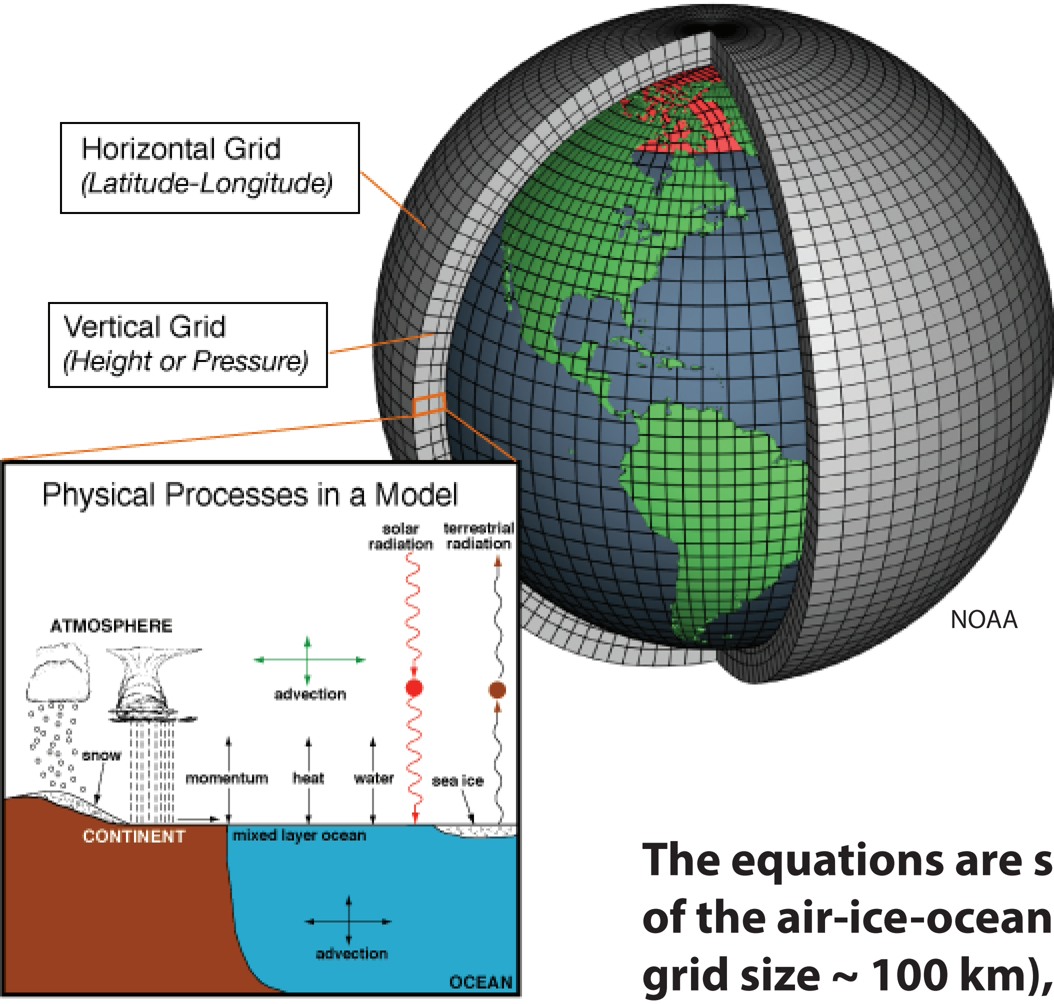
kilometers

(Perovich, 2005)

optical properties

composite geometry -- area fraction of phases, connectedness, necks

Global Climate Models



Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

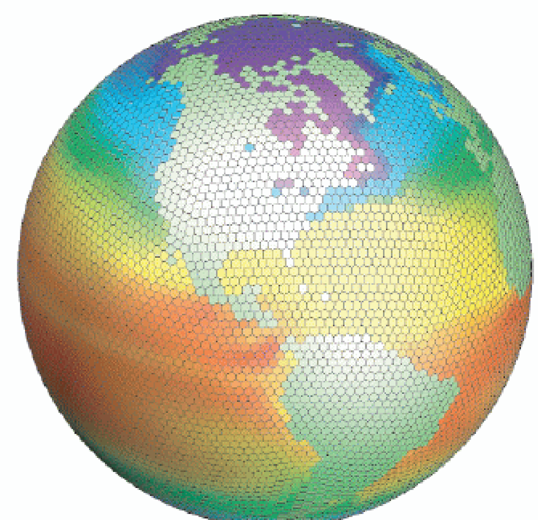
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

key challenge :

incorporating sub - grid scale processes

linkage of scales



Randall *et al.*, 2002

sea ice component of a Global Climate Model

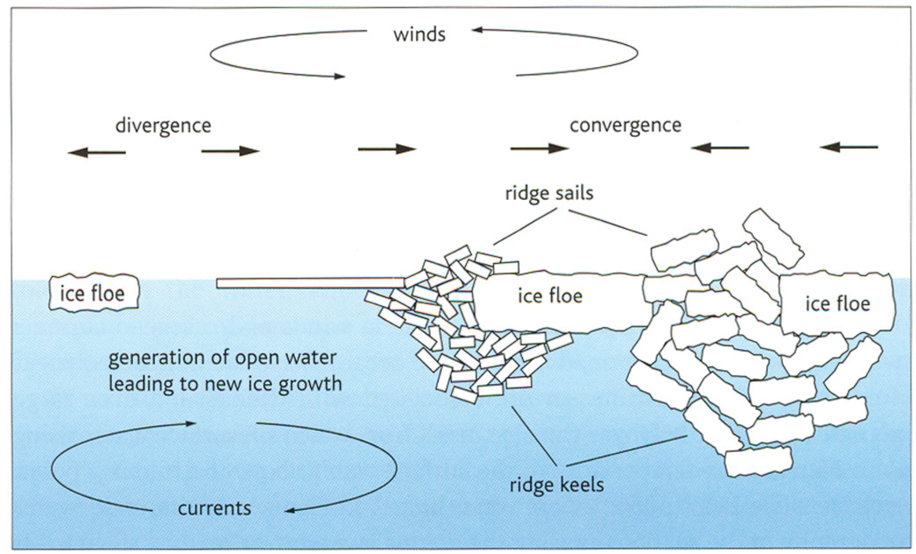
What are the key ingredients -- or ***governing equations***
that need to be solved on grids using powerful computers?

1. **Ice thickness distribution evolution equation**

(Thorndike *et al.* 1975)

dynamics
+
thermodynamics

PDE incorporating ice velocity field
ice growth and melting
mechanical redistribution
ridging and opening



2. **Conservation of momentum, stress *vs.* strain relation** (Hibler 1979)

$F = ma$ **for sea ice**

dynamics

3. **Heat equation of sea ice and snow**

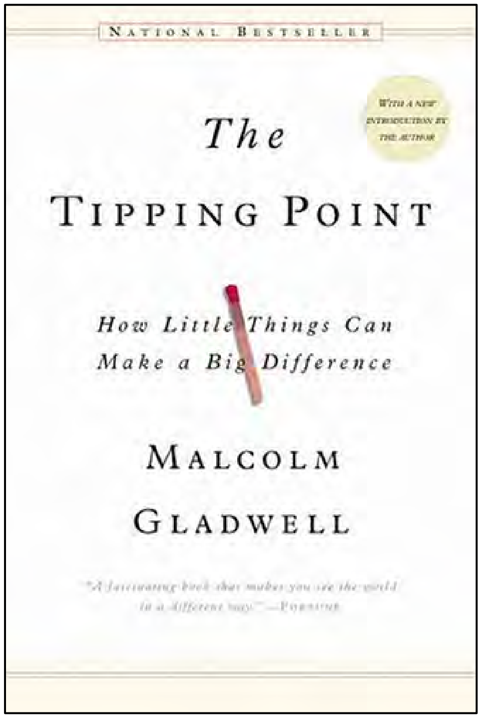
coupling ocean and atmosphere

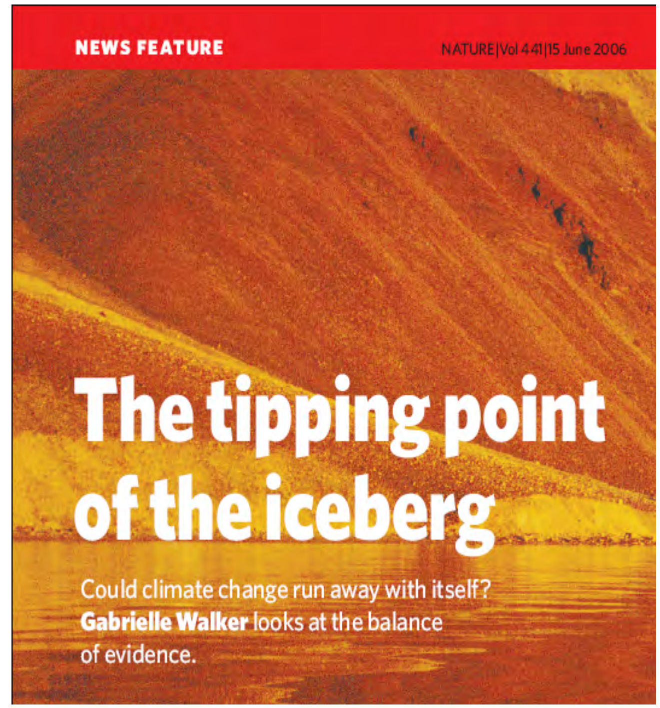
thermodynamics

(Maykut and Untersteiner 1971)

tipping points in the mainstream

climate tipping points – September Arctic sea ice cover





Melting of the Greenland ice sheet
Melting of the West Antarctic ice sheet
Permafrost and tundra loss, leading to the release of methane
Shutoff of N. Atlantic thermohaline conveyor (Gulf Stream) •••

Lenton, et al., PNAS 2008

active area of mathematical research on sea ice -- low order (toy) models of climate change:

Has Arctic sea ice loss passed through a "tipping point"?

an *irreversible* downward slide to ice-free Arctic summers, driven by ice-albedo feedback

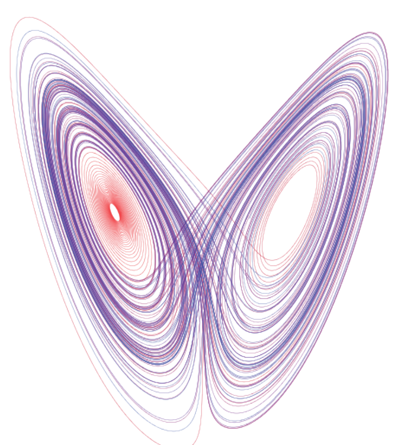
***Eisenman, Wettlaufer, PNAS 2009*:** nonlinear ODE for energy in upper ocean

look for "bifurcations" in solutions

multiple equilibria: ice-free, ice covered, ...

- tipping point unlikely in loss of summer ice

dynamical systems



Lorenz butterfly

further work

Abbot, Silber, Pierrehumbert, JGR 2011

bifurcations when include clouds, ice loss

Serreze, Nature 2011; Tietsche et al. GRL 2011

sea ice could recover quickly

Who cares if Arctic sea ice disappears?

Ralph (Malik) Ahkivgak, c. 20 Oct 1988

Drew Barrymore
John Krasinski
INSPIRED BY THE INCREDIBLE TRUE STORY
that united the world
BIG MIRACLE

Use of sea ice as a platform

- Walrus life cycle tied to sea-ice cycle
- Ice floes as diving platforms for feeding over shallow shelf

Photo: Marc Webber, US Fish & Wildlife Service

• The Arctic holds 25% of the world’s undiscovered oil & gas reserves

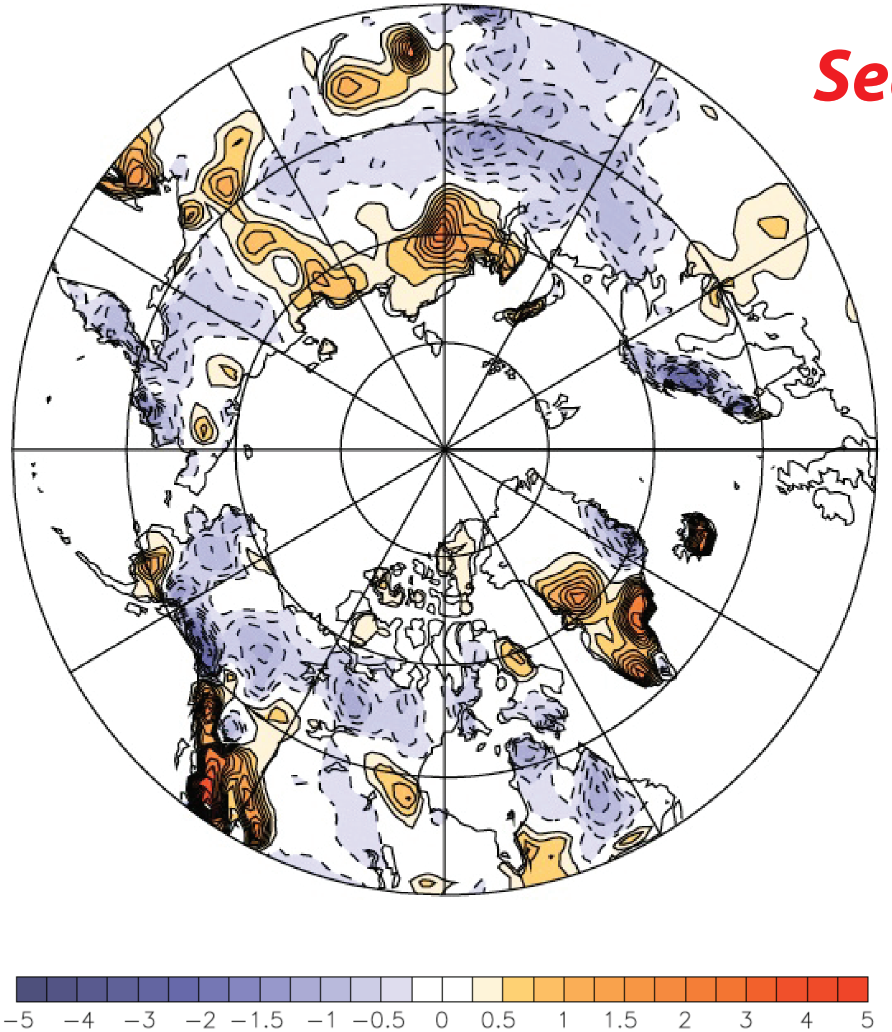
• Sea ice is both a hazard and a supporting feature for hydrocarbon exploration & production

oil companies care about Arctic sea ice loss

Source: UNEP/GRID-Arendal

Sea-ice loss: impacts beyond the Arctic

changes in precipitation and temperature patterns, storm tracks, ...



Orsolini et al., 2011

- One climate model projects reduced precipitation in American West (Sewall & Sloan, 2005)
  Utah - greatest snow on Earth?
- Analysis of 2007 ice minimum suggests above normal snow deposition in NW North America (Orsolini et al., 2011)
- Colder weather in SE Asia, possibly in Eastern US (Hondo et al., 2009)

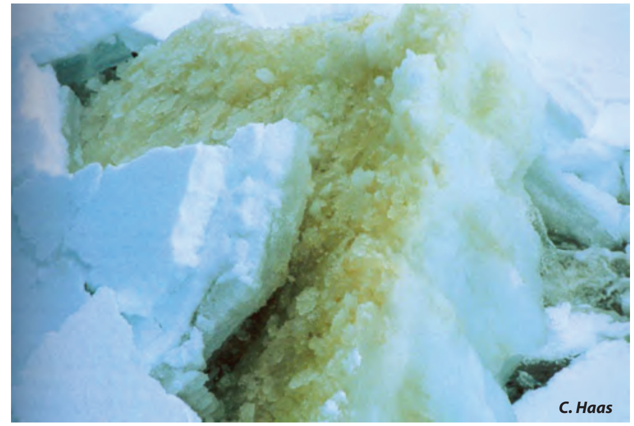
sea ice microphysics

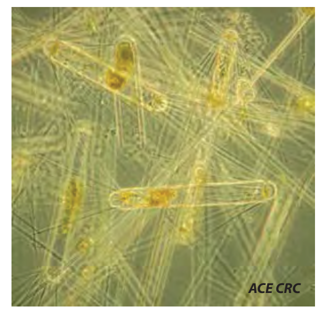
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo

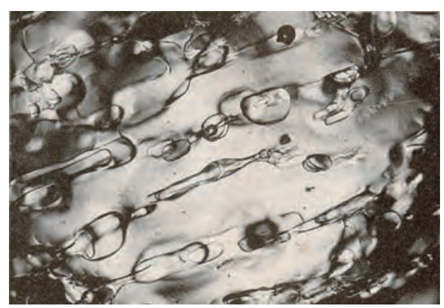
nutrient flux for algal communities





- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO_2*
- *Antarctic surface flooding and snow-ice formation*
- *evolution of salinity profiles*

↕ **linkage of scales**



Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

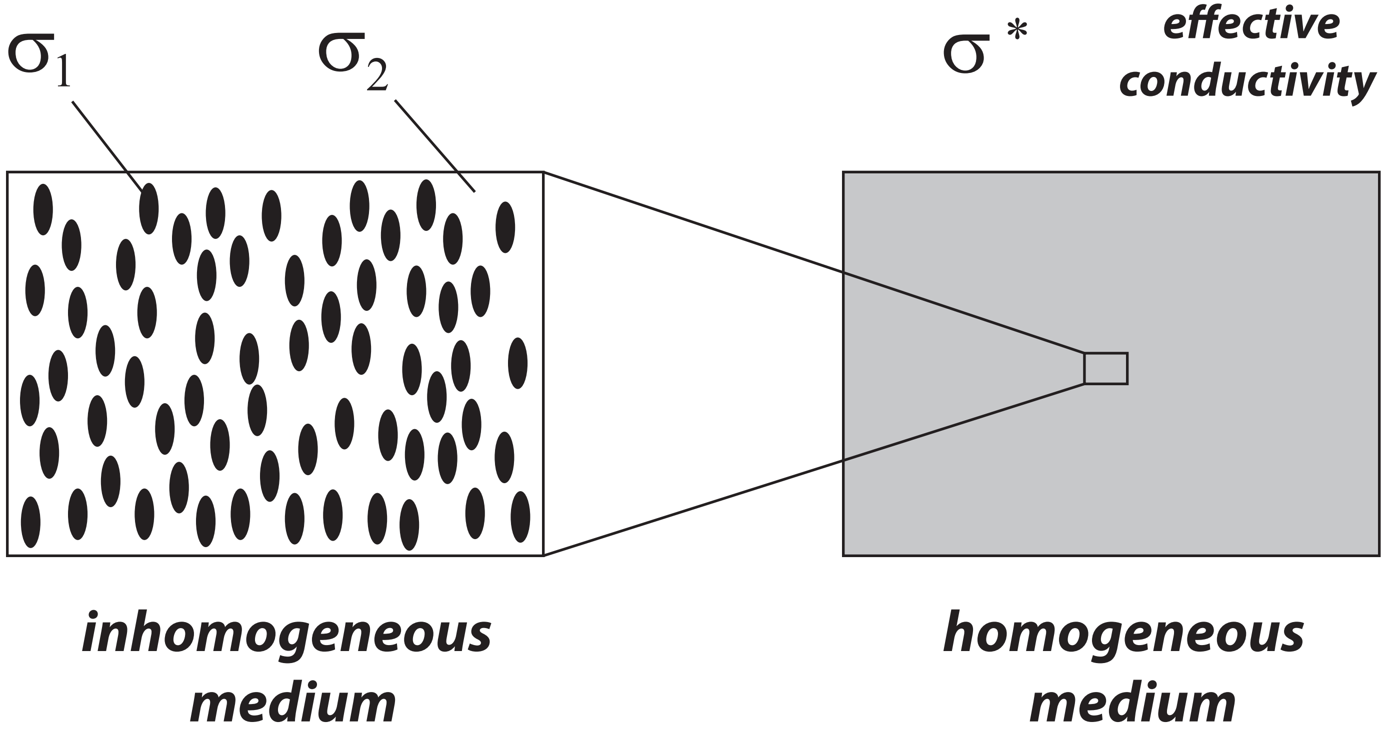
$\mathbf{k}$ = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

e.g. transport properties of composites - electrical conductivity, thermal conductivity, etc.

HOMOGENIZATION

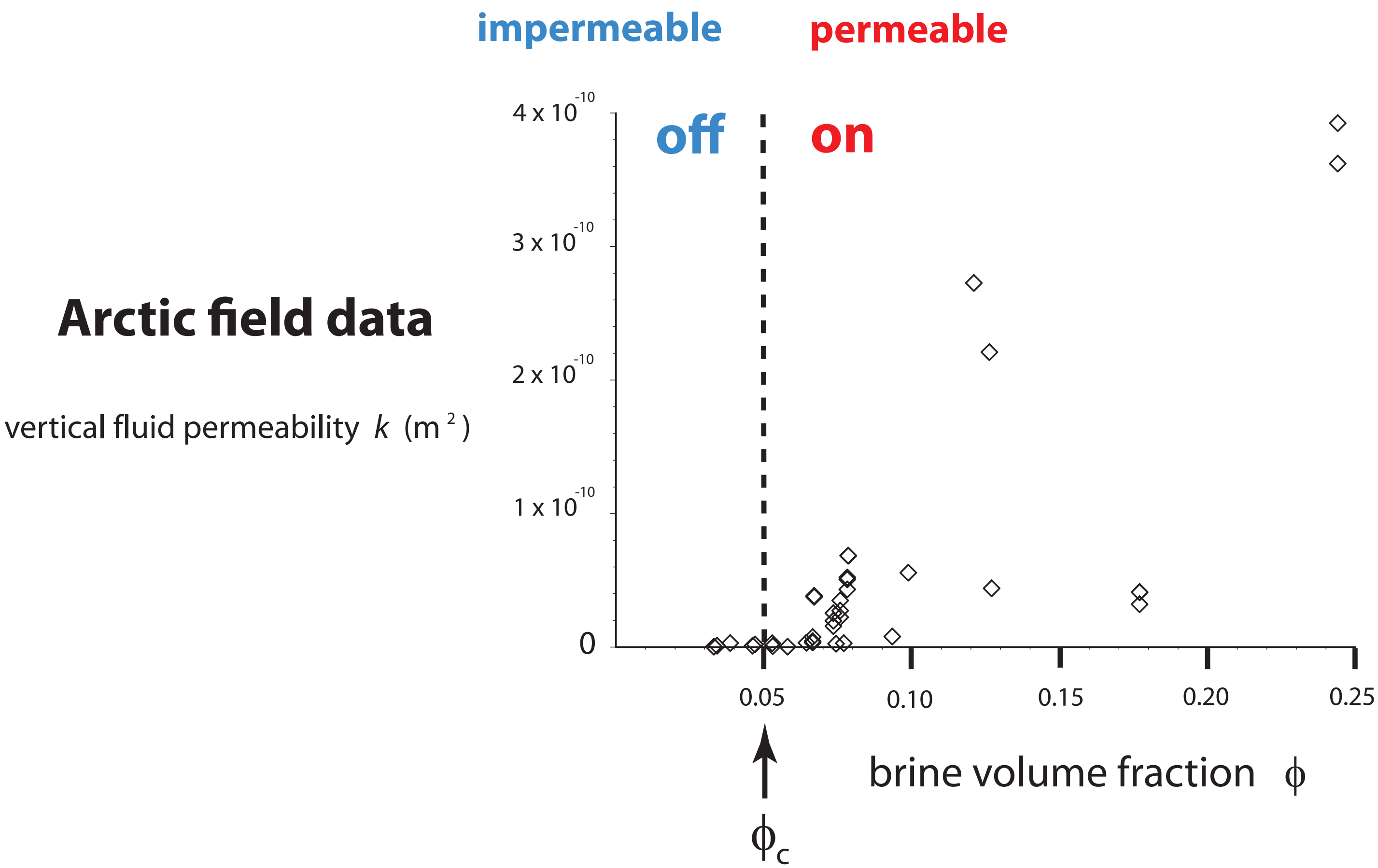


find the homogeneous medium which behaves macroscopically the same as the inhomogeneous medium

Maxwell 1873 : effective conductivity of a dilute suspension of spheres
Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*
*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

Critical behavior of fluid transport in sea ice



on - off switch for fluid flow

critical brine volume fraction $\phi_c \approx 5\%$ ⟷ $T_c \approx -5^\circ C$, $S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

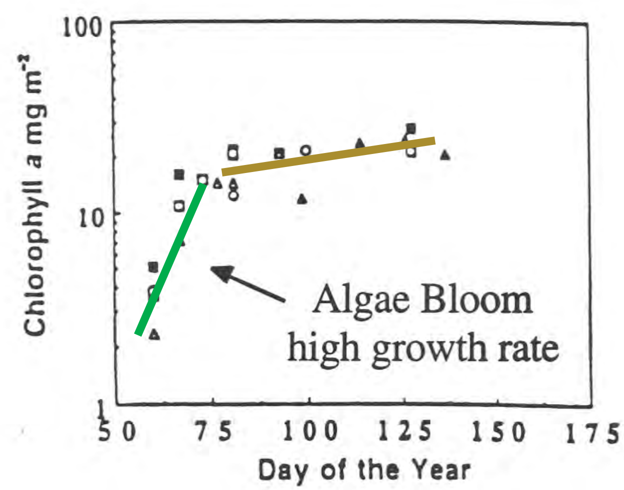
sea ice algal communities

D. Thomas 2004

nutrient replenishment controlled by ice permeability

biological activity turns on or off according to ***rule of fives***

critical behavior of microbial activity



Convection-fueled algae bloom
Ice Station Weddell

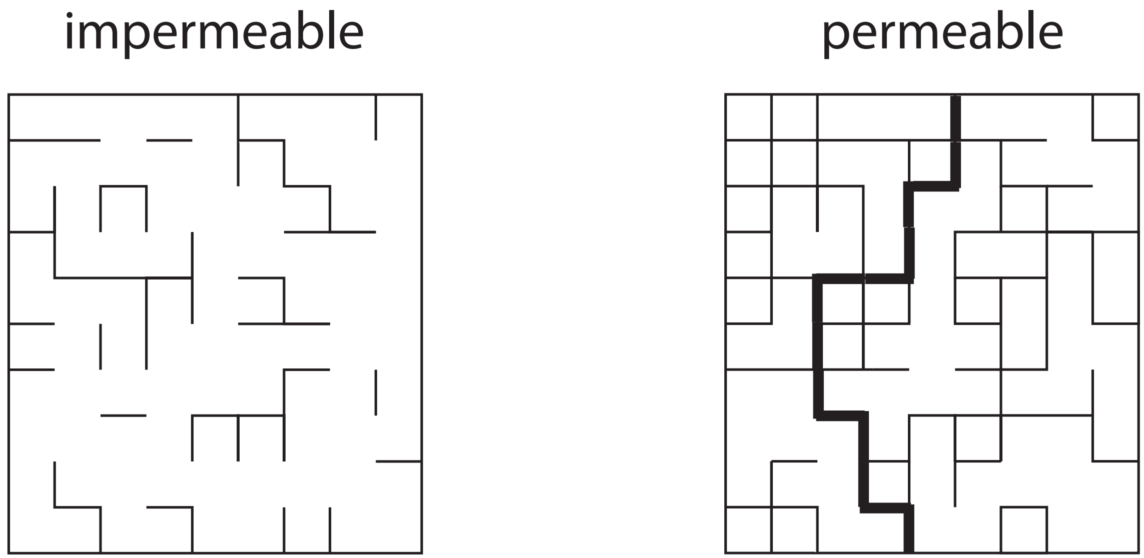
Golden, Ackley, Lytle ***Science 1998***

Fritsen, Lytle, Ackley, Sullivan ***Science 1994***

Why is the rule of fives true?

percolation theory

mathematical theory of connectedness



$p = 1/3$ $p = 2/3$

bond → *open* with probability p
closed with probability $1-p$

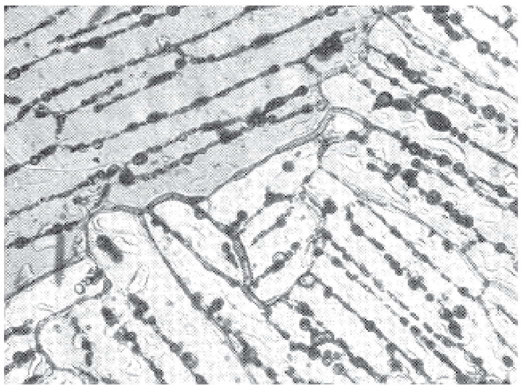
percolation threshold

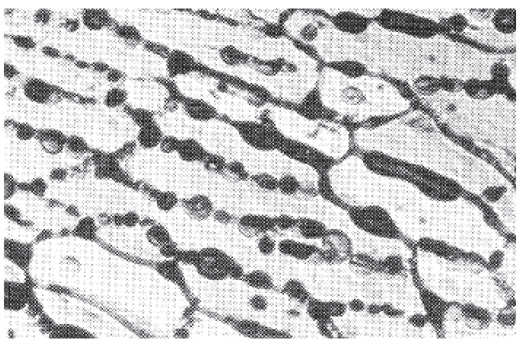
$p_c = 1/2$ for $d = 2$

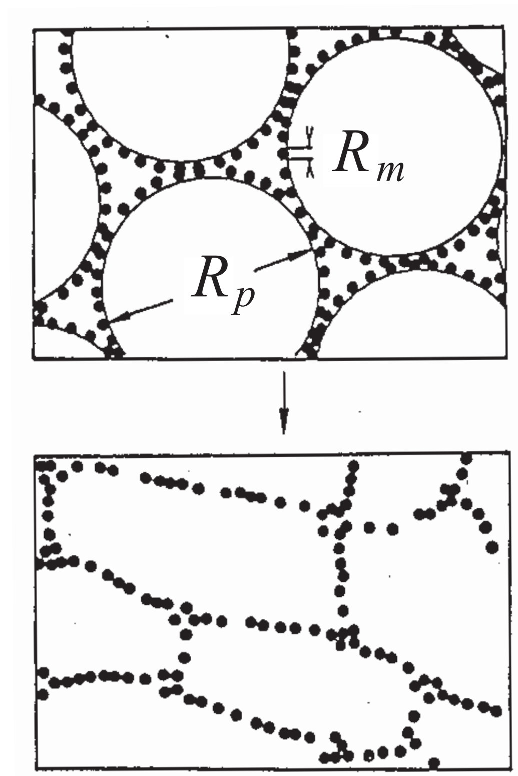
first appearance of infinite cluster ***"tipping point" for connectivity***

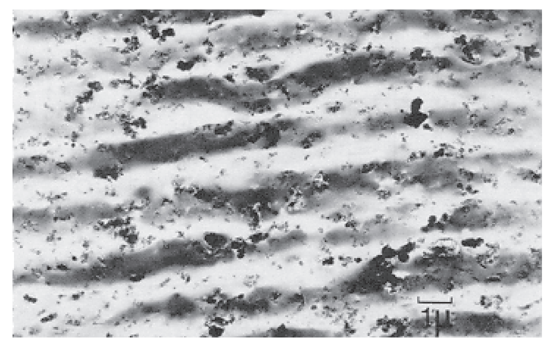
Continuum percolation model for ***stealthy*** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

$\phi_c \approx 5\,\%$ Golden, Ackley, Lytle, *Science,* 1998









sea ice

compressed powder

radar absorbing composite

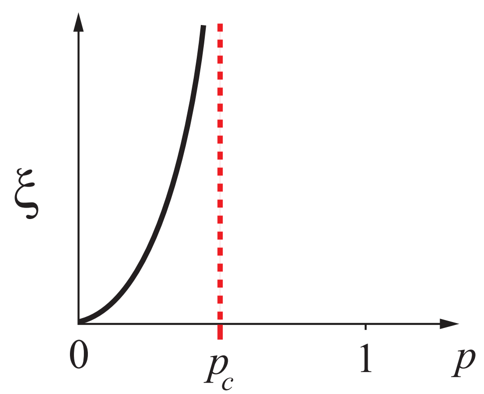
sea ice is radar absorbing

order parameters in percolation theory

geometry

correlation length

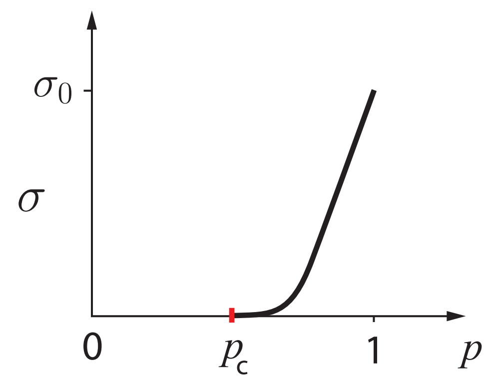
(characteristic scale of connectedness)



$$\xi(p) \sim |p - p_c|^{-\nu}$$

transport

effective conductivity
or fluid permeability



$$\sigma(p) \sim \sigma_0 (p - p_c)^t$$

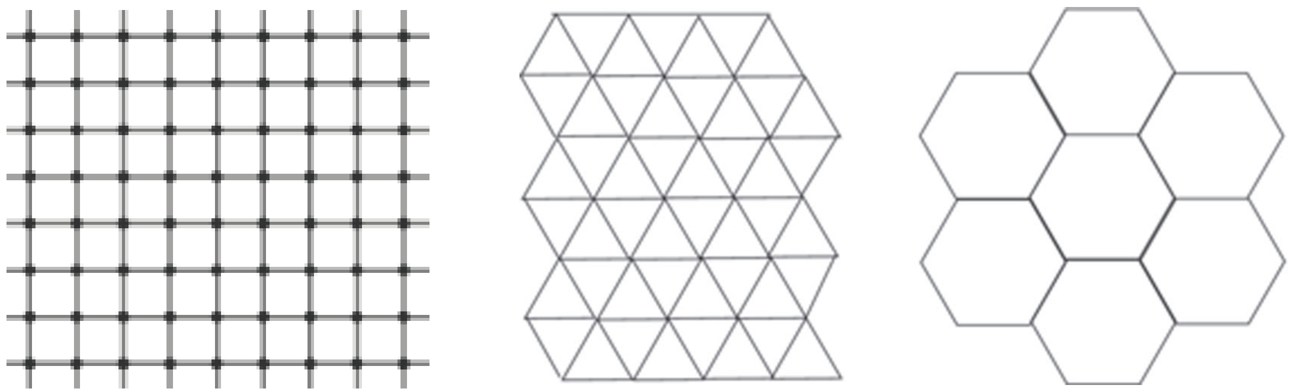
UNIVERSAL critical exponents for lattices -- depend only on dimension

(**$1 \leq t \leq 2$,** Golden, *Phys. Rev. Lett.* 1990 **;** *Comm. Math. Phys.* 1992)

non-universal behavior in continuum

two dimensional lattices

square triangular hexagonal



percolation thresholds change

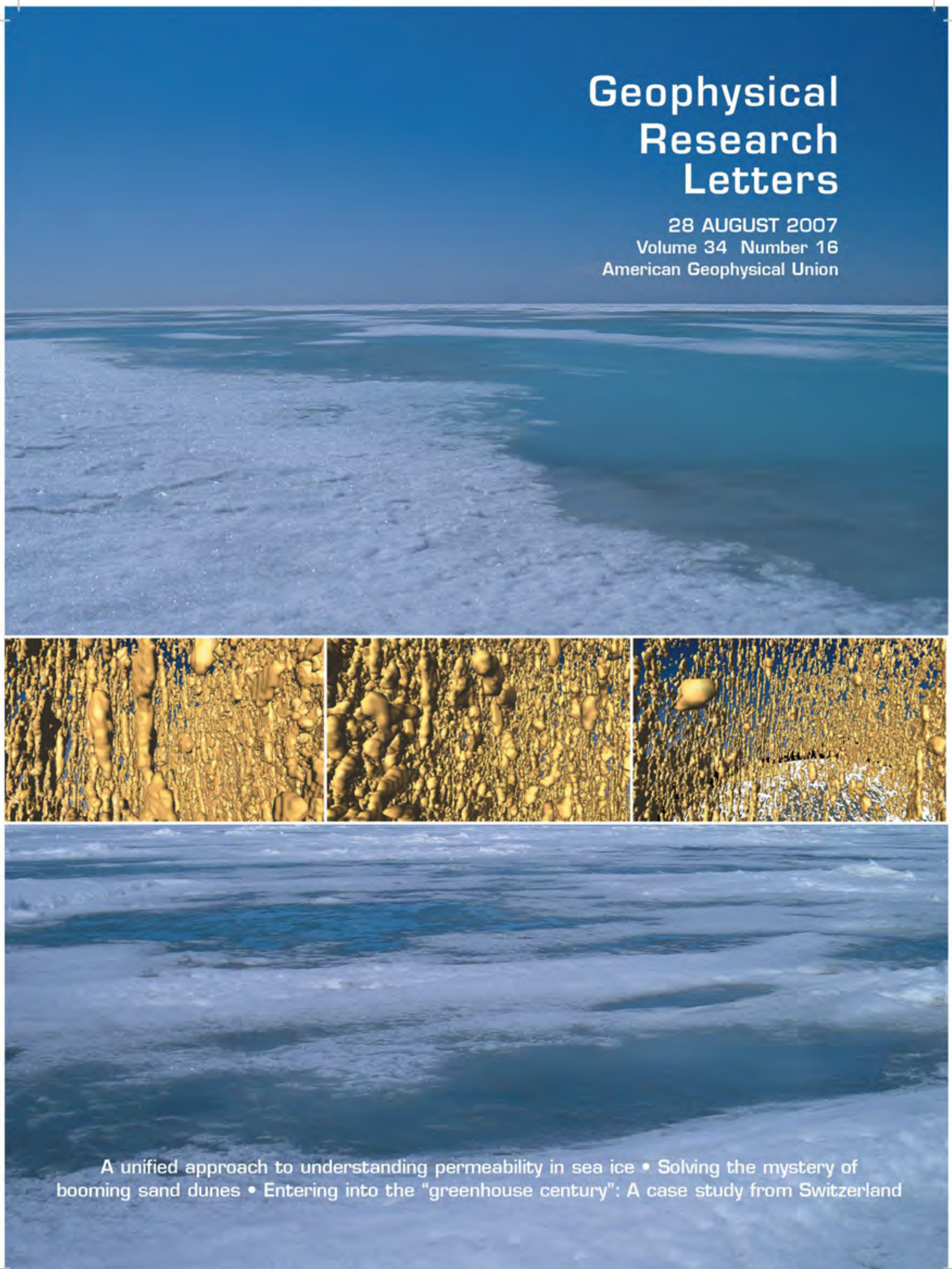
critical exponents are universal

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**

micro-scale

controls

macro-scale processes



rigorous bounds
percolation theory
hierarchical model
network model

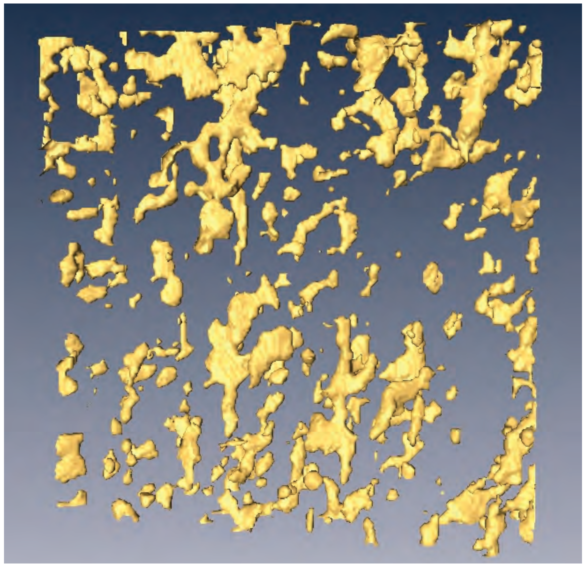
field data

X-ray tomography for brine inclusions

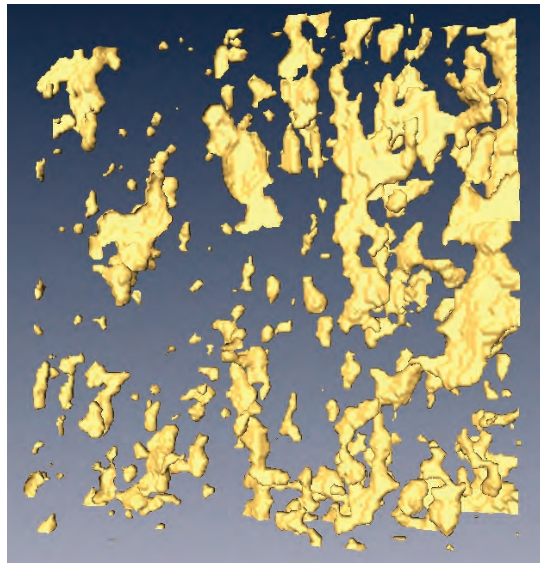
unprecedented look at thermal evolution of brine phase and its connectivity

brine connectivity (over cm scale)

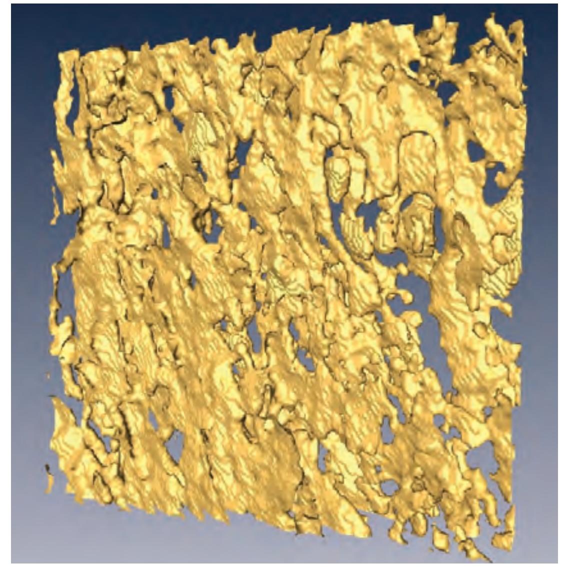
8 x 8 x 2 mm



-15 °C, $\phi = 0.033$



-6 °C, $\phi = 0.075$



-3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats → 3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

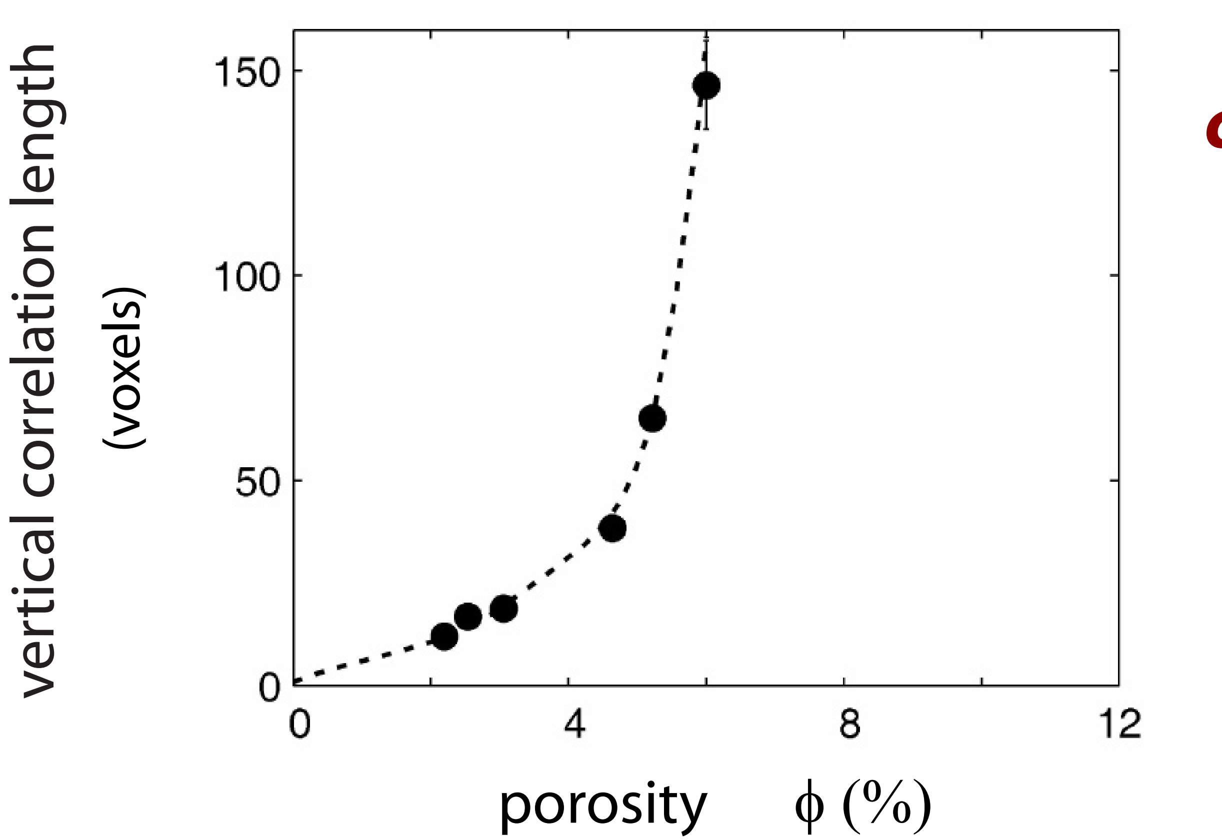
- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

The key connectivity functions of percolation theory have been computed extensively for many lattice models, but ***NOT*** for natural materials.

We have calculated them for sea ice single crystals and estimated anisotropic percolation thresholds.

Pringle, Miner, Eicken, Golden, JGR (Oceans) 2009



correlation length

characteristic scale of connectedness

divergence of vertical ***correlation length*** for single crystal data

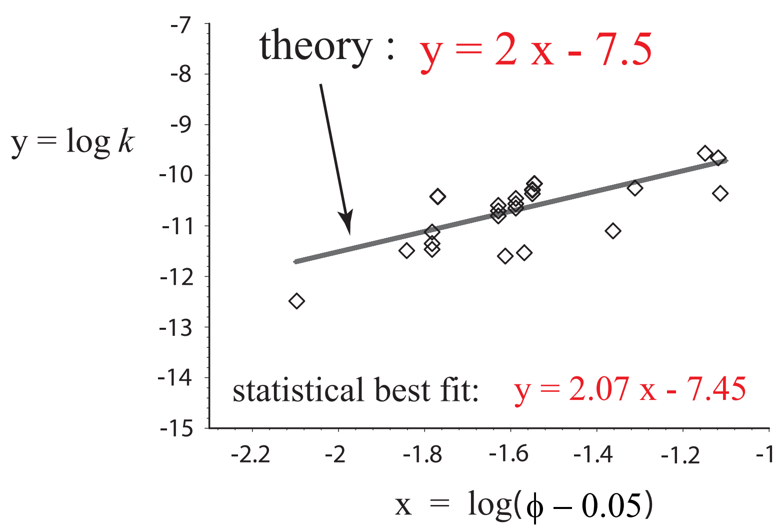
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0



Remote sensing of sea ice

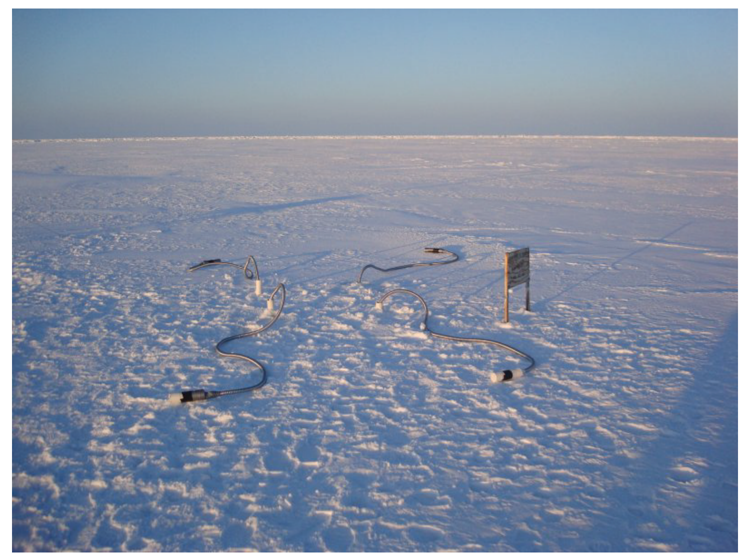
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice properties from electromagnetic (EM) data

$$\varepsilon^*$$

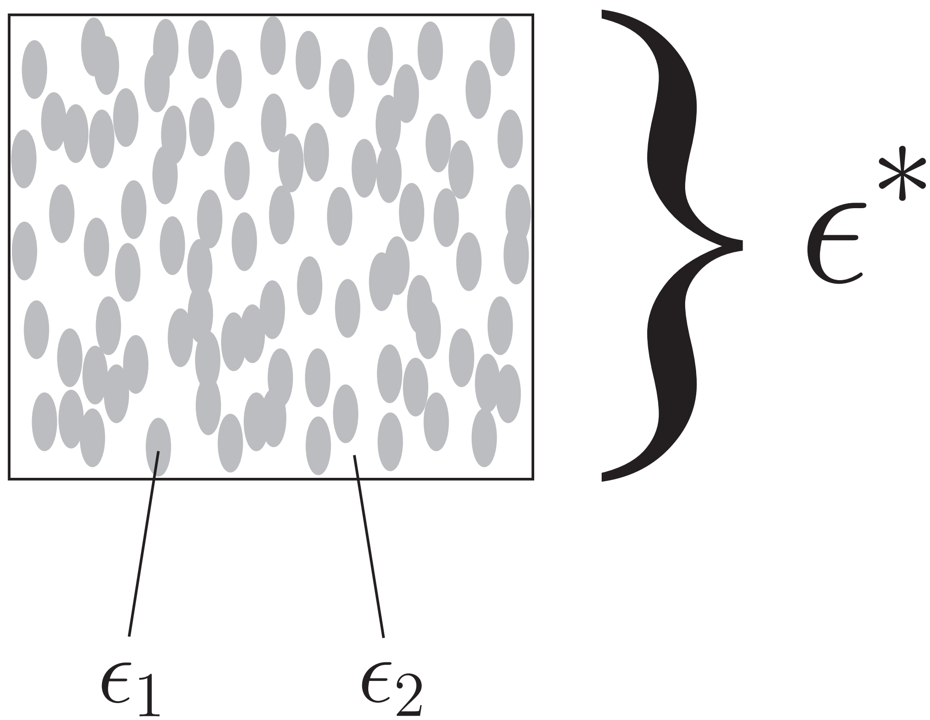
effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit

What are the effective propagation characteristics of an EM wave in the medium?



p_1 , p_2 = volume fractions of brine and ice

ocean swells propagating through a vast field of pancake ice

HOMOGENIZATION: long wave sees an effective medium, not individual floes

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method

Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

composite geometry (spectral measure μ) ⟶ ε^*

integral representations, rigorous bounds, approximations, etc.

$$1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z} \qquad s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

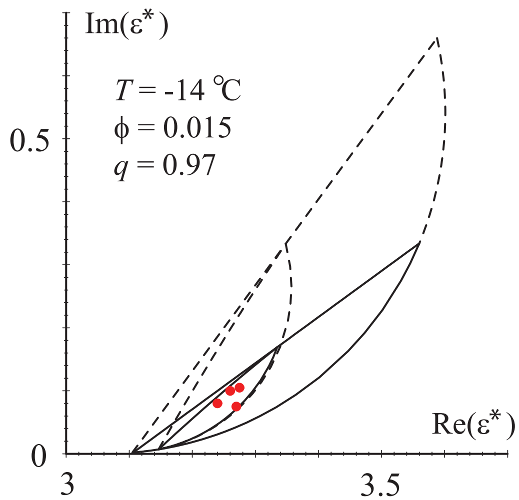
Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)
(McPhedran, McKenzie, and Milton, 1982)

ε^* ⟶ **composite geometry (spectral measure μ)**

recover brine volume fraction, connectivity, etc.

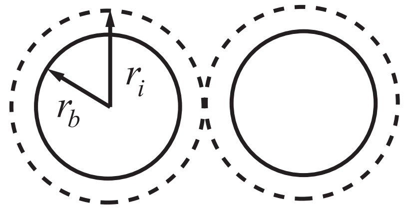
forward and inverse bounds for sea ice

forward bounds



4.75 GHz data

matrix particle

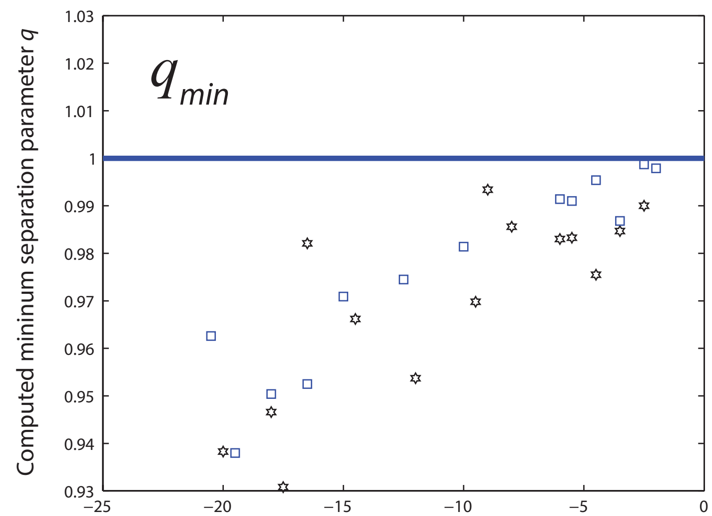


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



Slab temperature °C

inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

rigorous inverse bound on spectral gap

construct algebraic curves which bound admissible region in (p,q)-space

Orum, Cherkaev, Golden
Proc. Roy. Soc. A, 2012

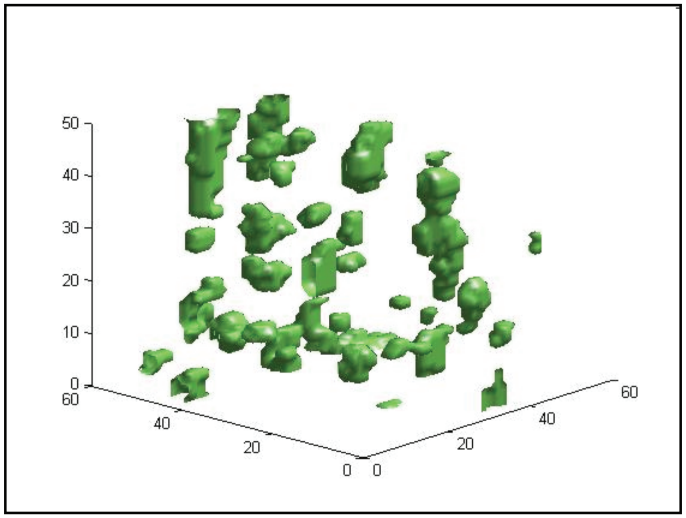
inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden
Physica B, 2007

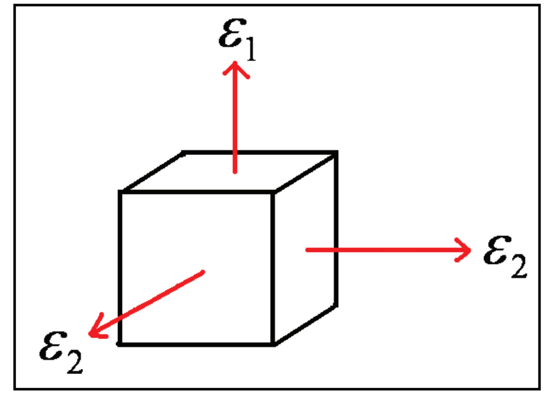
polycrystalline bounds two-scale homogenization

Gully, Lin, Cherkaev, Golden, 2014

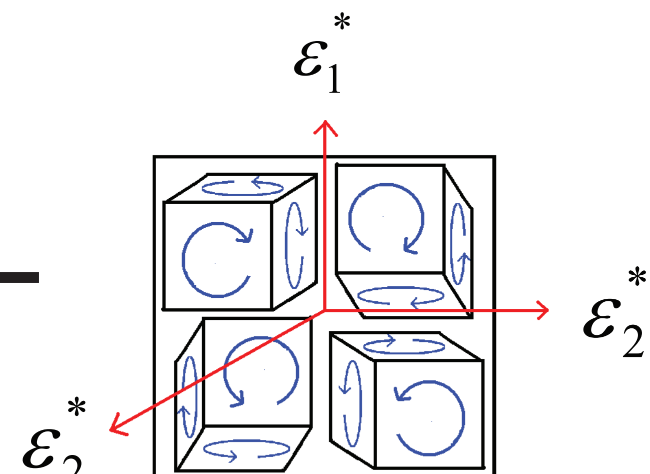
two scale homogenization for polycrystalline sea ice



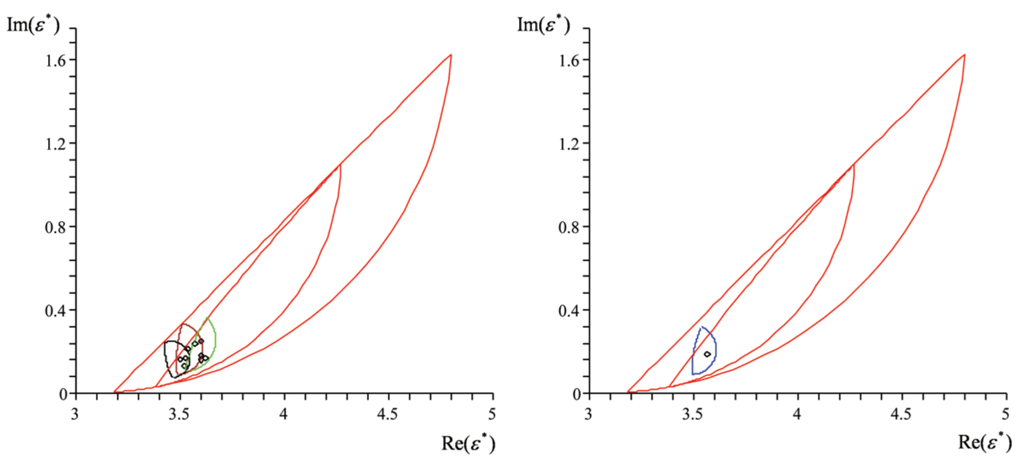
numerical homogenization for single crystal



analytic continuation for polycrystals



bounds

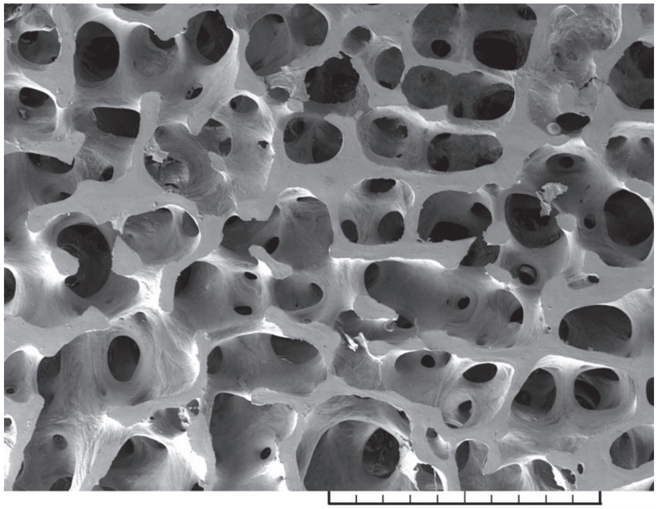


Gully, Lin, Cherkaev, Golden 2014

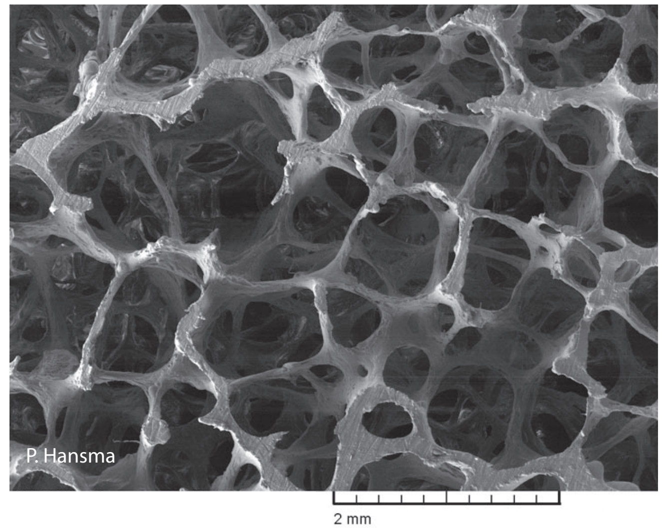
spectral characterization of porous microstructures in bone

Golden, Murphy, Cherkaev, J. Biomechanics 2011

(a) young healthy trabecular bone



(b) old osteoporotic trabecular bone

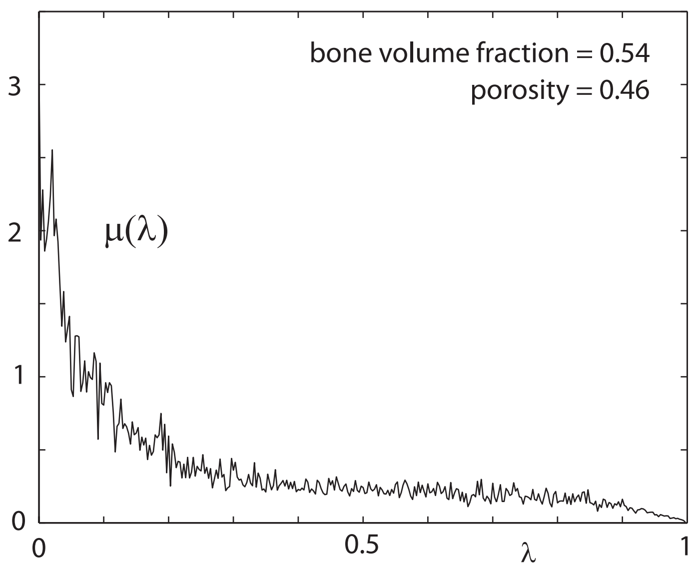


+

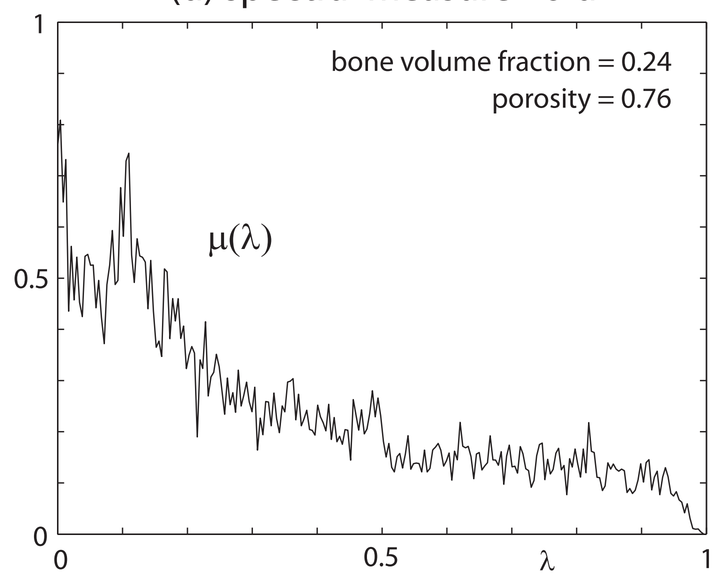
reconstruction of spectral measures from complex permittivity data

using regularized inversion scheme

(c) spectral measure - young



(d) spectral measure - old



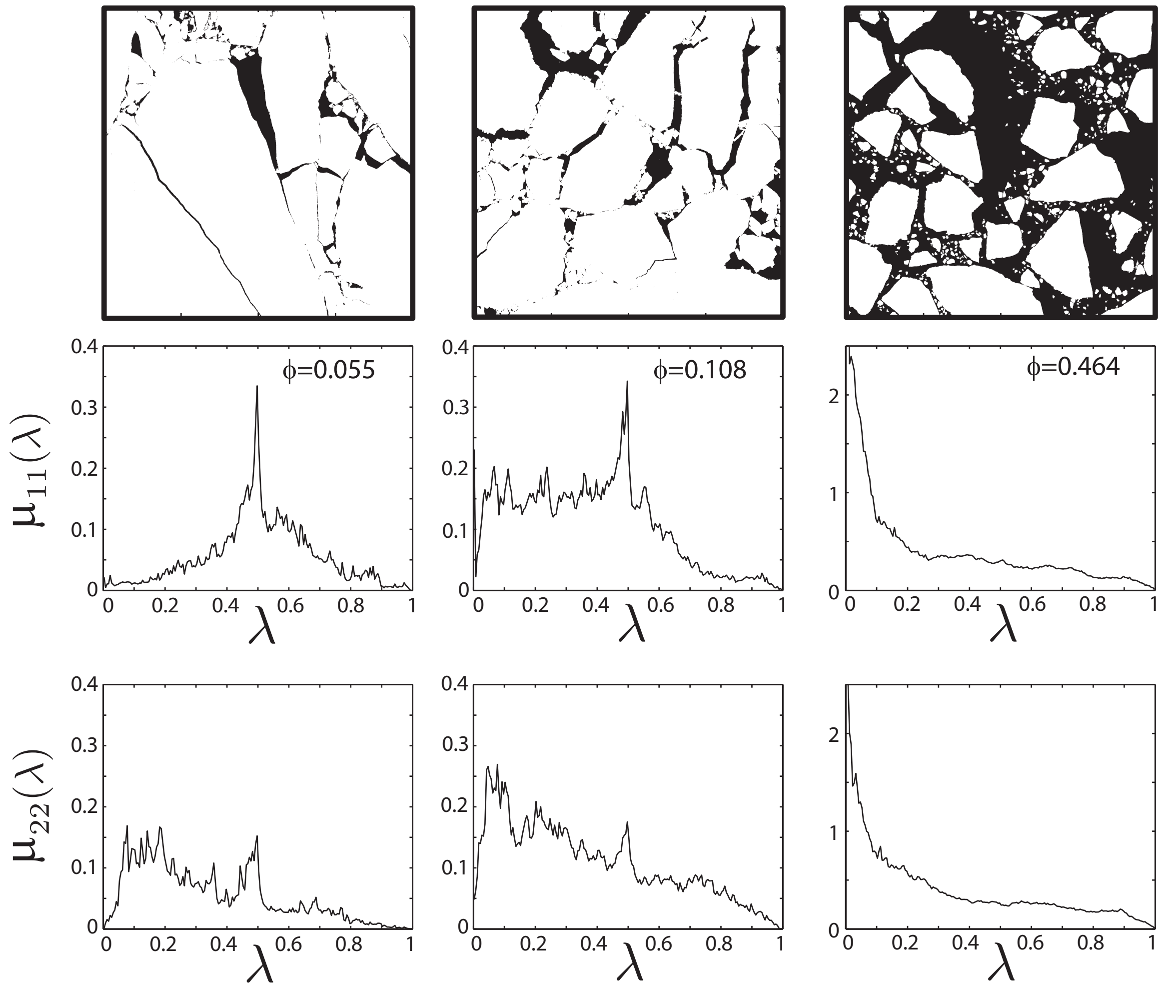
EM monitoring of osteoporosis

loss of bone connectivity

the math doesn't care if it's sea ice or bone!

spectral measures provide a path toward rigorously incorporating "composite microstructure" into calculations of effective behavior on larger scales

spectral measures for the Arctic sea ice pack



area under curve = φ = open water fraction

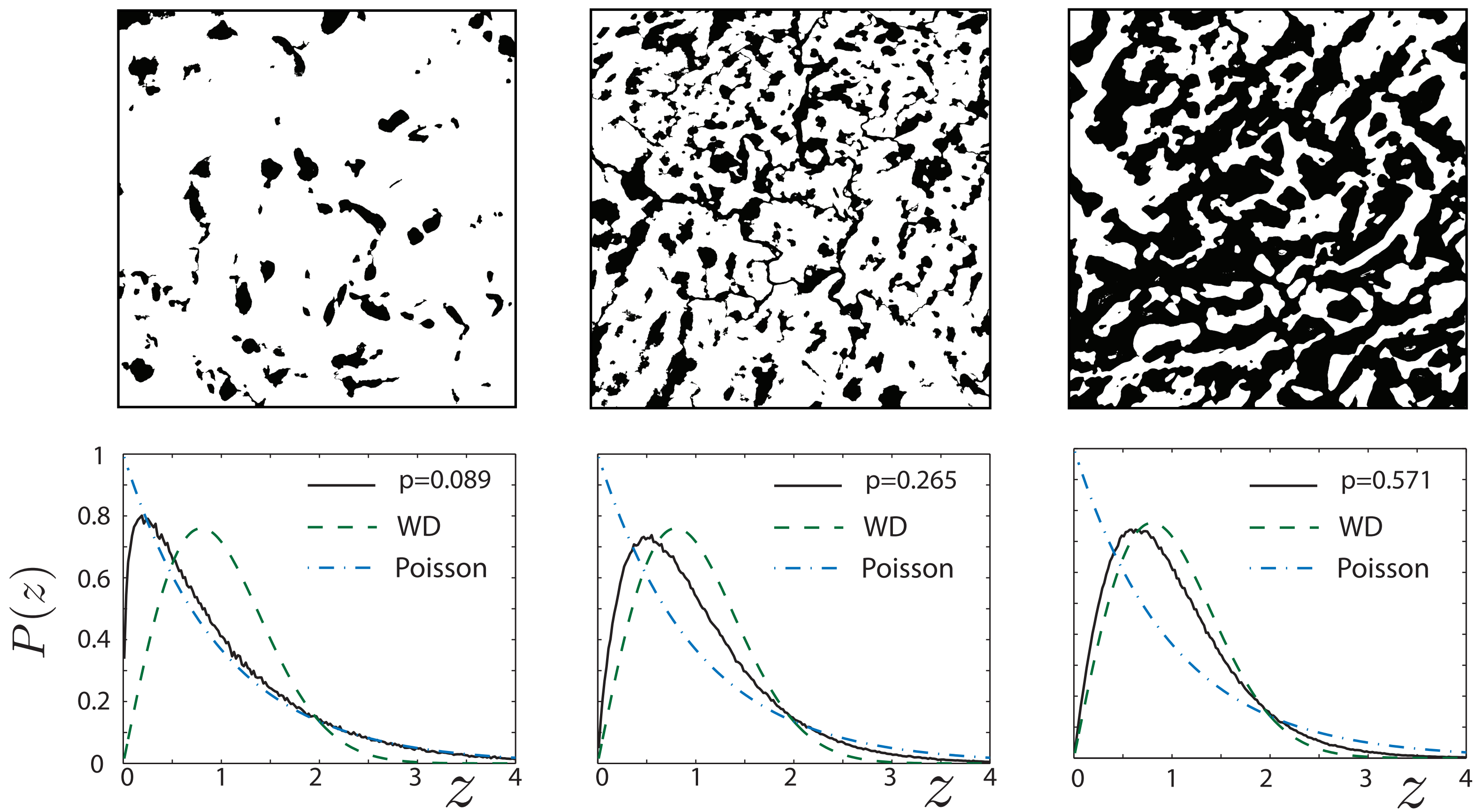
spectral gap closes as ocean phase becomes connected

N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden, 2014

random matrix characterization of connectedness transition -- discretization of $\chi\Gamma\chi$

Unfolded Eigenvalue Spacing Distribution

ARCTIC MELT PONDS



eigenvalue statistics for transport tend toward the UNIVERSAL Wigner-Dyson distribution as the "conducting" phase becomes connected over large scales

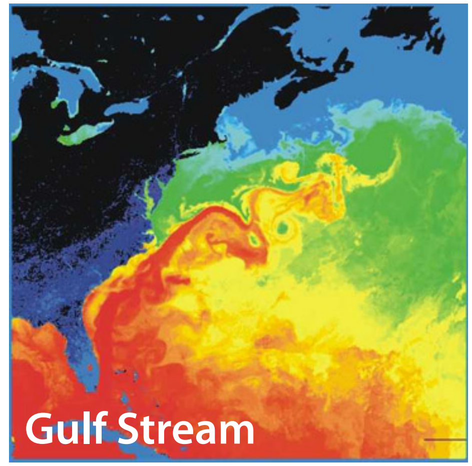
uncorrelated → *"level repulsion"*

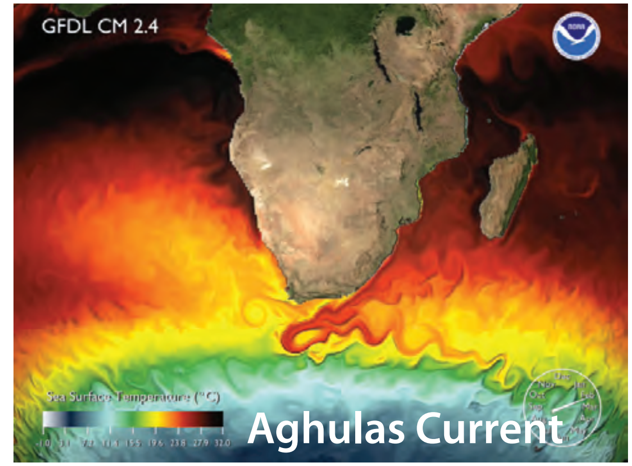
N. B. Murphy, K. M. Golden 2014

advection enhanced diffusion

effective diffusivity

tracers, buoys diffusing in ocean eddies
diffusion of pollutants in atmosphere
salt, heat, nutrient transport in ocean





advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

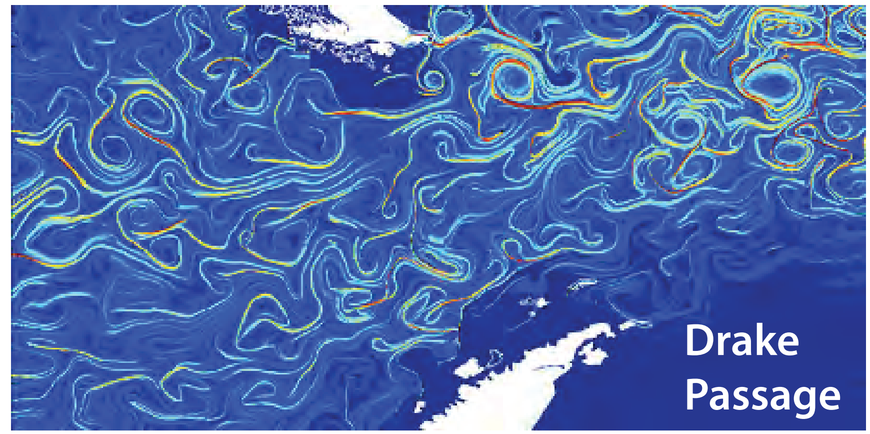
↓ ***homogenize***

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* ***effective diffusivity*** — *analytic function of Péclet number*

Stieltjes integral for κ^* ***with spectral measure***

Avellaneda and Majda, PRL 89, CMP 91 *Murphy, Zhu, Golden 2014*



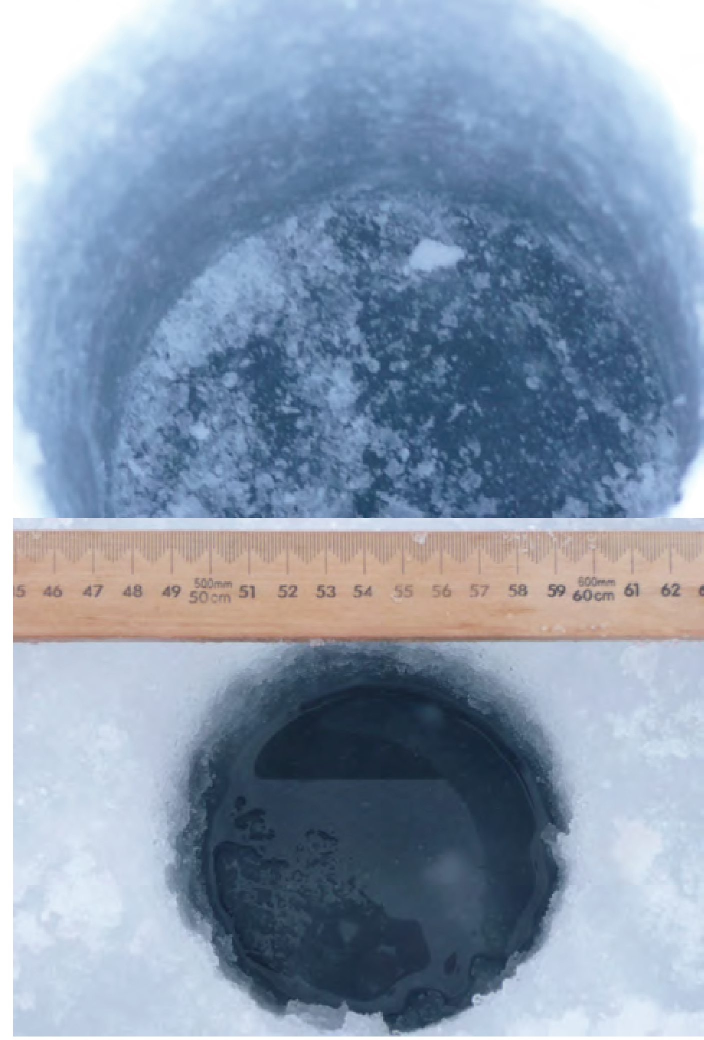
Arctic and Antarctic field experiments

develop electromagnetic methods of monitoring fluid transport and microstructural transitions

extensive measurements of fluid and electrical transport properties of sea ice:

2007	*Antarctic*	*SIPEX*
2010	*Antarctic*	*McMurdo Sound*
2011	*Arctic*	*Barrow AK*
2012	*Arctic*	*Barrow AK*
2012	*Antarctic*	*SIPEX II*
2013	*Arctic*	*Barrow AK*
2014	*Arctic*	*Chukchi Sea*

photo by Jan Lieser

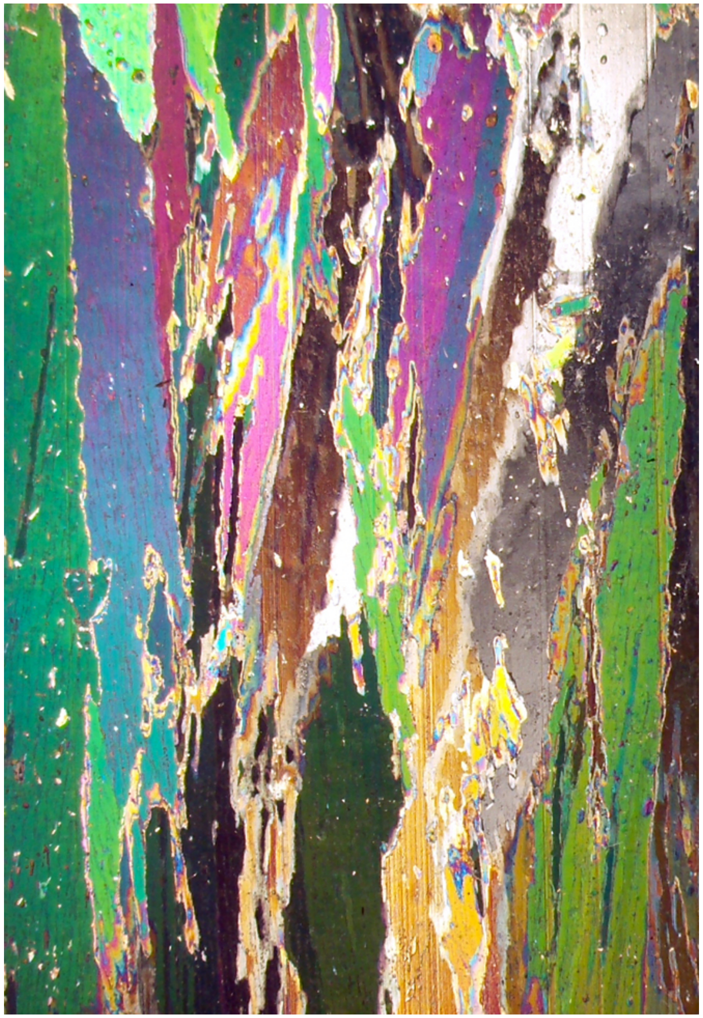


measuring fluid permeability of Antarctic sea ice

SIPEX 2007

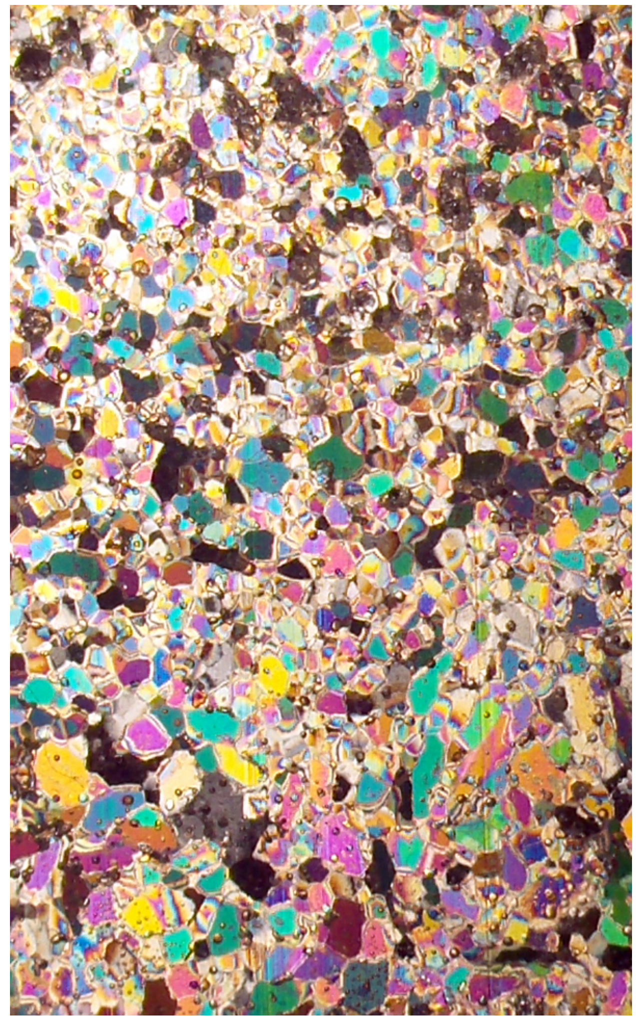
higher threshold for fluid flow in Antarctic granular sea ice

columnar



5%

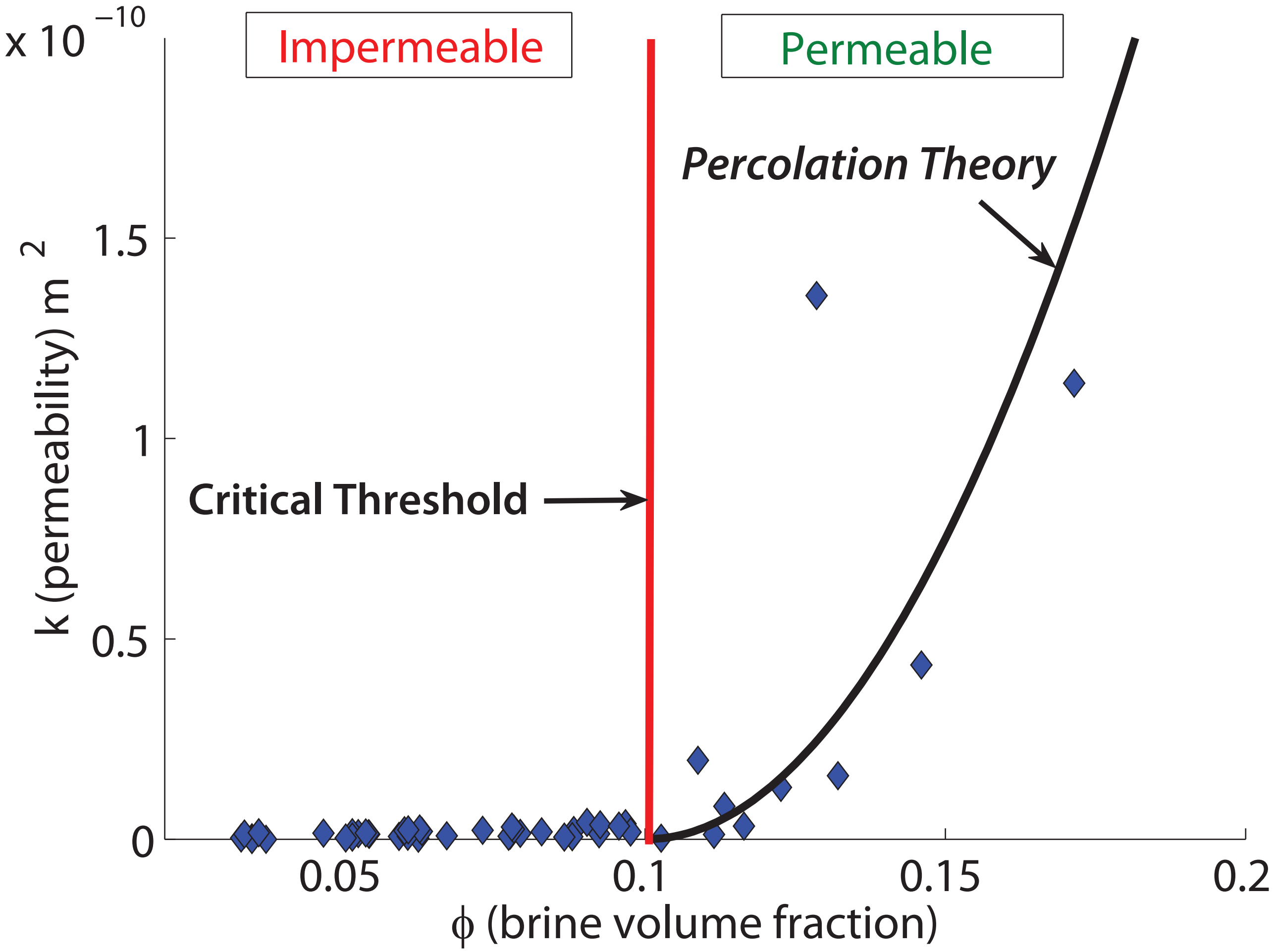
granular



10%

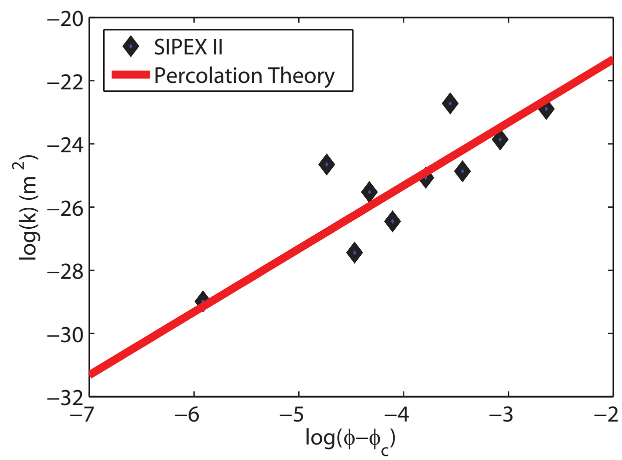
Golden, Gully, Lubbers, Sampson, Tison 2014

SIPEX II vertical permeability data



same universal critical exponent as lattice models

data above threshold

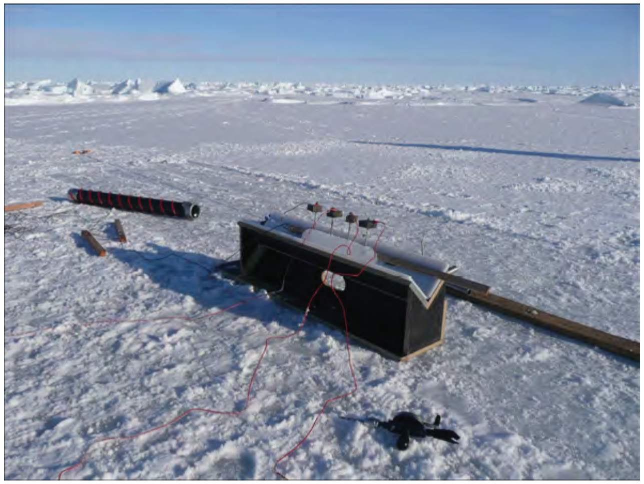


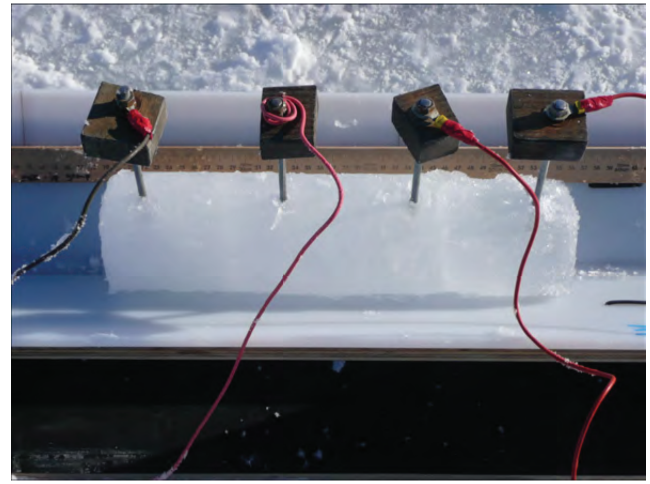
higher threshold in granular ice predicted with percolation theory by Golden, et al. (Science, 1998)

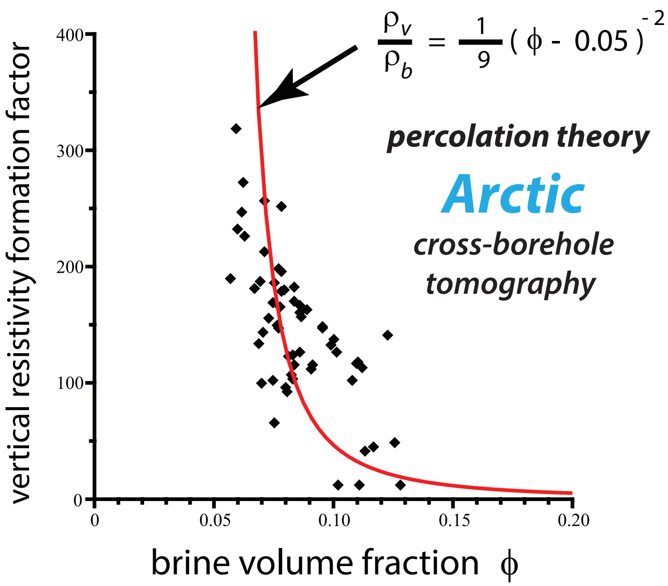
not confirmed experimentally until SIPEX I (2007) and SIPEX II (2012)

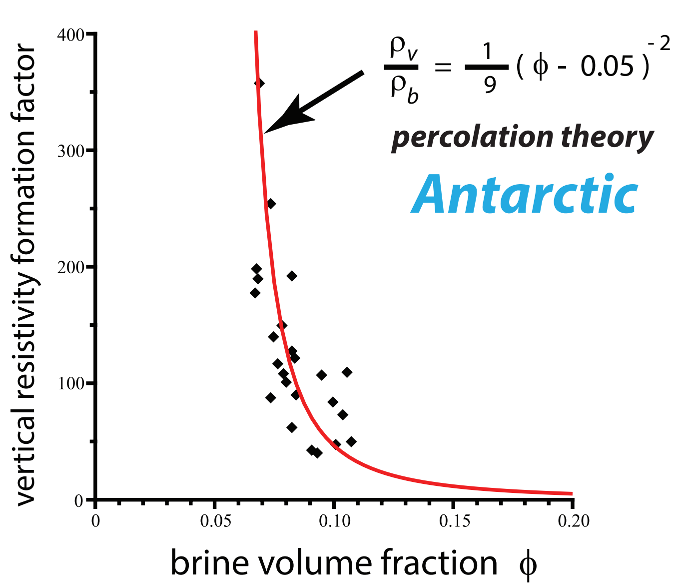
critical behavior of electrical transport in sea ice

electrical signature of the on-off switch for fluid flow





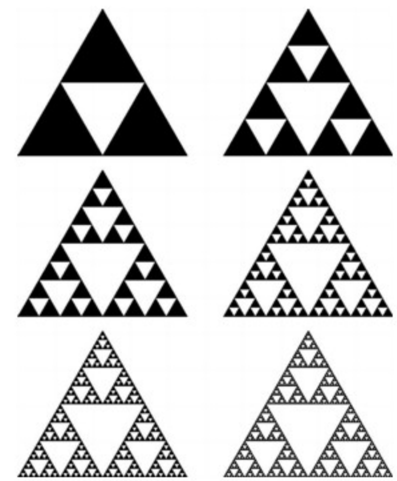




cross-borehole tomography - electrical classification of sea ice layers

Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2014

fractals and multiscale structure



melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

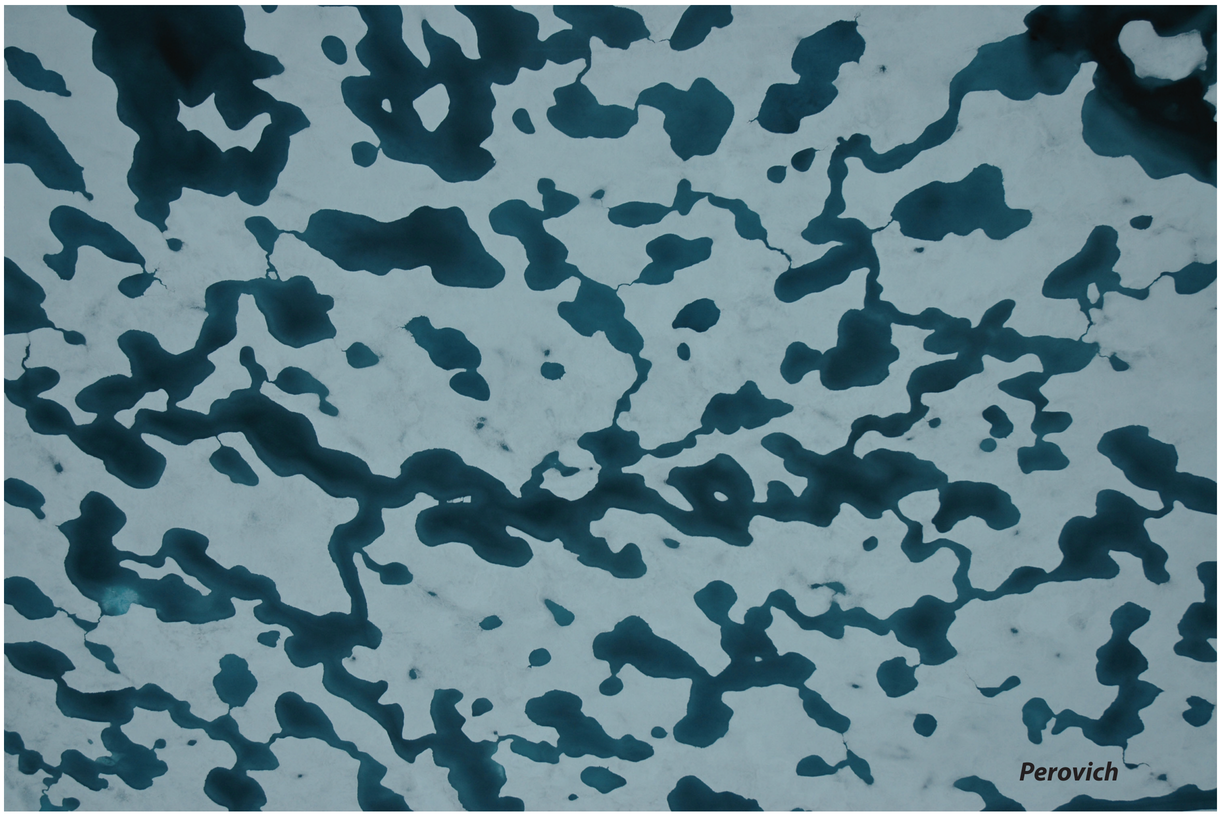
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

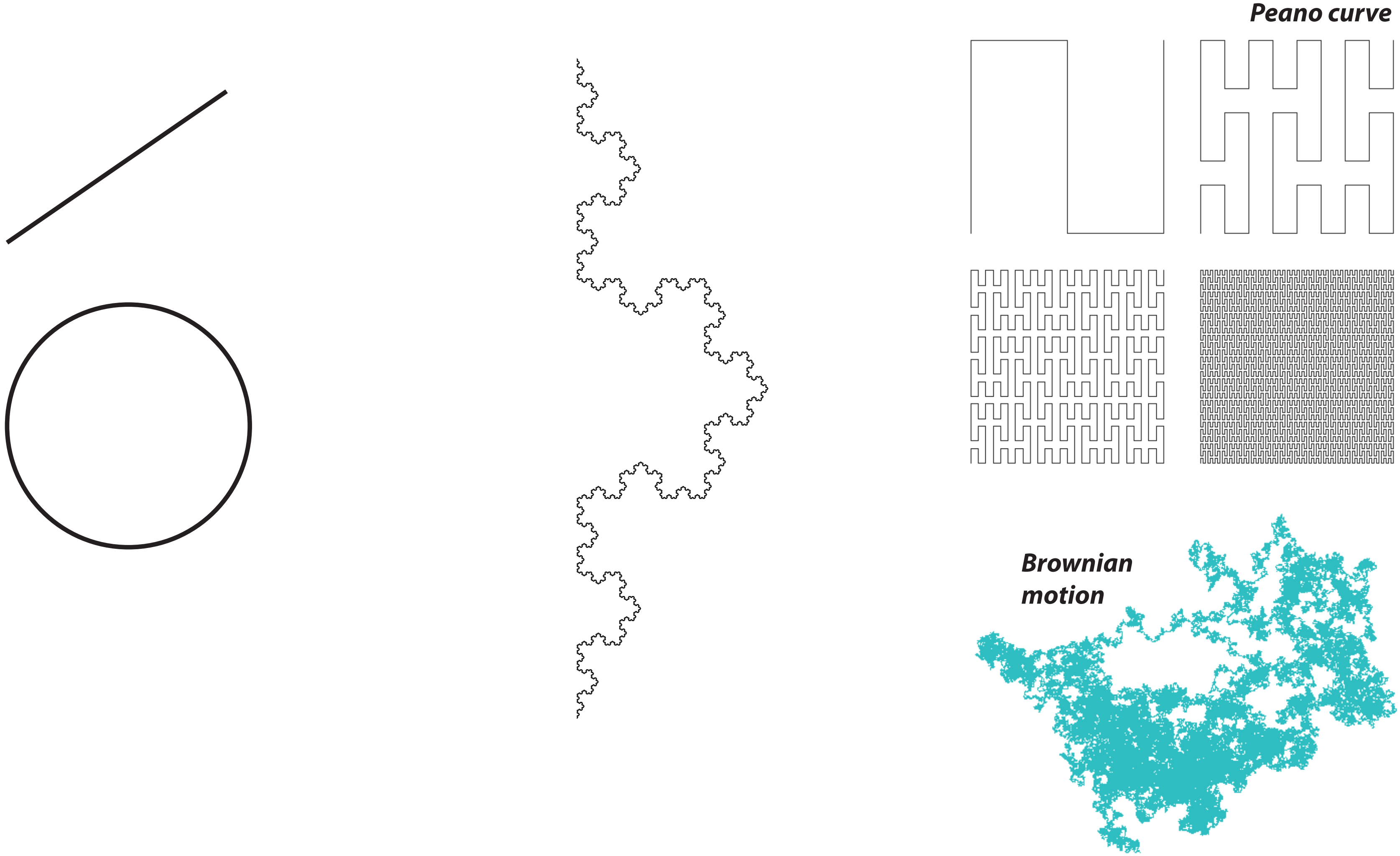
Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

fractal curves in the plane

they wiggle so much that their dimension is >1



simple curves — $D = 1$

Koch snowflake — $D = 1.26$

space filling curves — $D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

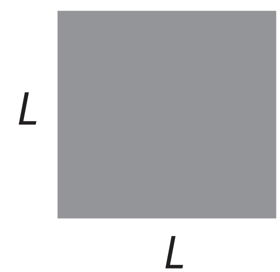
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

simple shapes



$A = L^2$

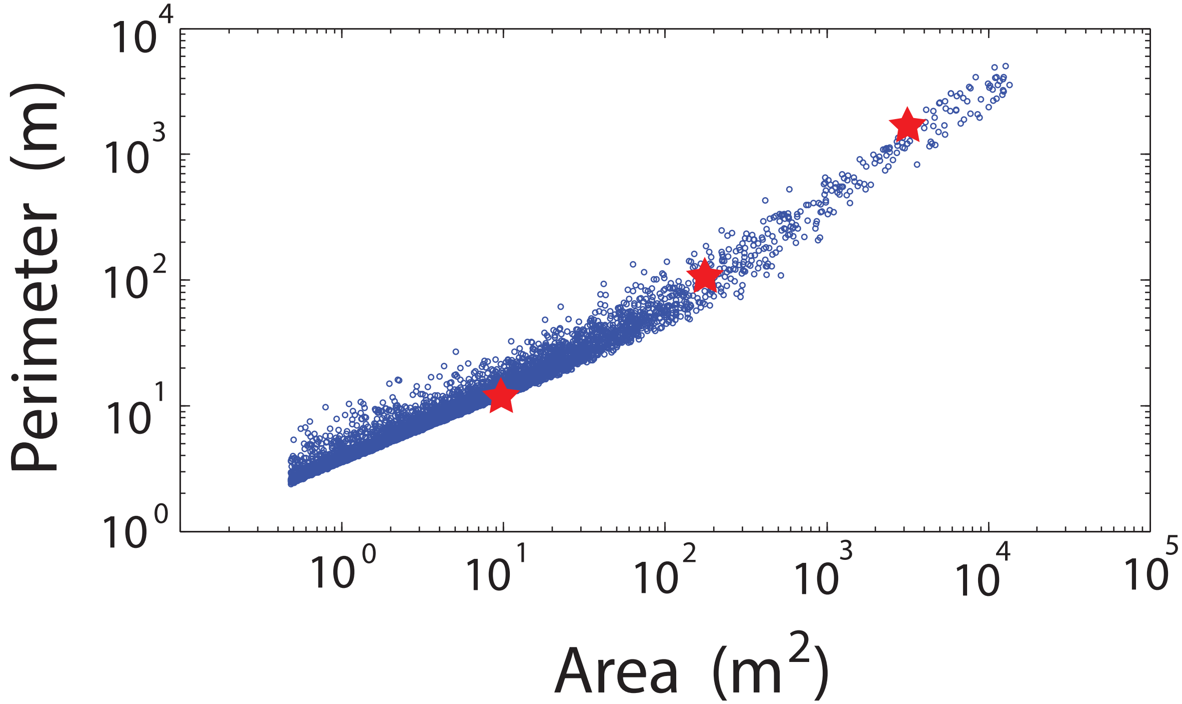
$P = 4L = 4\sqrt{A}$

$$P \sim \sqrt{A}^{\,D}$$

for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

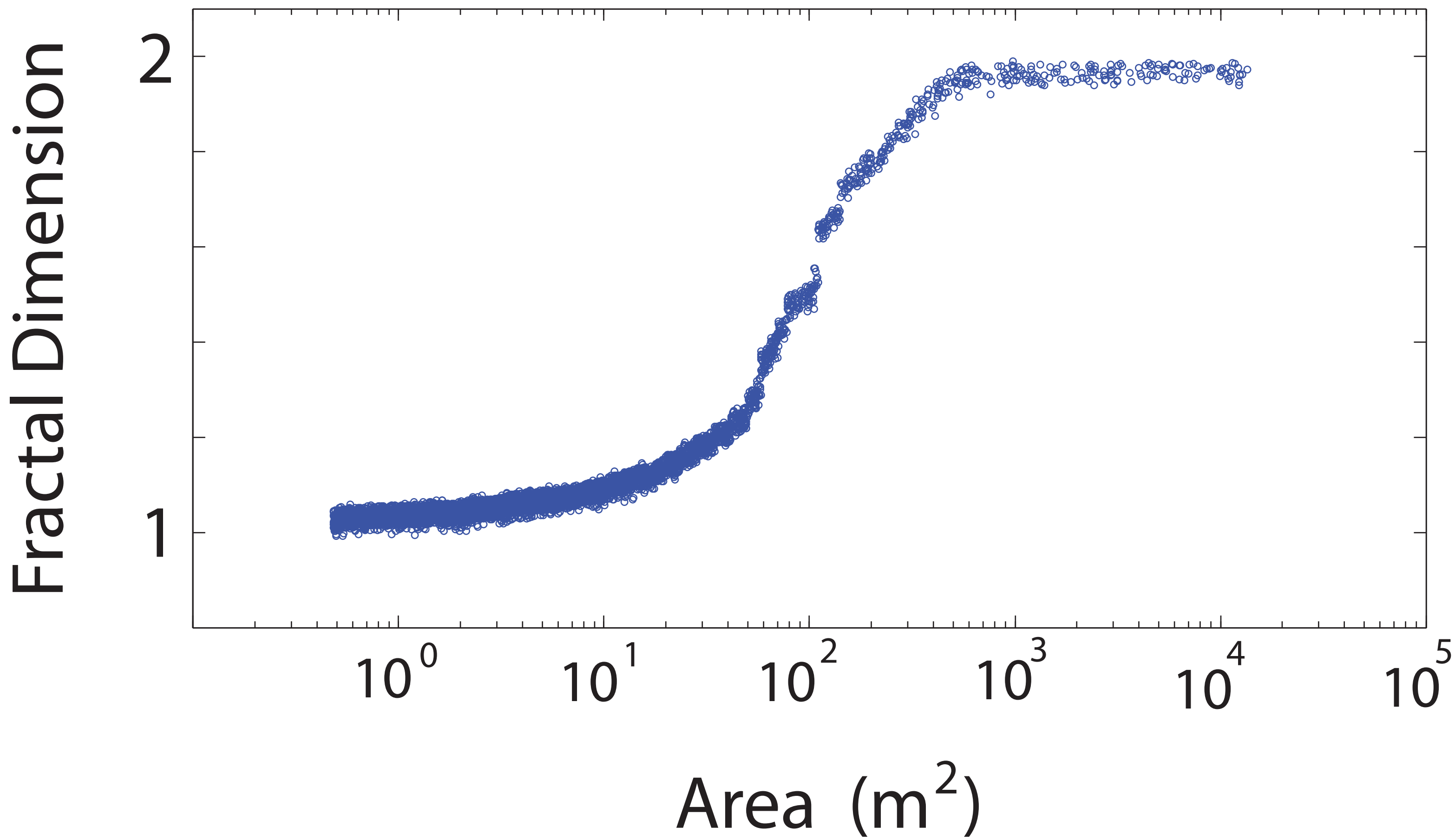


data from
5269 Arctic
melt ponds

simple pond ***transitional pond*** ***complex pond***

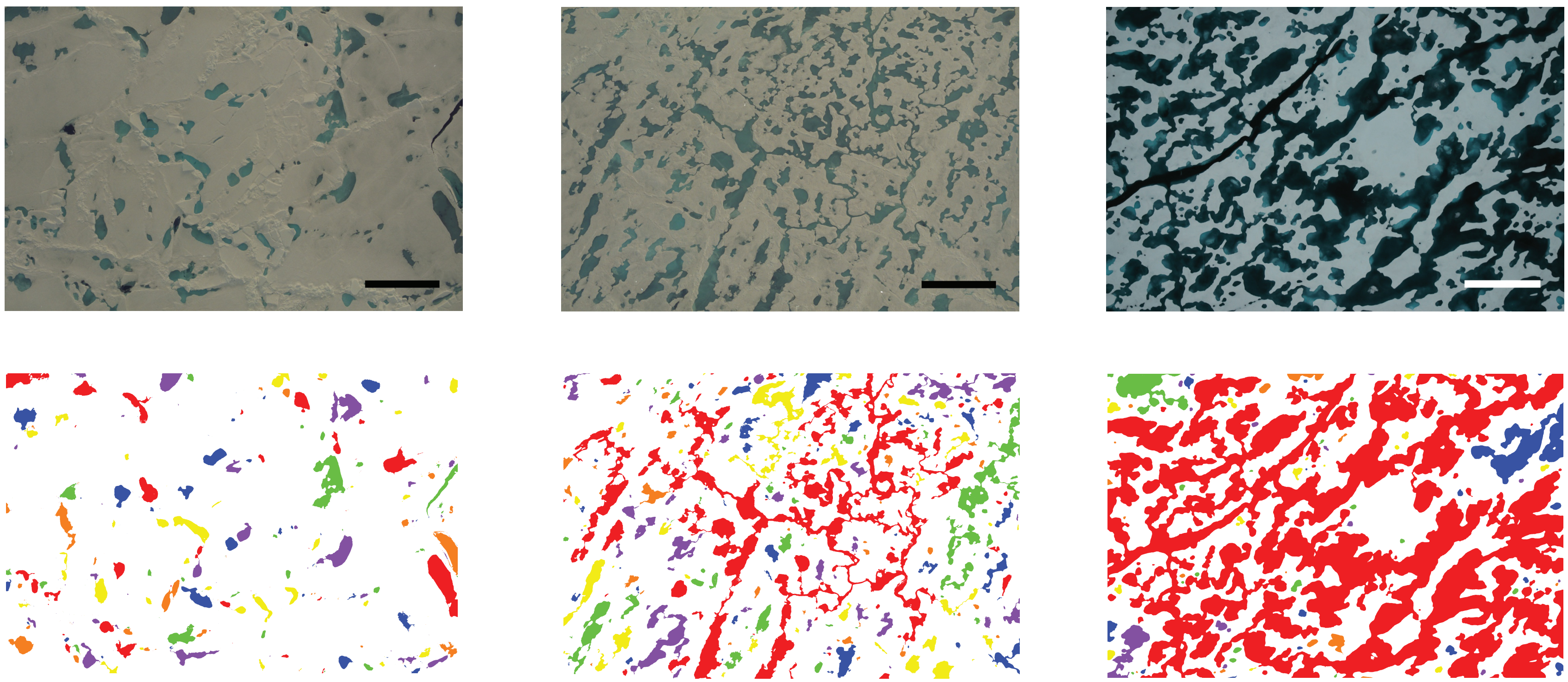
transition in the fractal dimension

complexity grows with length scale



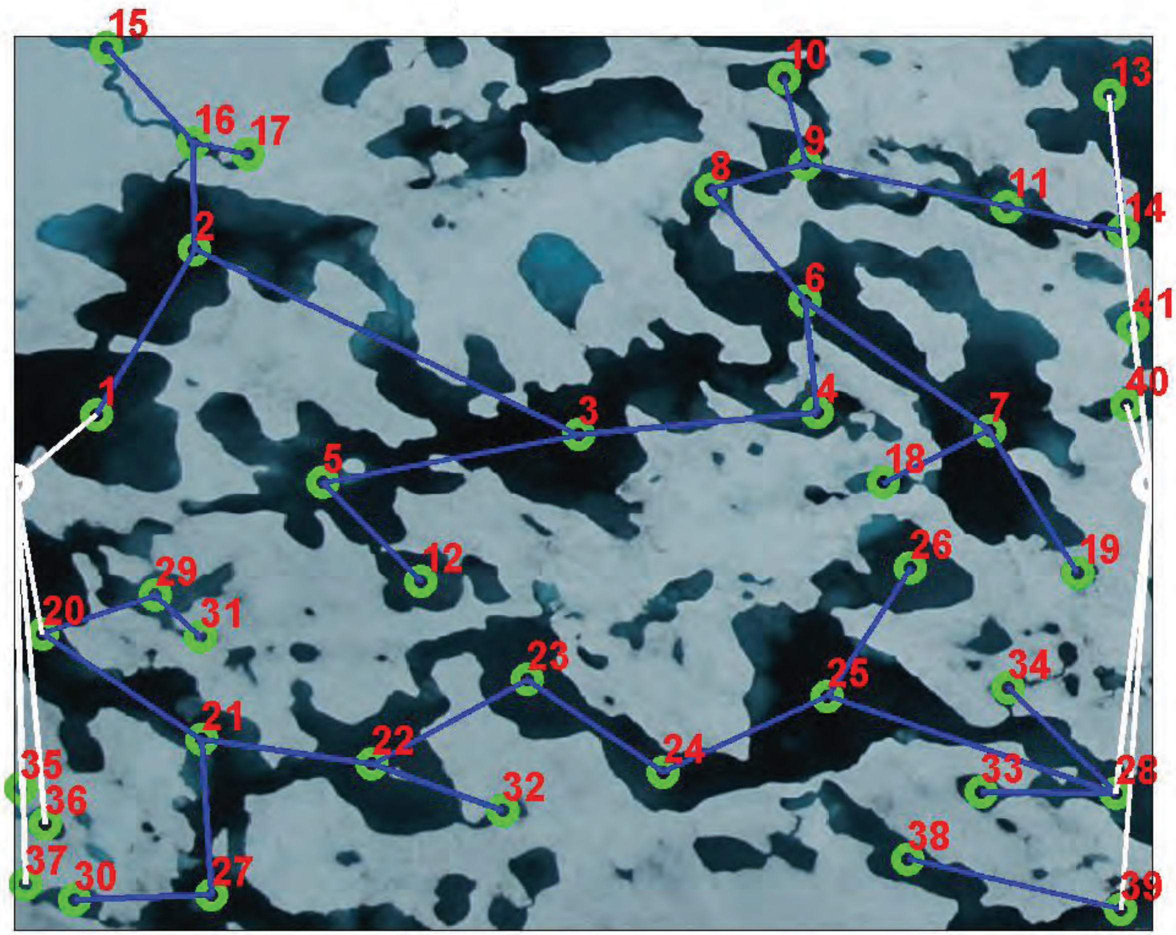
compute "derivative" of area - perimeter data

small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

map melt pond configurations onto resistor networks
compute horizontal fluid permeability

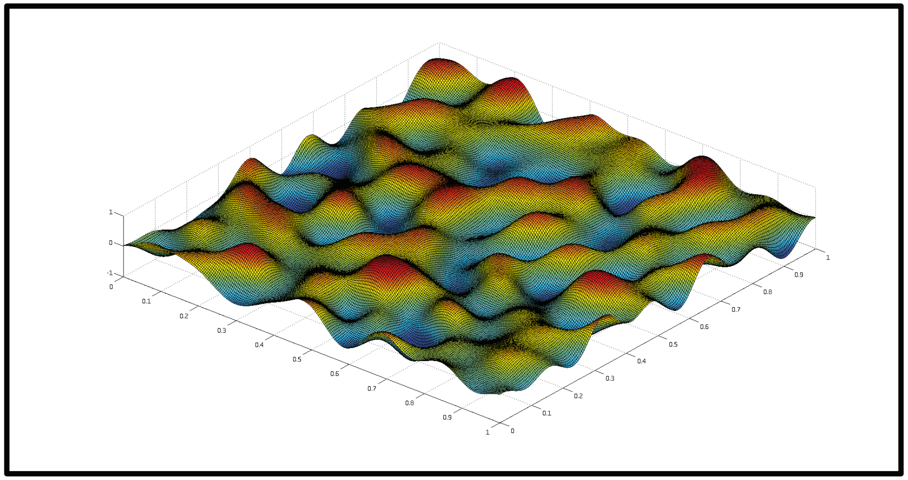


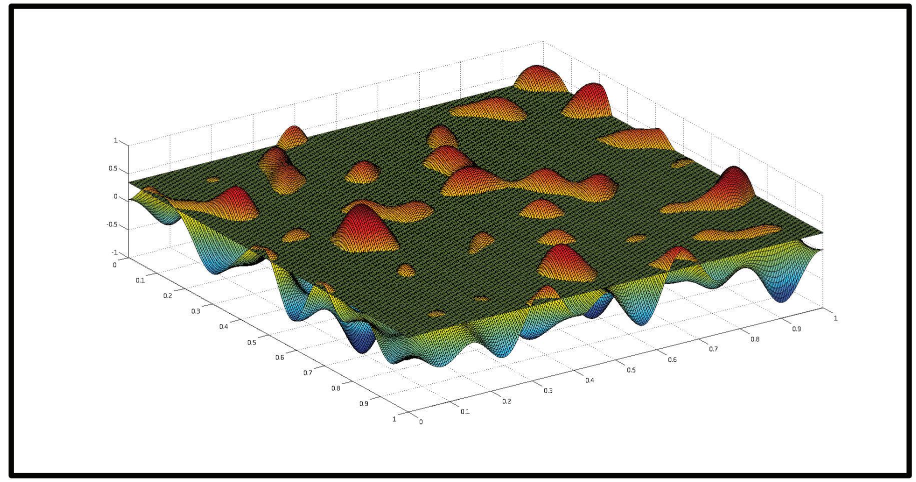
4 August 2005, Healy–Oden Trans Arctic Expedition (HOTRAX)

Barjatia, Song, Tasdizen, Golden 2014

Continuum percolation model for melt pond evolution

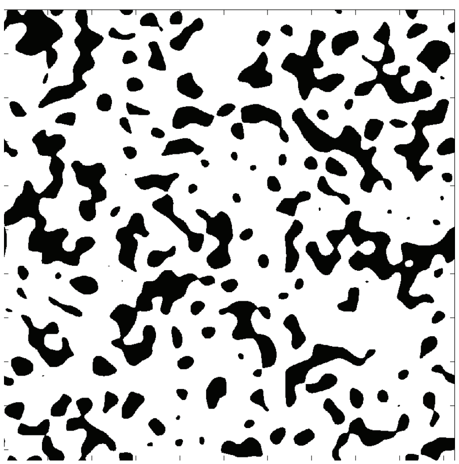
Brady Bowen, Court Strong, Ken Golden, 2014

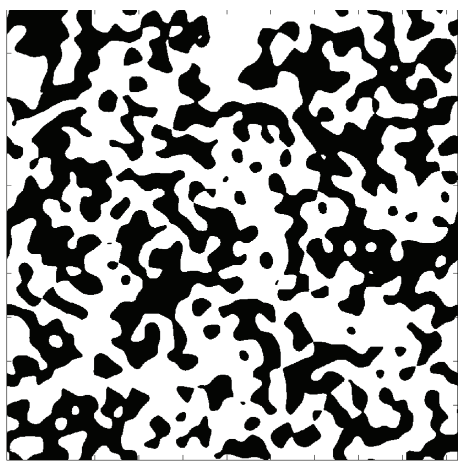


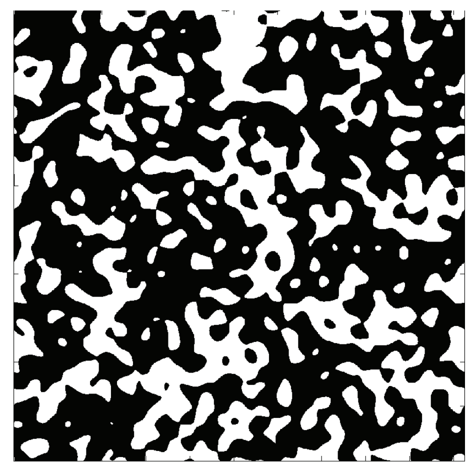


random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds



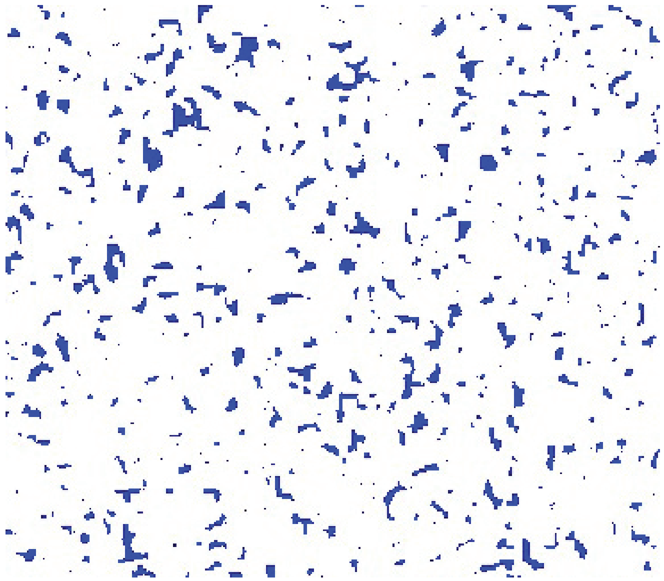




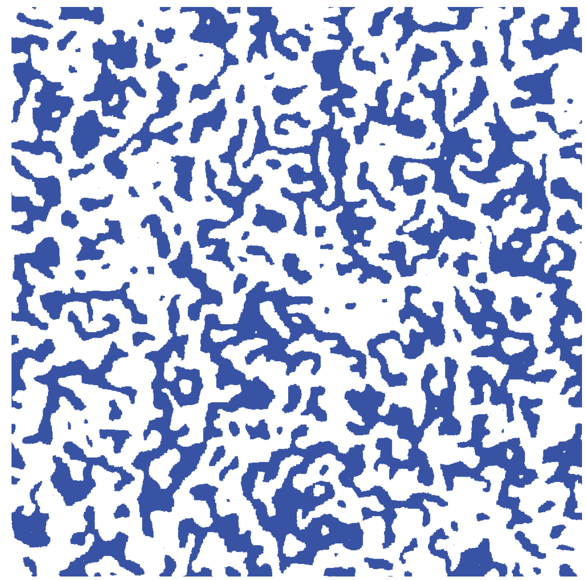
electronic transport in disordered media *diffusion in turbulent plasmas* *(Isichenko, Rev. Mod. Phys., 1992)*

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{ice} \\ \downarrow & -1 \quad \text{water} \end{cases} \qquad M = \lim_{N \to \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$



COLD



WARM

"melt ponds" are clusters of magnetic spins that align with the applied field

clusters exhibit transition in fractal dimension

Thekkedath, Alali, Strong, Golden
Sudakov, Ma, Golden

Conclusions

1. Summer Arctic sea ice is melting rapidly.

2. Fluid flow through sea ice mediates many processes of importance to understanding climate change and the response of polar ecosystems.

3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes, and provide a path toward rigorous representation of sea ice in climate models .

4. Field experiments are essential to developing relevant mathematics.

5. Our research will help to improve projections of climate change and the fate of the Earth sea ice packs.

THANK YOU

National Science Foundation

Division of Mathematical Sciences

Arctic Natural Sciences

Office of Polar Programs

CMG Program

(Collaboration in Mathematical Geosciences)

Office of Naval Research

Applied Computational Analysis Program

Arctic and Global Prediction Program

Mathematics and Climate Research Network

Australian Government
Department of the Environment and Water Resources
Australian Antarctic Division

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Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999