Sea ice, climate, and multiscale composites

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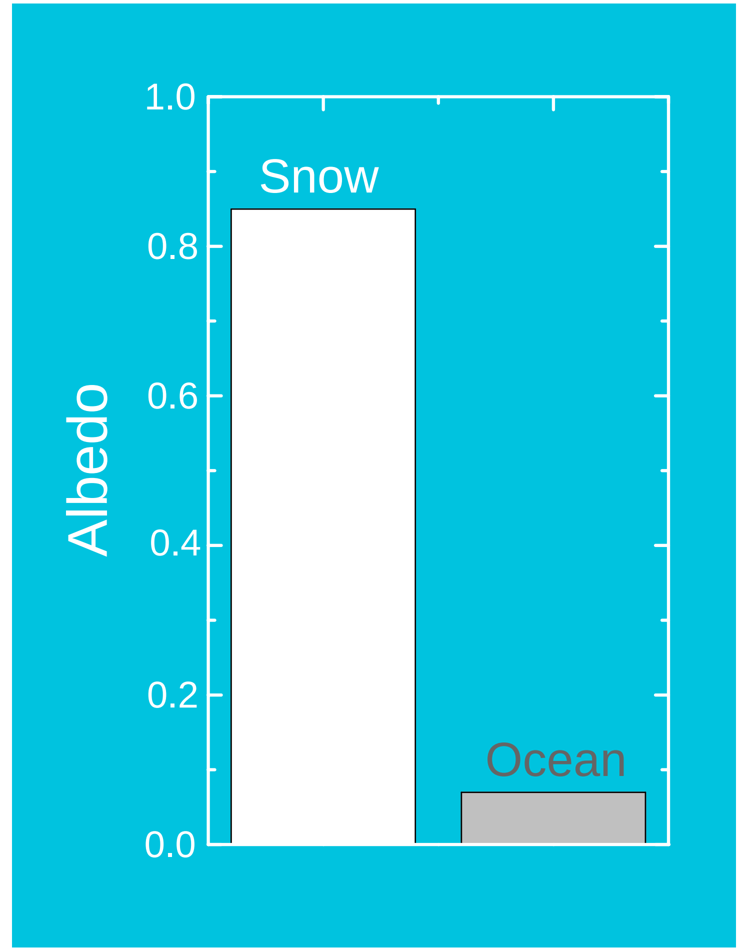
IMA Workshop on Stochastic Modeling of the Oceans and Atmosphere 12 March 2013


SEA ICE covers 7 - 10% of earth's ocean surface
• boundary between ocean and atmosphere
• mediates exchange of heat, gases, momentum
• global ocean circulation
• indicator and agent of climate change

polar ice caps critical to global climate in reflecting incoming solar radiation

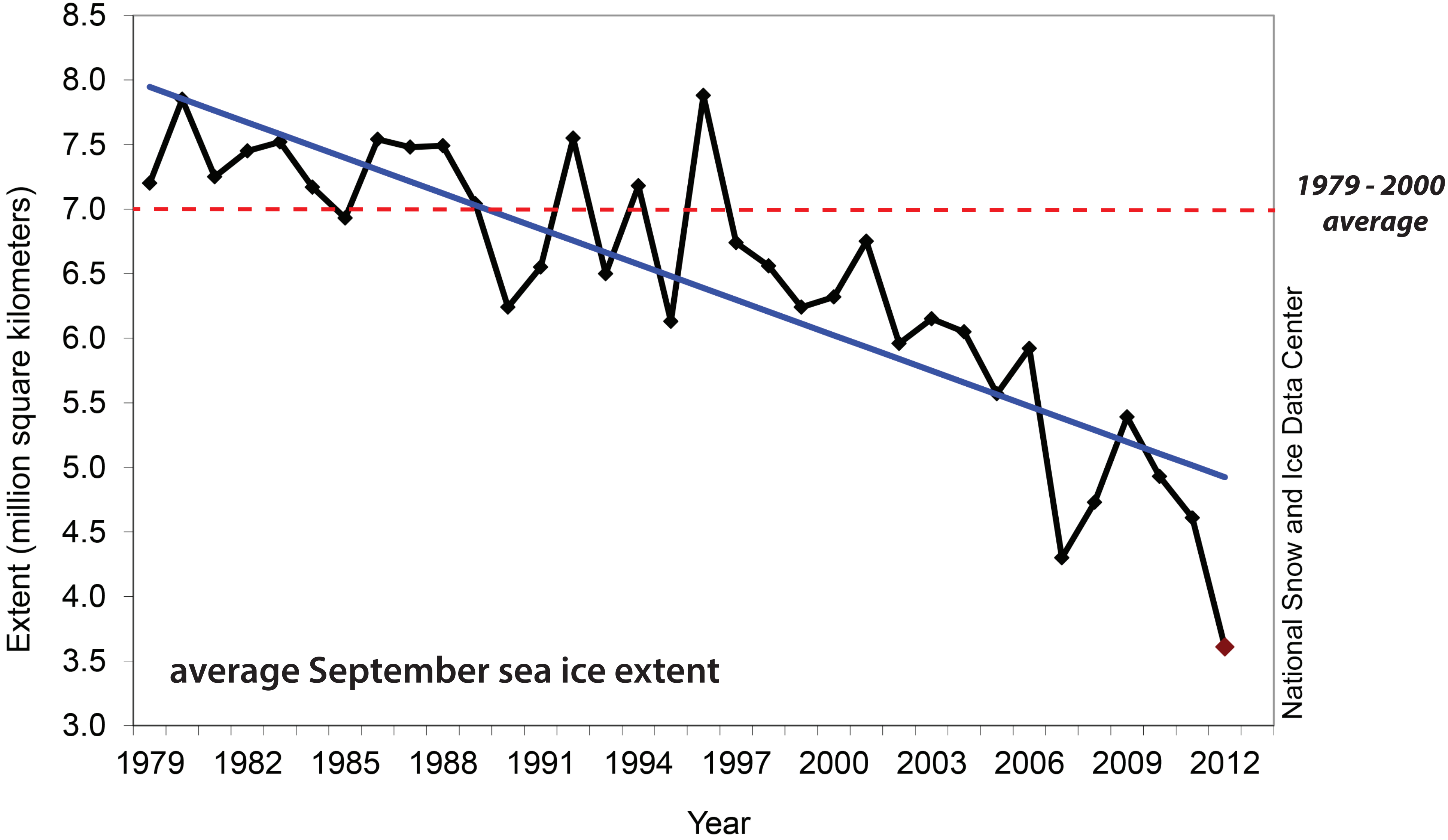
white snow and ice reflect



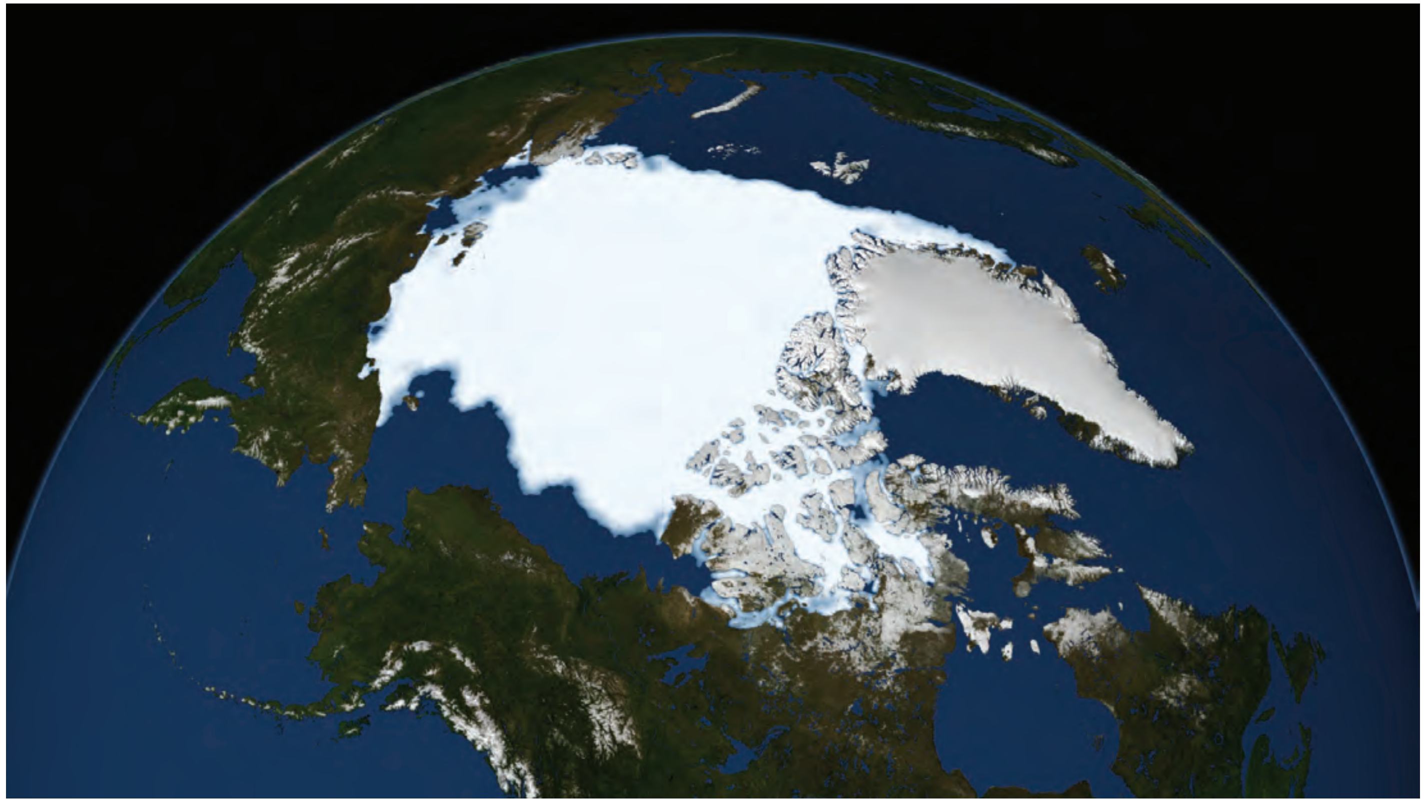
dark water and land absorb

$$\textbf{\textit{albedo}}\ \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

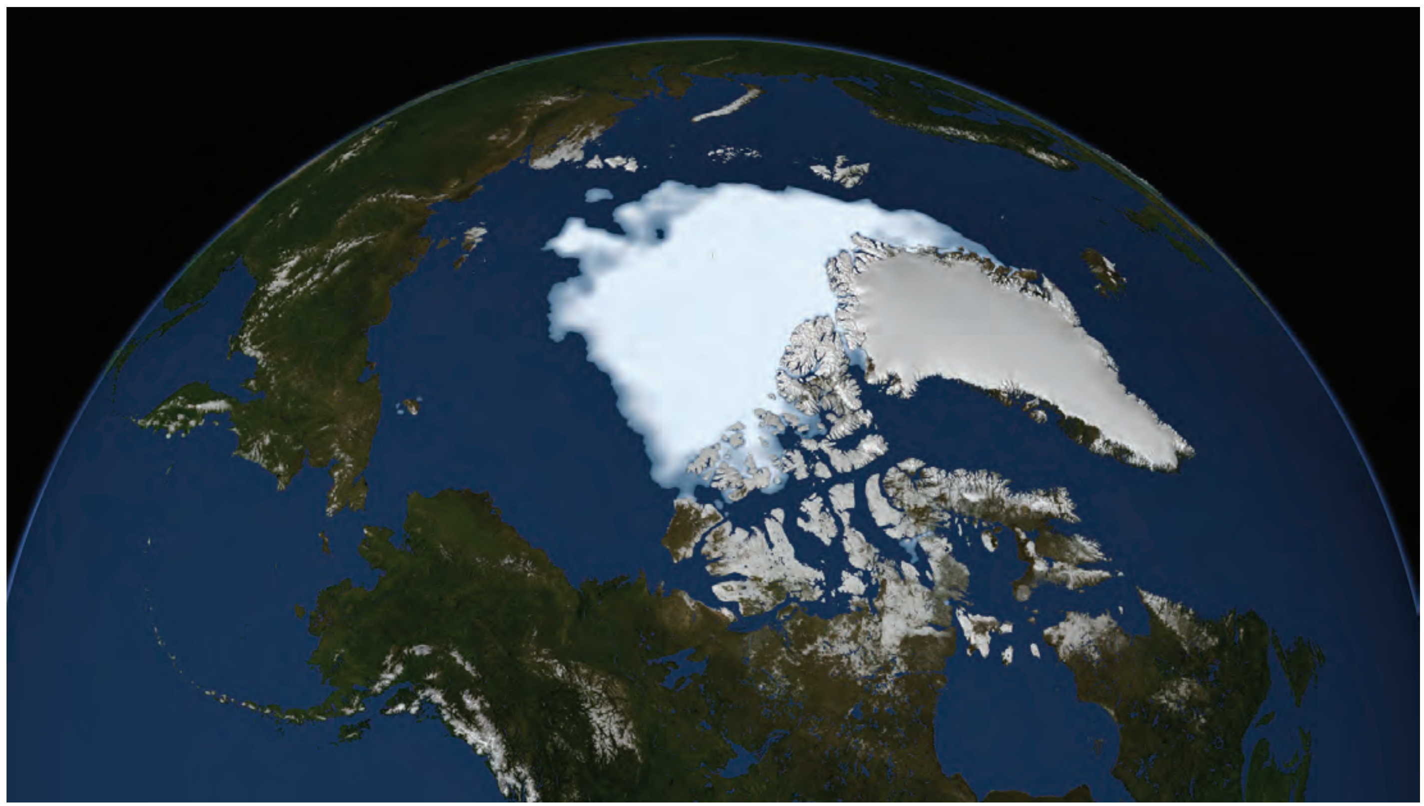
the summer Arctic sea ice pack is melting



21 September 1979

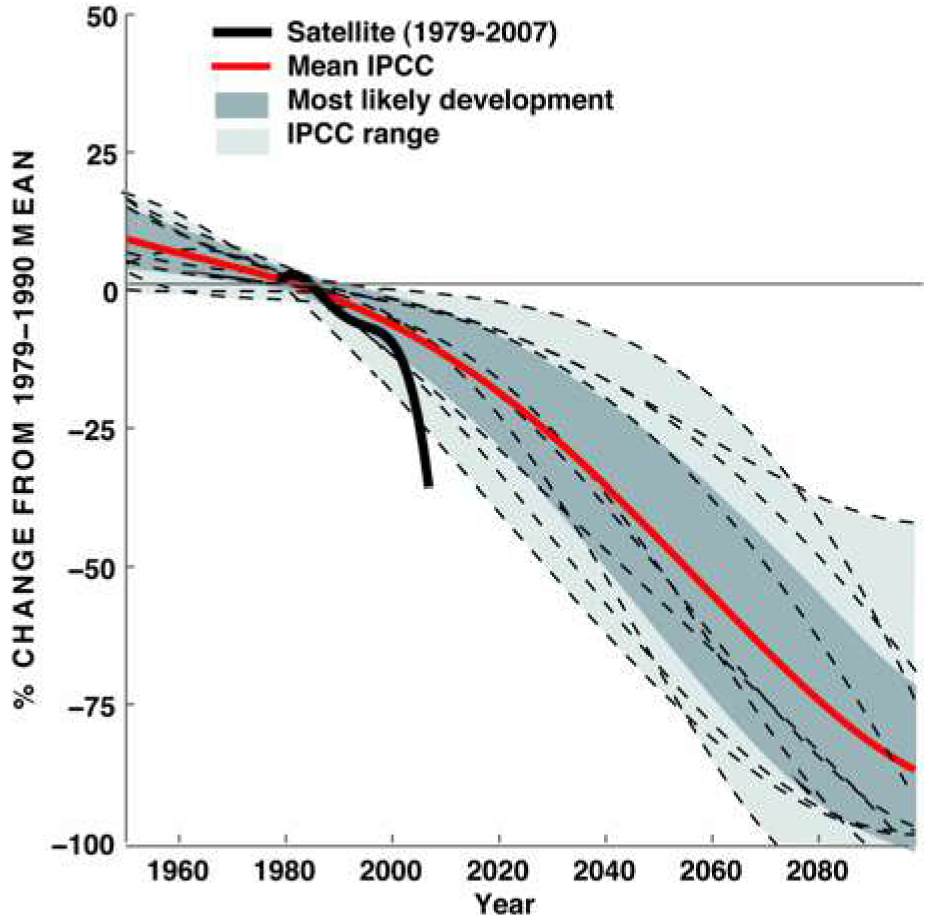


13 September 2012

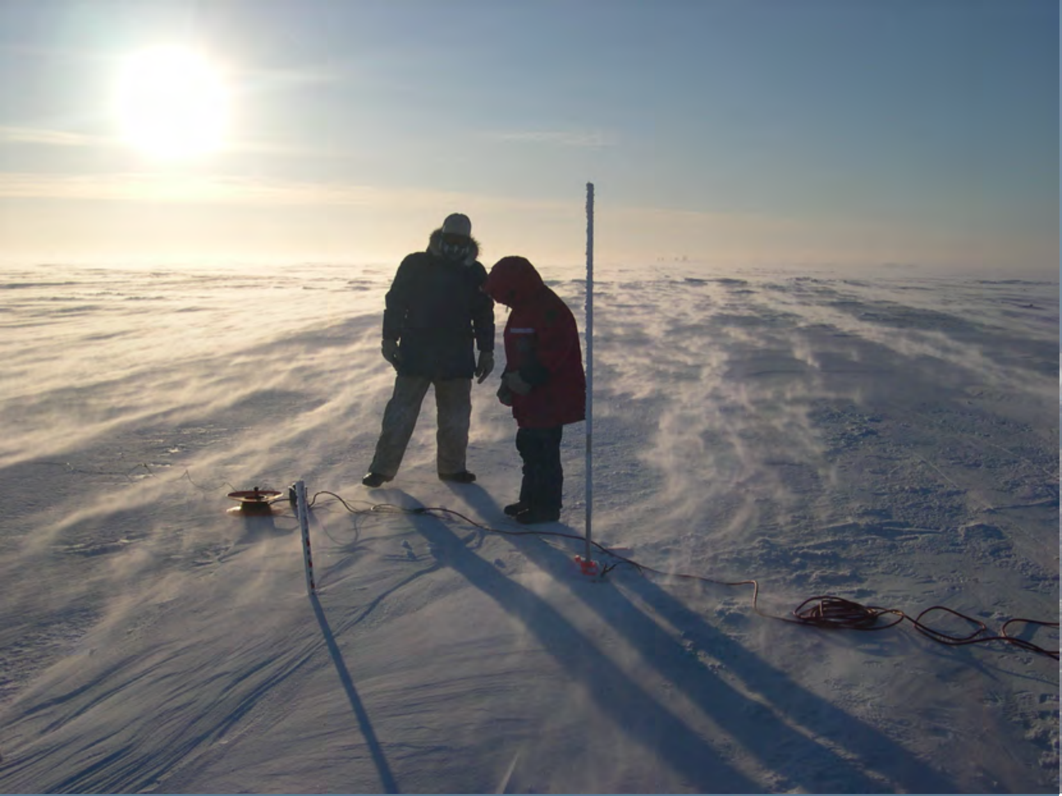


Intergovernmental Panel on Climate Change (IPCC) 2007 projections

observed decline in summer Arctic sea ice outpacing global climate models

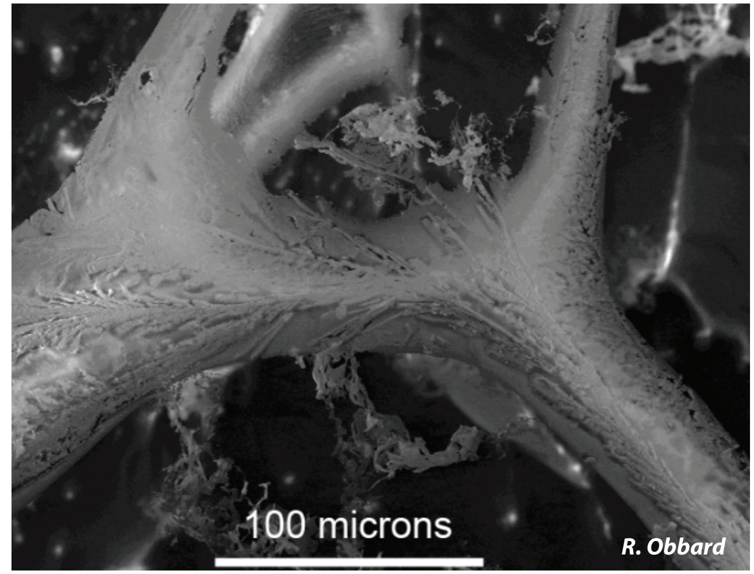


Arctic sea ice loss compared to IPCC models



sea ice may appear to be a barren, impermeable cap ...

brine inclusions in sea ice (mm)



micro - brine channel (SEM)

brine channels (cm)

sea ice is a porous composite

pure ice with brine, air, and salt inclusions

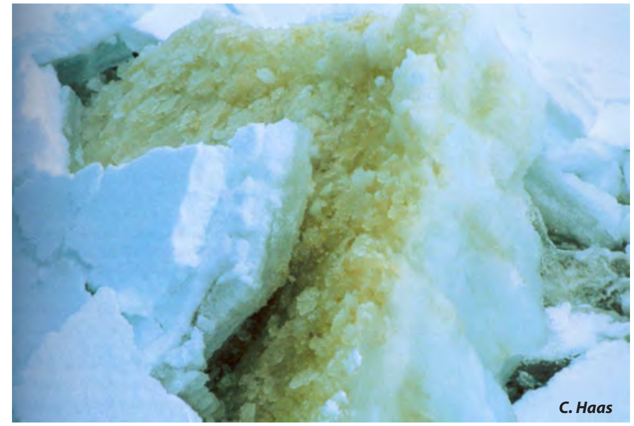
horizontal section

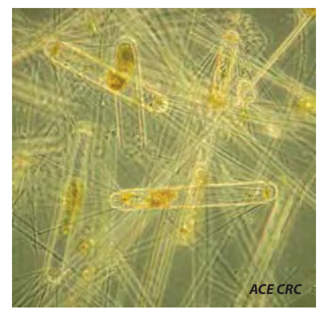
vertical section

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems:

evolution of Arctic melt ponds and sea ice albedo

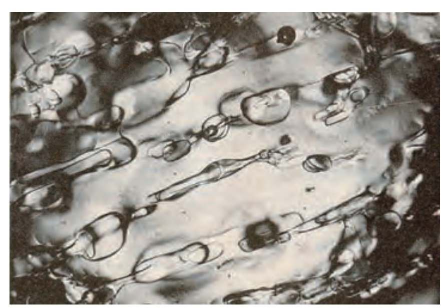
nutrient flux for algal communities





- drainage of brine and melt water
- ocean-ice-air exchanges of heat, CO_2

linkage of scales



What is this talk about?

Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.

small scales

1. ***Fluid flow through sea ice***
2. ***Arctic and Antarctic experiments***
3. ***Fractal melt ponds***
4. ***Multiscale homogenization***

large scales

critical behavior ***linkage of scales***

cross-pollination

.... develop rigorous representations of sea ice in climate models.

sea ice microphysics

fluid transport

Darcy's Law for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

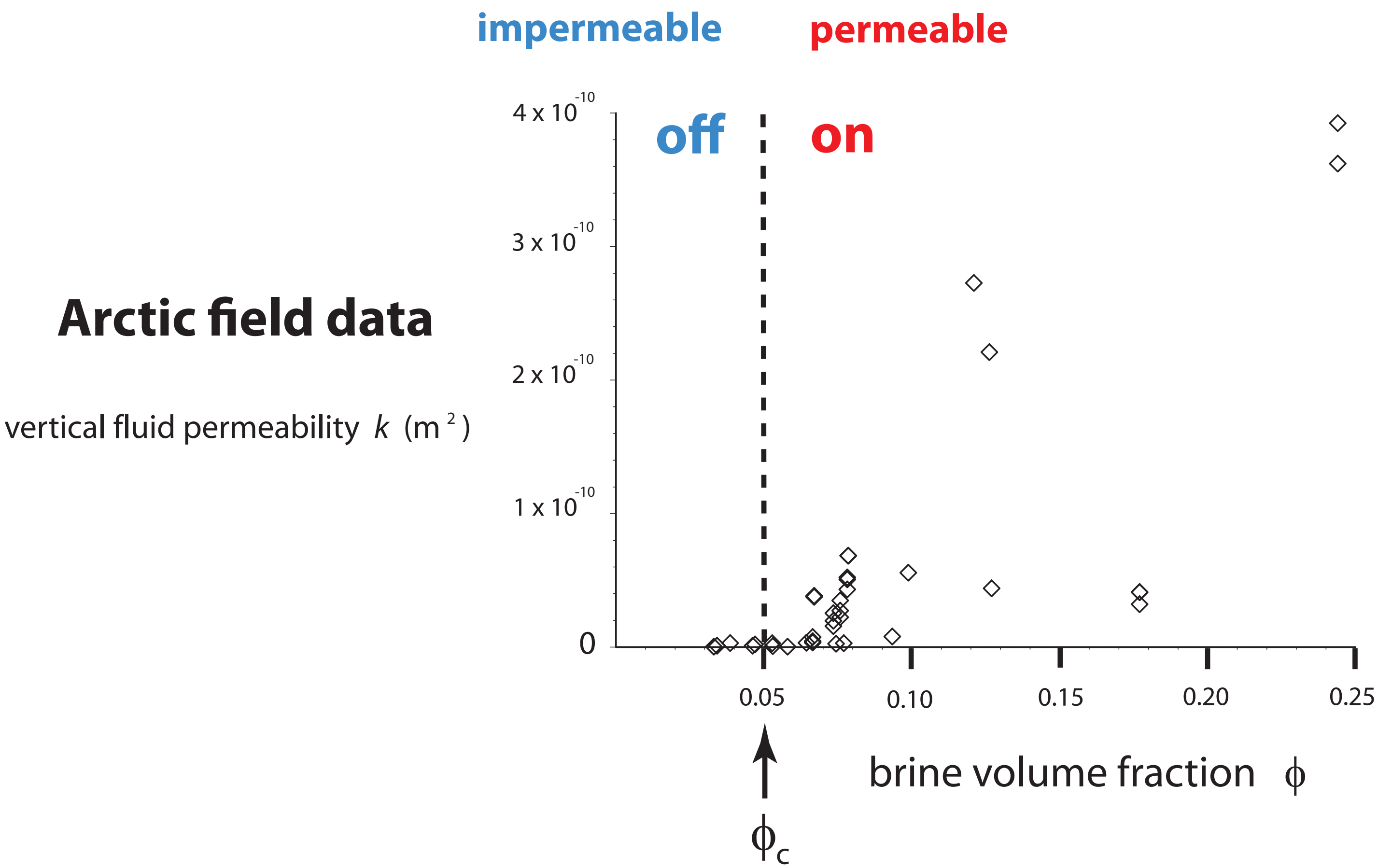
viscosity

k = fluid permeability tensor

example of *homogenization*

mathematics for analyzing effective behavior of heterogeneous systems

Critical behavior of fluid transport in sea ice



on - off switch for fluid flow

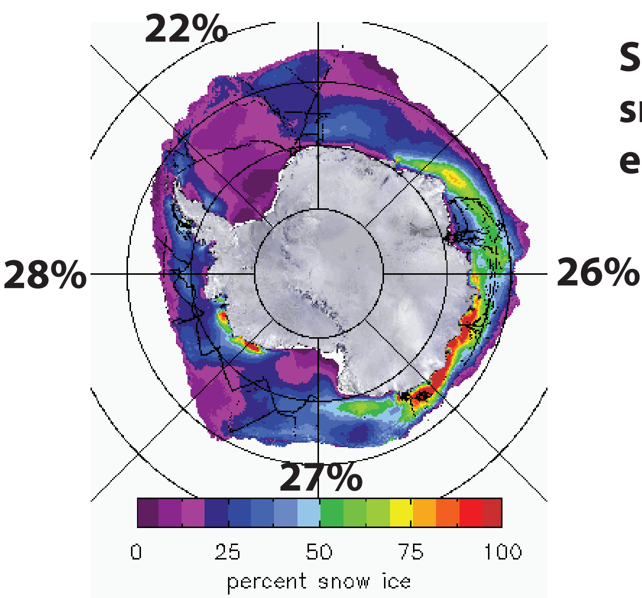
critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ C,\ S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007
Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

rule of fives constrains:

Antarctic surface flooding and snow-ice formation

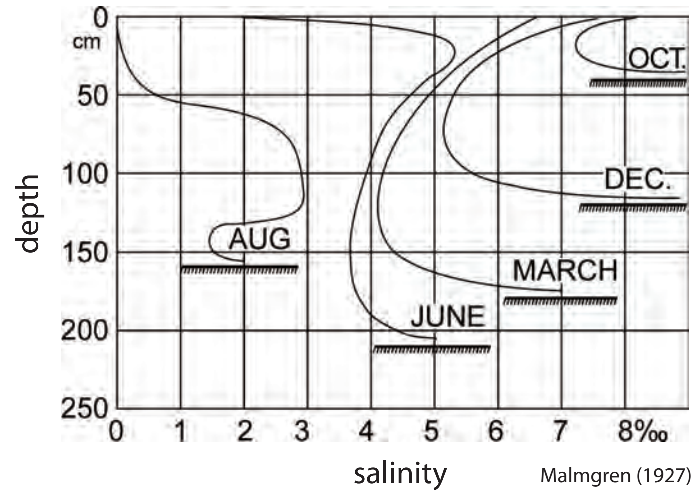


September snow-ice estimates

Antarctic snow-to-ice conversion from passive microwave imagery

T. Maksym and T. Markus, 2008

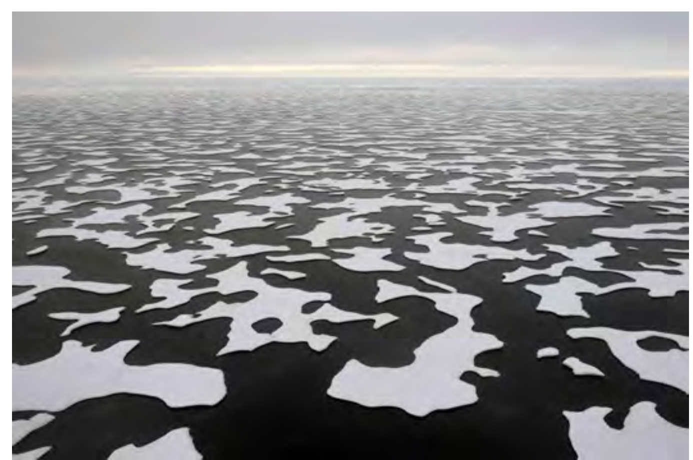
evolution of salinity profiles



currently assumed constant in climate models

convection - enhanced thermal conductivity

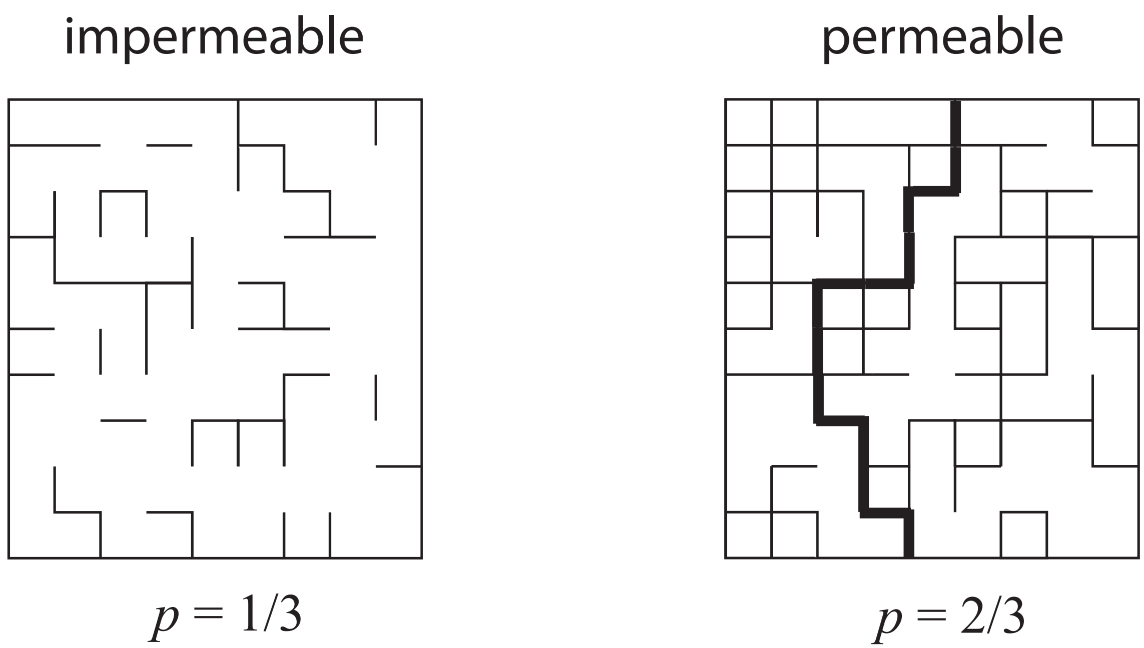
Lytle and Ackley, 1996
Trodahl, et. al., 2000, 2001
Wang, Zhu, Golden, 2012



Why is the rule of fives true?

percolation theory

mathematical theory of connectedness



bond → *open* *with probability p*
closed *with probability 1-p*

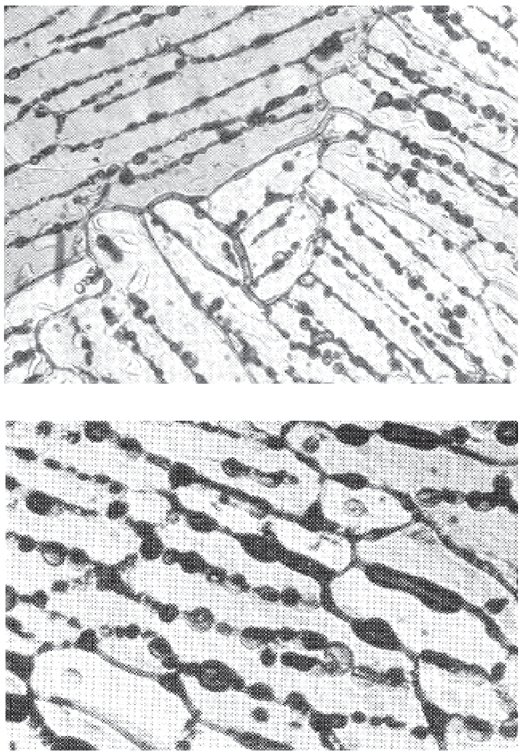
percolation threshold

$p_c = 1/2 \quad \text{for} \quad d = 2$

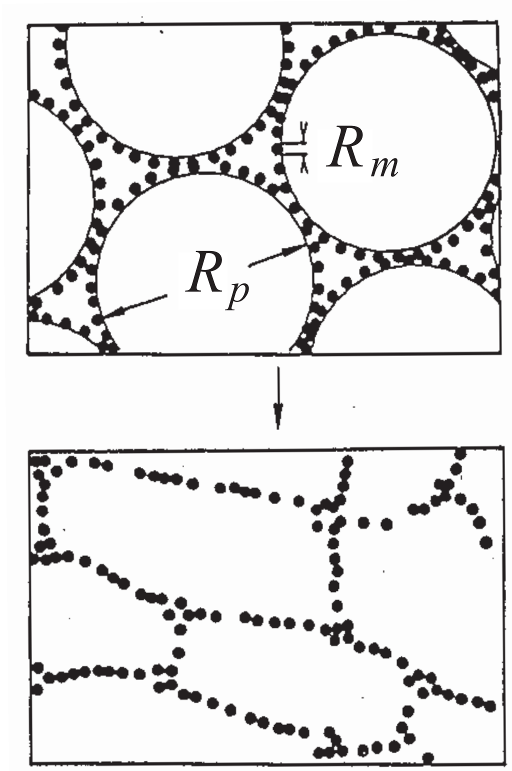
first appearance of infinite cluster ***"tipping point" for connectivity***

Continuum percolation model for stealthy materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

$\phi_c \approx 5\,\%$ Golden, Ackley, Lytle, *Science,* 1998



sea ice

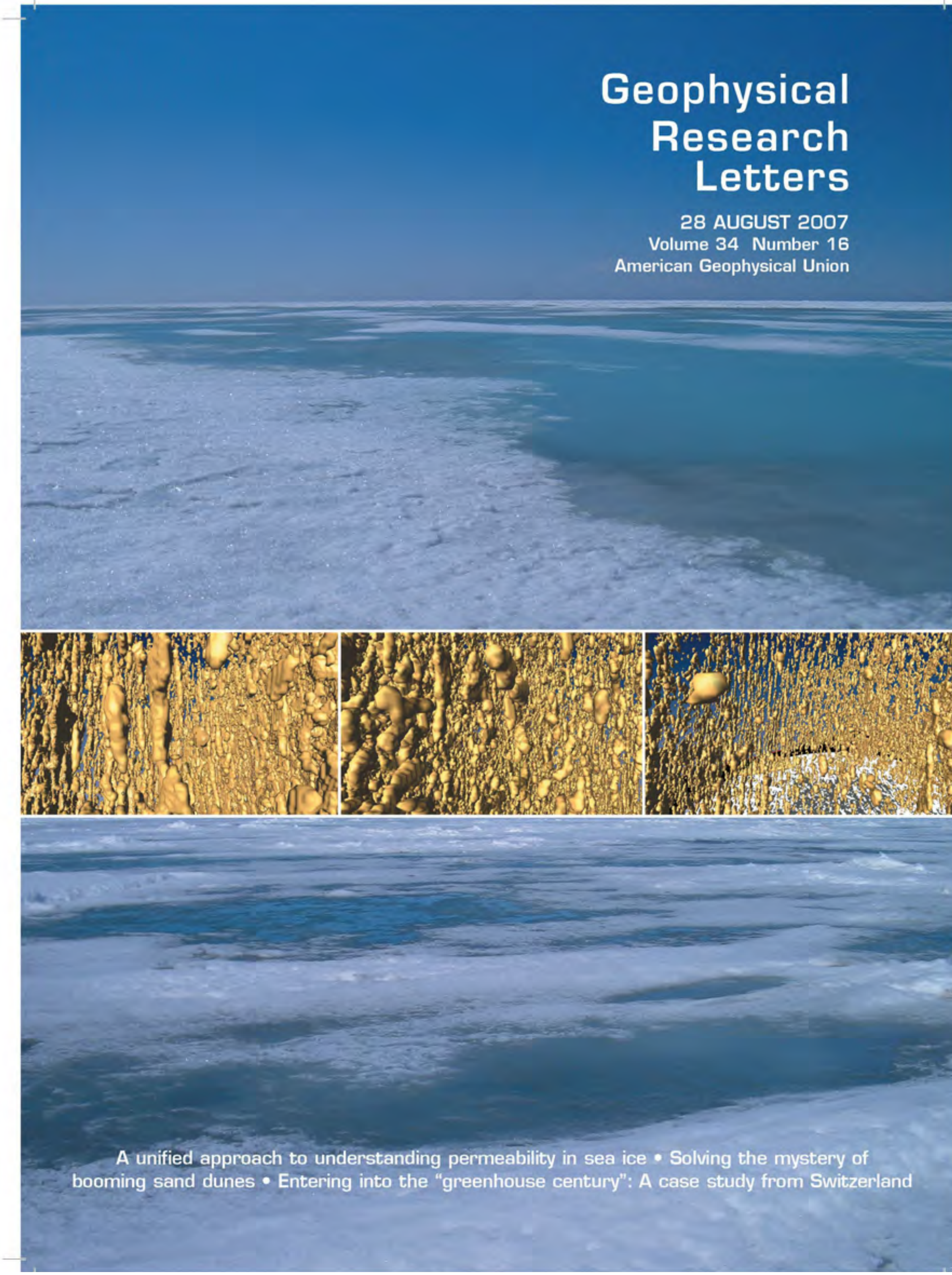


compressed powder

radar absorbing composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice **Golden, Eicken, Heaton, Miner, Pringle, Zhu**



rigorous bounds
percolation theory
hierarchical model
network model

field data

micro-scale

controls

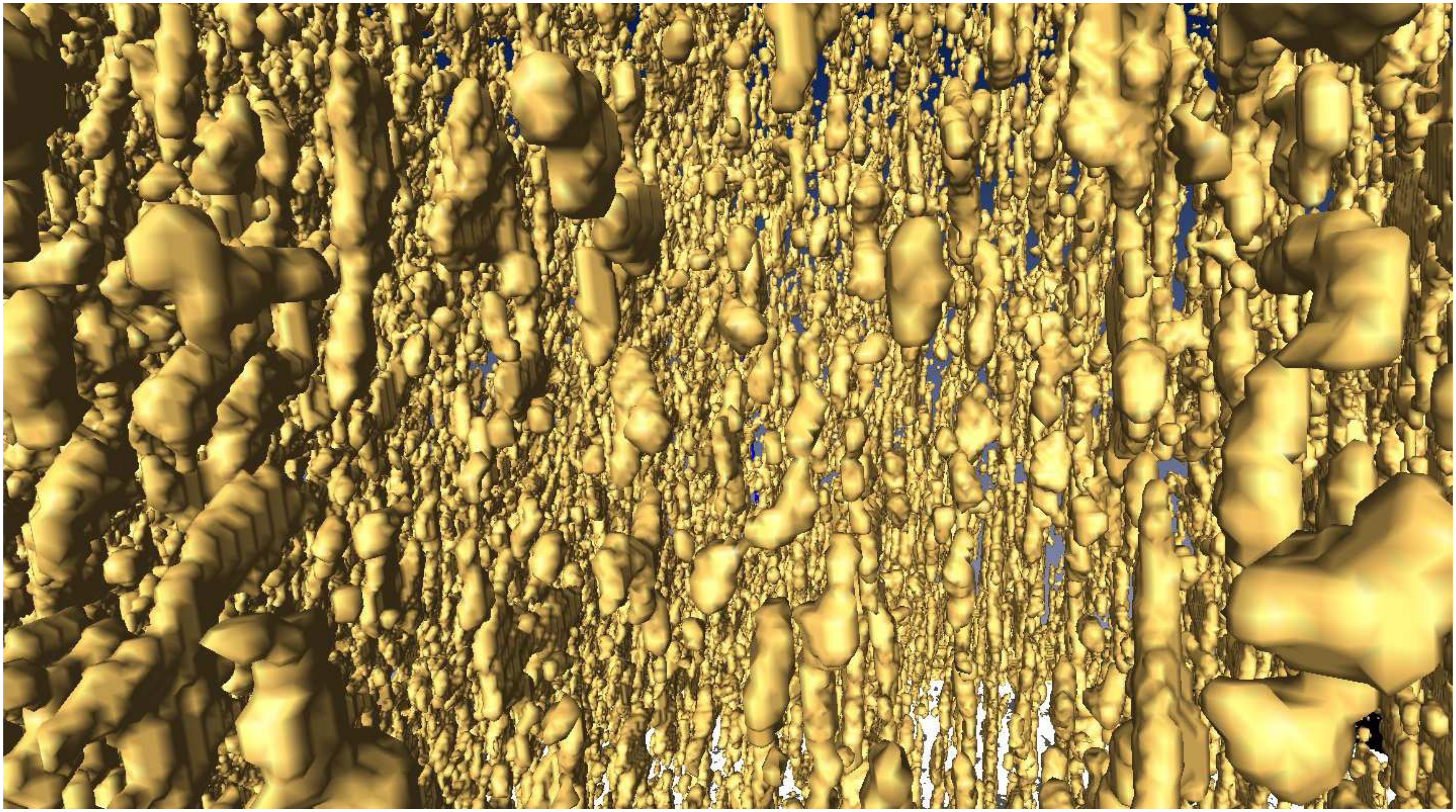
macro-scale processes

X-ray tomography for brine inclusions

unprecedented look at thermal evolution of brine phase and its connectivity

X-ray computed tomography of brine inclusions in sea ice

~ 1 cm across

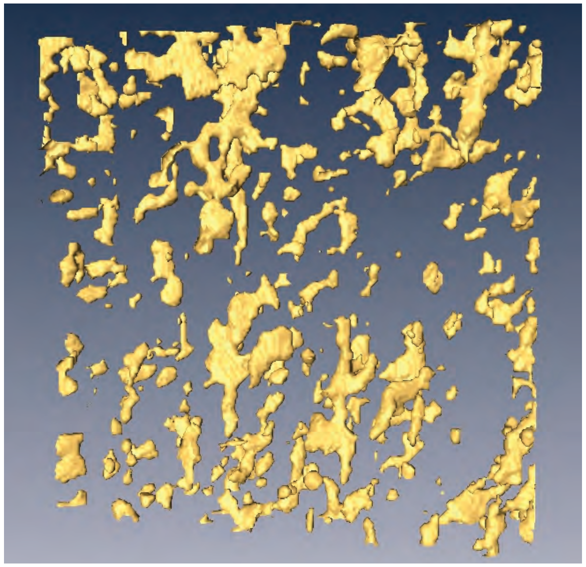


brine volume fraction $\phi = 5.7\ \%$ $T = -8^{\circ}C$

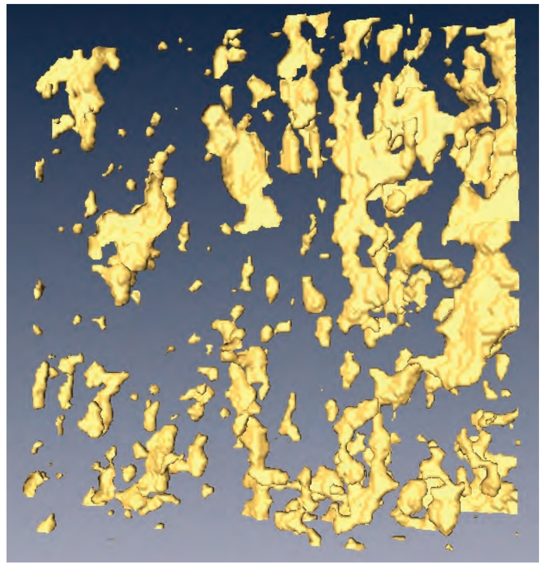
Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

brine connectivity (over cm scale)

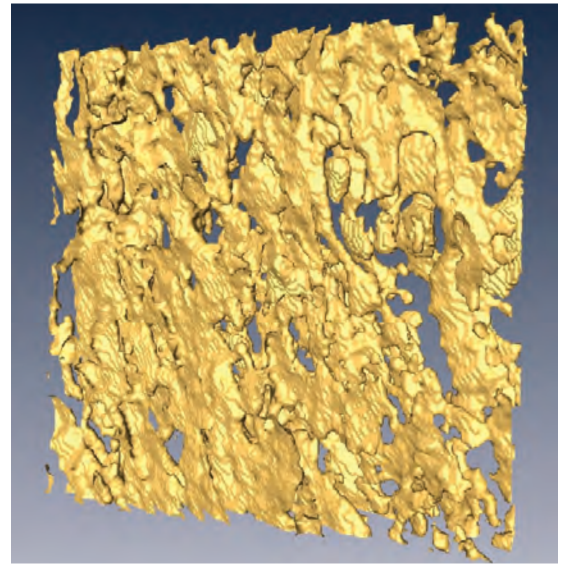
8 x 8 x 2 mm



-15 °C, $\phi = 0.033$



-6 °C, $\phi = 0.075$



-3 °C, $\phi = 0.143$

X-ray tomography confirms percolation threshold

3-D images
pores and throats → 3-D graph
nodes and edges

analyze graph connectivity as function of temperature and sample size

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

Pringle, Miner, Eicken, Golden, J. Geophys. Res. 2009

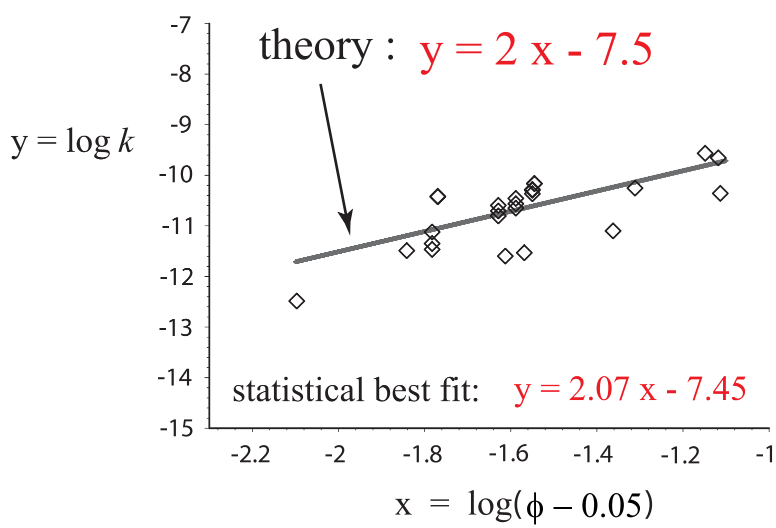
lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical exponent t

$$k_0 = 3 \times 10^{-8}\ \mathrm{m}^2$$

- exponent is UNIVERSAL lattice value $t \approx 2.0$
- sedimentary rocks like sandstones also exhibit universality
- critical path analysis -- developed for electronic hopping conduction -- yields scaling factor k_0

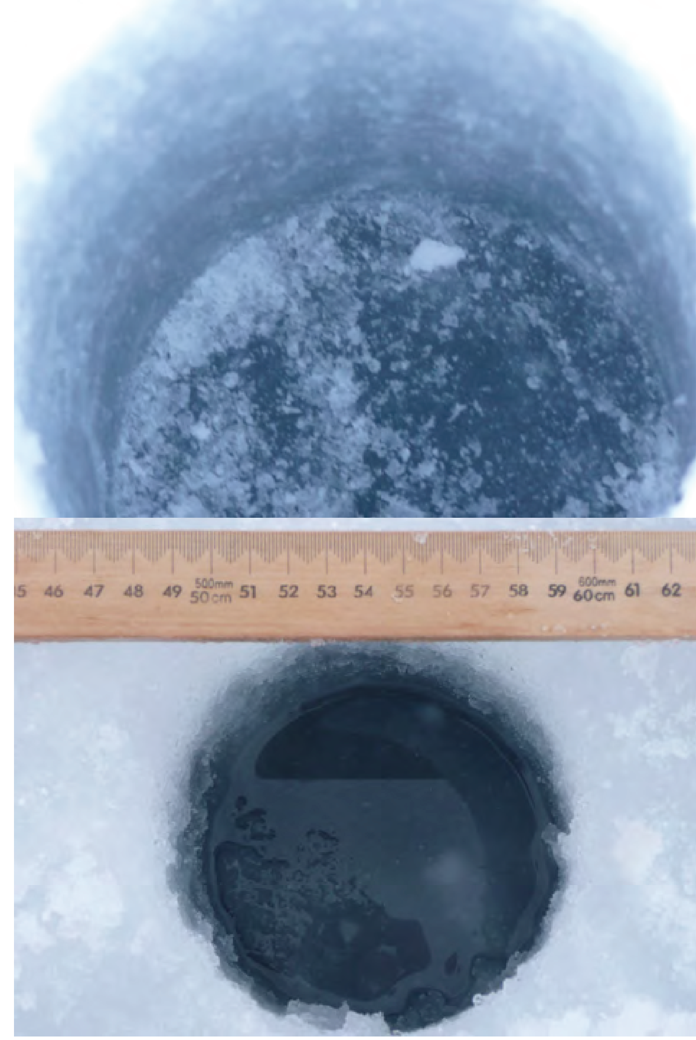


develop electromagnetic methods of monitoring fluid transport and microstructure

extensive measurements of fluid and electrical transport properties of sea ice:

2007	***Antarctic***	***SIPEX***
2010	***Arctic***	***Barrow AK***
2010	***Antarctic***	***McMurdo Sound***
2011	***Arctic***	***Barrow AK***
2012	***Arctic***	***Barrow AK***
2012	***Antarctic***	***SIPEX II***

photo by Jan Lieser

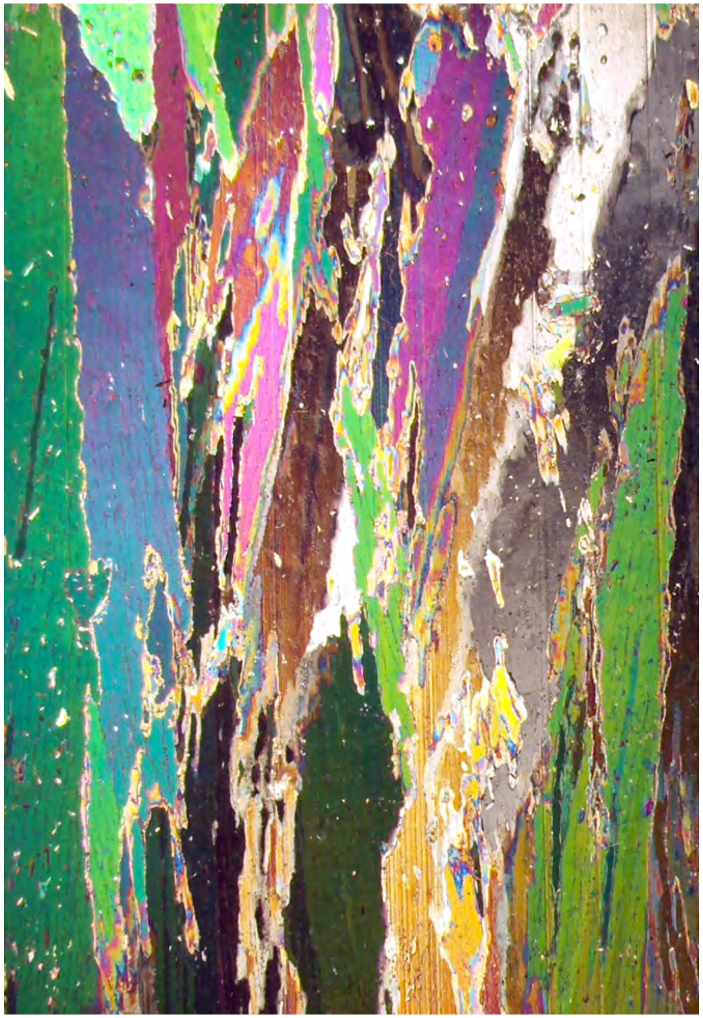


measuring fluid permeability of Antarctic sea ice

SIPEX 2007

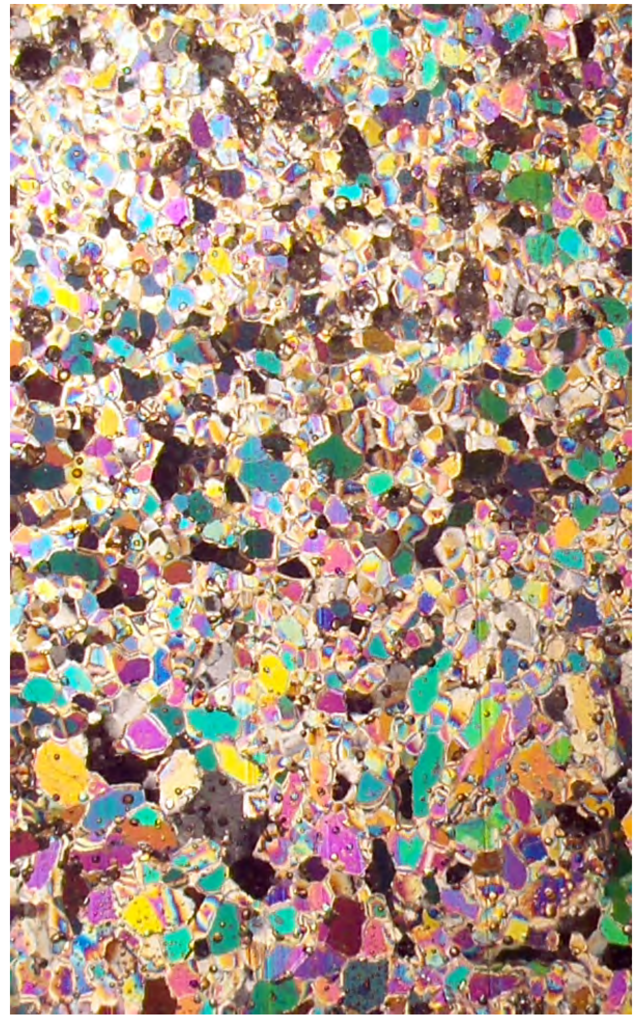
higher threshold for fluid flow in Antarctic granular sea ice

columnar



5%

granular

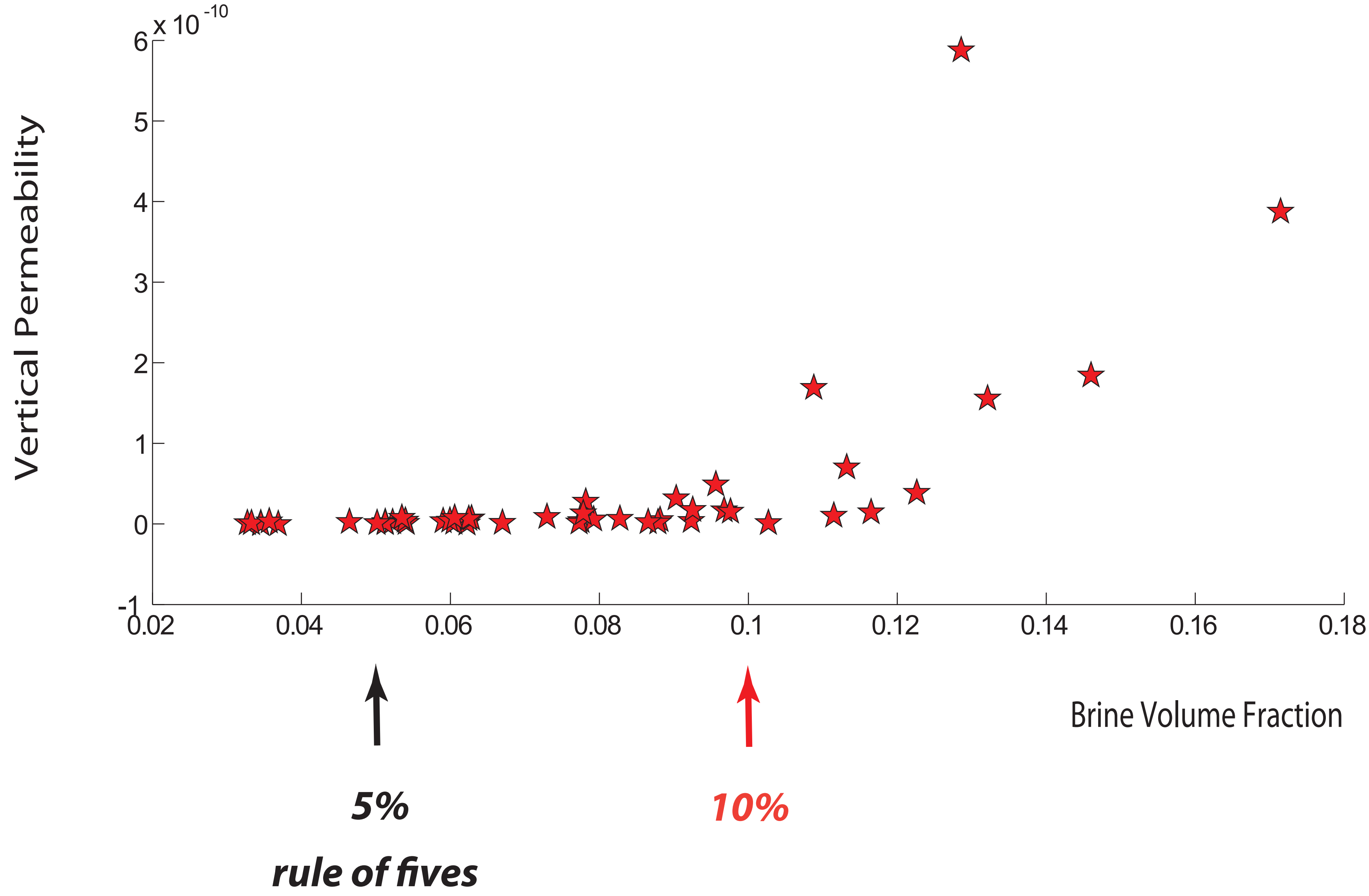


10%

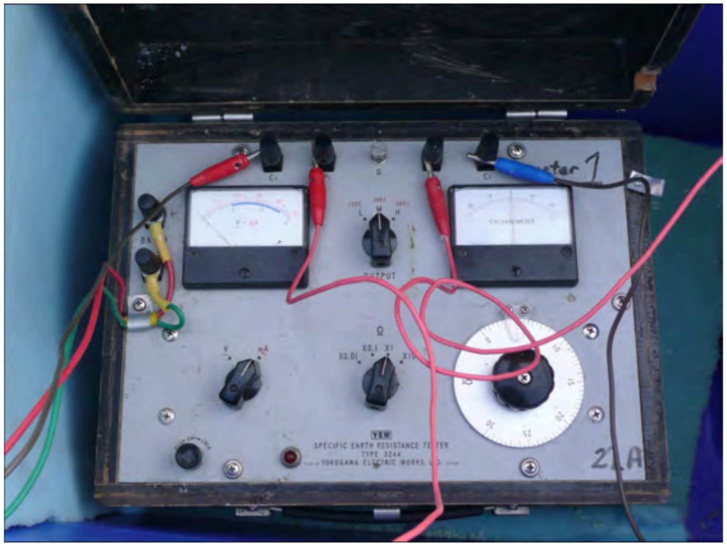
different microstructure -- different threshold

granular ice common in Arctic surface layer

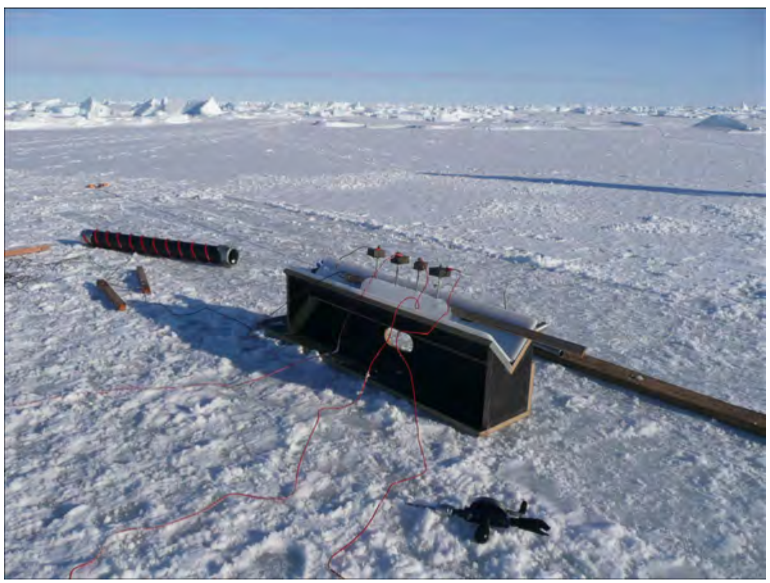
SIPEX II **preliminary vertical fluid permeability data** **September - November 2012**

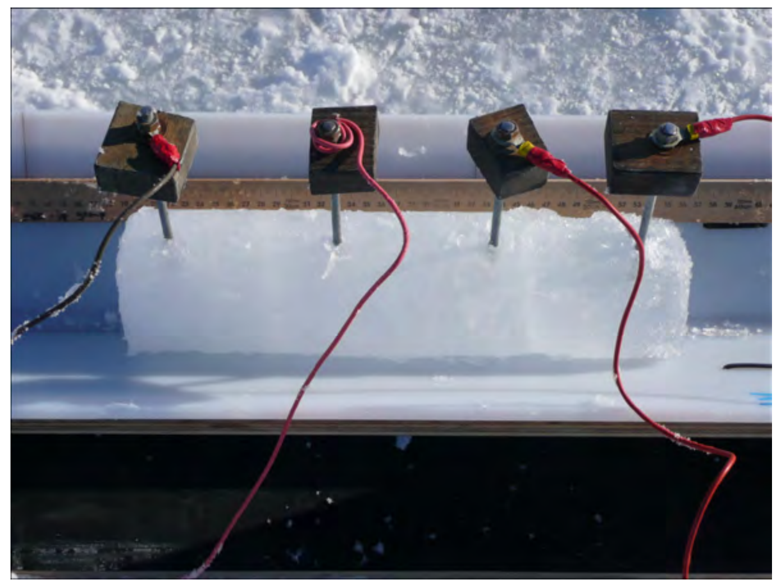


electrical measurements



Wenner array





vertical conductivity

Zhu, Golden, Gully, Sampson ***Physica B*** 2010

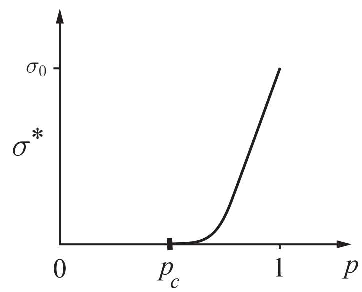
Sampson, Golden, Gully, Worby ***Deep Sea Research*** 2011

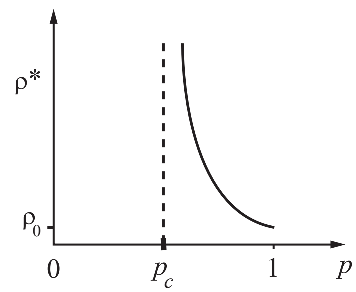
critical behavior of electrical transport in sea ice

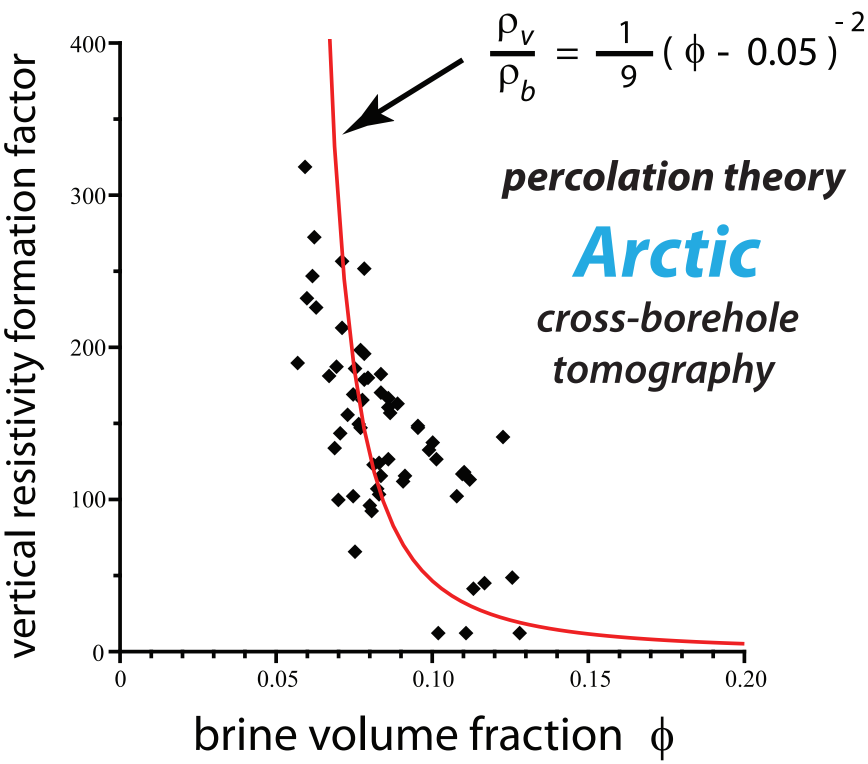
electrical signature of the on-off switch for fluid flow

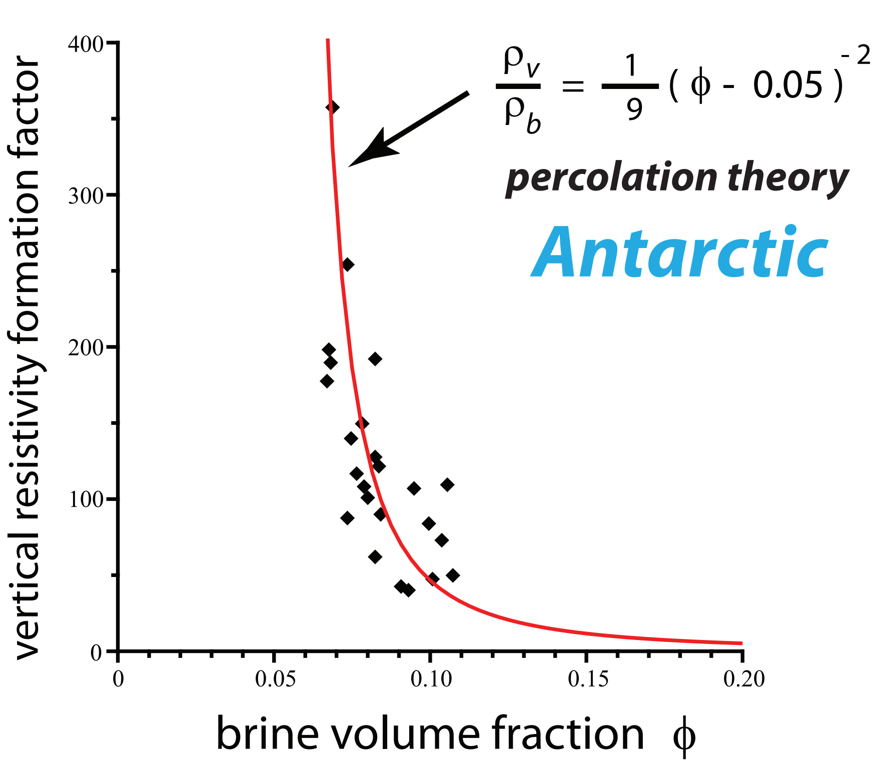
same universal critical exponent as for fluid permeability

studied for over 50 years but no previous observations or theory of critical behavior



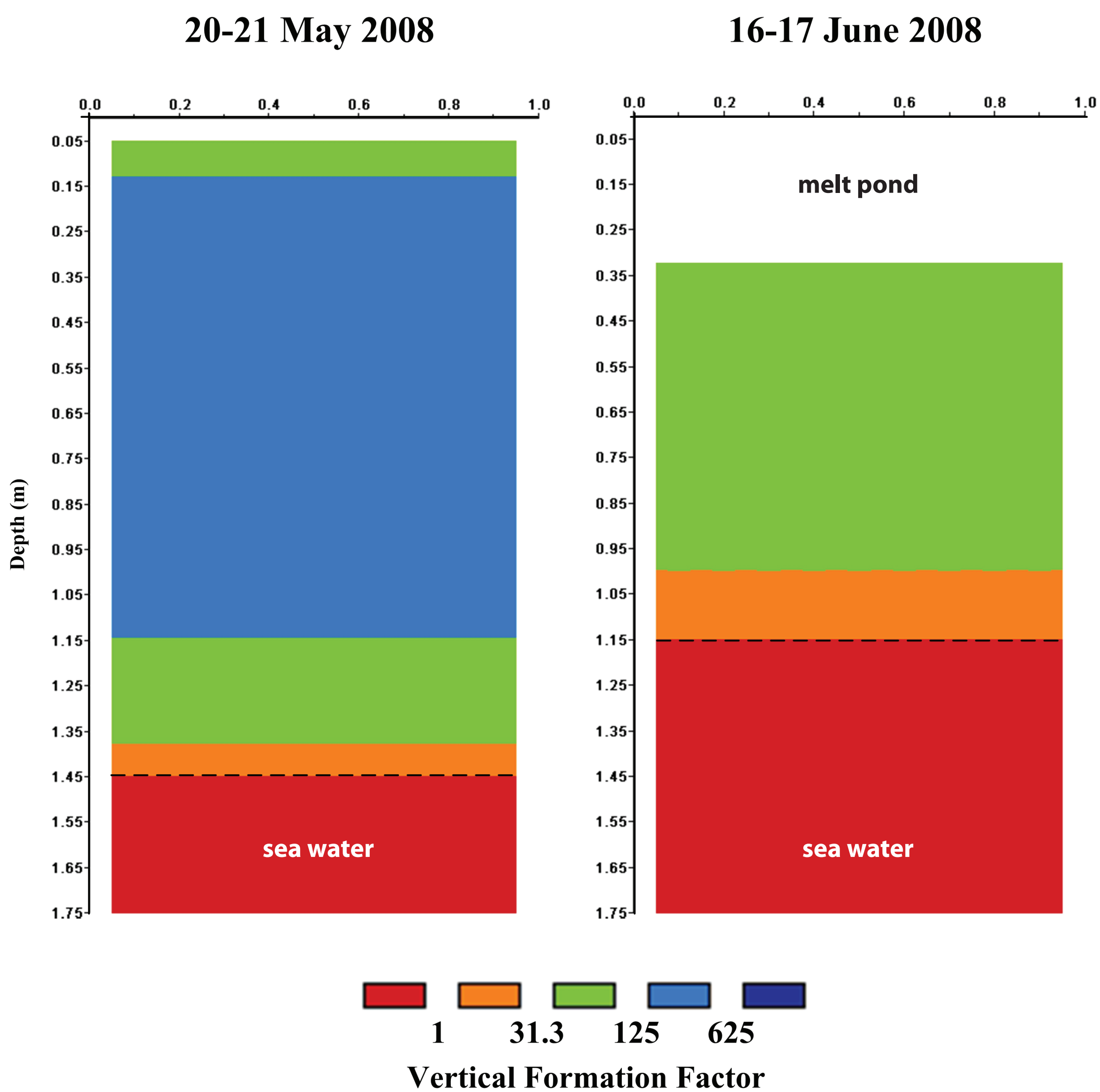






Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, Worby 2013

Cross-borehole tomographic reconstructions of the vertical resistivity formation factor for Arctic sea ice before and after melt pond formation



impermeable enough to allow ponds to grow

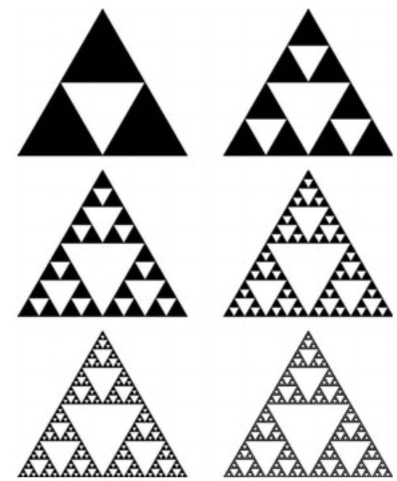
impermeable from the perspective of CO2 exchange and build-up of nutrients and biomass in the ice, and sufficiently impermeable to fluid drainage to support surface ponding

highly permeable ice that allows for CO2 pumping and build-up of nutrients and biomass

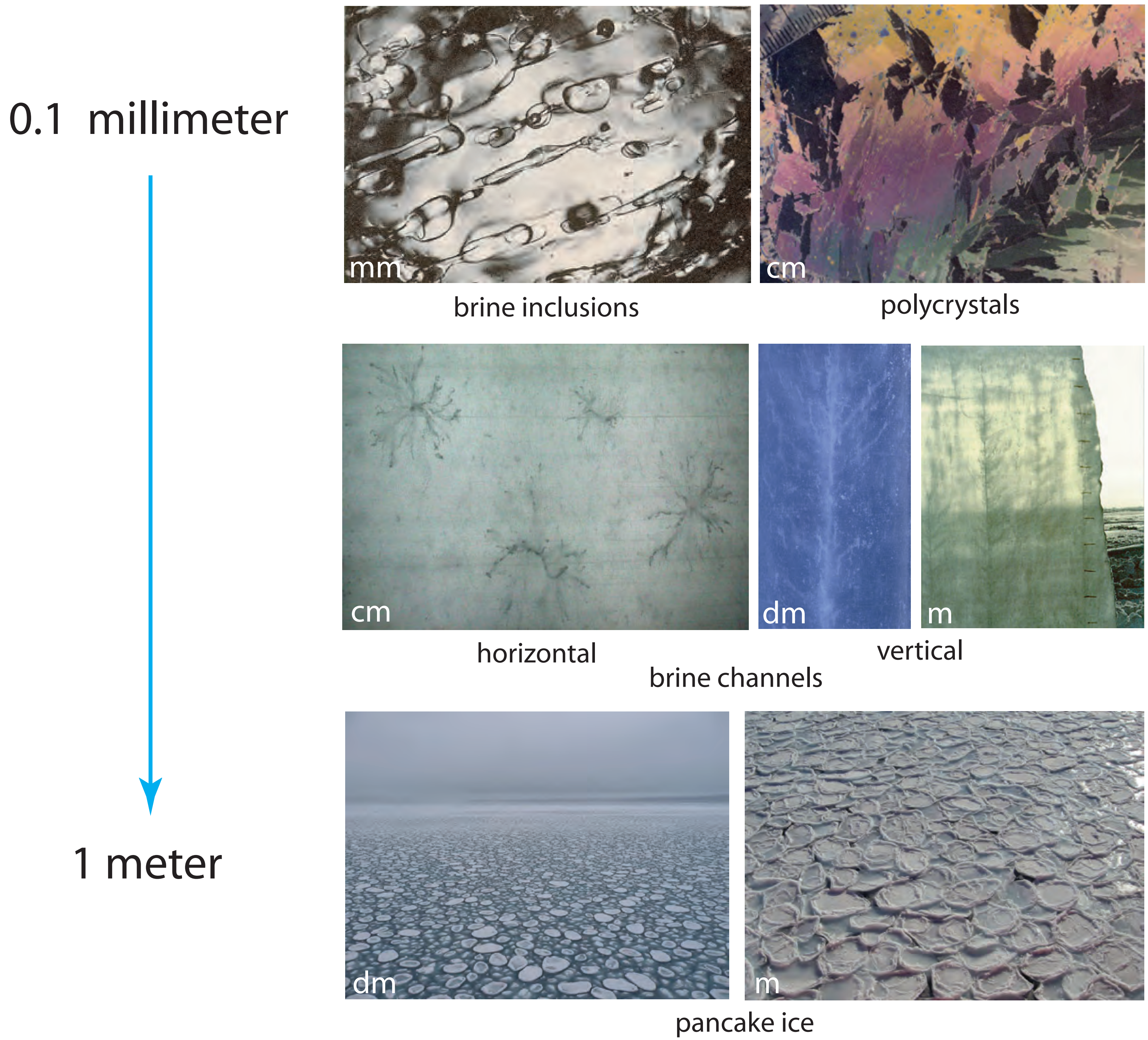
The evolution of resistivity structure seen here is consistent with warming of the ice, thus increasing the fluid permeability.

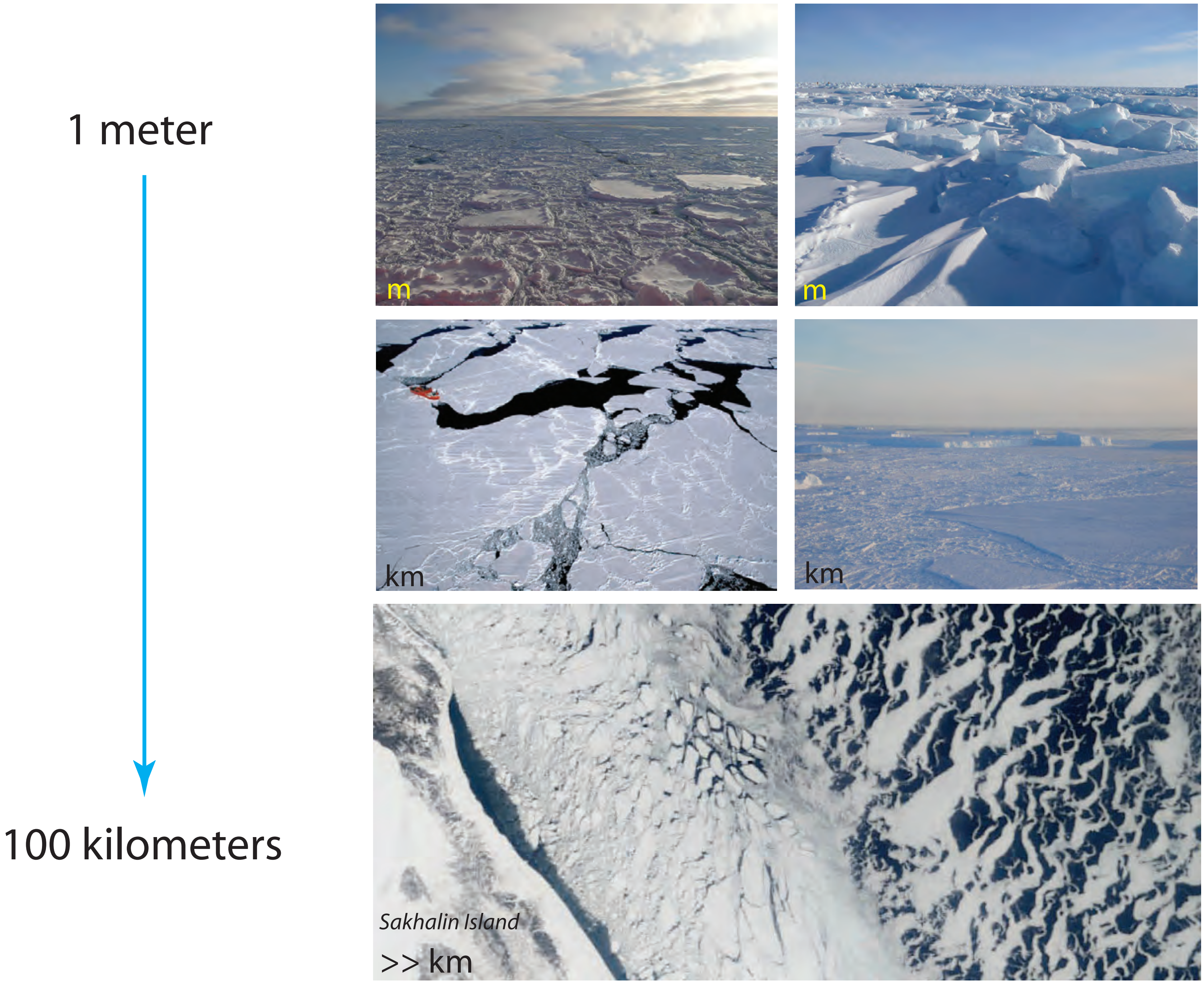
Golden, Eicken, Gully, Ingham, Jones, Lin, Reid, Sampson, and Worby 2013

fractals and multiscale structure



sea ice displays ***multiscale*** structure over 10 orders of magnitude




1 meter
100 kilometers
m
m
km
km
Sakhalin Island
>> km

melt pond formation and albedo evolution:

- ***major drivers in polar climate***
- ***key challenge for global climate models***

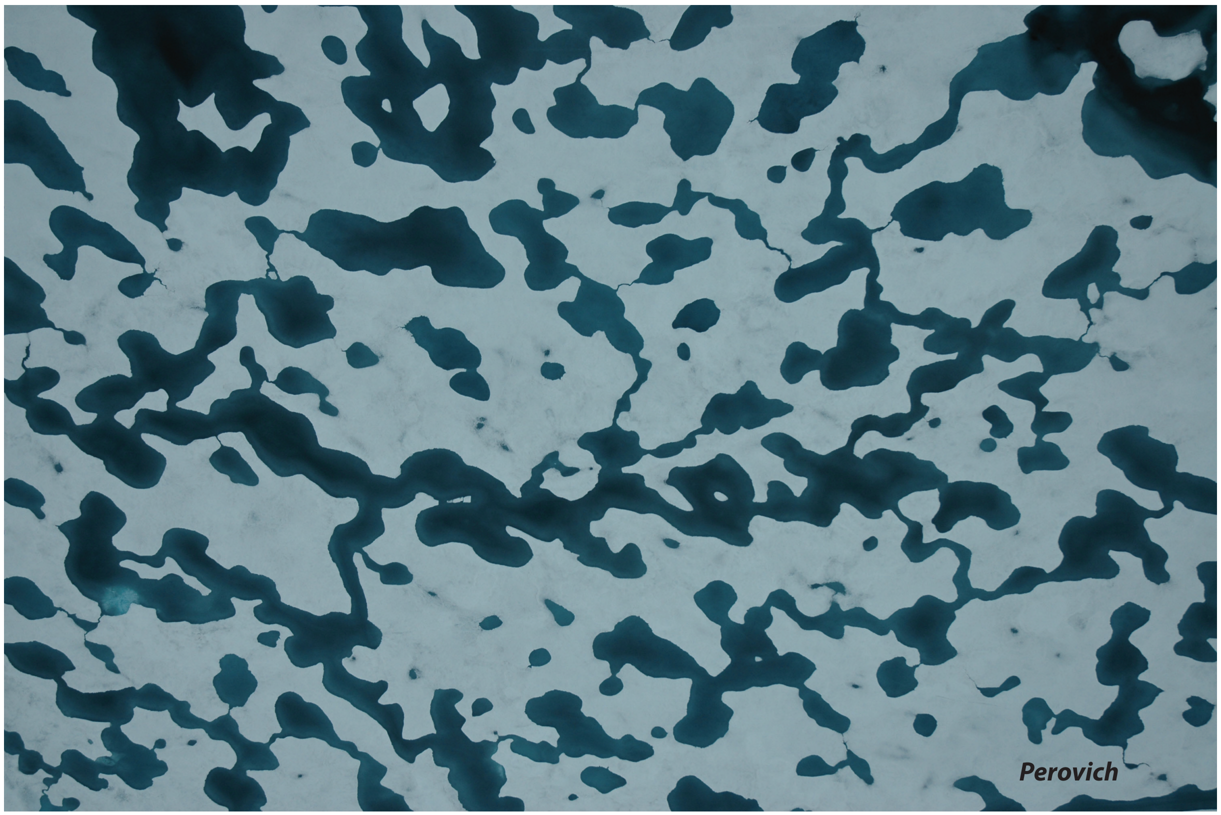
numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson, Perovich 2009

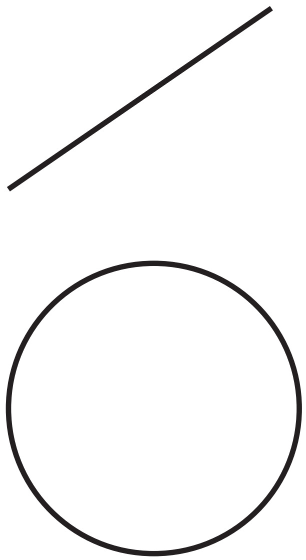
Flocco, Feltham, Hunke 2012

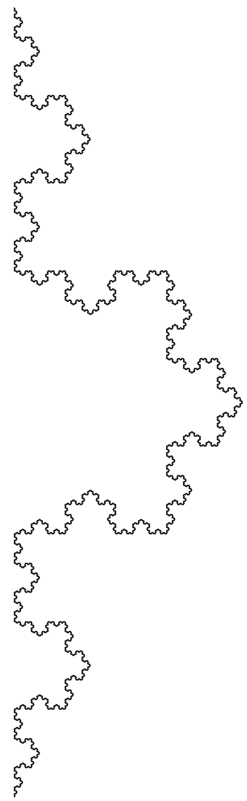


Are there universal features of the evolution similar to phase transitions in statistical physics?

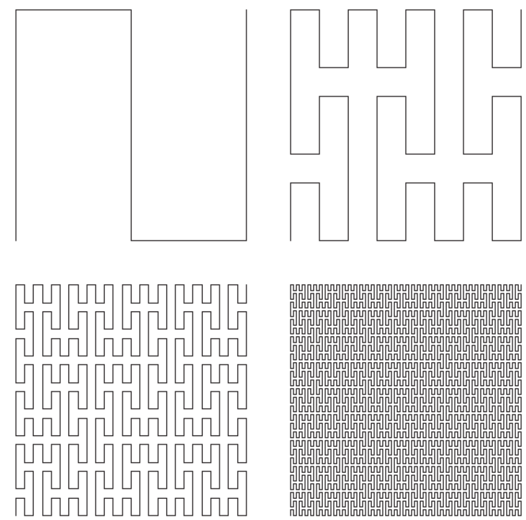
fractal curves in the plane

they wiggle so much that their dimension is >1





Peano curve



Brownian motion

simple curves	*Koch snowflake*	*space filling curves*
$D = 1$	$D = 1.26$	$D = 2$

clouds exhibit fractal behavior from 1 to 1000 km

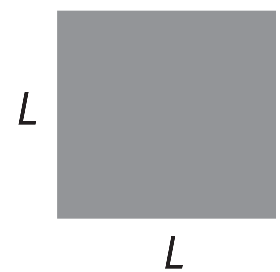
use ***perimeter-area*** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982

$$P \sim \sqrt{A}$$

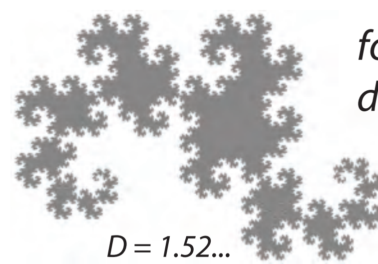
simple shapes



$A = L^2$

$P = 4L = 4\sqrt{A}$

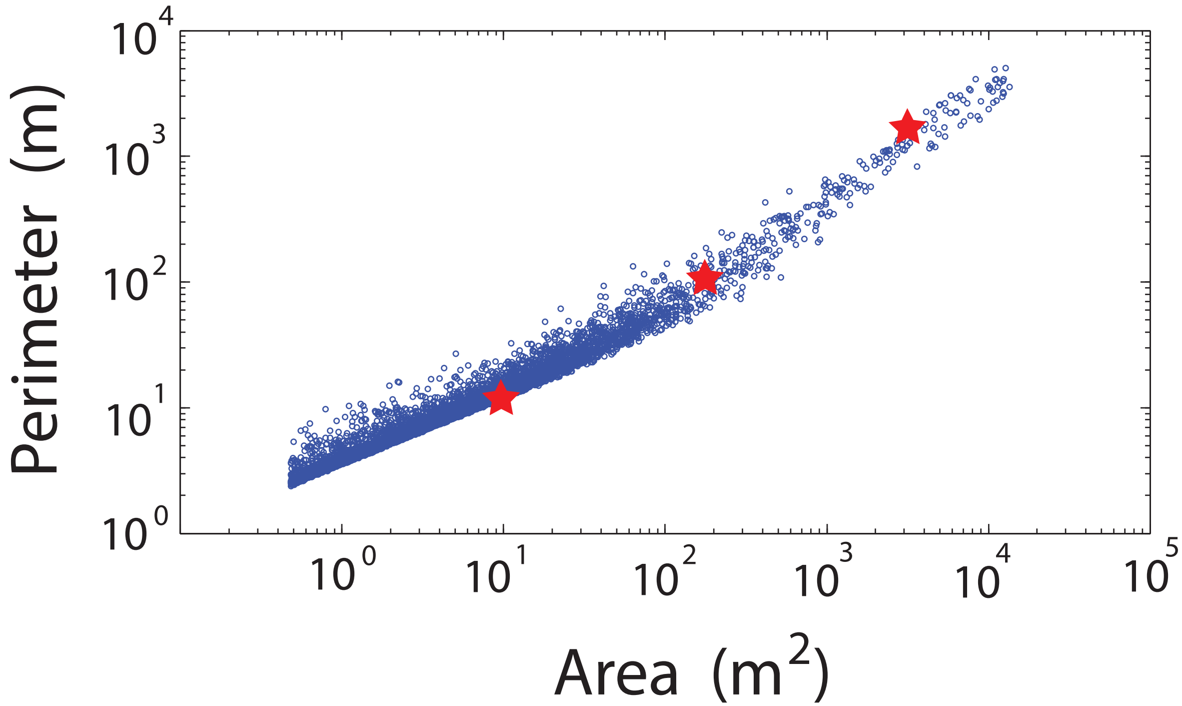
$$P \sim \sqrt{A}^{\,D}$$



for fractals with dimension D

Transition in the fractal geometry of Arctic melt ponds — *The Cryosphere, 2012*

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

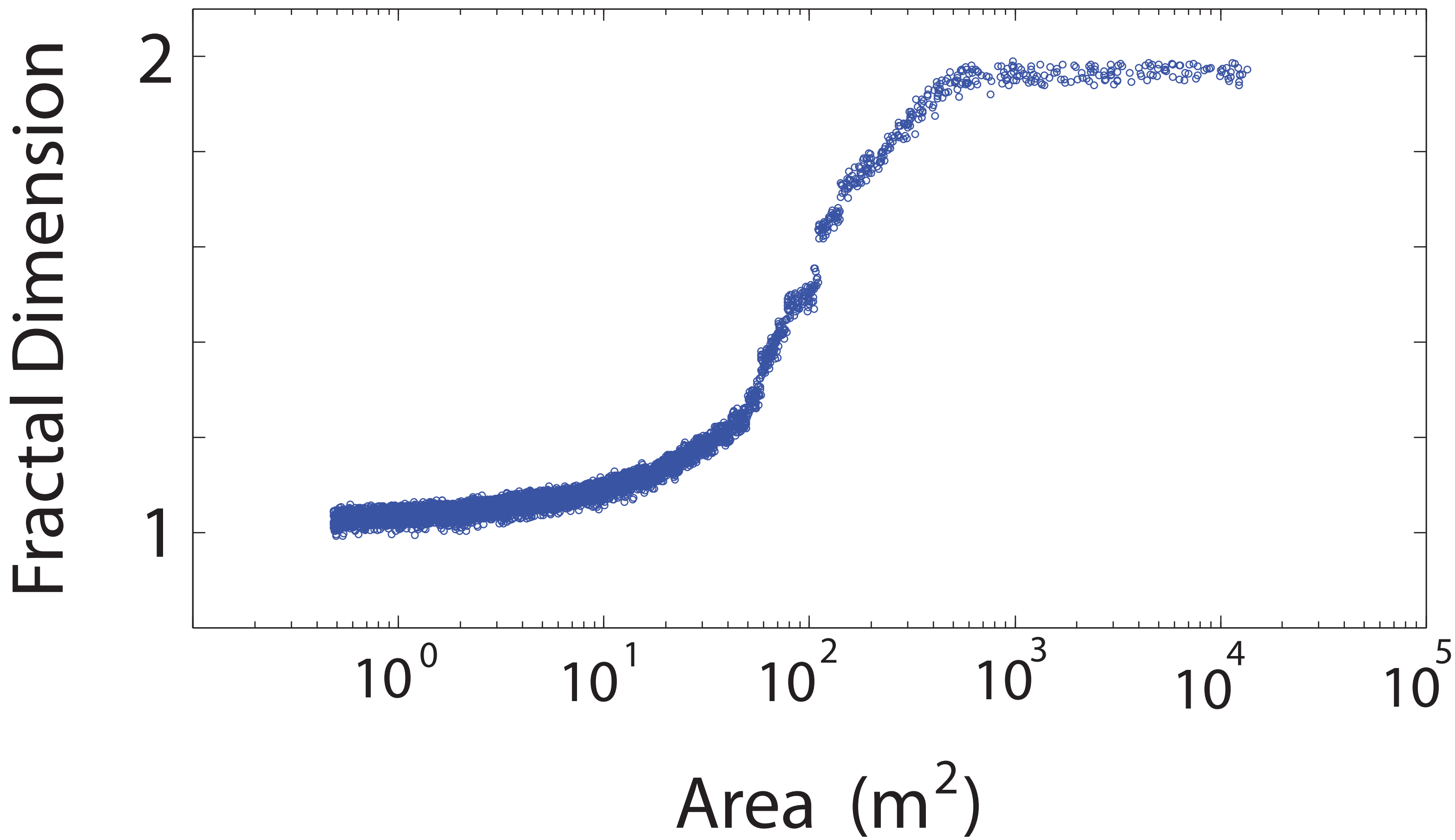


data from
5269 Arctic
melt ponds

simple pond ***transitional pond*** ***complex pond***

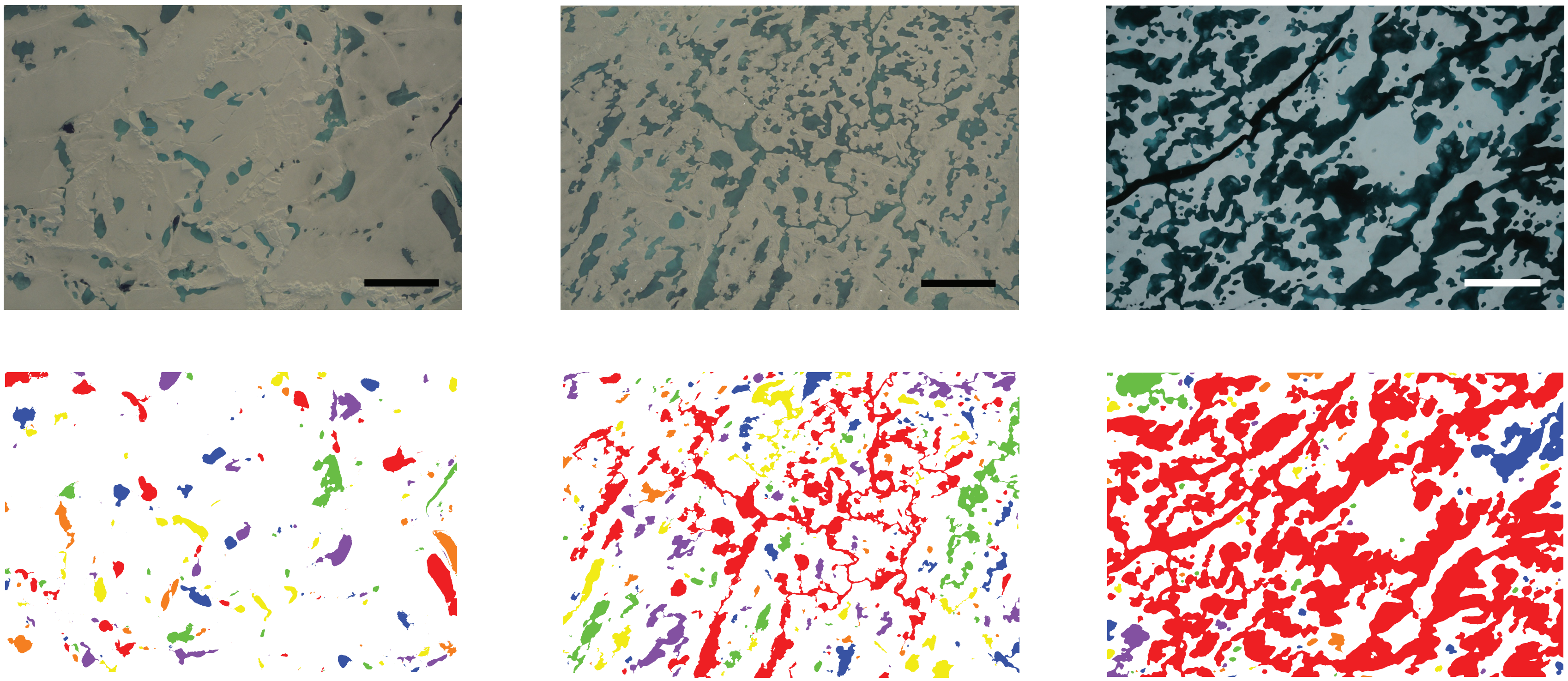
transition in the fractal dimension

complexity grows with length scale



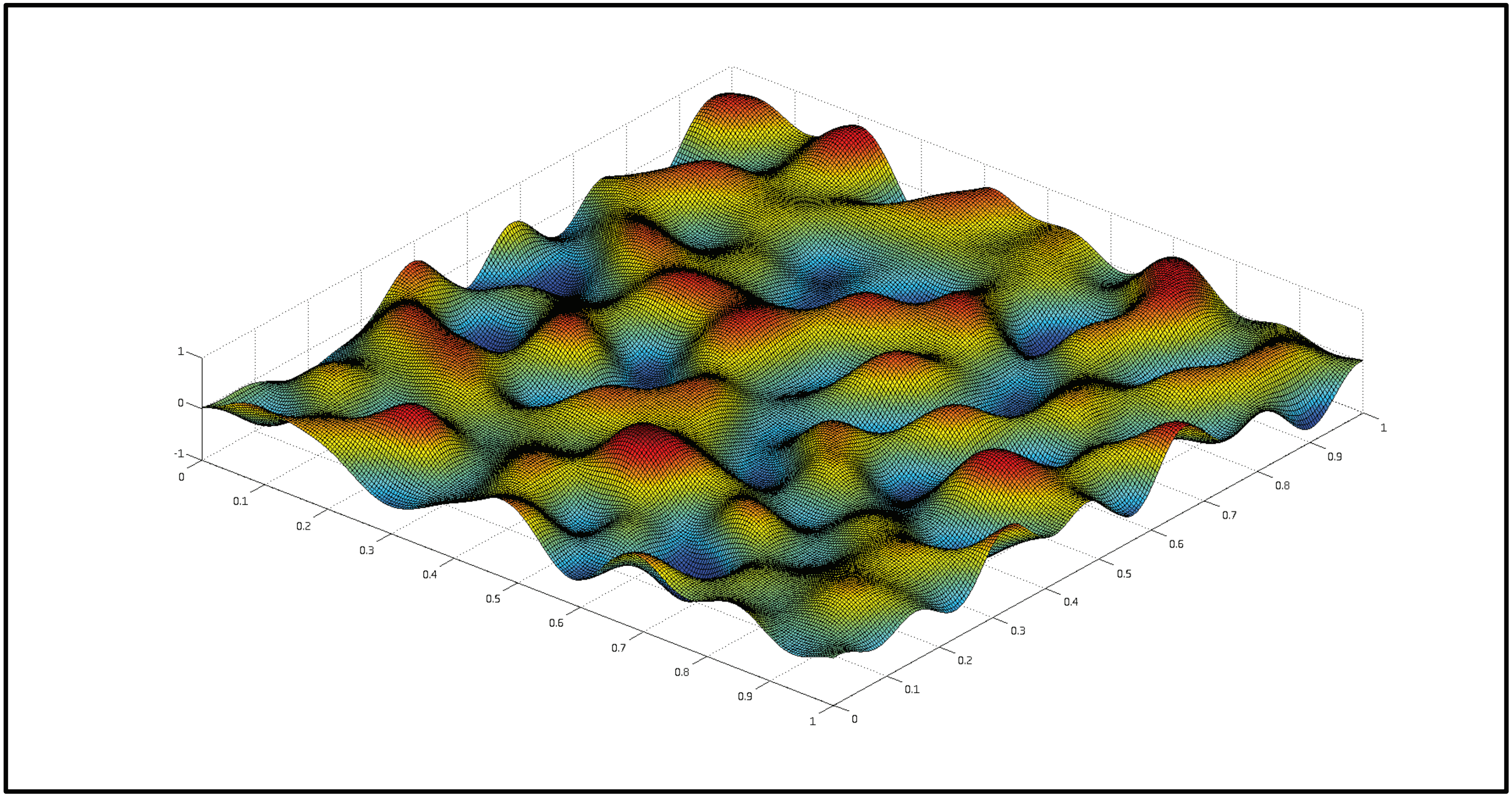
compute "derivative" of area - perimeter data

small simple ponds coalesce to form large connected structures with complex boundaries



melt pond percolation

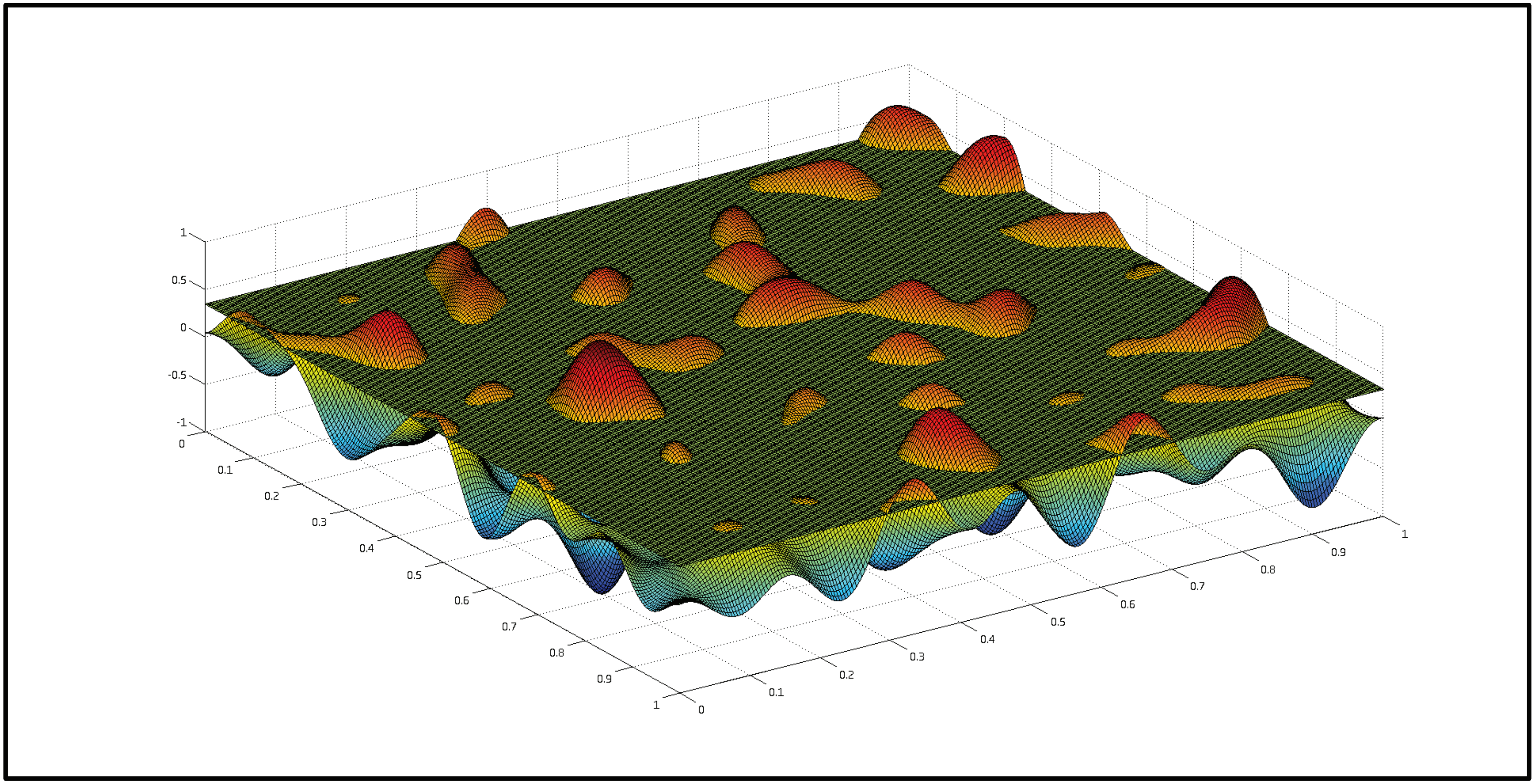
Continuum percolation model for melt pond evolution



random Fourier surface

Brady Bowen and Ken Golden

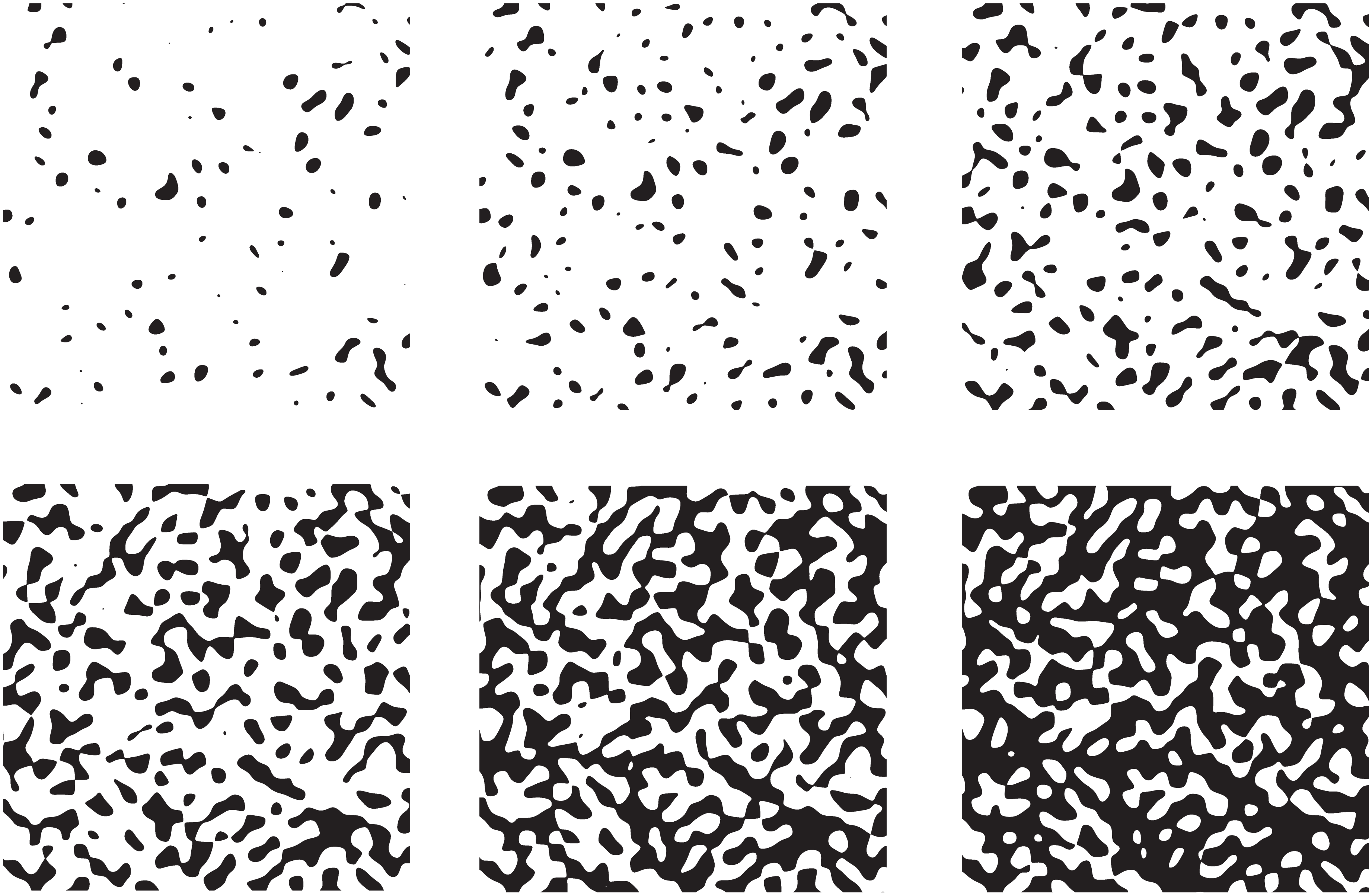
intersections of a plane with the surface define melt ponds



as the plane varies in height the regions evolve like melt ponds

at a critical height h_c ponds percolate and form an infinite ocean

electronic transport in disordered media *diffusion in turbulent plasmas* *(Isichenko, Rev. Mod. Phys., 1992)*

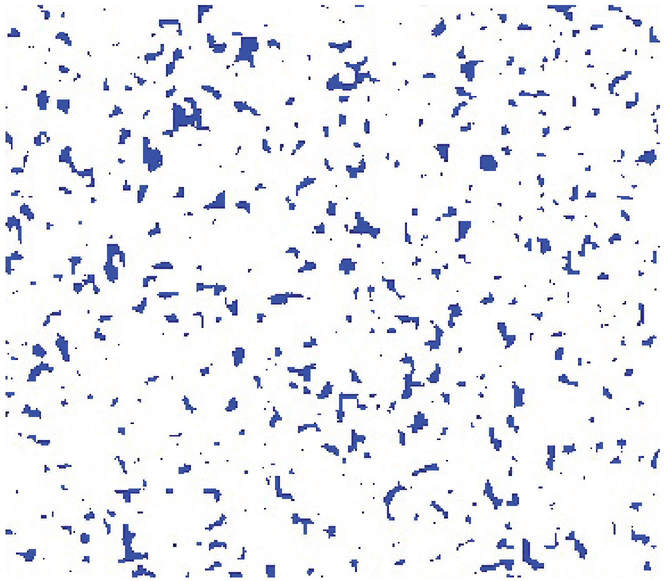


h_c

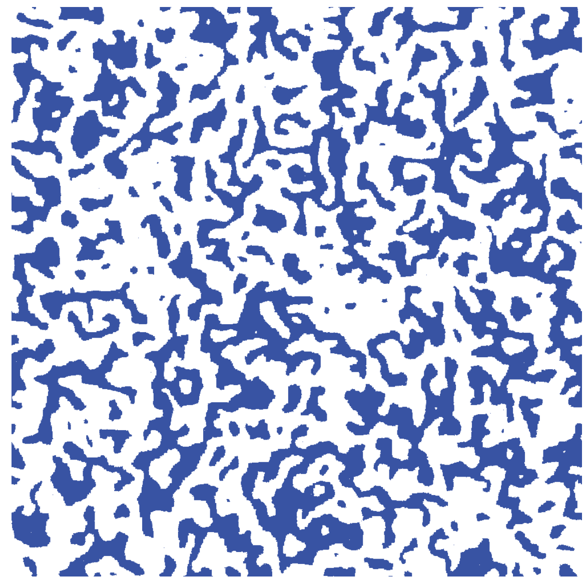
percolation threshold

Ising model for ferromagnets → Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{<i,j>}^{N} s_i s_j - H \sum_{i}^{N} s_i \qquad s_i = \begin{cases} \uparrow & +1 \quad \text{ice} \\ \downarrow & -1 \quad \text{water} \end{cases} \qquad M = \lim_{N\to\infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

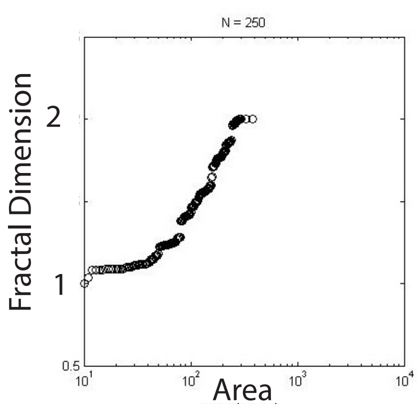


COLD



WARM

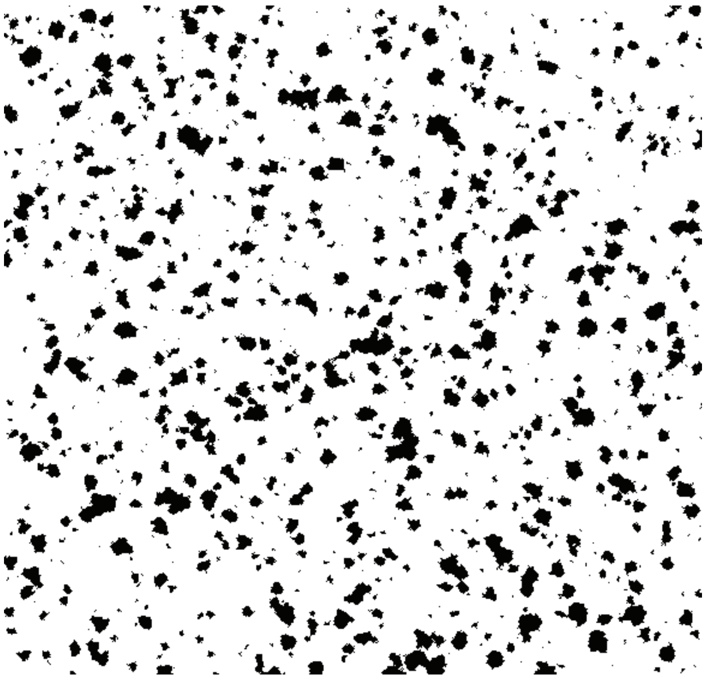
"melt ponds" are clusters of magnetic spins that align with the applied field

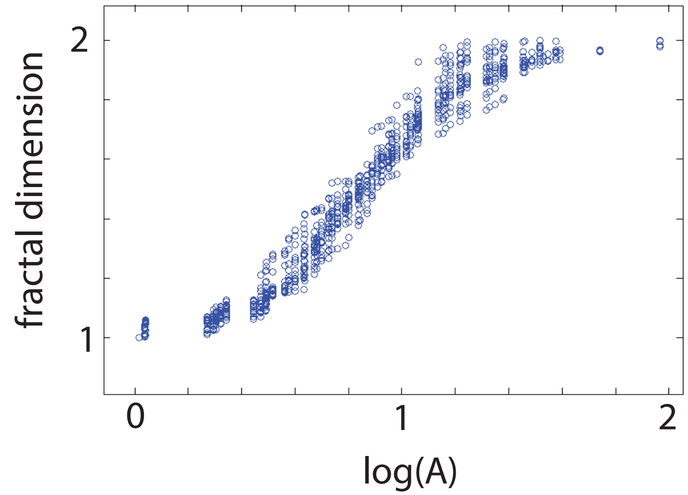


clusters exhibit transition in fractal dimension

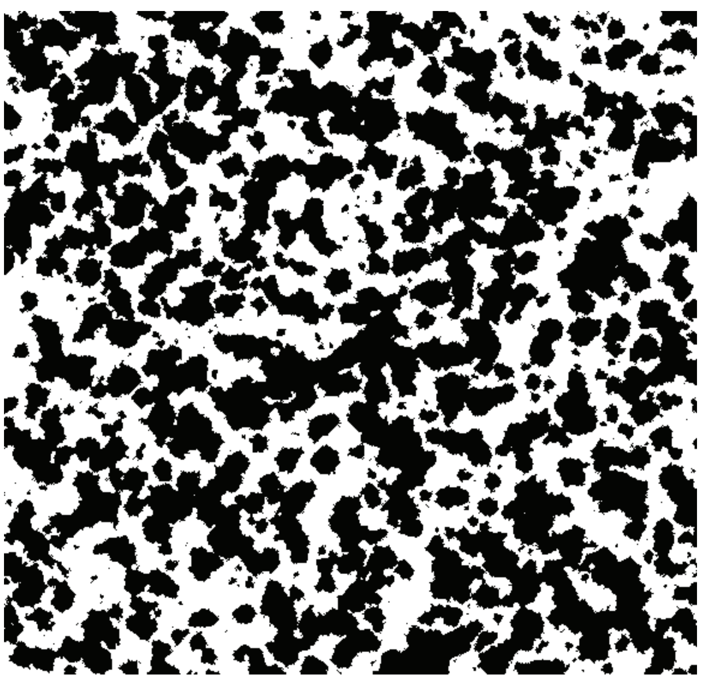
Thekkedath, Alali, Golden

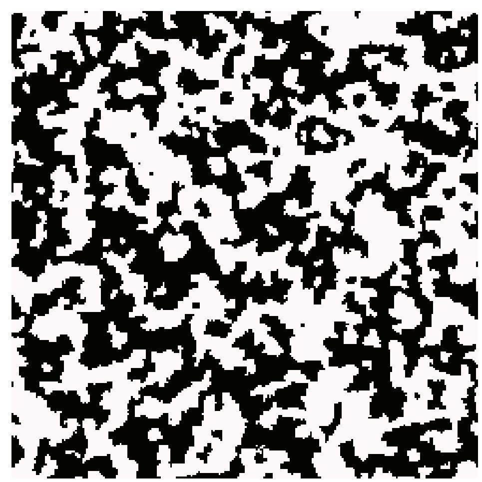
simple stochastic growth model of melt pond evolution





a square is more likely to melt
if its neighbors have melted

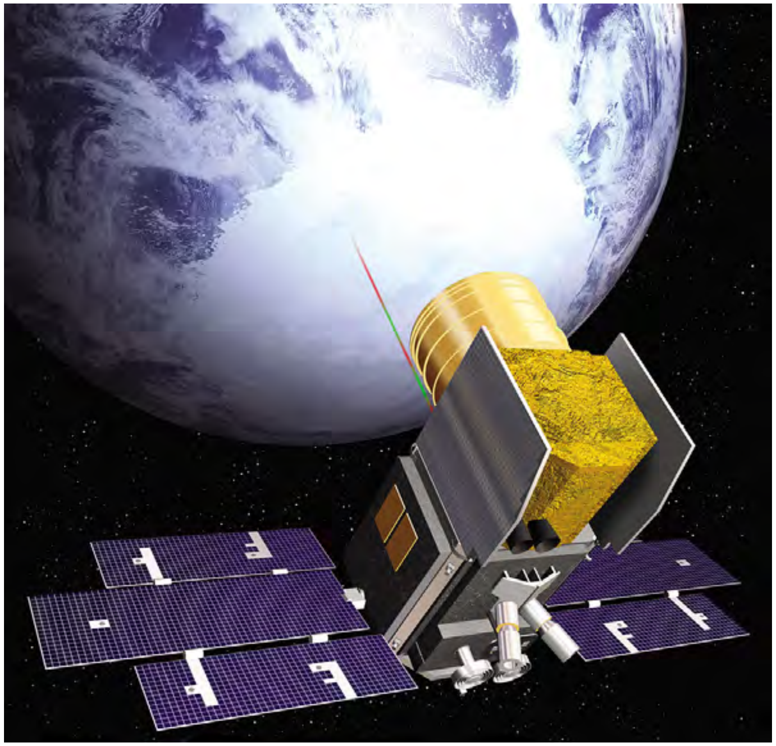




voter
model

Rebecca Nickerson (high school student) and Ken Golden

multiscale homogenization



NASA's Ice, Cloud and Land Elevation Satellite (ICESat)

The Worbot - a low frequency EM induction instrument for measuring sea ice thickness

The key parameter in modeling the response of sea ice to an EM field is its

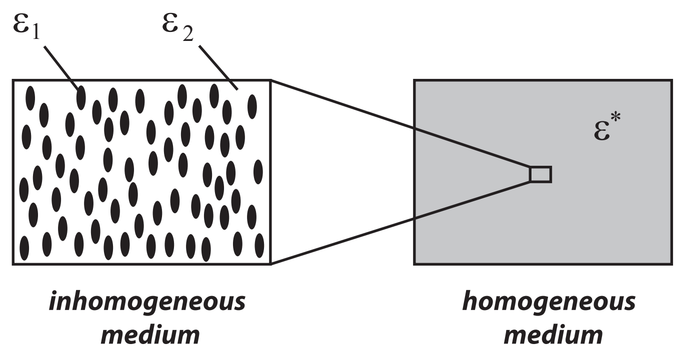
complex *permittivity* or *dielectric constant* ε^*

which depends strongly on the brine microstructure

e.g., interpretation of EM thickness data depends on knowledge of ε^*

Theory of Effective Electromagnetic Behavior of Composites

analytic continuation method



Forward Homogenization Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)

composite geometry
(spectral measure μ) → ε^*

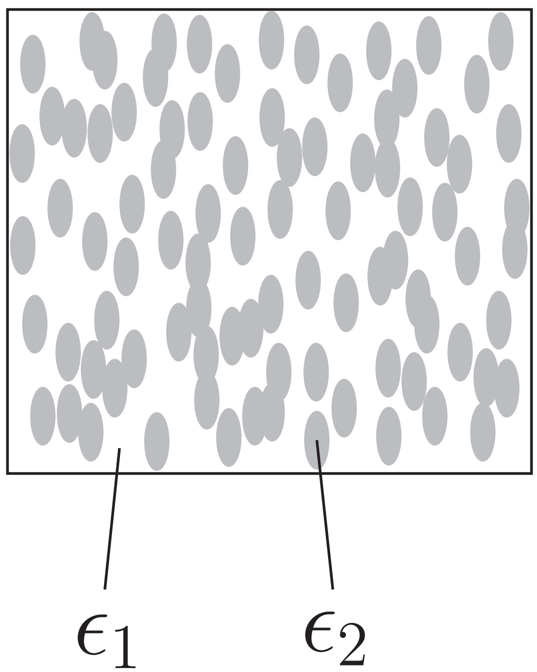
integral representations, rigorous bounds, approximations, etc.

Inverse Homogenization Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)
(McPhedran, McKenzie, and Milton, 1982)

ε^* → ***composite geometry***
(spectral measure μ)

recover brine volume fraction, connectivity, etc.

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



$\Big\} \; \epsilon^*$

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

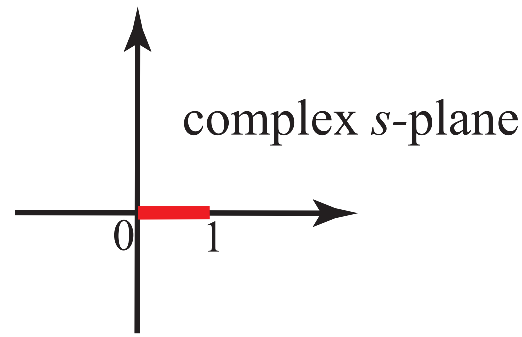
$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1 , p_2 = volume fractions of the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2} \; , \text{ composite geometry} \right)$$

Stieltjes integral representation

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

μ

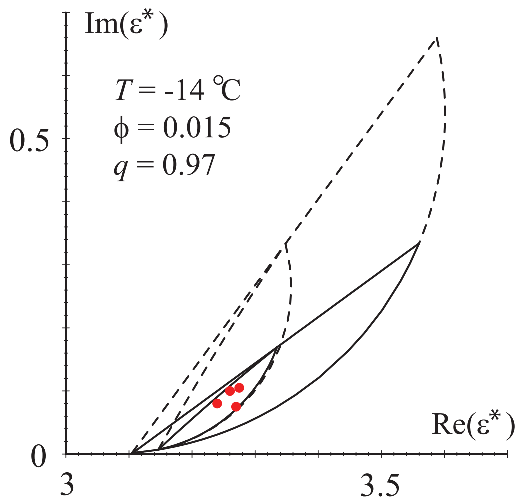
- spectral measure of self adjoint operator $\Gamma\chi$
- mass = p_1
- higher moments depend on n-point correlations

separation of geometry from parameters

Golden and Papanicolaou, Comm. Math. Phys. 1983

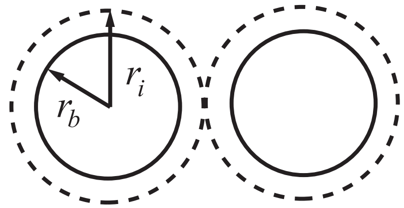
forward and inverse bounds for sea ice

forward bounds



4.75 GHz data

matrix particle

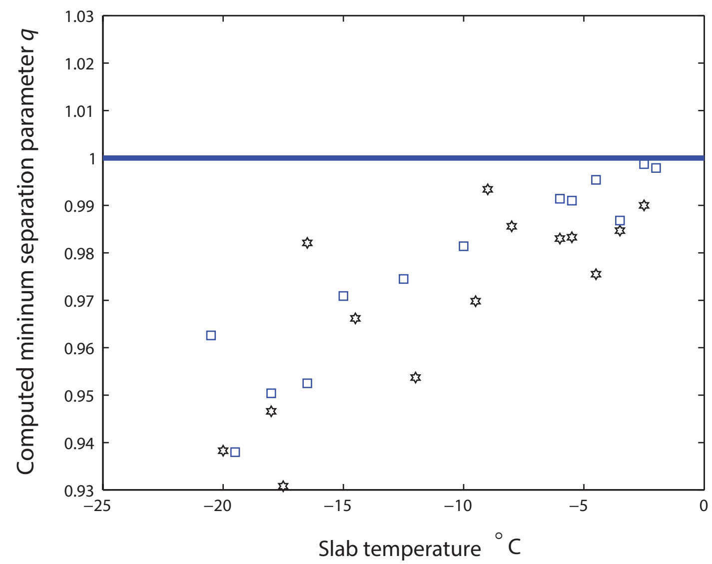


$$q = r_b / r_i$$

$$0 < q < 1$$

Golden 1995, 1997

inverse bounds



inverse bounds and recovery of brine porosity

Gully, Backstrom, Eicken, Golden Physica B, 2007

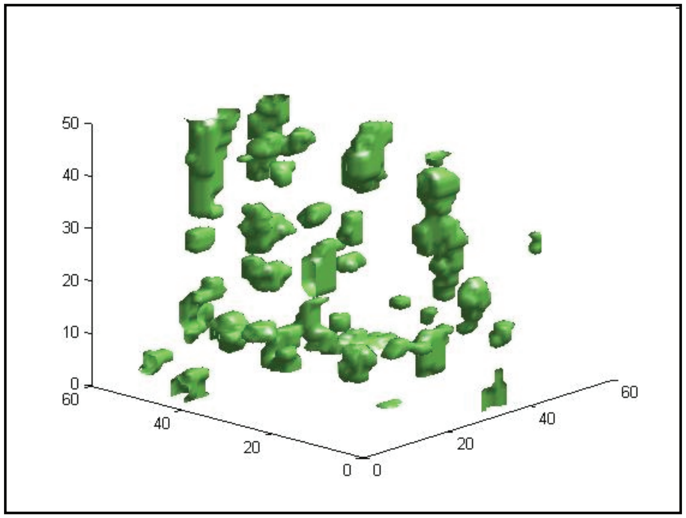
polycrystalline bounds

Gully, Lin, Cherkaev, Golden, 2013

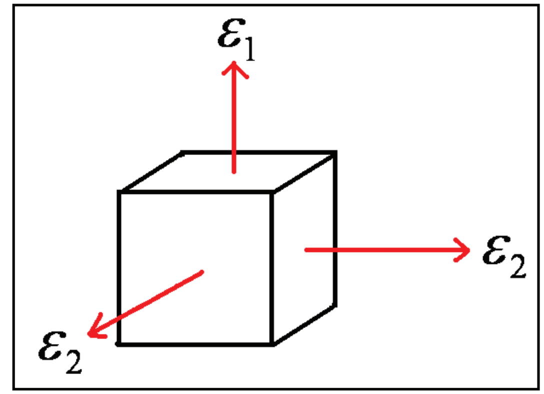
inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity ε^*

Orum, Cherkaev, Golden Proc. Roy. Soc. A, 2012

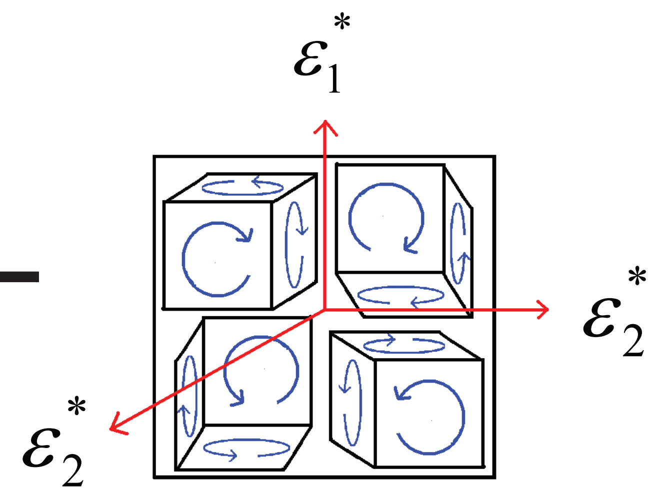
two scale homogenization for polycrystalline sea ice



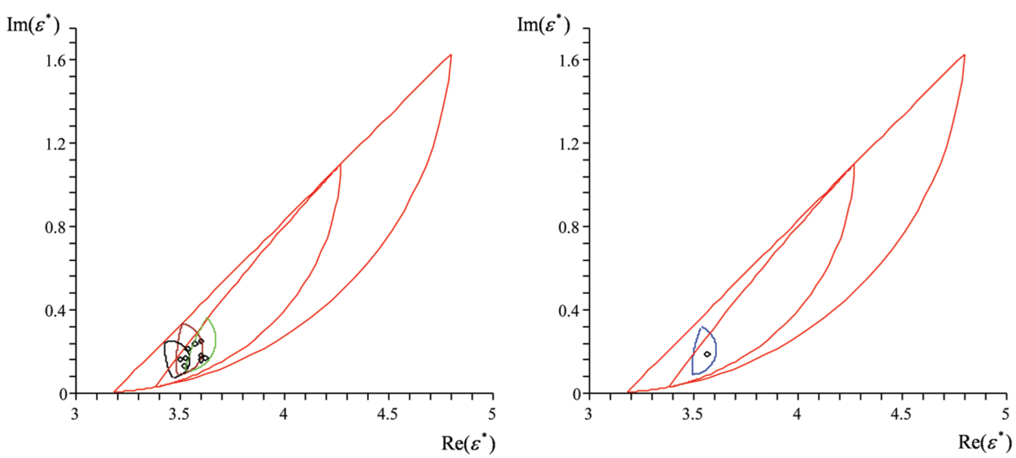
numerical homogenization for single crystal



analytic continuation for polycrystals



bounds



Gully, Lin, Cherkaev, Golden 2013

direct calculation of spectral measure

1. Discretization of composite microstructure gives lattice of 1's and 0's (random resistor network).

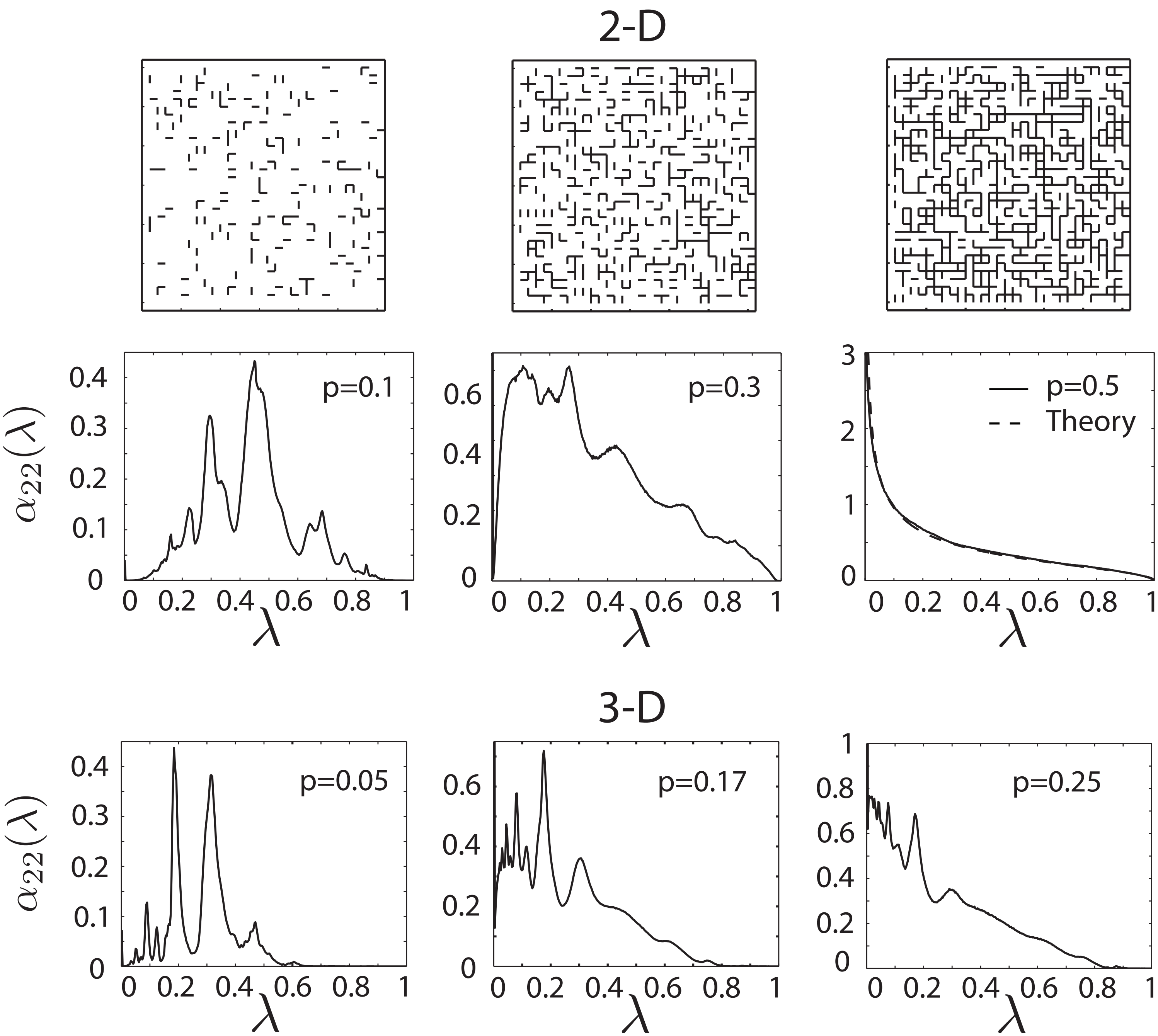
2. The fundamental operator $\Gamma\chi$ becomes a random matrix depending only on the composite geometry.

3. Compute the eigenvalues λ_i and eigenvectors of $\Gamma\chi$ with $(\text{length})^2 = \alpha_i$

$$\mu(\lambda) = \sum_i \alpha_i \, \delta(\lambda - \lambda_i)$$

Dirac point measure (Dirac delta)

The Spectral Measures for Random Resistor Networks



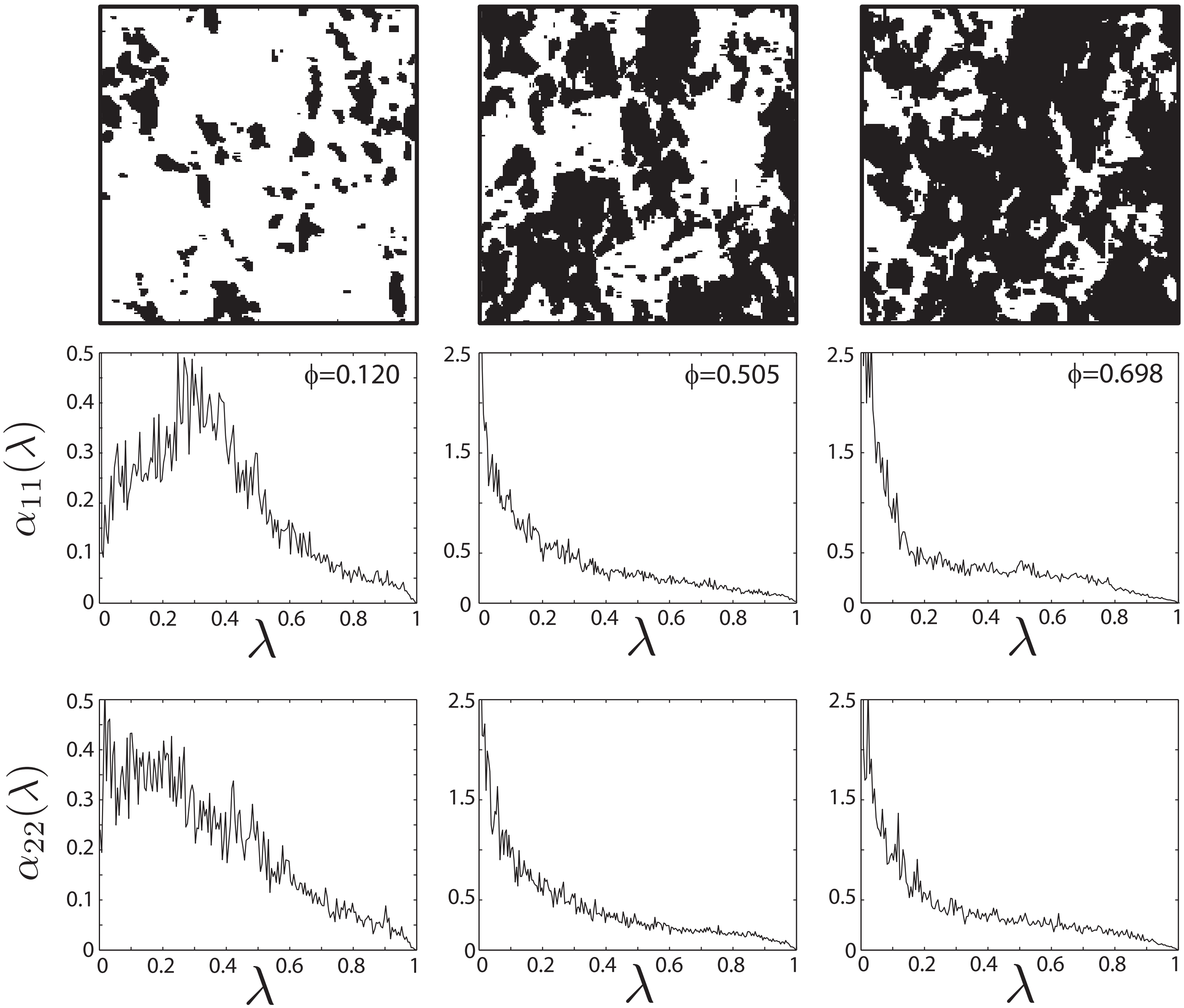
$p_c = 0.5$

$p_c \approx 0.25$

spectral gaps collapse at the percolation transitions

Murphy and Golden, J. Math. Phys. (2012)

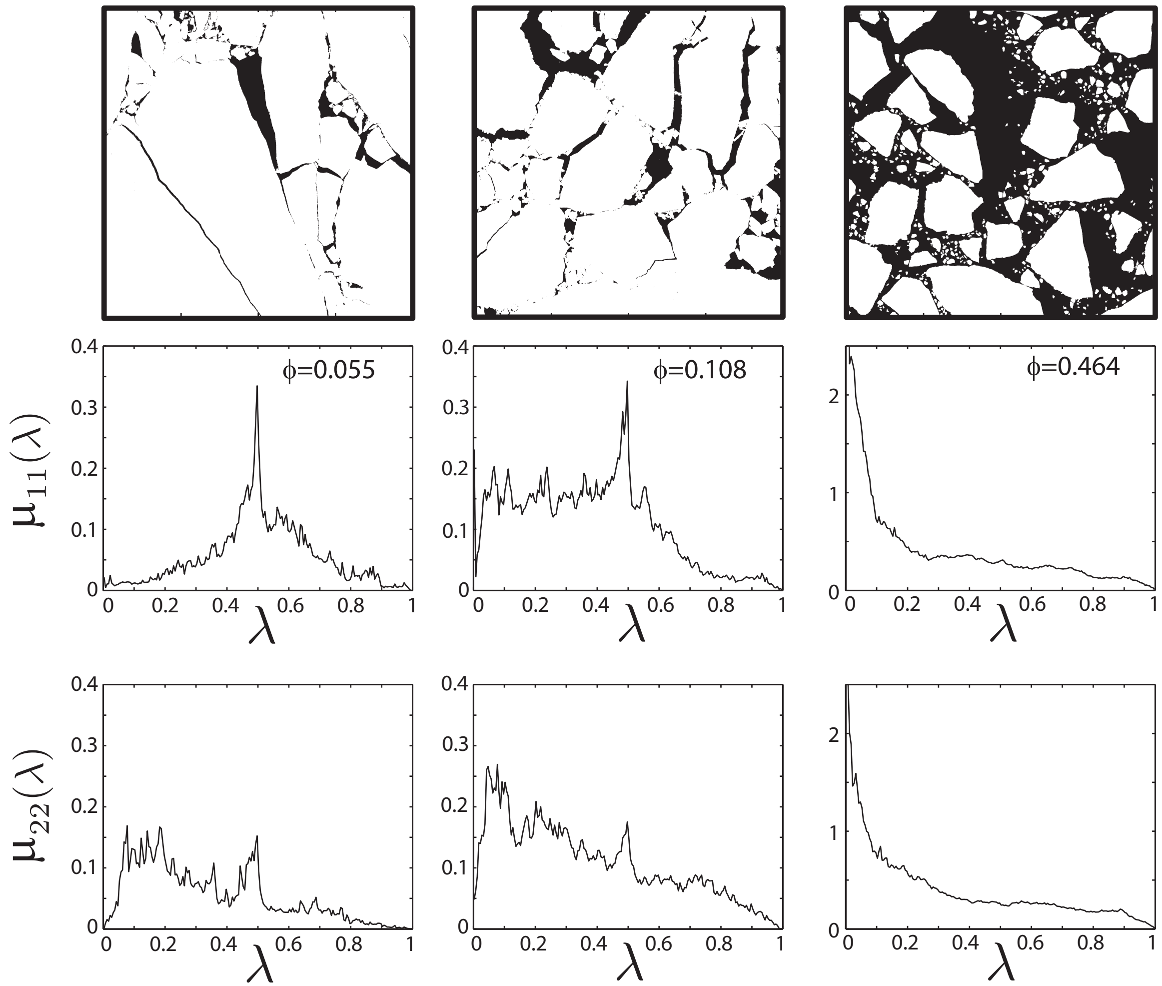
Spectral Measures for Sea Ice Structures: Brine Inclusions



N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden, 2013

spectral measures provide a path toward rigorously incorporating "composite microstructure" into calculations of effective behavior on larger scales

spectral measures for the Arctic sea ice pack



area under curve = φ = open water fraction

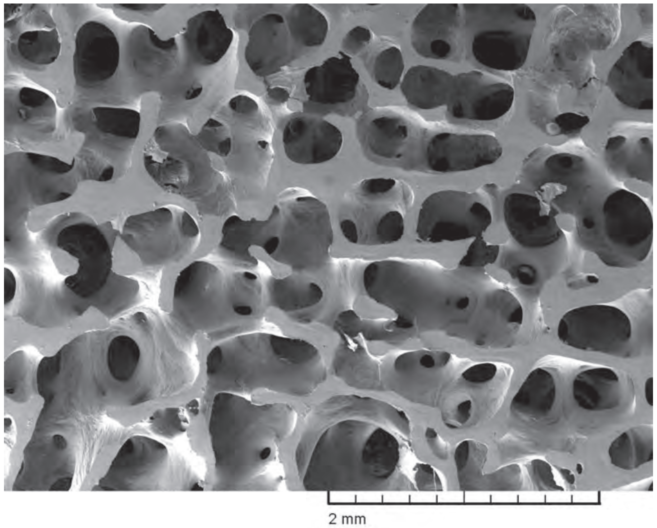
spectral gap closes as ocean phase becomes connected

N. B. Murphy, C. Hohenegger, C. S. Sampson, D. K. Perovich, H. Eicken, E. Cherkaev, B. Alali, and K. M. Golden, 2013

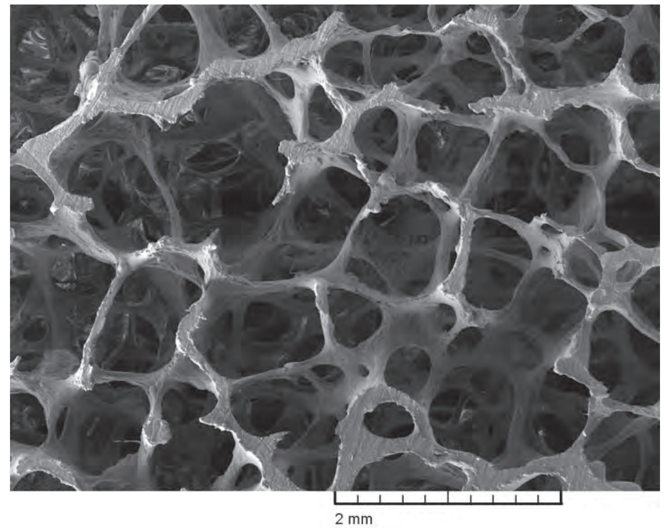
spectral characterization of porous microstructures in bone

Golden, Murphy, Cherkaev, J. Biomechanics 2011

(a) young healthy trabecular bone

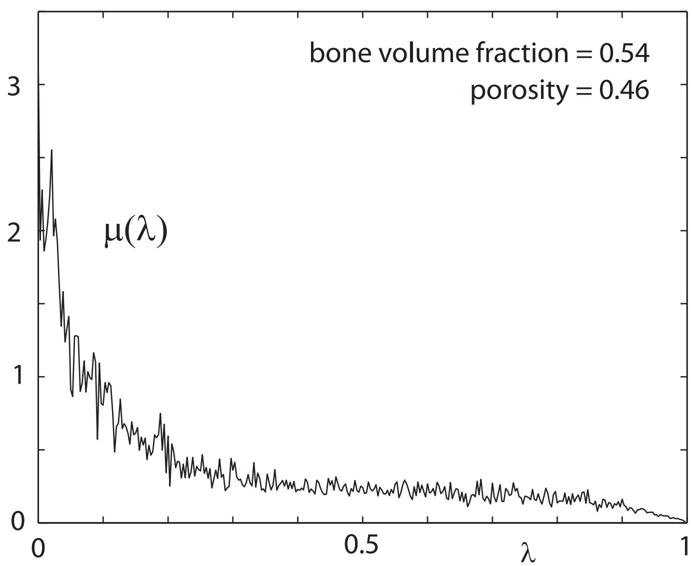


(b) old osteoporotic trabecular bone

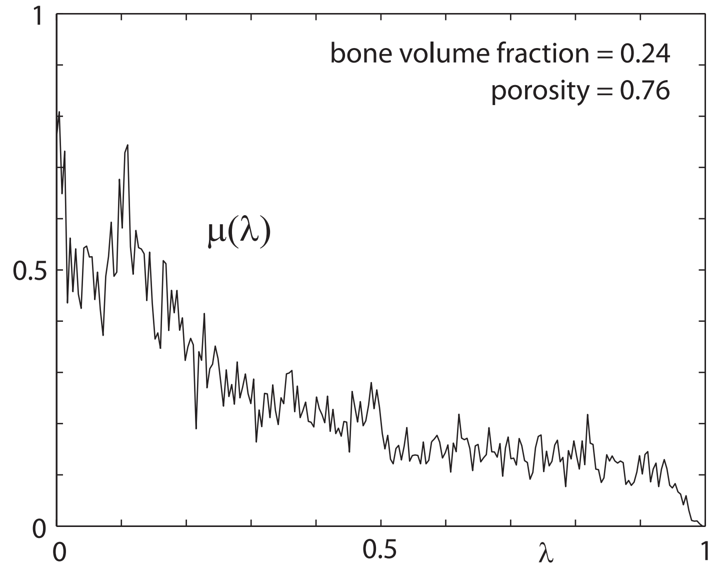


P. Hansma

(c) spectral measure - young

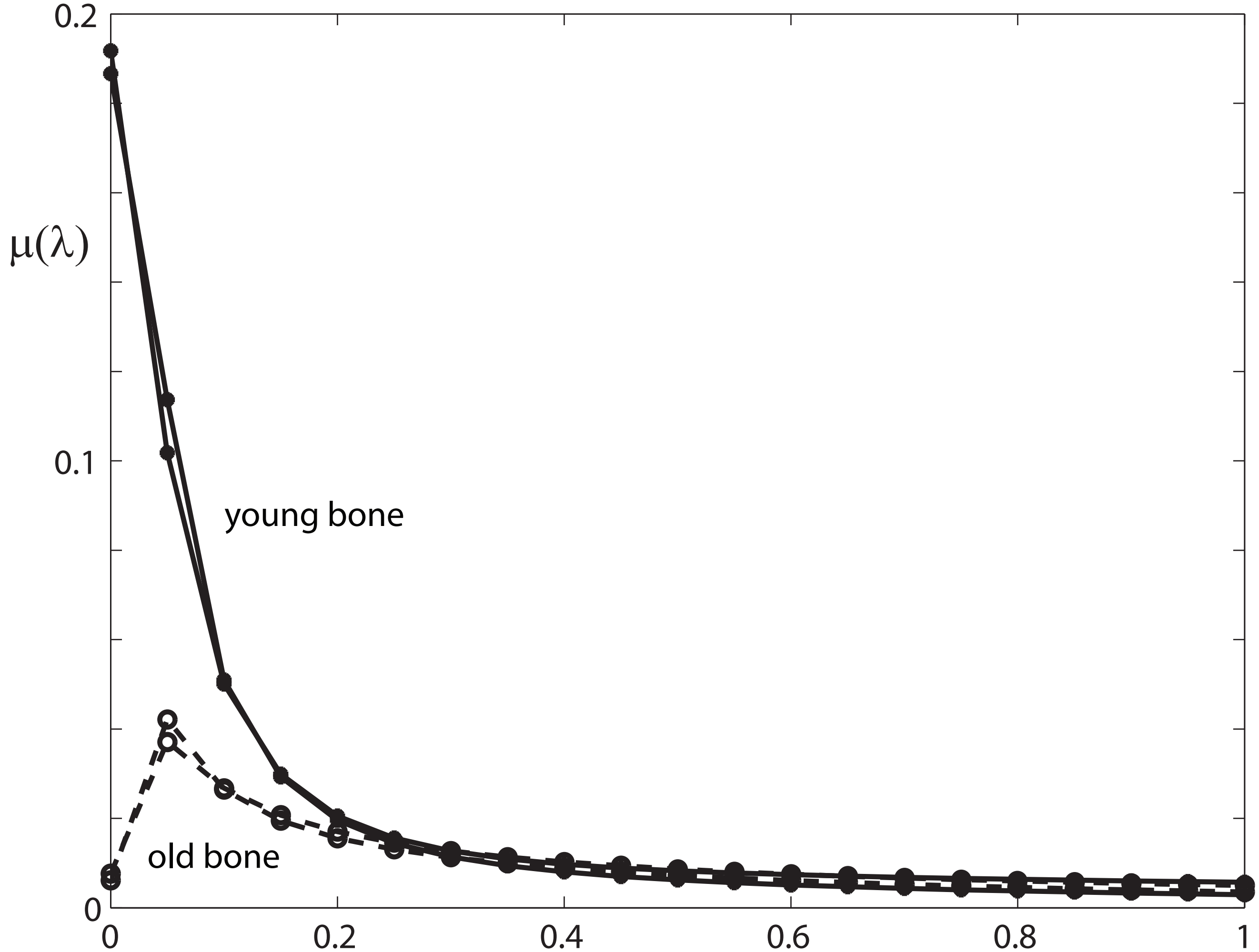


(d) spectral measure - old



the math doesn't care if it's sea ice or bone!

reconstruction of spectral measures from simulated complex permittivity data



regularized inversion scheme

Golden, Murphy, Cherkaev, J. Biomech. 2011

Random Matrix Theory Characterization of Phase Transitions

$$\chi_2 \Gamma \chi_2$$ } ← ***Real Symmetric Random Matrix***

Random Diagonal Projection Matrix → χ_2

Γ ← ***Non-Random*** Projection Matrix

- The elements of a random matrix are determined by a probability law.

- Wigner (1951) and Dyson (1953) first used random matrix theory (RMT) to describe quantized energy levels of heavy atomic nuclei.

- RMT has since been used to characterize: phase transitions in disordered mesoscopic conductors, quantum chaos, neural networks, random graphs, etc.

- ***In composites***, connectedness transitions can be characterized by transitions in the short and long range correlations of eigenvalues of the matrix $\chi_2 \Gamma \chi_2$.

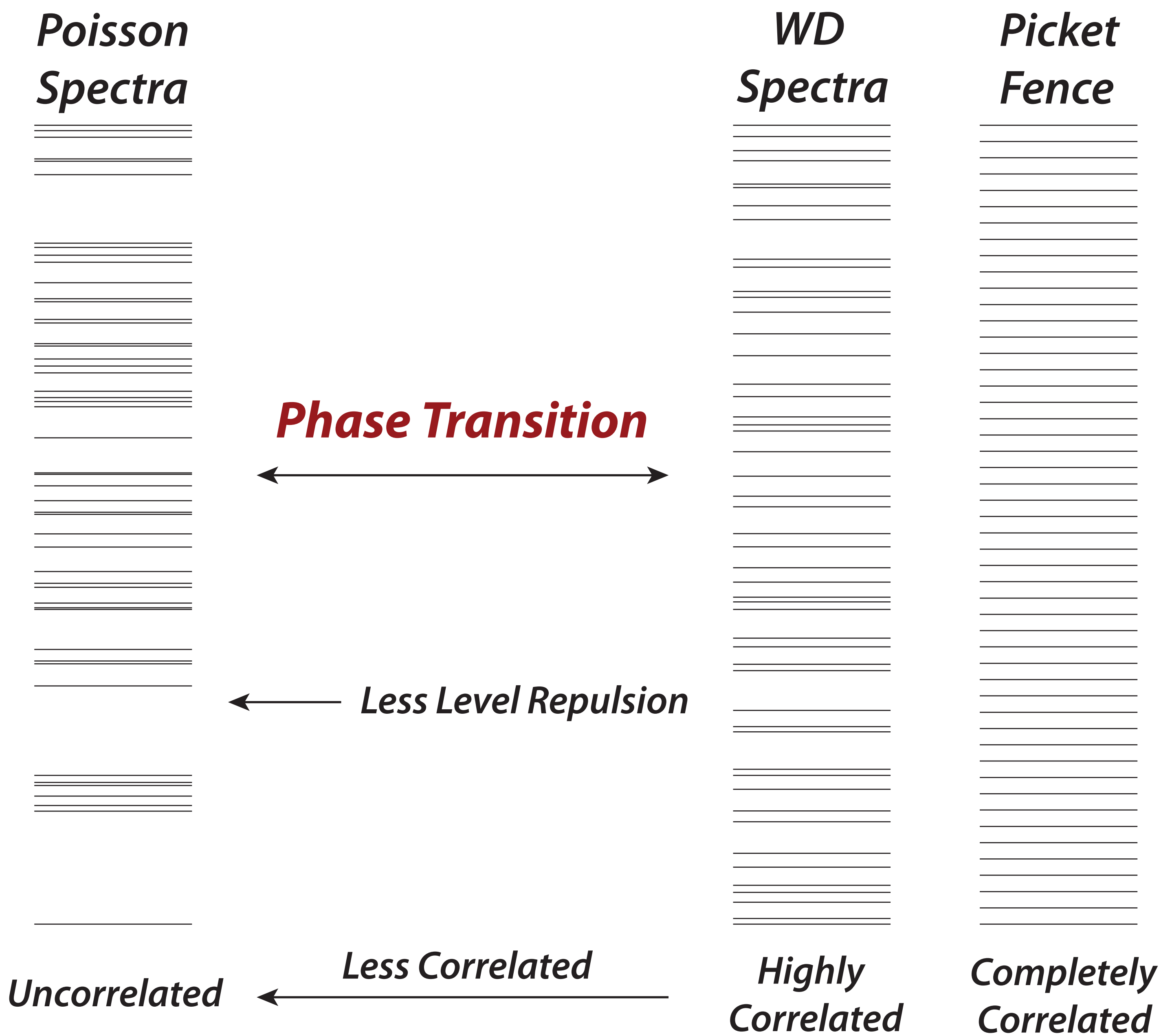
Transitions in Eigenvalue Correlations

$$P(z) = \exp(-z)$$

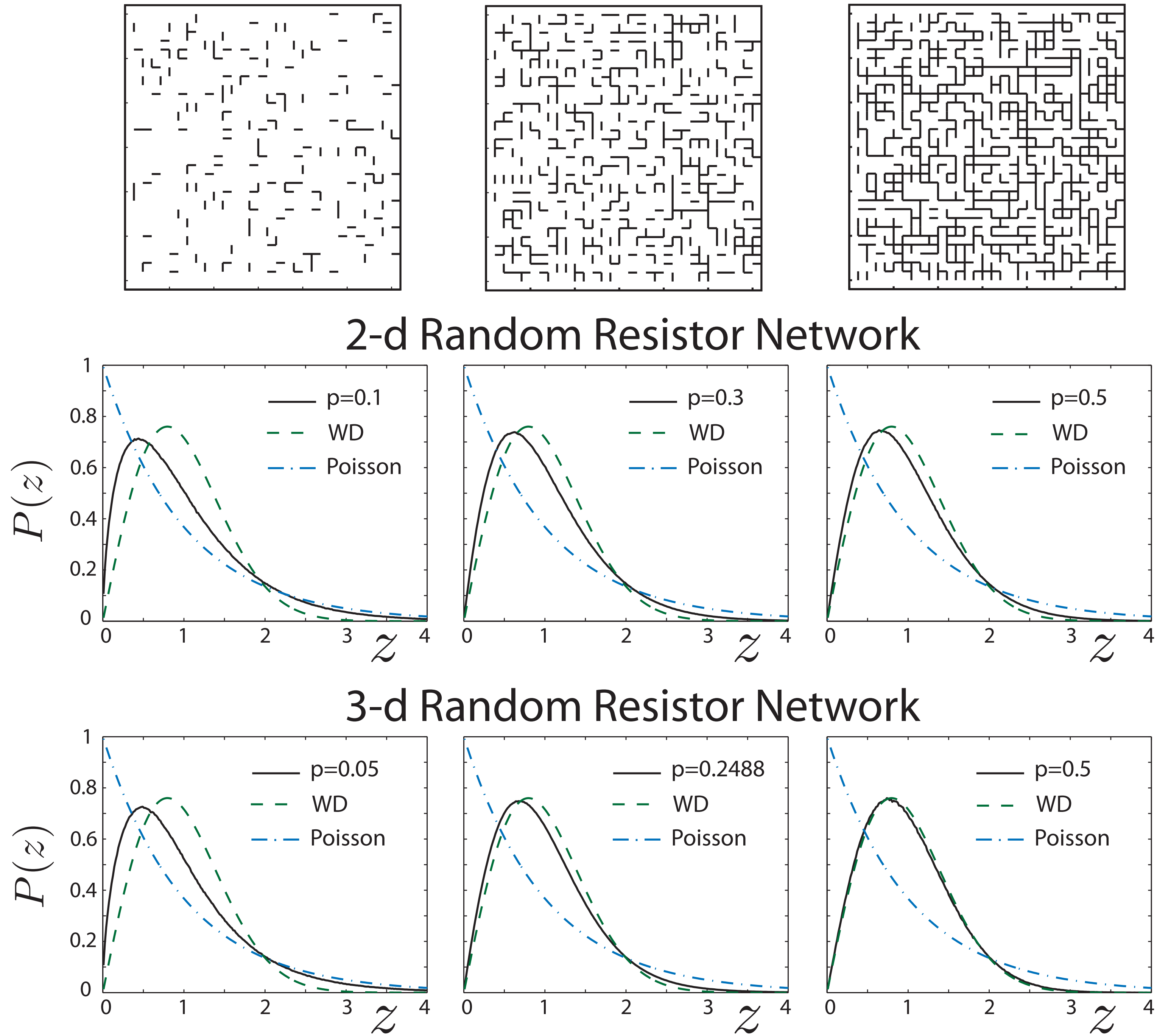
Eigenvalue Spacing Distribution

$$P(z) \approx \frac{\pi z}{2} \exp(-\pi z^2/4)$$

Eigenvalue Spacing Distribution



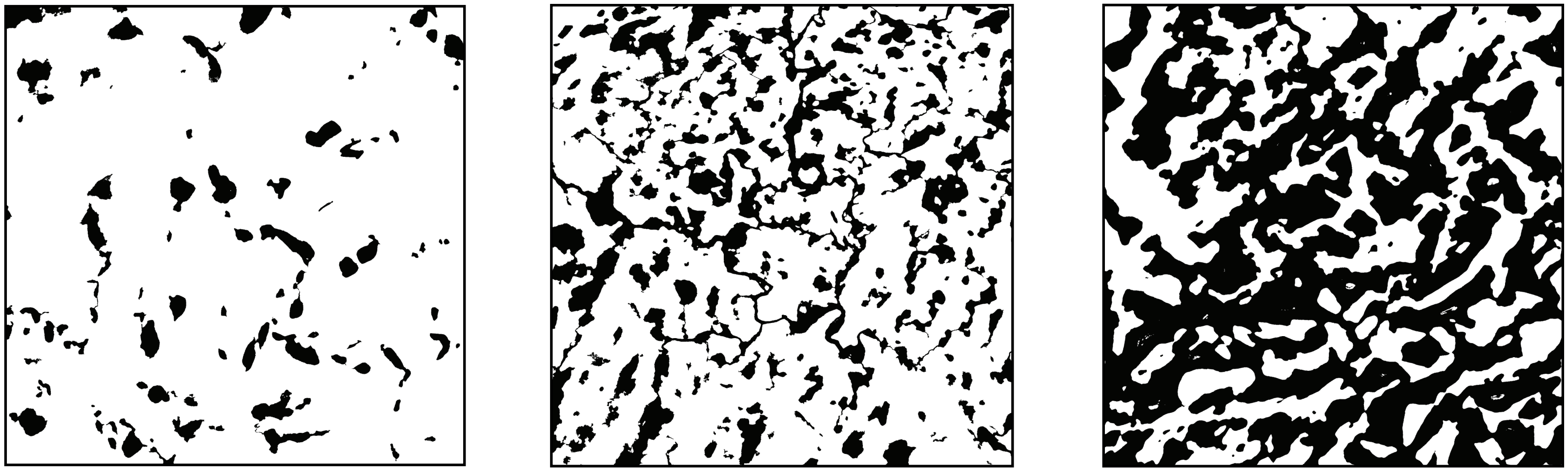
Unfolded Eigenvalue Spacing Distribution

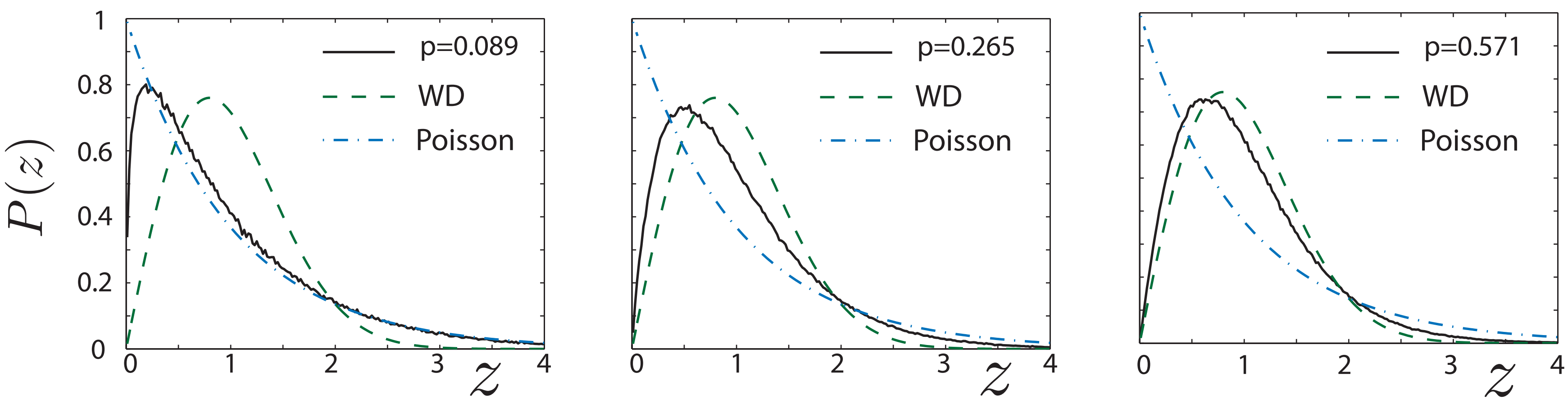


N. B. Murphy, K. M. Golden 2013

Unfolded Eigenvalue Spacing Distribution

ARCTIC MELT PONDS





eigenvalue statistics for transport exhibit UNIVERSALITY as the "conducting" phase becomes connected over large scales

N. B. Murphy, K. M. Golden 2013

advection enhanced diffusion

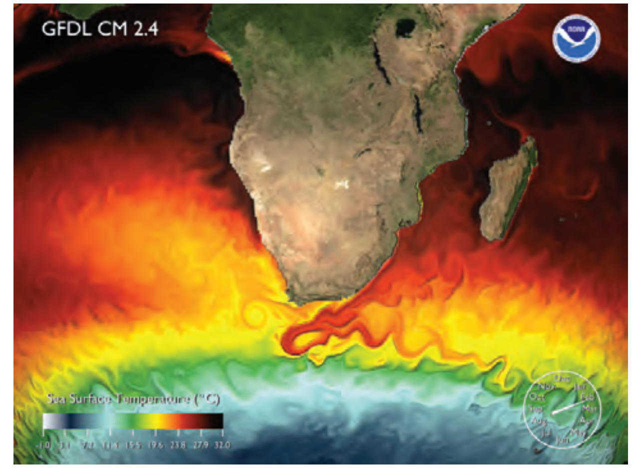
effective diffusivity

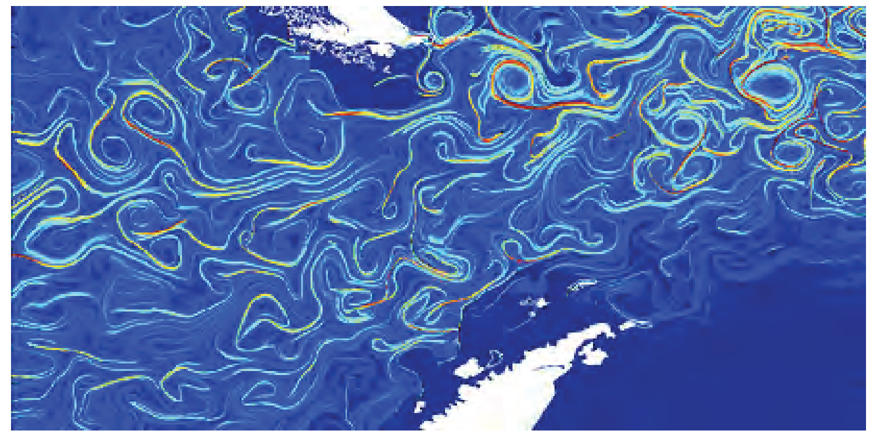
tracers, buoys diffusing in ocean eddies

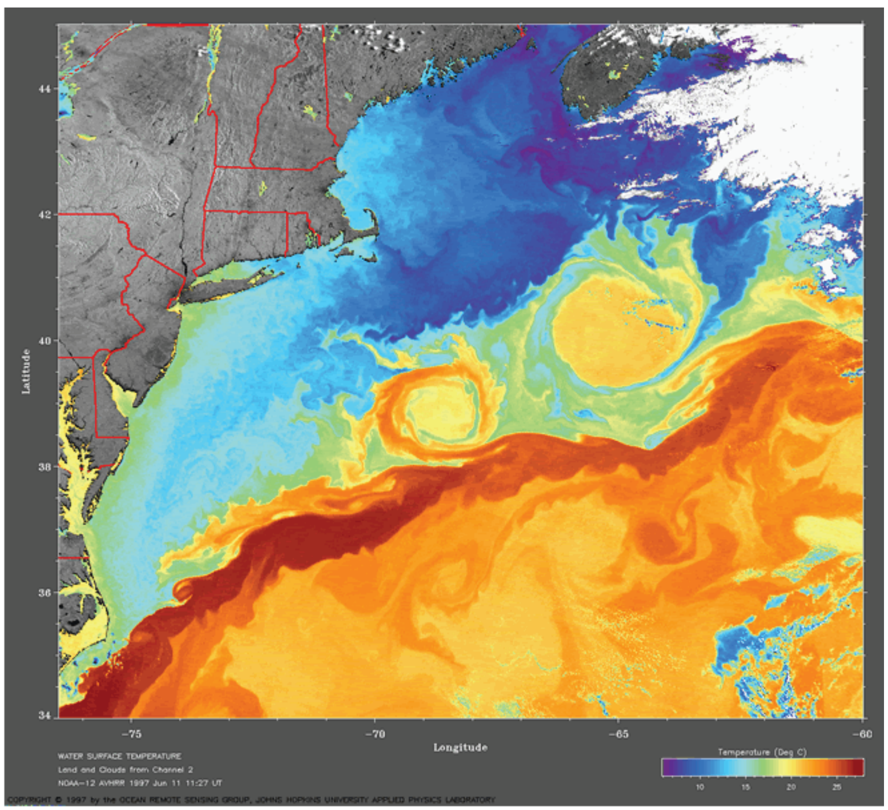
pollutants

enhanced heat and salt transport

enhanced sea ice thermal conductivity







advection diffusion equation with a velocity field $\vec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

homogenize →

$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

composites

$$\frac{\epsilon^*}{\epsilon_2} = 1 - \int_0^1 \frac{d\mu(\lambda)}{s - \lambda}$$

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

μ spectral measure of $\chi\Gamma\chi$

advection diffusion

$$\frac{\kappa^*}{\kappa_0} = 1 + \xi^2 \int_0^\infty \frac{d\phi(\tau)}{1 + \xi^2 \tau^2}$$

ξ = Péclet number

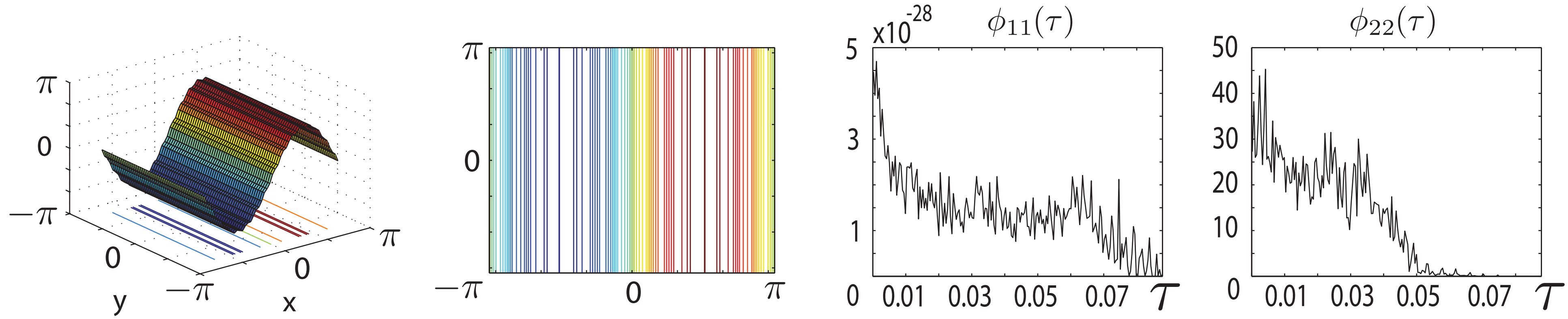
ϕ spectral measure of $\mathrm{i}\Gamma\mathbf{H}\Gamma$

$\vec{u} = \kappa_0 \, \xi \, \vec{\nabla} \cdot \mathbf{H}, \quad \mathbf{H}$ antisymmetric vector potential

spectral measures for sample flows

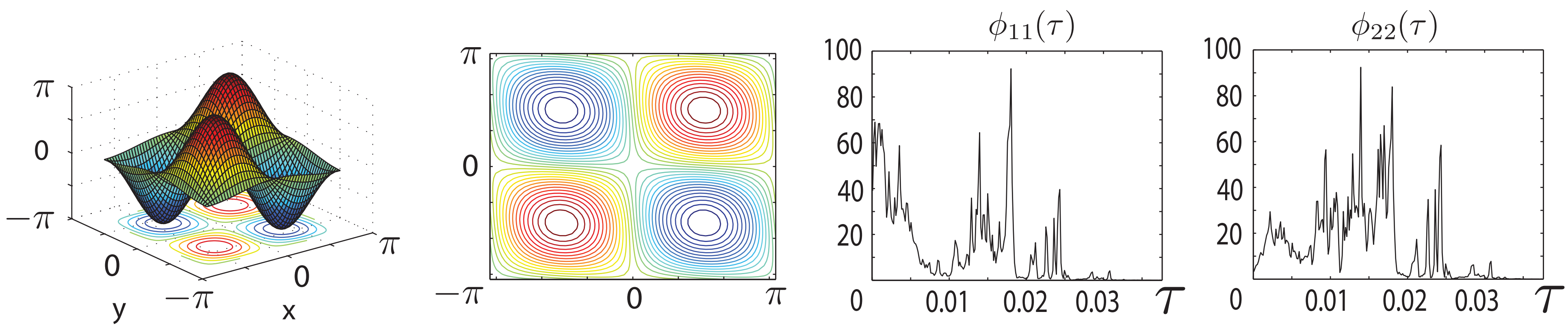
Shear Flow: $H(x,y) = \sin x + (0.5+\eta)\sin(15x)/15, \quad \eta \sim Uniform(-0.1, 0.1)$

stream function **streamlines** **spectral functions**



Modified Cat's Eye Flow: $H(x,y) = \sin x \sin y + \eta \cos x \cos y, \quad \eta \sim U(-0.1, 0.1)$

stream function **streamlines** **spectral functions**

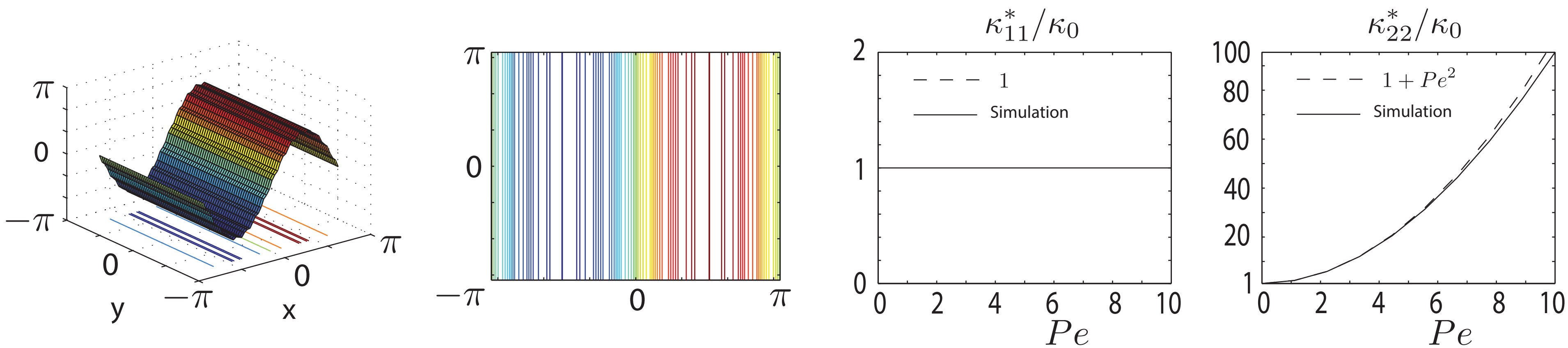


Murphy, Zhu, Golden, 2013

effective diffusivities for sample flows

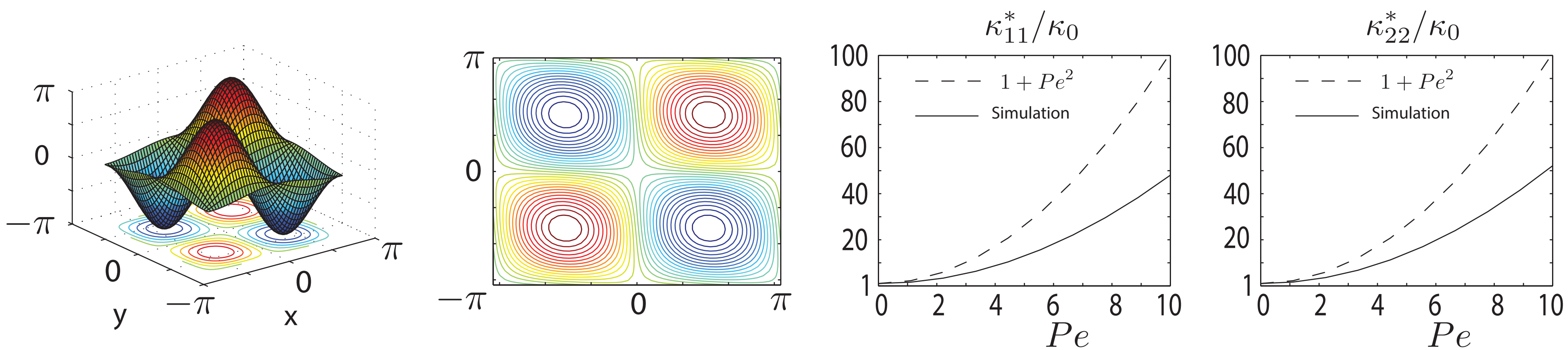
Shear Flow: $H(x,y) = \sin x + (0.5+\eta)\sin(15x)/15, \quad \eta \sim Uniform(-0.1, 0.1)$

stream function **streamlines** **effective diffusivities**



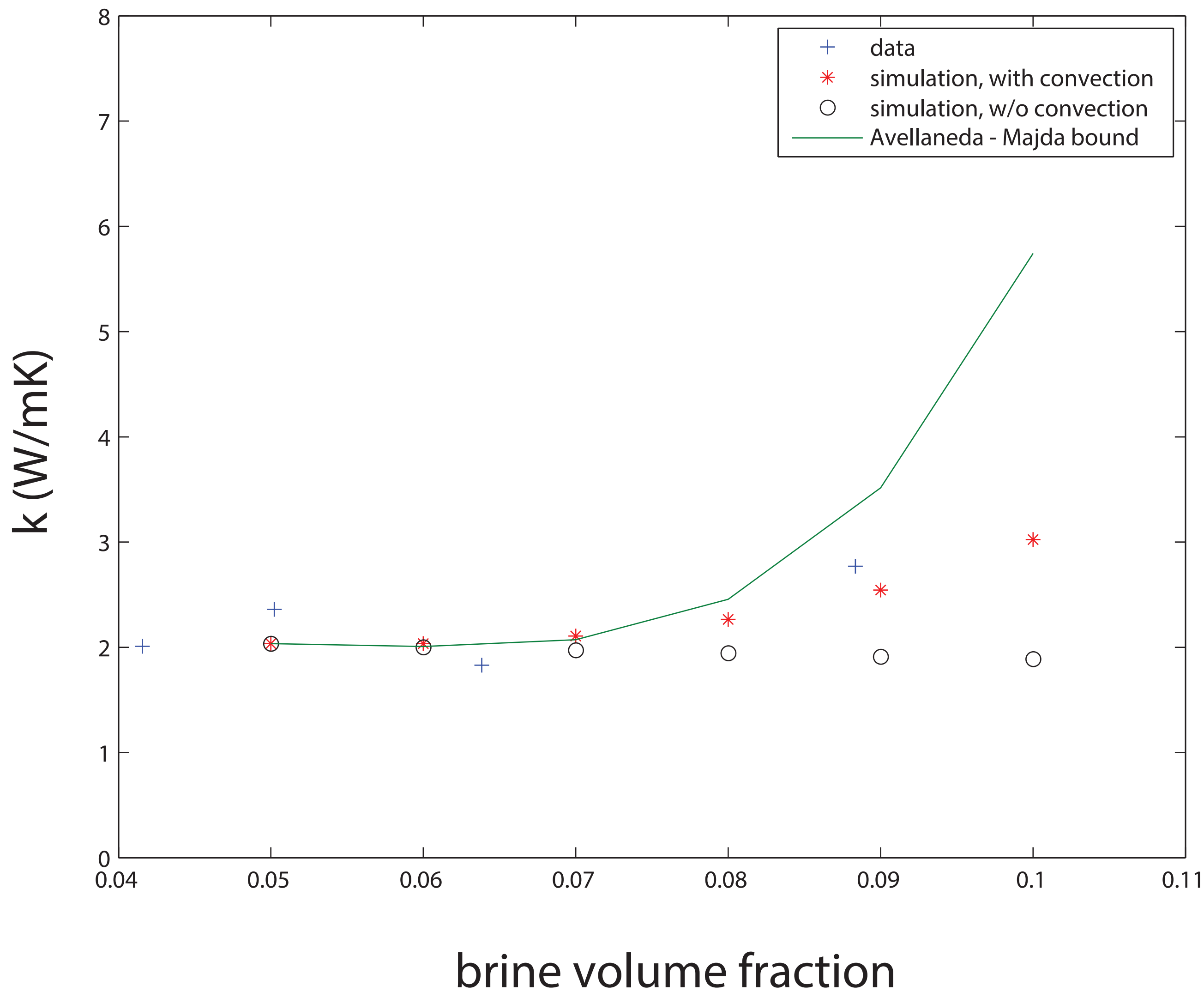
Modified Cat's Eye Flow: $H(x,y) = \sin x \sin y + \eta \cos x \cos y, \quad \eta \sim Uniform(-0.1, 0.1)$

stream function **streamlines** **effective diffusivities**



Murphy, Zhu, Golden, 2013

convection enhanced thermal conductivity of sea ice for shear flow



Wang, Zhu, Golden 2013

Conclusions

1. Sea ice exhibits composite structure on many length scales.

2. Fluid flow through sea ice governs many processes of importance to understanding climate change and the response of polar ecosystems.

3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes.

4. Homogenization theory and upscaling methods can provide a rigorous path to representing large scale effective behavior in coarse models.

5. Random matrix theory can help characterize transitions important for climate science and composite materials.

THANK YOU

National Science Foundation

Division of Mathematical Sciences

Arctic Natural Sciences

Office of Polar Programs

CMG Program

(Collaboration in Mathematical Geosciences)

Office of Naval Research

Applied Computational Analysis Program

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Buchanan Bay, Antarctica   Mertz Glacier Polynya Experiment   July 1999