

A CELEBRATION OF THE CONTRIBUTIONS OF THE  
STATISTICAL SCIENCES TO SOCIETY

<http://statistics2013.org>





## **Recipe for a Bernoulli Sponsored\* 2013 Regional Public Event**

1. Form a small local committee from universities in your region.
2. Develop a small budget for promotion of a 2013 public event to include local professionals, students, teachers and legislators.
3. Secure official sponsorship of the Bernoulli Society at <http://www.bs2013.org/>, and join the list of institutions on the international web page <http://www.statistics2013.org/>
4. Request funding from local participating universities.
5. Select a speaker, location and time for your 2013 event.
6. Invite local news media.
7. Don't miss this historic\* opportunity to communicate the importance of education and research in probability and statistics to the world.

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\* 300<sup>th</sup> Anniversary of the publication of Jacob Bernoulli's Ars Conjectandi and 250<sup>th</sup> Anniversary of the publication of Bayes' theorem.

# Mathematics and the Melting Polar Ice Caps

Kenneth M. Golden  
Department of Mathematics  
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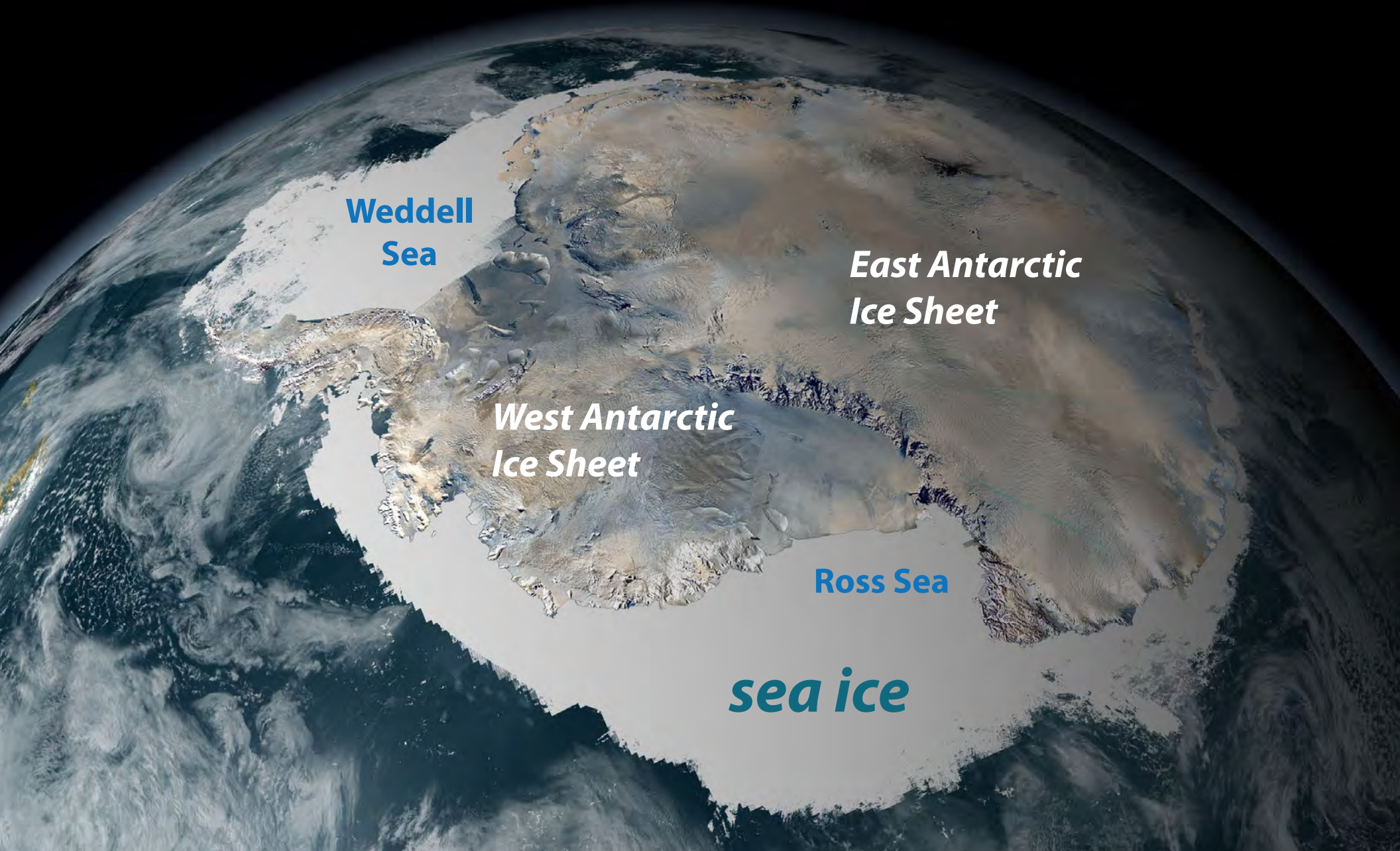
*Alison Kohout  
September 2012*

***36th Conference on Stochastic Processes and Their Applications  
Bernoulli Society Open Lecture    UC Boulder    August 1, 2013***



# ***ANTARCTICA***

southern cryosphere



**Weddell  
Sea**

***East Antarctic  
Ice Sheet***

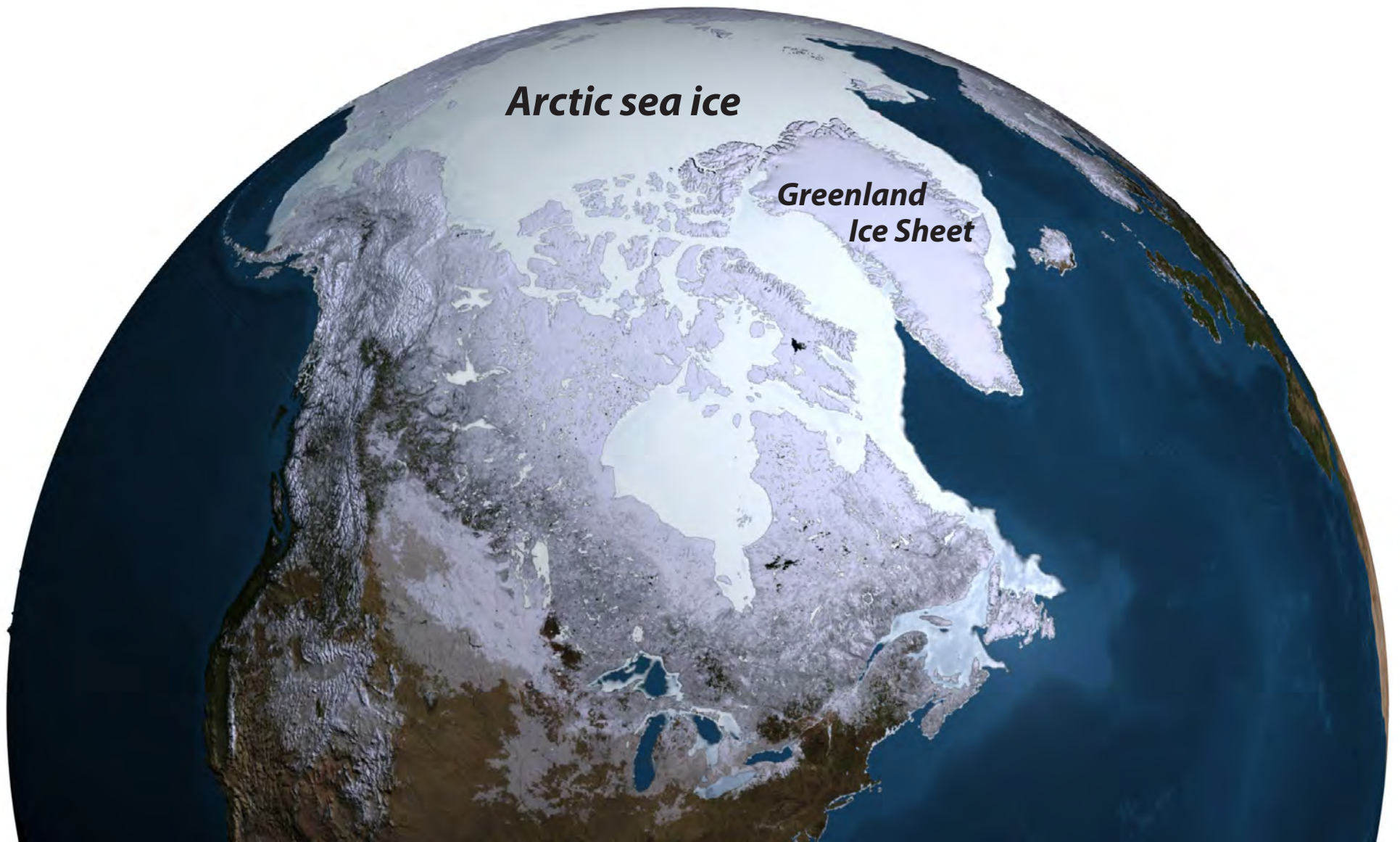
***West Antarctic  
Ice Sheet***

**Ross Sea**

***sea ice***



# **northern cryosphere**





# SEA ICE covers 7 - 10% of earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- indicator and agent of **climate change**





# polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice  
reflect

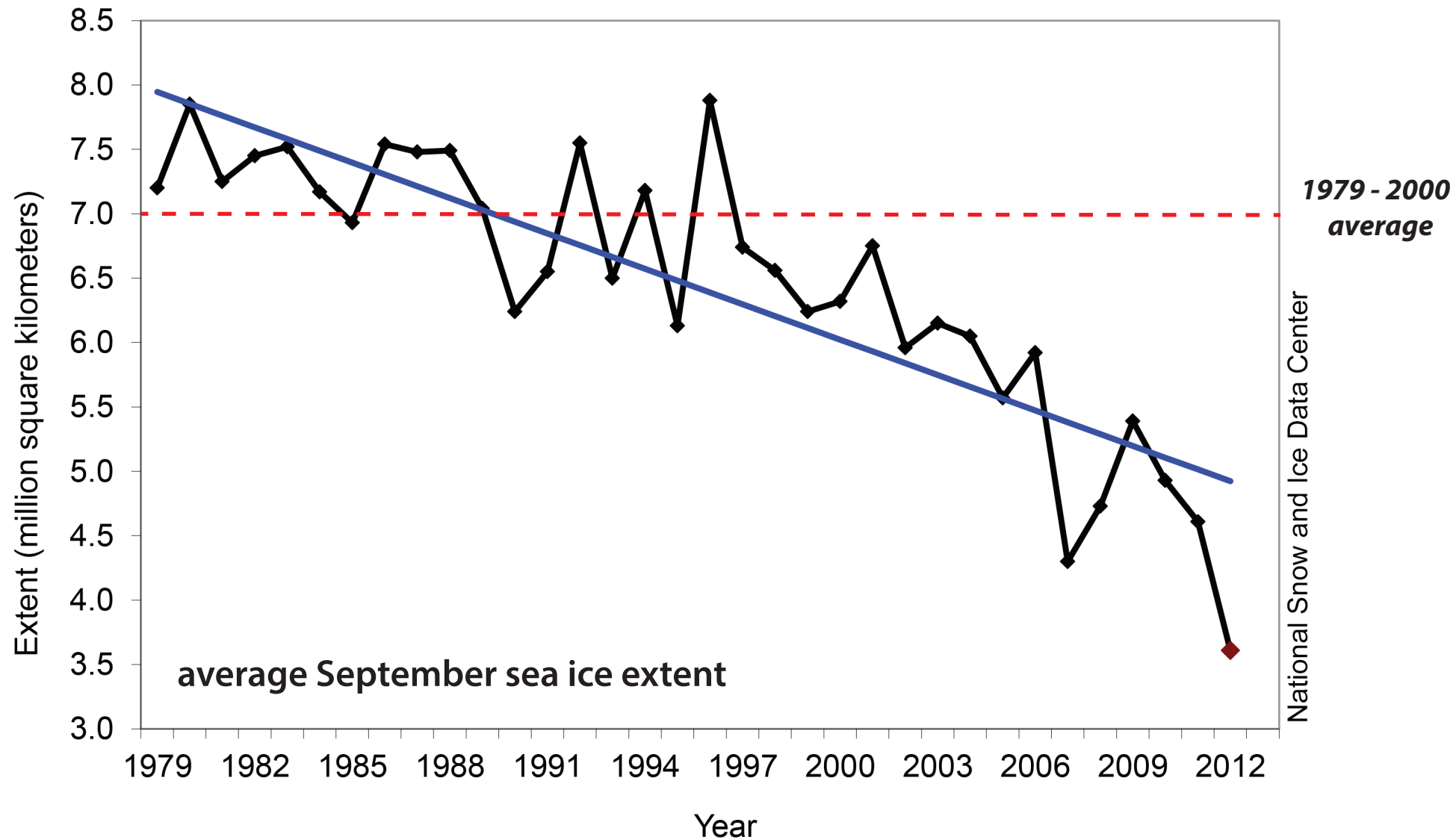


dark water and land  
absorb

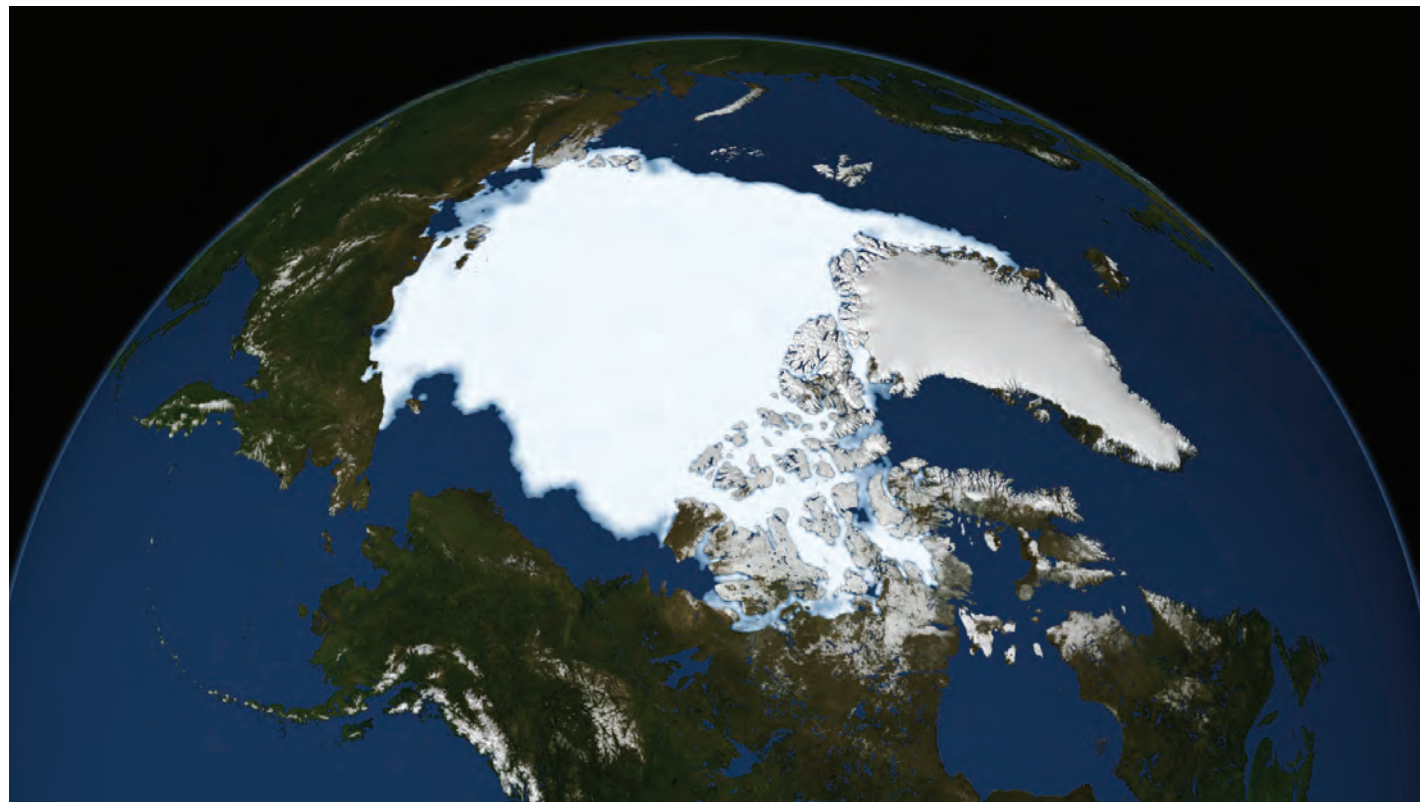
$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$



# *the summer Arctic sea ice pack is melting*

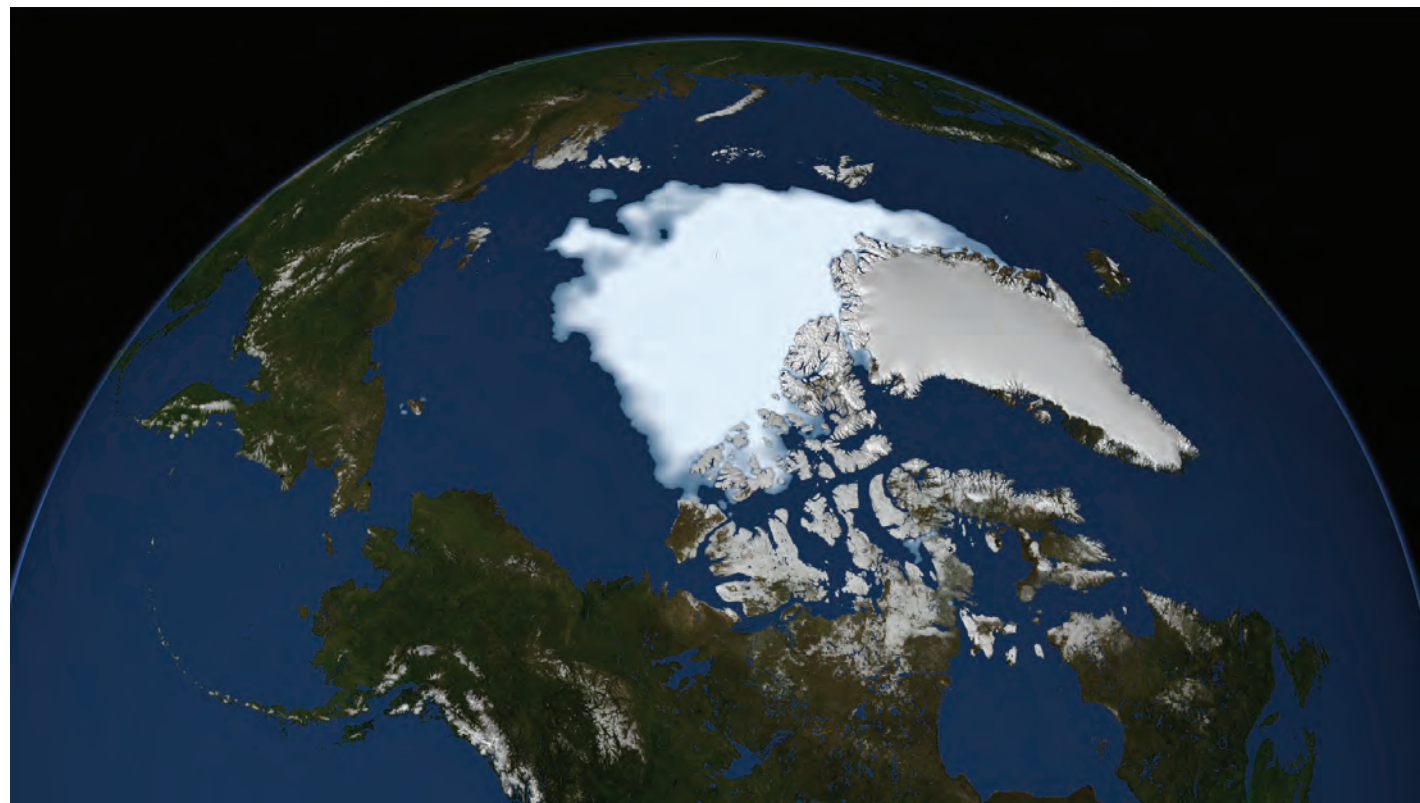


**21 September 1979**



**Polar Amplification**

**13 September 2012**





# Ice albedo feedback

*Melting*

+

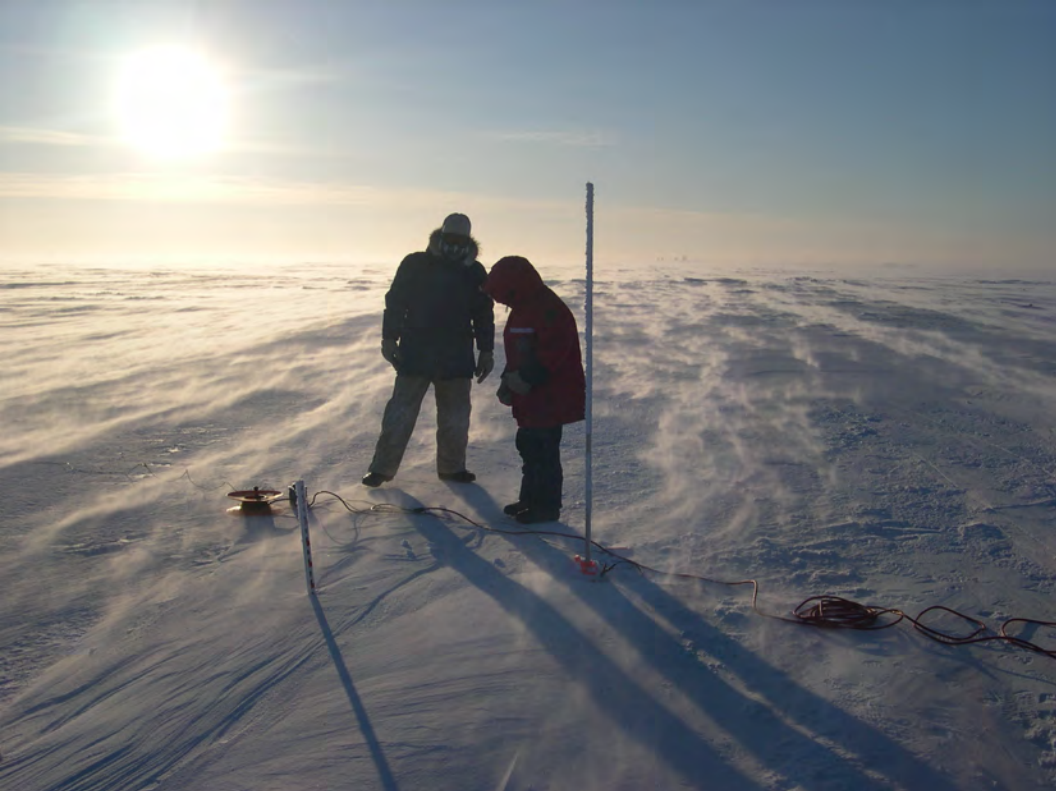
*Absorbed  
sunlight*



+

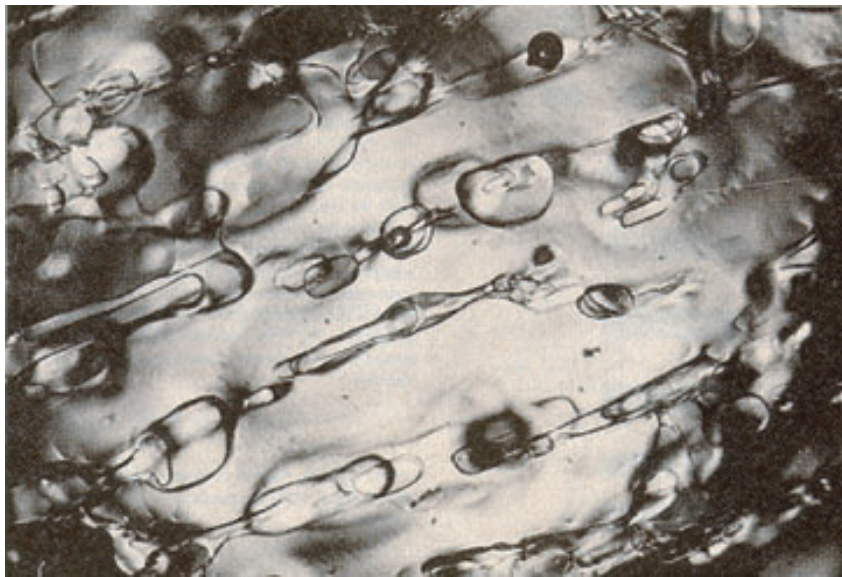
*Lower  
albedo*

+

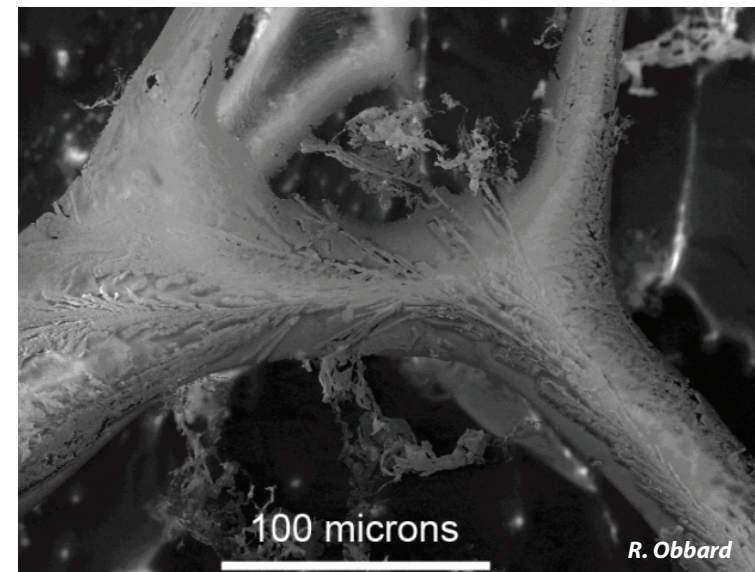


*sea ice may appear to be a  
barren, impermeable cap ...*





**brine inclusions in sea ice (mm)**



**micro - brine channel (SEM)**

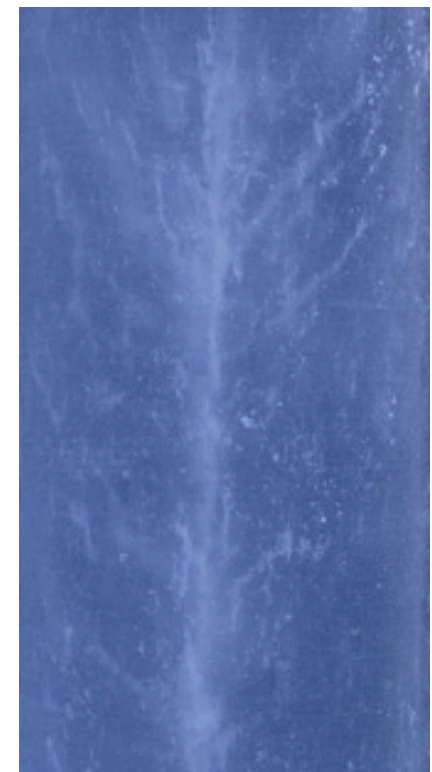
***sea ice is a  
porous composite***

pure ice with brine, air, and salt inclusions

**brine channels (cm)**



horizontal section



vertical section

# fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems:

*evolution of Arctic melt ponds and sea ice albedo*



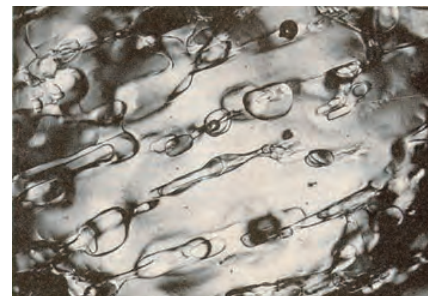
*nutrient flux for algal communities*



- *drainage of brine and melt water*
- *ocean-ice-air exchanges of heat, CO<sub>2</sub>*



linkage of scales





# ***What is this talk about?***

## **SPA in climate science**

(Stochastic Processes and Applications)

***Using the mathematics of composite materials and statistical physics to study sea ice structures and processes ... to improve projections of climate change.***

### ***1. Fluid flow through sea ice - percolation***

homogenization for composite materials

### ***2. Electromagnetic monitoring of sea ice***

homogenization for larger scale structures

### ***3. Arctic and Antarctic experiments***

### ***4. Fractal geometry of Arctic melt ponds***

***critical behavior***

***linkage of scales***

***cross-pollination***

**Develop rigorous representations of sea ice in climate models.**

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Tony Worby, Klaus Meiners

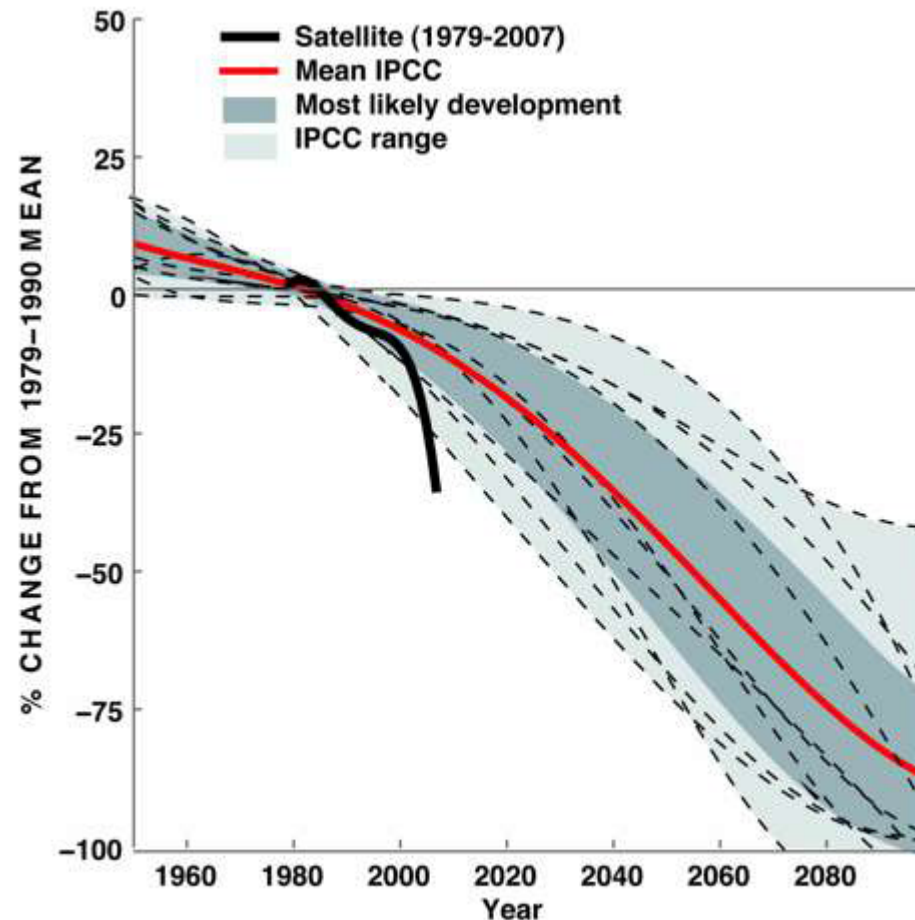
## CRREL

Don Perovich, Chris Polashenski



# Intergovernmental Panel on Climate Change (IPCC) 2007 projections

*observed decline in summer Arctic sea ice  
outpacing global climate models*



Arctic sea ice loss compared to IPCC models

# Global Climate Models

Climate models are systems of partial differential equations (PDE) derived from the basic laws of physics, chemistry, and fluid motion.

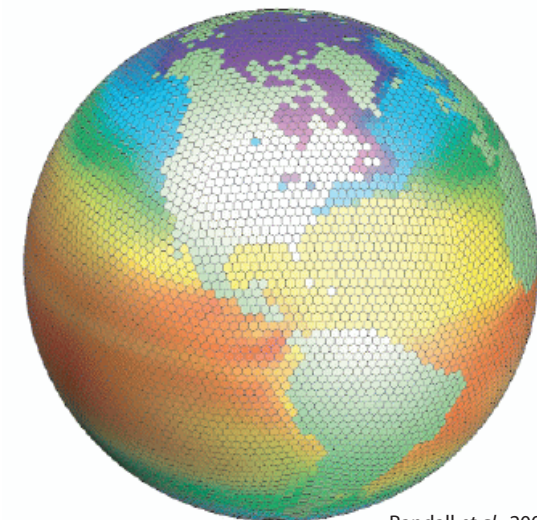
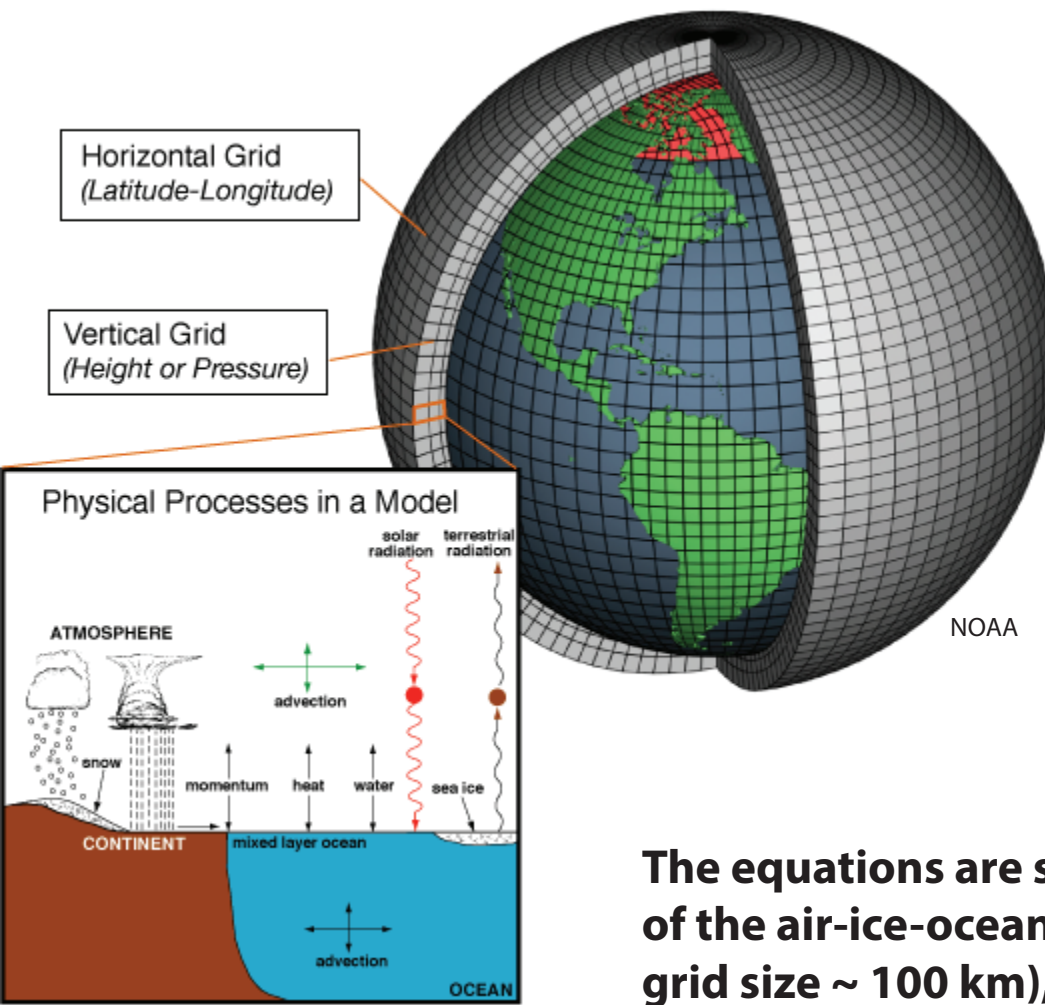
They describe the state of the ocean, ice, atmosphere, land, and their interactions.

The equations are solved on 3-dimensional grids of the air-ice-ocean-land system (with horizontal grid size ~ 100 km), using very powerful computers.

*sub - grid scale processes*

## *sea ice components of GCM:*

- *ice thickness distribution*
- *balance of forces ( $F=ma$ )*
- *heat and radiation fluxes*







**Who cares if  
Arctic sea ice  
disappears?**



**Ralph (Malik) Ahkivgak, c. 20 Oct 1988**

© Bill Hess – Running Dog Publications; <http://wasillaalaskaby300.squarespace.com/>



Drew Barrymore

John Krasinski



**INSPIRED BY THE INCREDIBLE TRUE STORY**  
that united the world

# BIG MIRACLE

[seaice.alaska.edu/gi](http://seaice.alaska.edu/gi)

- The Arctic holds 25% of the world's undiscovered oil & gas reserves
- Sea ice is both a hazard and a supporting feature for hydrocarbon exploration & production

## *oil companies care about Arctic sea ice loss*



Source: UNEP/GRID-Arendal



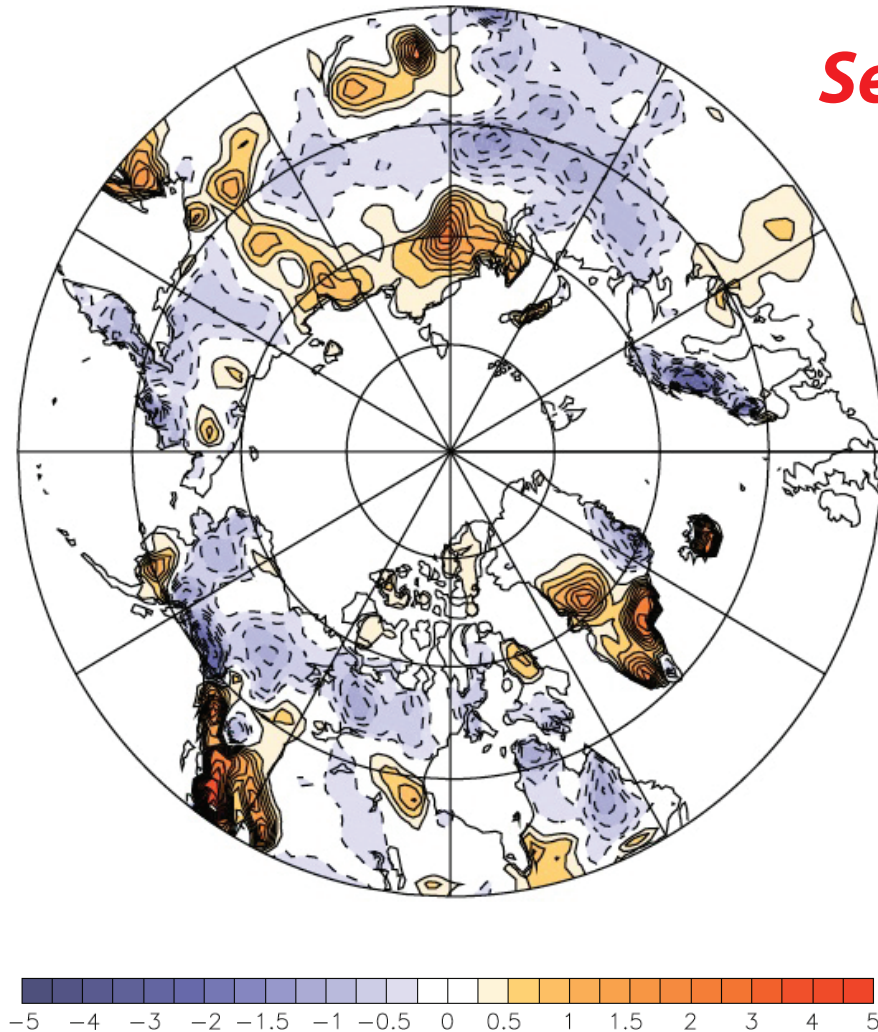
## ***Sea-ice loss: impacts beyond the Arctic***

***changes in precipitation and temperature patterns, storm tracks, ...***

- One climate model projects reduced precipitation in American West (Sewall & Sloan, 2005)

***Utah - greatest snow on Earth?***

- Analysis of 2007 ice minimum suggests above normal snow deposition in NW North America (Orsolini et al., 2011)
- Colder weather in SE Asia, possibly in Eastern US (Hondo et al., 2009)



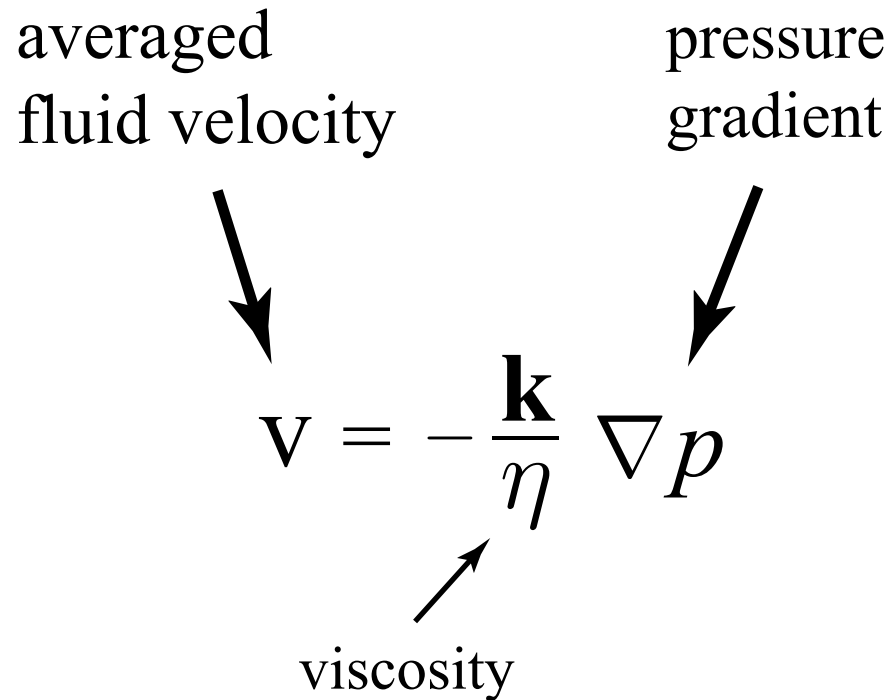
*Orsolini et al., 2011*

***sea ice microphysics***

***fluid transport***



# *Darcy's Law* for slow viscous flow in a porous medium



The diagram shows the equation  $\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$  with three labels and arrows pointing to its components: 'averaged fluid velocity' points to  $\mathbf{v}$ , 'pressure gradient' points to  $\nabla p$ , and 'viscosity' points to  $\eta$ .

averaged  
fluid velocity

pressure  
gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

viscosity

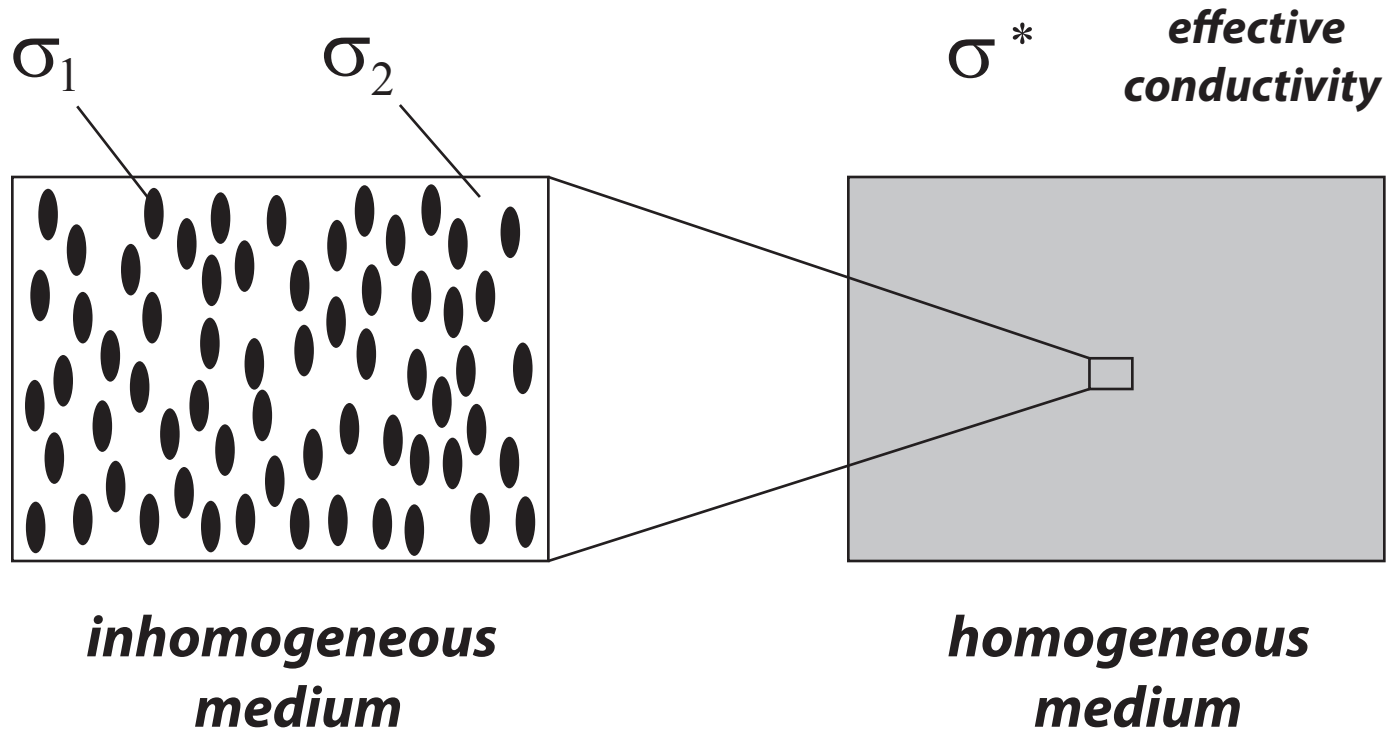
$\mathbf{k}$  = fluid permeability tensor

example of *homogenization*

**mathematics for analyzing effective behavior of heterogeneous systems**

e.g. transport properties of composites - electrical conductivity, thermal conductivity, etc.

# HOMOGENIZATION



**find the homogeneous medium which  
behaves macroscopically the same as  
the inhomogeneous medium**

*Maxwell 1873 : effective conductivity of a dilute suspension of spheres*

*Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid*

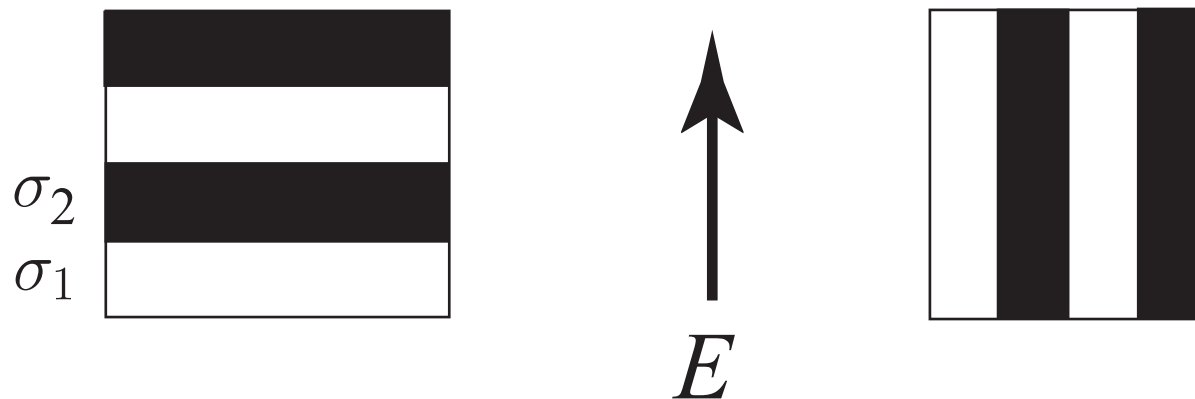


# arithmetic and harmonic mean bounds on transport properties

effective electrical conductivity  $\sigma^*$  for two phase composite of  $\sigma_1$  and  $\sigma_2$

*optimal bounds* on  $\sigma^*$  for known volume fractions  $p_1$  and  $p_2$ :

$$\frac{1}{\frac{p_1}{\sigma_1} + \frac{p_2}{\sigma_2}} \leq \sigma^* \leq p_1\sigma_1 + p_2\sigma_2$$

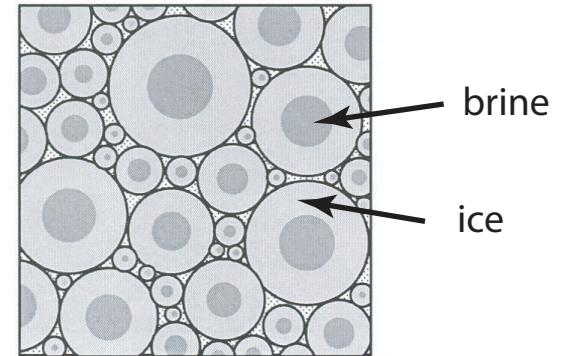
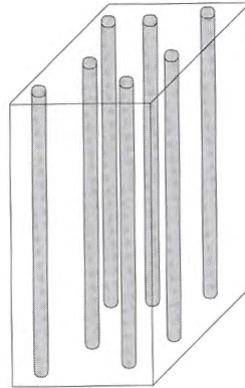


applied electric field

Wiener, 1912

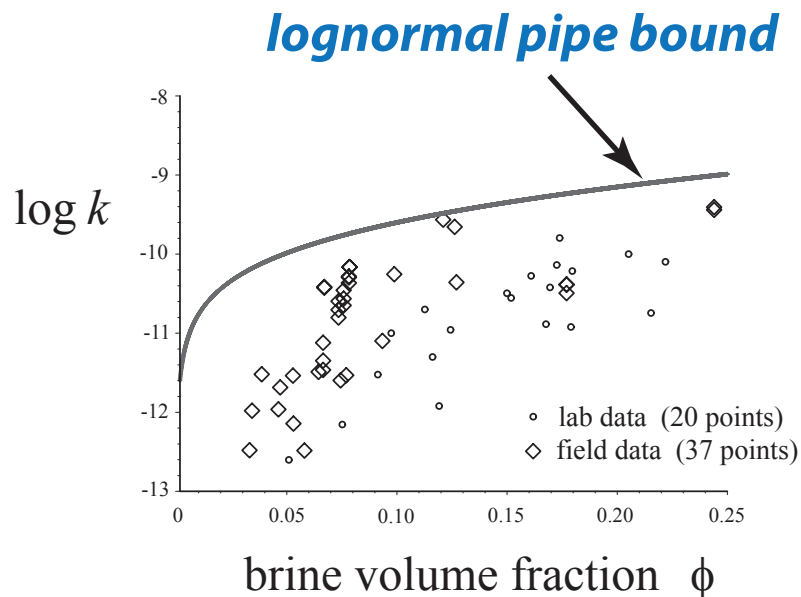
# PIPE BOUNDS on vertical fluid permeability $k$

vertical pipes  
with appropriate radii  
maximize  $k$



optimal coated  
cylinder geometry

**fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)**



inclusion cross sectional areas  $A$   
lognormally distributed

$$k \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

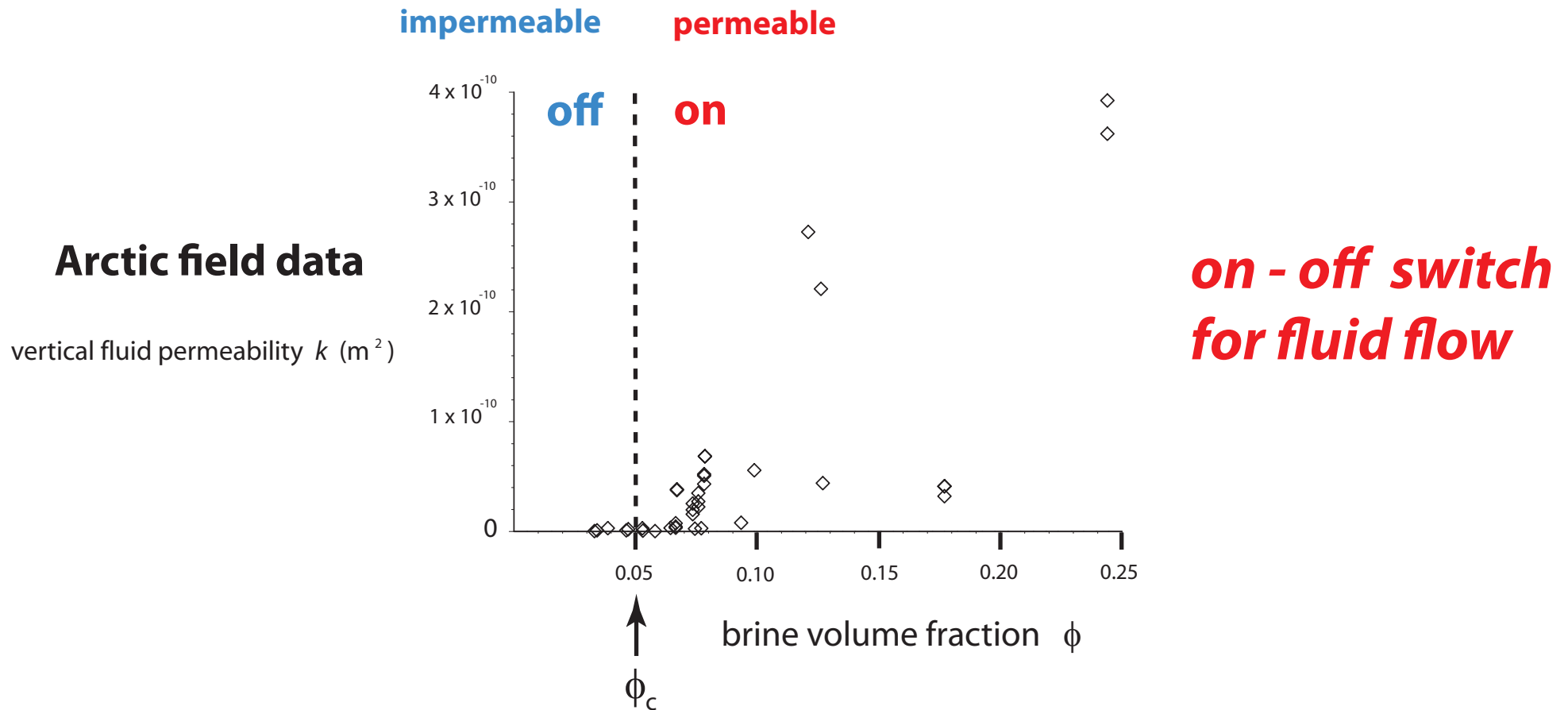
get bounds through variational analysis of  
**trapping constant**  $\gamma$  for diffusion process  
in pore space with absorbing BC

$$\mathbf{k} \leq \gamma^{-1} \mathbf{I} \quad \text{for any ergodic porous medium (Torquato 2002, 2004)}$$

**Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007**  
**Golden, Heaton, Eicken, Lytle, Mech. Materials 2006**



# Critical behavior of fluid transport in sea ice



critical brine volume fraction  $\phi_c \approx 5\%$   $\longleftrightarrow$   $T_c \approx -5^\circ \text{C}$ ,  $S \approx 5 \text{ ppt}$

## RULE OF FIVES

Golden, Ackley, Lytle *Science* 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

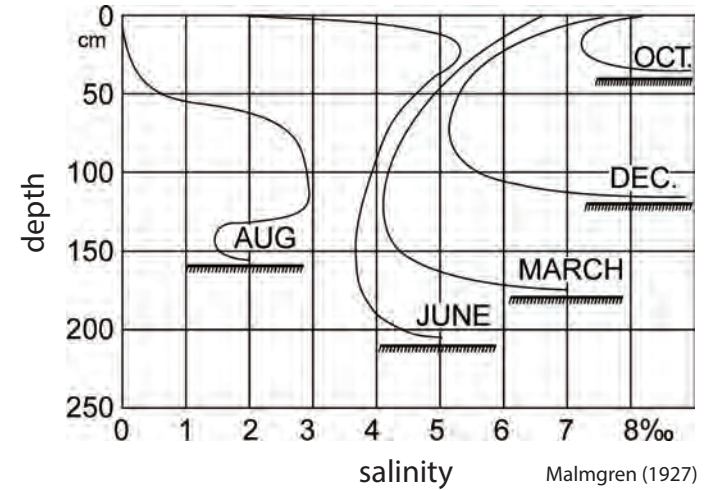
Pringle, Miner, Eicken, Golden *J. Geophys. Res.* 2009

## rule of fives constrains:

### Antarctic surface flooding and snow-ice formation



### evolution of salinity profiles



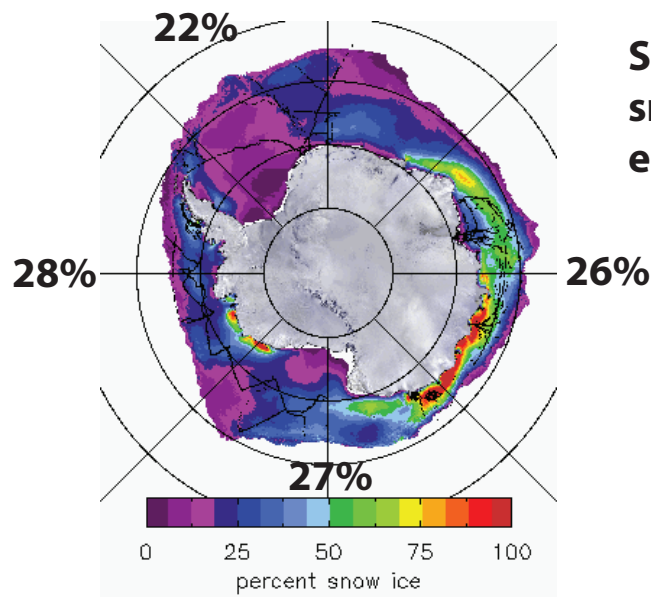
*currently assumed constant in climate models*

### convection - enhanced thermal conductivity

*Lytle and Ackley, 1996*

*Trodahl, et. al., 2000, 2001*

*Wang, Zhu, Golden, 2012*



September  
snow-ice  
estimates

Antarctic snow-to-ice conversion from passive microwave imagery

T. Maksym and T. Markus, 2008

### evolution of melt ponds and sea ice albedo







# sea ice algal communities

D. Thomas 2004

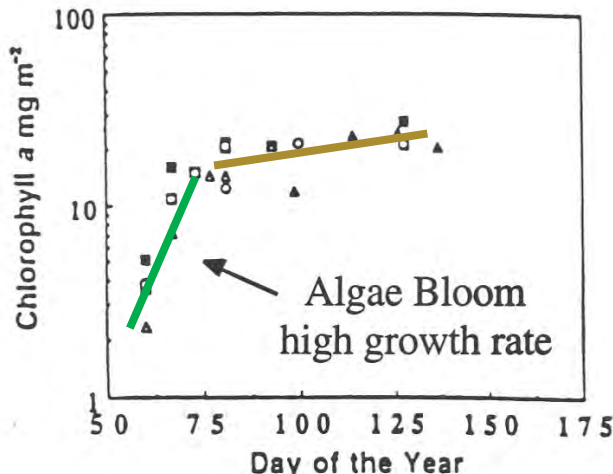
nutrient replenishment  
controlled by ice permeability

biological activity turns on  
or off according to  
*rule of fives*

Golden, Ackley, Lytle      Science 1998

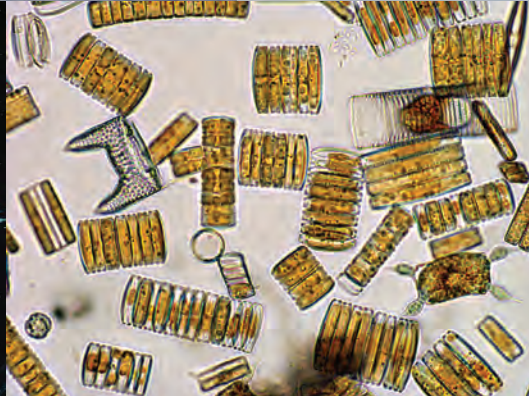
Fritsen, Lytle, Ackley, Sullivan      Science 1994

## critical behavior of microbial activity



Convection-fueled algae bloom  
Ice Station Weddell

# sea ice ecosystem



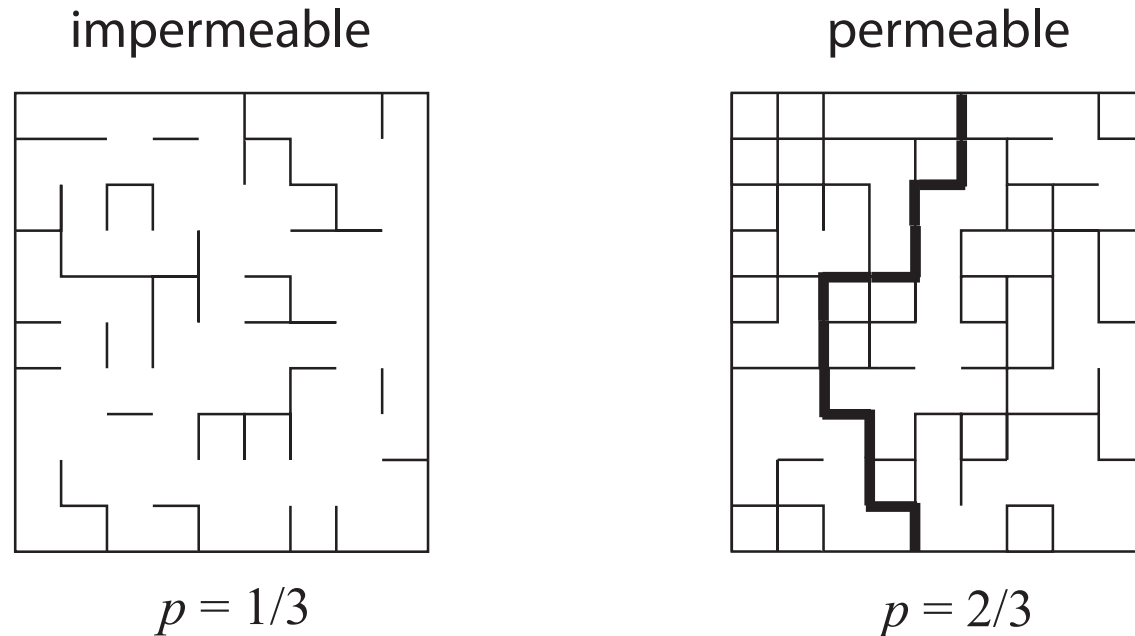
sea ice algae  
support life in the polar oceans



***Why is the rule of fives true?***

# percolation theory

*mathematical theory of connectedness*



bond  $\longrightarrow$  *open* with probability  $p$   
*closed* with probability  $1-p$

**percolation threshold**

$$p_c = 1/2 \quad \text{for } d = 2$$

*first appearance of infinite cluster*

*“tipping point” for connectivity*

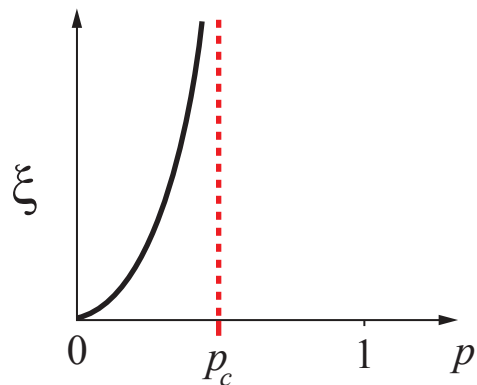


# order parameters in percolation theory

## geometry

correlation length

characteristic scale  
of connectedness

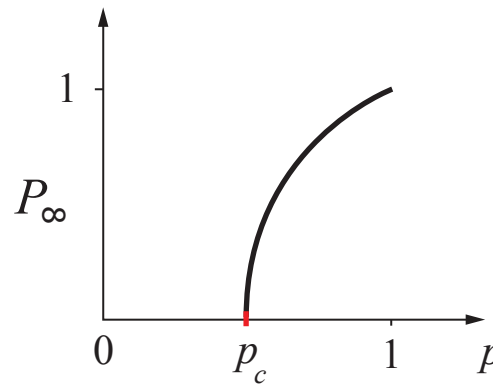


$$\xi(p) \sim |p - p_c|^{-\nu}$$

$$p \rightarrow p_c$$

infinite cluster density

probability the origin  
belongs to infinite cluster

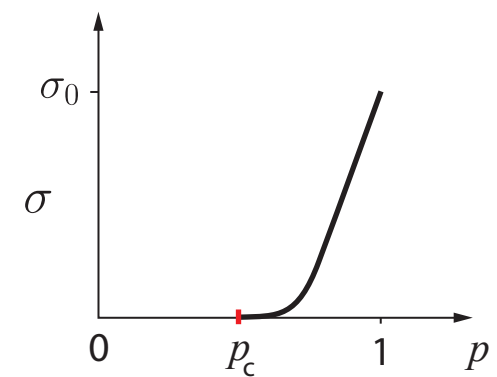


$$P_\infty(p) \sim (p - p_c)^\beta$$

$$p \rightarrow p_c^+$$

## transport

effective conductivity  
or fluid permeability



$$\sigma(p) \sim \sigma_0 (p - p_c)^t$$

$$p \rightarrow p_c^+$$

**UNIVERSAL critical exponents for lattices -- depend only on dimension**

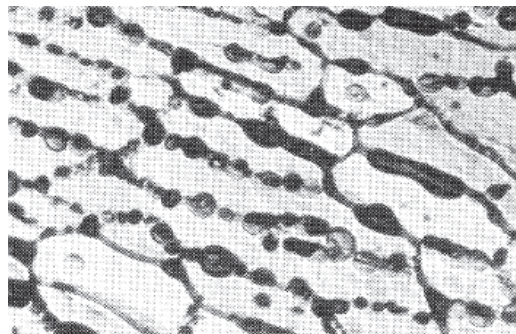
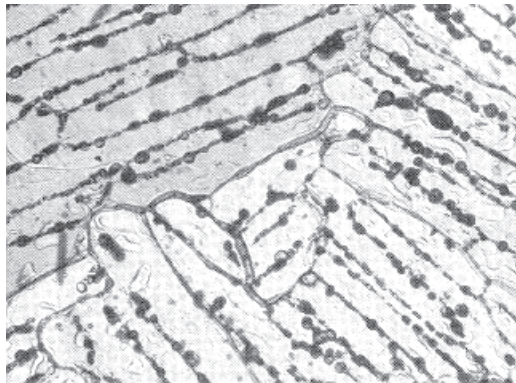
( $1 \leq t \leq 2$ , Golden, *Phys. Rev. Lett.* 1990 ; *Comm. Math. Phys.* 1992)

**non-universal behavior in continuum**

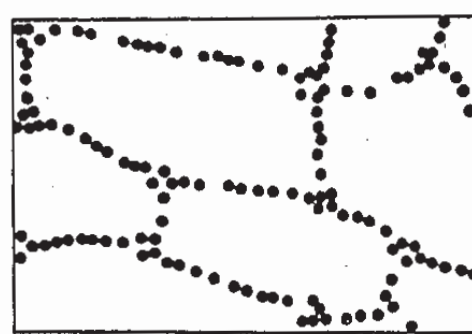
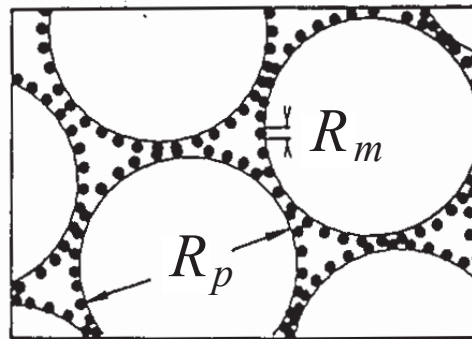
*Continuum* percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

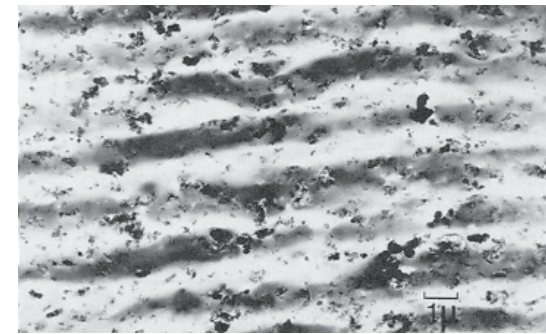
Golden, Ackley, Lytle, *Science*, 1998



sea ice

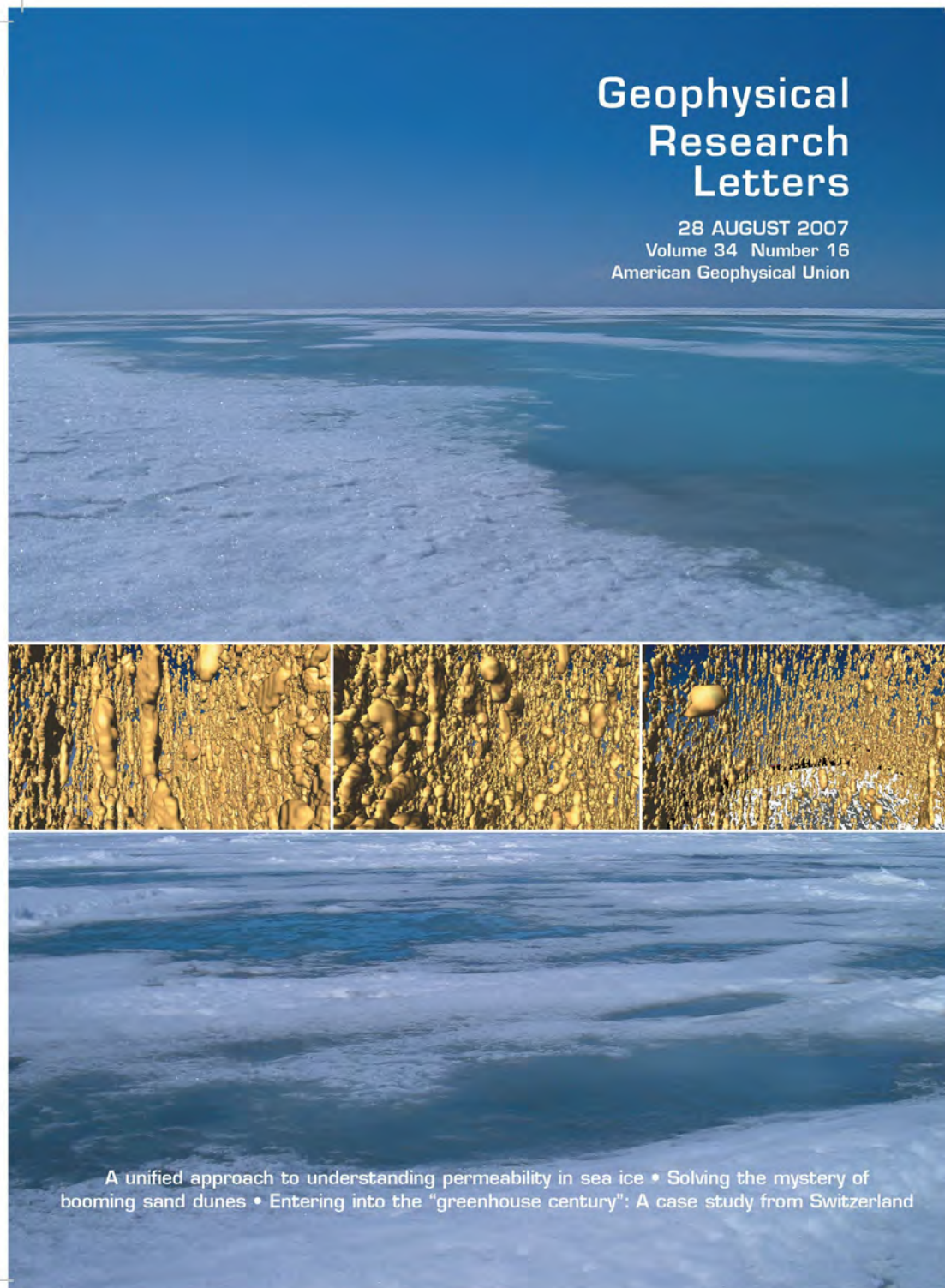


compressed  
powder



radar absorbing  
composite

**sea ice is radar absorbing**



***rigorous bounds  
percolation theory  
hierarchical model  
network model***

***field data***

X-ray tomography for  
brine inclusions

***unprecedented look  
at thermal evolution  
of brine phase and  
its connectivity***

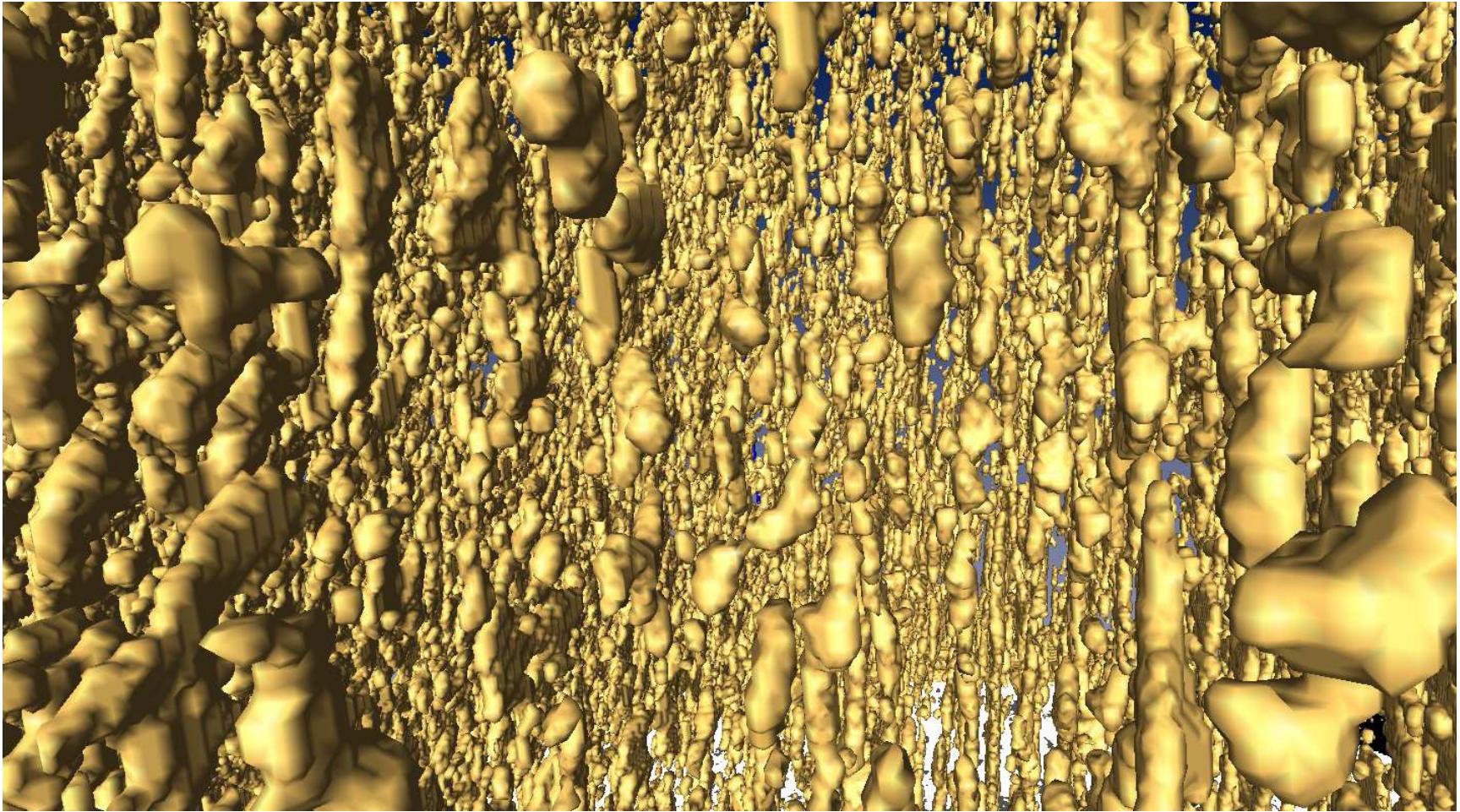
micro-scale  
controls  
macro-scale  
processes

A unified approach to understanding permeability in sea ice • Solving the mystery of  
booming sand dunes • Entering into the “greenhouse century”: A case study from Switzerland



# X-ray computed tomography of brine inclusions in sea ice

*~ 1 cm across*



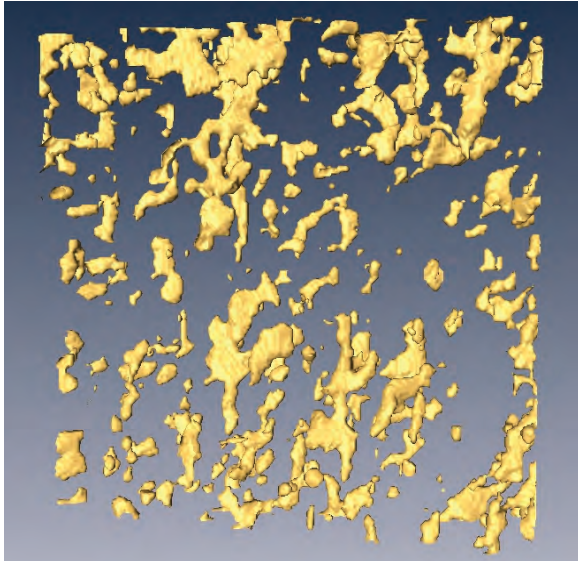
brine volume fraction  $\phi = 5.7 \%$        $T = -8^{\circ}\text{C}$

Golden, Eicken, Heaton, Miner, Pringle, Zhu, *Geophys. Res. Lett.* 2007

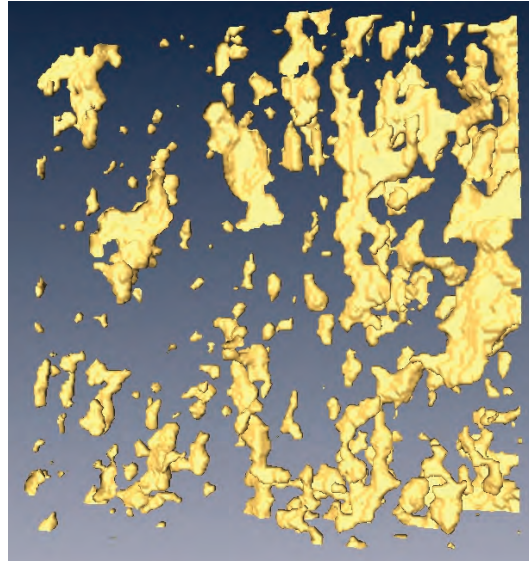


# brine connectivity (over cm scale)

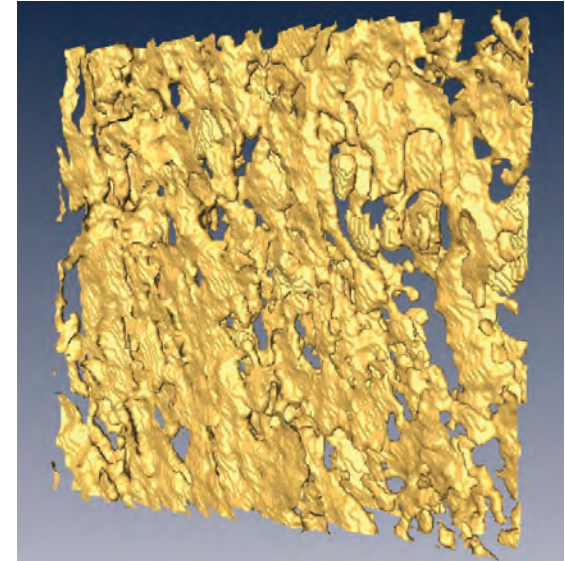
8 x 8 x 2 mm



-15 °C,  $\phi = 0.033$



-6 °C,  $\phi = 0.075$



-3 °C,  $\phi = 0.143$

## X-ray tomography confirms percolation threshold

3-D images  
pores and throats



3-D graph  
nodes and edges

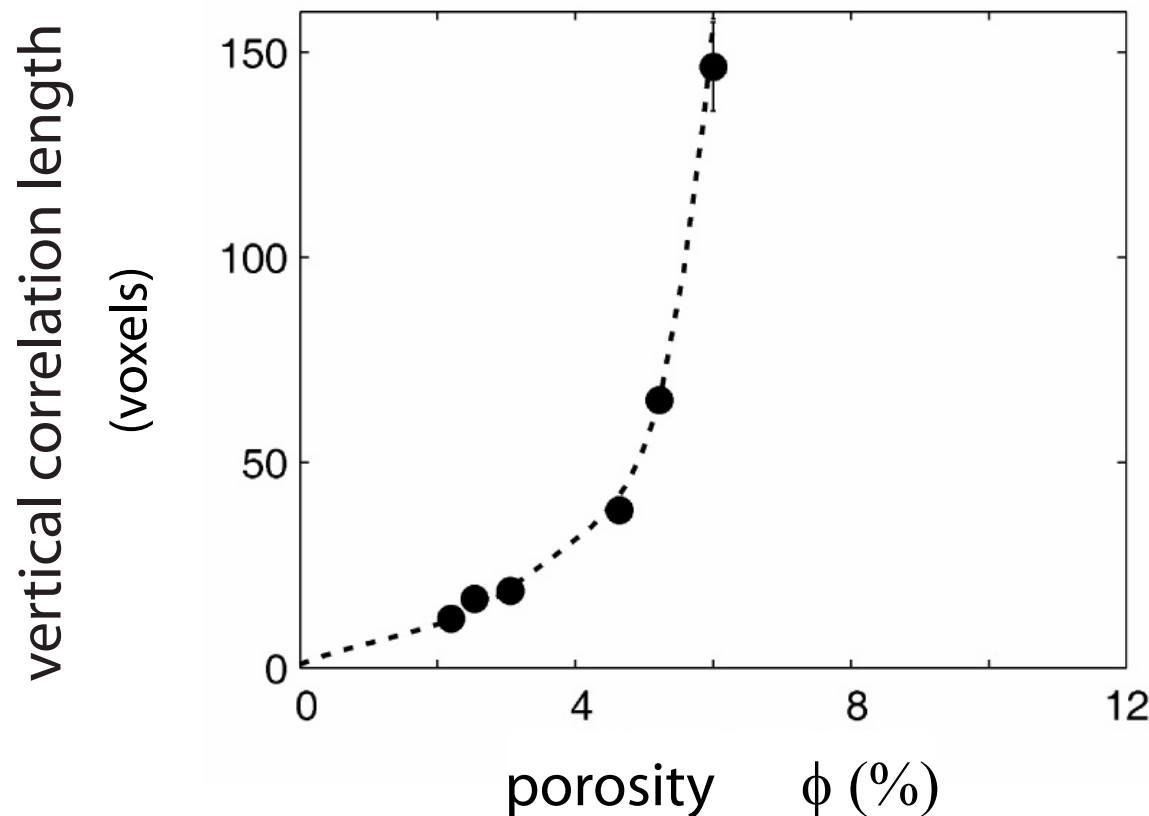
***analyze graph connectivity as function of temperature and sample size***

- ***use finite size scaling techniques to confirm rule of fives***
- ***order parameter data from a natural material***

The key connectivity functions of percolation theory have been computed **extensively** for many lattice models, but **NOT** for natural materials.

***We have calculated them for sea ice single crystals and estimated anisotropic percolation thresholds.***

Pringle, Miner, Eicken, Golden, JGR (Oceans) 2009



divergence of  
correlation length  
for single crystal data



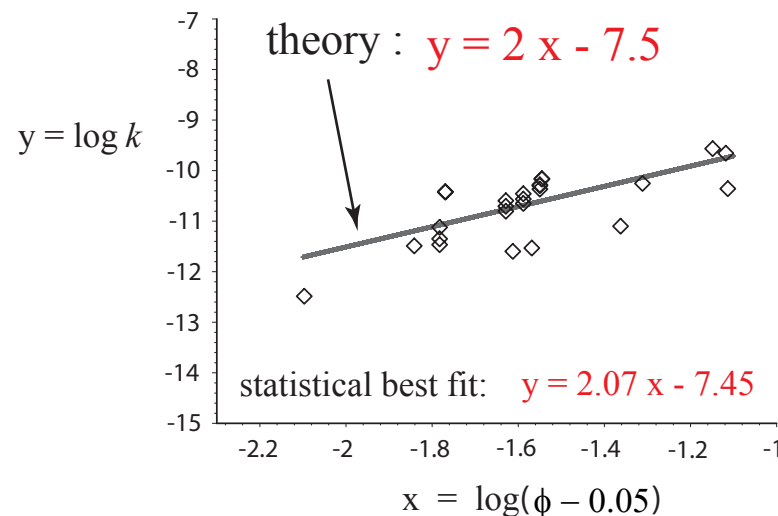
# lattice and continuum percolation theories yield:

$$k(\phi) = k_0 (\phi - 0.05)^2$$

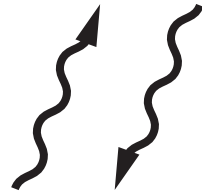
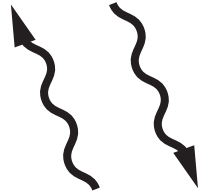
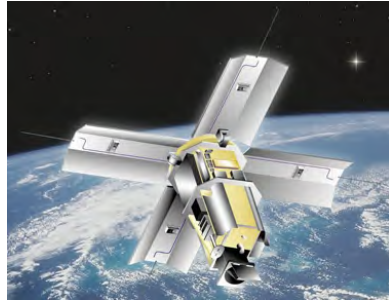
critical  
exponent  
 $t$

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

- exponent is **UNIVERSAL** lattice value  $t \approx 2.0$
- **sedimentary rocks** like sandstones also exhibit universality
- **critical path analysis** -- developed for electronic hopping conduction -- yields scaling factor  $k_0$



# Remote sensing of sea ice



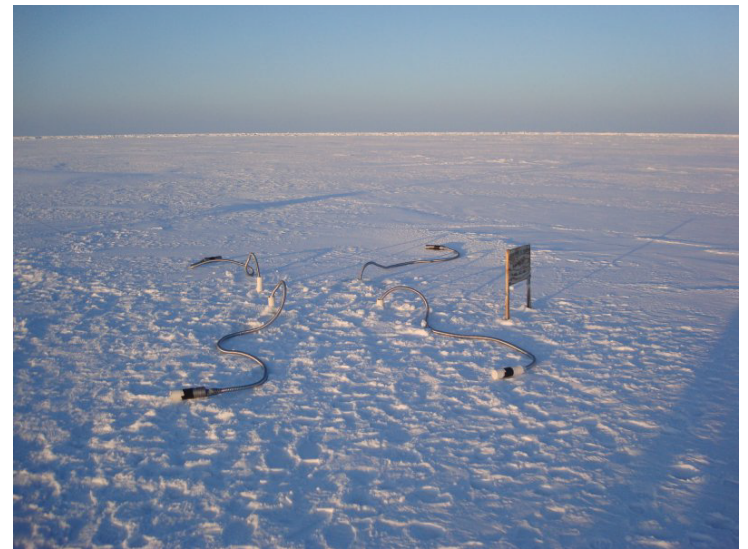
*sea ice thickness*  
*ice concentration*

## **INVERSE PROBLEM**

Recover sea ice  
properties from  
electromagnetic  
(EM) data

$$\epsilon^*$$

effective complex permittivity  
(dielectric constant, conductivity)



*brine volume fraction*  
*brine inclusion connectivity*

# ***Theory of Effective Electromagnetic Behavior of Composites***

## ***analytic continuation method***

***Forward Homogenization*** Bergman (1978), Milton (1979), Golden and Papanicolaou (1983)



integral representations, rigorous bounds, approximations, etc.

***Inverse Homogenization*** Cherkaev and Golden (1998), Day and Thorpe (1999), Cherkaev (2001)  
(McPhedran, McKenzie, and Milton, 1982)



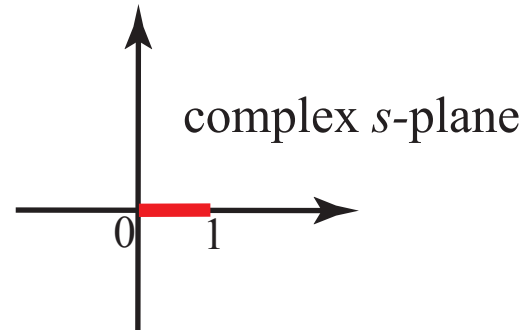
recover brine volume fraction, connectivity, etc.



# Stieltjes integral representation

separation of geometry from parameters

$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$



$$F(s) = 1 - \frac{\epsilon^*}{\epsilon_2} = \int_0^1 \frac{d\mu(z)}{s - z}$$

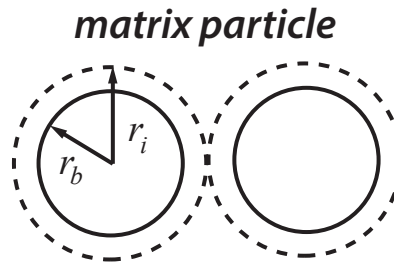
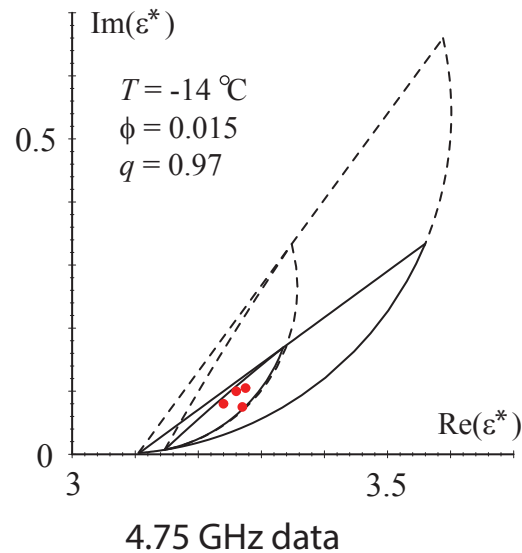
- $\mu$  /
- spectral measure of self adjoint operator  $\Gamma \chi$
  - mass =  $p_1$
  - higher moments depend on  $n$ -point correlations

$$\Gamma = \nabla(-\Delta)^{-1}\nabla.$$

$\chi$  = characteristic function of the brine phase

# forward and inverse bounds for sea ice

## forward bounds

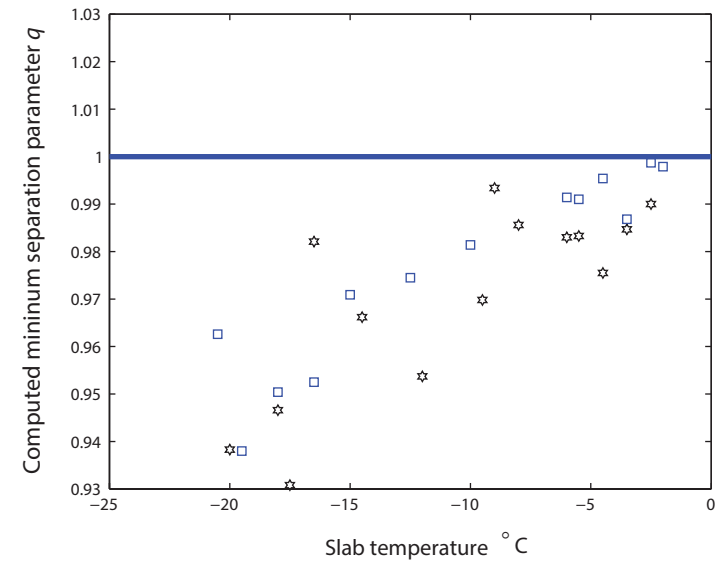


$$q = r_b / r_i$$

$$0 < q < 1$$

**Golden 1995, 1997**

## inverse bounds



## inverse bounds and recovery of brine porosity

**Gully, Backstrom, Eicken, Golden  
Physica B, 2007**

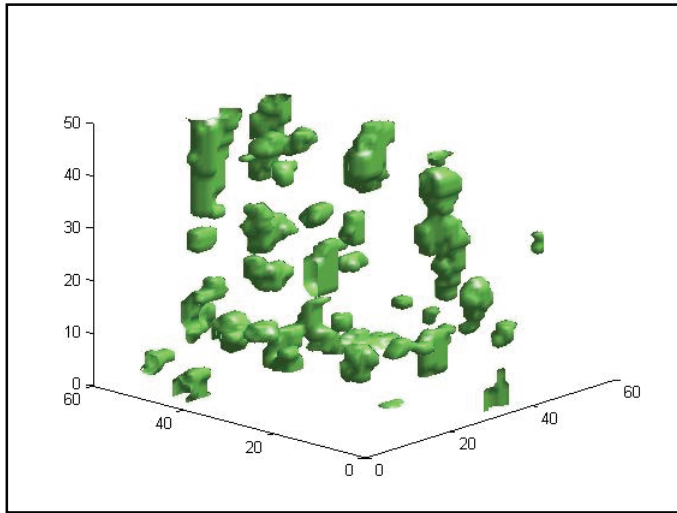
## polycrystalline bounds

**Gully, Lin, Cherkaev, Golden, 2013**

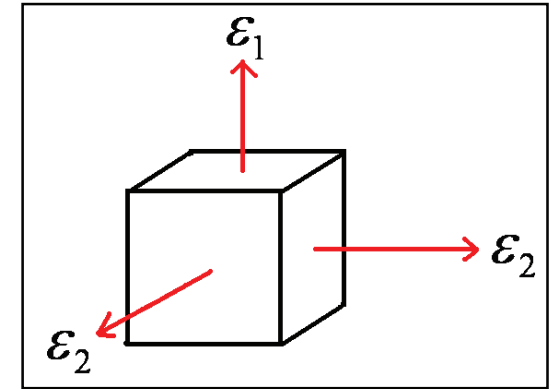
## inversion for brine inclusion separations in sea ice from measurements of effective complex permittivity $\epsilon^*$

**Orum, Cherkaev, Golden  
Proc. Roy. Soc. A, 2012**

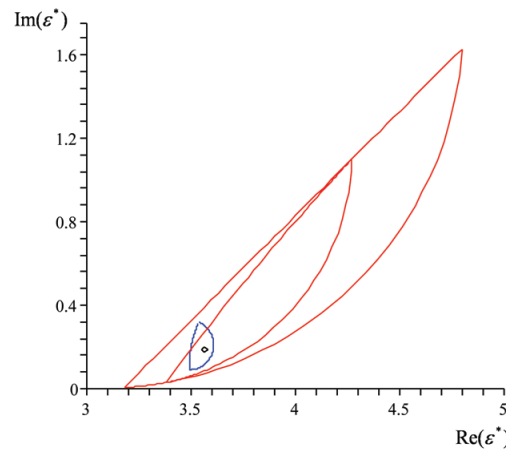
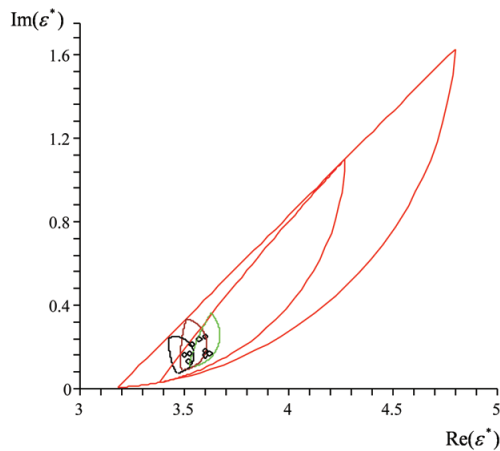
# two scale homogenization for polycrystalline sea ice



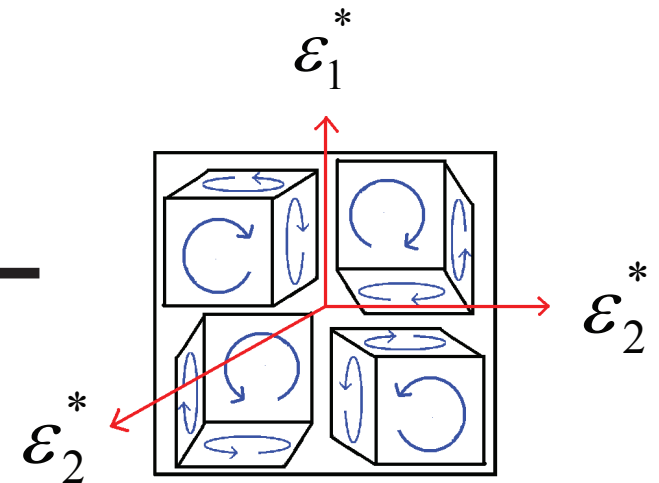
numerical homogenization  
for single crystal



analytic continuation  
for polycrystals



bounds

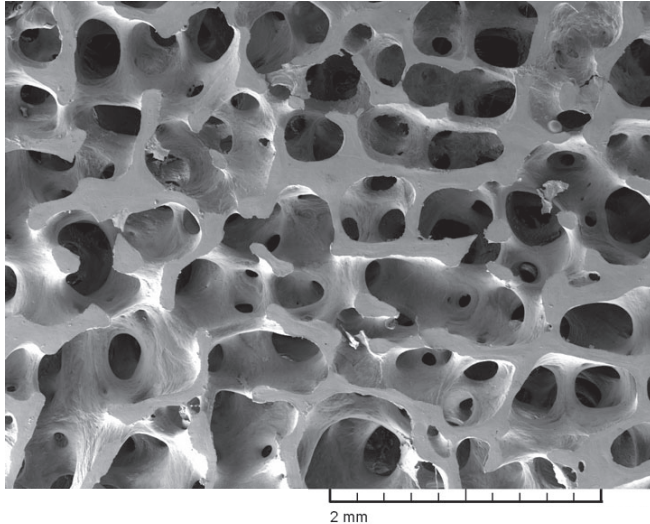




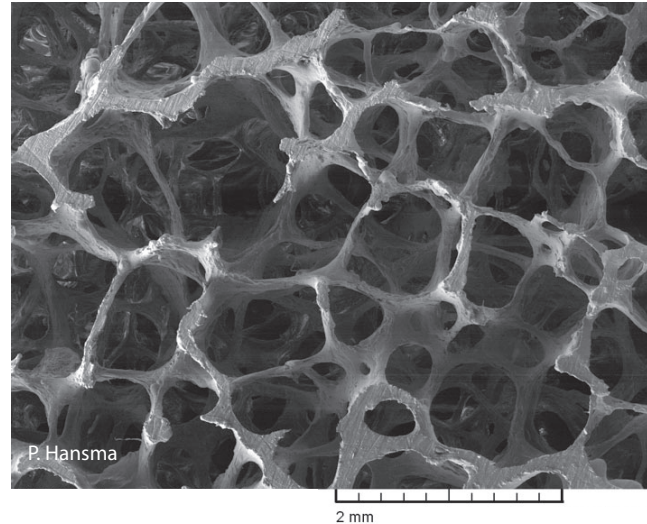
# spectral characterization of porous microstructures in bone

Golden, Murphy, Cherkaev, J. Biomechanics 2011

(a) young healthy trabecular bone



(b) old osteoporotic trabecular bone



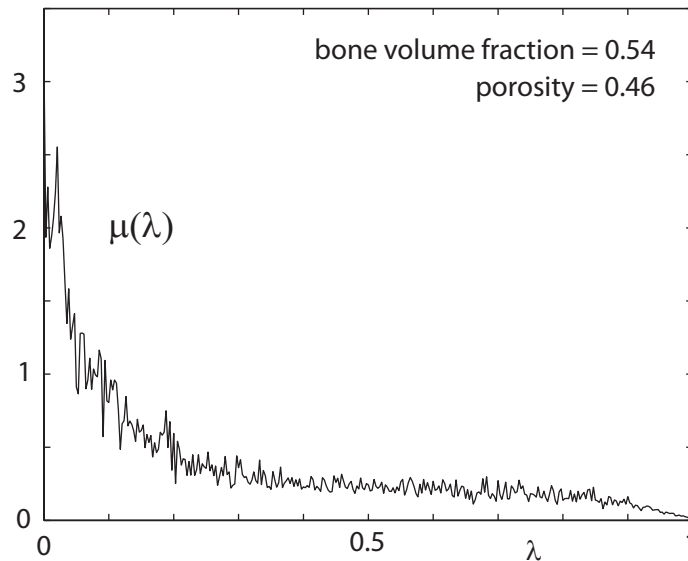
+

reconstruction of spectral  
measures from complex  
permittivity data

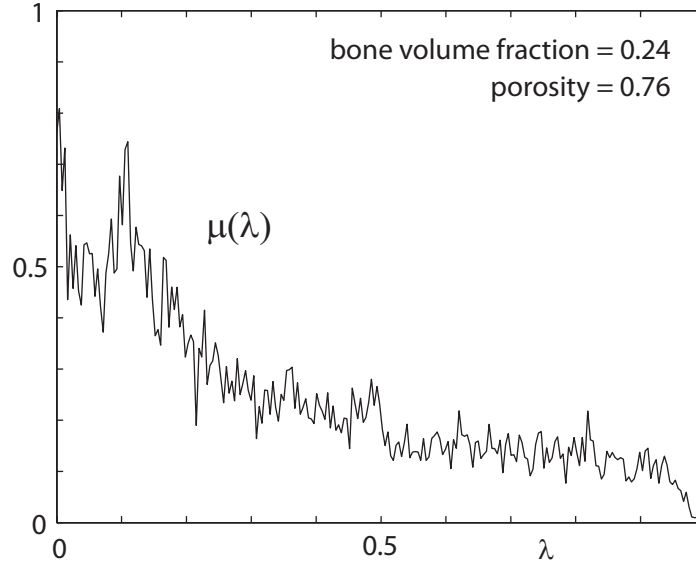
*using regularized  
inversion scheme*



(c) spectral measure - young



(d) spectral measure - old



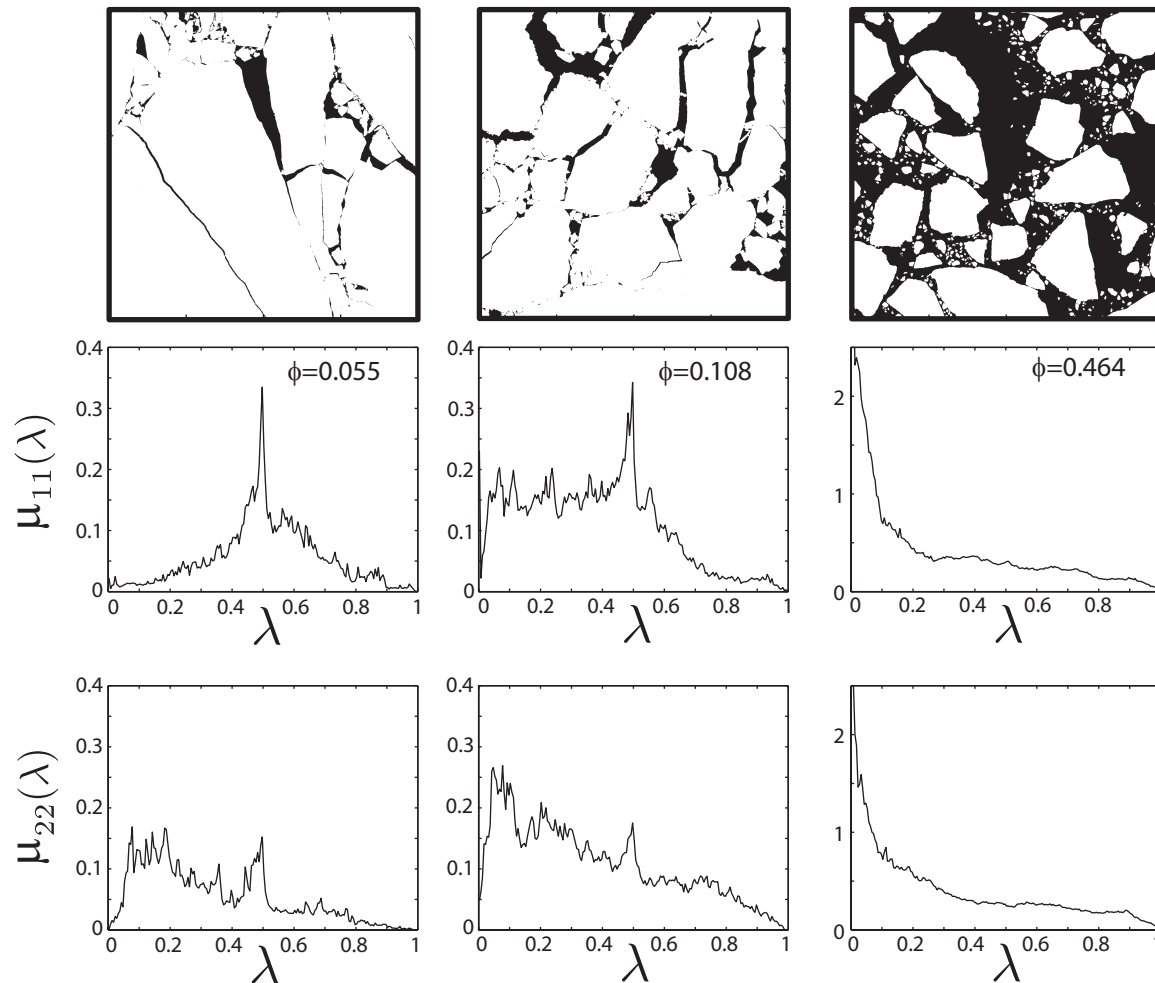
***EM monitoring  
of osteoporosis***

***loss of bone  
connectivity***

***the math doesn't care if it's sea ice or bone!***

spectral measures provide a path toward rigorously incorporating  
“composite microstructure” into calculations of effective behavior on larger scales

*spectral measures for the Arctic sea ice pack*



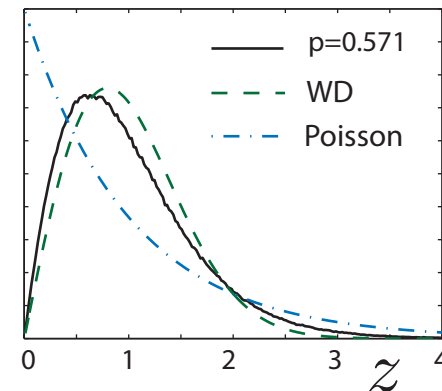
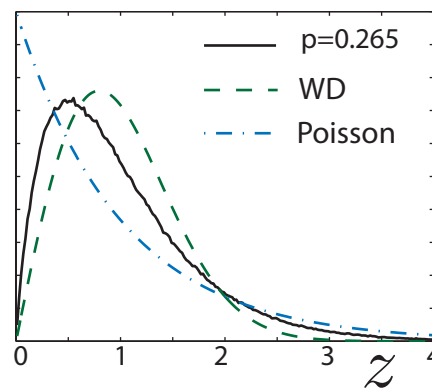
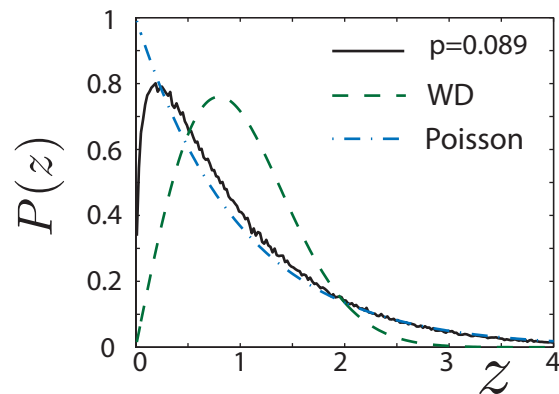
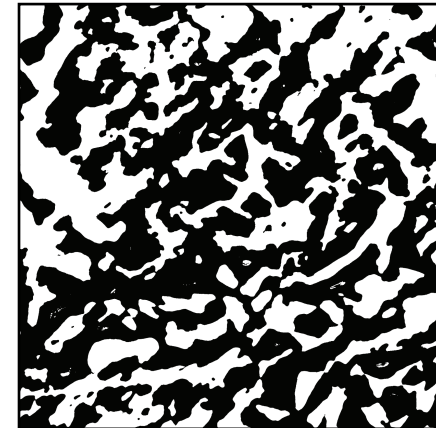
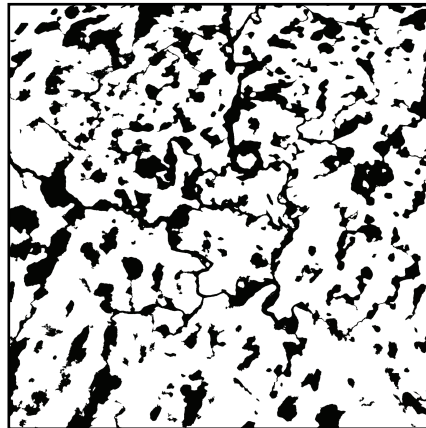
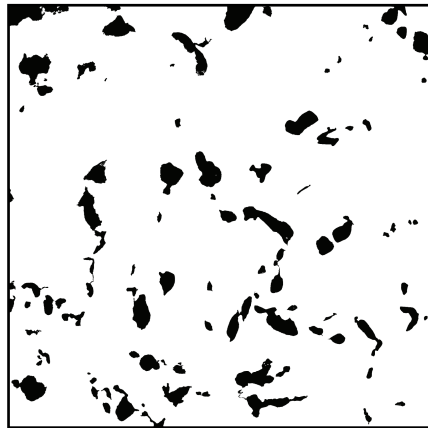
*area under curve =  $\phi$  = open water fraction*

*spectral gap closes as ocean phase becomes connected*

*random matrix characterization of connectedness transition -- discretization of  $\chi\Gamma\chi$*

Unfolded Eigenvalue Spacing Distribution

## ARCTIC MELT PONDS



*eigenvalue statistics for transport tend toward the **UNIVERSAL Wigner-Dyson distribution** as the “conducting” phase becomes connected over large scales*

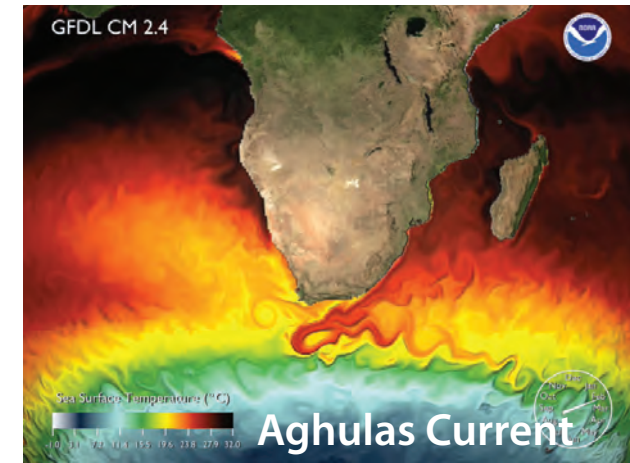
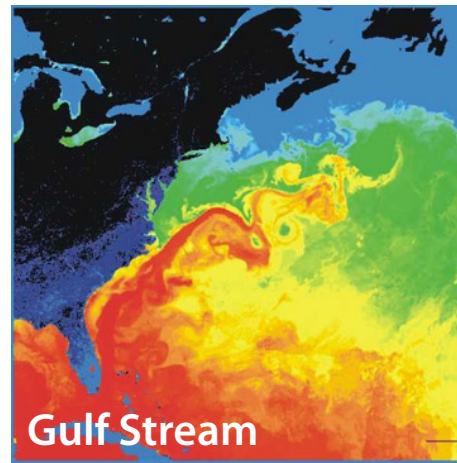
*uncorrelated  $\longrightarrow$  “level repulsion”*



# *advection enhanced diffusion*

## effective diffusivity

tracers, buoys diffusing in ocean eddies  
diffusion of pollutants in atmosphere  
salt and heat transport in ocean



advection diffusion equation with a velocity field  $\vec{u}$

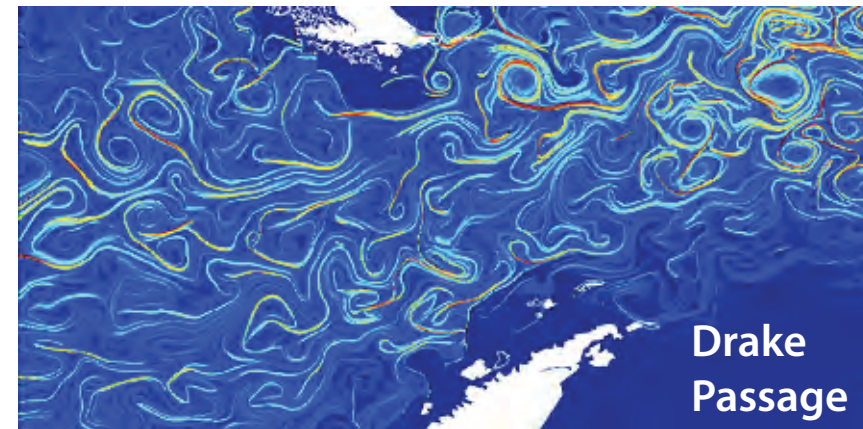
$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

*homogenize*

$$\frac{\partial \bar{T}}{\partial t} = \kappa^* \Delta \bar{T}$$

$\kappa^*$  *effective diffusivity*



# Stieltjes integral for $\kappa^*$ with spectral measure

## composites

*Golden and Papanicolaou, CMP 1983*

$$\frac{\epsilon^*}{\epsilon_2} = 1 - \int_0^1 \frac{d\mu(\lambda)}{s - \lambda}$$
$$s = \frac{1}{1 - \epsilon_1 / \epsilon_2}$$

## advection diffusion

*Avellaneda and Majda, PRL 89, CMP 91*

$$\frac{\kappa^*}{\kappa_0} = 1 - \int_0^\infty \frac{d\rho(z)}{t - z}$$

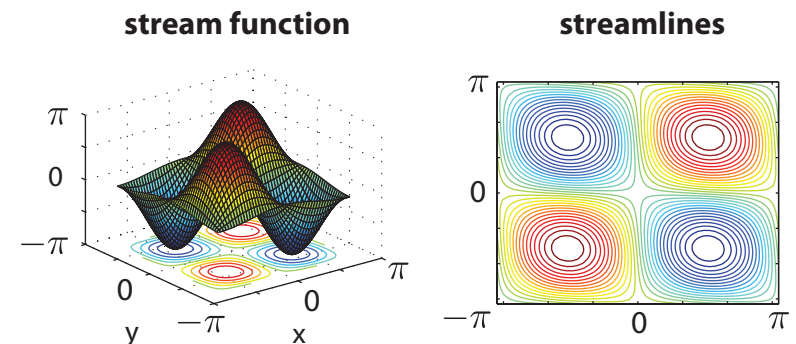
$$t = -1/\xi^2, \quad \xi = \text{Péclet number}$$

- computations of spectral measures and effective diffusivity for model flows

$$i\Gamma H \Gamma \quad \vec{u} = \kappa_0 \xi \vec{\nabla} \cdot \mathbf{H}$$

- rigorous bounds and computations on convection enhanced thermal conductivity of sea ice

Murphy, Zhu, Golden 2013



Wang, Zhu, Golden 2013

# Arctic and Antarctic field experiments

*develop electromagnetic methods  
of monitoring fluid transport and  
microstructural transitions*

extensive measurements of fluid and  
electrical transport properties of sea ice:

**2007    Antarctic    SIPEX**

**2010    Antarctic    McMurdo Sound**

**2011    Arctic        Barrow AK**

**2012    Arctic        Barrow AK**

**2012    Antarctic    SIPEX II**

**2013    Arctic        Barrow AK**





# Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and  
the Mathematics of  
Transport in Sea Ice

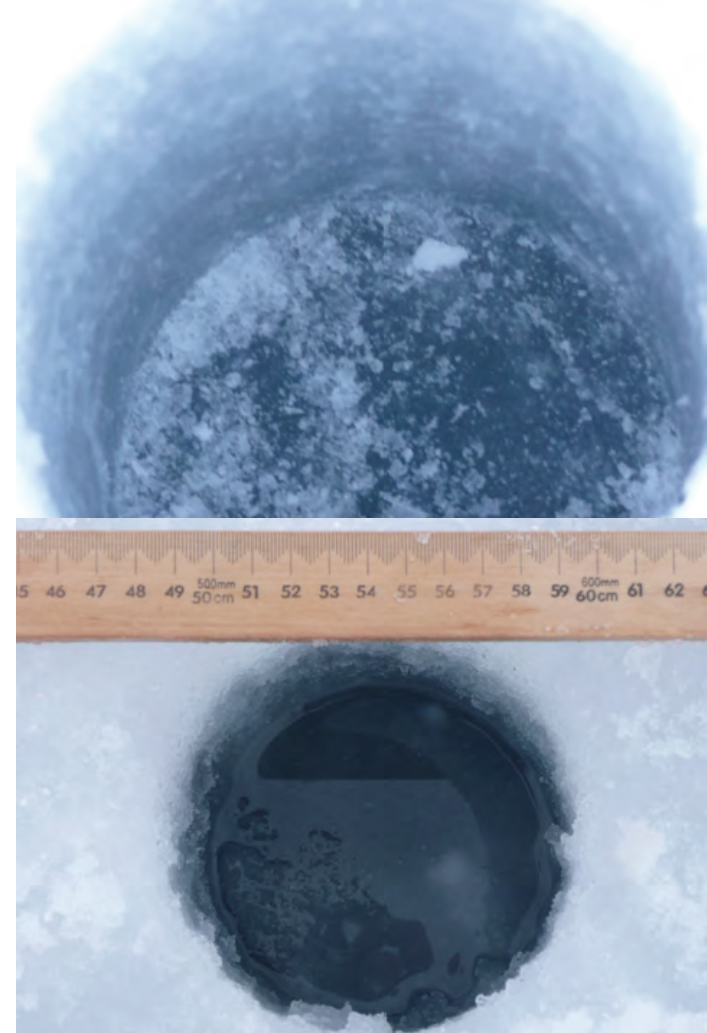
page 562

Mathematics and the  
Internet: A Source of  
Enormous Confusion  
and Great Potential

page 586

*photo by Jan Lieser*

*Real analysis in polar coordinates (see page 613)*



***measuring  
fluid permeability  
of Antarctic sea ice***

***SIPEX 2007***

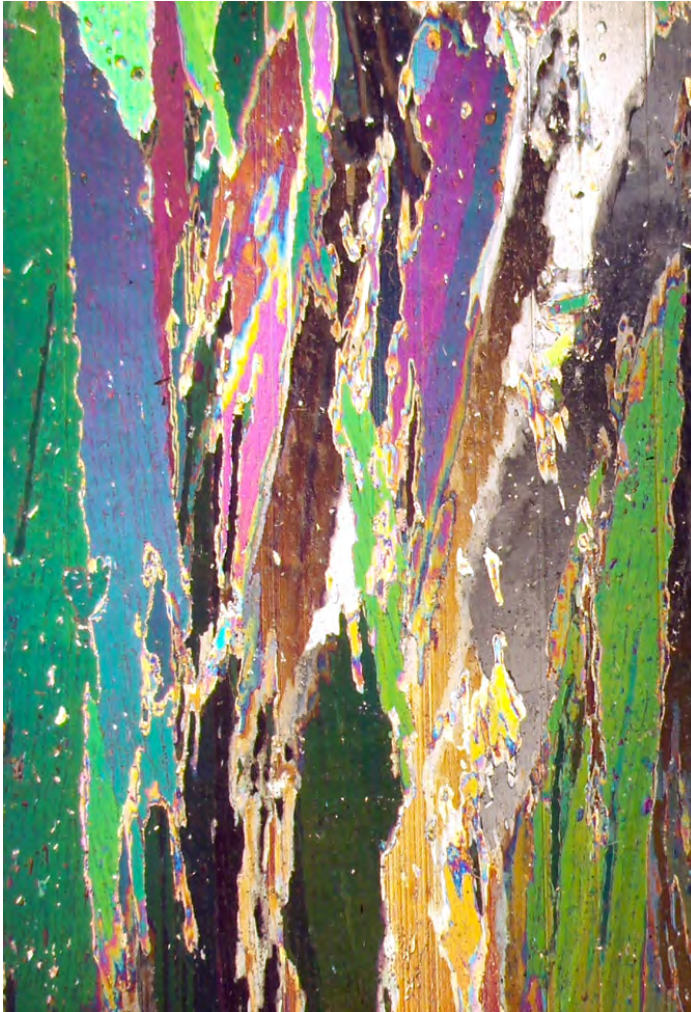


# ***higher threshold for fluid flow in Antarctic granular sea ice***

columnar

granular

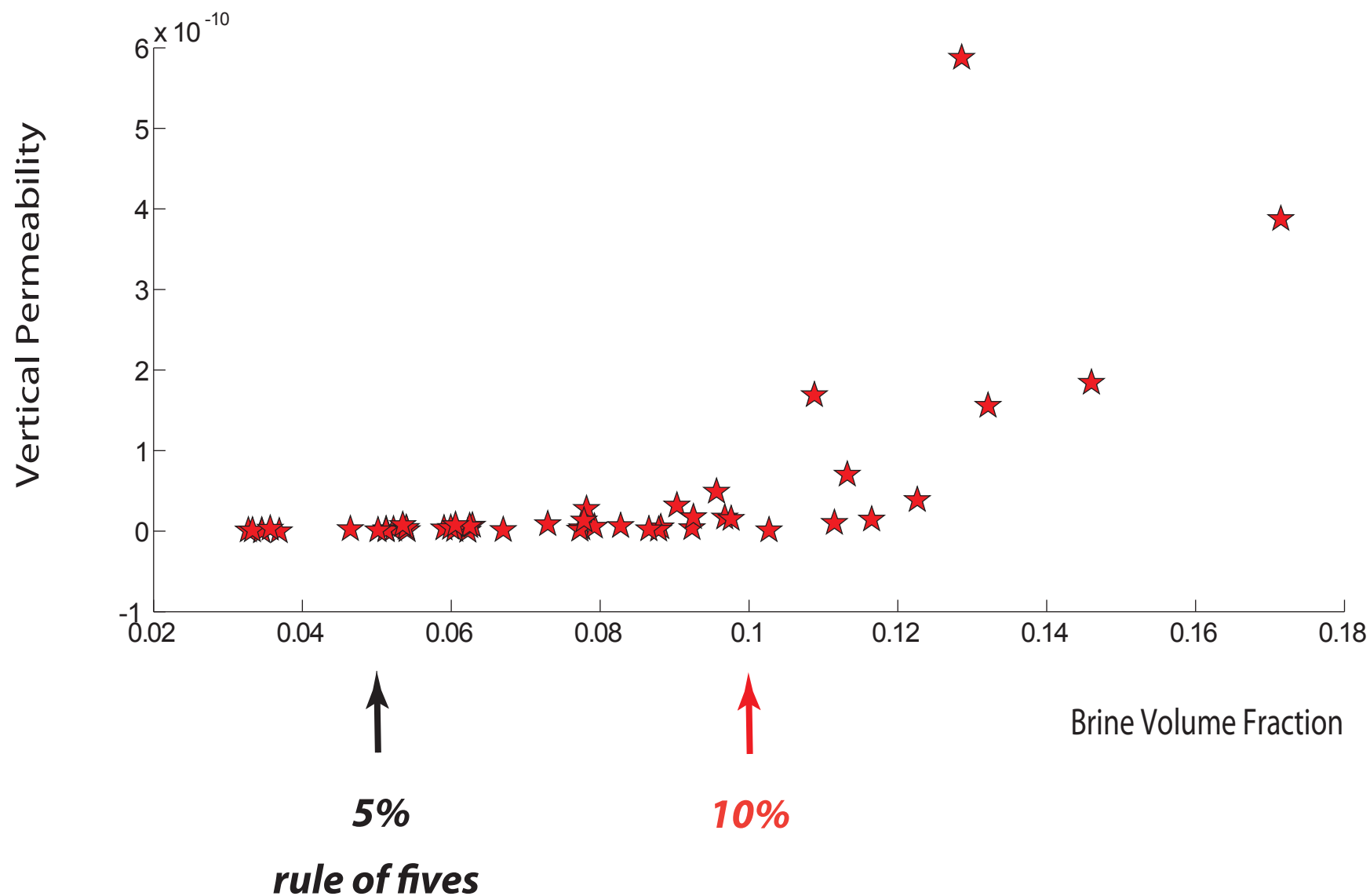
**5%**



**10%**



***Golden, Gully, Lubbers, Sampson, Tison 2013***





# tracers flowing through inverted sea ice blocks

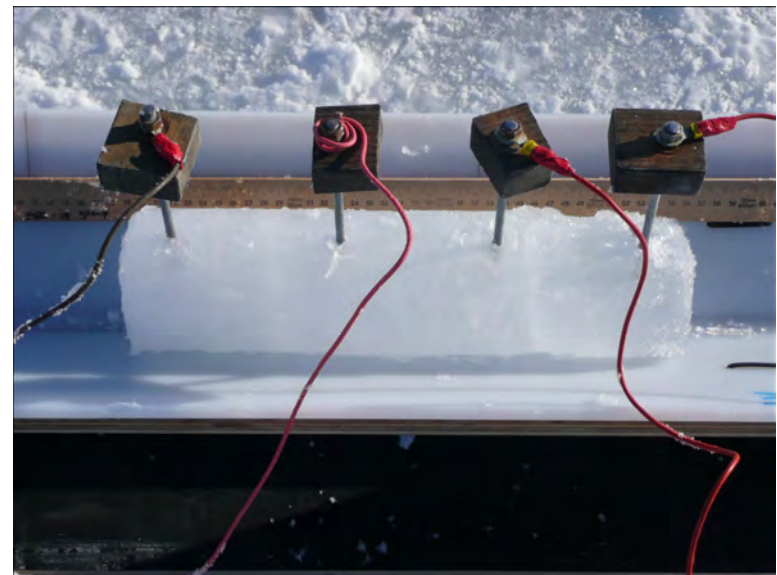
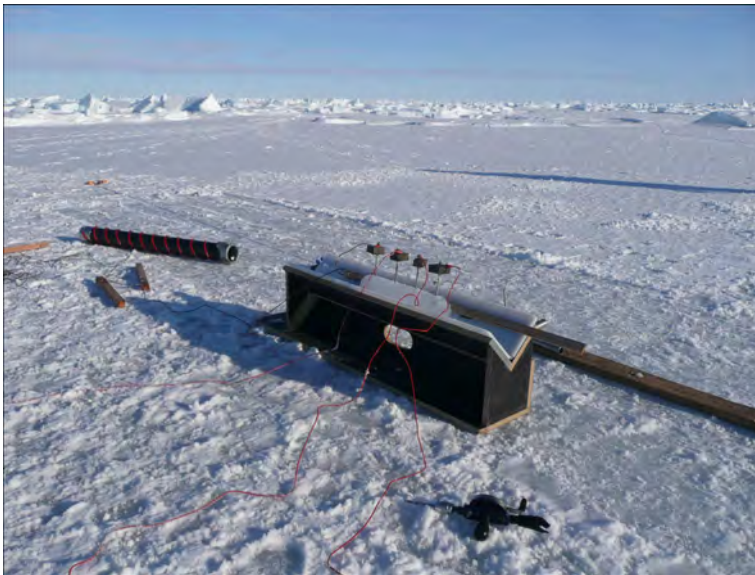




## electrical measurements



## Wenner array



## vertical conductivity

Zhu, Golden, Gully, Sampson *Physica B* 2010

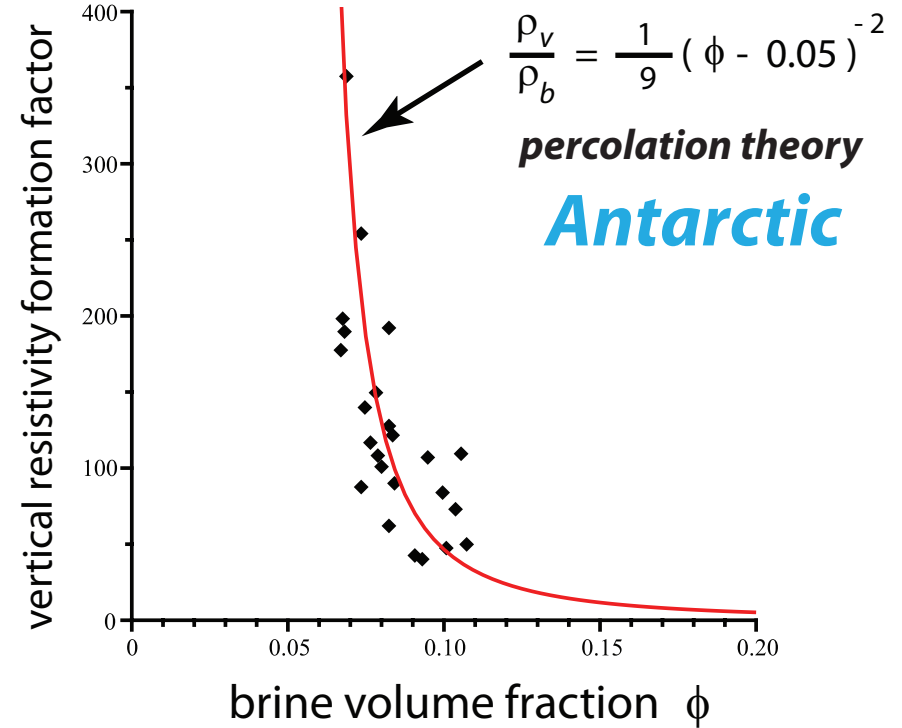
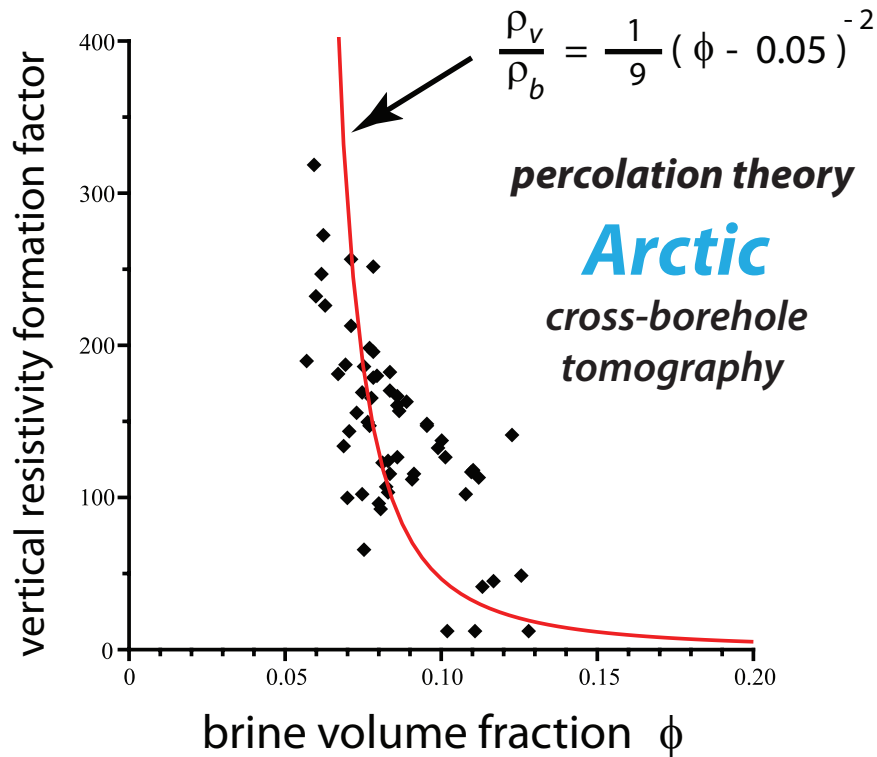
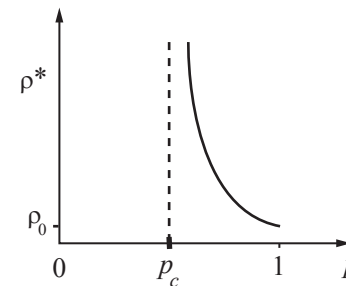
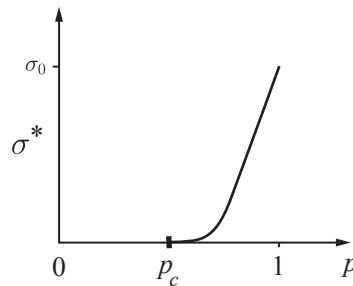
Sampson, Golden, Gully, Worby *Deep Sea Research* 2011

# critical behavior of electrical transport in sea ice

## electrical signature of the on-off switch for fluid flow

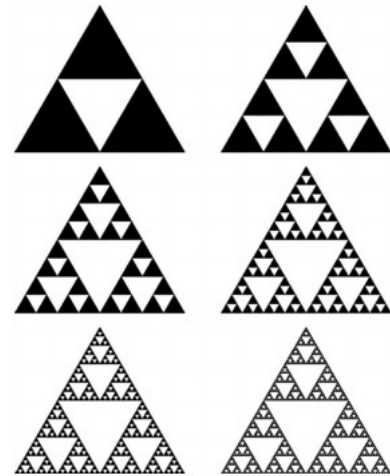
same universal critical exponent as for fluid permeability

studied for over 50 years but no previous observations or theory of critical behavior





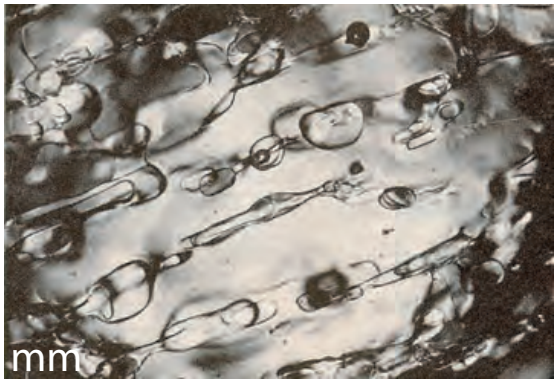
# ***fractals and multiscale structure***





sea ice displays *multiscale* structure over 10 orders of magnitude

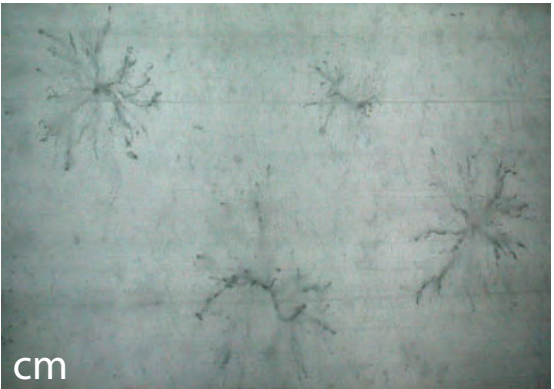
0.1 millimeter



brine inclusions



polycrystals



horizontal

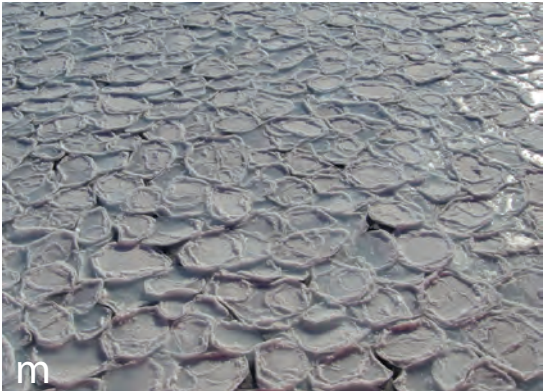


brine channels



vertical

1 meter

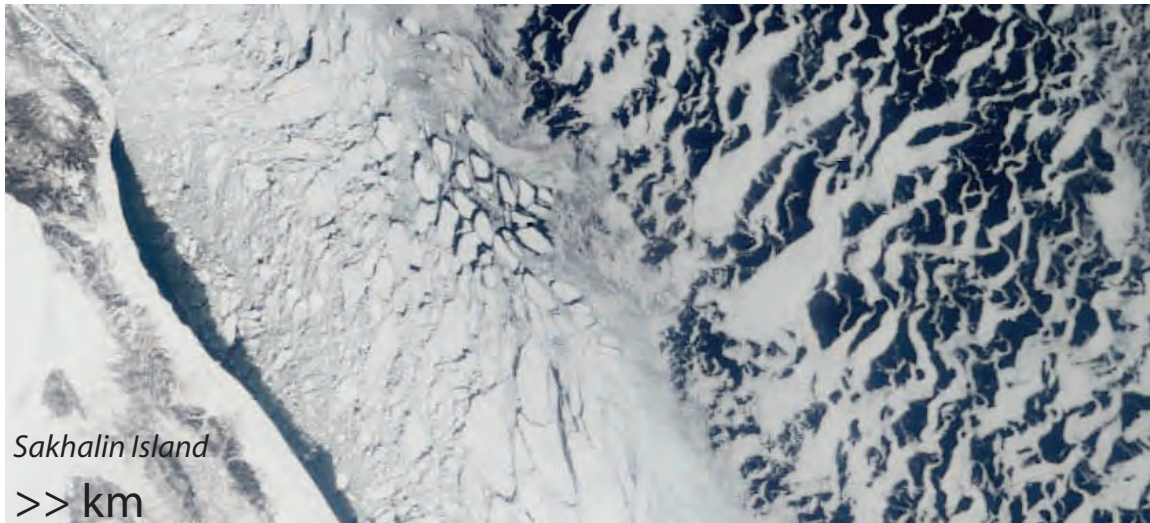


pancake ice

1 meter



100 kilometers





# ***melt ponds on the surface of Arctic sea ice***



# *melt pond formation and albedo evolution:*

- *major drivers in polar climate*
- *key challenge for global climate models*

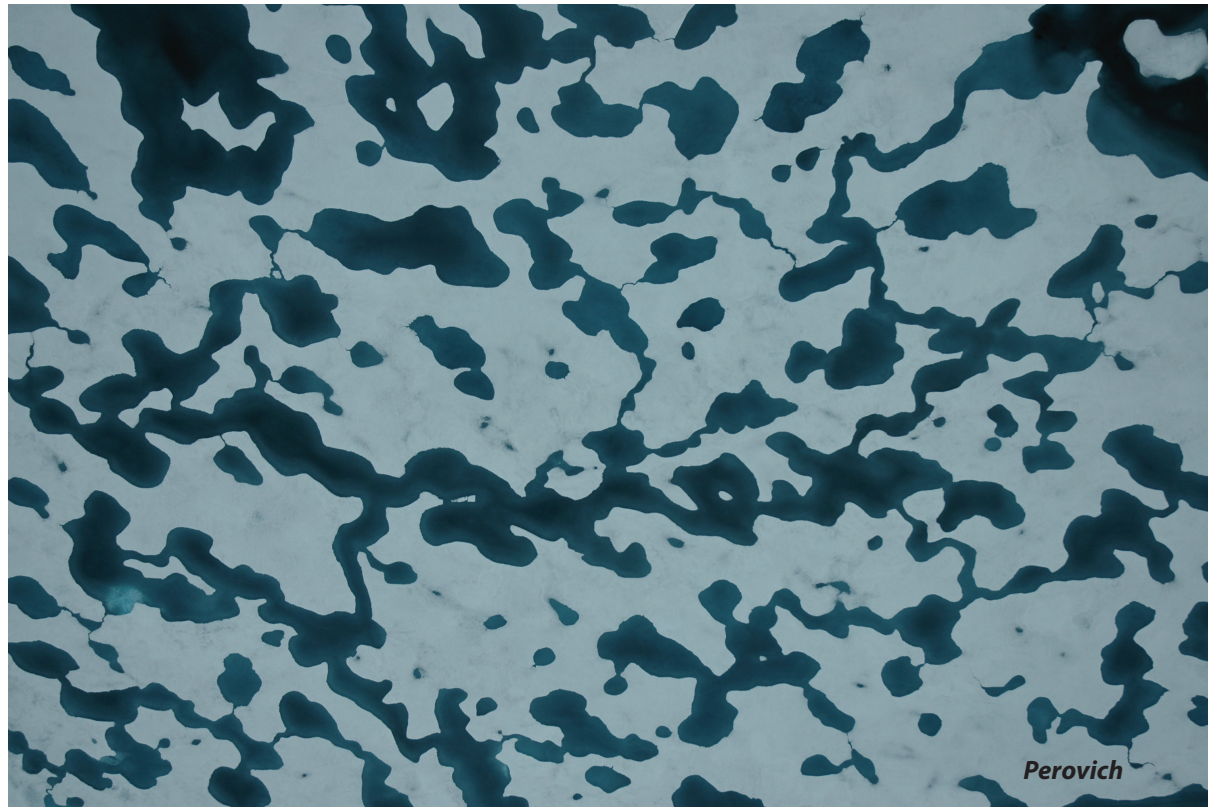
**numerical models of melt pond evolution, including topography, drainage (permeability), etc.**

Lüthje, Feltham,  
Taylor, Worster 2006

Flocco, Feltham 2007

Skyllingstad, Paulson,  
Perovich 2009

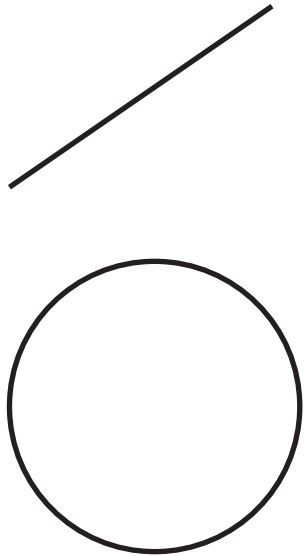
Flocco, Feltham,  
Hunke 2012



**Are there universal features of the evolution similar to phase transitions in statistical physics?**

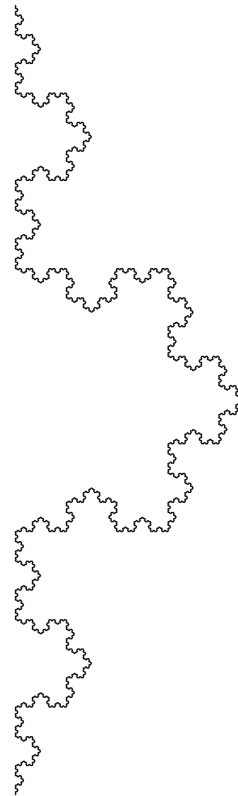
# *fractal curves in the plane*

*they wiggle so much that their dimension is  $>1$*



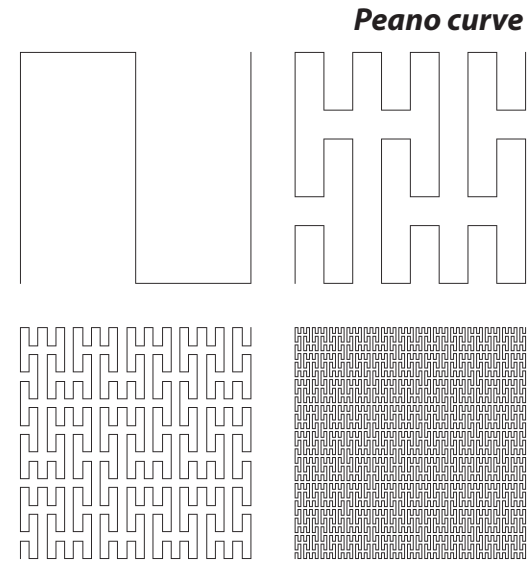
*simple curves*

$$D = 1$$



*Koch snowflake*

$$D = 1.26$$



*Peano curve*

*Brownian motion*

*space filling curves*

$$D = 2$$



# clouds exhibit fractal behavior from 1 to 1000 km

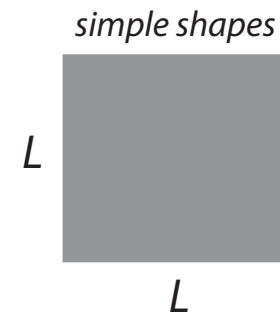
use **perimeter-area** data to find that cloud and rain boundaries are fractals

$$D \approx 1.35$$

*S. Lovejoy, Science, 1982*

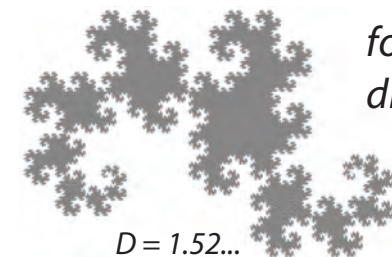


$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

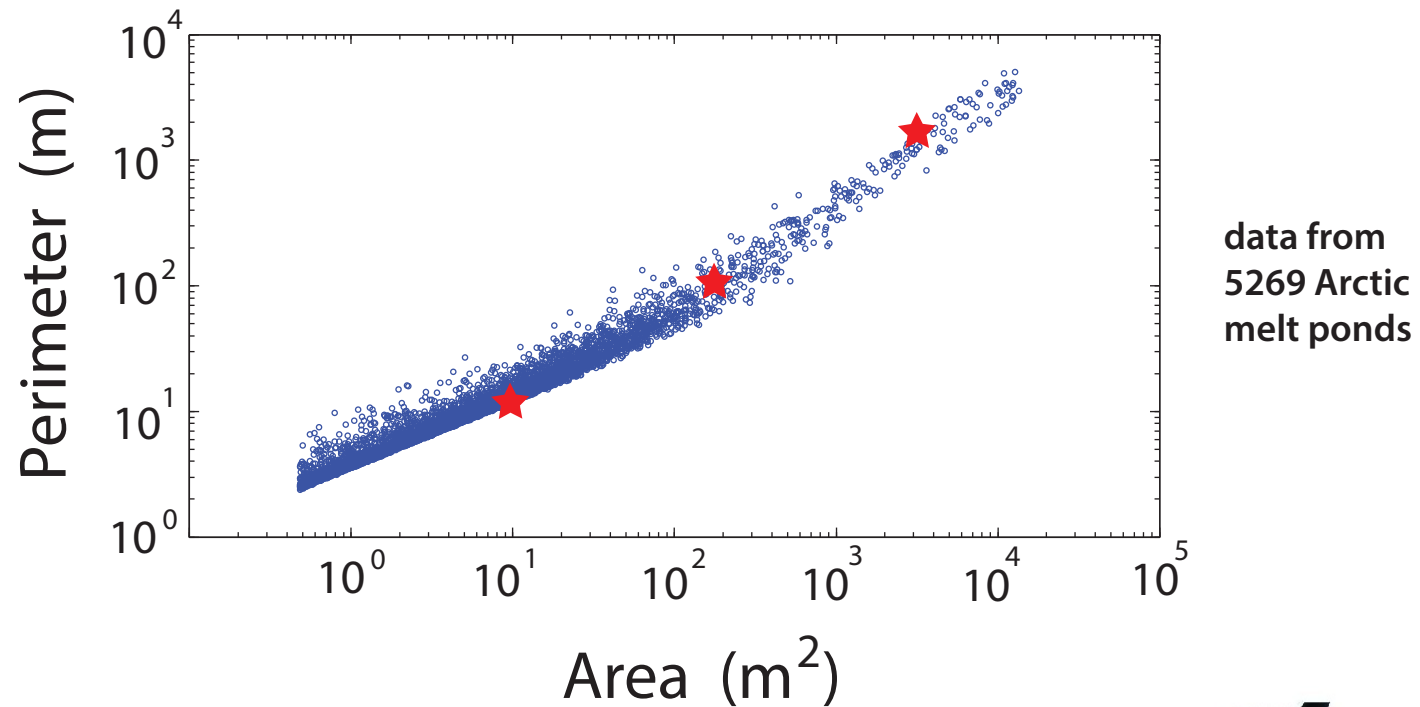
$$P \sim \sqrt{A}^D$$



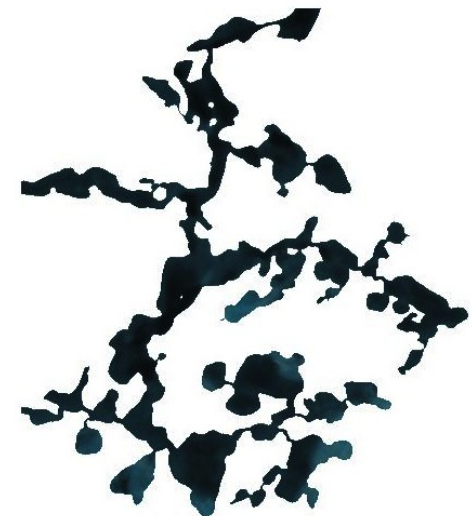
for fractals with dimension  $D$

$D = 1.52...$

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden



~ 30 m



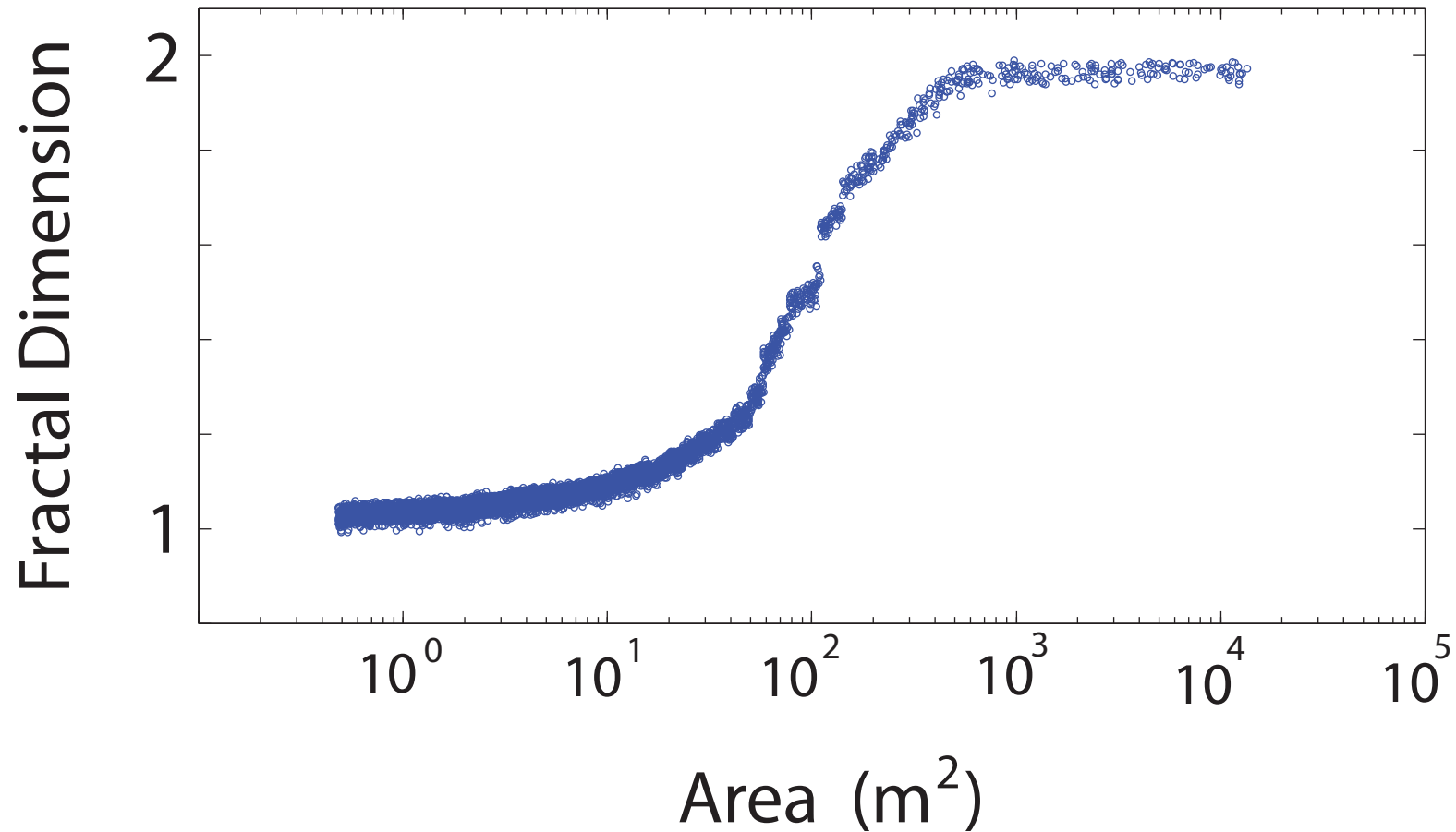
***simple pond***

***transitional pond***

***complex pond***

## *transition in the fractal dimension*

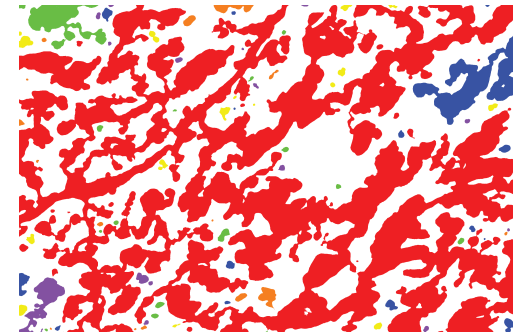
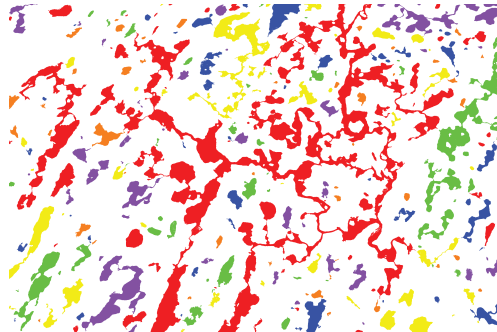
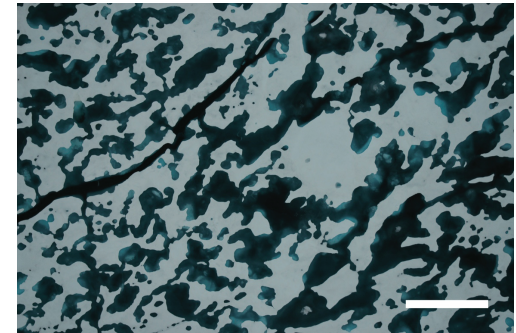
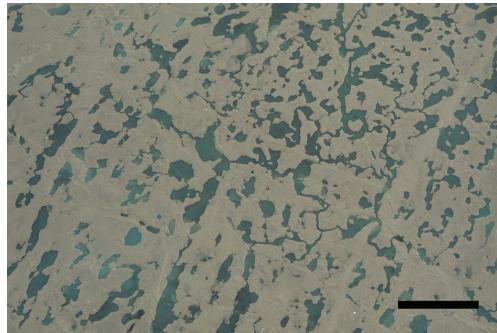
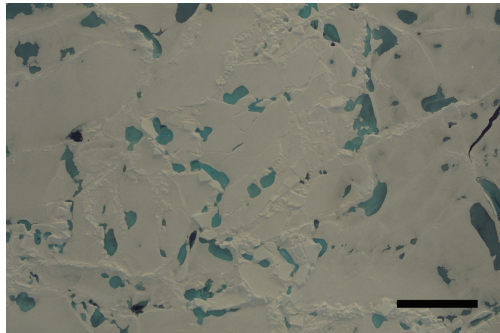
complexity grows with length scale



compute “derivative” of area - perimeter data

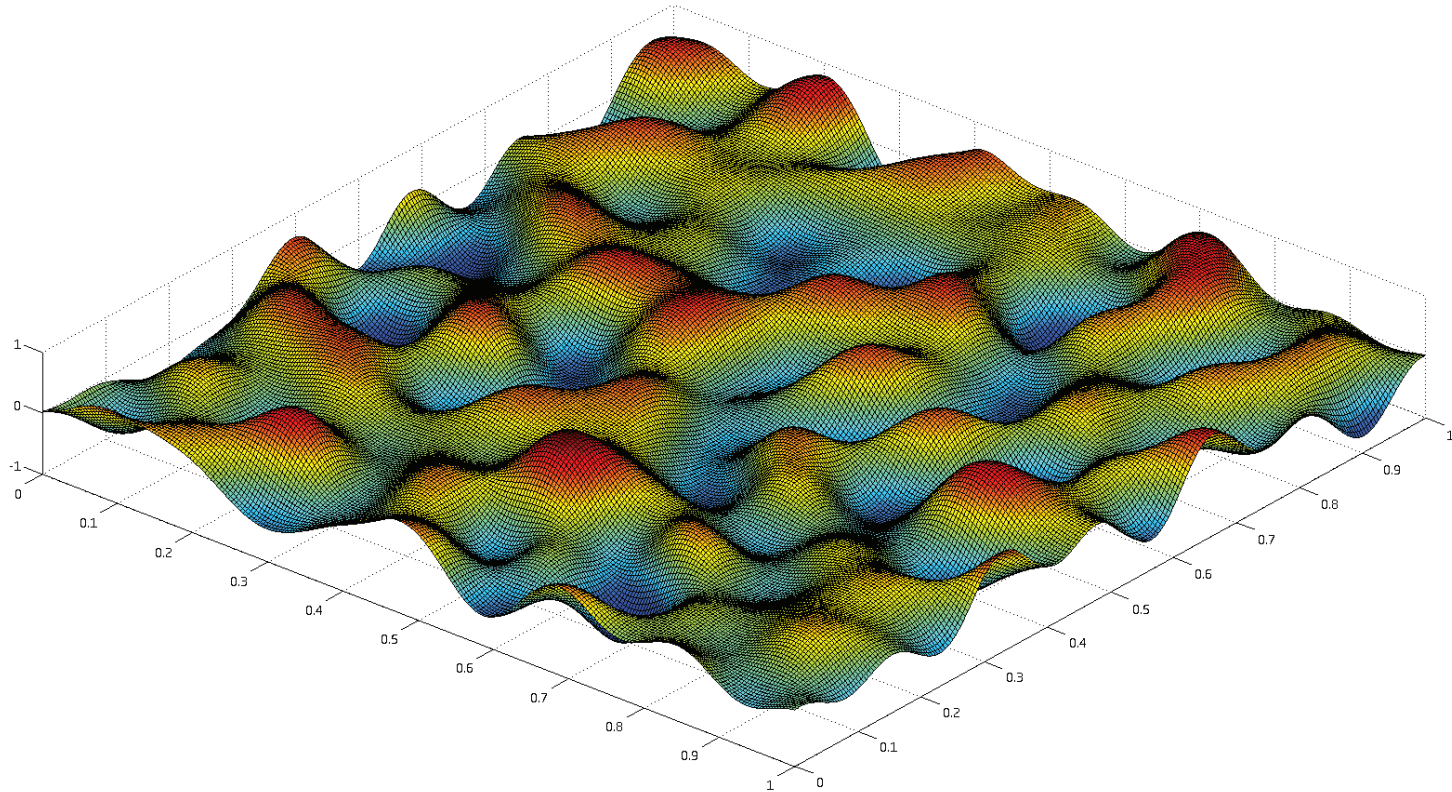


***small simple ponds coalesce to form  
large connected structures with complex boundaries***



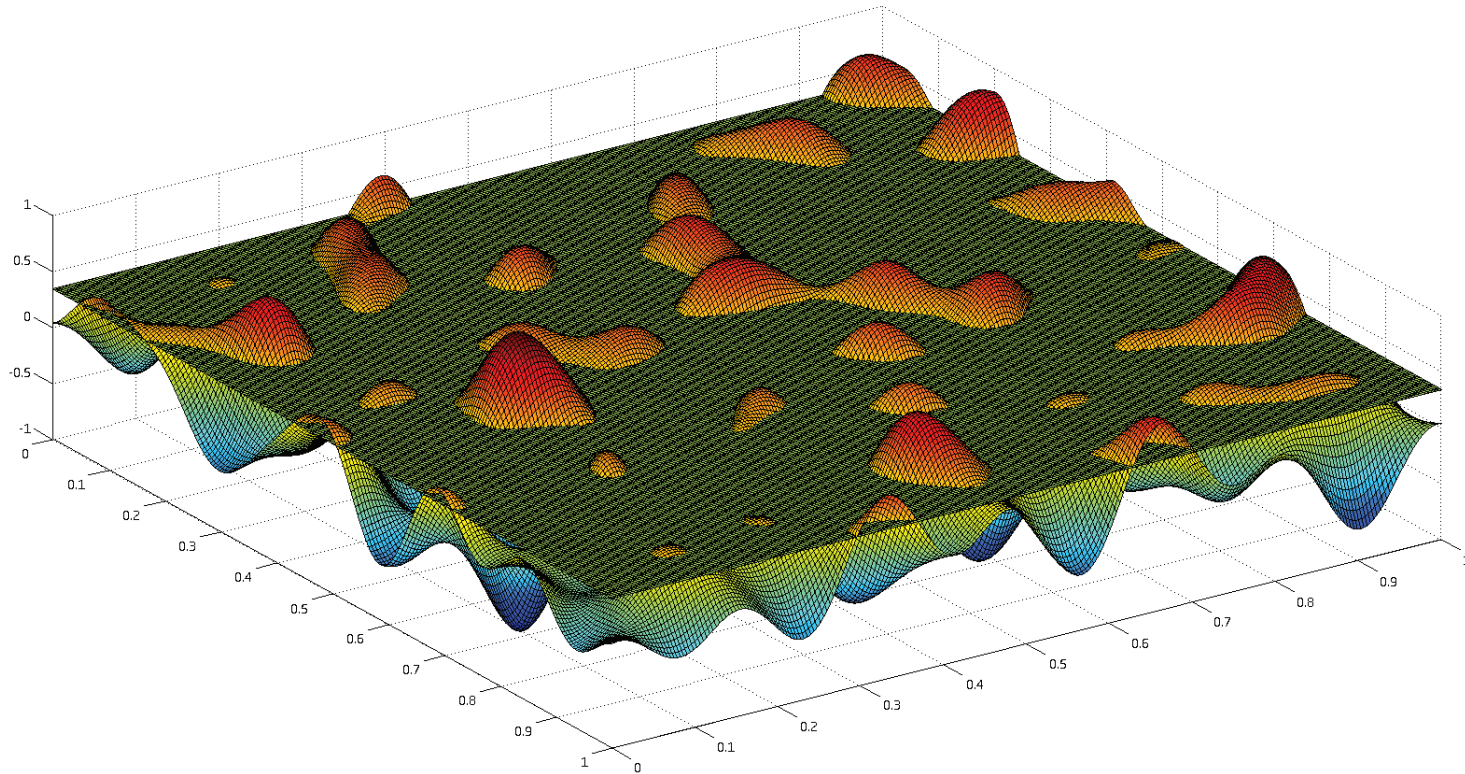
**melt pond percolation**

# Continuum percolation model for melt pond evolution



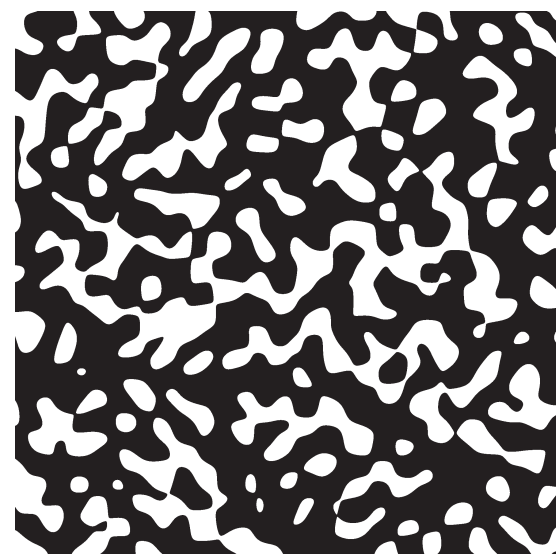
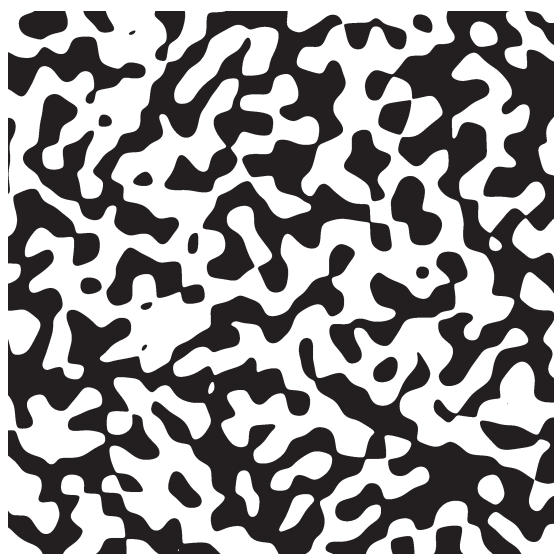
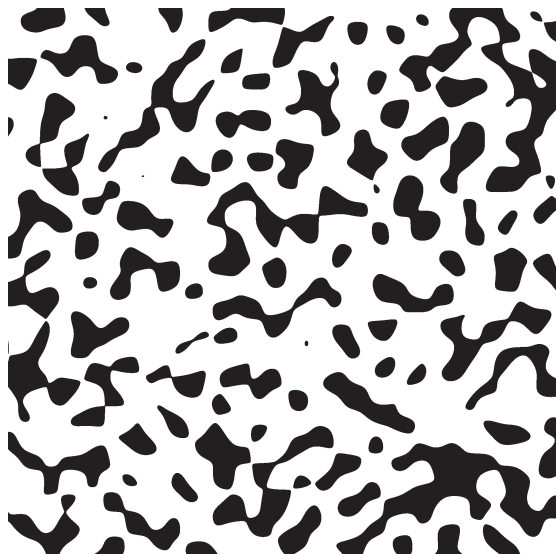
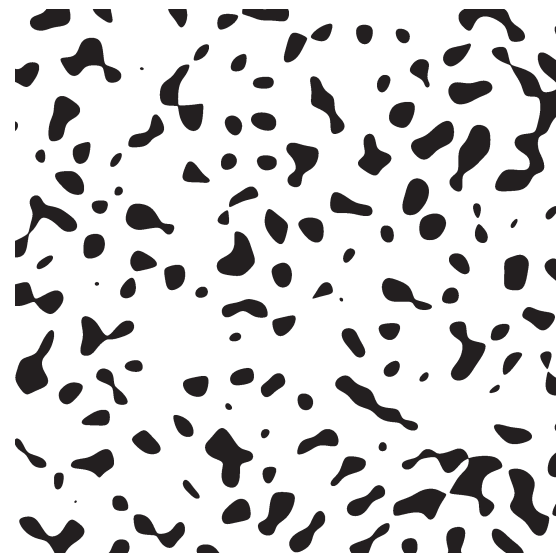
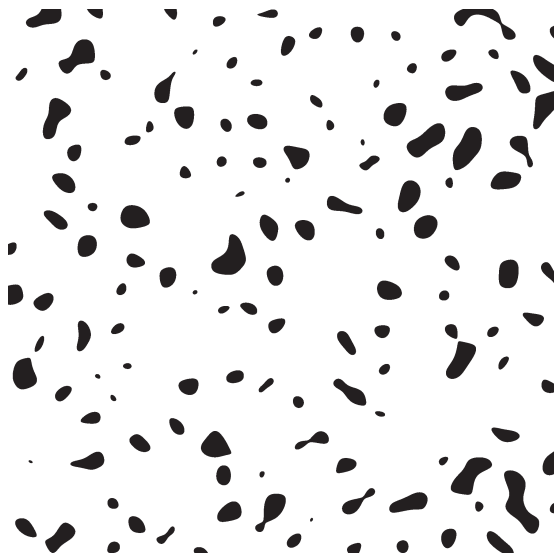
*random Fourier surface*

intersections of a plane with the surface define melt ponds



as the plane varies in height the regions evolve like melt ponds  
at a critical height  $h_c$  ponds **percolate** and form an infinite ocean

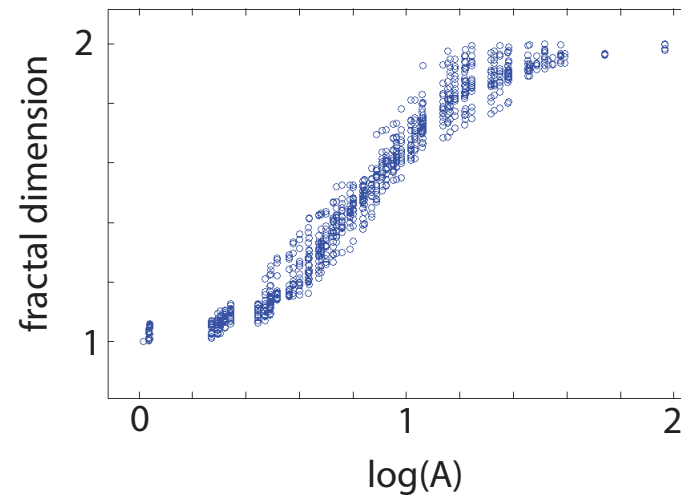
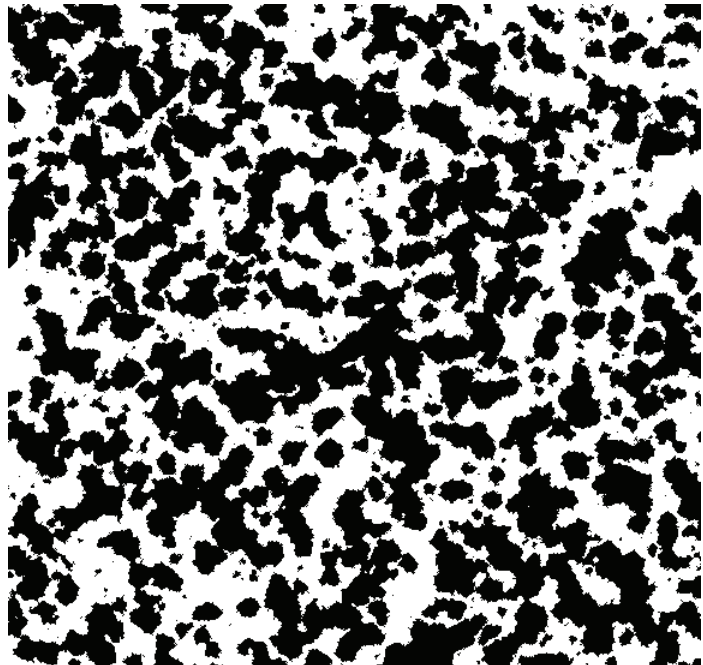
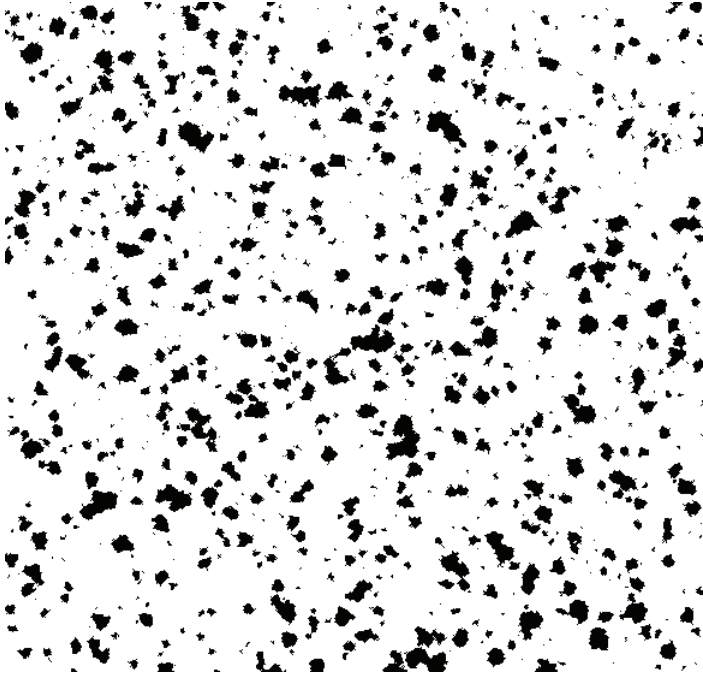




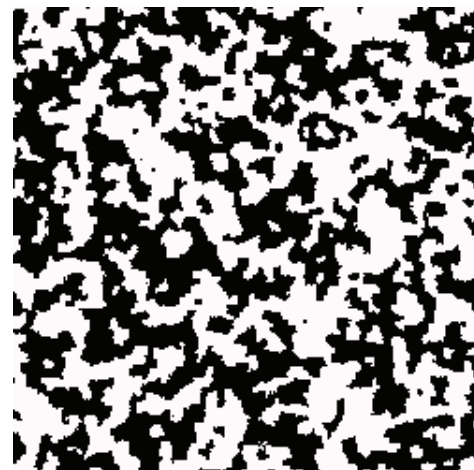
$h_c$

*percolation threshold*

# simple stochastic growth model of melt pond evolution



a square is more likely to melt  
if its neighbors have melted



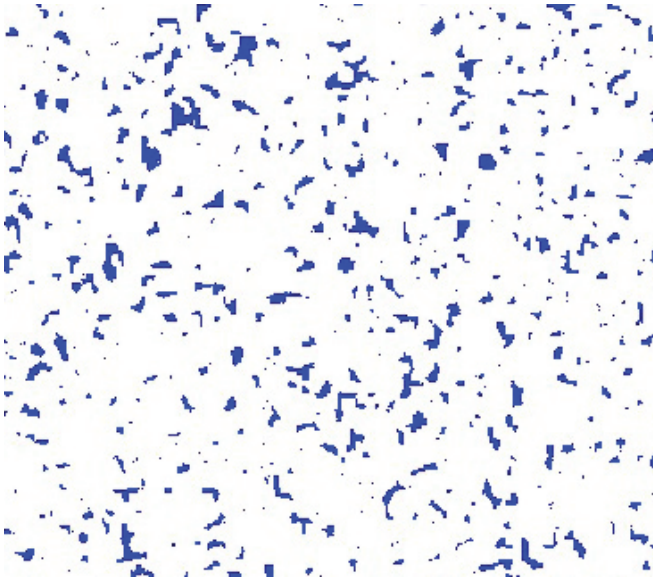
voter  
model

## Ising model for ferromagnets

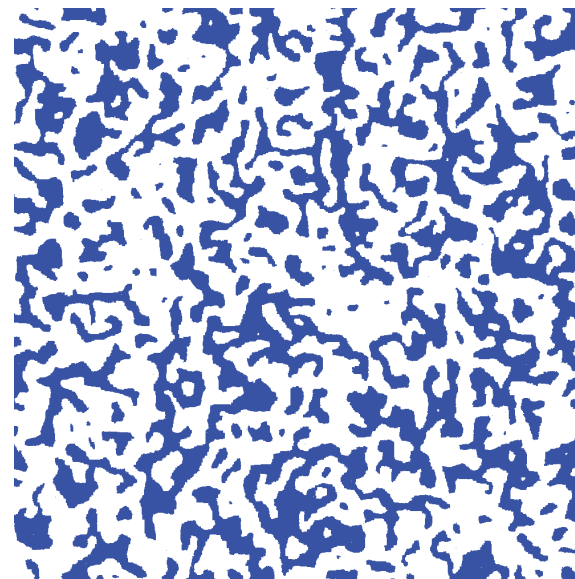


## Ising model for melt ponds

$$\mathcal{H}_\omega = -J \sum_{\langle i,j \rangle}^N s_i s_j - H \sum_i^N s_i \quad s_i = \begin{cases} \uparrow & +1 & \text{ice} \\ \downarrow & -1 & \text{water} \end{cases} \quad M = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$



COLD



WARM

***“melt ponds” are clusters of magnetic spins that align with the applied field***

***clusters exhibit transition in fractal dimension***



# ***Conclusions***

- 1. Summer Arctic sea ice is melting rapidly.**
- 2. Fluid flow through sea ice mediates many processes of importance to understanding climate change and the response of polar ecosystems.**
- 3. Mathematical models of composite materials and statistical physics help unravel the complexities of sea ice structure and processes, and provide a path toward rigorous representation of sea ice in climate models .**
- 4. Field experiments are essential to developing relevant mathematics.**
- 5. Our research will help to improve projections of climate change and the fate of the Earth sea ice packs.**

# THANK YOU

## National Science Foundation

Division of Mathematical Sciences

Arctic Natural Sciences

Office of Polar Programs

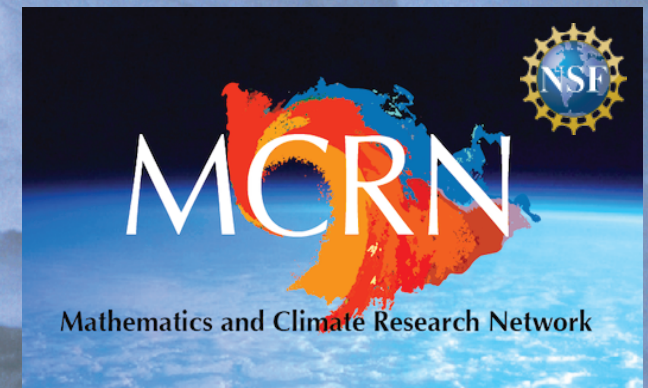
CMG Program

(Collaboration in Mathematical Geosciences)

## Office of Naval Research

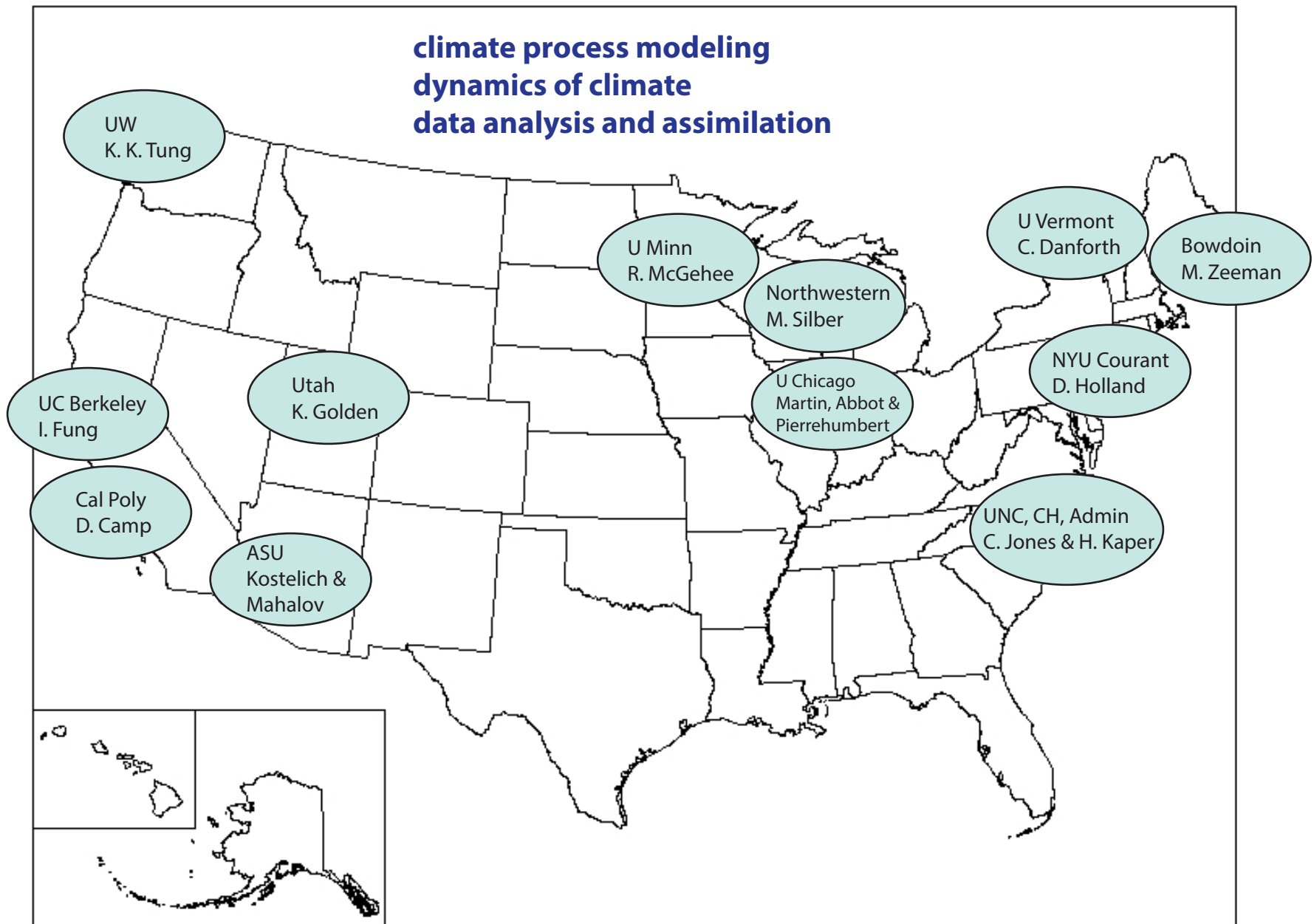
Applied Computational Analysis Program

Arctic and Global Prediction Program



***Buchanan Bay, Antarctica    Mertz Glacier Polynya Experiment    July 1999***

# ***Mathematics and Climate Research Network (MCRN)***



***NSF DMS 2010-2015, Lorenz postdocs, grad, undergrad, polar expeditions***

***Jones, Golden, Kaper, Zeeman***



# Fire endangers Hobart's ice ship

By DAVID CARRIGG

AN engine-room fire has left the Hobart-based Antarctic research ship *Aurora Australis* without power in dangerous sea ice off the Antarctic coast.

None of the 79 people on board was injured in the blaze, which broke out early yesterday morning while the ship was in deep water 185km off the coast.

The extent of the damage is not known.

Australian Antarctic Division director Rex Moncur said the fire was extinguished by flooding the engine room with an inert gas.

The gas had to be cleared before crew wearing breathing apparatus could enter and assess the situation.

He said it could be some time before the extent of damage was known.

The 25 crew and 54 expeditioners, mostly from Hobart, would wear thermal clothing and stay below decks to keep warm.

"There is always a risk of becoming ice-bound in these waters at this time of the year but at this stage we don't expect to launch a rescue mission from Hobart," Mr Moncur said.

The ship was in regular radio contact with the Antarctic Div-



A file photo of the *Aurora Australis* in Antarctica.

ision's Hobart office.

He expected the expeditioners and crew to abandon the pioneering winter voyage and return the ship to Hobart for repairs in about a week.

The Antarctic Division, which hires the ship from P&O Australia, would not be hiring another vessel for the expedition.

"It's a pretty specialist vessel so you couldn't get the sort of research capability that this ship has got readily available," Mr Moncur said.

"We hope the next voyage can still proceed on schedule, which is early September."

The *Aurora Australis* is owned by P&O Australia and chartered by the Antarctic Div-

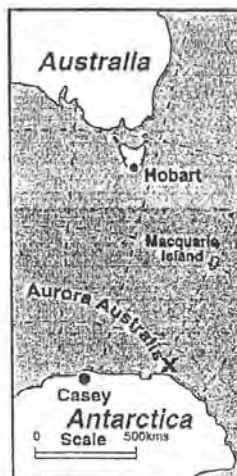
ision for about \$11 million a year.

P&O Australia managing director Richard Hein said yesterday the company was assessing the situation and a number of rescue options were being considered.

It was too early to say whether P&O would be liable for the cost of the aborted mission.

The vessel left Hobart last Wednesday for a seven-week voyage mainly to study a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

The ship was nearing the polynya when the fire broke out.



Oceanographers believe a closer study of the phenomenon will lead to a better understanding of climate change.

CSIRO Marine Research oceanographer Steve Rintoul said the dense bottom water, created only in a few places in Antarctica and to a lesser extent in the North Atlantic, was critical to the chemistry and biology of the world's oceans.

## Fire strands Antarctic ship in sea ice

AN engine room fire has disabled the icebreaker *Aurora Australis* in sea ice, deep in Antarctic waters.

There were no injuries and the ship was not in danger after Tuesday night's fire.

Australian Antarctic Division director Mr Rex Moncur said. But Mr Moncur said he expected it would have to abandon its pioneering mid-winter voyage to the edge of the Ant-

arctic continent and return to Hobart for repairs.

The cause of the fire was not known but the engines have been turned off, with the ship 100 nautical miles from the Antarctic coast.

### THE CANBERRA TIMES

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## Antarctic voyage stopped by fire

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Australian Antarctic Division director Rex Moncur said there were no injuries and the ship was not in danger after Tuesday night's fire.

But Mr Moncur said he expected *Aurora Australis* would have to abandon its pioneering mid-winter voyage to the edge of the Antarctic continent to return to Hobart for repairs.

The fire had been extinguished and the engines were turned off, leaving the ship in sea ice about 100 nautical miles from the Antarctic coast, he said. The weather was good.

Crew had to wear breathing apparatus to enter the engine room and it was likely to be 24 hours before the damage could be fully assessed.

The *Aurora*, with 54 expeditioners and 25 crew, left Hobart last Wednesday for a seven-week voyage which was to have focused on a polynya, an area where savage winds break up the sea ice and cause heavy, salt-laden water to sink to the bottom.

Mr Moncur said, the cause of the fire was not yet known.

2:45 am July 22, 1998

"Please don't be alarmed but we have an uncontrolled fire in the engine room ...."

about 10 minutes later ...

"Please don't be alarmed but we're lowering the lifeboats ...."

*Sydney Morning Herald*  
23 July, 1998

### ICEBREAKER BURNS

A pioneering \$2-million Australian scientific voyage to the mid-winter Antarctic polynya is expected to be scrapped following an engine room fire on the *Aurora Australis* yesterday. The 54 people on board were forced on deck in the

