Microbial Ecology and the Physics of Sea Ice

Kenneth M. Golden Department of Mathematics University of Utah



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of climate change

polar ice caps critical to climate in reflecting sunlight during summer

the summer Arctic sea ice pack is melting



National Snow and Ice Data Center

Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million square kilometers September 2012 -- 3.4 million square kilometers



Arctic sea ice decline: faster than predicted by climate models



challenge

represent sea ice more rigorously in climate models

account for key processes such as melt pond evolution



Impact of melt ponds on Arctic sea ice simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, JGR Oceans 2012

For simulations with ponds September ice volume is nearly 40% lower.

... and other sub-grid scale structures and processes *linkage of scales*

Sea Ice is a Multiscale Composite Material

sea ice microstructure

brine inclusions



Weeks & Assur 1969

millimeters

polycrystals



Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole

K. Golden

sea ice mesostructure

H. Eicken

Golden et al. GRL 2007

sea ice macrostructure

centimeters

Arctic melt ponds



Antarctic pressure ridges

sea ice floes



sea ice pack



K. Frey

K. Golden

J. Weller



NASA

meters

What is this talk about?

1. LIFE IN THE ICE

sea ice microphysics and fluid transport

2. LIFE UNDER THE ICE

melt ponds, under-ice light field, algal blooms

3. Species competition - resources depend on climate

Solving problems in physics and biology of sea ice drives advances in theory of composite materials and ecological systems.

Microbial life IN sea ice

sea ice microphysics

fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities







Antarctic surface flooding and snow-ice formation

evolution of salinity profiles
ocean-ice-air exchanges of heat, CO₂

sea ice ecosystem



sea ice algae support life in the polar oceans

fluid permeability k of a porous medium



porous

concrete

how much water gets through the sample per unit time?

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Darcy's Law for slow viscous flow in a porous medium



 $\mathbf{k} =$ fluid permeability tensor

Critical behavior of fluid transport in sea ice



critical brine volume fraction $\phi_c \approx 5\%$ \checkmark $T_c \approx -5^{\circ}C, S \approx 5$ ppt

RULE OF FIVES

Golden, Ackley, Lytle Science 1998 Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007 Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

nutrient replenishment controlled by ice permeability

biological activity turns on or off according to *rule of fives*

Golden, Ackley, Lytle

Science 1998

Fritsen, Lytle, Ackley, Sullivan Science 1994

critical behavior of microbial activity



Why is the rule of fives true?

percolation theory

probabilistic theory of connectedness



bond \longrightarrow *open with probability p closed with probability 1-p*

percolation threshold $p_c = 1/2$ for d = 2

smallest *p* for which there is an infinite open cluster

Continuum percolation model for *stealthy* materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on ice production and algal growth

 $\phi_c \approx 5\%$ Golden, Ackley, Lytle, *Science*, 1998



sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



PIPE BOUNDS on vertical fluid permeability k

Golden, Heaton, Eicken, Lytle, Mech. Materials 2006 Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophys. Res. Lett. 2007

> vertical pipes with appropriate radii maximize k





fluid analog of arithmetic mean upper bound for effective conductivity of composites (Wiener 1912)

optimal coated cylinder geometry



$$x \leq \frac{\phi \langle R^4 \rangle}{8 \langle R^2 \rangle} = \frac{\phi}{8} \langle R^2 \rangle e^{\sigma^2}$$

inclusion cross sectional areas A lognormally distributed

In(A) normally distributed, mean μ (increases with T) variance $\sigma^{_2}(\mbox{Gow and Perovich 96})$

get bounds through variational analyis of **trapping constant** γ for diffusion process in pore space with absorbing BC

Torquato and Pham, PRL 2004

 $\mathbf{k} \leq \gamma^{-1} \mathbf{I}$

for any ergodic porous medium (Torquato 2002, 2004)

BACTERIAL FORAGING

Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



 $R_{i,j}^{h} \xrightarrow{R_{i,j}^{v}} R_{i,j}^{h}$

Zhu, Jabini, Golden, Eicken, Morris *Ann. Glac*. 2006

- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution;
 Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k.
- Rigorous bound on *k* for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden Multiscale Modeling and Simulation, 2018

How does the biology affect the physics?

Notices Anterior Mathematical Society

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



measuring fluid permeability of Antarctic sea ice

SIPEX 2007

higher threshold for fluid flow in Antarctic granular sea ice

columnar

5%

granular



10%

Golden, Sampson, Gully, Lubbers, Tison 2020

tracers flowing through inverted sea ice blocks







advection enhanced diffusion

effective diffusivity

nutrient and salt transport in sea ice heat transport in sea ice with convection sea ice floes in winds and ocean currents tracers, buoys diffusing in ocean eddies diffusion of pollutants in atmosphere

advection diffusion equation with a velocity field $ec{u}$

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa_0 \Delta T$$
$$\vec{\nabla} \cdot \vec{u} = 0$$
$$homogenize$$
$$\frac{\partial \overline{T}}{\partial t} = \kappa^* \Delta \overline{T}$$

κ^* effective diffusivity

Stieltjes integral for κ^* with spectral measure

Avellaneda and Majda, PRL 89, CMP 91

Murphy, Cherkaev, Xin, Zhu, Golden, Ann. Math. Sci. Appl. 2017 Murphy, Cherkaev, Zhu, Xin, Golden, J. Math. Phys. 2020









Stieltjes Integral Representation for Advection Diffusion

Murphy, Cherkaev, Zhu, Xin, Golden, J. Math. Phys. 2020

$$\kappa^* = \kappa \left(1 + \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2} \right), \quad F(\kappa) = \int_{-\infty}^{\infty} \frac{d\mu(\tau)}{\kappa^2 + \tau^2}$$

- μ is a positive definite measure corresponding to the spectral resolution of the self-adjoint operator $i\Gamma H\Gamma$
- H = stream matrix , $\kappa =$ local diffusivity
- $\Gamma :=
 abla (-\Delta)^{-1}
 abla \cdot$, Δ is the Laplace operator
- $i\Gamma H\Gamma$ is bounded for time independent flows
- $F(\kappa)$ is analytic off the spectral interval in the κ -plane

separation of material properties and flow field spectral measure calculations

Rigorous bounds on convection enhanced thermal conductivity of sea ice

Kraitzman, Hardenbrook, Murphy, Zhu, Cherkaev, Strong, Golden 2020



cat's eye flow model for brine convection cells

similar bounds for shear flows

rigorous bounds assuming information on flow field INSIDE inclusions

Kraitzman, Cherkaev, Golden SIAM J. Appl. Math (in revision), 2020



rigorous Padé bounds from Stieltjes integral + analytical calculations of moments of measure



Ice floe diffusion in winds and currents

Anomalous diffusion and sea ice dynamics

sub- and super-diffusive behavior of motion of sea ice floes as tracked by buoy data

Jennifer Lukovich, Jennifer Hutchings, David Barber, Ann. Glac. 2015

Huy Dinh, Elena Cherkaev, Ken Golden, 2019

Home ranges in moving habitats: polar bears and sea ice

"diffusive" polar bear motion superimposed with drifting sea ice

Marie Auger-Méthé, Mark Lewis, Andrew Derocher, Ecography, 2016



Microbial life UNDER sea ice

melt ponds, algal blooms

melt pond formation and albedo evolution:

- major drivers in polar climate
- key challenge for global climate models

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham, Taylor, Worster 2006 Flocco, Feltham 2007 Skyllingstad, Paulson, Perovich 2009 Flocco, Feltham, Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012



Continuum percolation model for melt pond evolution level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography



intersections of a plane with the surface define melt ponds







electronic transport in disordered media

diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

Ising Model for a Ferromagnet



Curie point critical temperature

Baker, PRL 1968

Ising model for ferromagnets —> Ising model for melt ponds

Ma, Sudakov, Strong, Golden, New J. Phys., 2019

 $\mathcal{H} = -\sum_{i}^{N} H_{i} s_{i} - J \sum_{\langle i,j \rangle}^{N} s_{i} s_{j} \qquad s_{i} = \begin{cases} \uparrow & +1 & \text{water (spin up)} \\ \downarrow & -1 & \text{ice (spin down)} \end{cases} \quad \begin{array}{c} \text{random mathematication mathematication} \\ \text{represents} \\ \hline & \text{albedo} \\ \hline & \text{albedo} \\ \end{array}$

random magnetic field represents snow topography

only nearest neighbor patches interact

Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system "flows" toward metastable equilibria.

Order from Disorder



ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



2011 massive under-ice algal bloom

Arrigo et al., Science 2012

melt ponds act as *WINDOWS*

allowing light through sea ice



bloom

no bloom

Have we crossed into a new ecological regime?

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, *Science Advances*, 2017

The distribution of solar energy under ponded sea ice Horvat, Flocco, Rees Jones, Roach, Golden, 2020

(2015 AMS MRC @ Snowbird)





melt ponds are WINDOWS

light reaches the upper ocean



Perovich

The frequency and extent of sub-ice phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder, Flocco, Feltham, Science Advances, 2017

- Model for initiation of light-limited phytoplankton bloom (depth of mixed layer, ice thickness, melt pond area fraction, ...)
- Thinner summertime Arctic sea ice is increasingly covered in melt ponds, which permit more light penetration.
- Marked increase in light conditions conducive to sub-ice blooms.
- As little as 20 years ago, conditions for sub-ice blooms may have been uncommon; frequency has increased so that nearly 30% of the ice-covered Arctic Ocean in July permits sub-ice blooms.

Recent climate change may have significantly altered the ecology of the Arctic Ocean.

The distribution of solar energy under ponded first-year sea ice

Horvat, Flocco, Rees Jones, Roach, Golden, Geophys. Res. Lett. 2020

- Model for 3D light field under ponded sea ice.
- Distribution of solar energy at depth influenced by *shape and connectivity* of melt ponds, as well as area fraction.
- Aggregate properties of the sub-ice light field, such as a significant enhancement of available solar energy under the ice, are controlled by parameter closely related to pond fractal geometry.
- Model and analysis explain how melt pond geometry *homogenizes* under-ice light field, affecting habitability.

Pond geometry affects the ecology of the Arctic Ocean.

The Melt Pond Conundrum:

How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

J. Geophys. Res. Oceans 2017

2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE) aboard USCGC Healy





Large ecosystems in transition: bifurcations and mass extinction

I. Sudakov, S. Vakulenko, D. Kirievskaya, K. M. Golden, Ecological Complexity, 2017

model of multispecies populations competing for distributed resources

coupling climate and population dynamics via resources

feedback between species abundances and resources through temperature

resource competition model

Leon & Tumpson, *J. Theor. Biol.* 1975 Tilman, *Ecology*, 1977 Huisman & Weissing, *Nature*, 1999

(solved "plankton paradox")

$$\frac{dx_i}{dt} = x_i(-r_i + \phi_i(v) - \sum_{j=1}^N \gamma_{ij} x_j), \quad 1 \le i \le N$$

$$\frac{dv}{dt} = D(S-v) - \sum_{j=1}^{N} c_j x_j \phi_j(v)$$

$$\phi_j(v) = \frac{a_j v}{K_j + v} , \quad a_j, \ K_j > 0$$

- x_i species abundance
- ϕ_i species growth rate
- r_i species mortality rate
- γ_{ij} describe competition (e.g. toxic compounds)
 - ^{*i*} diagonals ~ self-regulation restricting abundances
- *S* supply concentration of resource
- v resource availability
- *D* resource turnover rate
- *c*_j determine how species share resource
- K_j saturation constants

Extend model to *M* resources, whose supplies depend on temperature *T*, which depends on species abundances.

$$T = ar{T} + \Delta T, \quad \Delta T = \sum_{k=1}^{N} \mu_{kj} x_j$$

 $S_k = ar{S}_k(ar{T}) + \Delta S_k + O(\Delta T^2)$
 $\Delta S_k = \sum_{k=1}^{N} b_{kj}(ar{T}) x_j, \quad k = 1, ..., M$

 $b_{kj} = \frac{dS_k(\bar{T})}{d\bar{T}} \mu_{kj} \begin{cases} > 0 \text{ positive feedback} \\ < 0 \text{ negative feedback} \end{cases}$

reduction to Lotka-Volterra system

$|b_{ik}| \ll \gamma$ weak climate coupling

close to 'competitive systems' - exhibit no stable periodic or chaotic regimes: almost all trajectories converge to equilibria

$|b_{ik}| \gg \gamma$ strong climate coupling

- M = 1, possible that all N species survive in equilibrium, or coexistence of many equilibria
- M = 2, feedback coeffs have different signs, periodic sols
- *M* > 2, system can produce time chaotic solutions

when the number of species increases, so does the likelihood of a sharp drop in species number as the climate changes and feedback processes grow stronger

The model exhibits coexistence of many species, yet also displays the possibility of catastrophic bifurcations, where all species become extinct under the influence of abiotic factors (strong climate coupling).



Conclusions

- 1. The physics of fluid transport in porous composites regulates microbial life inside sea ice.
- 2. The geometry of melt ponds controls light in the upper ocean, and initiation of under-ice algal blooms.
- 3. Resource competition models provide tools to study the complexity of these microbial communities.

THANK YOU

Office of Naval Research

Arctic and Global Prediction Program Applied and Computational Analysis Program

National Science Foundation

Division of Mathematical Sciences Division of Polar Programs







Mathematics and Climate Research Network



Australian Government

Department of the Environment and Water Resources Australian Antarctic Division











Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999