

PROJECT SUMMARY: Homogenization for Sea Ice

Overview: The precipitous losses of summer Arctic sea ice observed in the past few decades have far reaching impacts on Earth's climate system, and have significantly outpaced the projections of most climate models. One of the fundamental challenges of climate science is to develop more rigorous representations of sea ice in climate models, and account for important sub-grid scale processes and their projection onto larger scale patterns of change. Sea ice exhibits composite structure over many orders of magnitude, from the mm scale brine inclusions laced throughout the ice, to km scale sea ice floes which form the "microstructure" of the ice pack, viewed as a composite of ice and ocean. We propose to apply powerful methods of homogenization for composites to analyzing effective behavior of sea ice on scales relevant to climate models. We consider several key issues in sea ice modeling where the mathematics of homogenization and statistical physics can provide a rigorous framework for analysis and computation of sea ice properties.

1. Dense pack ice transitions to open ocean over a region of broken ice termed the *marginal ice zone* (MIZ). The "width" of the MIZ is an important climatic length scale. A recent advance by co-PI Strong in objectively identifying MIZ width and geometry and then finding striking trends as the climate has warmed is based on an idealized concentration field satisfying Laplace's equation. Here we will generalize this analysis to include an inhomogeneous "diffusivity" in the transport equation, which can then capture the *actual* satellite-derived concentration field. We will investigate how this effective coefficient is then related to smaller scale information about ice pack *microgeometry*. This approach will provide a basis for analytical investigation of MIZ dynamics including its susceptibility to deformation.
2. The *analytic continuation method* (ACM) yields Stieltjes integral representations for the effective transport coefficients of two phase composites, which involve the spectral measures of a self adjoint operator. This operator depends only on the microgeometry, and is a random matrix in discretizations of the medium. We will investigate how the spectral measures and eigenvalue statistics are related to floe geometry and percolation properties of floe configurations. There are similar formulas for the effective parameters of polycrystals and for the effective diffusivity in advection diffusion equations. We will use these representations to investigate transport in sea ice velocity fields, advection-enhanced thermal transport in sea ice, and the properties of sea ice as a polycrystalline material.
3. Recently a polydisc integral formula for the elasticity of two-phase composites was obtained, involving spectral measures on the torus whose moments are related to microstructural statistics. We will explore similar representations for ice pack rheology, and investigate the structure of the spectral measures and their dependence on composite microgeometry.

Intellectual Merit: This research addresses one of the most critical scientific challenges of our time – to understand how Earth's climate is changing and improve projections of its future trajectory. Our approach brings sophisticated mathematics to key problems where more rigor is needed.

Broader Impacts: The effects of planetary warming are far reaching, and a better understanding of sea ice and its role in climate will improve our ability to predict changes in storm tracks, precipitation and temperature patterns, etc. affecting large populations. The PI has had extensive interactions with the media, and gives a large number of public lectures, so that the results of this research will likely be widely distributed to the public, beyond usual academic channels. Results of this research will be integrated into graduate and undergraduate mathematical climate and sea ice courses taught by the PI and co-PI's.