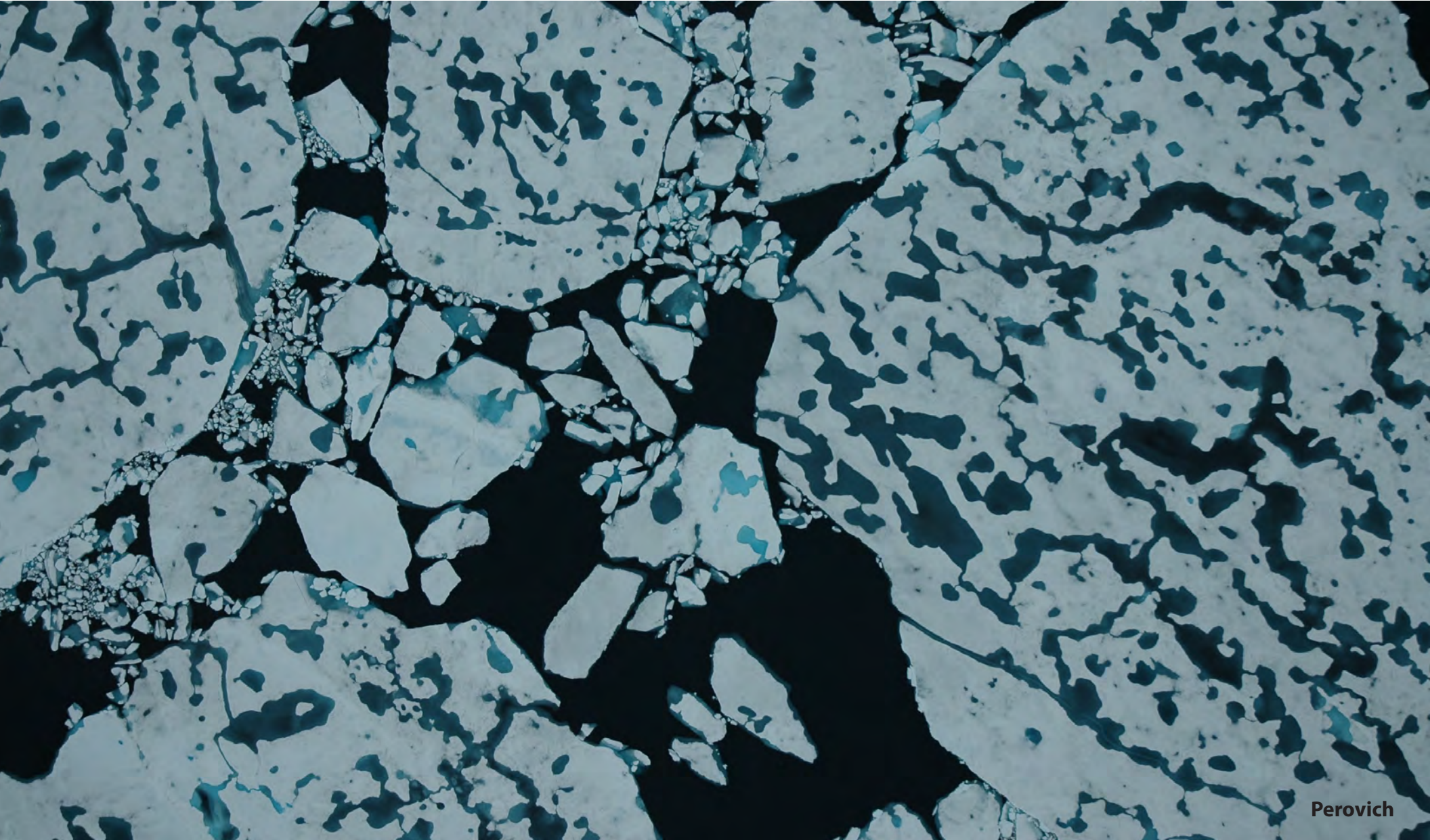


Modeling Sea Ice

Kenneth M. Golden University of Utah



SEA ICE covers ~12% of Earth's ocean surface

- boundary between ocean and atmosphere
- mediates exchange of heat, gases, momentum
- global ocean circulation
- hosts rich ecosystem
- indicator of **climate change**

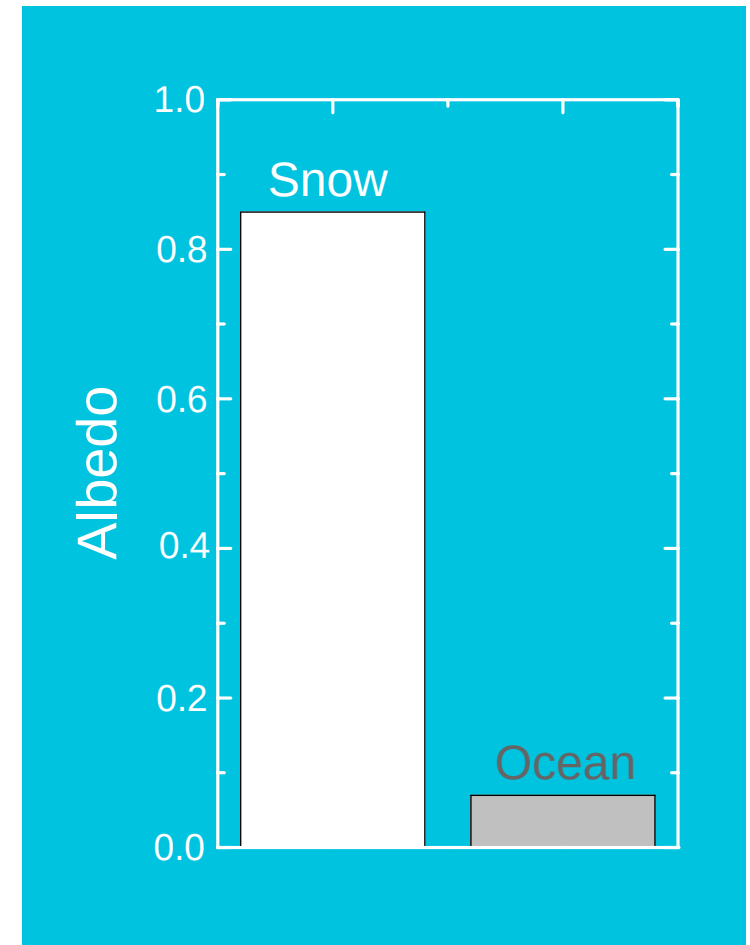


polar ice caps critical
to climate in reflecting
sunlight during summer

polar ice caps critical to global climate in reflecting incoming solar radiation



white snow and ice
reflect



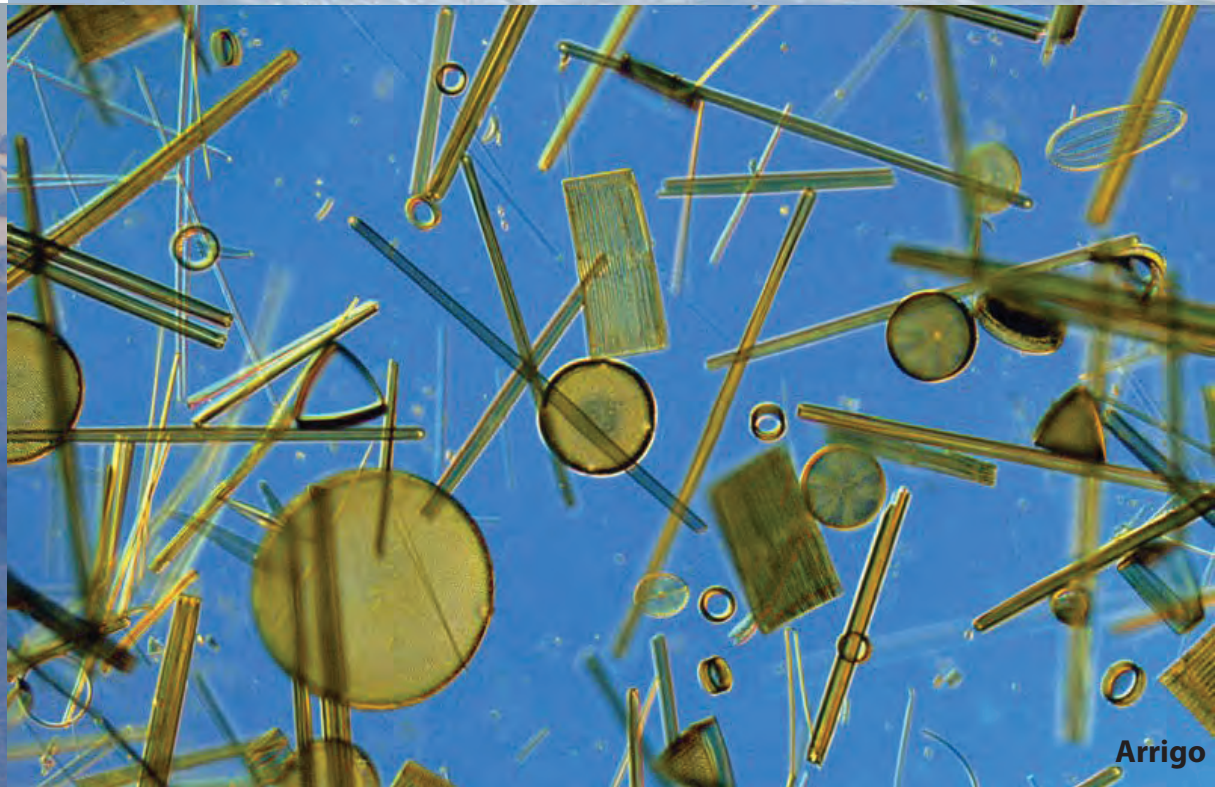
dark water and land
absorb

$$\text{albedo } \alpha = \frac{\text{reflected sunlight}}{\text{incident sunlight}}$$

Microbial Ecology and the Physics of Sea Ice

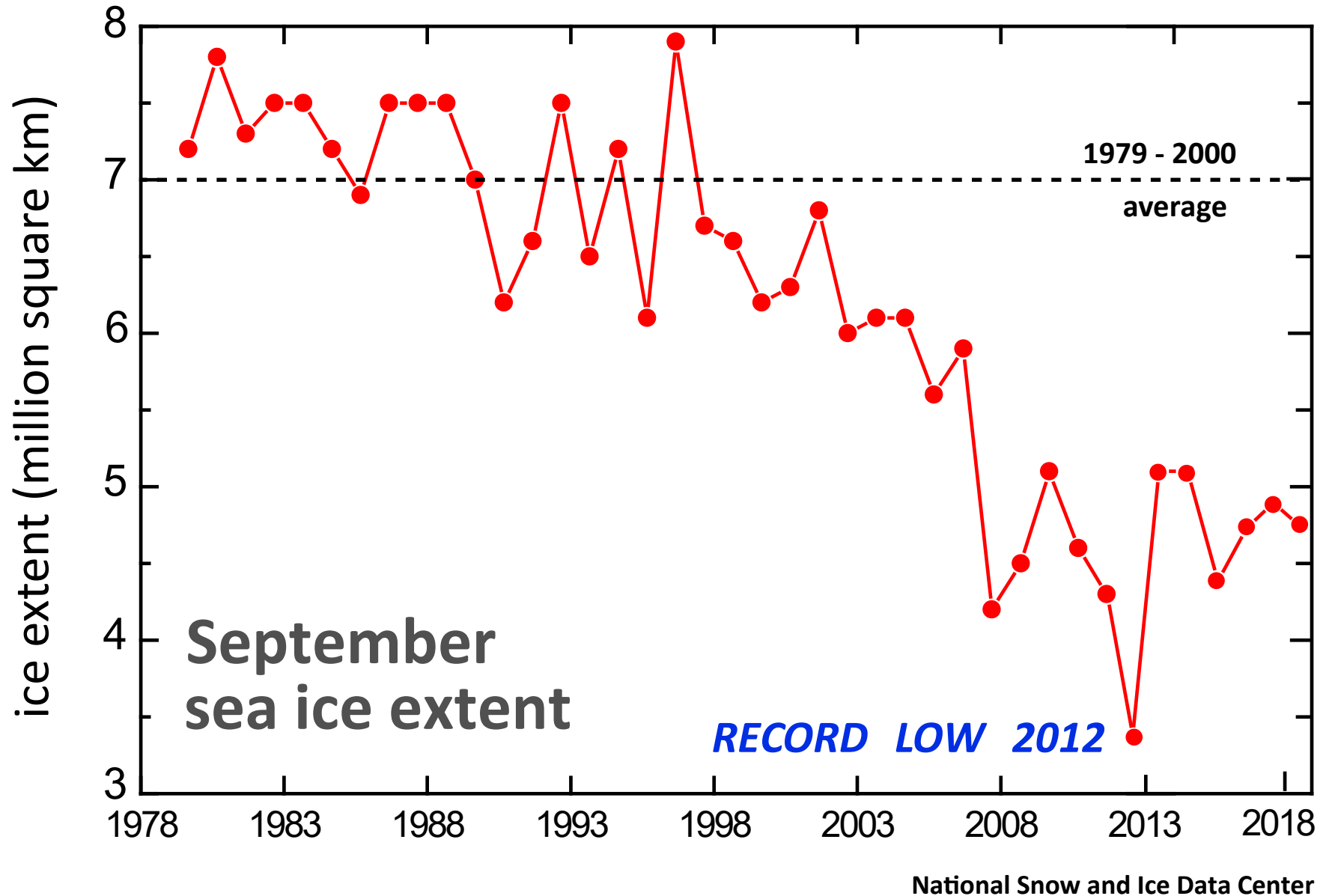


Golden



Arrigo

the summer Arctic sea ice pack is melting



Change in Arctic Sea Ice Extent

September 1980 -- 7.8 million km²

September 2012 -- 3.4 million km²



*recent losses
in comparison to
the United States*

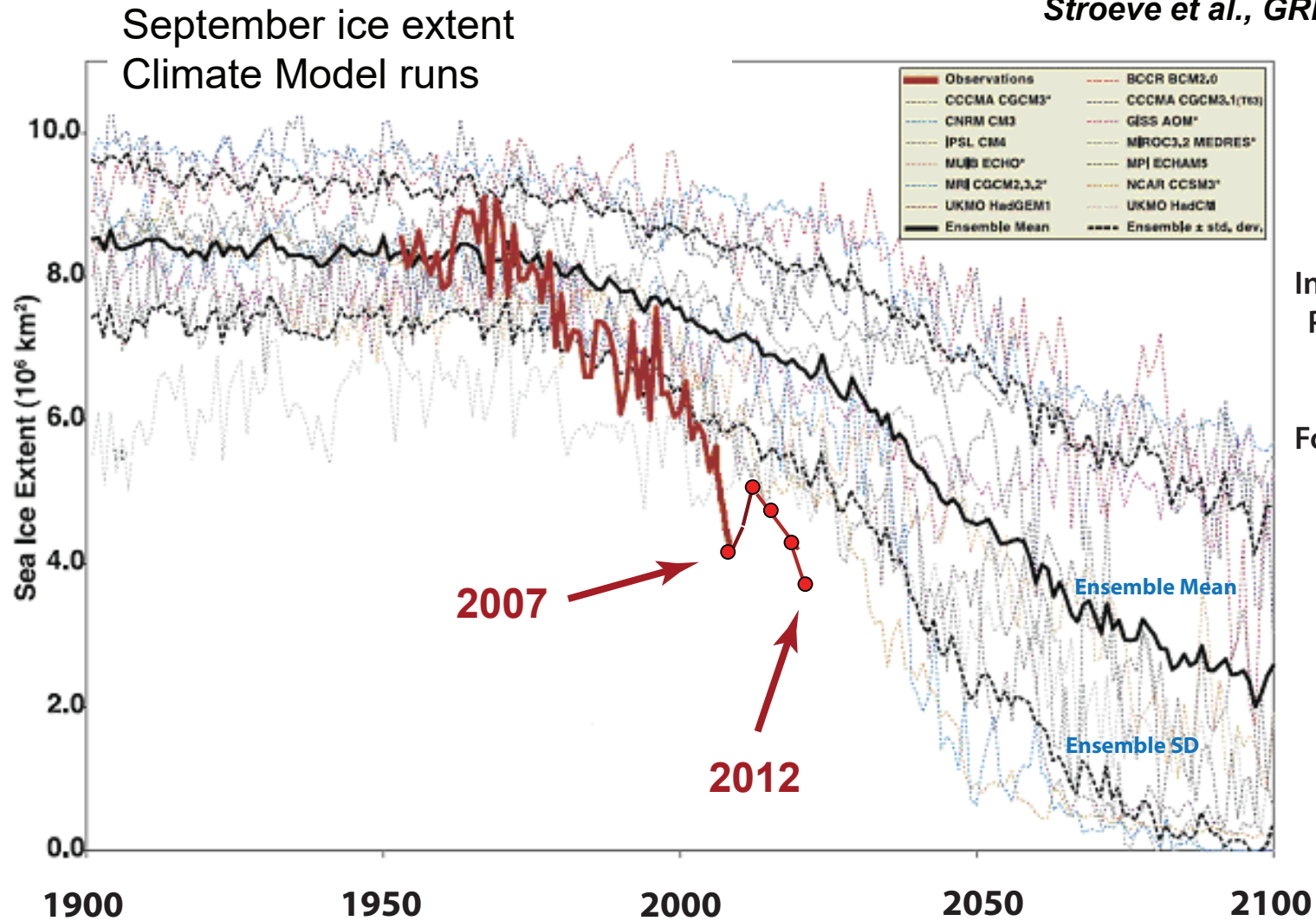
Perovich



Arctic sea ice decline: faster than predicted by climate models

Stroeve et al., GRL, 2007

Stroeve et al., GRL, 2012



**IPCC AR4
Models**

Intergovernmental
Panel on Climate
Change (IPCC)

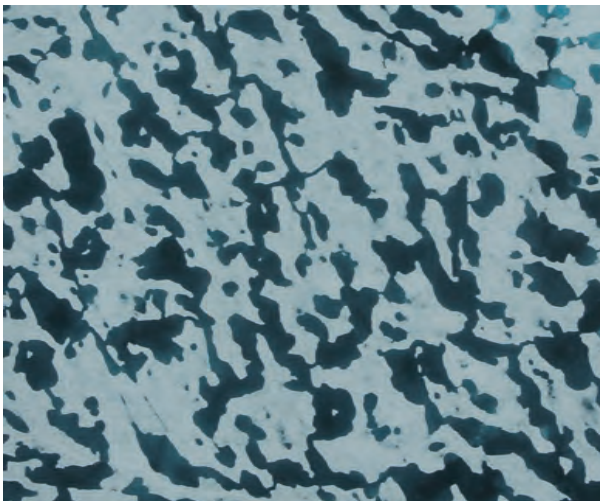
Fourth Assessment
AR4, 2007

challenge

represent sea ice more rigorously in climate models

account for key processes

such as melt pond evolution



Impact of melt ponds on Arctic sea ice
simulations from 1990 to 2007

Flocco, Schroeder, Feltham, Hunke, *JGR Oceans* 2012

**For simulations with ponds
September ice volume is nearly 40% lower.**

... and other sub-grid scale structures and processes

linkage of scales

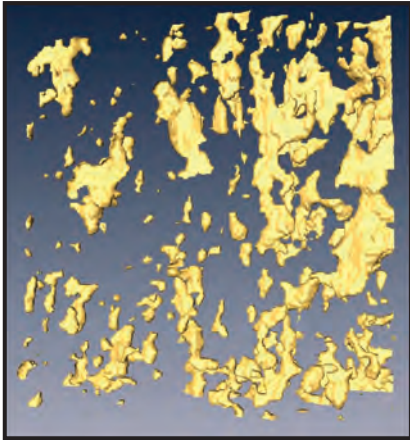
Sea Ice is a Multiscale Composite Material

sea ice microstructure

brine inclusions

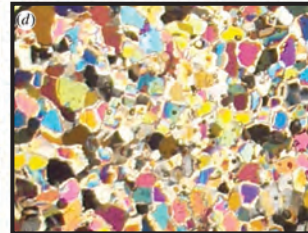
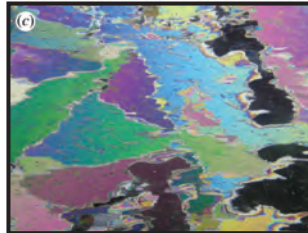


Weeks & Assur 1969



H. Eicken
Golden et al. GRL 2007

polycrystals

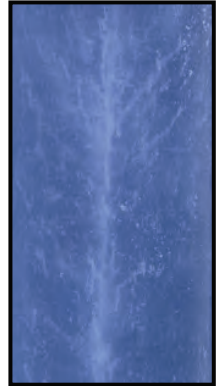


Gully et al. Proc. Roy. Soc. A 2015

brine channels



D. Cole



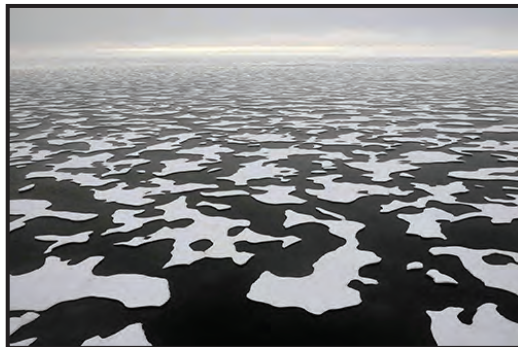
K. Golden

millimeters

centimeters

sea ice mesostructure

Arctic melt ponds



K. Frey

Antarctic pressure ridges



K. Golden

meters

sea ice macrostructure

sea ice floes



J. Weller

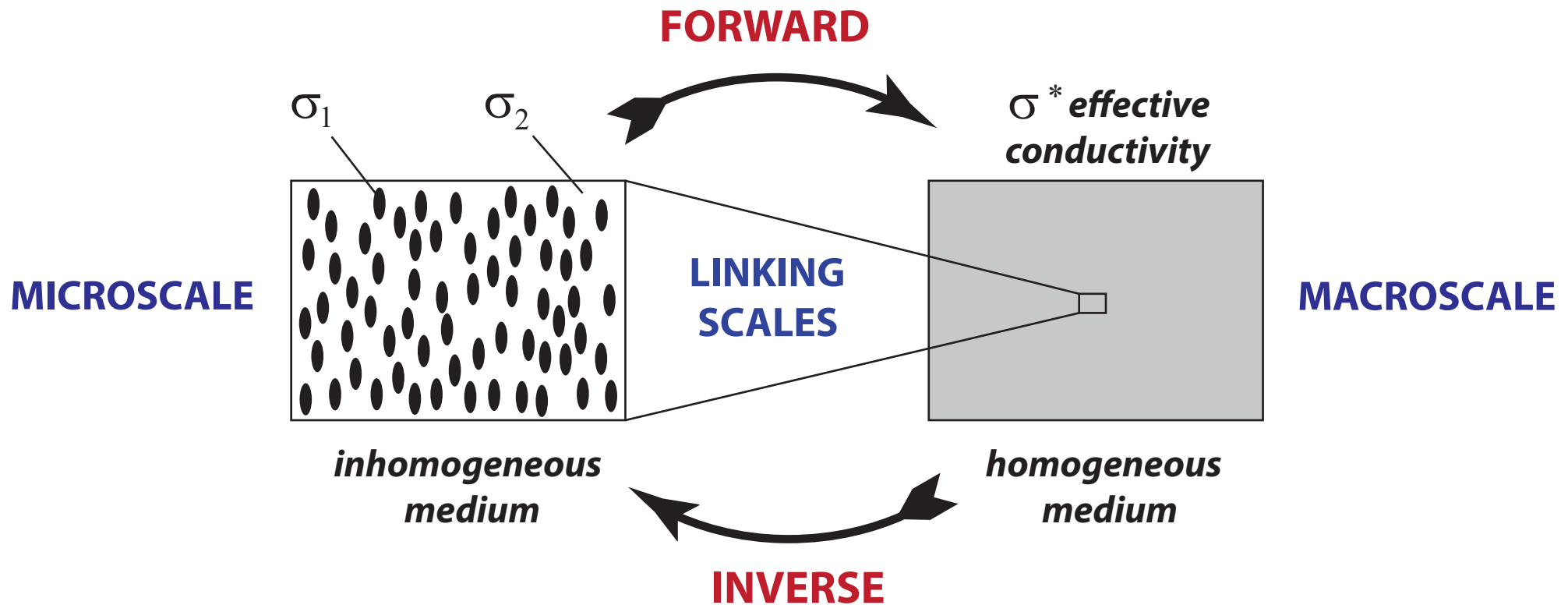
sea ice pack



NASA

kilometers

HOMOGENIZATION for Composite Materials



Maxwell 1873 : effective conductivity of a dilute suspension of spheres

Einstein 1906 : effective viscosity of a dilute suspension of rigid spheres in a fluid

*Wiener 1912 : arithmetic and harmonic mean **bounds** on effective conductivity*

*Hashin and Shtrikman 1962 : variational **bounds** on effective conductivity*

widespread use of composites in late 20th century due in large part to advances in mathematically predicting their effective properties

What is this talk about? **homogenization for multiscale composites**

the role of “microstructure” in determining sea ice effective properties

Using methods of **homogenization and statistical physics** to LINK SCALES
in the sea ice system ... compute effective behavior on scales relevant to
coarse-grained sea ice and climate models, process studies, ...

MICROSCALE: brine + polycrystalline structure; EM and fluid transport

MESOSCALE: advection diffusion, thermal transport, waves, melt ponds

MACROSCALE: ice transport, MIZ width and location, low order models

A tour of Stieltjes integrals in the study of sea ice and its role in climate.

***Solving problems in physics of sea ice drives
advances in theory of composite materials.***

cross - pollination

**bone, stealthy coatings
magnets, rat brains, RMT**

How do scales interact in the sea ice system?

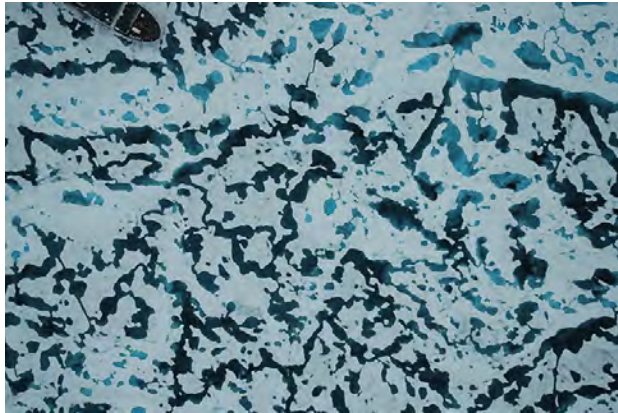


basin scale -
grid scale
albedo

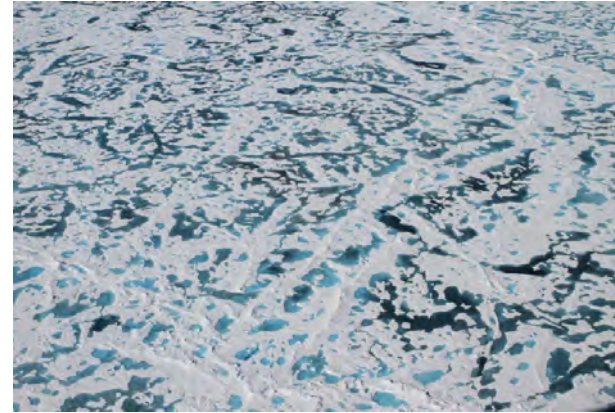
NASA

Linking Scales

km
scale
melt
ponds



km
scale
melt
ponds

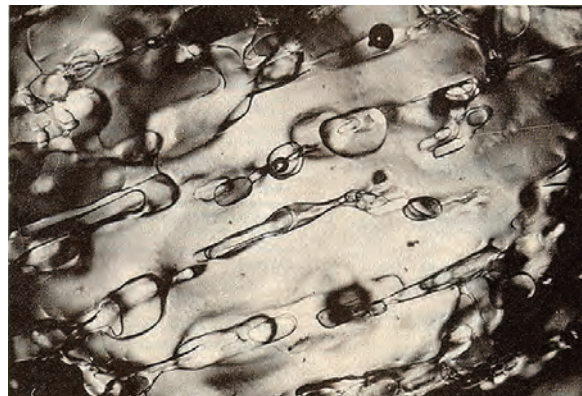


Perovich

Linking

Scales

mm
scale
brine
inclusions



meter
scale
snow
topography



sea ice microphysics

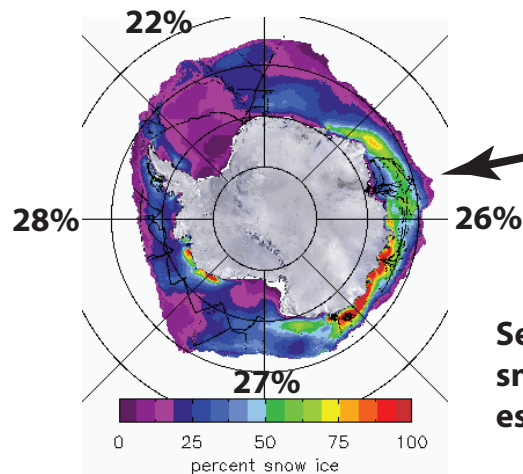
fluid transport

fluid flow through the porous microstructure of sea ice governs key processes in polar climate and ecosystems

evolution of Arctic melt ponds and sea ice albedo



nutrient flux for algal communities



T. Maksym and T. Markus, 2008

*Antarctic surface flooding
and snow-ice formation*

September
snow-ice
estimates

- evolution of salinity profiles
- ocean-ice-air exchanges of heat, CO_2

sea ice ecosystem



sea ice algae
support life in the polar oceans

fluid permeability of a porous medium



how much water gets through the sample per unit time?

Darcy's Law

for slow viscous flow in a porous medium

averaged fluid velocity

pressure gradient

$$\mathbf{v} = -\frac{\mathbf{k}}{\eta} \nabla p$$

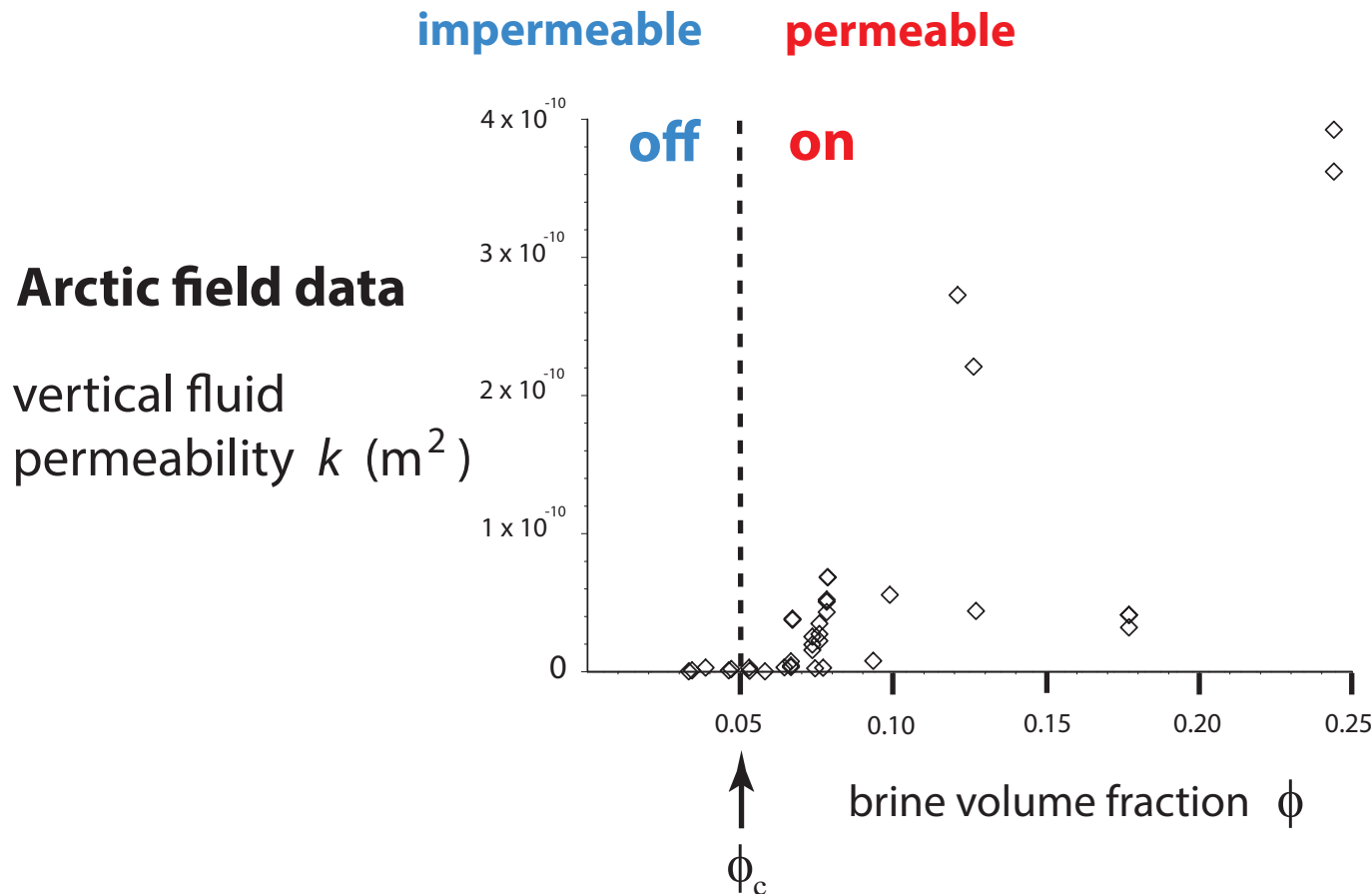
viscosity

$\mathbf{k}$ = fluid permeability tensor

HOMOGENIZATION

mathematics for analyzing effective behavior of heterogeneous systems

Critical behavior of fluid transport in sea ice



***“on - off” switch
for fluid flow***

critical brine volume fraction $\phi_c \approx 5\% \longleftrightarrow T_c \approx -5^\circ \text{C}, S \approx 5 \text{ ppt}$

RULE OF FIVES

Golden, Ackley, Lytle Science 1998

Golden, Eicken, Heaton, Miner, Pringle, Zhu GRL 2007

Pringle, Miner, Eicken, Golden J. Geophys. Res. 2009



sea ice algal communities

D. Thomas 2004

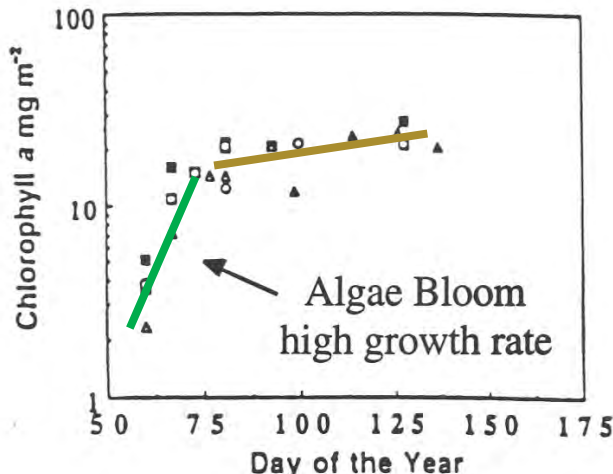
nutrient replenishment
controlled by ice permeability

biological activity turns on
or off according to
rule of fives

Golden, Ackley, Lytle **Science 1998**

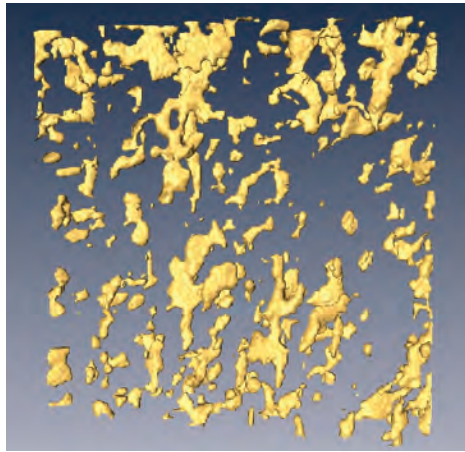
Fritsen, Lytle, Ackley, Sullivan **Science 1994**

critical behavior of microbial activity

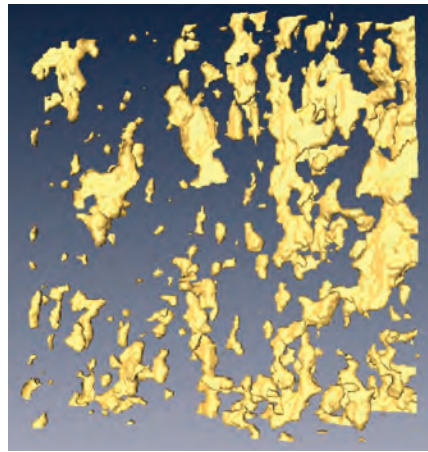


Convection-fueled algae bloom
Ice Station Weddell

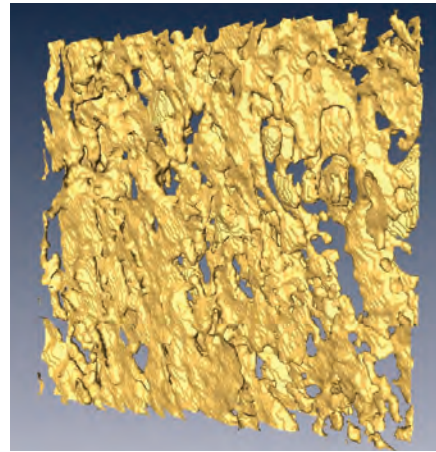
brine volume fraction and **connectivity** increase with temperature



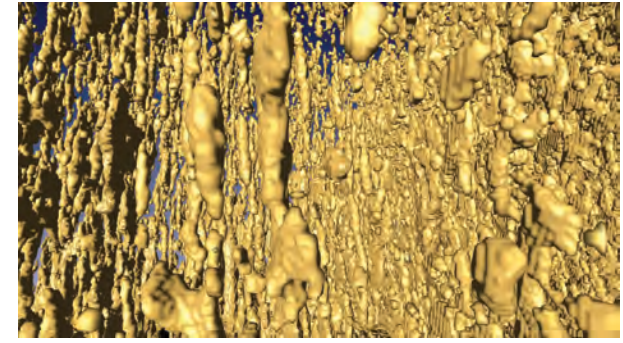
$T = -15\text{ }^{\circ}\text{C}$, $\phi = 0.033$



$T = -6\text{ }^{\circ}\text{C}$, $\phi = 0.075$



$T = -3\text{ }^{\circ}\text{C}$, $\phi = 0.143$



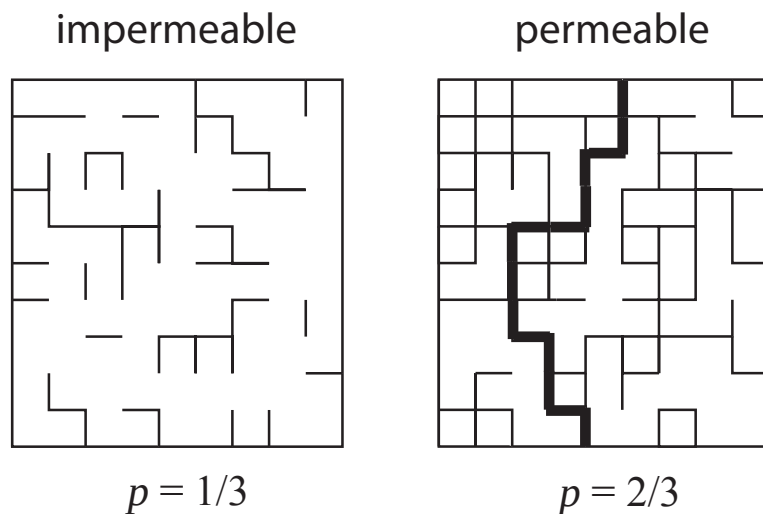
$T = -4\text{ }^{\circ}\text{C}$, $\phi = 0.113$

X-ray tomography for brine phase in sea ice

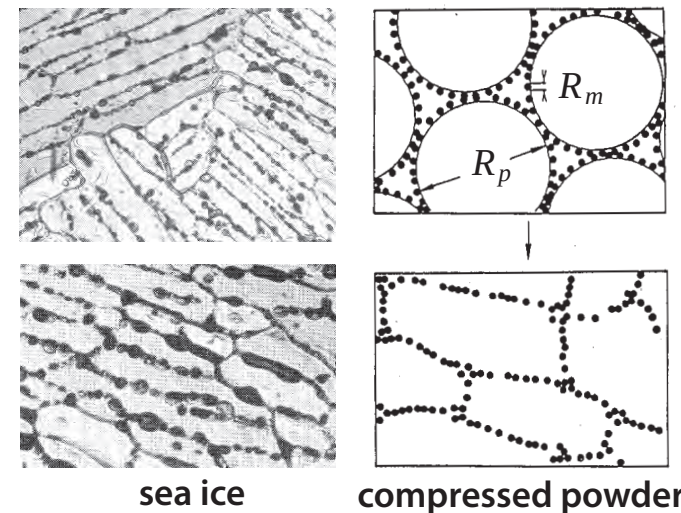
Golden, Eicken, *et al.*, *Geophysical Research Letters* 2007

PERCOLATION THRESHOLD $\phi_c \approx 5\%$

Golden, Ackley, Lytle, *Science* 1998



lattice percolation



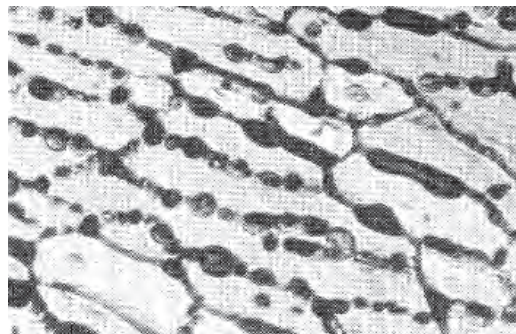
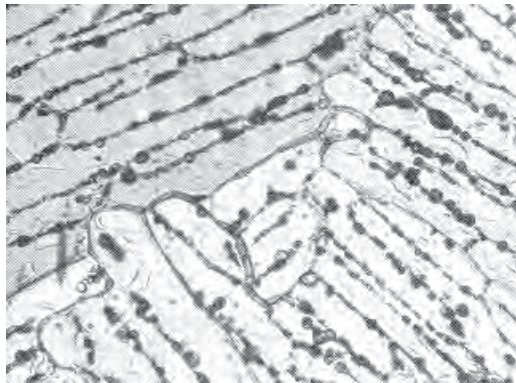
continuum percolation

Kusy, Turner
Nature 1971

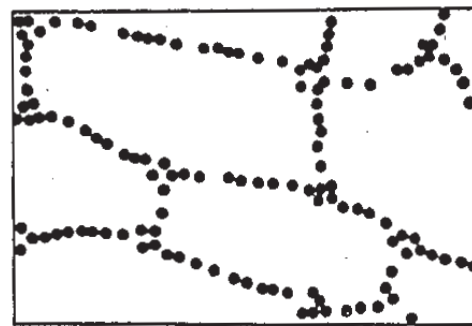
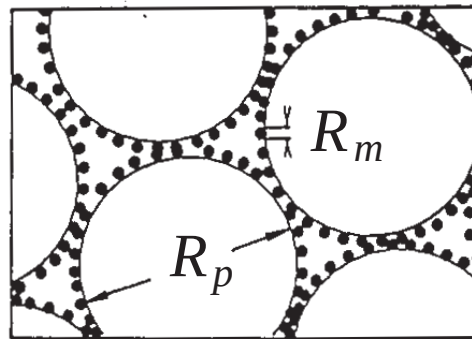
Continuum percolation model for **stealthy** materials applied to sea ice microstructure explains **Rule of Fives** and Antarctic data on **ice production** and **algal growth**

$$\phi_c \approx 5 \%$$

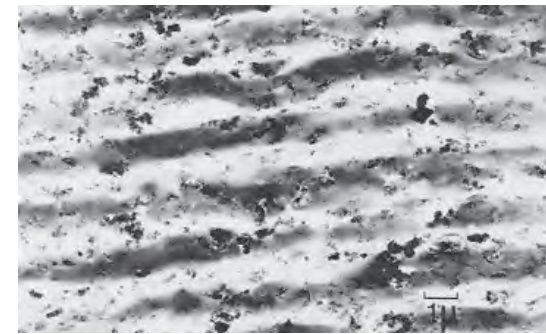
Golden, Ackley, Lytle, *Science*, 1998



sea ice



compressed
powder

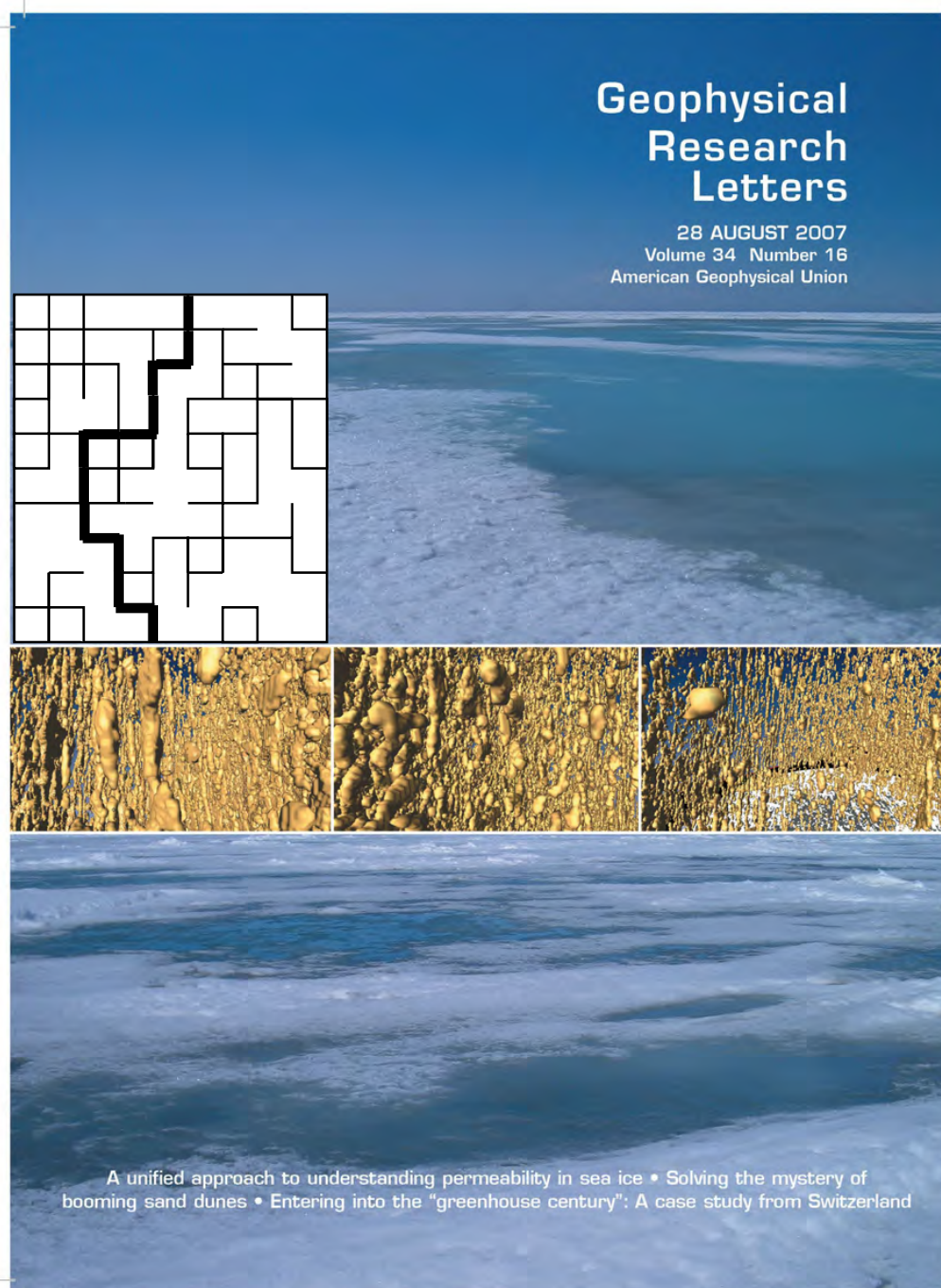


radar absorbing
composite

sea ice is radar absorbing

Thermal evolution of permeability and microstructure in sea ice

Golden, Eicken, Heaton, Miner, Pringle, Zhu, Geophysical Research Letters 2007



micro-scale
controls
macro-scale
processes

percolation theory

$$k(\phi) = k_0 (\phi - 0.05)^2$$

critical
exponent
t

$$k_0 = 3 \times 10^{-8} \text{ m}^2$$

***hierarchical model
network model
rigorous bounds***

agree closely
with field data

***X-ray tomography for
brine inclusions***

***unprecedented look
at thermal evolution
of brine phase and
its connectivity***

confirms rule of fives

***Pringle, Miner, Eicken, Golden
J. Geophys. Res. 2009***

Notices

of the American Mathematical Society

May 2009

Volume 56, Number 5

Climate Change and
the Mathematics of
Transport in Sea Ice

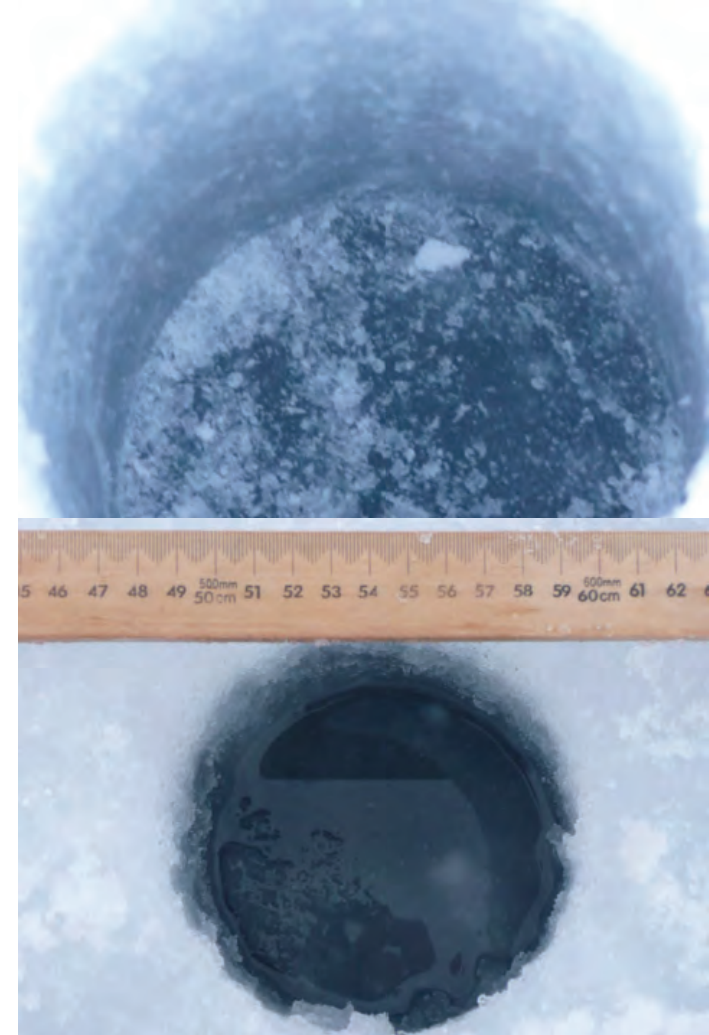
page 562

Mathematics and the
Internet: A Source of
Enormous Confusion
and Great Potential

page 586

photo by Jan Lieser

Real analysis in polar coordinates (see page 613)



***measuring
fluid permeability
of Antarctic sea ice***

SIPEX 2007

The Melt Pond Conundrum:

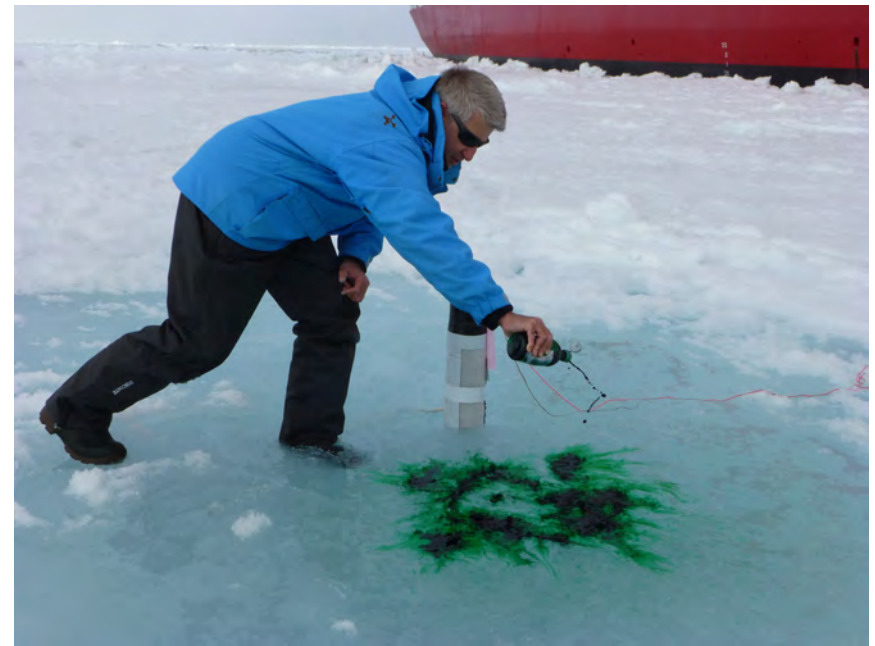
How can ponds form on top of sea ice that is highly permeable?

C. Polashenski, K. M. Golden, D. K. Perovich, E. Skyllingstad, A. Arnsten, C. Stwertka, N. Wright

Percolation Blockage: A Process that Enables Melt Pond Formation on First Year Arctic Sea Ice

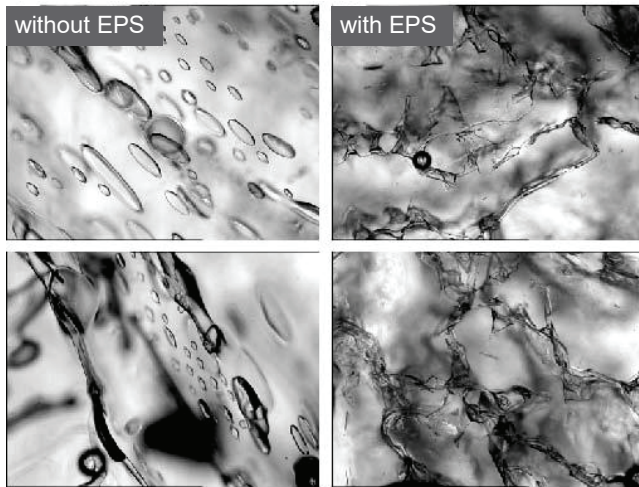
J. Geophys. Res. Oceans 2017

*2014 Study of Under Ice Blooms in the Chuckchi Ecosystem (SUBICE)
aboard USCGC Healy*

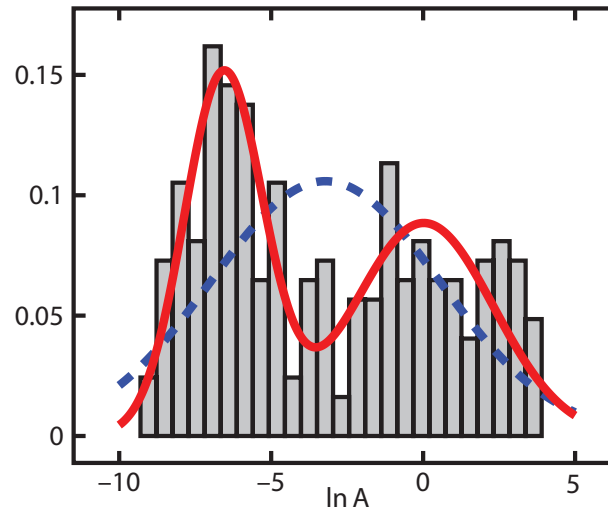


Sea ice algae secrete extracellular polymeric substances (EPS) affecting evolution of brine microstructure.

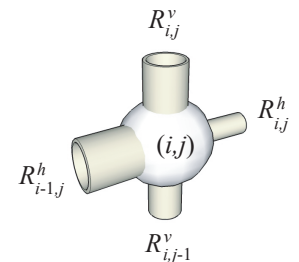
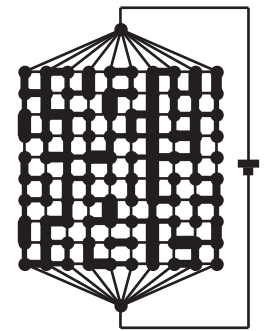
How does EPS affect fluid transport?



Krembs, Eicken, Deming, PNAS 2011



RANDOM PIPE MODEL



- **Bimodal** lognormal distribution for brine inclusions
- Develop random pipe network model with bimodal distribution; Use numerical methods that can handle larger variances in sizes.
- Results predict observed drop in fluid permeability k .
- Rigorous bound on k for bimodal distribution of pore sizes

Steffen, Epshteyn, Zhu, Bowler, Deming, Golden
Multiscale Modeling and Simulation, 2018

Zhu, Jabini, Golden,
Eicken, Morris
Ann. Glac. 2006

How does the biology affect the physics?

higher threshold for fluid flow in granular sea ice

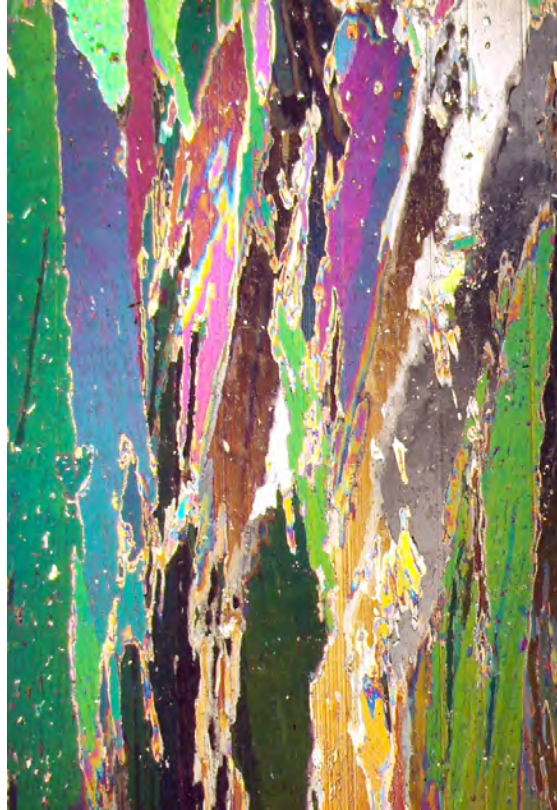
microscale details impact “mesoscale” processes

nutrient fluxes for microbes
melt pond drainage
snow-ice formation

columnar

granular

5%



10%

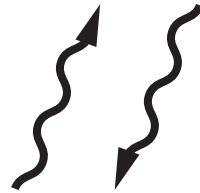
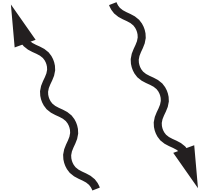


Golden, Sampson, Gully, Lubbers, Tison 2020

electromagnetically distinguishing ice types

Kitzel Lusted, Elena Cherkaev, Ken Golden

Remote sensing of sea ice



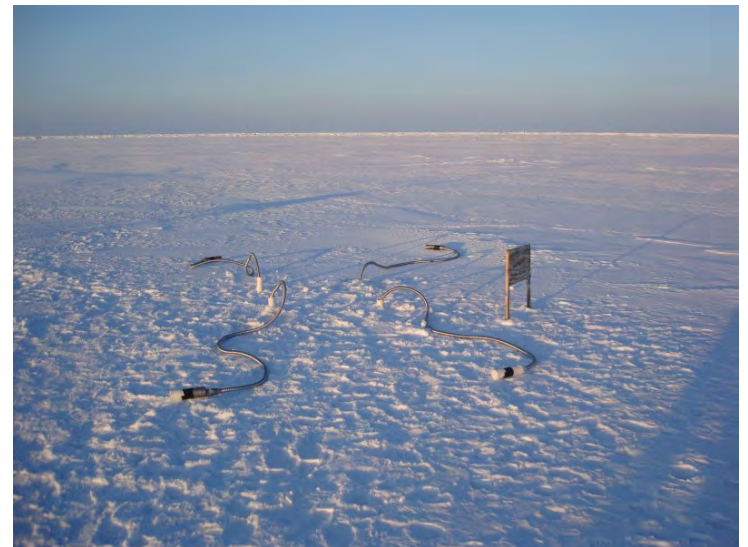
sea ice thickness
ice concentration

INVERSE PROBLEM

Recover sea ice
properties from
electromagnetic
(EM) data

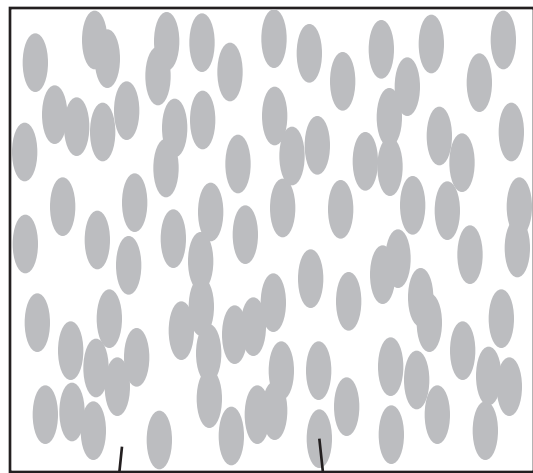
$$\epsilon^*$$

effective complex permittivity
(dielectric constant, conductivity)



brine volume fraction
brine inclusion connectivity

Effective complex permittivity of a two phase composite in the quasistatic (long wavelength) limit



ϵ_1

ϵ_2



ϵ^*

$$D = \epsilon E$$

$$\nabla \cdot D = 0$$

$$\nabla \times E = 0$$

$$\langle D \rangle = \epsilon^* \langle E \rangle$$

p_1, p_2 = volume fractions of
the components

$$\epsilon^* = \epsilon^* \left(\frac{\epsilon_1}{\epsilon_2}, \text{ composite geometry} \right)$$

**What are the effective propagation characteristics
of an EM wave (radar, microwaves) in the medium?**

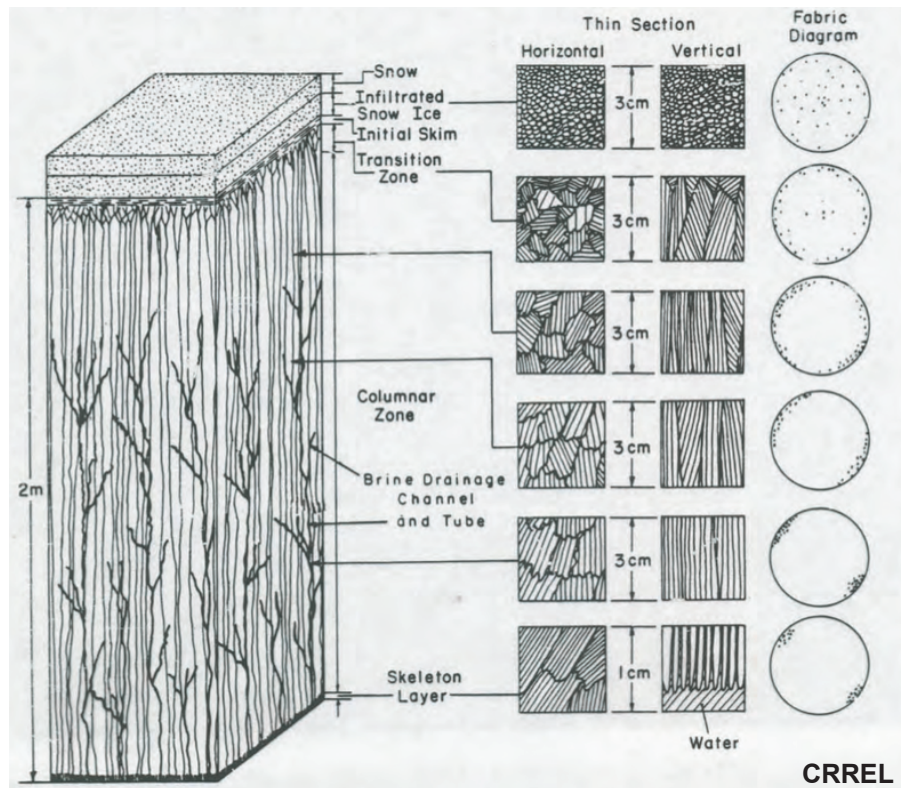
Rigorous bounds on the complex permittivity tensor of sea ice with polycrystalline anisotropy within the horizontal plane

Kenzie McLean, Elena Cherkaev, Ken Golden 2020

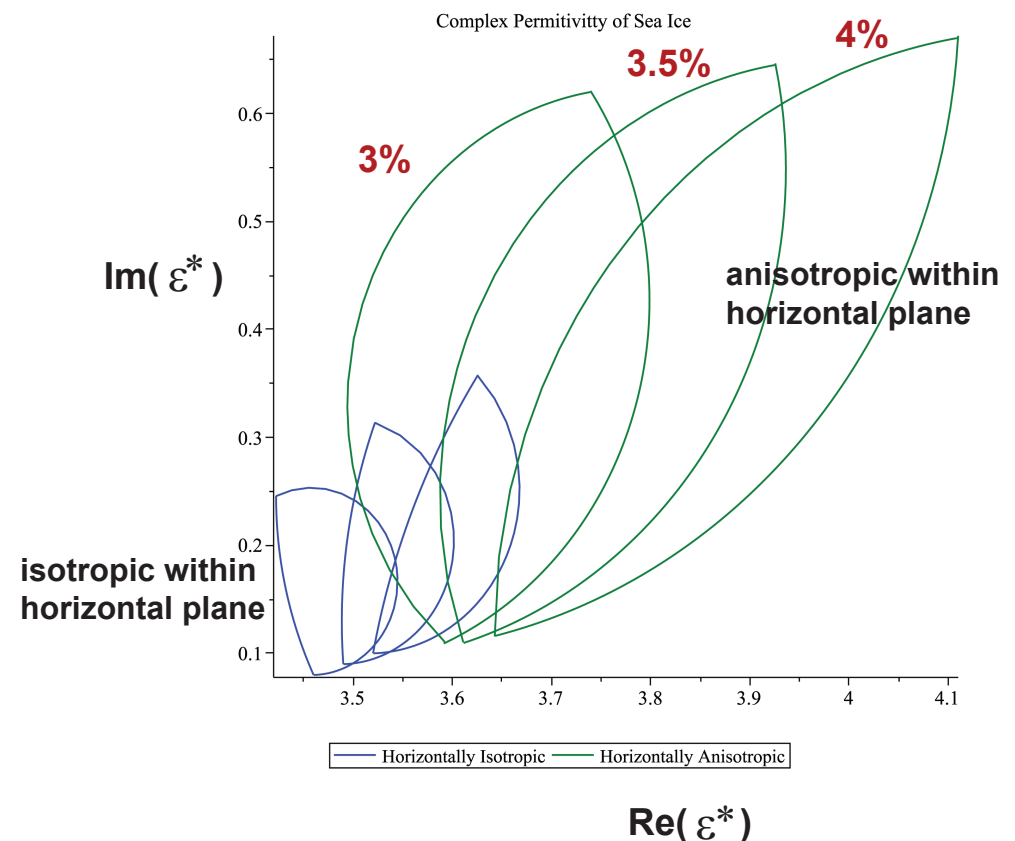
motivated by **Weeks and Gow, *JGR* 1979: c-axis alignment in Arctic fast ice off Barrow**

Golden and Ackley, *JGR* 1981: radar propagation model in aligned sea ice

input: orientation statistics



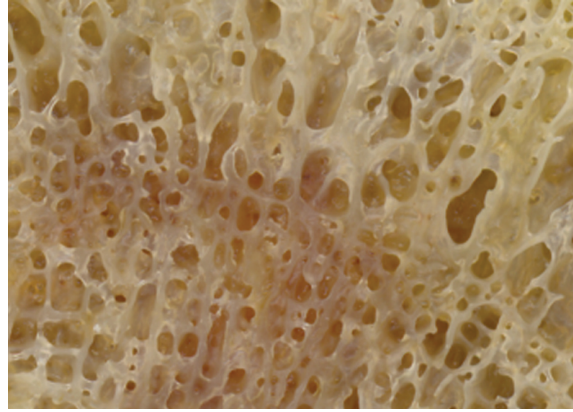
output: bounds



SEA ICE

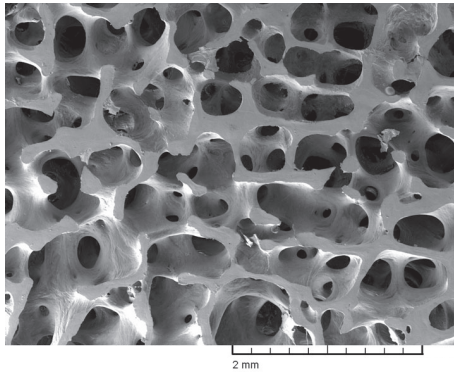


HUMAN BONE

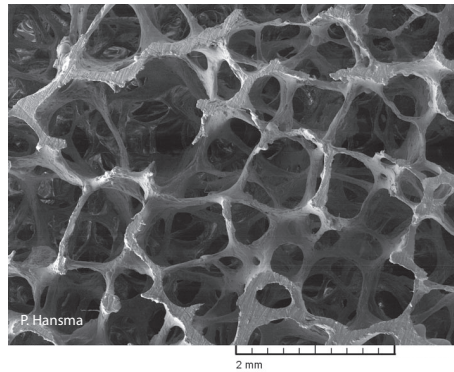


*spectral characterization
of porous microstructures
in human bone*

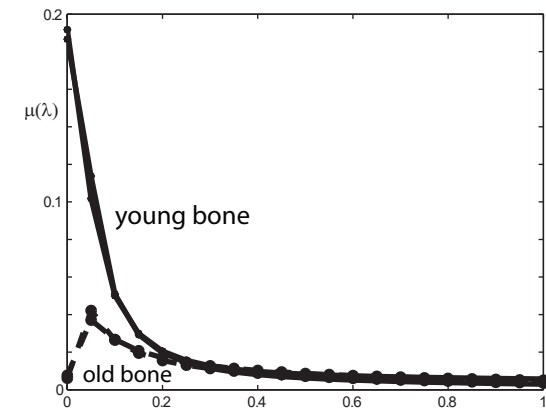
young healthy trabecular bone



old osteoporotic trabecular bone



reconstruct spectral measures
from complex permittivity data



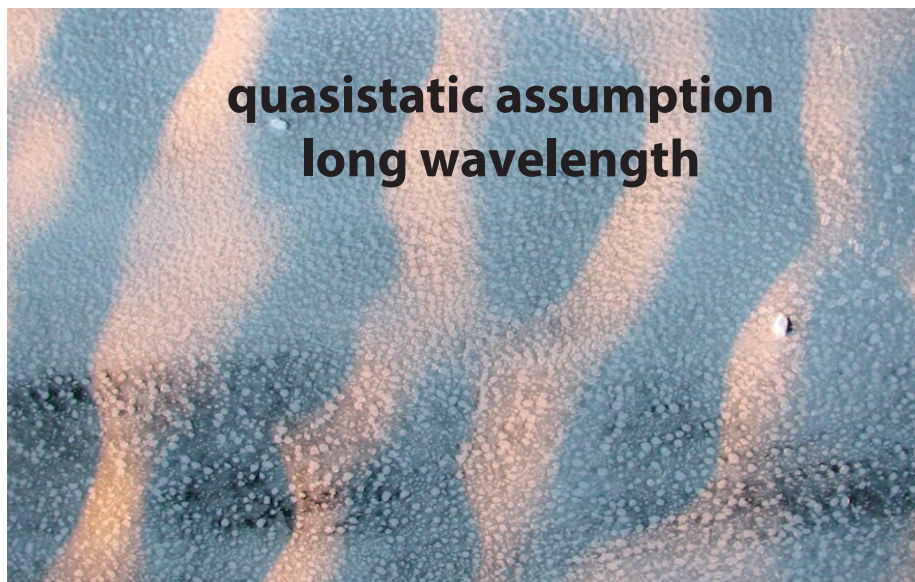
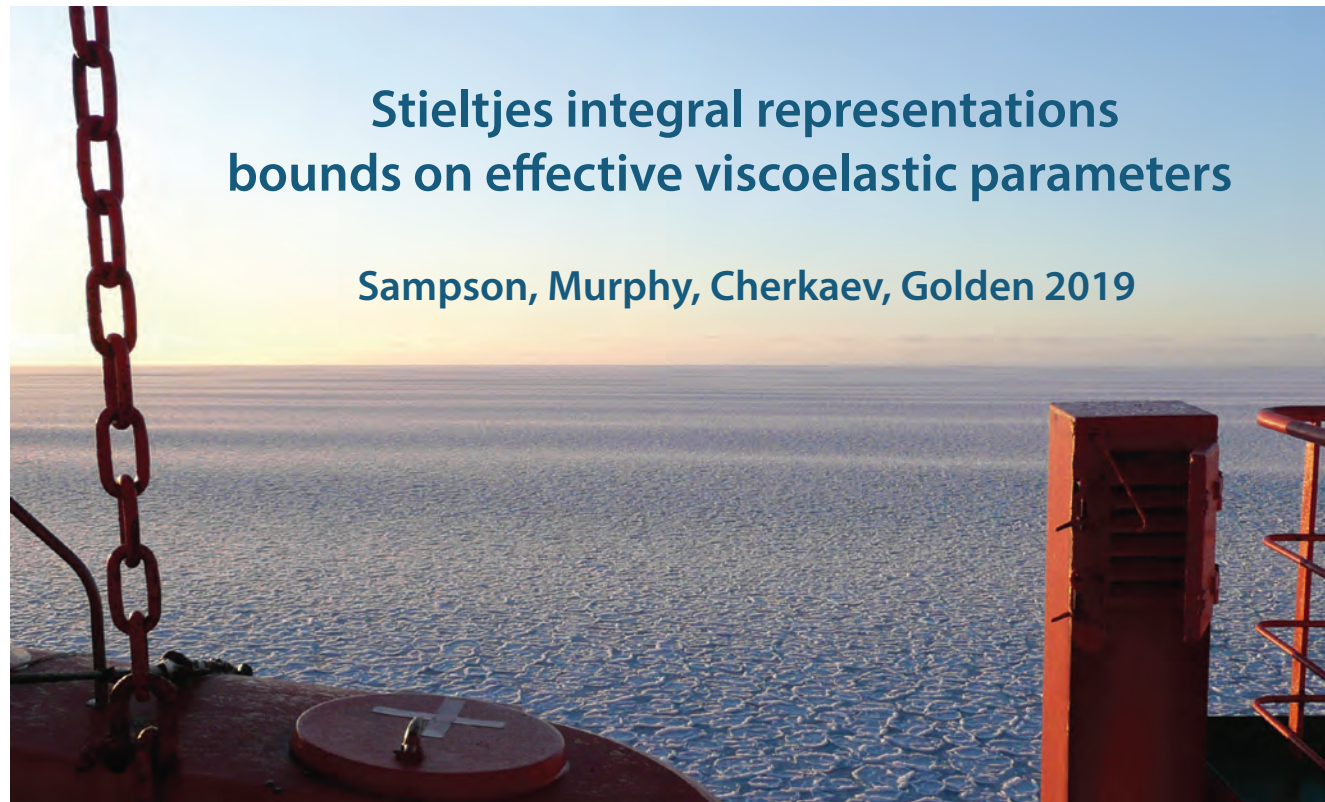
use regularized inversion scheme

*apply spectral measure analysis of brine connectivity and
spectral inversion to electromagnetic monitoring of osteoporosis*

Golden, Murphy, Cherkaev, J. Biomechanics 2011

the math doesn't care if it's sea ice or bone!

wave propagation in the marginal ice zone





Ice floe diffusion in winds and currents

Anomalous diffusion and sea ice dynamics

sub- and super-diffusive behavior of motion of sea ice floes as tracked by buoy data

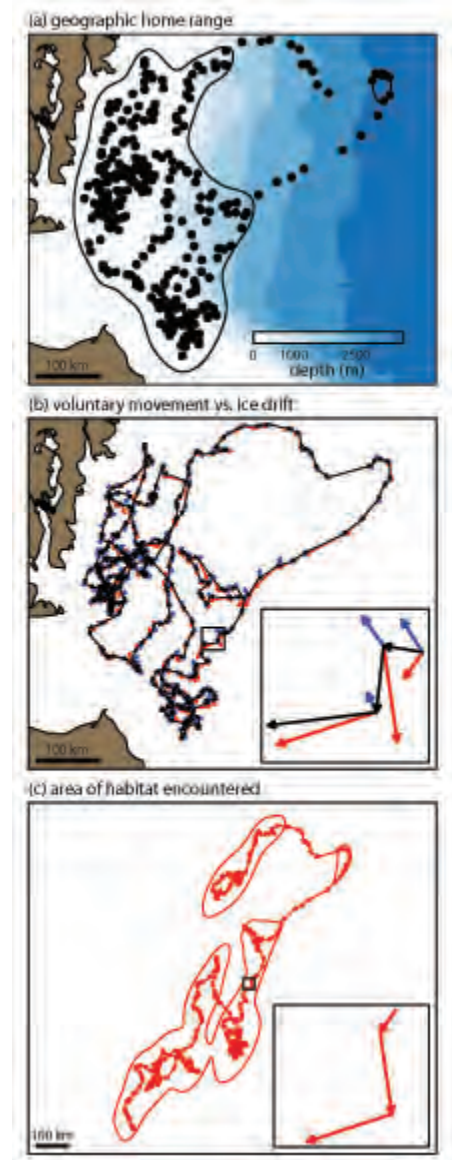
Jennifer Lukovich, Jennifer Hutchings, David Barber, *Ann. Glac.* 2015

Huy Dinh, Elena Cherkaev, Ken Golden, 2019

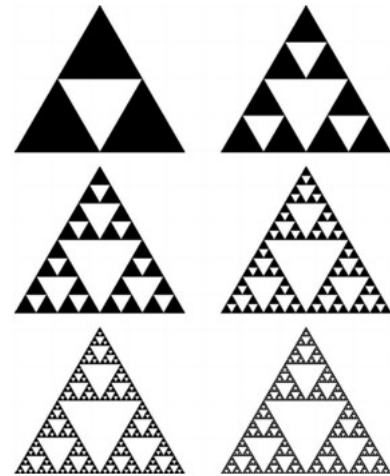
Home ranges in moving habitats: polar bears and sea ice

“diffusive” polar bear motion superimposed with drifting sea ice

Marie Auger-Méthé, Mark Lewis, Andrew Derocher, *Ecography*, 2016



fractals and multiscale structure



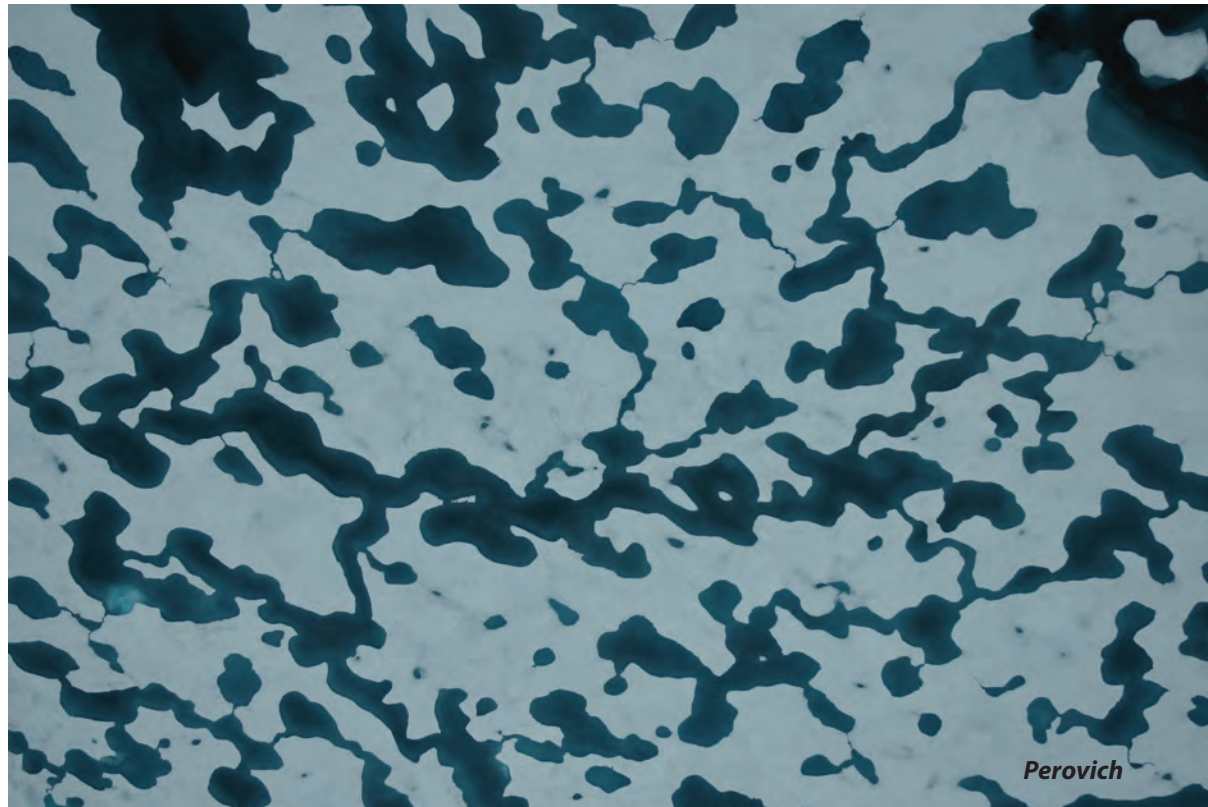
melt pond formation and albedo evolution:

- *major drivers in polar climate*
- *key challenge for global climate models*

numerical models of melt pond evolution, including topography, drainage (permeability), etc.

Lüthje, Feltham,
Taylor, Worster 2006
Flocco, Feltham 2007

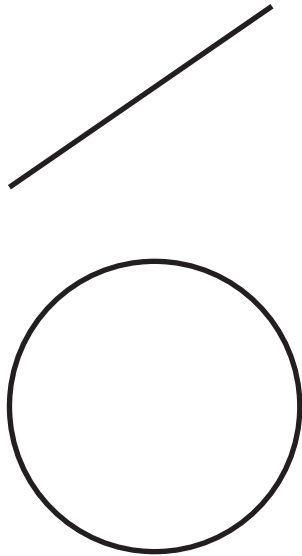
Skyllingstad, Paulson,
Perovich 2009
Flocco, Feltham,
Hunke 2012



Are there universal features of the evolution similar to phase transitions in statistical physics?

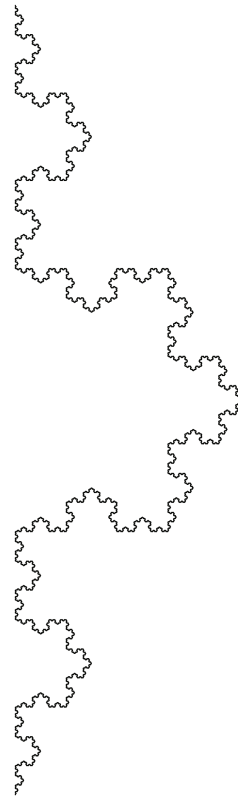
fractal curves in the plane

they wiggle so much that their dimension is >1



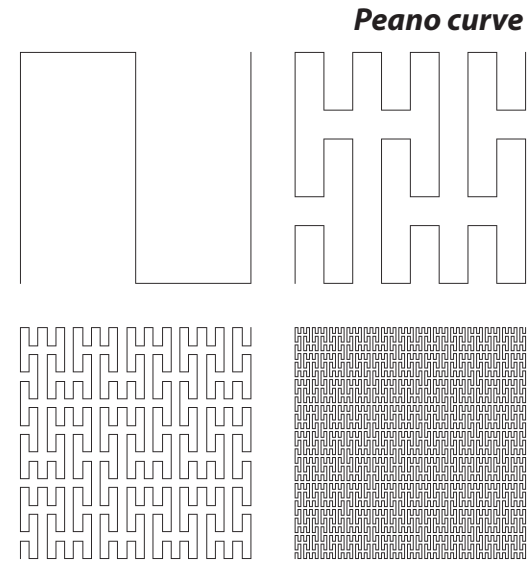
simple curves

$$D = 1$$



Koch snowflake

$$D = 1.26$$



Peano curve

Brownian motion

space filling curves

$$D = 2$$

clouds exhibit fractal behavior from 1 to 1000 km

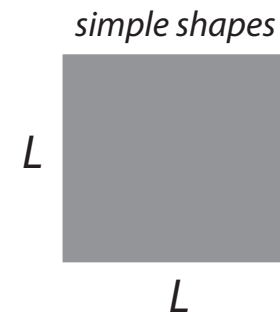
use **perimeter-area** data to find that
cloud and rain boundaries are fractals

$$D \approx 1.35$$

S. Lovejoy, Science, 1982



$$P \sim \sqrt{A}$$



$$A = L^2$$
$$P = 4L = 4\sqrt{A}$$

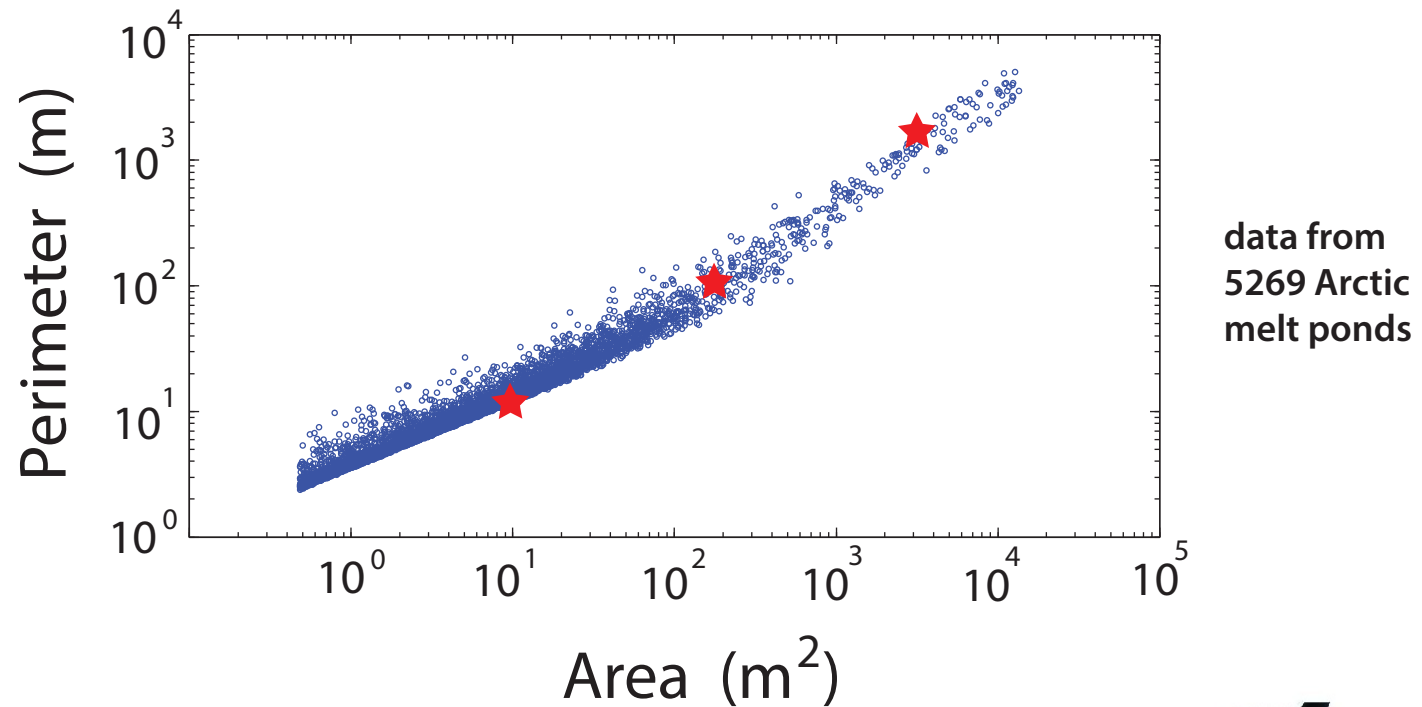
$$P \sim \sqrt{A}^D$$



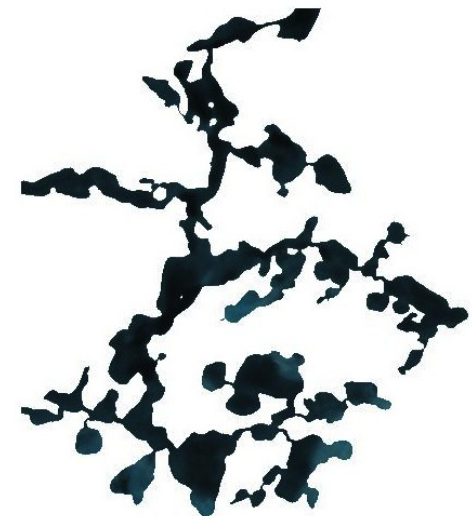
for fractals with
dimension D

$D = 1.52...$

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden



~ 30 m



simple pond

transitional pond

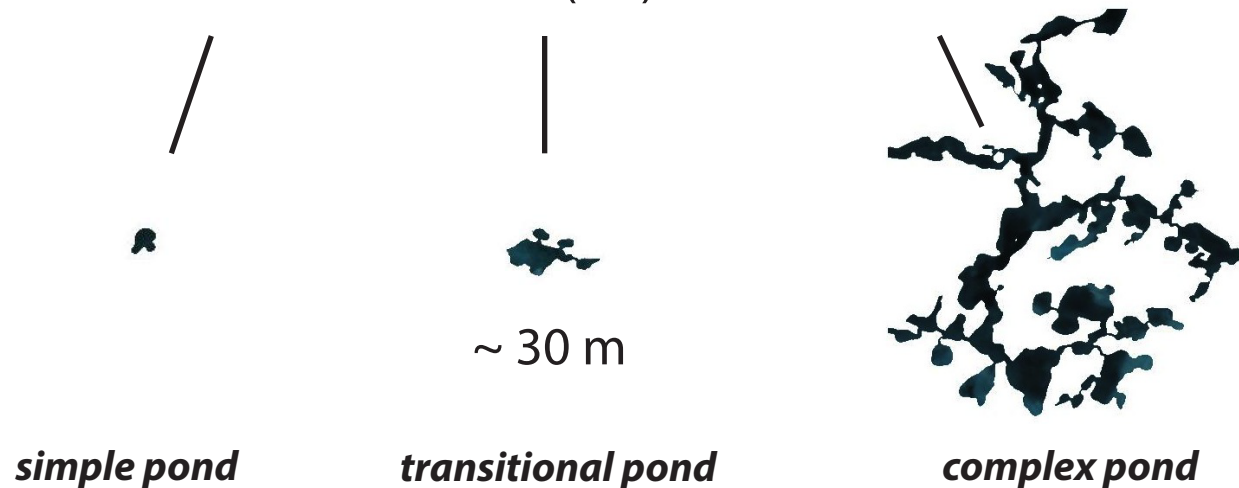
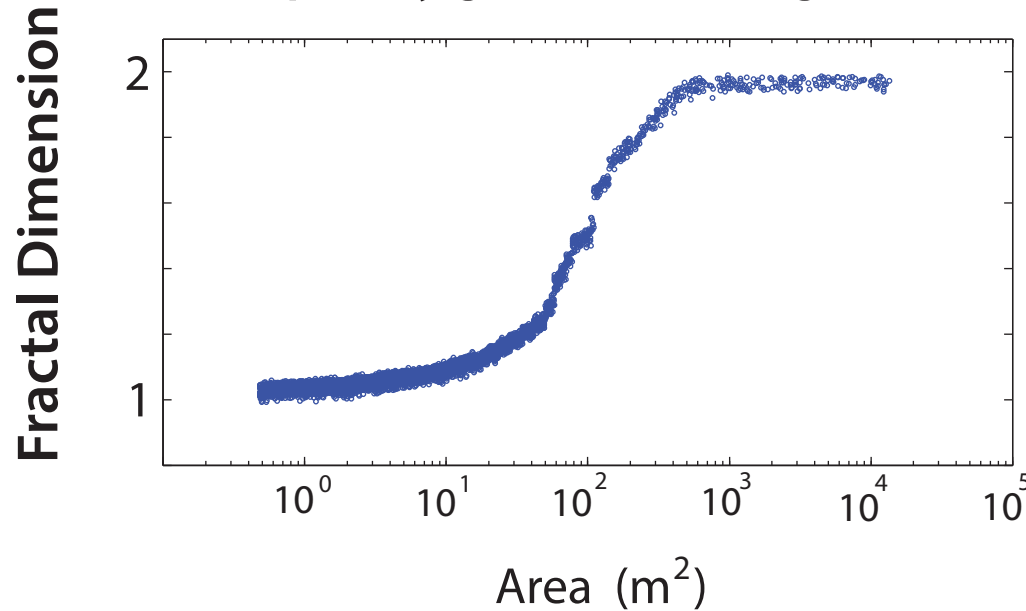
complex pond

Transition in the fractal geometry of Arctic melt ponds

Christel Hohenegger, Bacim Alali, Kyle Steffen, Don Perovich, Ken Golden

The Cryosphere, 2012

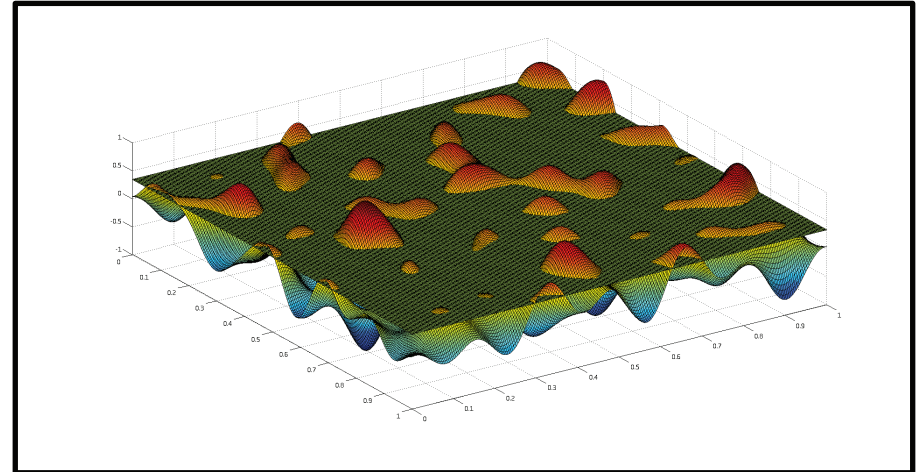
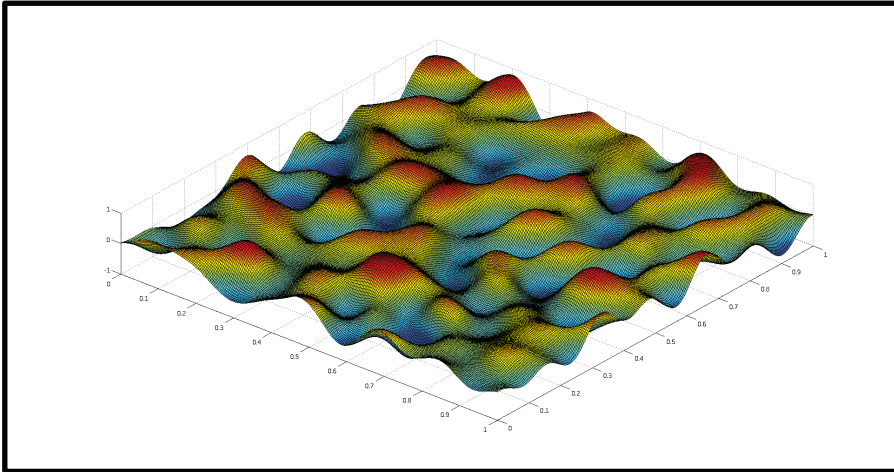
complexity grows with length scale



Continuum percolation model for melt pond evolution

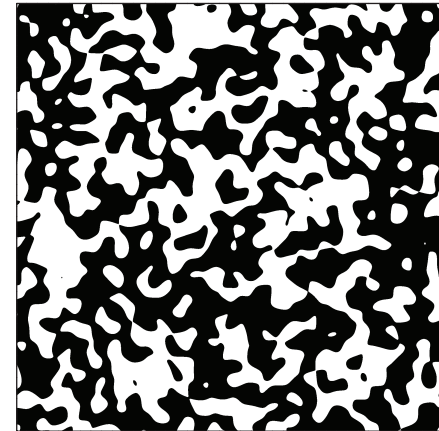
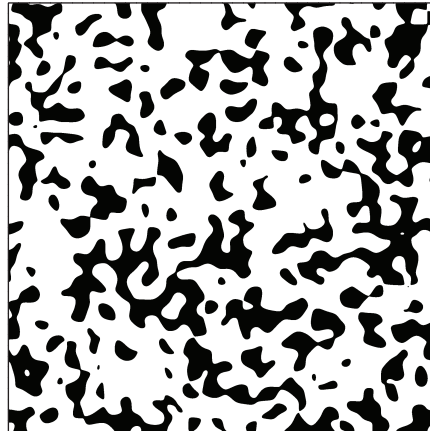
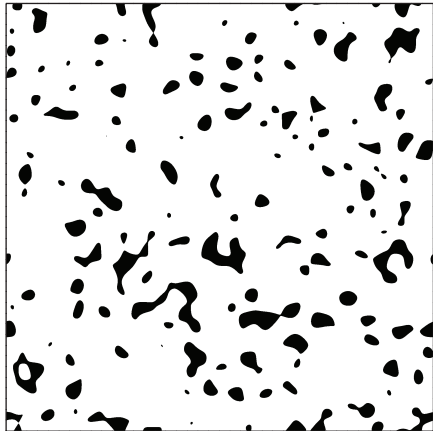
level sets of random surfaces

Brady Bowen, Court Strong, Ken Golden, J. Fractal Geometry 2018



random Fourier series representation of surface topography

intersections of a plane with the surface define melt ponds

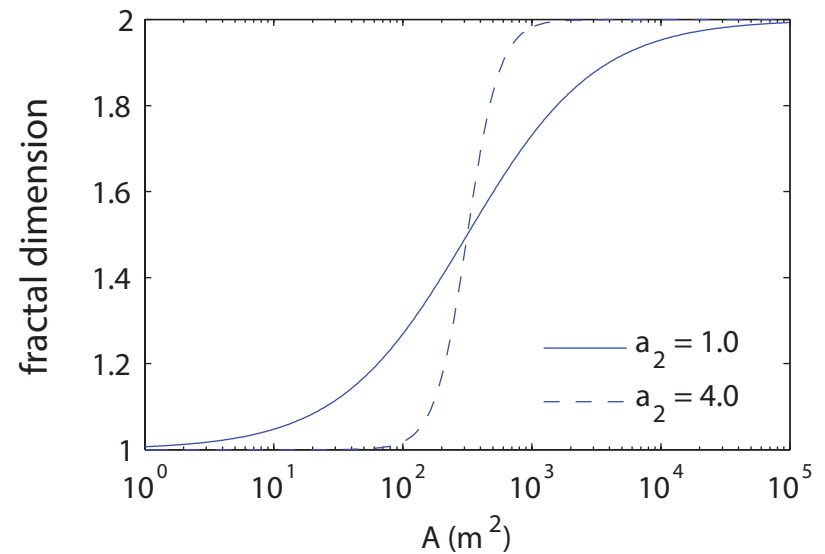
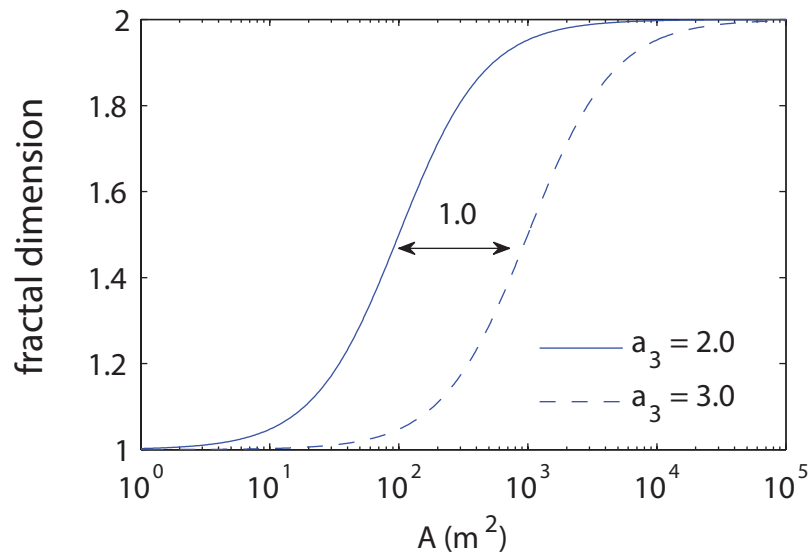


electronic transport in disordered media

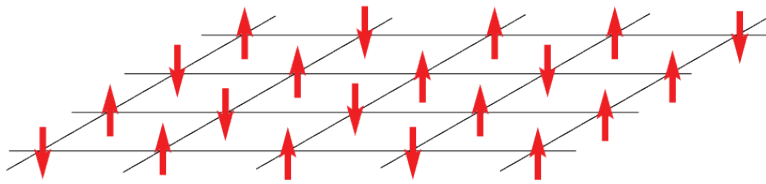
diffusion in turbulent plasmas

Isichenko, Rev. Mod. Phys., 1992

fractal dimension curves depend on statistical parameters defining random surface



Ising Model for a Ferromagnet



applied
magnetic
field

$\uparrow$ H

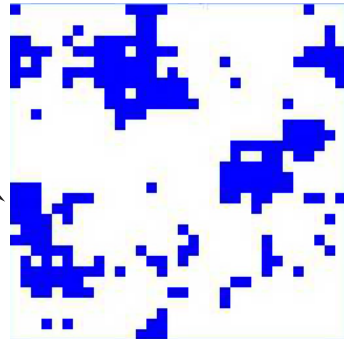
$$s_i = \begin{cases} +1 & \text{spin up} & \text{blue} \\ -1 & \text{spin down} & \text{white} \end{cases}$$

$$\mathcal{H} = -H \sum_i s_i - J \sum_{\langle i,j \rangle} s_i s_j$$

nearest neighbor Ising Hamiltonian

ferromagnetic interaction $J \geq 0$

*islands of
like spins*

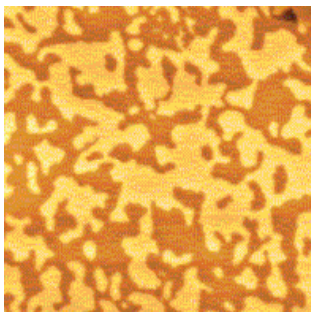


magnetization

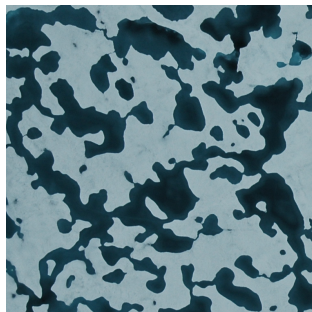
homogenized parameter
like effective conductivity

$$M(T, H) = \lim_{N \rightarrow \infty} \frac{1}{N} \left\langle \sum_j s_j \right\rangle$$

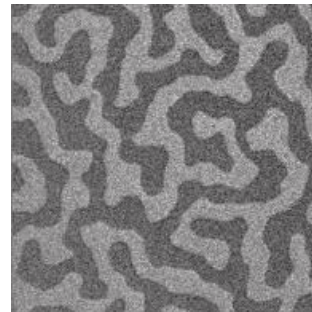
energy is lowered when nearby spins align
with each other, forming **magnetic domains**



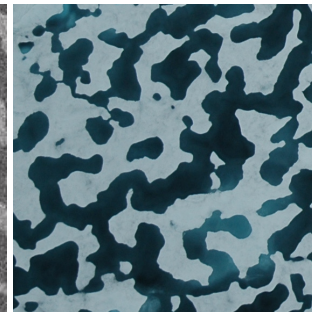
magnetic domains
in cobalt



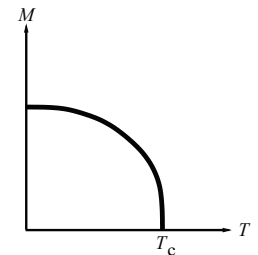
melt ponds (Perovch)



magnetic domains
in cobalt-iron-boron



melt ponds (Perovch)



Curie point
critical temperature

Ising model for ferromagnets $\longrightarrow$ Ising model for melt ponds

Ma, Sudakov, Strong, Golden, *New J. Phys.*, 2019

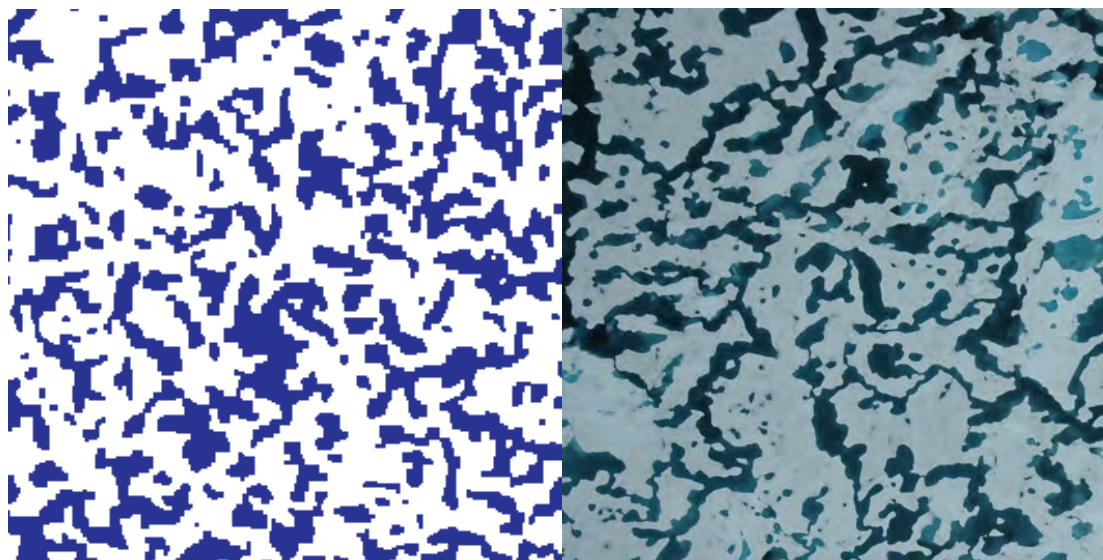
$$\mathcal{H} = - \sum_i^N H_i s_i - J \sum_{\langle i,j \rangle}^N s_i s_j \quad s_i = \begin{cases} \uparrow & +1 \text{ water (spin up)} \\ \downarrow & -1 \text{ ice (spin down)} \end{cases}$$

random magnetic field represents snow topography

magnetization M pond area fraction $F = \frac{(M+1)}{2}$ only nearest neighbor patches interact
 $\sim$ albedo

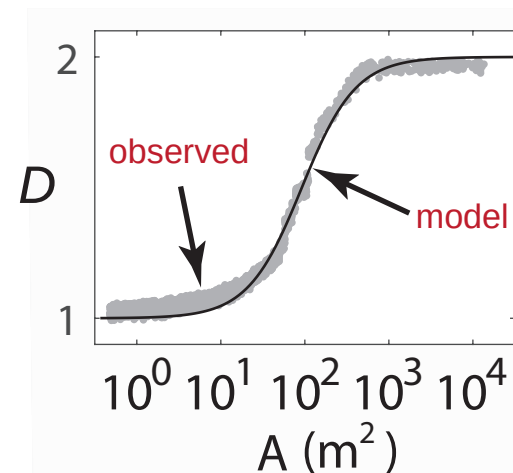
Starting with random initial configurations, as Hamiltonian energy is minimized by Glauber spin flip dynamics, system “flows” toward metastable equilibria.

Order from Disorder



Ising
model

melt pond
photo (Perovich)



pond size
distribution exponent

observed -1.5

(Perovich, et al. 2002)

model -1.58

*Scientific American
EOS, PhysicsWorld, ...*

ONLY MEASURED INPUT = LENGTH SCALE (GRID SIZE) from snow topography data



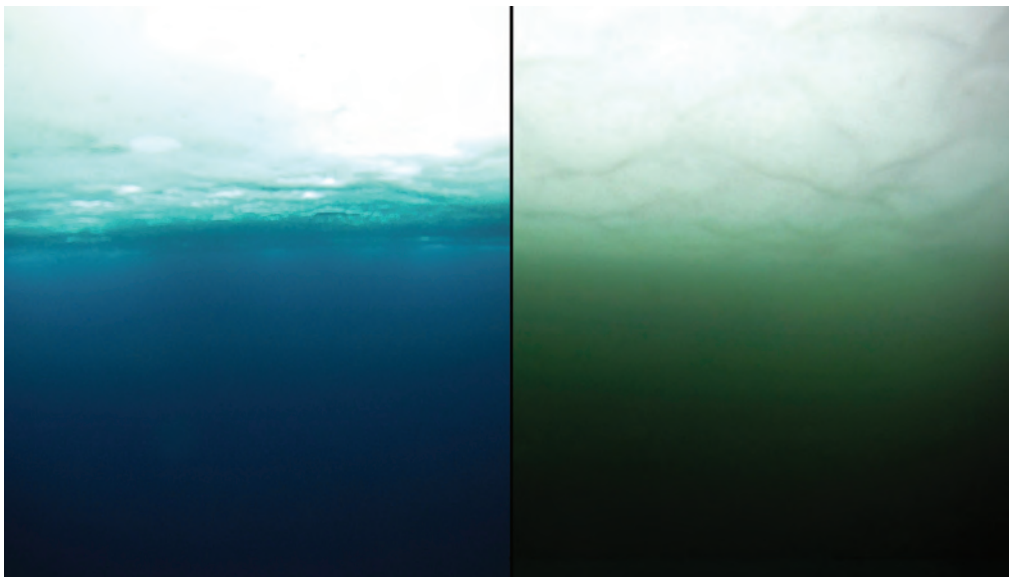
2011 massive under-ice **algal bloom**

Arrigo et al., *Science* 2012

melt ponds act as

WINDOWS

allowing light
through sea ice



no bloom

bloom

***Have we crossed into a
new ecological regime?***

The frequency and extent of sub-ice
phytoplankton blooms in the Arctic Ocean

Horvat, Rees Jones, Iams, Schroeder,
Flocco, Feltham, *Science Advances*, 2017

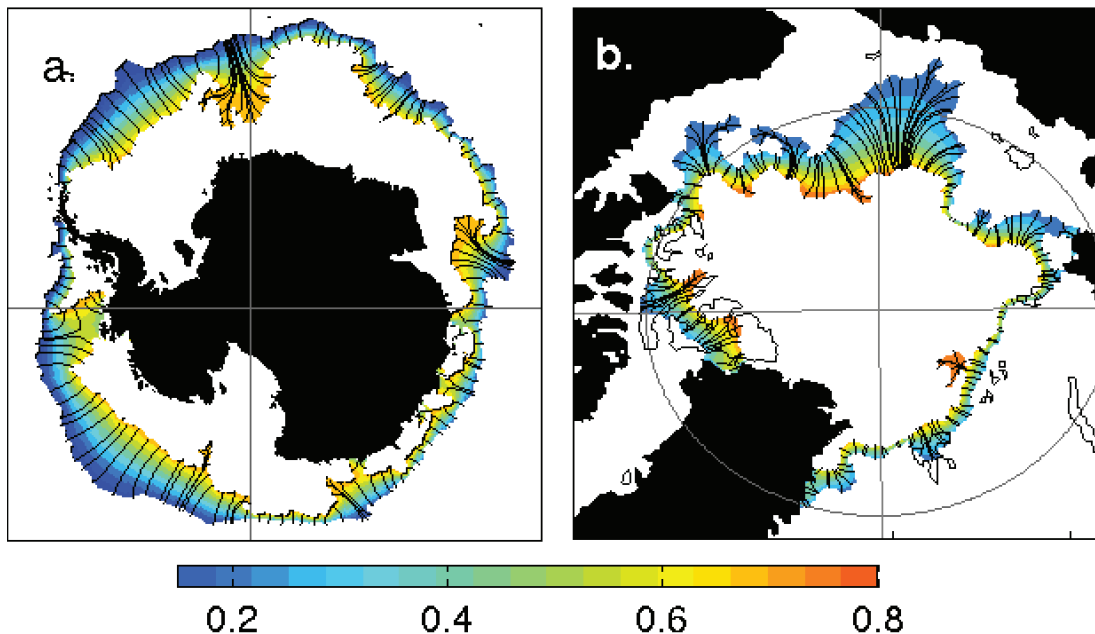
(2015 AMS MRC, Snowbird)

macroscale

Marginal Ice Zone

MIZ

- biologically active region
- intense ocean-sea ice-atmosphere interactions
- region of significant wave-ice interactions



MIZ WIDTH

fundamental length scale of
ecological and climate dynamics

Strong, *Climate Dynamics* 2012

Strong and Rigor, *GRL* 2013

transitional region between
dense interior pack ($c > 80\%$)
sparse outer fringes ($c < 15\%$)

**How to objectively
measure the “width”
of this complex,
non-convex region?**

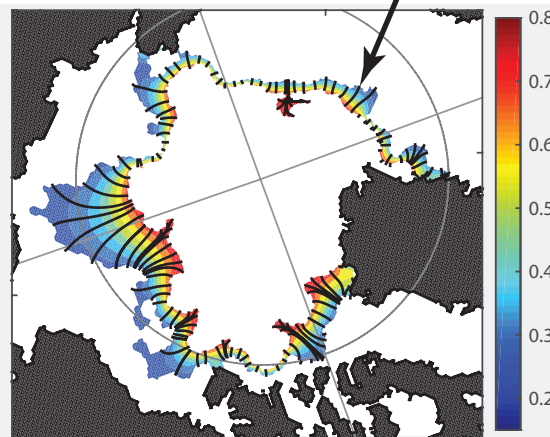
Objective method for measuring MIZ width motivated by medical imaging and diagnostics

Strong, *Climate Dynamics* 2012
Strong and Rigor, *GRL* 2013

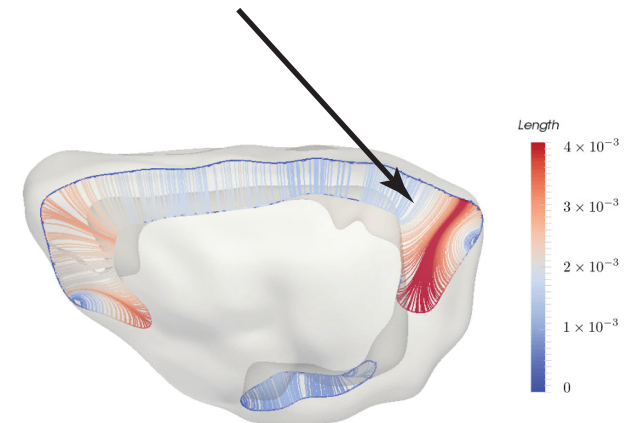
**39% widening
1979 - 2012**

“average” lengths of streamlines

streamlines of a solution
to Laplace’s equation



Arctic Marginal Ice Zone



**crosssection of the
cerebral cortex of a rodent brain**

analysis of different MIZ WIDTH definitions

Strong, Foster, Cherkaev, Eisenman, Golden
J. Atmos. Oceanic Tech. 2017

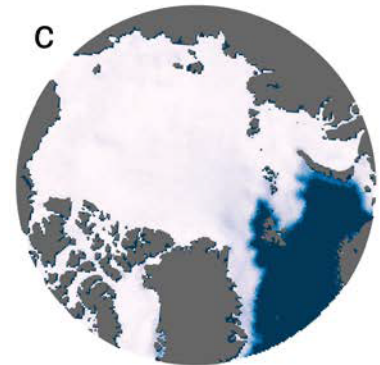
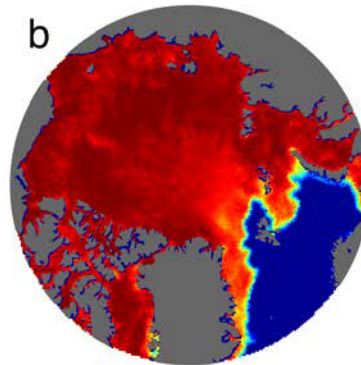
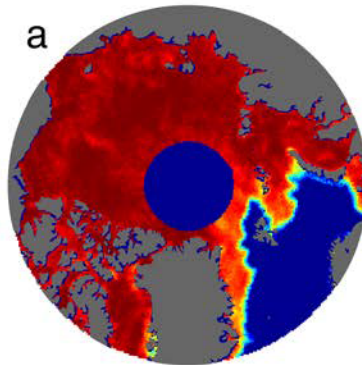
Strong and Golden
Society for Industrial and Applied Mathematics News, April 2017

Filling the polar data gap with partial differential equations

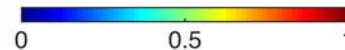
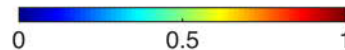
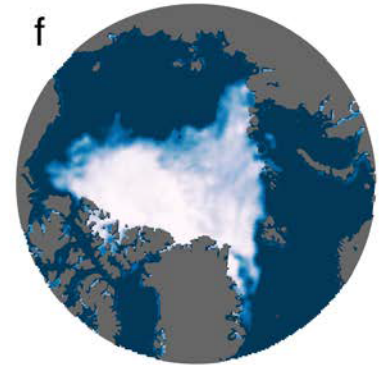
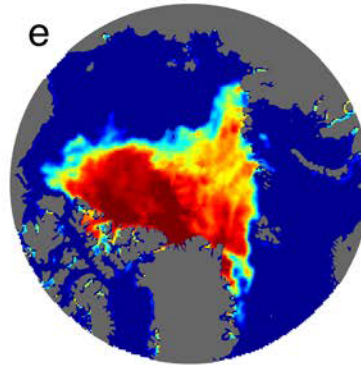
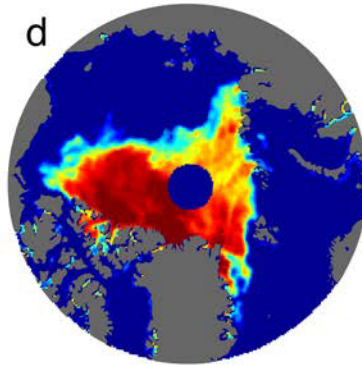
hole in satellite coverage
of sea ice concentration field

previously assumed
ice covered

Gap radius: 611 km
06 January 1985



Gap radius: 311 km
30 August 2007



$$\Delta\psi=0$$

fill with harmonic function satisfying
satellite BC's plus stochastic term

Strong and Golden, *Remote Sensing* 2016
Strong and Golden, *SIAM News* 2017

NOAA/NSIDC Sea Ice Concentration CDR
product update will use our PDE method.

Partial differential equation models for sea ice concentration

Delaney Mosier, Court Strong, Jingyi Zhu, Elena Cherkaev, Ken Golden

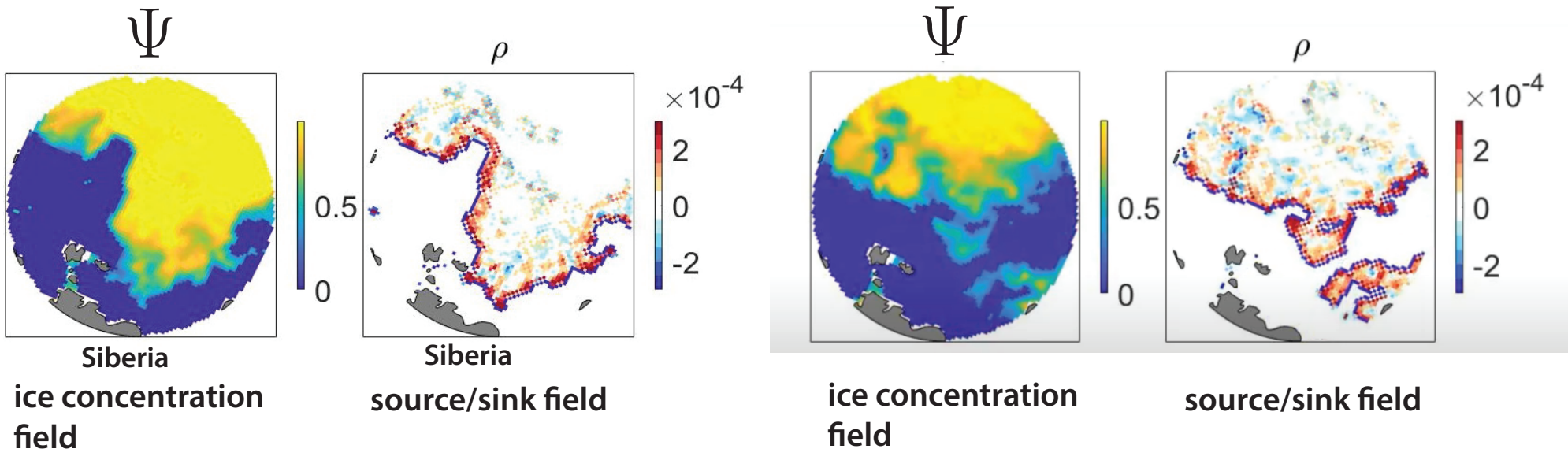
$$\Delta \Psi = 0$$

Generalize simplistic Laplace equation (steady state heat equation) model for Ψ

$$\nabla \cdot (D \nabla \Psi) = 0 \quad \Delta \Psi = \rho$$

Advection diffusion equation model to generate more realistic dynamics of Ψ

$$\frac{\partial \Psi}{\partial t} = \nabla \cdot (D \nabla \Psi) - \vec{v} \cdot \nabla \Psi \quad \nabla \cdot \vec{v} = 0$$



NSIDC

Poisson equation to identify **sources** and **sinks** of ice concentration

University of Utah Sea Ice Modeling Group (2017-2020)

Senior Personnel: Ken Golden, Distinguished Professor of Mathematics
Elena Cherkaev, Professor of Mathematics
Court Strong, Associate Professor of Atmospheric Sciences
Ben Murphy, Ph.D.

Postdoctoral Researcher: Noa Kraitzman (now at Australian National University)

Graduate Students: Kyle Steffen (now at UT Austin with Clint Dawson)
Christian Sampson (now at UNC Chapel Hill with Chris Jones)
Huy Dinh (starting sea ice MURI Postdoc at NYU/Courant)
Rebecca Hardenbrook
David Morison (Physics Department)
Ryleigh Moore
Delaney Mosier, Daniel Hallman

Undergraduate Students: Kenzie McLean, Jacqueline Cinella Rich, Dane Gollero,
Samir Suthar, Anna Hyde, Kitsel Lusted, Ruby Bowers
Kimball Johnston, Jerry Zhang

High School Students: Jeremiah Chapman, Titus Quah, Dylan Webb

Sea Ice Ecology Group Postdoc Jody Reimer, Grad Student Julie Sherman,
Undergrads Anna Hyde, Kayla Stewart + incoming

Conclusions

1. Sea ice is a fascinating multiscale composite with structure similar to many other natural and man-made materials.
2. Mathematical methods developed for sea ice advance the theory of composites in general.
2. **Homogenization and statistical physics help *link scales in sea ice and composites***; provide rigorous methods for finding effective behavior; advance sea ice representations in climate models.
3. **Fluid flow** through sea ice mediates **melt pond evolution** and many processes important to climate change and polar ecosystems.
5. Field experiments are essential to developing relevant mathematics.
6. Our research will help to **improve projections of climate change**, the fate of Earth's sea ice packs, and the ecosystems they support.

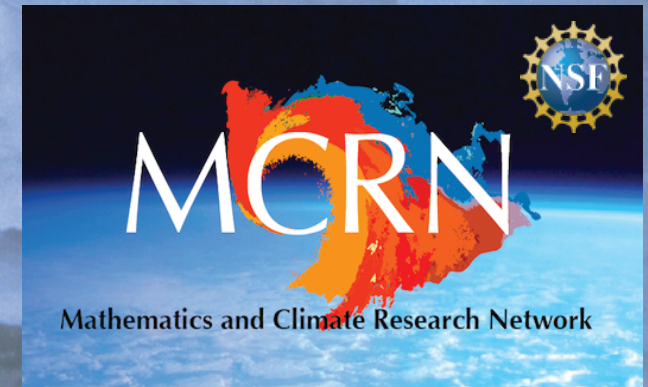
THANK YOU

Office of Naval Research

Applied and Computational Analysis Program
Arctic and Global Prediction Program

National Science Foundation

Division of Mathematical Sciences
Division of Polar Programs



Buchanan Bay, Antarctica Mertz Glacier Polynya Experiment July 1999

Notices

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Modeling Sea Ice



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